Effort, Unemployment And Business Cycle

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Acknowledgements

As an engineering student before coming to Humboldt University one year ago, I did not know much about economics and never expected that I will someday write something on the very academic business cycle theory. As a result, I am really thankful to my advisor, Prof. Harald Uhlig. PhD for all the things I've learnt from him, from IAMA lectures, QMNM seminars and thesis advices, not only for his algorithm of loglinearization and methodology of Toolkit, but also the interest to economics and the attitude toward academic. I am also very thankful to two PhD students of him: Alexander Kriwoluzky and Stephan Ried. They've given me precious advices and suggestions concerning my thesis. And, I thank Siemens for the financial support during my studies in Germany. Last but not least, I will always be grateful to my parents and girlfriend, Baihe.

Abstract

This thesis targets on the problem: if efficiency wage considerations can help our understanding of Real Business Cycle theory.

Based on the idea from Uhlig and Xu (1996): effort as a continuously adjustable variable, I develop a new model to analysis effort and the cycle. I find the result that variability of effort due to efficiency wage considerations help in explaining the rather large cyclical movements in employment as well as the rather low cyclical movements in real wages. In my model, technology also fluctuates bigger than that in Hansen's model, but only 1.5 times bigger. Because of that, all the other variables fluctuate slightly bigger than that in the benchmark model.

As a result, I think that efficiency wage theories help understanding real business cycles in some extent, but more research should be done on the modeling of effort. I suggest that we may use two variables in stead of one: "effort of workers", which correlates negatively with technology and "quality of workers" which correlates positively.

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1. Introduction

One of the main outstanding challenges to economic research is: what main factor influences and subsequently changes the decisions of all actors in an economy? Real Business Cycle Theory (RBC Theory), which is introduced by Finn Kydland and Edward Prescott, is the dominant theories over the last few decades. Unlike other theories of the business cycle, it sees recessions and periods of economic growths as the efficient response of output to exogenous variables (see Kydland and Prescott (1982) for the origin Cooley (1995) and Kydland (1995), for the summary of this line of research).

One of the main critics of RBC theory is on the efficiency market. The original Nobel Prize winning research by Kydland and Prescott modeled economic fluctuations with efficient markets, where shirking, moral hazard, etc. are not possible, and unemployment compensation is complete, i.e. employment is voluntary. Many researchers felt that this downplayed the role of market inefficiencies, and minimized the importance of unemployment.

The efficiency wage theory, in the other hand, explains the nature of involuntary unemployment. As early as in 1984, Shapiro-Stiglitz famously proposed an answer to these puzzles:

1. Why is there unemployment?
2. In the absence of minimum wages, why aren't wages bid down sufficiently by job seekers so that everyone who wants a job finds one?
3. Can the neoclassical paradigm explain involuntary employment?

Concerning the above questions, two basic observations undergrid their analysis:

1. Unlike other forms of capital, humans can choose their level of effort.
2. It is costly for firms to determine how much effort workers are exerting.

Shapiro and Stiglitz (1984) views unemployment as a device which threatens hired workers into providing the effort their employer seeks. For the threat to be effective, it must be more attractive to be a worker than to be unemployed. Firms will not lower their wages because wages is the threat to prevent workers from shirking.

Many researchers have studied, if efficiency wage considerations can help understanding business cycle fluctuations. One of the interesting studies is from Uhlig and Xu (1996). They develop a new real business cycle style model with efficiency wage features. Their paper shows that increasing the variability of effort due to efficiency wage considerations helps in explaining the rather large cyclical employment movements as well as the rather low cyclical movements in real wages, supporting the argument of Solow (1979), but requires unplausibly large movements in the technology parameter.

After implementing Uhlig and Xu's model in my seminar of Quantitative Macroeconomics and Numerical Methods (QMNM), I believe that something more can be done following this track of thinking. In my thesis work, based on the idea of RBC and efficiency wage theory, I modify the model developed by Uhlig and Xu. My mode shows that with the idea of rather large cyclical employment movements unchanged, fluctuation of technology shock is reduced to a much smaller amount. The standard deviation of technology is only around 2.1, compared with the value of 36.1 in Uhlig and Xu's model.

However, there is still some unsolved puzzle in my model. Firstly, the standard deviation in my model is still bigger than that in Hansens' Benchmark. Secondly, with efficiency wage considerations, after a positive capital deviation, output drops slightly. I think that these problems stems from the modeling of effort. In the efficiency wage model, effort drops when good time comes, i.e. a positive technology shock. While technology plays a positive role in influencing Solow residual, which drives the neoclassical growth model, effort plays a negative role. In order to keep the standard deviation of Solow residual close to the value from real world, a higher fluctuation of technology is unavoidable. Also, the negative fluctuation of effort causes the negative influence on output.

As a result, I make the conclusion that efficiency wages considerations can help understanding RBC theory in some extend, but can not be regard as a totally complement to RBC theory.

In the last part of my thesis, I've made some more analysis on "effort" concerning its relationship between technology. I think that the variable that normally we call "effort of workers" or "quality of workers" should be split into two variables: effort and quality. That is to say, effort is different from quality. Both of them influence Solow residual as a human factor, but quality of workers increases when good time comes while effort drops at the same time. As a result, I think that there should be more things done by splitting the present variable effort into two variables, which may help us to have a better understanding of RBC theory.

2. Literature

This chapter provides a review of literatures, especially explains the result of Uhlig and Xu (1996) which directly caused my further study on effort and business cycle.

2.1. Classical theories

Real Business Cycle Theory (or RBC Theory) is a macroeconomic school of thought that the business cycle is caused by different kind of exogenous shocks, including technology innovation, weather, war, etc, but not total supply and demand. It sees recessions and periods of economic growth as the efficient response of output to these exogenous variables. According to RBC theory, business cycles are therefore "real" in that they do not represent a failure of markets to clear, but rather reflect the most efficient possible operation of the economy. It differs in this way from other theories of the business cycle, like Keynesian economics and Monetarism, which see recessions as the failure of some market to clear.

One of the critics is the classic wage-employment variability puzzle (see, especially Prescott, 1986). Any model that proposes to explain business cycles must also explain the fact that wage varies much less than employment. Standard neoclassical RBC model displays the opposite property.

Several modifications have been proposed concerning the research labor. An important development in the understanding of labor is provided by Hansen (1985). He argues that labor is indivisible: individuals either enter or exit the labor force. If somebody is in the labor force, he has to work in response to technology shocks for a fixed number of hours rather than simply adjusting the working hours themselves.

In another paper by Kydland and Prescott (1990), both changes in hours per worker and changes in number or workers vary. They use this theory to estimate the importance of Solow technology shocks and find hat they are a major contributor, approximately 70% of the total. Still, it is significantly smaller than that obtained by Hansen (1988).

Moreover, the problem of “involuntary employment” can not be totally solved in the neoclassical paradigm. In the classical model, agents are either employed or unemployed. They run in a lottery in the job market: the “unlucky” ones are asked to work, while the “lucky” ones can enjoy their leisure. Solow residual is used to explain the procyclicality of the average productivity of labor, which drives the model. Using standard Solow residual accounting, the size of these exogenous shocks can be calibrated to actual data.

It has been argued that employment should be “involuntary”, i.e. unemployed workers would prefer to accept work at current wages rather than remaining unemployed. Efficient risk sharing as assumed in the real business cycle does not preclude involuntary unemployment (Rogerson and Wright 1988).

In having a good understanding of “involuntary employment”, theory of efficiency wages has been proposed. In a model introduced by Shapiro and Stiglitz (1984), workers can choose to either providing effort, or not doing so. The conclusion is that, wage is more than some money; it is also the threat to prevent workers from shirking. As a result, even in bad times, firms will not reduce wages because reducing the wage to being unemployed reduces its threat and may provide the workers to shirk more.

2.2. Existing studies

Because of the seemingly complementarity of both theories, Real business cycle models with efficiency wages have been studied. Danthine and Bonaldson (1990, 1995) studied RBC models with efficiency wages and demonstrated that efficiency wages considerations, at least when motivated by the gift exchange paradigm, can improve out understanding of business cycles. Critics say that, effort turns out to be constant in the equilibrium of their models, and the quantitative nature of fluctuations induced by technology shocks changed little. Uhlig and Xu (1996) have done some further research on this point. In their work, they improved the modeling of effort from binary variable (As in the Shapiro and Stiglitz model, workers either provide effort or not doing so) into a continuous adjustable variable. Their work shows, that increasing the variability of effort due to efficiency wage considerations helps in explaining the rather large cyclical employment movements as well as the rather low cyclical movements in real wages, supporting the point made by Solow (1979), but require unplausibly large movements in the technology parameter, “the standard deviation of the technology innovation required to explain the observed output fluctuations is easily seven times as large as the value used by Hansen (1985)”. Because of the latter aspect, they argue that adjustable effort due to efficiency wage considerations is unlikely to play an important role for understanding business cycles. Because of the creative idea of the modeling of effort, I would give some more analysis on their model.

The author introduce a variable ν, which allows to continuously vary the effort response between very small (at ν close to zero) and very large (at ν close to 1). ν =1 means that wages age efficient.

Then, the authors change the value of ν from very small number to a big number close to 1, and find that the bigger value of ν, the larger the cyclical movement of employment compared to the movement of wages.

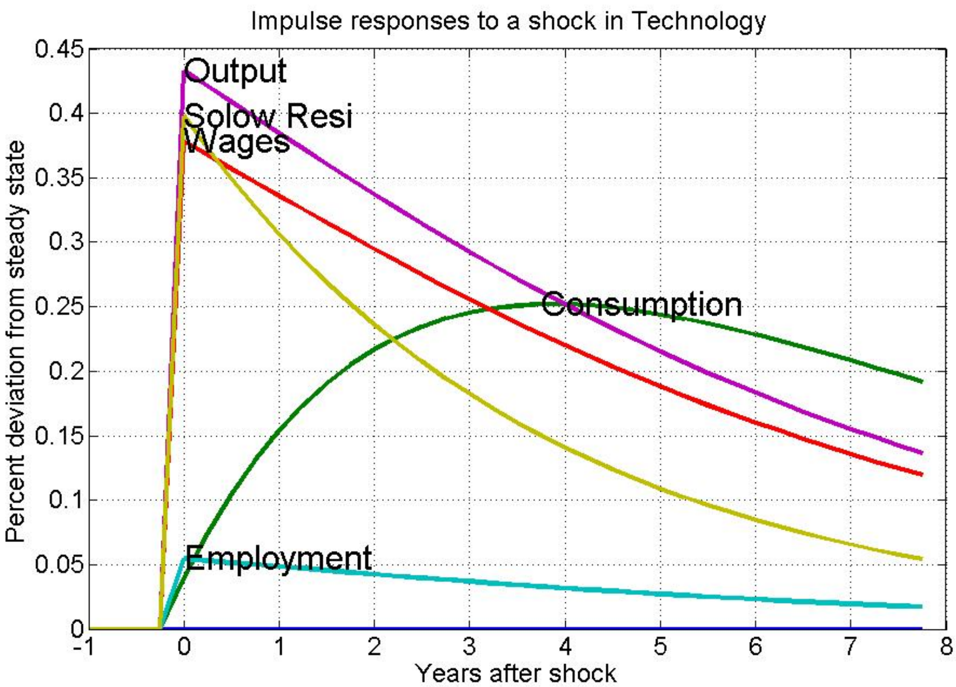

Figure1: Impulse response to a one percent deviation in technology, by setting ν =0.5 (efficiency wage theory does not take effect).

In Figure 1, ν is set to a small number, 0.5. It shows that, when efficiency wage theory does not take effect, employment is below wage, which means there are rather large cyclical wage movement as well as the rather low cyclical movement in employment. This is not true.

Figure 2 is the result by setting ν =0.95. We can see that after a positive technology shock, employment and wage go up. And the movement of employment is significantly higher than that of wage. This is just the case that the cyclical movement of employment is high and that of wage is low. The responses are more or less similar to the benchmark RBC model

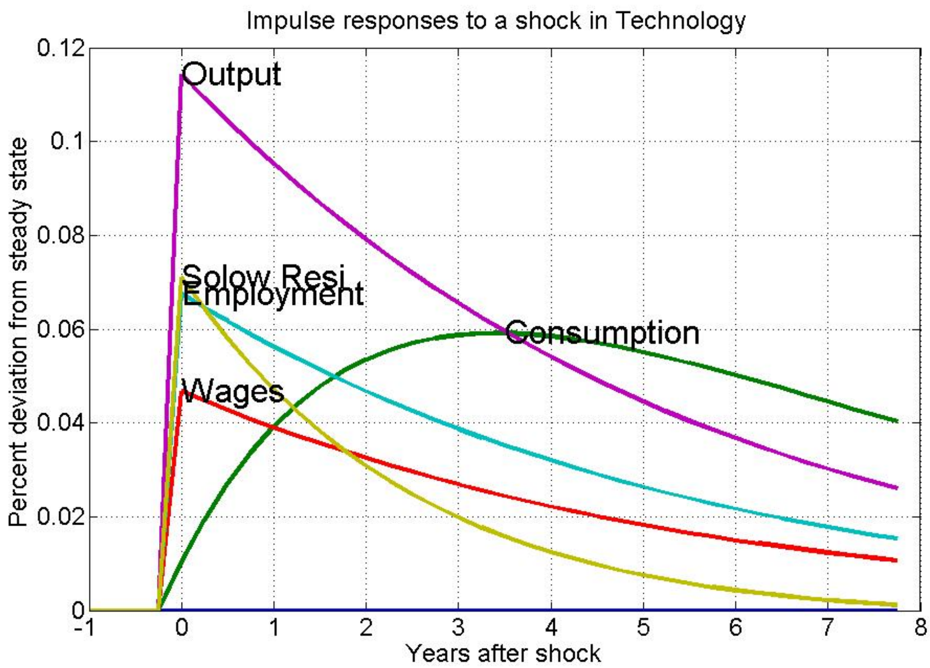

Figure 2 Impulse response to a one percent deviation in technology, by setting ν =0.95 (efficiency wage theory take effect).

