

**On the heterogeneity of price
stickiness and its consequences for
European monetary policy**

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Abstract

An extensive and recently concluded evidence on micro price data indicates that heterogeneity in price stickiness is a widely found feature of price setting throughout the euro area. The thesis at hand therefore aims at incorporating this evidence in an elaborated New Keynesian DSGE model by extending the production from one sector to four sectors. Each sector is endowed with a certain price stickiness drawn from empirical data. The dynamics produced by such a model are stronger and shorter in comparison with a multi-sector model equipped with an equal average price rigidity in every sector, which is supposed to mirror the rigidity caused by heterogeneous price stickiness.

In addition, a Taylor rule is considered which allows the monetary authority to add more weight to the sticky sectors in their considerations to set the interest rate.

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1 Introduction

Are prices sticky? If so, how sticky are they? What are the implications of sticky prices? These questions were at the centre of an extensive discussion, which reaches back to John M. Keynes¹. Today empirical evidence leaves no doubt, that prices are sticky and the classical school have merged with Keynesian features to form the New Neoclassical Synthesis². Important features in these New Keynesian Models are the intertemporal optimization, rational expectations, imperfect competition and short-run rigidities in prices and wages which cause distortions in the markets. These distortions lead to welfare losses, which can be contained by a monetary authority through an appropriate setting of the nominal interest rate or a modification of the volume of money. However, a successful monetary policy requires a profound understanding of the size and character of the rigidities and implies the incorporation of these rigidities in macro modelling.

The two most widely applied mechanisms to include rigidities in theoretical models are based on Taylor (1980) and Calvo (1983). Whereas the Taylor model is based on fixed price durations, the Calvo model includes some uncertainty in a model, as it sets a probability to determine the rate of rigidity. Each price or wage setter is endowed with a predefined probability to be able to adjust its price or wage in every period. Price or wage setters, regulated by only a low probability, may find their power restricted by this probability and may not be able to react to an exogenous shock, as they would like to do. Furthermore the group of price or wage setter could also differ by the degree of probability, where some could adjust much faster to a shock, while others are bound to their prior value lacking the possibility to set a new value.

These theoretical considerations have been investigated extensively with empirical data. The first comprehensive study on price changes was published by Mark Bills and Peter Klenow³ and presented data concerning consumer

¹Keynes (1935)

²Goodfriend and King(1997)

³Bils and Klenow (2004)

products in the United States. Shortly afterwards results for Europe were published⁴. One of the main findings in these studies was, that the price rigidity is indeed heterogeneous across firms. For example, prices for energy or food products are quite volatile across time, whereas sophisticated durable goods show a relative high degree of price rigidity. But although this heterogeneity in price rigidity is well documented, most New Keynesian DSGE models only incorporate one sector endowed with a certain price rigidity. This approach may only be justified, if the loss of information arising from not including all sectors can be diminished by endowing the only included sector with a rigidity, which approximates the dynamics implied by the heterogeneous price rigidity. Previous research on this topic has indicated, that including a broader range of price rigidities instead of a single one, leads to stronger and shorter reactions in the model⁵. This thesis aims at contributing to this research field by extending a widely applied and more general DSGE model to a four-sector model and calibrating it with price rigidities which are based on the recent published facts on micro product price data.

Obviously, not every product or product category with its specific rigidity can be included in a model and so, a somewhat random cutting in the sector accumulation has to be made. In this context Álvarez et al. (Álvarez et al. (2005)) have published a paper with specifications for a four-sector model, which reflect the more diverse reality of heterogeneity in price stickiness according to the authors well. The authors had concentrated their efforts to find a set of sectors, which could together account for the decreasing hazard function, the probability of firm to set a new price over time⁶.

These specifications are to incorporate in a general New Keynesian DSGE model based on Smets and Wouters (2003). Due to its wide scope, this model has gained popularity in research and its elaboration will result in more real-

⁴An overview is given in Altissimo et al. (2006)

⁵refer to Bils and Klenow (2004) or Carvalho (2006)

⁶A detailed evaluation of the results based on empirical data for Spanish producer prices can be found in Álvarez et al. (2005), pages pp. 23-27

istic responses to exogenous shocks. It therefore facilitates the understanding of the dynamics brought into the model by heterogeneous price rigidities.

The results obtained in this paper show, that by including only a single sector in a model, endowed with an average of the whole rigidity, a loss of dynamics is very likely, as this sector, presuming to reflect the whole range of heterogeneity in price stickiness, will not lead to the same dynamics. This conclusion is drawn in two steps. First a one-sector model is extended to an four-sector model and secondly two four-sector model are compared, whereas one model incorporates the whole range of heterogeneity and the other only an average degree of heterogeneity.

The first step, the extension of a one-sector model, shows, that through including three additional sectors, the dynamics in the model are altered. However, this mostly concerns the interaction between investment and capital accumulation and can be explained by the modified structure of the model. As deviations in total investment will by definition only influence the single sectors partially, the reaction of capital will be less profound to deviations in investment than before. In the next step a comparison between a heterogeneous price rigid four-sector model and a similar price rigid four-sector model is conducted. As mentioned before, the average price rigid model cannot reflect the dynamics of the heterogeneous model, as its reactions to a shock are less extensive and more persistent. These findings suggests that heterogeneity in price stickiness is an important feature and its omission would lead to an artificial restriction of the dynamics in most models.

The altered dynamics have also implications for the monetary authority, which aims at minimising the negative consequences, which arise from price stickiness. Therefore a modified Taylor rule is implemented, which allows the monetary authority to apply different weights to single sectors. Focusing on the sticky sector the implementation leads to more profound reactions after a interest rate shock and less extensive reactions after a technology shock. However, these results show to be not very robust.

The thesis will be organised as follows: Next, in section 2, a brief overview of relevant literature on price rigidities is given. Afterwards in section 3, the main empirical facts about heterogeneity of price stickiness are presented and the calculations of average price stickiness are explained. Section 4 introduces the model, which is analysed in section 5 and results are presented in section 6. Furthermore, in section 7 a modified Taylor rule is introduced and the resulting consequences reported. All results are discussed in section 8 and section 9 concludes.

2 Literature

Broad literature on price heterogeneity has only been published in recent years. Previously research had been concentrated on single products⁷ or on a restricted basket of products⁸. This limited scope can be explained by a lack of data, as the collection and evaluation of data on prices covering the whole range of all or nearly all products produced in a economy is an extensive task. Furthermore only recently access has been granted to appropriate data⁹.

The first comprehensive study on price changes and its implications was published by Mark Bills and Peter Klenow¹⁰ and is based on data from the American Bureau of Labour Statistics (BLS). About 70% of consumer spending in the United States, adding up to 80,000 single goods and services classified in 350 categories, are covered and the authors report a median price duration of 4.3 months. But the comparison of average price duration of the different product and service categories reveals profound differences among the categories and so the authors build a multi-sector model with up to 30 sectors, which differ in their time-dependent price setting, to examine the consequences of this heterogeneity¹¹. They conclude that a one-sector model with a mean duration of the data's empirical distribution approximates the multi-sector model best.

Carvalho (2006) extends the idea of a multi-sector sticky-price model and assigns a sector to every of the BLS's 350 categories distinguishing the sectors by the frequency of price change reported by Bills and Klenow (2004).

⁷Mussa(1981) for example studied newspaper prices during the German hyperinflation.

⁸Kashyap(1995) reviewed prices in mail order catalogues between 1953 and 1987.

⁹In the first research on price changes Frederick Mills (Mills 1927) could only rely on prices for 200 goods and today data from national statistical offices may only be investigated under strict confidentiality agreements (Vermeulen et al. 2007).

¹⁰Bills and Klenow (2004)

¹¹Bills and Klenow (2002), working paper version of Bills and Klenow (2004).

He finds that the heterogeneity leads to stronger and more persistent effects after a monetary shock, if compared to a single-sector model with a similar price rigidity. In contrast to Bils and Klenow, he models the price rigidity with a Calvo¹² mechanism and argues that a single-sector model has to include an up to three times lower than average frequency of price changes in order to match the responses of the multi-sector model.

In Europe, research on prices on a micro level has mainly been published in the framework of the ECB's Inflation Persistence Network (IPN)¹³. As in Bils and Klenow (2004) for the first time data from the National Statistical Offices was analysed and Vermeulen et al. (Vermeulen et al. 2007, p. 5) present the main findings for producer prices as following¹⁴:

"First, producer prices change infrequently: each month around 21% of prices change. Second, there is a substantial cross-sector heterogeneity in the frequency of price changes: prices change very often in the energy sector, less often in food and intermediate goods and least often in non-durable non-foods and durable goods. Third, countries have a similar ranking of industries in terms of frequencies of price changes. Fourth, there is no evidence of downward nominal rigidity: prices changes are for about 45% decreases and 55% increases. Fifth, price changes are sizeable compared to the inflation rate."

The resulting implications for micro-founded macro models are discussed by Angeloni et al. (2006). They argue that many assumptions of these models have to be reviewed and they propose a multi-sector Calvo model to include some of the micro stylised facts, as for example the to a great extent random

¹²Calvo (1983)

¹³An overview of data is presented in Dhyne et al. (2005) for consumer prices and in Vermeulen et al. (2007) for producer prices. For a list of the IPN's research papers refer to http://www.ecb.int/home/html/researcher_ipn.en.html

¹⁴Most New Keynesian DSGE models include price stickiness on a producer price level; Altissimo et al. (2006) present a more general overview on finding from the IPN

number of price adjustments, a declining hazard function and the sectoral heterogeneity. Standard one-sector Calvo models result in a constant hazard function and therefore predict the probability of price changes to be equal across time. Micro evidence suggests in contrast a decreasing probability of the firm to set a new price. Álvarez et al. (2005) account for this decreasing hazard function in a multi-sector Calvo model and explain the empirical observation by its aggregating character: As time passes after a shock, flexible firms are the first to revise their prices and the firms who are still due to change their price are increasingly dominated by sticky firms with their lower probability of price changes. The authors support their approach by calculating four groups of firms with different probability of price adjustment based upon Spanish producer and consumer price data. The results presented show that this approach works well as it accounts to manage for the decreasing hazard function drawn from actual data.

Another implication arising from heterogeneity in prices relates to the monetary rule. In an environment, characterised by heterogeneity, each sector reacts differently to shocks and a monetary authority aiming at a target inflation or minimising the output gap faces the question, if placing more emphasis on a certain sector, instead of the aggregate, will lead to improved results for the economy as a whole. Aoki (2001) computes a two-sector model with one flexible-price sector and one sticky-price sector and observes, that a better result is archived, if the monetary authority targets the inflation in the sticky sector, rather than the total inflation. This conclusion is drawn measuring the welfare loss implied by the two strategies. The same approach is applied by Benigno (2003), although he considers a two-region model with greater independence between the regions. A welfare criterion, the discounted sum of the households' utility, is calculated to evaluate the deadweight loss caused by either strategy. He concludes, that weighted average, where the weights equal the economic size, is only optimal, if the price stickiness across the sectors equal. In any other case, emphasis should be placed on the more

rigid sector in order to archive a better result for the economy as a whole. Following this rule, the overall adjustment after a shock would burden the flexible sector more, as it would react stronger than usual to a shock, and welfare losses in the sticky sector, due to its low ability to adapt to the shock, could be downsized.

3 Facts

In standard New Keynesian DSGE models, only one overall sector is included and endowed with a certain Calvo price stickiness. First micro price evidence, however, indicated that heterogeneity in price stickiness across sectors is common and extensive research on prices has been conducted to shed light on this matter. In order to limit this thesis to a feasible framework, emphasis is placed on data concerning the euro area¹⁵ and at this research conducted by the ECB's IPN sticks out. In this section an overview over gained insights concerning general price setting and producer prices in particular is given which relies on Altissimo et al. (2006) and Vermeulen et al. (2007). Producer prices are highlighted as these are the ones usually incorporated in macro model building.

Additionally the issue of mirroring the price rigidity of a multi-sector model in a single-sector model is addressed.

3.1 Empirical evidence on heterogeneity of price stickiness

First the methodology of the data accumulation is explained, afterwards the main empirical insights are presented and at last factors which led to price heterogeneity are explained.