By comparing figure 1 and figure 2, we find that efficiency wage theory does help to understand the rather large cyclical movement in employment. However, we can also see that the quantitative nature of fluctuations induced by technology shocks changed little, though the curves are in very good shapes. This brings out the question: is the technology shock fluctuates reasonably?

Then, the paper the paper presents a detailed comparison of the standard deviations.

	US Data	Hansens Benchmark	With Efficiency Wages				
ν			0	0.5	0,9	0,95	1.0
σ_y	1.72	1.62	1.25	1.30	1.63	1.92	3.26
σ_c	0.86	0.86	0.39	0.40	0.49	0.58	0.98
σ_h	1.69	1.30	0.08	0.16	0.68	1.13	3.26
σ_w	0.76	1.23	1.16	1.14	0.95	0.79	0.01
σ_q			0.00	1.13	8.53	14.92	44.24
σ_{SR}	1.18	1.18	1.18	1.18	1.18	1.18	1.18
σ_ε		1.4	1.4	2.3	8.0	12.9	36.1

Table 1: Standard Deviations comparison between Uhlig and Xu (1996), Hansen's Benchmark model and real US data.

In table 1, the standard deviation of technology has been chosen so that the standard deviation of Solow residual is kept constant in the US level. All data are Hodrick-Prescott-filtered[1]. The cyclical average deviation from trend in percent is given by σ_y for output, by σ_c for consumption, by σ_h for hours worked, by σ_w for real wages, while σ_ε is the standard deviation of the technology innovation. The standard deviation of the shock σ_ε has always been chosen so as to have the observed Solow residual fluctuations (σ_{SR} =1.18).

As one can see, the efficiency wages help in explaining the rather large cyclical variation in employment (σ_h) as well as the low cyclical variation in wages (σ_w), if

[1] Please refer to appendix for more information

effort is sensible, say $\nu \approx 0.95$.

However, the problem is that, the standard deviation of technology shock is very big. $\sigma_\varepsilon = 12.9$ compared to the Solow residual of 1.18. Because of the unplausible large technology shock deviation, this doesn't seem like a fruitful explanation.

In order to have a further investigate, I also simulate and plot the figure of deviations of variables as follows:

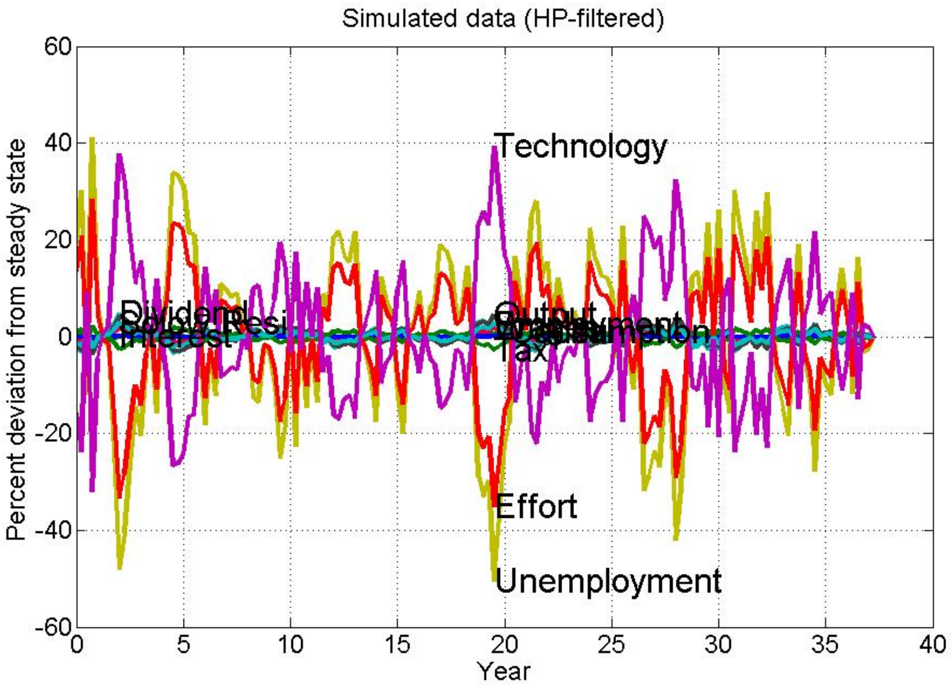

Figure 3: The HP-filtered percent deviation from steady state of the variables in Uhlig and Xu (1996) model.

From this figure, we can see clearly that technology fluctuate dramatically, and is exactly the opposite of variable effort.

Talking about the work by Uhlig and Xu, I think that they've introduced a very practical and convincing way to analyze effort, and some of their results are really interesting and delightful. In the other hand, I believe that there may be some other methods that we can use to help understanding real business cycles using efficient wage theory. With some of the ideas from Uhlig and Xu concerning the modeling of effort, I develop a new model.

3. The Model

In this chapter, I present my model, a real business cycle model with efficiency wage features.

Briefly, time is discrete, $t = 0,1,\ldots$ There are workers, competitive firms, representative households and a government. In each period of time, firms maximize their discounted present utility by reaping output, paying wages to workers and dividends to capital owners, and spending on monitoring workers. Households, by providing labor and choosing consumption and investment, also maximize their discounted present utility. The role of the government is to pay unemployment compensation to the jobless workers, with the finance of the tax charged from wage payments as well as unemployment compensation payments

The detailed explanation of the model is as follows:

3.1. The Autarky model of (un)employment

Firstly, I focus on the problem of workers. If employed, they receive wage from firms; if unemployed, they receive unemployment compensation from the government. I assume that all workers are identical except the effort they exert in their work. Although they provide effort in working, their effort can not be observed. As a result, the status of employment and unemployment are changed regardless of their working effort. Here I just introduce two variables concerning the employment status as follows:

x_1: The probability to lose a job automatically for an employed worker, i.e. the separation rate.

x_2: The probability to find a job for an unemployed worker

Then, I define e = employment rate, u = unemployment rate, and thus can have the relationship between e/u and x1/x2 as follows:

$$e = (1 - x_1)e + x_2 u \tag{3.1}$$

$$u = x_1 e + (1 - x_2) u \tag{3.2}$$

$$e + u = 1 \tag{3.3}$$

Solving for the above equations and I have:

$$x_1 e = x_2 u \tag{3.4}$$

With the steady state value of e and u, which can be find in other literatures, i.e. Uhlig and Xu (1996), I will be able to find the relationship between x_1 and x_2. But I still need some other information concerning the two values.

Note that x_1 is the "Separation rate", i.e. the probability of loosing job automatically of any employed people, which should be some value concerning the society and not fluctuating so much. As a result, I go to the website of "US Department of Labor" (http://www.dol.gov/) and finally find out the data. I set the mean value of the real dataset to be $x_1 (x_1 = 0.033)$. Please see Appendix for details.

With the value of the above parameters, I am able to study the utility of the employed/unemployed workers as a benchmark:

Let V^u be the sum of discounted utility of unemployed ones. For an unemployed worker, he receives unemployment compensation Z in the current period. In the next period, he can receive utility V^e with probability x_2, and V^u with a probability of $(1 - x_2)$.

Let V^e be the expected present value of utility of an employed worker. . For an employed worker, he receives wage less disutility from effort $(W - G(q))$ in the current period. In the next period, he will receive utility V^u with probability x_1, and V^e with probability $(1 - x_1)$.

The above two situations can be explained in the following figure (Figure 4)

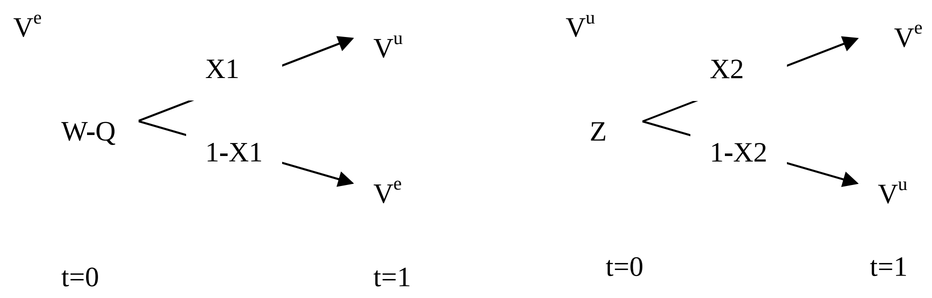

Figure 4: The discounted present utility (t=0) of an employed worker (left) and an unemployed worker (right).

We have:

$$V^u = Z + \beta[x_2 V^e + (1 - x_2)V^u] \tag{3.5}$$

$$V^e = (W - G(q) + \beta[x_1 V^u + (1 - x_1)V^e] \tag{3.6}$$

With the above two equations, I am able to solve for two variables V^u and V^u :

$$V^e = \frac{x_1\beta Z - (\beta - x_2\beta - 1)(W - G(q))}{(\beta - x_1\beta - 1)(\beta - x_2\beta - 1) - x_1 x_2 \beta^2} \tag{3.7}$$

$$V^u = \frac{x_2\beta(W - G(q)) - (\beta - x_1\beta - 1)Z}{(\beta - x_1\beta - 1)(\beta - x_2\beta - 1) - x_1 x_2 \beta^2} \tag{3.8}$$

Then I assume that workers "passive" in the model: they can not decide the value of wage and unemployment compensation. As a result, they can only calculate their utility using the steady state value of W and Z. Also, as an expected measurement of themselves, they use the steady state value of q to calculate the expected $G(q)$. In other words, W, Z and $G(q)$ are constants, and thus V^e and V^u are constant.

Then, I make an assumption that in the first period of time, everybody, including shirkers and non-shirkers has a job and thus utility $u(w_t)$. With V^u and V^e, we are able to calculate the sum of discounted utility of non-shirkers and shirkers recursively.

$$U_t^{NS} = u(w_t) - G(q_t) + \beta[x1 * V^u + (1 - x1)V^e] \tag{3.9}$$

$$U_t^S = u(w_t) + \beta[(x1 + m_t) * V^u + (1 - x1 - m_t)V^e] \tag{3.10}$$

Note that, here I made an assumption that there already exist "the first period", as a "standard" starting point of the whole calculation. By using this "starting condition", the whole recursive calculation holds. This is, in all, a dynamic analysis concerning effort. As a complement and another way of investigating effort q, in Chapter 3.6, I will provide another way of thinking concerning effort, which is mainly the idea from Uhlig and Xu (1996). In that deduction, effort is investigated within each period of time, and the relationship relating effort and other variables are found.

3.2. The production Function

Then I want to explain my extension in the Cobb-Douglas production function. Here, I make a little modification and define the production function as follows:

$$y_t = A_t k_{t-1}^{\alpha} (q_t n_t e_t)^{1-\alpha} \tag{3.11}$$

Where, y_t is output, A_t is technology, q_t is effort, $n_t e_t$ is real labor. α is capital share.

Note that, in my model, I define n_t as the labor provided by households. This is the amount of labor households are willing to provide in the market. However, not all the willing labor can find a job in the market. Firms are also in the market. They can not choose n_t, but they will choose the employment rate e_t in order to maximize their utility. As a result, the total "real working labor" is $n_t e_t$ and thus the unemployed labor is $n_t u_t$.

Also, I want to say something about q_t. q_t here is defined as the effort (or quality) of work. Concerning the relationship between effort and output, e.g. $y(q)$, I think that this should be a positive function and a twice differentiable concave function. Figure 5 can be an example of the relationship between y_t and q_t.

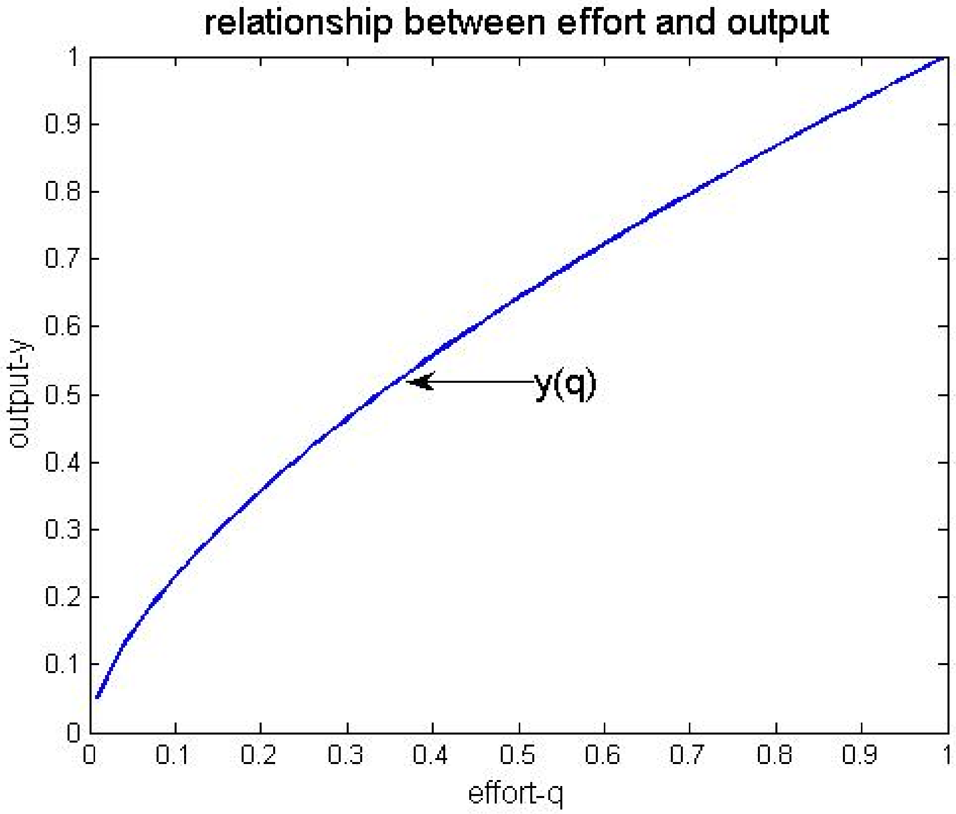

Figure 5: The relationship between effort and output.

Changes in A_t are stochastically driving the model, i.e. A_t fluctuates stochastically around some long-run mean and all other variables are functions of it.

Now I am going to compare the views with respect to effort q_t. In the benchmark view, q_t is fixed for anybody who works. Another point of view is from efficiency wage considerations. This view sees effort movements due to the cyclically varying threat of unemployment. According to this point of view, effort should be higher in recession, because the threat of unemployment is larger then.

Applying standard Solow Residual accounting to equation (1.11) leads to :

$$s_t = A_t q_t^{1-\alpha} \tag{3.12}$$

Movements in s_t are synonymous with movements in the exogenous technology parameter A_t and effort parameter q_t.

The technology autoregressive function is defined as:

$$\log A_t = \rho \log A_{t-1} + \varepsilon_t \tag{3.13}$$

$$\varepsilon_t \sim \mathrm{N}(0, \sigma_\varepsilon^2)$$

3.3. Firms

Assume that there are a large number of identical firms. Firms reap output (y_t), pay investor dividends (d_t), workers wages (w_t), and spend some money on monitoring workers ($m_t^2 y_t$) so that there will be no shirkers e.g. the incentive constraint holds. By choosing monitoring fee (m_t), employment rate (e_t), and capital (k_{t-1}) firms maximize their discounted present value by:

$$\max E\{\sum_{n_t, k_{t-1}, m_t} \beta^t [y_t - w_t n_t e_t - d_t k_{t-1} - m_t^2 y_t]\}$$

$$\text{s.t. } U^{NS} \geq U^S$$

By assumption about all the bargaining power resting within the firms, the inequality will be finally satisfied with equality, e.g. $U^{NS} = U^S$

Note, here the cost of monitoring is m_t^2, and in the previous section, the chance to lose job because of monitoring for the shirkers are m_t. This means that the function of "monitor cost" and "monitor effect" is a positive, continuous and concave function. Figure 6 gives a better look of the relationship.

We can solve firm's problem by setting up and solving firms' Lagrangian function. Please check the details in the next chapter.

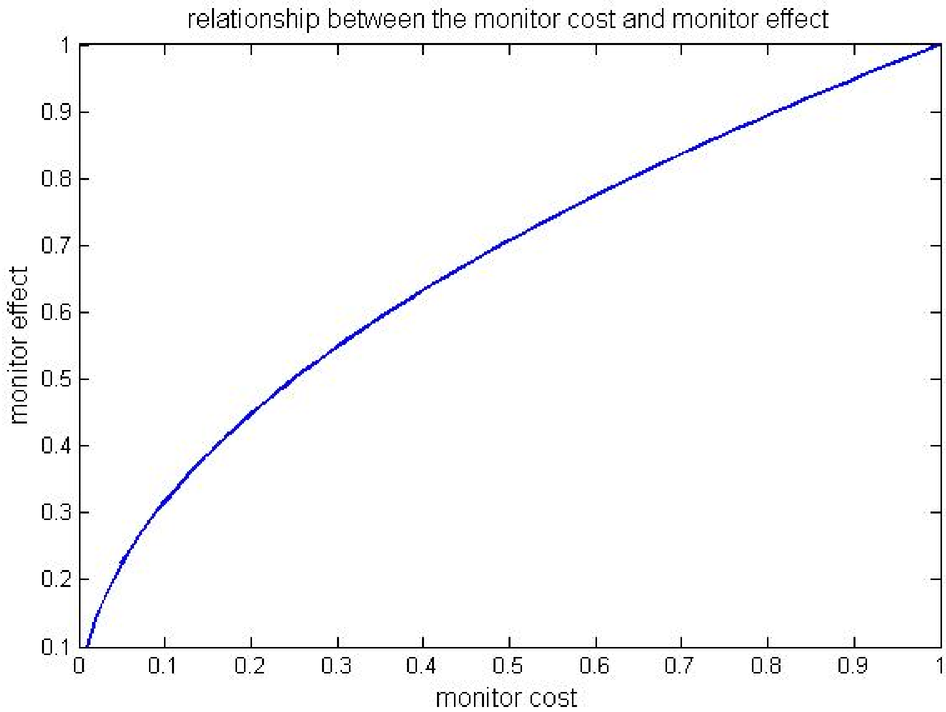

Figure 6: The relationship between the monitor cost and monitor effect.

3.4. Households

Representative agents wish to maximize the discount sum of utilities over an infinite lifespan. Each period, the agent obtains utility from consumption c_t, and disutility from labor supply n_t. Agent has the production function as firms, i.e. $y_t = A_t k_{t-1}^{\alpha}(q_t n_t e_t)^{1-\alpha}$. In each period, the agent must respect an initial condition and a budget constraint: $c_t + k_t + \tau_t(w_t n_t e_t + z_t n_t u_t) = y_t + (1-\delta)k_{t-1}$. This constraint guarantees that each period, what agents receive (production y_t and the capital left over from period t-1, with a depreciation rate of $1-\delta$) is equal to the spending. (consumption c_t, capital k_t and tax on wages $\tau_t(w_t n_t e_t + z_t n_t u_t)$)

The date t state variable is the capital stock available for production, k_{t-1}. As the problem is written now, the date t control variables are consumption c_t, and labor supply n_t. The numerical equations are as follows:

$$\max_{\{c_t, k_t, n_t\}} E\{\sum_t \beta^t [\log c_t - Bn_t]\},$$

$$\text{s.t.} \quad c_t + k_t + \tau_t(w_t n_t e_t + z_t n_t u_t) = y_t + (1-\delta)k_{t-1}$$

We can solve household's problem by setting up the Lagrangian. Please check the details in the next chapter.

3.5. Government:

In the model there is also a government, that pays the unemployment compensation Z, which in turn finances it via a proportional tax rate τ_t on wage payments as well as unemployment compensation payments. The government runs a balanced budget each period:

$$z_t n_t u_t = \tau_t(w_t n_t e_t + z_t n_t u_t).$$

Note that we can both sides by n_t and yield a simpler form:

$$z_t u_t = \tau_t(w_t e_t + z_t u_t) \tag{3.14}$$

3.6. Effort:1

Now, let's move on to the very heart of effort modeling.

In Chapter 3.2, I've already provide some dynamic way of thinking concerning effort q. Here, I will present an investigation on effort in any period of time, and find out the relationship between effort and other variables. The idea of this chapter is from Uhlig and Xu (1996). I will present the main idea of their paper because I think that this analysis is very important to make my model integrated as a whole.

To provide the description of effort, the time subscript is dropped for this section only, because the calculations can be used in any period of time, e.g. independent of t.

Assume that, there is some large number of workers and an even larger number of firms. The market mechanism is described by the following sequence of steps (these four steps are the quote from Uhlig and Xu (1996):

[1] The main idea of this section is from Uhlig and Xu (1996) and most of the contents of this section are quoted from it.

1. **Shirking choice**

   By working and receiving wage w, each worker i commits to a positive effort-for-wage-function $q_i(w)$. $q_i(w)$ is a non-negative and concave function, with $q_i(w) > 0$ and $q_i''(w) < 0$

2. **First round of entry**

   Each firm decides to enter into the market or not. If the firm enters the market, it chooses one worker. If several firms choose the same worker, a lottery is held to determine the "lucky" firms, and the "unlucky" firms have to choose another one. The first round ends until either all workers are matched with one firm or all entering firms are matched with one worker so that no firm and no worker is a member of two matches.

   Also, firms monitor workers. With some given probability p_m, firms observe the effort-for-wage function $Q_i(w)$ for worker i and can decide to fire the worker.

3. **Second round of entry**

   In this round, each firm that did not enter the market can choose to enter. If they enter the market, they will choose a remaining unemployed worker. They do not know this worker is previously fired or not. A lottery similar to the first round is held here.

4. **The work day**

   Now we have pairs of firm and worker. Firms pay workers wage w, rent some capital k at the market rate d, and produces final output $Y_i = f(k_i, q_i(w_i))$.

   Workers, supply effort $q(w)$ and receive wage w. After paying the tax, they keep $(1-\tau)w$

Also, Uhlig and Xu let p_1 be the chance for a worker to be matched to some firm in the first round, and p_2 be the chance for a remaining worker to be matched to some firm in the second round. As a result, the unemployment rate for a non-shirker is $p_{ns} = (1-p_1)(1-p_2)$, which is also the unemployment rate then.

With the set up of the above assumptions, Uhlig and Xu solve the following problems:

In equilibrium, the expected utility for a shirker is the same as that of a non-shirker, i.e. $V_{ns} = V_s$

As a result, the following equation is derived:

$$V((1-\tau)w) - V((1-\tau)z) = (\frac{1-p_1}{p_1 p_m})\frac{1-p_{ns}}{p_{ns}}G(q) \tag{3.15}$$

Where: $$\kappa = \frac{1-p_1}{p_1 p_m}$$

Here κ does not depend on p_2. Assume that only p_2 fluctuate with total employment, then κ is independent of time and is a constant. Applying log-utility function of $V(\cdot)$ yields:

$$\kappa G(q_t) = \frac{u_t}{e_t}\log(\frac{w_t}{z_t}) \tag{3.16}$$

Uhlig and Xu implicitly analysis the above equation as follow: effort $q_t = q(w_t, u_t, z_t)$, is a function of wages w_t, the unemployment rate $u_t = 1-e_t$, and the unemployment compensation z_t. The disutility of effort $G(q)$ can be defined as a function of wages (w_t), the unemployment rate ($u_t = 1-e_t$), and the (before-tax) unemployment compensation (z_t), e.g. $G(q_t) = G(w_t, u_t, z_t)$.

For the numerical calculations, Uhlig and Xu assume a particular functional form for the disutility of providing effort:

$$G(q) = \begin{cases} 0 & q = 0 \\ \theta(1-\nu+\nu q^{\phi}) & q > 0 \end{cases} \tag{3.17}$$

The parameter ν is the "sensitivity of effort". By varying the value of ν continuously, effort turns out to be an adjustable. $\nu \in (0,1)$, by choosing its value, we can vary the effort response between very small (ν close to zero) and very large (ν

close to 1). A Graph of the disutility function for several parameters is given in figure 7.