From 2003 to 2005 the IPN undertook an extensive study analysing price setting behaviour of firms. For the first time data from the National Statistical Institutes could be investigated in various countries¹⁶. Additionally surveys were conducted to become aware of the motivation of firms to modify

¹⁵For the United States see Bils and Klenow (2005), for Canada Amirault (2004), for the United Kingdom Hall (2000) or for Japan Bank of Japan (2000)

¹⁶Producer price data was gained from Belgium, France, Germany, Italy, Portugal and Spain, refer to Vermeulen et al. (2007)

their prices¹⁷.

The National Statistical Institutes collect price information on a monthly basis via a statistical survey. Staff member of the Statistical Institutes gather price information on a representative sample of products and services, which are identified by a product code. On this basis national indices are constructed, which may further be aggregated to calculate an euro area wide index.

The percentage of the producer products, for which data was made available by the National Statistical Offices for the national studies, ranges from 83% for Belgium to 100% for Germany with the exception of Italy, where only 44% of all producer products were included. Furthermore the respective products can be grouped into 6 categories: energy and capital products, durable products, intermediate products, non-durable non-food products and food products. The IPN studies measured the frequencies of price changes, the percentage of prices which change in a given sector between two observations, and the implied price duration. Price changes were analysed concerning their magnitude and direction and the frequencies were tested among others for the type of product, the type of outlet, the time span since the last price change, macroeconomic conditions and seasonal factors.

Concerning the main findings Vermeulen et al. (2007) report an average frequency of price changes of 21%, which exceeds the frequency for consumer prices of 15% calculated by Dhyne et al. (Dhyne et al. 2005) for consumer prices. However, a direct comparison is restricted by the varying composition of the two product baskets, but a comparison of similar sub-baskets or single goods still hints at more flexible producer prices. An allocation of all single products to the sectors food products, non-durable non-food products, durable products, intermediate products energy products and capital prod-

¹⁷National Central Banks in Austria, Belgium, France, Germany, Italy, Luxembourg, the Netherlands, Portugal and Spain collected the information, refer to Fabiani et al. (2005)

ucts reveals that prices change heterogeneously across sectors: prices for capital goods change least frequently with a frequency of 9%. Next are durable products, which change on average with a frequency of 10% and non-durable non-food goods, of which 11% change in a given period. This frequency is doubled to 22% considering intermediate goods. Prices change most often in food products (27%) and energy products (72%). The high frequency for energy goods can be explained by its unsteady supply. Additionally energy products or many food products are not highly processed goods, but have rather undergone only few transformations and a general pattern, raw goods contain a higher frequency of price changes than processed goods, can be noticed. A classification of all goods concerning the investigated countries exposes country-specific differences, but these are dominated by the sectoral allocation discussed before¹⁸.

Concerning the direction of price changes the national studies report that on average 45% of all price changes are caused by price cuts and a downward price rigidity cannot be detected. The same holds for consumer prices, as Dhyne et al. (2006) find that 40% of all consumer prices changes are decreases. Separating price increases from decreases, the average frequency of producer price decreases is lower (10%) than the average frequency of producer price increases (12%). Alike the magnitude of the median price increase is bigger (3% of the former price) than the magnitude of the median price decrease (2% of the former price). But the distribution of price changes is not restricted to low values and changes of 10% and more are not uncommon.

Several reasons to explain the price stickiness have been analysed: the cost structure, degree of competition, the use of attractive pricing, the level of inflation, seasonality, price regulations and changes in the VAT rates. Re-

¹⁸Vermeulen et al. (2007) compute country-pair correlations of frequencies of price changes, which are always positive and range from 0.3 between Italy and Belgium to 0.89 for Spain and France

search to this end has made use of surveys and regression analysis¹⁹.

The influence of cost structure on the frequency of price changes was generally investigated by Blanchard (1982). He points out that the price setting is largely affected by the number of manufacturing steps. The volatility in prices for raw materials will influence the first production steps due to their dependence on these raw materials. Transferring this idea to a monopolistic competition framework, where the price charged by a firm is a markup over marginal costs, less volatile input costs will lead to less price changes, whereas input cost characterised by a high volatility will entail a high number of price changes.

Findings concerning the role of aggregate inflation suggests that a higher inflation rate will be reflected in a higher frequency of price increases. But the same increase in inflation will not also raise the frequencies of price decreases, but rather lower their frequency²⁰. The size of such a propulsive effect is sizeable²¹ and the difference between the frequency of price increases and decrease may indicate that this difference is a crucial factor driving aggregate inflation. In addition sectoral inflation was also found to raise (lower) the frequency of price increases (decreases).

The role of competition in a market for price changes has been discussed extensively, although with mixed results. In a perfectly competitive market straight thinking would suggest that firms modify their frequencies often, as the opportunity costs of leaving the prices unaffected would be high (Martin 1993). Limiting the perfect competition condition, these opportunity costs would decrease with the market power of a firm and in an oligopoly market suppliers would probably ignore incentives to modify their price to secure

¹⁹The methods included a conditional logit model (Gautier (2006)), time series models (Álvarez et al. (2006)) and a cross-country regression analysis (Aucremanna and Dhyne (2005))

²⁰These results were obtained by means of a correlation analysis (Cornille and Dosche (2006)) and time series models (Stahl (2006))

²¹Evidence for Germany (Spain) reveals that a one point increase in aggregate inflation raises the frequency of price increases by 9% (6%) resulting in a rise of average frequency of price increases from 0.12 to 0.131 (0.12 to 0.127.)

the mutual tacit understanding (Stiglitz 1984). The results of the country studies, regressing the frequency of price changes on proxies for the degree of market competition, underline the causality of a higher rate of price changes in a more competitive environment. This is also supported by the evaluation of surveys, which emphasise the same relation.

Seasonal patterns of price changes are found throughout all countries. A peak of price changes can be noted in January, whereas only few prices are modified during the summer months or December. But seasonality may not be a feature of price setting, but rather of the determinants of price setting. Survey evidence reported by Fabiani et al. (2006) hint at the statement by firms, that explicit or implicit contracts prohibit them from modifying a price to a desired level. The process of wage changing, which takes mainly place in January, may therefore explain the price changing peak in this particular month.

Furthermore attractive pricing may influence the firm's pricing decisions. Price ending in 0, 5 or 9 are very common²², as they ease transactions and attract consumers psychologically. In order to stick to such an attractive price firms may ignore incentives to modify a price and therefore increase the degree of price stickiness for their product.

Another source of price rigidity is found in price regulation by the state. Dexter et al. (2002) held the institutional process to alter product prices in the sphere of the state responsible for a higher degree of price stickiness concerning these products. This is supported by Lünemann and Mathä (2005), who find empirical evidence for more rigid prices in sectors regulated by the state.

Finally the examination of time series²³ suggests that changes in the value adding tax (VAT) increase temporary the frequency of price changes.

²²Sabbatini et al. (2005), Alvarez et al. (2005) and Stahl (2006) report a share of attractive pricing to range from 19% in Germany to 43% in Italy.

²³Álvarez et al. (2005) and Stahl(2005)

3.2 Average price stickiness

To account for the heterogeneity in price stickiness presented above a comparison between a multi-sector model and a one-sector model will be conducted and in order to extract only the differences caused by the increase of sectors, the single-sector model has to be endowed with a price stickiness similar to the multi-sector one. The price stickiness of the single-sector model, expressed as a Calvo parameter, has therefore to match some kind of average of the multi-sector model, and several computations are thinkable. First a weighted average of the price duration can be calculated by multiplying the economic weights of each sector with the average price spell in each sector. Second the weighted average can be applied to the single Calvo parameters of the diverse sectors. Third the Calvo parameter can be calculated from the median of single price spells or, finally, the median of the diverse Calvo parameters can be computed. An overview of these calculations based on the specifications computed by Álvarez et al. (2005)²⁴ can be found in table 1, and with the exception of the weighted Calvo parameter there is strong evidence to suppose that a single-sector Calvo parameter of 0.8 should be able to mimic the multi-sector model.

The transformation of price spells expressed in months into a Calvo parameter is facilitated by the relationship between the periodical probability of price alterations θ and the underlying rate of price changes in continuous time δ : $\theta = 1 - e^{-\delta\mu}$, where μ defines the applied unit of time²⁵. This leads to a formula to convert the duration of price spells to a Calvo parameter:

$$d = \frac{-1}{\ln(1 - \theta)},$$

²⁴Álvarez et al. (2005) propose 4 groups: a flexible group (economic weight: 22%; average price spell: 1 month), a intermediate group (economic weight: 50%, average price spell: 10 months), an annual group (economic weight: 7%, average price spell: 18 months) and a sticky group (economic weight: 21%, average price spell: 36*months*); further details are given in section 6.1

²⁵refer to Carvalho (2006)

Description	Formula	Calvo Parameter
Weighted monthly average	$\sum_{i=1}^n \Omega_i \text{month}_i$	0.8076
Weighted Calvo parameter	$\sum_{i=1}^n \Omega_i \xi_{p,i}$	0.633832
Monthly Median	$\frac{1}{2}(\text{month}_{i/2} + \text{month}_{(i/2)+1})$	0.8071
Calvo Median	$\frac{1}{2}(\text{Calvo}_{i/2} + \text{Calvo}_{(i/2)+1})$	0.79365

Table 1: *Calculations for an average price spells based on Álvarez et. al (2005) where Ω_i sets the economic weight of each sector i , $\xi_{p,i}$ denotes the Calvo parameter for sector i and month_i reflects the average monthly price spell in a sector i . Calvo parameter are computed on a quarterly basis.*

where d denotes the duration of price spells in the applied unit of time μ and $(1 - \theta)$ states the corresponding Calvo parameter.

4 The Model

The model presented in this thesis is based on Smets and Wouters (2003) extended for n sectors to disclose the consequences of sectoral heterogeneity in price stickiness. Together with Cristano, Eichenbaum and Evans (2005) this is one of the elaborated New Keynesian Models, which are rich in detail, while at the same time manage to mimic the main characteristics of empirical data. Price stickiness is introduced via a Calvo parameter and the model's elaboration allows to examine its consequences to a greater extent. Sticky wages are also included via a Calvo parameter, but a possible heterogeneity due to for example different professions or labour unions is not included in the scope of this thesis.

The households maximise their utility over an infinite horizon choosing the adequate amount of consumption and labour. Consumption is qualified by a time-varying habit formation²⁶ and labour is provided in a monopolistic competition. This form of the labour market originates from a definition of a continuum of households, where the labour skills of every household slightly differ and the households therefore can exert some monopoly power over their type of labour. The monopoly power is, however, limited by the degree of substitutability and the household set their wages only under terms of a Calvo parameter reducing their abilities to react to changes in the environment in a timely manner. Capital is also accumulated by the households and rented to firms. An increase in the capital stock leads to adjustment costs, which can be avoided by modifying the capital utilization of already existing capital. But this modification also bears costs in form of foregone consumption.

On the side of production, the market is segmented in an upstream market with a continuum of intermediate goods firms in a monopolistic competition, and a perfectly competitive downstream market. Each of the intermediate

²⁶The habit consists of lagged aggregate consumption, which is not affected by one agent's consumption decision (Abel 1990).

firms is assigned to one of the n sectors. The monopolistic power in this upstream market is caused by the somewhat diverse products and limited by the degree of substitutability. Prices may be set by these firms only in accordance with a Calvo mechanism and the only inputs in this part of the market are labour and capital provided by the households. Firms that may not modify their price in a certain period are assumed to index their price according to the prior inflation rate. This leads to a backward-looking component in the inflation equation. The n sectors differ only insofar as a certain share of the continuum of intermediate firms and diverse Calvo parameters are assigned to each of them. In the downstream market, a final good made out of the intermediate goods is produced and used for consumption and capital accumulation. The government finances its consumption by a lump sum tax and the monetary authority sets the nominal interest rate by reacting to the target inflation, the actual inflation and the output gap between actual and under flexible prices and wages potential output. 10 different shocks are incorporated.