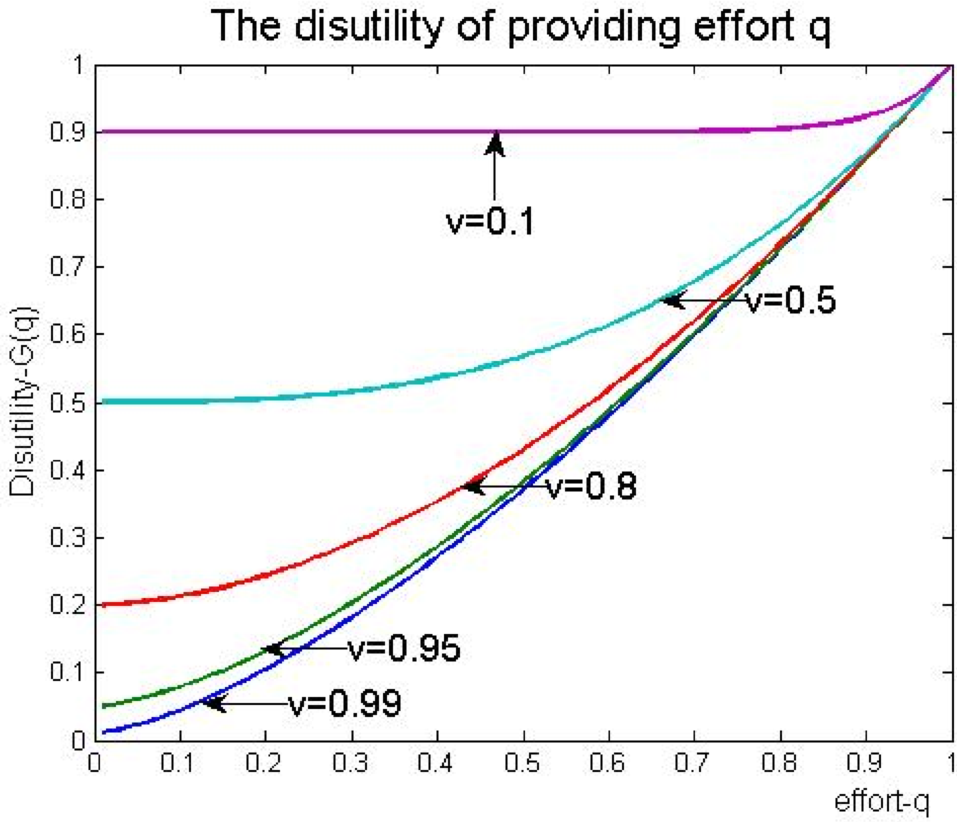

Figure 7: The disutility of providing effort, by varying the adjustable variable ν.

Note that, here, θ is set to be 1, and by checking the steady state, we can have the following equation:

$$\log(\frac{\overline{w}}{z})\phi\nu = 1 \tag{3.18}$$

Combine equation (3.19) and (3.20) yields:

$$\kappa\frac{e_t}{u_t}(1-\nu)+\frac{1}{\phi} = \log w_t - \log z_t \tag{3.21}$$

This equations can be used in numerical calculations.

After that, Uhlig and Xu go back to the firms' problem, with the production function $Y_i = f(k_i, q_i(w_i))$. By differentiating firm's problem with respect to k_t and w_t, sorting terms and obtain the usual Solow condition.

$$1 = \frac{q'(w)}{q(w)/w} \tag{3.22}$$

With this equation and differentiate equation (3.23) with respect to w, one obtains:

$$1 = \kappa \frac{e_t}{u_t} \theta \phi \nu q_t^{\phi} \tag{3.24}$$

This equation shows that the relationship between effort and unemployment claimed in the efficiency wage theory is clear: when employment goes up, effort must go up as well so as to keep this equations satisfied.

3.7. Others

Apart from the above equations, there are some other important ones in the model as follow:

Employment rate plus unemployment rate is equal to 1:

$$e_t + u_t = 1 \tag{3.25}$$

Total real labor (l_t), equals to the total labor provided by household (n_t) times the employment rate (e_t) chosen by firms.

$$n_t e_t = l_t \tag{3.26}$$

Applying standard Solow residual accounting to the production function $y_t = A_t k_{t-1}^{\alpha}(q_t n_t e_t)^{1-\alpha}$. Leads to:

$$s_t = A_t q_t^{1-\alpha} \tag{3.27}$$

4. Model Analysis

This chapter provides a detailed algebra analysis of my model, including calculation of derivatives, steady states and loglinearization. All the calculations are prerequisite to Toolkit[1] implementation.

4.1. Collect Equations

Now I list all the equations I derived from my model:

Household:

Households solve the following problem:

$$\max_{\{c_t, k_t, n_t\}} E\{\sum_t \beta^t [\log c_t - Bn_t]\} ,$$

$$\text{s.t.} \quad c_t + k_t + \tau_t (w_t n_t e_t + z_t n_t u_t) = y_t + (1-\delta) k_{t-1}$$

Setting Lagrangian:

$$L = \max_{\{c_t, k_t, n_t\}} E\sum_t \beta^t \{[\log c_t - Bn_t] - \lambda_t [c_t + k_t + \tau_t (w_t n_t e_t + z_t n_t u_t) - y_t - (1-\delta) k_{t-1}]\}$$

FONCs:

w.r.t c_t: $$\frac{1}{c_t} - \lambda_t = 0 \qquad (4.1)$$

w.r.t. n_t $$B + \lambda_t (\tau_t w_t e_t + \tau_t z_t u_t) = \lambda_t (1-\alpha) \frac{y_t}{n_t} \qquad (4.2)$$

w.r.t. k_t $$\lambda_t = \beta E_t [\lambda_{t+1} R_{t+1}]^2 \qquad (4.3)$$

w.r.t. λ_t $$c_t + k_t + \tau_t (w_t n_t e_t + z_t n_t u_t) = y_t + (1-\delta) k_{t-1} \qquad (4.4)$$

Firm:

Firms solve the following problem:

$$\max E\{ \sum_{n_t, k_{t-1}, m_t} \beta^t [y_t - w_t n_t e_t - d_t k_{t-1} - m_t^2 y_t]\}$$

$$\text{s.t.} \quad U^{NS} = U^S$$

1 Please refer to the appendix as a detailed introduction
2 This is also the Euler equation

Setting Lagrangian:

$$L = \max E \sum \beta^t \{ y_t - w_t n_t e_t - d_t k_{t-1} - m_t^2 y_t - \psi_t (U_t^{NS} - U_t^S) \}$$

$$= \max E \sum \beta^t \{ a_t k_{t-1}^{\alpha} (q_t n_t e_t)^{1-\alpha} - w_t n_t e_t - d_t k_{t-1} - m_t^2 y_t - \psi_t [(\beta m_t (V^e - V^u) - G(q_t)] \}$$

FONCs:

w.r.t. e_t:
$$(1-\alpha)\frac{y_t}{n_t} - w_t n_t - \psi_t \frac{1}{\kappa \phi e_t^2} = 0 \tag{4.5}$$

w.r.t: k_{t-1}:
$$d_t = \alpha \frac{y_t}{k_{t-1}} \tag{4.6}$$

w.r.t. m_t :
$$2 m_t y_t + \psi_t \beta (V^e - V^u) = 0 \tag{4.7}$$

w.r.t. ψ_t
$$G(q_t) = \beta m_t (V^e - V^u) \tag{4.8}$$

Market:

$$y_t = A_t k_{t-1}^{\alpha} (q_t n_t e_t)^{1-\alpha} \tag{4.9}$$

$$R_t = \alpha \frac{y_t}{k_{t-1}} + 1 - \delta \tag{4.10}$$

Government:

$$z_t u_t = \tau_t (w_t e_t + z_t u_t) \tag{4.11}$$

Effort:

$$\kappa \frac{e_t}{u_t}(1-\nu) + \frac{1}{\phi} = \log w_t - \log z_t \text{ [1]} \tag{4.12}$$

$$1 = \kappa \frac{e_t}{u_t} \theta \phi \nu Q_t^{\phi} \tag{4.13}$$

[1] Please see Appendix for details of loglinearization of this equation

Technology:

$$\log A_t = \rho \log A_{t-1} + \varepsilon_t \tag{4.14}$$

Solow residual:

$$s_t = A_t q_t^{1-\alpha} \tag{4.15}$$

Others:

$$e_t + u_t = 1 \tag{4.16}$$

$$n_t e_t = l_t \tag{4.17}$$

For the unemployment compensation, according to the paper of Uhlig-Xu, it is set to be constant. $z_t = z$

As a result, there are in total 1equations, and 17 variables. There is enough information to solve the model.

The variables are: $c_t, k_t, \tau_t, w_t, n_t, y_t, \lambda_t, R_t, d_t, u_t, A_t, q_t, \psi_t, m_t, s_t, e_t, l_t$

Figure in the next page is a list of all the variables.

Variable	Description
c_t	Consumption
k_t	Capital
τ_t	Tax Rate
w_t	Wage
n_t	The “Willing” Labor provided by households
y_t	Output
λ_t	Lagrangian multiplier in household’s problem
R_t	Return rate
d_t	Dividend
u_t	Unemployment rate
A_t	Technology
q_t	Effort
ψ_t	Lagrangian Multiplier in firm’s problem
m_t	Monitor cost
s_t	Solow Residual
e_t	Employment Rate
l_t	Real working labor in the market

Table 2: A list of all the variables in my model.

4.2. Steady State

According to the equations in Chapter 4.1, the steady state of the variables is:

$$\frac{1}{\bar{c}} - \bar{\lambda} = 0 \tag{4.18}$$

$$B + \bar{\lambda}\bar{\tau}(\overline{we} + \overline{zu}) = \lambda(1-\alpha)\frac{\bar{y}}{\bar{n}} \tag{4.19}$$

$$\bar{c} + \bar{k} + \bar{\tau}(\overline{wne} + \overline{znu}) = \bar{y} + (1-\delta)\bar{k} \tag{4.20}$$

$$1 = \beta\bar{R} \tag{4.21}$$

$$(1-\alpha)\frac{\bar{y}}{\bar{n}} - \overline{wn} - \bar{\psi}\frac{1}{\kappa\phi\bar{e}^2} = 0 \tag{4.22}$$

$$\bar{d} = \alpha\frac{\bar{y}}{\bar{k}} \tag{4.23}$$

$$2\overline{my} + \bar{\psi}\beta(V^e - V^u) = 0 \tag{4.24}$$

$$1 - \nu + \nu\bar{q}^{\phi} = \beta\bar{m}(V^e - V^u) \tag{4.25}$$

$$\bar{y} = \bar{A}\bar{k}^{\alpha}(\overline{qne})^{1-\alpha} \tag{4.26}$$

$$\bar{R} = \alpha\frac{\bar{y}}{\bar{k}} + 1 - \delta \tag{4.27}$$

$$\overline{zu} = \bar{\tau}(\overline{we} + \overline{zu}) \tag{4.28}$$

$$\kappa\frac{\bar{e}}{\bar{u}}(1-\nu) + \frac{1}{\phi} = \log\bar{w} - \log\bar{z} \tag{4.29}$$

$$1 = \kappa\frac{\bar{e}}{\bar{u}}\theta\phi\nu\bar{q}^{\phi} \tag{4.30}$$

$$\bar{A} = 1 \tag{4.31}$$

$$\bar{s} = \bar{A}\bar{q}^{1-\alpha} \tag{4.32}$$

$$\bar{e} + \bar{u} = 1 \tag{4.33}$$

$$\overline{ne} = \bar{l} \tag{4.34}$$

4.3. Loglinearization

By now, I've got several valuable equations concerning my model. Because they are non-linear equations, it is difficult to analyze and interpret them. By the process of loglinearization, I replace the original variables by the "percent deviations from the steady state" and thus replace the dynamic nonlinear equations by dynamic linear equations. These linear equations made interpretation and further calculations much easier.

I loglinearized all the equations around the steady state and get the following linear equations:

$$\hat{c}_t + \hat{\lambda}_t = 0 \tag{3.1}$$

$$\bar{\lambda}\bar{\tau}\,\overline{we}(\hat{\lambda}_t + \hat{\tau}_t + \hat{w}_t + \hat{e}_t) + \bar{\lambda}\bar{\tau}\,\overline{zu}(\hat{\lambda}_t + \hat{\tau}_t + \hat{u}_t) - \frac{\bar{\lambda}\bar{y}(1-\alpha)}{\bar{n}}(\hat{\lambda}_t + \hat{y}_t - \hat{n}_t) = 0 \tag{3.2}$$

$$\bar{c}\hat{c}_t + \bar{k}\hat{k}_t + \bar{\tau}\,\overline{wne}(\hat{\tau}_t + \hat{w}_t + \hat{n}_t + \hat{e}_t) + \bar{\tau}\,\overline{znu}(\hat{\tau}_t + \hat{n}_t + \hat{u}_t) - \bar{y}\hat{y}_t - (1-\delta)\bar{k}\hat{k}_{t-1} = 0 \tag{3.3}$$

$$\hat{d}_t + \hat{k}_{t-1} - \hat{y}_t = 0 \tag{3.4}$$

$$(1-\alpha)\overline{ye}(\hat{y}_t + \hat{e}_t) - \overline{wne}^2(\hat{w}_t + \hat{n}_t + 2\hat{e}_t) - \frac{\bar{\psi}}{\kappa\phi}\hat{\psi}_t = 0 \tag{3.5}$$

$$\bar{R}\hat{R}_t - \alpha\frac{\bar{y}}{\bar{k}}(\hat{y}_t - \hat{k}_{t-1}) = 0 \tag{3.6}$$

$$\bar{\tau}\,\overline{zu}(\hat{\tau}_t + \hat{u}_t) + \bar{\tau}\,\overline{we}(\hat{\tau}_t + \hat{w}_t + \hat{e}_t) - \overline{zu}\hat{u}_t = 0 \tag{3.7}$$

$$\hat{y}_t - \hat{a}_t - (1-\alpha)\hat{q}_t - \alpha\hat{k}_{t-1} - (1-\alpha)\hat{n}_t - (1-\alpha)\hat{e}_t = 0 \tag{3.8}$$

$$\hat{y}_t + \hat{m}_t - \hat{\psi}_t = 0 \tag{3.9}$$

$$\phi v\hat{q}_t - \hat{m}_t = 0 \tag{3.10}$$

$$\hat{e}_t - \hat{u}_t - \frac{\hat{w}_t}{\log(\bar{w}/\bar{z})} = 0\ ^{1} \tag{3.11}$$

$$\hat{s}_t - \hat{a}_t - (1-\alpha)\hat{q}_t = 0 \tag{3.12}$$

[1] Please see Appendix for details of this loglinearization

$$\bar{u}\hat{u}_t + \bar{e}\hat{e}_t = 0 \tag{3.13}$$

$$\hat{e}_t + \phi\hat{q}_t - \hat{u}_t = 0 \tag{3.14}$$

$$\hat{n}_t + \hat{e}_t - \hat{l}_t = 0 \tag{3.15}$$

And:

$$E_t[\hat{\lambda}_{t+1} + \hat{R}_{t+1}] - \hat{\lambda}_t = 0 \tag{3.16}$$

$$\hat{a}_{t+1} = \rho\hat{a}_t \tag{3.17}$$

Here, equation (3.11) is a little bit difficult to loglinearized. I attach the detailed algebra in the Appendix as an example of loglinearization.

4.4. Rough analysis on the loglinearized equations

According to my experience when developing my model, I think that the loglinearized equations are as important as the model theories. By analysis on these linear equations, I am able to find the trend of variable changes and thus have a better understanding of the model. I would like to call the loglinearized equations as a double check of the model. With the help of it, I am not only able to study the problem from the "theory side" but also from the "equation side"

For example, if there is a technology shock, then, normally more workers are employed and output increases. From equation: $\hat{e}_t + \phi\hat{q}_t - \hat{u}_t = 0$ (3.14), we can find that the increase in employment cause the decrease of effort. Because effort decreases, according to equation $\hat{y}_t - \hat{a}_t - (1-\alpha)\hat{q}_t - \alpha\hat{k}_{t-1} - (1-\alpha)\hat{n}_t - (1-\alpha)\hat{e}_t = 0$ (3.8), output will decrease. As a result, the positive effect of technology shock will be partly counteracted by the negative effect of effort. As a result, in order to keep the standard deviation Solow residual constant, there may be a bigger technology shock.

However, in the other hand, if output increase, according to $\hat{y}_t + \hat{m}_t - \hat{\psi}_t = 0$ (3.9), the monitoring fee also increases, then according to $\phi\nu\hat{q}_t - \hat{m}_t = 0$ (3.10), effort increases as well. As a result, the change in effort may not fluctuate so much and there might not be such big technology shock.

4.5. Setting calibrations

To calibrate the model, several additional parameters had to be chosen.

In order to be able to compare my results with Hansen (1985) and Uhlig-Xu (1996), I try to use the same calibration values, such as α and β as them (Calibrations are almost the same in these two models). Also, I normalize $\bar{q} = 1$, and find some other steady state data from the US Department of Labor (See Appendix for details). The calibration values are as in Table 3:

Parameter	Value	Description
R	1.01	Interest rate, the same as in Hansen's model
β	$1/R$	Discount factor
α	0.36	Capital share, the same as in Hansen's model
δ	0.025	Capital depreciate rate, the same as in Hansen's model
ρ	0.95	Autoregressive parameter on technology shock
$\bar{u}$	0.05	Steady state value of unemployment rate Data from Uhlig-Xu (1996)
$\bar{e}$	$1-\bar{u}$	Steady state value of employment rate
$\bar{l}$	1/3	Steady state value of real labor The same value as in Hansen's model
$\bar{w}/\bar{z}$	2	Replacement ratio, data from Uhlig-Xu (1996)
x_1	0.033	Separation Rate, Data from US Department. of Labor
x_2	$x_1 \times 19$	Probability to find a job for the separated people
ν	$\in (0,1)$	Sensitivity of effort
σ_ε		Standard deviation of technology shocks Change this value so that to keep σ_{SR} constant
$\bar{a}$	1	Steady state value of technology is normalized to 1
$\bar{q}$	1	Steady state value of effort is normalized to 1

Table 3: The list of the calibration in my model

5. Results

After the complicated loglinearization, I am able to calculate all impulse responses in my model using Toolkit[1]. In this chapter, I will present numeral results as well as graphic results, and examine the cyclical consequences of an efficiency wage theory, when effort is an adjustable variable.

5.1. Graphical results

A graphic figure of impulse response helps people understand and analyze the model a lot. Here I make use of Toolkit and calculate the impulse response after a one-percent deviation shock of technology.

By varying the value of ν, I find the following different results:

First, setting effort sensitivity to a very small number, $\nu = 0.1$, and the result is as follows:

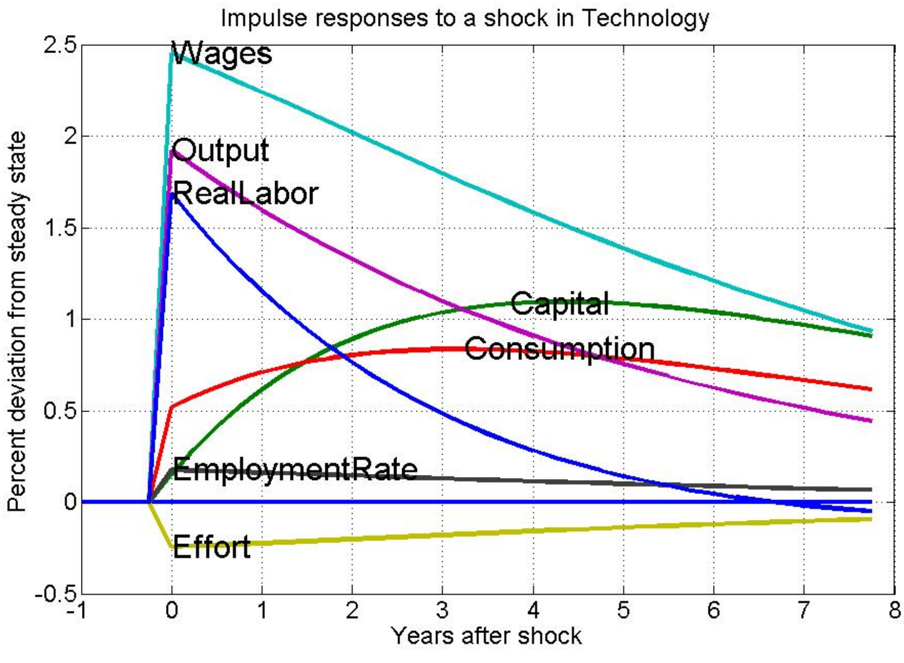

Figure 8: Impulse responses to a one percent shock in Technology in my model, with ν =0.1.

We can easily find that the impulse response of wage is weird: it is too big! Other

[1] Please refer to Appendix for a Short introduction of Toolkit

impulse responses are reasonable.

Then, following the comparison methodology of Uhlig and Xu (1996), increase the effort sensitivity, say, $\nu = 0.5$, and the result is as follows:

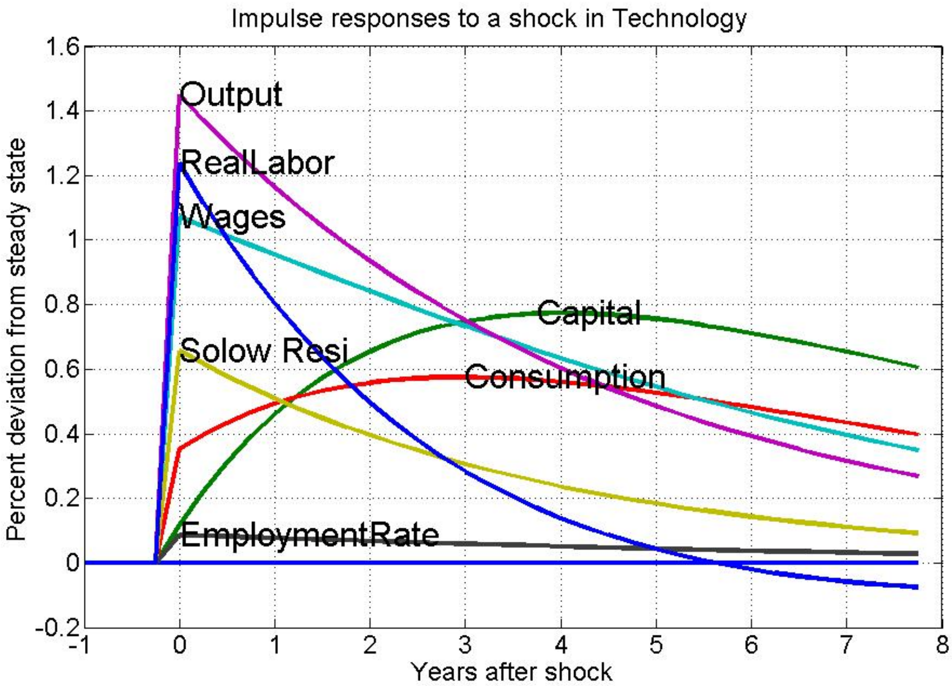

Figure 9: Impulse responses to a one percent shock in Technology in my model, with ν =0.5.

In Figure 9, we can find that the impulse response of wage is smaller than that in Figure 8. Still, it is too big; it fluctuates more than real labor.

Then, I set the effort sensitivity to a big value $\nu = 0.95$, and the result is as follows:

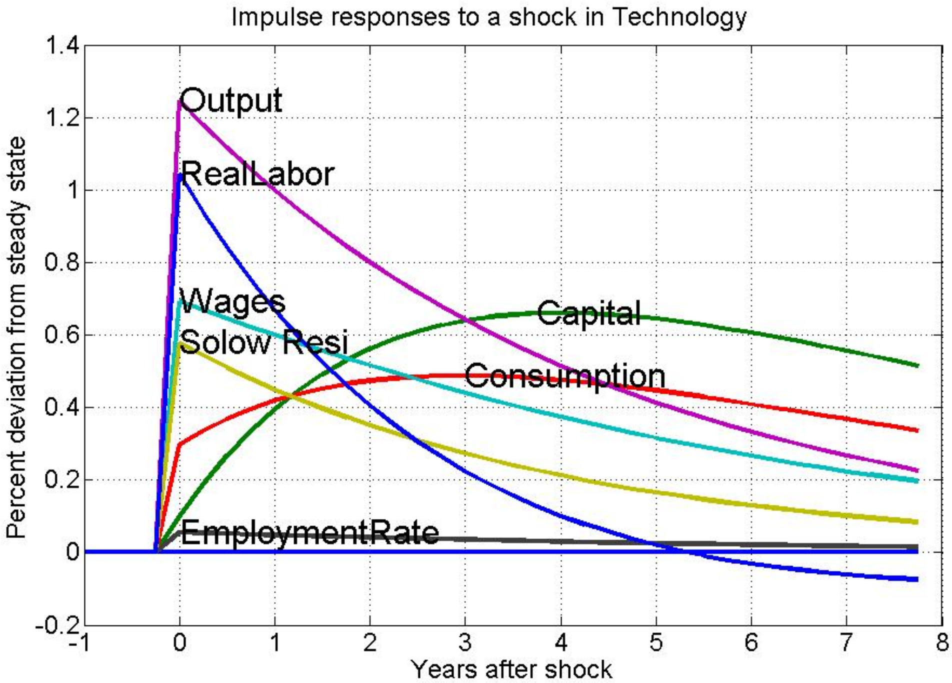

Figure 10: Impulse responses to a one percent shock in Technology in my model, with ν =0.95.

Now the whole figure looks nice. As a result, I think that efficient wage consideration does help explain the rather large cyclical employment movement made as well as the rather low cyclical movements in real wages.