4.1 The Households

A continuum of households maximizes their utility function over an infinite time horizon. The household τ faces the utility function

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \varepsilon_t^b \left(\frac{(C_t^\tau - H_t)^{1-\sigma_c}}{1-\sigma_c} - \varepsilon_t^L \frac{(l_t^\tau)^{1+\sigma_l}}{1+\sigma_l} \right), \quad (1)$$

where C_t^τ refers to household τ 's consumption and l_t^τ states the amount of labour the household provides. H_t defines the external habit and depends on the aggregated consumption in the prior period $H_t = hC_{t-1}$. The shock ε_t^b and ε_t^L affect the intertemporal substitution, respectively the labour supply. Both are defined as a first-order autoregressive process with an i.i.d.-normal error: $\varepsilon_t^b = \rho_b \varepsilon_{t-1}^b + \eta_t^b$ and $\varepsilon_t^L = \rho_L \varepsilon_{t-1}^L + \eta_t^L$. The households maximise their objective function subject to the intertemporal budget constraint

$$b_t \frac{B_t^\tau}{P_t} + C_t^\tau + I_t^\tau = \frac{B_{t-1}^\tau}{P_t} + Y_t^\tau. \quad (2)$$

Household τ can acquire nominal one-period bonds B_t^τ at the market price b_t . The current income Y_t^τ and the financial wealth resulting from bonds bought in the prior period may be used to consume, to invest in capital or to buy further bonds. The current income

$$Y_t^\tau = w_t^\tau l_t^\tau + A_t^\tau + (r_t^k z_t^\tau K_{t-1}^\tau - \Psi(z_t^\tau K_{t-1}^\tau)) + Div_t^\tau - tax_t \quad (3)$$

consists of five parts: the labour income, state contingent security payments A_t^τ , return on capital depending of the rate of utilization z_t^τ and subtracted by the cost for varying this rate Ψ , dividends paid by the intermediate monopolistic firms, which are owned by the households, and taxes collected by the government.

As in Cristano, Eichenbaum and Evans (2003) a perfect insurance market is established. The security payments lead to the equality of the labour income of an individual household τ and the aggregate labour income. Thereby the individual consumption equals the aggregate. As a consequence the same holds for the marginal utility of wealth and the level of capital holdings, bond holdings and share of firm dividends.

Each household differs in the type of labour, which is not perfectly substitutable and provides the households with some monopoly power in the labour market. The total labour supply takes the form of an integral over all households²⁷

$$L_t = \left(\int_0^1 (l_t^\tau)^{1/(1+\lambda_{w,t})} d\tau \right)^{1+\lambda_{w,t}}, \quad (4)$$

where $\lambda_{w,t}$ is an independent and identically-distributed shock setting the degree of substitutability of the different kinds of labour. The households supply labour in the market clearing quantity at the wage level w_t^τ . Their ability to adapt to exogenous shocks is limited by a Calvo mechanism. In every period the probability to not be able to adjust one's wage is set by a Calvo parameter ξ_w . In this case wages are modified with respect to the prior inflation π_{t-1} by the following indexation rule:

$$w_t^\tau = (\pi_{t-1})^{\gamma_w} w_{t-1}^\tau. \quad (5)$$

²⁷Dixit-Stiglitz-type aggregator, refer to Dixit-Stiglitz (1977)

The parameter γ_w states the rate of indexation. With the probability $(1 - \xi_w)$ the household sets a new price $w_t^\tau = \tilde{w}_t^\tau$, which will be derived in section 5.1.

4.2 The Firms

The economy produces a single final good in a perfectly competitive market, which can be used for consumption and investment. The only input in the production process is a continuum of intermediate goods indexed by j . Each of the intermediate goods is produced with capital and labour by a single firm j . Due to a not-perfect substitutability the intermediate firms exert some monopolistic power and any profits arising thereby are paid as dividends Div_t to the households. In addition the continuum from 0 to 1 of intermediate firms is split into sectors, which differ only in economic weight and in their abilities to adapt prices to exogenous shocks.

The technology of the intermediate firms j in sector i is based on the effective utilization of the capital stock $\tilde{K}_{i,j,t} = z_t K_{i,j,t-1}$ and firm specific combination of individual households labour $L_{i,j,t}$:

$$y_{i,j,t} = \varepsilon_t^a \tilde{K}_{i,j,t}^\alpha L_{i,j,t}^{1-\alpha} - \Phi. \quad (6)$$

α states the division of capital and labour in the production function, Φ sets the fixed costs and $\varepsilon_t^a = \rho \varepsilon_{t-1}^a + \eta_t^a$ defines a productivity shock. These three parameter do not differ across sectors.

The level of capital depends in each period on the rate of depreciation, which is constant across time, and new investment:

$$K_t = (1 - \tau)K_{t-1} + \left(1 - S\left(\varepsilon_t^i \frac{I_t}{I_{t-1}}\right)\right) I_t. \quad (7)$$

I_t is the gross investment, from which the cost of modifying the level of investment S have to be deducted to obtain the net investment. $\varepsilon_t^i = \rho_i \varepsilon_{t-1}^i + \eta_t^i$ states a shock to these costs and τ denotes the rate of depreciation.

The intermediate firm meets the demand for their product j , which arise due to the price $p_{i,j,t}$ chosen by the firm. But in every period a new optimal price $p_{i,j,t} = \tilde{p}_{i,j,t}$ may only be set with a probability $(1 - \xi_{p,i})$, which differs across

the sectors. This Calvo mechanism is extended by an indexation for prices, which may not be adjusted freely. In this case with a probability of $\xi_{p,i}$ a new price is set according to the prior inflation rate:

$$p_{i,j,t} = (\pi_{t-1})^{\gamma_p} p_{i,j,t-1}. \quad (8)$$

The parameter γ_p defines the rate of indexation, which applies to all sectors equally.

The final good is produced according to a sectioning, where the weight of each sector Y_i is predefined. The production function in each sector i

$$Y_{i,t} = \left[\int_{\kappa_{i,1}}^{\kappa_{i,2}} (y_t^j)^{1/(1+\lambda_{p,t})} dj \right]^{1+\lambda_{p,t}} \quad (9)$$

is restricted to the economic weight of each particular sector, the integral between $\kappa_{i,1}$ and $\kappa_{i,2}$. The rate of substitutability between the slightly different products is described by $\lambda_{p,t}$ and thereby the price markup over marginal costs set by firm j is defined. $\lambda_{p,t}$ is defined as an i.i.d. normal "cost-push" shock.

The total production

$$Y_t = \left(\sum_{i=1}^n (Y_{i,t})^{1/(1+\lambda_{p,t})} \right)^{1+\lambda_{p,t}} \quad (10)$$

is a sum of the n sectors taking into account the rate of substitutability $\lambda_{p,t}$ between the sectors. This rate equals the intra-sectoral one mentioned before.

4.3 The Government

The government spends taxes tax_t on government consumption G_t , which will have no effect neither on the households utility nor on the firm's profit. The level of government consumption is denoted by an $AR(1)$ process: $\varepsilon_t^G = \rho_G \varepsilon_{t-1}^G + \eta_t^G$. Furthermore a monetary authority sets the nominal interest rate $R_t = 1 + i_t = 1/b_t$ according to a generalized Taylor rule (Taylor 1993),

which will be described in detail in section 5.2. In this context it's worthwhile to mention that the model does not include the usage of money, and the monetary authority influences the market only through setting the nominal interest rate. Seigniorage and other implications resulting from the introduction of money are not considered.

4.4 Market Equilibrium

In the equilibrium demand and supply in the various markets match, while the representative household maximises his utility and the firm its profit. Taking into account the government spending rule, which results in taxes, and the Taylor rule an equilibrium is an allocation of $B_t, C_t, H_t, (l_t^\tau)_{t \in [0,1]}, (L_{j,t})_{j \in [0,1]}, L_t, (\tilde{K}_{j,t})_{j \in [0,1]}, (K_{j,t})_{j \in [0,1]}, I_t, (y_{j,t})_{j \in [0,1]}, Y_t, Div_t$ and prices $b_t, r_t^k, (W_t^\tau)_{\tau \in [0,1]}, W_t, (P_{j,t})_{j \in [0,1]}$ and P_t , which solves the maximisation problem of the representative household and the maximisation problem of the firm, clears the markets and follows the constraints imposed by the policy rules. In the final goods market Consumption, Investment, Government spending and the costs for changing the capital utilization must match the output of the economy:

$$C_t + I_t + G_t + \Psi(z_t)K_{t-1} = Y_t. \quad (11)$$

The capital market clears, if the capital demanded by the firms equals the capital supplied by the households:

$$\int_0^1 \tilde{K}_{j,t-1} dj = \tilde{K}_{t-1} \quad (12)$$

and the labour market equilibrium is achieved, if the demand for labour by the firms coincides with the supply of labour by the households

$$\int_0^1 L_{j,t} dj = \left(\int_0^1 (l_t^\tau)^{1/(1+\lambda_{w,t})} d\tau \right)^{1+\lambda_{w,t}}. \quad (13)$$

5 Model Analysis

In this section the first-order conditions resulting from the maximisation problem of the household and the firm are derived. In order to solve for the recursive law of motion in the Toolkit²⁸, the log-linearised versions of this optimality conditions have to be calculated.

5.1 The Household

The maximisation of the utility function of the household (1) subject to the budget constraint (2) with respect to consumption yields the marginal utility of consumption

$$\Lambda_t = \varepsilon_t^b (C_t - H_t)^{\sigma_c}. \quad (14)$$

Equation (14) extends a standard first-order condition for consumption by the habit formation H_t defined in the subsection 4.1 and is part of the following optimality conditions for holding bonds and capital. The Lucas asset pricing equation for bonds defines that

$$E_t \left[\beta \frac{\Lambda_{t+1}}{\Lambda_t} \frac{R_t P_t}{P_{t+1}} \right] = 1, \quad (15)$$

where $P_t = \left(\int_0^1 (p_{j,t})^{-1/\lambda_{p,t}} dj \right)^{-\lambda_{p,t}}$ states the level of aggregate prices. This Dixit-Stiglitz aggregation can be derived by calculating first-order conditions for aggregate output. The Lucas asset pricing equation for capital

$$E_t \left[\beta \frac{\Lambda_{t+1}}{\Lambda_t} (Q_{t+1}(1 - \tau) + z_{t+1} r_{t+1}^k - \psi(z_{t+1})) \right] = Q_t \quad (16)$$

introduces the variable Q denoting the value of installed capital. In order to derive equation (16) the capital accumulation equation (7) has to be included in the maximisation. In the same way the optimality condition for the utilization rate for capital is calculated and it states the equality of the rental rate with the marginal costs of capital utilization:

$$r_t^k = \Psi'(z_t). \quad (17)$$

²⁸Uhlig 1999

If the rental price exceeds the marginal costs, the rate of utilization of capital will be increased up to the point where marginal costs for modifying the utilization matches the rental price.

The optimality condition for investment is also derived by maximising the objective function of the household subject to the budget constraint and the capital accumulation equation:

$$\begin{aligned} Q_t S' \left(\frac{\varepsilon_t^I I_t}{I_{t-1}} \right) \frac{\varepsilon_t^I I_t}{I_{t-1}} & - E_t \left[\beta Q_{t+1} \frac{\Lambda_{t+1}}{\Lambda_t} S' \left(\frac{\varepsilon_{t+1}^I I_{t+1}}{I_t} \right) \frac{\varepsilon_{t+1}^I I_{t+1}}{I_t} \frac{I_{t+1}}{I_t} \right] + 1 \\ & = Q_t \left(1 - S \left(\frac{\varepsilon_t^I I_t}{I_{t-1}} \right) \right), \end{aligned}$$

$S'(\cdot)$ denotes the marginal cost for modifying the investment level.

As a wage setter the household τ chooses a wage level to maximise its utility subject to the budget constraint and the demand for its labour

$$l_t^\tau = \left(\frac{w_t^\tau}{W_t} \right)^{-(1+\lambda_{w,t})/\lambda_{w,t}} L_t. \quad (18)$$

The demand depends on the ratio of household's wage w_t^τ and the aggregate wage level W_t taking into account the substitutability and total labour supply (4). The aggregate wage level is like the total labour supply a Dixit-Stiglitz aggregator function:

$$W_t = \left[\int_0^1 (w_t^\tau)^{-1/\lambda_{w,t}} d\tau \right]^{-\lambda_{w,t}}. \quad (19)$$

It results from first-order conditions for aggregate labour. As described before, the household may only set a new wage, if it receives a random wage changing signal. Therefore with a probability of $(1 - \xi_w)$ in each period the household may solve its maximisation problem and will set a new wage \tilde{w}_t^τ according to

$$\frac{\tilde{w}_t^\tau}{P_t} E_t \sum_{v=0}^{\infty} \beta^v \xi_w^v \left(\frac{(P_t/P_{t-1})^{\gamma_w}}{P_{t+v}/P_{t+v+1}} \right) \frac{l_{t+v}^\tau U_{C,t+v}}{1 + \lambda_{w,t+v}} = E_t \sum_{v=0}^{\infty} \beta^v \xi_w^v l_{t+v}^\tau U_{l,t+v}. \quad (20)$$

This equation states that the wage is set so that the marginal return to working, the additional utility gained through a marginally increased consumption

U_C , is a markup over the marginal cost U_l expressed in the disutility caused by working marginally more.