We can have a look of the responses of other variables:

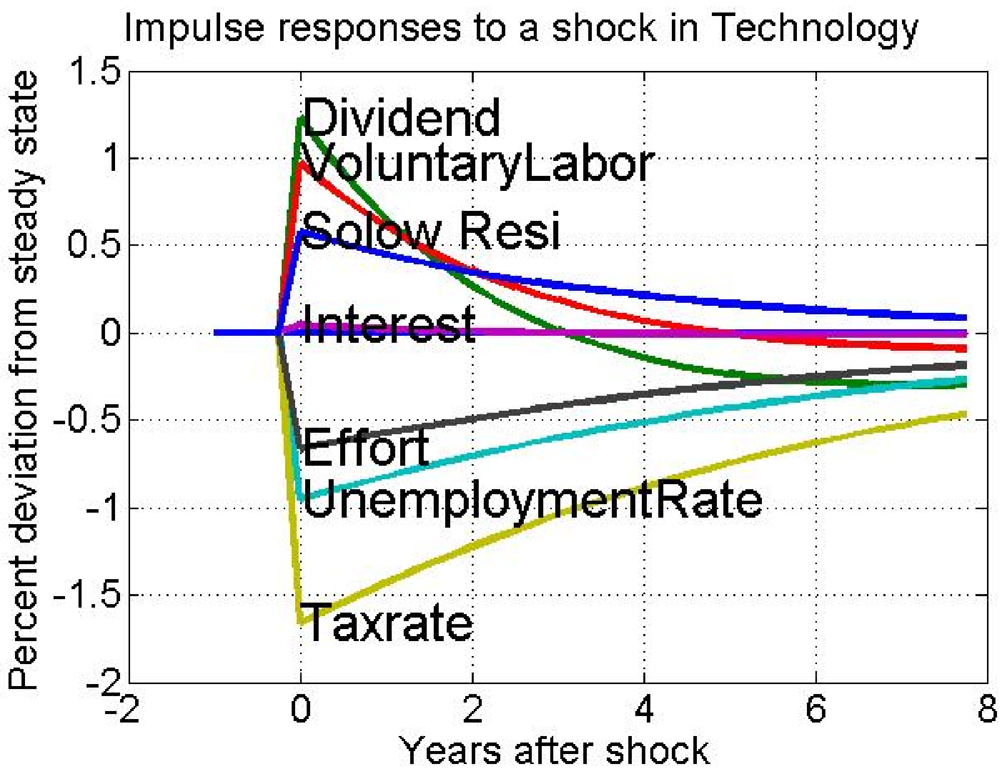

Figure 11: Impulse responses of other variables to a one percent deviation shock in Technology in my model, with adjustable variable ν =0.95.

We can find that after a positive technology shock, employment increases, and also more households are willing to work. These together make the real labor increases. And, just as I've predicted, effort drops.

5.2. Numeral Results

After an intuitive comparison based on graphics, let's move on to the numeral results. I use the Toolkit again, turning on Hodrick-Prescott-filter[1], investigate the standard deviations of all the variables, and compare them to the results from Uhlig and Xu (1996), classical RBC model, and real US data. All the data here are HP filtered.

[1] See Appendix for more information on HP Filter

	US data	Hansens Benchmark	With Efficiency Wages							
ν			0.1[1]		0.5		0.95		1	
			Mine[2]	Paper[3]	Mine	Paper	Mine	Paper	Mine	Paper
σ_y	1.72	1.62	2.701	1.25	2.64	1.30	2.53	1.92	2.51	3.26
σ_c	0.86	0.86	0.852	0.39	0.790	0.40	0.743	0.58	0.738	0.98
σ_h	1.69	1.3	2.373	0.08	2.29	0.16	2.12	1.13	2.09	3.26
σ_w	0.76	1.23	3.477	1.16	2.63	1.14	1.59	0.79	1.52	0.01
σ_q			0.348	0.00	1.31	1.13	1.51	14.92	1.52	44.24
σ_{SR}	1.18	1.18	1.18	1.18	1.18	1.18	1.18	1.18	1.18	1.18
σ_a		1.4	1.405	1.4	2.13	2.3	2.14	12.9	2.15	36.1

Table 4: Standard deviation comparison between my model, Uhlig and Xu's model, RBC model, and real US data.

Table 4 give us a detailed comparison of the main variables in my model, Uhlig and Xu's model, RBC model and real US data.

First of all, let's compared my results to the results from Uhlig and Xu (1996).

Both models show that, when ν is small, standard deviation of real labor (σ_h) is smaller than the standard deviation of wage (σ_w); when ν is big and close to 1, σ_w is bigger than σ_h. This proves the idea that efficiency wage considerations help explain the rather large cyclical employment movement as well as the rather low cyclical movement in real wages. The main difference between the two models is the

[1] According to the paper, the valued are calculated by setting v=0. However, I argue that, according to equation (3.18) in chapter3.6, v should not be 0. As a result, I set v to a small value, v = 0.1
[2] These are the results from my model.
[3] These are the results from Uhlig –Xu (1996), These data are also provided in Table 1

value of σ_a and σ_q. In both models, the technology movements is getting higher when ν changes from small to big values. But in my model, the technology shocks do not fluctuate that big. In my model, when $\nu = 1$, σ_a has the biggest value of 2.15, around 1.5 times as big as that from Hansen's model. As a result, I think that my model make sense at least in the extent that it helps to have a better understanding of RBC theory with efficiency wage considerations.

There are also some other differences between my model and Uhlig and Xu's model. In Uhlig and Xu's model, the author set the optimal value of ν to be 0.95, which makes the whole model has a closer result with Hansen's. In my model, I would say $\nu = 1$ is the best choice, which means that the workers are totally sensitive to efficiency wages.

Then, let's compare the results of my models with Hansen's benchmark model and US data.

As I've mentioned before, I still can not avoid a bigger fluctuation of technology compared with the benchmark. What's more, I've got bigger fluctuations in all other variables, including output, working hours and wages. The good thing is that, when ν increases, the differences is getting smaller. And, the positive gap between σ_h and σ_w is bigger than that in the benchmark model, which is closer to real US data.

I would like to discuss the critics of my model in the next chapter.

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6. Discussion

I've got a not bad result in chapter 5, but there are still lots of things to analysis. In this chapter, I will present the shortcomings of my model, give my explanation of them, and finally give a further analysis on the variable "effort".

6.1. Unexplained results

Despite the above results, there are also shortcomings of this model.

From table 3, we can find that though the standard deviation of technology drops significantly comparing the results from the results from Uhlig and Xu (1996), it is still 1.5 times bigger than that from the classical RBC model. Also, the standard deviations of other variables are bigger than the benchmark values. Please read figure 5 for details:

	US data	Hansens Benchmark	With Efficiency Wages ($\nu=1$)
σ_y	1.72	1.62	2.51
σ_c	0.86	0.86	0.738
σ_h	1.69	1.3	2.09
σ_w	0.76	1.23	1.52
σ_q			1.52
σ_{SR}	1.18	1.18	1.18
σ_a		1.4	2.15

Table 5: Standard deviation comparison between my model, US data and Hansen's benchmark model, with $\nu=1$

I think that the reason is because of efficiency wage considerations. When good time comes, the threat of unemployment drops and working effort drops. That is to say, effort correlates negatively with technology in the efficiency wage considerations. Because of that, technology has to fluctuate more in order to keep the Solow residual constant. The bigger fluctuation of technology causes the bigger fluctuation of other variables.

Also, there are some other things that are implausible.

See figure 12. by setting $v=1$, after a positive one percent capital deviation, output drops slightly, which should not be true in the real world.

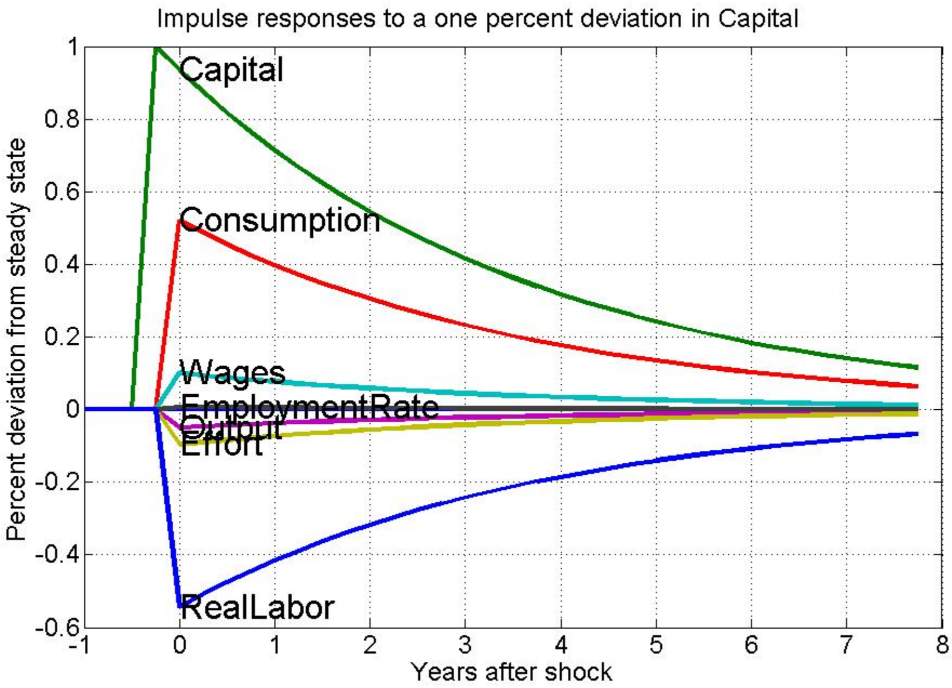

Figure 12: Impulse responses to a one percent deviation in capital, with efficiency wage considerations ($v=1$).

If I do not have the efficiency wage considerations and set v to a small value, i.e. $v=0.1$, the impulse response to a one percent deviation in capital is in Figure 13.

We can see that now the deviations of the variables are reasonable.

I think this is also a result of the efficiency wage considerations. If workers are more sensitive to wages, their effort drop more in good times, and thus cause the drop of output.

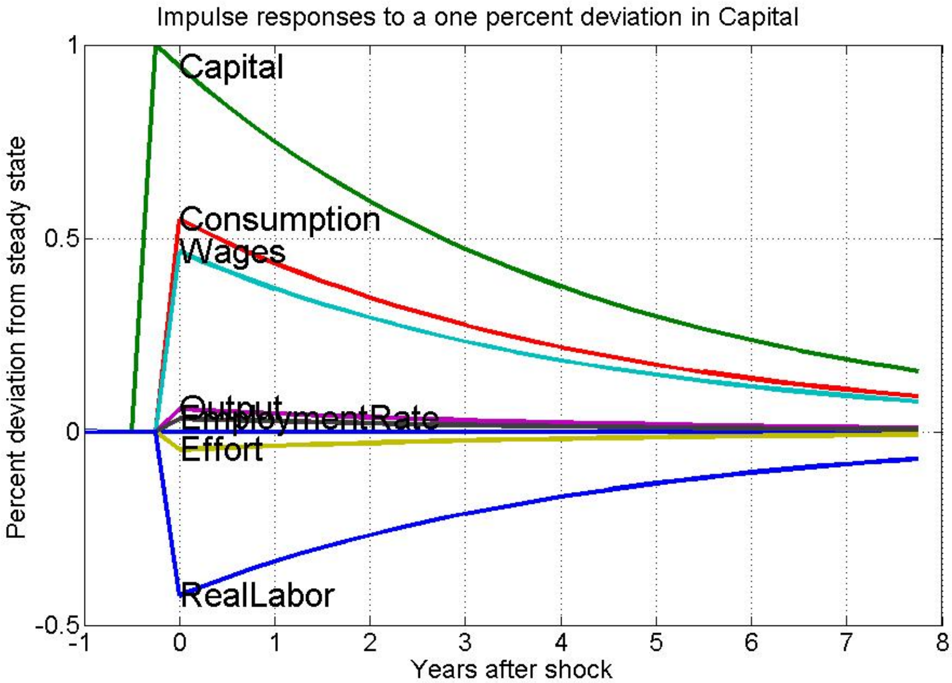

Figure 13: Impulse responses to a one percent deviation in capital, with ($v = 0.1$).

In all, I think that the efficiency wage considerations help to explain the rather large employment movement and rather low wage movement. However, because effort correlates negatively with technology, other variables fluctuate more than in the Benchmark model and there is even a negative deviate of output when capital deviates positively.

6.2. Further discussion on "effort"

In order to solve this problem, I believe that further improvements should be made on the modeling of effort.

At the very heart of my paper, according to the efficiency wage theory, when bad time comes, unemployment is the threat to prevent workers from shirking. As a result, if good time comes, effort drops, i.e. an increase in technology result to be a decrease in effort.

In fact, there are some other models regarding effort as something positively correlated with technology. For example, in the Labor hoarding model, firm-worker relationships are characterized by labor hoarding. With labor hoarding, firms hesitate to fire workers even during recessions e.g. because it is costly to find new workers in the next upswing, see e.g. Burnside, Eichenbaum and Rebelo (1993). As a result, effort increases when technology increases. Figure 14 shows the relationship between technology and effort in the above three models.

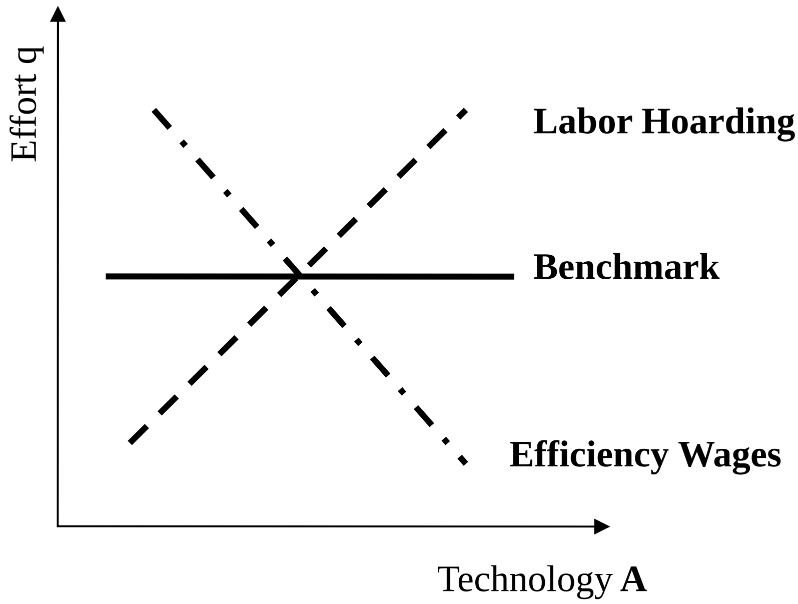

Figure 14: The relationship between technology and effort in the three models: RBC model, efficiency wage model, and labor hoarding model.

Why all the three theories make sense? Especially, the efficiency wage mode contradicts the labor hoarding model. I think the problem lies in the modeling of effort. In all of the three models, effort can also been seen as the quality of workers. Both “effort of workers” and “quality of workers” are human factors that contribute to growth. By providing it, workers receive disutility, and output increase. However, I think that we should do something to distinguish them.

Quality is something that objectively exists and can not be changed by workers in a short time. For example, college students are more compatible in labor markets, because these labors have better “quality”. In the other hand, effort is something that subjectively exists and fluctuates a lot according to time, weather, people’s mood, etc.

For example, a cup of coffee helps a sleepy person to provide better working effort.

Effort is something negatively correlated with technology shocks, but quality is something positively correlated. For example, in some society, every farmer cultivate using some cultivate cattle. One day, somebody invented a new cultivator, a more productive machine. The economic will boom and output will increase. For the farmers, they are more qualified by using machines, but in the other hand, they will be very like to consume more leisure than he used to be.

In all, I think that in order to have a better understanding of RBC theory, a better way of model effort should be proposed and there is still a lot of work to do.

7. Conclusions

Understanding the causes of aggregate fluctuations is a central goal of macroeconomics. The leading theory concerning the sources and the nature of macroeconomics fluctuations is the Real Business Cycle Theory.

Because the benchmark Real Business Cycle model is criticized for its “voluntary employment” and the Efficiency Wage Theory is well-known in understanding “involuntary employment”. Many researchers have worked hard to see if efficiency wage considerations can improve our understanding of business cycle.

One interesting work is done by Uhlig and Xu (1996). They proposed a model and examine the cyclical consequences of an efficiency wage theory, when effort is an adjustable variable. They find that increase the variability of effort due to efficiency wage consideration helps to explain the rather large cyclical employment movement as well as the rather low cyclical wage movement, but require unplausibly large movements in the technology parameter, seven times as large as that in Hansen’s benchmark model.

Following their track of thinking, I made use of their way of modeling effort as an adjustable variable, change the other part of the model, and propose a new model. With this model, I am able to explain the rather large employment movements compared to wage movements, and reduce the fluctuation of technology significantly.

However, there are still some unexplained results in the model, the standard deviation of technology is still around 1.5 times bigger than that in the benchmark model. Also, standard deviations of other variables are bigger than the benchmark values.

I think that these problems are mainly caused by the modeling of effort. In the efficiency wage theories, effort correlates negatively with technology. In order to keep the Solow residual constant, technology shocks have to fluctuate more.

As a result, I think that efficiency wage theory help to explain RBC theory, but only in some extent. More things concerning the modeling of effort should be made.
I think that the original view of “effort” should be divided into two variables, “effort

of workers” and “quality of workers”. Effort of workers is negatively correlated with technology, while quality of workers is positively correlated. They explain the human contribution in production together. I think that more things can be done in this track of thinking and that will help have a better understanding of RBC theory.

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9. Appendix

A. The Toolkit

Toolkit is some template MATLAB code designed by Prof. H Uhlig. Based on a through understanding of loglinearization and matrix transformation, it takes advantage of matrix calculation by MATLAB and makes the complicated impulse responses calculation in Real Business Cycle theory easier and more understandable. Also, it integrates Fourier transformation and can transform and calculate data in the frequency domain. By using frequency domain algorithms such as the Hodrick-Prescott (HP) Filter[1], the randomness of shocks can be eliminated and the "true" deviations can be calculated regardless of random samples.

Here is an example of using Toolkit to calculate Hansen's benchmark model.

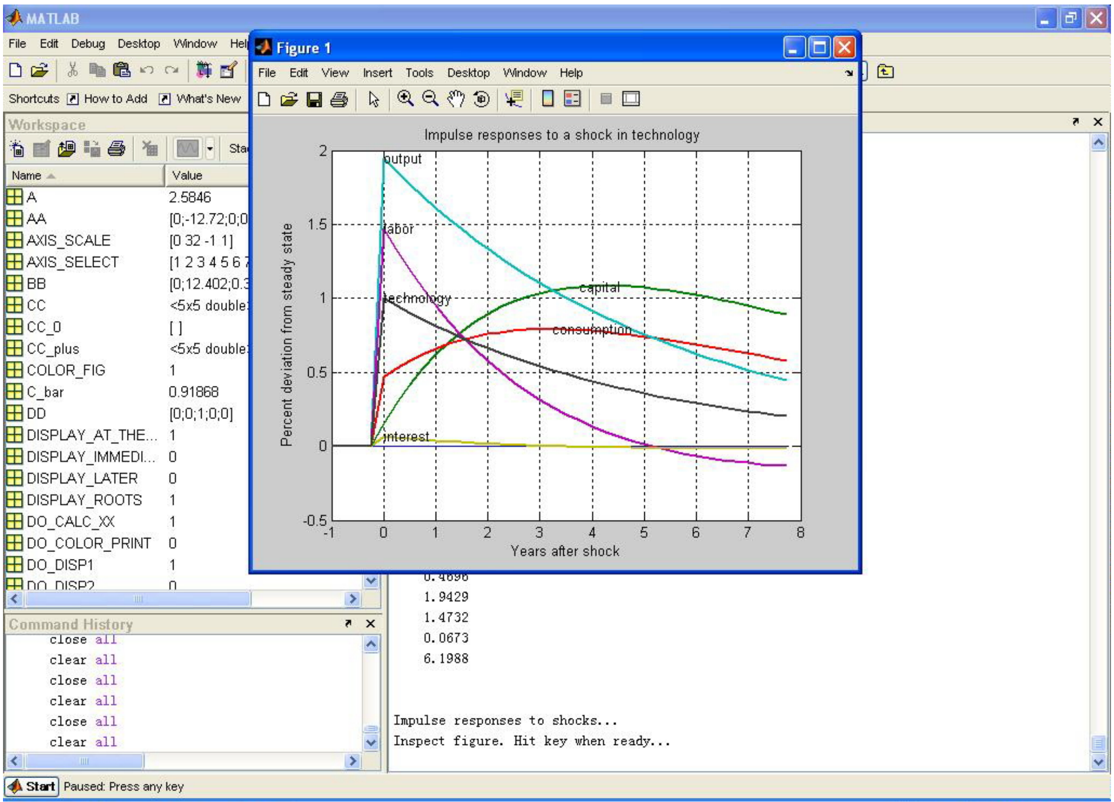

Figure 15: An example of Toolkit by Prof. H Uhlig PhD.

In my thesis work, all my models are implemented using Toolkit. I think that this is one of the best tools in calculation shock responses in Real Business Cycle.

[1] Please refer to Appendix for more information of HP Filter

B. Hodrick-Prescott Filter

HP filter was first applied by economists Robert J. Hodrick and Edward C. Prescott. It functions similar to a bypass filter. The aim of the HP Filter is to remove long run trend from a desensonalized time series.

Because of the randomness of shocks, variables in time domain may change slightly in different calculations. HP Filter transforms the data from time domain into frequency domain and is able to get the fixed standard deviation results.

Here is an example of using HP Filter.

The left figure in Figure 1 shows the time series of real GNP for the United States from 1954-2005. We can find that this is a continuous but not steady increase. The right figure transforms these levels into growth rates of real GNP and extracts a smoother growth trend by using HP Filter. The HP filter identifies the longer term fluctuations as part of the growth trend while classifying the more jumpy fluctuations as part of the cyclical component

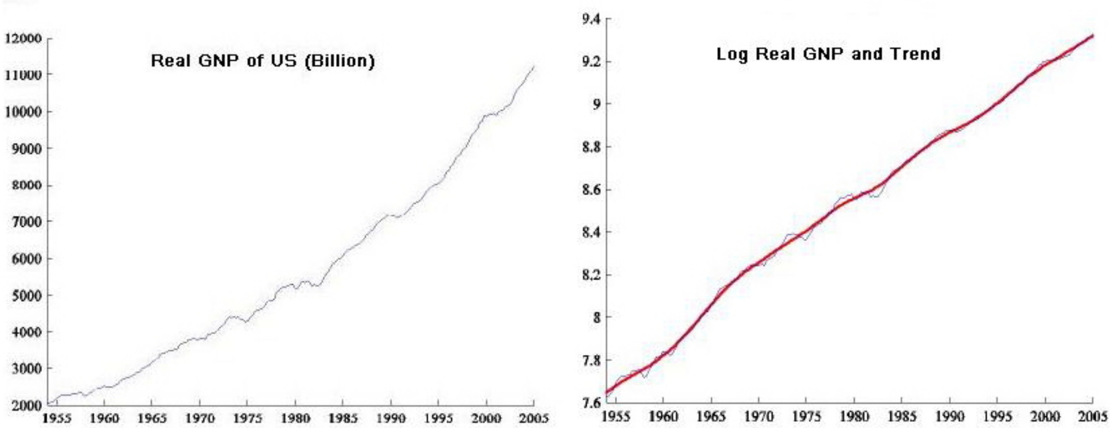

Figure 16: An example of Hodrick-Prescott Filter.

C. An example of Loglinearization

As I've mentioned in chapter 4, equation: $\kappa \frac{e_t}{u_t}(1-\nu)+\frac{1}{\phi}=\log w_t-\log z_t$ is a little bit difficult to loglinearized. Here I will provide the details of it, as an example of loglinearization.

First, I define: $H_t=\log w_t-\log z_t$, and transform the original equation to :

$$\frac{e_t}{u_t}\kappa(1-\nu)+\frac{1}{\phi}=H_t \tag{9.1}$$

And the steady state of this equation is:

$$\kappa \frac{\overline{e}}{\overline{u}}(1-\nu)+\frac{1}{\phi}=\log \overline{w}-\log \overline{z} \tag{9.2}$$

Then, we replace the original variables by the variable denoting percent deviations from the steady states

$$\frac{\overline{e}\exp(\hat{e}_t)}{\overline{u}\exp(\hat{u}_t)}\kappa(1-\nu)+\frac{1}{\phi}=\overline{H}\exp(\hat{H}_t) \tag{9.3}$$

Applying the approximation, (when x is small)

$$\exp(x)\approx 1+x \tag{9.4}$$

We can rewrite equation (3.20) to be:

$$\frac{\overline{e}(1+\hat{e}_t)}{\overline{u}(1+\hat{u}_t)}\kappa(1-\nu)+\frac{1}{\phi}=\overline{H}(1+\hat{H}_t) \tag{9.5}$$

Use equation (3.19), and finally we have:

$$\hat{e}_t-\hat{u}_t=\hat{H}_t \tag{9.6}$$

Then, for the equation: $H_t=\log w_t-\log z_t$, we do the same thing.

Steady state: $\overline{H}=\log \overline{w}-\log \overline{Z}$, approximation: $\overline{H}e^{\hat{H}_t}=\log \overline{W}e^{\hat{W}_t}-\log \overline{Z}e^{\hat{Z}_t}$, and we have: $\hat{H}_t=\frac{\hat{w}_t-\hat{z}_t}{\overline{H}}$

So the loglinearization for equation (4.12) is: $\hat{e}_t-\hat{u}_t-\frac{\hat{w}_t}{\log(\overline{w}/\overline{z})}=0$

D. US Separation Rate

In order to have the separation rate, I go to the website of "US Department of Labor" (http://www.dol.gov/) and get the data. Note that here the data is only available from 2000 to 2007. Because separation rate does not fluctuate much during different periods, I just calculate the mean value of all the data and set x_1 to it. Please see Table 6 for the data.