With a probability of ξ_w a household may only adjust its wage according to the indexation rule (5). Given the total labour supply (19) this leads to the law of motion of the aggregate wage level:

$$(W_t)^{-1/\lambda_{w,t}} = \xi_w (W_{t-1}(\pi_{t-1})^{\gamma_w})^{-1/\lambda_{w,t}} + (1 - \xi_w)(\widetilde{W}_t)^{-1/\lambda_{w,t}}. \quad (21)$$

5.2 The Firm

Based on the production function of an individual firm (6) cost minimisation leads to

$$\frac{W_t L_{j,t}}{r_t^k \widetilde{K}_{j,t}} = \frac{1 - \alpha}{\alpha}. \quad (22)$$

The stated capital-labour ratio will not vary across firms and mirror the aggregate capital labour ratio. The same holds for the marginal costs, which also do not differ across the individual firms:

$$MC_t = \frac{1}{\varepsilon_t^a} W_t^{1-\alpha} (r_t^k)^\alpha (\alpha^{-\alpha} (1 - \alpha)^{\alpha-1}) \quad (23)$$

In the downstream market cost minimisation of the firms producing the final good in a specific sector i leads to demand of individual products

$$y_t^{i,j} = \left(\frac{p_t^j}{P_t^i} \right)^{-(1+\lambda_{p,t})/\lambda_{p,t}} Y_t^i. \quad (24)$$

This demand is determined by the ratio between the price of the individual product p_t^j and the price of the final good P_t^i produced by the corresponding sector, the substitutability of the different intermediate goods $\lambda_{p,t}$ and sector i 's production Y_t^i stated in equation (9). Due to the perfect competition in each sector the price of the final good i can be denoted by

$$P_t^i = \left[\int_{\kappa_{i,1}}^{\kappa_{i,2}} (p_t^j)^{-1/\lambda_{p,t}} dj \right]^{-\lambda_{p,t}}. \quad (25)$$

With this information the profit $\varpi_t^{i,j}$ of firm j in sector i resulting from turnover minus costs and paid as dividends div_t to the households can be stated as:

$$\varpi_t^{i,j} = (p_t^{i,j} - MC_t)y_t^{i,j} - MC_t\Phi. \quad (26)$$

A derivation with respect to $p_t^{i,j}$ reveals the profit maximising price, but firms may only modify their price, if they receive a random price-change signal, which occurs in every period with a probability of $(1 - \xi_{p,i})$. This probability differs across the n sectors. A new optimal price \tilde{p}^j is set considering the possibility of not being able to modify the price in the future and therefore discounting expected profits with the discount rate $\beta\rho_t$. This rate bears analogy to the pricing kernel for nominal returns used by the households: $\rho_{t+k} = (\Lambda_{t+k}/\Lambda_t)(1/P_{t+k})$. An actual profit optimization reveals the following condition

$$E_t \sum_{v=0}^{\infty} \beta^v \xi_{p,i}^v \Lambda_{t+v} y_{t+v}^j \left(\frac{\tilde{p}_t^{i,j}}{P_t^i} \left(\frac{(P_{t-1+v}^i/P_{t-1})^{\gamma_p}}{P_{t+v}/P_t} \right) - (1 + \lambda_{P,t+v}) \frac{MC_t}{P_t^i} \right) = 0, \quad (27)$$

where the new price $\tilde{p}_t^{i,j}$ is set to obtain a markup over the expected future marginal costs. In a market without price rigidities the markup would equal the rate of substitutability $1 + \lambda_{p,t}$. But if firms cannot adjust their prices in every period, the markup will vary with the exogenous shocks causing the differences between a fully flexible economy and a price rigid one.

If a price with a probability of $\xi_{p,i}$ may not be adjusted freely, the particular firm j adopts a price according to the indexation rule (8). Given the definition of the final sector good i 's price, the law of motion for the price level in a specific sector can be derived:

$$(P_t^i)^{-1/\lambda_{p,t}} = \xi_{p,i} (P_{t-1}^i (\pi_{t-1})^{\gamma_p})^{-1/\lambda_{p,t}} + (1 - \xi_{p,i}) (\tilde{p}_t^j)^{-1/\lambda_{p,t}}. \quad (28)$$

5.3 The log-linearised Model

In this subsection the log-linearised versions of the optimality conditions are presented. The equations are therefore log-linearised around their non-stochastic steady state according to $X_t = \bar{X}e^{\hat{x}_t}$, where $\hat{x}_t = \log(X_t/\bar{X})$.

Hence the $\hat{\cdot}$ above a variable states the log-deviation from its steady state. As the differing Calvo parameter for prices effects the reaction of a sector to an exogenous shock, the supply side of the economy is divided into the n sectors. Thereby the response of each sector may be followed separately to explain the aggregate response of a economy marked by price heterogeneity. The aggregate response is calculated as a sum of the sectors weighted with the steady state share of each sector.

The linearised production function in each sector i defines the relationship between output and the inputs labour and capital

$$\hat{Y}_{i,t} = \phi \hat{\varepsilon}_t^a + \phi \alpha \hat{K}_{i,t} + \phi \alpha \psi r_t^k + \phi (1 - \alpha) \hat{L}_{i,t}, \quad (29)$$

where $\psi = \psi'(1)/\psi''(1)$ states the inverse of the elasticity of the capital utilization cost function Ψ and ϕ is a sum of 1 and the share of the fixed cost facing each firm. The deviation of capital in sector i from its steady state is conventional and depends on last period's deviation and a share of the deviation in investment. This share reflects the economic weight of sector i :

$$\hat{K}_{i,t} = (1 - \tau) \hat{K}_{i,t-1} + \tau \Omega_i \hat{I}_t. \quad (30)$$

The labour demand in sector i in the production function will decrease, if wage rises, and increase, if the cost for renting capital rises. Due to the relation between capital and labour in production it also depends positively on deviations in capital:

$$\hat{L}_{i,t} = -\hat{W}_t + (1 + \psi) \hat{r}_t^k + \hat{K}_{i,t}. \quad (31)$$

As the households are not restricted to a specific sector, they may offer their labour to different sectors. Given that the disutility of labour is the same in all sectors, this leads to an equal wage level across the n sectors. The same holds for capital and its rental rate.

The law of motion for the price $P_{i,t}$ combined with the first-order condition

for setting a new optimal price (27) leads to the inflation equation of sector i :

$$\begin{aligned}\widehat{\pi}_{i,t} = & \frac{\beta}{1 + \beta\gamma_p} E_t \widehat{\pi}_{i,t+1} + \frac{\gamma_p}{1 + \beta\gamma_p} \widehat{\pi}_{i,t-1} \\ & + \frac{1}{1 + \beta\gamma_p} \frac{(1 - \beta\xi_{i,p})(1 - \xi_{i,t})}{\xi_{i,t}} [\alpha \widehat{r}_t^k + (1 - \alpha) \widehat{w}_t - \widehat{\varepsilon}_t^a] + \widehat{\lambda}_t^p\end{aligned}$$

The inflation in a sector depends on its future value and marginal costs. The indexation of prices, which may not be set freely in a certain period, leads to a backward looking part dependent on $\widehat{\pi}_{i,t-1}$. The weight of this component is defined by the degree of indexation γ_p . The probability of not being able to set a new price freely and therefore rely on the indexation rule, ξ_i , determines the impact of deviations in marginal costs, \widehat{r}_t^k and \widehat{w}_t , onto inflation. As this probability differs across sectors, it is the starting point of any effect of price heterogeneity on aggregate fluctuations and causes the results presented in section 6.

The aggregate responses of the above variables is calculated as a weighted sum of each sector. The weights are defined as the steady state ratio between the n sectors and set exogenously. As an example the following definition for aggregated capital K_t is given:

$$\widehat{K}_t = \Omega_1 \widehat{K}_{1,t} + \Omega_2 \widehat{K}_{2,t} + \dots \Omega_n \widehat{K}_{n,t}, \quad (32)$$

where the sum of the weights $\sum_{i=1}^n \Omega_i = 1$. Exchanging the n sectoral deviations $\widehat{K}_{i,t}$ with the equivalent of sectoral labour deviation, sectoral output deviation or sectoral inflation deviation leads to the corresponding aggregate deviation.

The deviation in investment, which affects equation (30), depends on its weighted previous and future value and the deviations in the value of capital \widehat{Q}_t , which obviously does not differ across sectors:

$$\widehat{I}_t = \frac{1}{1 + \beta} \widehat{I}_{t-1} + \frac{\beta}{1 + \beta} E_t \widehat{I}_{t+1} + \frac{\varphi}{1 + \beta} \widehat{Q}_t + \widehat{\varepsilon}_t^I. \quad (33)$$

$\varphi = 1/\bar{S}''$ describes the inverse of the second derivative of the steady state cost for capital adjustment.

The deviation of total consumption from its steady state depends on its past and future value. Furthermore it decreases with rising nominal interest rate and increases, if the expected future inflation increases:

$$\widehat{C}_t = \frac{h}{1+h}\widehat{C}_{t-1} + \frac{1}{1+h}E_t C_{t+1} - \frac{1-h}{(1+h)\sigma_c}(\widehat{R}_t - E_t \widehat{\pi}_{t+1}) + \frac{1-h}{(1+h)\sigma_c}\widehat{\varepsilon}_t^b \quad (34)$$

Without any habit formation, the consumption equation would be strictly forward-looking. The habit formation also influences the interest rate elasticity of consumption, as a high habit persistence decreases the effect of a deviation in interest rates on consumption. The deviation of consumption also interferes in the goods market equilibrium equation and the wage equation. Deviations in the total production depend besides consumption on investment and government spending:

$$\widehat{Y}_t = (1 - \tau k_y - g_y)\widehat{C}_t + \tau k_y \widehat{I}_t + \widehat{\varepsilon}_t^G, \quad (35)$$

where g_y denotes the government spending-output ratio in the steady state and k_y the capital-output ratio also in the steady state. The wage equation is derived by combining the optimality condition for a new wage (20) with the law of motion of the aggregate wage level:

$$\begin{aligned} \widehat{W}_t &= \frac{\beta}{1+\beta}E_t \widehat{W}_{t+1} + \frac{1}{1+\beta}\widehat{W}_{t-1} + \frac{\beta}{1+\beta}E_t \widehat{\pi}_{t+1} + \frac{1+\beta\gamma_w}{1+\beta}\widehat{\pi}_t + \frac{\gamma_w}{1+\beta}\widehat{\pi}_{t-1} \\ &\quad - \frac{1}{1+\beta} \frac{(1-\beta\xi_w)(1-\xi_w)}{(1+\frac{(1+\lambda_w)\sigma_L}{\lambda_w})\xi_w} \left[\widehat{w}_t - \sigma_L \widehat{L}_t - \frac{\sigma_c}{1-h}(\widehat{C}_t - h\widehat{C}_{t-1}) + \widehat{\varepsilon}_t^L \right] + \widehat{\lambda}_t^W \end{aligned}$$

The wage deviation depends on its past and future value and the past, present and future value of total inflation. An increase in labour demand \widehat{L}_t or consumption net of habit formation $\widehat{C}_t - h\widehat{C}_{t-1}$ raises the wage.

The equation for the value of installed capital

$$\widehat{Q}_t = -(\widehat{R} - E_t \widehat{\pi}_{t+1}) + (1-\tau)\beta E_t \widehat{Q}_{t+1} + \bar{r}^k \beta E_t \widehat{r}_{t+1}^k + \widehat{\varepsilon}_t^Q \quad (36)$$

introduces the shock $\widehat{\varepsilon}_t^Q$, which does not arise from the structure of the economy, but is introduced to reflect alterations in the cost of capital caused by stochastic variations in the external finance premium. Apart from this shock,

the deviations in the value are caused by the expected rental rate of capital, its own expected value and the difference between the nominal interest rate and the expected inflation rate.

In every period the nominal interest rate is set by a monetary authority according to a Taylor Rule. It aims at matching the interest rate with a target inflation $\bar{\pi}_t$ modelled as a first order autoregressive process. Besides the output gap arising from a perfectly flexible economy²⁹, differences between past and present inflation and past and present output gaps also influence the interest rate, as does the past nominal interest rate to smooth the fluctuations:

$$\begin{aligned}\hat{R}_t &= \rho \hat{R}_{t-1} + (1 - \rho) \left[\bar{\pi}_t + r_\pi (\hat{\pi}_{t-1} - \bar{\pi}_t) + r_y (\hat{Y}_t - \hat{Y}_t^P) \right] \\ &+ r_{\Delta\pi} (\hat{\pi}_t - \hat{\pi}_{t-1}) + r_{\Delta Y} \left[\hat{Y}_t - \hat{Y}_t^P - (\hat{Y}_{t-1} - \hat{Y}_{t-1}^P) \right] + \varepsilon_t^R.\end{aligned}$$

²⁹In the perfectly flexible economy there are no rigidities neither concerning prices nor wages. Further the three cost-push shocks ε_t^Q , λ_t^P and λ_t^W are per definition not existent.

6 Model Results and Answer

The equation presented in section 5.3 define the model in its log-linearised form, which are employed twice to calculate the sticky price part, which will be analysed later on, and to calculate a flexible part, which excludes the stickiness in wages and prices and serves as a reference point for the calculation of the output gap in the Taylor rule.

In total there are 27 variables in the sticky price part and 22 variables in the flexible price part, which are marked with a superscript p . The difference arise due to the output gap, which is only calculated in the sticky price part, and the sectoral computation of inflation, as there are no sectoral differences to be found in the flexible price part. Endogenous state variables are: K_1^p , K_2^p , K_3^p , K_4^p , C^p , I^p , π^p , W^p , Y_1^p , Y_2^p , Y_3^p , Y_4^p , R^p , Q^p , K_1 , K_2 , K_3 , K_4 , C , I , π_1 , π_2 , π_3 , π_4 , π , W , Y_1 , Y_2 , Y_3 , Y_4 , R , Q , r_k^p and r_k , where the rental rates of capital r_k^p and r_k had to be included in order to facilitate the computation in the Toolkit. Other endogenous variables are K^p , Y^p , L_1^p , L_2^p , L_3^p , L_4^p , L^p , K , Y , L_1 , L_2 , L_3 , L_4 , L and Y_g , where Y_g denotes the output gap. The model is driven by ten exogenous shocks: ϵ^L , ϵ^a , ϵ^b , ϵ^G , ϵ^I , ϵ^R , ϵ^Q , ϵ^{λ_P} , ϵ^{λ_W} and $\bar{\pi}$.

6.1 Calibration

The calibration of this model is for the most part taken from Smets and Wouter (2003), as the model at hand is based on their achievements. Furthermore the parameters applied in Smets and Wouter (2003) have become popular for calibrating DSGE models, because of their computation and the wide scope of the Smets and Wouters model. But in their particular model only one comprehensive sector is established and the calibration of the different sectors concerning their economic weight and price stickiness is therefore adopted from Álvarez et al. (2005). Although this paper deals mainly with the hazard function, the authors propose a four sector economy to account for the decreasing probability for a firm to set a new price over time. Based on Spanish producer prices they calculate the economic weight of each sector

and its price stickiness. Smets and Wouters (2003) also apply an empirical approach to deduct the parameters, as their estimation is based on time series for seven macroeconomic variables in the euro area. Obviously these two sets of parameters differ, as the prices in Smets and Wouter (2003) are stickier than in Álvarez (2005) but a combination can serve to introduce a somewhat realistic price heterogeneity in the Smets and Wouters (2003) model³⁰.

Smets and Wouters apply in their paper the strong econometric interpretation aiming at a full characterization of the observed data series. In contrast a weak interpretation would set the parameters to match the data with certain selected theoretical moments³¹. They make use of Bayesian techniques and set a prior distribution and prior mean for each parameter relying on calibration literature and earlier estimates. The actual estimation is based on time series for real GDP, real consumption, real investment, the GDP deflator, real wages, employment and nominal interest rate in the euro area. Employment had to be included to make up for the variable labour, as sufficient data was not available³². With ten structural shocks, but only seven time series an identification problem was avoided by declaring the shocks for the value of capital, price markup, wage markup and the monetary shock as pure white noise processes.

Before the actual estimation a few parameters where fixed as their value could be deduced from the means of the observed variables. To this group belongs the discount factor β with a value of 0.99 specifying the the annual

³⁰A new estimation of the parameters in a four sector model is another way to obtain a more realistic calibration for a such a model. But in the evaluation of the price heterogeneity an one-sector model will be compared to the four sector model and to secure the traceability of differences the same parameters have to be applied to both models. Thus a new estimate would result in a more realistic four sector model, but would force less realistic parameters upon the one-sector model.

³¹Gewerke (1999)

³²In Smets and Wouters (2003) an auxiliary equation for employment is introduced into the model, which is omitted in the model at hand, as its only purpose was the facilitate the estimation

steady state real interest rate to amount to 4%. The annual depreciation of capital is set to 10%, as $\tau = 0.025$ per quater. The division of capital and labour in the production function is set to $\alpha = 0.3$ resulting in an approximate steady-state share of capital in total output of 30%. Finally the steady state fraction of consumption (investment) in total output is set to 0.6 (0.22). Additionally the wage markup has to be fixed to $\lambda_w = 0.5$, because this parameter is not identified in the estimation procedure³³.

The main interest of Álvarez et al. (2005) is to model a decreasing hazard function by aggregating diverse sectors. The calculation of empirically based specifications for price stickiness and economic weight of four sectors is only conducted to underline their approach. However, due to the accuracy of their results concerning empirical observations these specifications may also serve to calibrate a full DSGE model with price heterogeneity. In their paper the authors deduct a relationship between single sector hazard functions and the aggregate economy wide hazard function and find that changes in the aggregate function are a linear combination of changes in the single functions plus an additional affect caused by the heterogeneity. Applying Calvo agents they observe that the aggregate function converges over time asymptotically to the hazard function of the stickiest sector. Next they set up several theoretical aggregate hazard functions with varying number of sectors and estimate these via a finite mixture model and the EM algorithm. The estimation is based on Spanish micro price data and model selection criteria help them to select the appropriate number of sectors. Finally each firm is assigned to a sector according to their pricing rule and the economic weight and price stickiness of each sector is calculated. This leads to four different sectors, where the most flexible sector possesses a price stickiness of 1 month and an economic weight of 22%. Prices in the intermediate sector change on average every 10 months and firms in this sector make up 50% of

³³A detailed explanation of the estimation methodology can be found in Smets and Wouters (2003), pages pp. 1137 - 1141.

the whole economy. The sticky sector contributes 21% to the total output and prices change only every 3 years. Additionally there is a small annual sector, where prices are modified every 18 months and its economic weights sums up to 7% of total production. According to these results, the model at hand is endowed with four sectors, which inherit the just specified economic weights and price stickiness.

An overview of all parameters is presented in table 2 and table 3 in the appendix. A discussion of the implications of the parameter choices is an interesting aspect, but will not automatically lead to a better understanding of price heterogeneity, as the comparison of a four-sector price heterogeneous model with a four-sector model with a similar average price stickiness is in the focus of interest and requires all but the Calvo price parameter to stay equal.

6.2 Impulse Responses

As the model is similar to Smets and Wouters (2003), which has been analysed by the authors in great detail, and as the focus in this thesis is on price heterogeneity, the analysis of the model is only secondary concentrated on the fluctuations of the diverse variables, but mainly concerned with the particularities arising from the introduction of price heterogeneity. The applied approach is as following: First a one-sector economy is extent to a four-sector economy, whereas the model stays basically the same, but the number of sectors is increased. This is implemented by allowing for equal weights, instead of full weight on only one-sector, in the aggregation equations (32). The next step consist of comparing a four-sector model with price heterogeneity to a four-sector model with an equal price stickiness in each sector, which is supposed to mimic the price heterogeneous model. The specifications for the four-sector heterogeneous model are taken from Álvarez (2005) and the price stickiness for the four-sector model with equal price rigidity are taken from table 1, namely a Calvo parameter of $\xi_p = 0.8$.

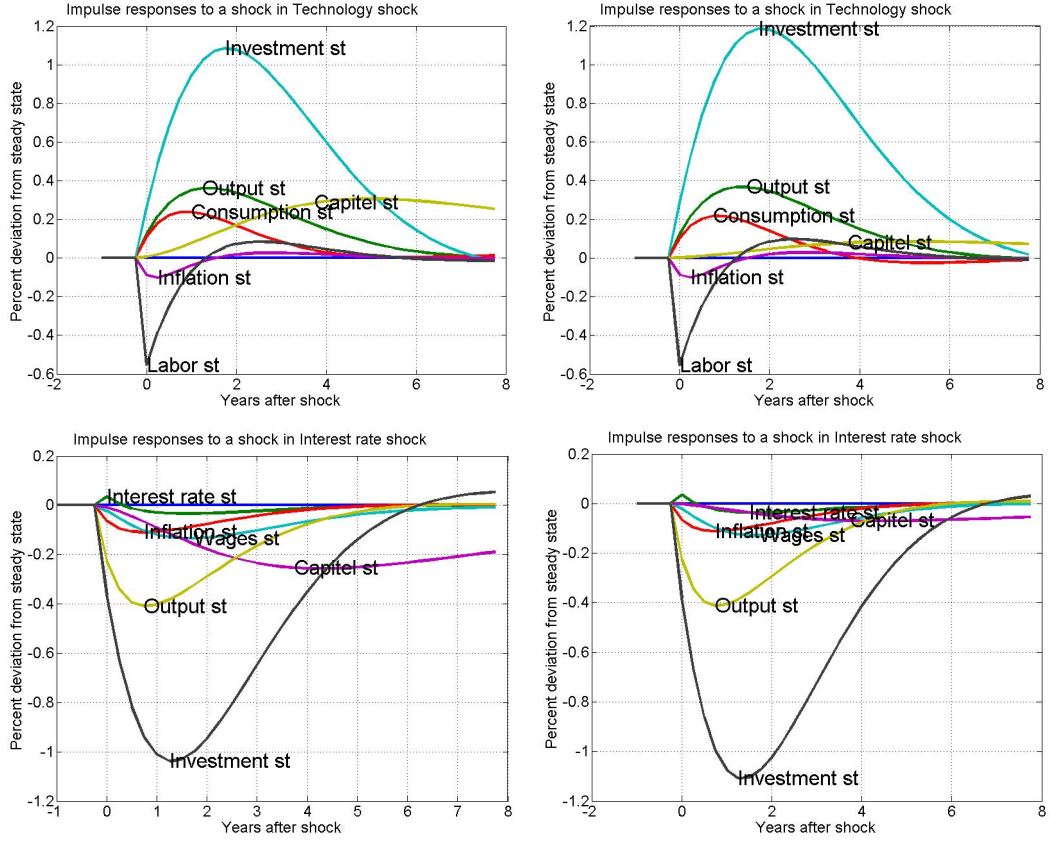


Figure 1: Top graphs: impulse responses to a technology shock in one-sector economy (left) and multi-sector economy (right); bottom graphs: impulse responses to a interest rate shock in a one-sector economy (left) and multi-sector economy (right); Calvo parameter $\xi_p = 0.8$ in all graphs

As it can be seen in figure 1, there are hardly any differences, if one extends a one-sector model to a four sector model. In both cases does the technology shock lead to rises in output, investment and consumption, but a fall in labour. Furthermore as marginal cost are decreased by the productivity shock and the monetary policy does not offset this fall completely, the inflation decreases. Discrepancies between the two graphs can be made out in the reactions of investment and capital. In the four sector model, investment reacts stronger to the shock, while deviations in capital are much lower. The reason for this are found in the structure of the model. Total

deviations in investment are not incorporated in the sectoral capital accumulation equation to their full extent, but rather weighted according to the economic share of the sector in total output. If there arises a productivity shock, firms will want to install more capital, as is more productive and this explains the rise in investment. However as only a certain share of the total investment ends up in the capital accumulation of each sector, the total investment deviates more from its steady state to account for the demand in capital in each sector. And although investment rises above its one-sector level, there is not enough deviation to cause an equal rise in capital, as there is in the one-sector model.