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
2000												3.3
2001	3.6	3.4	3.6	3.4	3.6	3.4	3.5	3.4	3.5	3.5	3.4	3.3
2002	3	3.3	3.1	3.2	3.2	3.2	3.2	3.2	3.1	3.1	3.3	3.1
2003	3.3	3.1	2.9	3	3	3.2	3	3	3.1	3.1	3.1	3.3
2004	3.1	3.2	3.3	3.3	3.2	3.3	3.3	3.3	3.3	3.4	3.3	3.4
2005	3.5	3.4	3.4	3.5	3.4	3.4	3.3	3.4	3.6	3.3	3.3	3.3
2006	3.3	3.4	3.6	3.2	3.6	3.4	3.4	3.3	3.3	3.4	3.5	3.3
2007	3.4	3.3	3.4	3.3	3.3	3.3	3.2					

Table 6: US national Separation Rate

E. Matlab Code

```
% VERSION 2.0, MARCH 1997, COPYRIGHT H. UHLIG.

% EXAMPL1.M calculates through Hansens benchmark real business

% cycle model in H. Uhlig, "A toolkit for solving nonlinear dynamic stochastic models easily".

% First, parameters are set and the steady state is calculated. Next, the matrices are

% declared.  In the last line, the model is solved and analyzed by calling DO_IT.M


% Copyright: H. Uhlig.  Feel free to copy, modify and use at your own risk.

% However, you are not allowed to sell this software or otherwise impinge

% on its free distri bution.

% This version uses all the updated equations


clc;        % totally reset MATLAB workspace

close all;

clear all;


disp('Chuanwen DONG Master Thesis coding: Effort, Unemployment and business cycle');
```

```
% disp('Hit any key when ready...');

% pause;


% Setting parameters: (values can be find in the paper)

q_bar    = 1;                          % normalize effort to be 1

a_bar    = 1;                          % normalize technology to be 1

wz_bar   = 2;                          % Replacement ratio, data from Uhlig-Xu (1996)

u_bar    = 0.05;                       % unemployment rate, data from U.S. Department of Labor

e_bar    = 1 - u_bar;                  % employment rate

l_bar    = 1/3;                        % Real labor, data from Cooley, Frontiers of...

n_bar    = l_bar/e_bar;                % labor provided by households

beta     = 1/1.01;                     % discount rate, Hansen's calibration

alpha    = 0.36;                       % capital share, Hansen's calibration

delta    = 0.025;                      % depreciation rate, Hansen's calibration

rho      = 0.95;                       % parameter of technology shock

v        = 0.95;                       % Sensitivity of effort between 0 (very small) and 1 (very large)

fai      = 1 / (log(wz_bar)*v) ;       % Parameter concerning effort, see paper concerning effort
```

```
ka       = u_bar / e_bar * log(wz_bar); % Parameter concerning effort, see paper concerning effort

G_bar    = 1 - v + v * q_bar;           % Disutility by providing effort q, with v given, see paper concerning effort

x1       = 0.033;                       % Total Separation Rate, data from U.S. Department of Labor

x2       = x1*19;                       % Probability of finding a job for the unemployed people

sigma_a  = 1;                           % Technology shock, changed it in order to keep the Solow residual fluctuation constant



% Calculating the steady state:

R_bar    = 1/beta;

yk_bar   = (R_bar - 1 + delta) / alpha;

k_bar    = (yk_bar / ((n_bar*e_bar)^(1-alpha)*a_bar*q_bar) ) ^ (1/(alpha - 1));

y_bar    = yk_bar * k_bar;

d_bar    = alpha * yk_bar;

% Following are the codes to calculate w_bar

T2       = (1-beta) / ((beta - x2*beta - 1)*(beta - x1*beta - 1) - x1*x2*beta^2);

t3       = n_bar * e_bar^2 * fai * beta^2;

t4       = (1-0.36) * y_bar * e_bar * fai * beta^2;
```

```
t5      = 2 * ka * y_bar;

t6      = T2^2*(1-1/wz_bar)^2;

t7      = 2*T2^2*(1-1/wz_bar);

t8      = T2^2;

p3      = t3 * t6;

p2      = -t3 * t7 - t4 * t6;

p1      = t3 * t8 + t4 * t7;

p0      = -t4 * t8 - t5;

p       = [p3, p2, p1, p0];

w       = roots(p)

w_bar   = w(1);        % other two roots are complex numbers, drop them

% End of w_bar calculation


z_bar     = w_bar / wz_bar;

% Expected discounted utility of unemployment

V_u = ( x2*beta* (w_bar-G_bar) - z_bar*(beta - x1*beta - 1)) / ((beta - x2*beta - 1)*(beta - x1*beta - 1) - x1*x2*beta^2);

% Expected discounted utility of employment
```

```
V_e = ( x1*beta*z_bar - (w_bar-G_bar)*(beta - x2*beta - 1)) / ((beta - x2*beta - 1)*(beta - x1*beta - 1) - x1*x2*beta^2);


tao_bar    = z_bar * u_bar / (z_bar * u_bar + w_bar * e_bar);   %taxrate on wages and z

c_bar      = y_bar - delta*k_bar - tao_bar*w_bar*n_bar*e_bar;   % Comsumption

lambda_bar = 1/c_bar;                                           % Lagrangian multiplier, set by household


m_bar      = ka*G_bar / beta /(V_e-V_u);                        % Monitor fee, as a propotion of output

persi_bar  = -2*m_bar*y_bar/beta/(V_e-V_u);                     % Lagrangian multiplier, set by firm


% Declaring the matrices.

%   1. endogenous states            k_t-1

%   2. other endogenous variables   c, w, d, l, u, y, R, tao, q, lambda, persi, m, s, e, l,

%   3. exogeous variables           a

VARNAMES = ['Capital          ',         % This is the Xt

            'Consumption      ',         % Below are all Yt

            'Wages            ',

            'Dividend         ',
```

```
            'Labor          ',
            'UnemploymentRate',
            'Output         ',
            'Interest       ',
            'Taxrate        ',
            'Effort         ',
            'Lambda         ',
            'persi          ',
            'Monitor        ',
            'Solow Resi     ',
            'EmploymentRate  ',
            'RealLabor      ',
            'Technology     '];
%Here, all the above variables should have the same length, otherwise, matlab synx error


% Check: intotal 17 equations, 17 variables;
% Endogenous state variables "X(t)": k(t)
```

```
% Endogenous other variables "Y(t)": c, w, d, l, u, y, R, tao, q, Lambda,Persi, m, s, e, l

% Exogenous state variables  "Z(t)": a(t) (which drives the model)

% Switch to that notation.  Find matrices for format

% 0 = AA x(t) + BB x(t-1) + CC y(t) + DD z(t)

% 0 = E_t [ FF x(t+1) + GG x(t) + HH x(t-1) + JJ y(t+1) + KK y(t) + LL z(t+1) + MM z(t)]

% 0 = NN z(t) + epsilon(t+1) with E_t [ epsilon(t+1) ] =


% for k(t-1):

AA = [ 0

       0

       k_bar

       0

       0

       0

       0

       0

       0
```

```
        0
        0
        0
        0
        0
        0];  %13*1

% for k(t):
BB = [ 0
        0
        -(1-delta)*k_bar
        1
        0
        alpha*yk_bar
        0
        -alpha
        0
```

```
        0
        0
        0
        0
        0
        0]; %13*1

% for Y(t)
cc_22  = lambda_bar * tao_bar * w_bar * e_bar;
cc_24  = lambda_bar * y_bar * (1-alpha) / (n_bar);
cc_25  = lambda_bar * tao_bar * z_bar * u_bar;
cc_28  = cc_22 + cc_25;
cc_210 = cc_22 + cc_25 - cc_24;
cc_32  = tao_bar * w_bar * n_bar * e_bar;
cc_35  = tao_bar * z_bar * n_bar * u_bar;
cc_34  = cc_32 + cc_35;
cc_52  = - (w_bar*n_bar*e_bar^2);
```

```
cc_56  = (1-alpha) * y_bar * e_bar;

cc_511 = (-1)*persi_bar/ka/fai;

cc_514 = cc_56 + 2*cc_52;

cc_66  = -(alpha * yk_bar);

cc_72  = tao_bar * w_bar * e_bar;

cc_75  = tao_bar * z_bar * u_bar - z_bar * u_bar;

cc_78  = tao_bar * z_bar * u_bar + cc_72;

cc_112 = 1 / (log(wz_bar));


% Below is the whole CC matrix:

%       C_t      W_t      D_t     n_t      u_t      y_t      R_t     tao_t    Q_t     la_t     persi_t m_t     s_t     e_t      l_t

CC = [ 1,       0,       0,      0,       0,       0,       0,      0,       0,      1,       0,       0,      0,      0,       0,

       0,       cc_22,   0,      cc_24,   0        -cc_24,  0,      cc_28,   0       cc_210,  0,       0,      0,      cc_22,   0,

       c_bar,   cc_32,   0,      cc_34,   cc_35,   -y_bar,  0,      cc_34,   0,      0,       0,       0,      0,      cc_32,   0,

       0,       0,       1,      0,       0,       -1,      0,      0,       0,      0,       0,       0,      0,      0,       0,

       0,       cc_52,   0,      cc_52,   0,       cc_56,   0,      0,       0,      0,       cc_511,  0,      0,      cc_514,  0,

       0,       0,       0,      0,       0,       cc_66,   R_bar,  0,       0,      0,       0,       0,      0,      0,       0,
```

```
        0,      cc_72,  0,      0,      cc_75,  0,      0,      cc_78,  0,      0,      0,      0,      0,      cc_72,  0,
        0,      0,      0,      alpha-1 0,      1,      0,      0,      alpha-1 0,      0,      0,      0,      alpha-1 0,
        0,      0,      0,      0,      0,      1,      0,      0,      0,      0,      -1,     1,      0,      0,      0,
        0,      0,      0,      0,      0,      0,      0,      0,      fai*v,  0,      0,      -1      0,      0,      0,
        0,      -cc_112 0,      0,      -1,     0,      0,      0       0       0,      0,      0,      0,      1,      0,
        0,      0,      0,      0,      0,      0,      0,      0,      alpha-1 0,      0,      0,      1,      0,      0,
        0,      0,      0,      0,      u_bar,  0,      0,      0,      0,      0,      0,      0,      0,      e_bar,  0,
        0,      0,      0,      0,      -1      0,      0,      0,      fai,    0,      0,      0,      0,      1       0,
        0,      0,      0,      -1      0,      0,      0,      0,      0,      0,      0,      0,      0,      -1      1];


%   Z(t)
DD = [ 0
       0
       0
       0
```

```
        0
        0
        0
        -1
        0
        0
        0
        -1
        0
        0
        0];
% 0 = E_t [ FF x(t+1) + GG x(t) + HH x(t-1) + JJ y(t+1) + KK y(t) + LLz(t+1) + MM z(t)]
FF = [ 0 ];

GG = [ 0 ];

HH = [ 0 ];
```

```
JJ = [ 0,   0,   0,   0,    0,   0,   1,   0,   0   1   0   0   0   0   0];
%      C_t   W_t          D_t    n_t     u_t     y_t     R_t     tao_t   Q_t    s_t
KK = [ 0,   0,   0,   0,    0,   0,   0,   0,   0,   -1  0   0   0   0   0];

LL = [ 0 ];

MM = [ 0 ];

% z(t+1) = NN z(t) + epsilon(t+1) with E_t [ epsilon(t+1) ] = 0,
NN = [rho];
Sigma = [ sigma_a^2  ];   % The variance of the shock
```

```
[l_equ,m_states] = size(AA);

[l_equ,n_endog ] = size(CC);

[l_equ,k_exog  ] = size(DD);




PERIOD     = 4;  % number of periods per year, i.e. 12 for monthly, 4 for quarterly

GNP_INDEX  = 3; % Index of output among the variables selected for HP filter, this is the third variable in the varnames matrix

%IMP_SELECT = [1:7];

   %  a vector containing the indices of the variables to be plotted


% 1,  2,  3,  4,  5,  6,  7,  8,  9,    10,  11,     12,     13,  14,  15,16,  17

% k,  c,  w,  d,  n,  u,  y,  R,  tao,  q,  lambda,  persi,  m,    s,  e,  l,   a


IMP_SELECT = [1,2,3,7,10,15,16];

%IMP_SELECT = [4,5,6,8,9,10,14];

%IMP_SELECT = [3,6,7,10,15,16];
```

```
DO_SIMUL   = 1; % Calculates simulations fourier-transforms-based calculations of moments

SIM_LENGTH = 150;

SIM_MODE = 2;

SIM_N_SERIES = 50;

DO_MOMENTS = 1; % Calculates moments based on frequency-domain methods

SIM_N_LEAD_LAGS = 6;

HP_SELECT  = 1:(m_states+n_endog+k_exog); % Selecting the variables for the HP Filter calcs.

DO_HP_FILTER = 1;

% DO_COLOR_PRINT = 1;

DO_ENLARGE=1;


% DO_COLOR_PRINT = 1;

 % Starting the calculations:

  do_it
```

Declarations of Authorship

I hereby confirm that I have authored this master's thesis independently and without use of others than the indicated resources.

Chuanwen DONG

Berlin 21st, Spetember, 2007.