The same intuition also holds for an interest rate shock. After an increase of the nominal interest rate, output, consumption (not included in the graph) and investment fall. In line with general assumptions, wages also fall. However, in the four-sector model investment deviates stronger and capital to a lesser extent than in the one-sector model. Due to the increased interest rate, costs for investing rise and the investment level falls. But as these changes in investment are not transferred one-to-one into the sectoral capital accumulation equations, a fall in capital is less profound than in the one-sector model.

In order to evaluate the implications of diverse Calvo parameters across the sectors, figure 2 shows a comparison between the impulse responses to a technology shock in a four-sector model with varying price stickiness according to Álvarez (2005), further on referred to as Álvarez parametrisation, and the impulse responses to the same shock in a four sector model with a equal price stickiness according to the calculations in section 3.2, referred to as average parametrisation.

Comparing the graphs on the left with the ones on the right it can be noticed that basically the reaction in the graphs resemble, but their are significant differences in the magnitude and persistence of the reaction to the technology shock. Only the deviation of labour, capital and the interest rate show no mayor variation. Striking differences are found in the lower graphs.

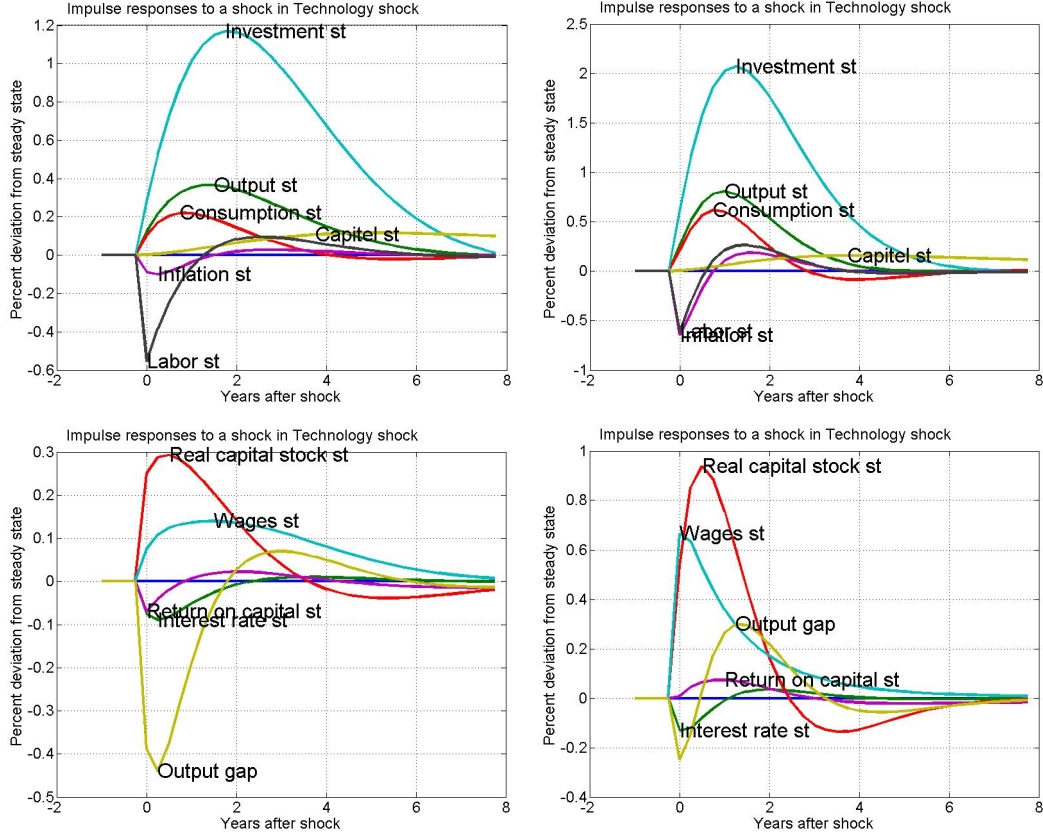


Figure 2: Left side: impulse responses with a Calvo parameter of $\xi_p = 0.8$ in all sectors; right side: impulse responses with Calvo parameters according to Álvarez (2005)

The deviation of the real capital stock is three times as large in the Álvarez parametrisation, than in the average parametrisation. The reaction of wage is much more fierce, but also shorter in the Álvarez parametrisation. In the same graph it can also be noticed, that the return on capital does not decrease before raising afterwards, but raises instantly. Also the decrease of the output gap is not too extensive and is later on accompanied by an increases more than three times as much as the rise of the counterpart in the average parametrisation. The differences in the upper two graphs are also noteworthy, especially the change in the inflation rate.

The changes in wage are especially interesting, as wages are also endowed

with a Calvo parameter, which defines their rigidity and stays constant, while only the Calvo parameters governing price rigidity are modified.

As we are comparing two multi-sector models, it is possible to break output, capital and labour down on a sectoral level. On this level deviations in the Álvarez parametrisation are also more volatile, although the differences in labour and capital are diminutive. In all three cases, the most rigid responses are shown by the sector three³⁴ with an intermediate price stickiness ($\xi_{p,3} = 0.8465$) and a small weight ($\Omega_3 = 0.07$), next comes sector four, which is the stickiest one ($\xi_{p,4} = 0.9201$) and a weight of about 20% ($\Omega_4 = 0.21$). It is followed by sector one, which is the most flexible one ($\xi_{p,1} = 0.0498$) with an weight of $\Omega_1 = 0.22$. Finally the most ample responses are shown by sector two, which is rather large ($\Omega_2 = 0.5$), but only medium price rigid ($\xi_{p,2} = 0.7408$). This ordering according to the magnitude of responses and especailly a comparison of the weight of a sector and its position suggests that this particular order of the sectors is not caused by the price rigidity, but rather by the econmic weight of each sector and assigning equal weights to the sectors reveals that there actually are no differences in the deviations in output, capital or labour between the single sectors, which result from price heterogeneity.

Another variable which can be broken down on a sectoral level is inflation and differences in the four sectors can be determined. This also holds in the case of applying equal weights to the sectors. Controlling for the weight the reactions of the sectors to a technology shock are of course equal in the case of the average parametrisation, as the four sectors are totally alike. In case of the Álvarez parametrisation the distinct Calvo parameters lead to different magnitudes in the reaction. It strikes out that sector one, the most flexible one, shows a deviation of nearly 3%, whereas the reactions of the other sectors are only marginal ranging from 0.015% for sector two to 0.001% for sector four. The most sticky sector shows the smallest deviation in inflation,

³⁴This indexation is conducted according to the rate of price stickiness. The most flexible sector is indexed by one and the most rigid sector by four

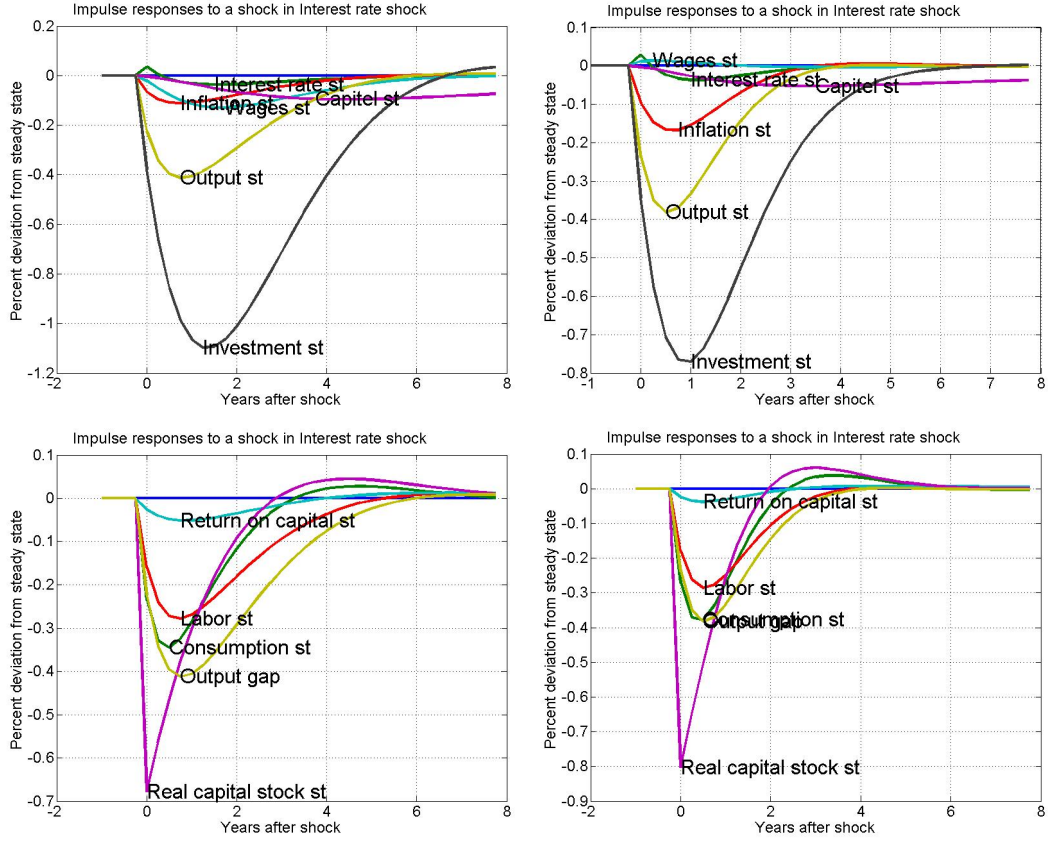


Figure 3: *Left side: impulse responses with a Calvo parameter of $\xi_p = 0.8$ in all sectors; right side: Impulse responses with Calvo parameters according to Álvarez (2005)*

as the probability of firms in this sector to react to exogenous shock by setting a new optimal price is more strict than in the other sectors.

Figure 3 gives the reaction of four-sector model with Álvarez parametrization and a four-sector model with average parametrization to a interest rate shock. As Smets and Wouter noticed in their paper³⁵ the interest objective shock does not account for much of the volatility in the model and the analyse of monetary policy is therefore concentrated on the interest rate shock affecting temporary the nominal interest rate set by the monetary authority.

³⁵Smets and Wouter (2003), pages pp. 1160-1162

As in the reactions to a technology shock, the general reaction to an interest rate shock in the Álvarez parametrisation looks similar to the responses in the average parametrisation. But in contrast to the technology shock, the magnitudes of certain variables do not only increase, but some also decrease if compared to the average parametrisation. Among them are the deviation of investment and the return to capital, in both cases the difference to the average parametrisation are not profound. But also the increases are not as distinct as in the reactions to a technology shock. The real capital stock deviates about 0.1% more in the Álvarez parametrisation than in the average parametrisation. The difference arising from different parametrisation in consumption is less than 0.05% and in total inflation a bit more than 0.05%. Splitting up output, capital, labour and inflation for their sectoral deviations reveals the same findings as in the technology shock. Configuring every sector with the same weight reveals that there are no differences between the sectors arising from price heterogeneity to be found in the deviations of output, capital or labour. Likewise the deviations of sectoral inflation discloses that the deviation of sector one, the most flexible one, is the largest with 0.75% and the others range from 0.00375% for sector two to 0.0003% for sector four.

A discussion of the other eight shocks is omitted as the general findings, differences in the deviations of most variables and no sectoral deviation in output, capital and labour, but for inflation, holds also for them. Instead an explanation for the observed deviations between the Álvarez and the average parametrization is presented.

Comparing the impulse responses of the Álvarez parametrisation with a totally flexible model, it seems that this parametrisation seems to be much more flexible than the proposed price stickiness in the average parametrisation³⁶. It is also interesting to notice that although in the Álvarez parametrisation

³⁶Graphically the average parametrisation looks similar to the Álvarez parametrisation with a Calvo parameter of $\xi_p = 0.2$, far lower than the calculated 0.8.

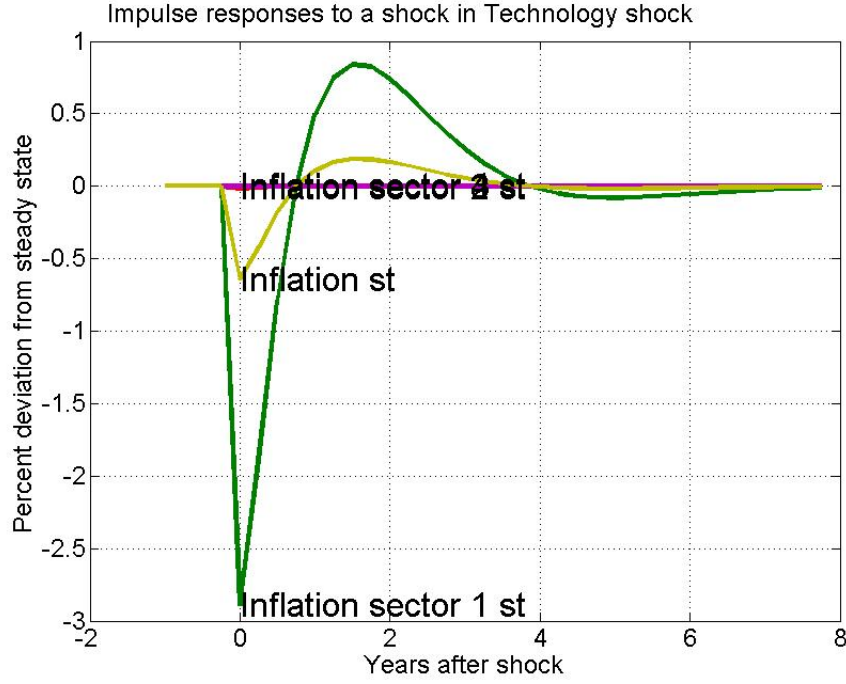


Figure 4: *Impulse responses to a technology shock with Álvarez Parametrisation*

78% of the economy is endowed with a price rigidity similar to the average price stickiness, the resulting impulse responses look very different to the average ones in magnitude. Also by restricting the weight of sector one or reducing its price flexibility, an Álvarez parametrisation modified in this way approximates the average parametrisation.

To explain these outcomes it is useful to reconsider the sectoral fluctuations in inflation after a technology shock which are presented in figure 4. As mentioned before, the inflation of sector one surpasses the others, including the total inflation, greatly. This large deviation is caused by the Calvo parameter assigned to sector one. It allows the firms in this sector to adapt their prices to exogenous shocks much faster than firms in the other sectors and leads to relative large deviations in inflation after a shock. But this sectoral

deviation in inflation has a linear influence on deviation in average inflation, as this variable is a weighted average of all sectoral deviations. Therefore the assigned Calvo parameter in sector one leads to an increased average inflation, which causes the stronger effects in other variables, observed in figures 2 and 3. Hence the profound differences between the Álvarez parametrisation and the average parametrisation are caused by the low price rigidity in sector one, which causes a large deviation in total inflation and dominates the influence of the inflation of the other sectors. An average parametrisation calculated with economic weights or medians is therefore not able to mimic the responses of a price heterogeneous multi-sector model.

This connection between the variables also explains the lower changes in the differences between the reaction to an interest shock, with its lower reaction of inflation in sector one, and the reaction to a technology shock.

7 Variations

The introduction of heterogeneity in price stickiness leads to varying reactions of inflation in the diverse sectors. The inflation in the most flexible sector shows a stronger reaction to a exogenous shock than the inflation in the most sticky sector, as firms in the diverse sector process different degrees of probability to be able to react to a shock. These degree have several implications for the reaction of the whole economy, as has been show for the Álvarez parametrisation, and the consequences are therefore of interest to the monetary authority, who aims at decreasing the out put gap between the actual and the potential output, whereas potential output is characterised by the absence of any form of rigidity. The monetary authority's motive to act is incorporated in the model by the Taylor rule, which gives the reaction of the nominal interest rate to deviations in the output gap, deviations in inflation from its desired level and finally deviations between the actual inflation and output gap and there prior values. A lower (higher) than desired inflation rate will lead to a negative (positive) deviation of the interest rate, which itself will result in higher (lower) output deviation rising (decreasing) prices till the inflation matches its desired level. A higher potential than actual output deviation, defined as a negative output gap deviation, will also lead to lowering of the interest rate encouraging investment and ultimately causing an increase in the actual output.

In the case of price heterogeneity the monetary authority not only faces an aggregate inflation deviation, but also sectoral inflation deviations, which contain different characteristics. Naturally the question rises, if paying attention to the aggregate inflation still results in the lowest possible output gap. General opinion suggests, that a better result can be achieved by concentrating on the rather sticky sectors and the inflation rate implied by them³⁷. The intuition behind this argument refers to the fact, that the output gap is caused by the rigidity. Therefore the distortions caused by a rather flex-

³⁷Refer to Aoki (2001), Benigno (2003) and Woodford (2003)

ible sector in a heterogeneous economy are smaller than the distortion of a more rigid sector and the monetary authority can reduce the total output gap more by concentrating on the rigid sector as the more flexible one is, by its flexibility, better endowed to react to exogenous shocks.

Out of this reason, the model variation presented is based on a modified Taylor rule, in which different weights can be assigned to the sectoral inflations:

$$\begin{aligned}\widehat{R}_t &= \rho \widehat{R}_{t-1} + (1 - \rho) \left[\bar{\pi}_t + r_\pi \left(\sum_{i=1}^n \Theta_i \widehat{\pi}_{i,t-1} - \bar{\pi}_t \right) + r_y (\widehat{Y}_t - \widehat{Y}_t^P) \right] \\ &+ r_{\Delta\pi} \left(\sum_{i=1}^n \Theta_i (\widehat{\pi}_{i,t} - \widehat{\pi}_{i,t-1}) \right) + r_{\Delta Y} [\widehat{Y}_t - \widehat{Y}_t^P - (\widehat{Y}_{t-1} - \widehat{Y}_{t-1}^P)] + \varepsilon_t^R,\end{aligned}$$

where Θ_i sets the weight assigned to the inflation deviation in each sector. Obviously there will only be an affect, if the this weights differs from the economic weight applied throughout the model. The rest of model does not vary from the details given in section 5.3 and the Álvarez parametrisation will be applied to define the economic weight and price stickiness in every sector. Concerning the weighting in the just outlined Taylor rule, more emphasis is placed on the sticky sectors and less on the flexible one.

The graphs in figure 5 present the impulse responses to a technology shock, where the graphs on the left are equal to the ones in section 6.2 and the graphs on the right display the impulse responses by a model with a modified Taylor rule³⁸. As it can be noticed, the graphs show the same basic reactions to the technology shock. However there are a few sizeable differences to explain. The level of deviation in the goods market is lowered, as output, consumption and investment deviate less from their steady state, if the modified Taylor rule is applied. The deviation in capital does not change much, whereas the decrease in labour and investment is a bit more profound. In the lower graphs, the output gap decreases initially more, but raises later

³⁸The following weights are applied: $\Theta_1 = 0.05$, $\Theta_2 = 0.3$, $\Theta_3 = 0.05$ and $\Theta_4 = 0.6$ for the modified Taylor rule and $\Omega_1 = 0.22$, $\Omega_2 = 0.5$, $\Omega_3 = 0.07$ and $\Omega_4 = 0.21$ for the economic weights.

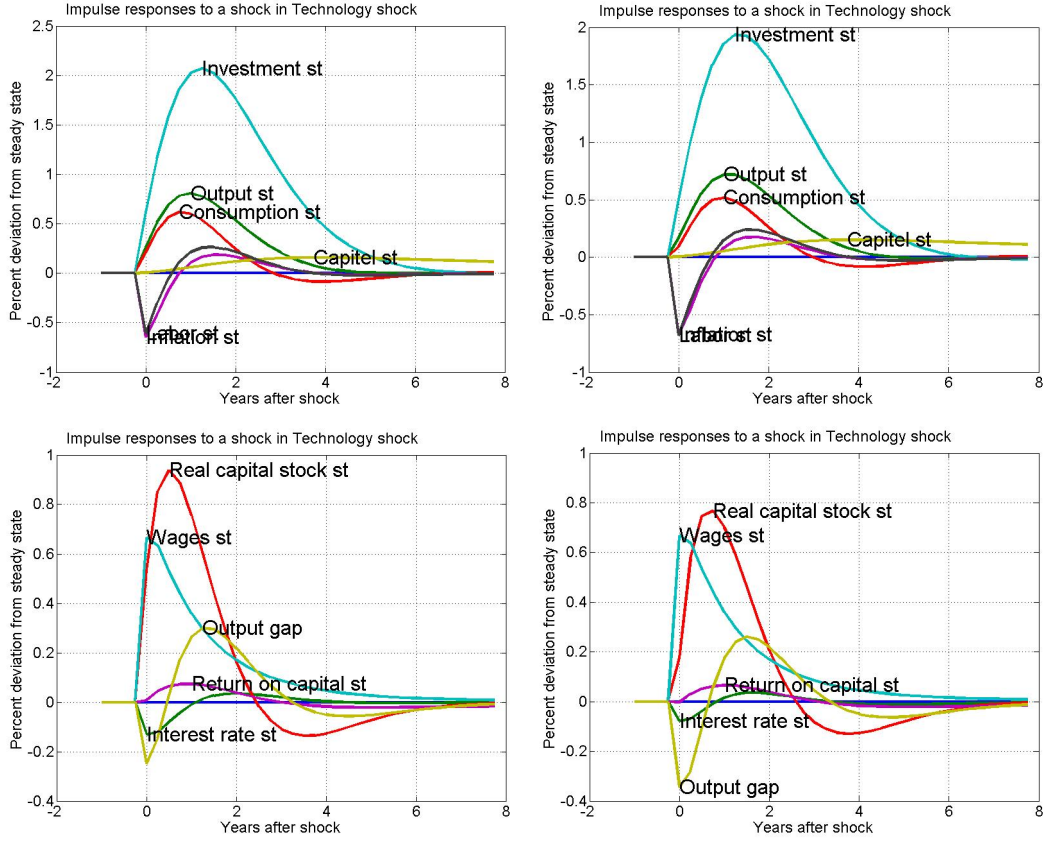


Figure 5: *Left side: Impulse responses to a technology shock; right side: impulse responses to a technology shock affected by a modified Taylor rule*

to a lesser extent. The interest rate and the real capital stock do not react as extensive as before, whereas the modified Taylor rule does not cause any difference in the reaction of wages or return on capital.

The source of the presented discrepancy is to be found in the nominal interest rate, as its composition has changed. Emphasis is placed on the inflation in the sticky sector, which is much lower than in the other sectors. Therefore the difference between the current and the prior inflation is smaller in size and as there is no inflation objective³⁹ its full, lower the average value

³⁹The inflation objective is modelled as a $AR(1)$ process activated by an inflation objective shock.

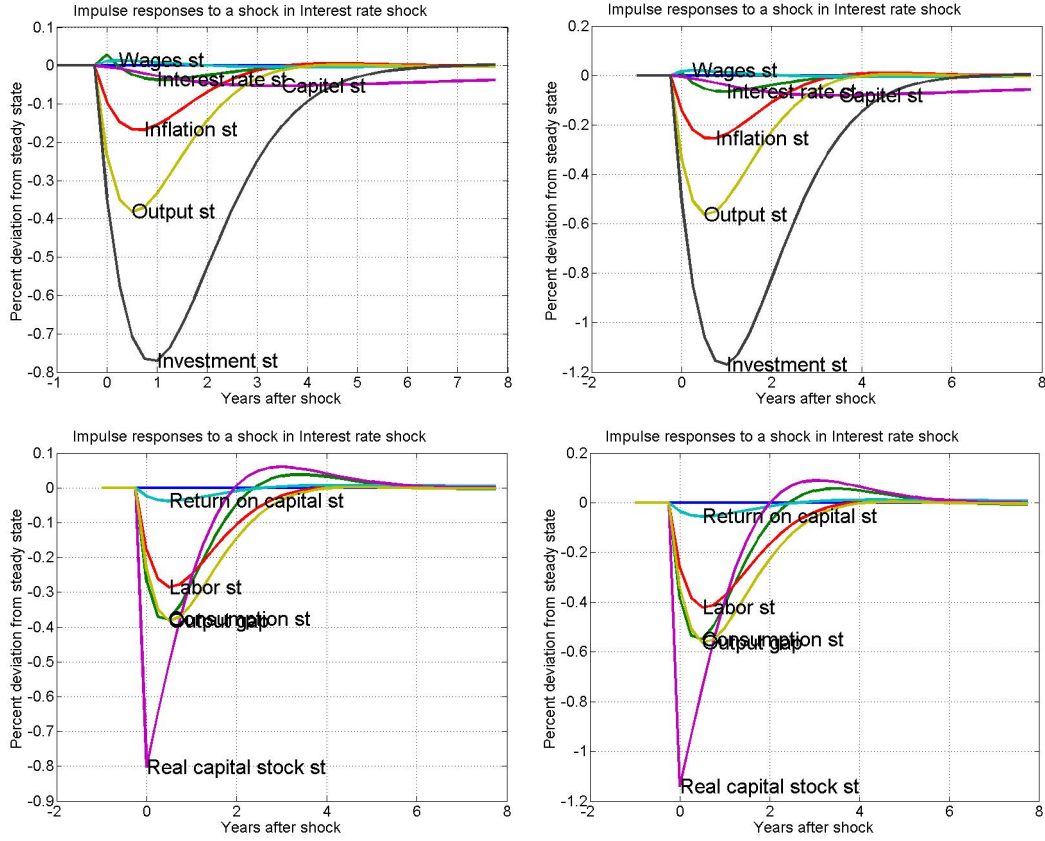


Figure 6: *Left side: Impulse responses to a interest rate shock; right side: impulse responses to a interest rate shock affected by a modified Taylor rule*

is absorbed by the interest rate resulting in a smaller deviation, as it can be observed in the graph in the lower right corner. This negative and smaller than before deviation has an alleviative effect on consumption and real capital deviation lowering ultimately the investment and output deviation. The slightly more profound deviation in total inflation is another consequence of modified weighting in the Taylor rule. Average inflation is mostly dominated by the inflation in the flexible sector one. As now a technology shock lowers the marginal costs, the monetary policy is to offset this, but, due to a lower weight on sector one, does not react as strongly as before allowing a stronger inflation in sector one and therefore a stronger deviation in aggregate inflation.

Figure 6 presents the impulse responses to a interest rate shock⁴⁰. Generally speaking this shock will rise the interest rate instantly and causes thus a negative deviation in investment. Furthermore capital and as a consequence output are likewise lowered and labour falls thereupon. Due to this fall in labour, the households cannot maintain the level of consumption. This overall reaction takes also place, if the modified Taylor rule is applied. However, in this case the reactions are more potent leading up to stronger negative deviations.

The stronger reaction in the model with the modified Taylor rule can be explained by the impact the shock has on the whole system. Due to the increased weight of the sticky sector and omitting the influence of the shock, the deviation in the interest rate is much lower in the model with the modified Taylor rule than in the baseline model. As a consequence the shock, which has the same magnitude in each model, has a much more profound effect on the interest rate in the modified Taylor rule model, than in the standard model, causing the same, but more extensive deviations in the other variables.

As these two shocks demonstrate the possible implications of a modified Taylor rule, the discussion of further shocks is omitted.

⁴⁰The following weights are applied: $\Theta_1 = 0.15$, $\Theta_2 = 0.3$, $\Theta_3 = 0.05$ and $\Theta_4 = 0.5$ for the modified Taylor rule and $\Omega_1 = 0.22$, $\Omega_2 = 0.5$, $\Omega_3 = 0.07$ and $\Omega_4 = 0.21$ for the economic weights. The weights Θ_1 and Θ_2 had to be altered in order to facilitate the computation in the Toolkit.

8 Discussion

In the preceding sections, the implications arising from the introduction of several sectors in a standard New Keynesian DSGE model and the consequences from endowing these sectors with a differing price stickiness, computed on the basis of empirical micro price data, are presented and discussed exemplarily by a technology and interest rate shock.

Concerning the extension to a multi-sector model an alteration of the model's dynamics has been exposed, in particular the interaction between total investment and sectoral capital accumulation has been shaken up leading to lower influence of investment on capital accumulation. In the next step, the comparison of a heterogeneous price sticky multi-sector model, with a similar multi-sector model equipped with an equal price stickiness in all sectors, which is supposed to implement the same degree of price stickiness, has revealed extensive differences. The findings can be generalised, as that the heterogeneous model displayed much stronger reactions to the shocks in nearly all variables, but sectoral differences in the responses could only be detected in the variable inflation. Finally, emphasising the rather sticky sectors in the monetary authority's policy rule also led to changes in the impulse responses, although the reaction was mixed, as one shock led to an increase in the deviation, whereas the other shock was followed by a stronger reaction of the endogenous variables.

The existence of differences resulting from the extension of a one-sector to a multi-sector model seem to stick out at first glance, as the splitting up of an economy into its sectors should not have any effect on the impulse responses of the whole economy. A possible explanation of this unexpected effect would be an aggregation effect, meaning that the combination of parts results in more than just the sum of the single parts. But as mentioned before the observed effect can be explained by the transformation of the single-sector model to a multi-sector model, as it was assumed that changes in total investment only have a weighted impact on the capital accumulation in the

single sectors, as common sense would suggest.

The comparison of the multi-sector model with heterogeneous price sticky sectors with a multi-sector model endowed with a equal price stickiness across the sectors, which is to supposed to mimic the total price stickiness in the heterogeneous model, reveals that an aggregate price sticky model fails in delivering the same impulse responses as the heterogeneous model and suggests that the heterogeneity leads to much stronger and short-lived effects, than supposed by the average price stickiness. The character of the responses of the heterogeneous model have more in common with a more flexible than average model. Unfortunately the applied implementation of the multi-sector model prevents the model from displaying possible differences in the reaction of the single sectors⁴¹ and a more detailed analysis of the dynamics caused by the introduction of price heterogeneity. This problem can be solved by dropping the assumptions that wage and the return on capital do not differ across sectors, which were imposed upon the model according to standard economic theory.

The implications of a modified Taylor rule, which leads to mixed reactions to the shocks, cannot be answered satisfyingly. Although an effect is clearly distinguishable, the computation of impulse responses reacts very sensitive to changes in parametrisation indicating that the model's robustness may not suffice for the implementation of specifications drawn from empirical micro price data.

⁴¹An attempt to include sectoral consumption and investment did not succeed, due to computational problems in the Toolkit.

9 Summary and Concluding Remarks

In the thesis at hand an elaborated DSGE model focused on the euro area was combined with recent insights from micro price data to evaluate the impact of heterogeneity in price stickiness on the model's dynamics. Furthermore, a possible reaction of a monetary authority to this modifications was discussed. During the last years an ample evidence on micro price data had been published motivated by a lack of knowledge about actual price rigidity. One of the main findings was, that price stickiness does not stay constant across the products, but rather varies. It has been denoted, that for example prices of raw materials fluctuate more than sophisticated durable goods and several factors driving the price rigidity have been named. Among them are for example the cost structure, the degree of competition in a market or attractive pricing. Despite this evidence most New Keynesian DSGE models are endowed with only one overall sector and this thesis has followed infrequent previous work and extent a DSGE model to a multi-sector one. But in contrast of applying a rather simple DSGE model, an elaborated DSGE model was extended resulting in detailed impulse responses of various variables.

The obtained results display the influence of heterogeneity in price stickiness on the dynamics of a model. But in order to distinguish effects arising from heterogeneity in price stickiness and possible effects from extending a one-sector model to a multi-sector model, a comparison had been drawn between a one sector model and a multi sector model. The obtained dynamics indicated a somewhat different reaction to a shock and an explanation has been proposed pointing to the structural computation of the four-sector model, which bore a different interaction between deviations in investment and deviations in capital. Next, two four-sector models were compared, whereas one was calibrated according to micro price evidence and the other according to an average price rigidity, which was supposed to mimic the reactions of the heterogeneous model. Exemplary the reactions to a technology shock and to an interest rate shock were presented, and it was noticed, that a heteroge-

neous model reacts much stronger and with less persistence in the responses. However, due to the applied structure of the model, sectoral variations in the variables output, capital, investment or labour were not noticeable and a detailed explanation of the observed changes must be left for future work. Finally, the Taylor rule was modified to facilitate a stronger weighting on the more sticky sectors, as most distortion is caused by these sectors. Such a modified Taylor rule altered the dynamics resulting in an increase of the reactions to an interest rate shock and to a decrease of the deviations after a technology shock. But with the modified Taylor rule included, the computation became much more sensitive to parameter changes, a possible indication that the model's robustness may not suffice to incorporate all parameter changes drawn from empirical micro price data.

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A Parameter values

Paramters	Value	Description
ρ_{ε^L}	0.889	autoregressive parameter of labour supply shock
ρ_{ε^a}	0.823	autoregressive parameter of productivity shock
ρ_{ε^b}	0.855	autoregressive parameter of preference shock
ρ_{ε^G}	0.949	autoregressive parameter of government spending shock
ρ_{ε^I}	0.927	autoregressive parameter of investment shock
$\rho_{\bar{\pi}}$	0.924	autoregressive parameter of inflation objective shock
ρ_{ε^R}	0	autoregressive parameter of interest rate shock
ρ_{ε^Q}	0	autoregressive parameter of value of capital shock
ρ_{λ_W}	0	autoregressive parameter of wage markup shock
ρ_{λ_P}	0	autoregressive parameter of price markup shock
σ_{ε^L}	3.52	standard error of labour supply shock
σ_{ε^a}	0.598	standard error of productivity shock
σ_{ε^b}	0.336	standard error of preference shock
σ_{ε^G}	0.325	standard error of government spending shock
σ_{ε^I}	0.085	standard error of investment shock
$\sigma_{\bar{\pi}}$	0.017	standard error of interest objective shock
σ_{ε^R}	0.081	standard error of interest rate shock
σ_{ε^Q}	0.604	standard error of value of capital shock
σ_{λ_W}	0.289	standard error of wage markup shock
σ_{λ_P}	0.16	standard error of price markup shock

Table 2: *Parameter estimates defining for the shocks*

A PARAMETER VALUES

Paramters	Value	Description
β	0.99	discount factor
τ	0.025	depreciation rate of capital
α	0.3	division in production function
ψ	1/0.169	inverse of the elasticity of the capital utilization cost function
γ_p	0.469	degree of partial price indexation
γ_w	0.763	degree of partial wage indexation
λ_w	0.5	markup in wage setting
σ_L	2.4	inverse of work effort elasticity
σ_c	1.353	inverse of intertemporal elasticity of substitution
h	0.573	habit persistence of past consumption
ϕ	1.408	1 plus share of fixed cost in production
φ	1/6.771	inverse of investment adjustment cost
\bar{r}^k	0.35	steady state return on capital
k_y	8.8	capital output ratio
C_y	0.6	share of consumption in output
I_y	0.22	share of investment in output
G_y	0.18	share of government spending in output
r_π	1.684	inflation coefficient
r_y	0.099	output gap coefficient
$r_{\Delta\pi}$	0.14	inflation growth coefficient
$r_{\Delta Y}$	0.159	output gap growth coefficient
ρ	0.961	autoregressive parameter on prior interest rate
ξ_f^P	0.000000001	Calvo prices, flexible model
ξ_f^W	0.000000001	Calvo wages, flexible model
ξ_s^W	0.737	Calvo wages, sticky model
$\xi_{s,1}^P$	0.0498	Calvo prices, sticky model, sector 1
$\xi_{s,2}^P$	0.7408	Calvo prices, sticky model, sector 2
$\xi_{s,3}^P$	0.8465	Calvo prices, sticky model, sector 3
$\xi_{s,4}^P$	0.9201	Calvo prices, sticky model, sector 4
ξ_s^P	0.8	Calvo prices, sticky one-sector model
Ω_1	0.22	Economic weight sector 1
Ω_2	0.5	Economic weight sector 2
Ω_3	0.07	Economic weight sector 3
Ω_4	0.21	Economic weight sector 4

Table 3: *Parameter for the baseline model minus the ones affecting the shocks*

Erklärung zur Urheberschaft

Hiermit erkläre ich, dass ich die vorliegende Arbeit allein und nur unter Verwendung der aufgeführten Quellen und Hilfsmittel angefertigt habe.

Stephan Stahlschmidt

Berlin, den 31. August 2007