What is the dynamic impact of fickle investors?

A Master Thesis Presented
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Abstract

Over the past two decades, several emerging market economies have liberalized their financial markets. However, while the idea that capital market liberalization contributes to growth is widely accepted - and constitutes one of the main points of the Washington consensus - the volatility of capital inflows also increase financial fragility and cause short term costs (e.g., bank failures, market failure, currency crises, and hyperinflation). Indeed, after the Asian financial crisis in the late nineties, many academics argue that because emerging markets lack modern financial institutions, they are particularly vulnerable to the volatility of global financial markets and as this vulnerability is expected to be higher in countries with a more open capital account, some prudence and caution in the liberalization of the capital market is due in these countries. The current debate is exacerbated by the lack of clear empirical results which could support the position in favor either of unconditional capital liberalizations or of the more prudential point of view.

Motivated by the current discussion on the topic this master thesis focuses on the model by Scott and Uhlig (1999) on volatile investments and long run growth. The model analyzed finds some evidence that the volatility associated with foreign capital inflows can have relevant negative effects on growth. More specifically, as the amount of foreign investments increases, the growth effect, spurred by the increased amount of capital at disposal in the economy, is dampened by the increasing volatility associated with the overseas capital inflows. In the steady state Scott and Uhlig (1999) derive a mean-variance trade-off for the growth rate of the economy.

Aim of my thesis is to analyze the model in detail and in particular its dynamic behavior, and trying to verify its validity also off the steady state. A small approximation problem is analyzed concluding that the results of the model are not affected by it. Impulse - response analysis and simulations are performed. The results confirm the existence of a mean-variance trade-off also in the dynamics. The relevant parameter for the interpretation of the results is nonetheless not much the amount of capital flows alone but rather the ratio given by the amount of flows to their volatility. The political implications drawn by Scott and Uhlig (1999) can therefore be confirmed.

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1 Introduction

The recent wave of financial markets liberalization, followed by the surge in international capital flows in particular towards emerging market economies and the repeated occurrence of financial and currency crises in the last decade raised some doubts about the beneficial role of capital flows for developing countries and specifically about the so called Washington consensus. The widespread concept that capital inflows help an emerging market economy to increase investments and finally growth has been challenged by the constatation that the volatility international investments display can be a harmful and even disruptive factor for a country without a well established and solid domestic banking and financial sector. Several empirical studies on this topic have indeed found a negative relationship between capital account liberalization/ capital inflows and long run growth[1]. Nevertheless, the question is not settled yet: the methodology, the data, the countries, and the indeces used in the empirical studies vary widely and consequently the results are controversial as well.

In the wake of the Asian crisis the discussion focused on capital controls, regulation of the financial and banking sector in emerging markets, the role international organizations like the IMF should have in setting prudential rules on capital mobility. Eventually the introduction of a Tobin tax became a topic of discussion in the European Parliament[2]. Motivated by the recent discussion on volatile international investments this thesis will analyze the dynamic aspects of the Scott and Uhlig (1999) model on fickle investors, i.e. volatile investment flows, and long-run growth. The paper considered presents two models which explore the mechanism through which investments' volatility affects long-run growth presenting a possible explanation for a negative repercussion of volatility on growth. Of the two models I will extend the analysis of the first one to investigate its dynamic behavior and to study its sensitivity to different parameters and stochastic processes.

The model chosen does not explain the behavior of international investors but simply takes their volatility as given and relates different levels of inflows and investments' volatility with domestic growth levels. This implies that the model is quite

[1] See Chapter 2 for a more comprehensive review of the literature on this topic

[2] See Eichengreen, Tobin, and Wysplosz (1995) for an argumentation in favor of a Tobin tax

versatile and can be applied to different situations where the source of outside financing is volatile. Agénor and Aizeman (1998), Aizeman (1997), Chang and Velasco (2001), Levine and Zervos, also consider volatility exogenous and focus instead on the domestic capital market imperfections to explain negative effects of capital inflows on domestic growth. Moreover, the assumption of exoneity of investment inflows is plausible according to the empirical work of Dumas (1994), Dumas and Solnik (1995), Eichengreen and Rose (1998), and Frankel and Rose (1996). Their work suggests that factors like the level of interest rates in developed countries, the consideration of the portfolio risk or of the foreign exchange risk, play a more important role in the decisions of international investors. Nevertheless a bit of caution is due in interpreting these studies. Empirical studies on this topic could be misleading for there might be a feedback relation between growth and investor behavior not captured in empirical investigations. In general, countries with certain growth experiences, or at particular levels of development, may be more prone to liberalize their capital accounts, implying the potential for reverse causality. In fact, Aghion, Bacchetta and Banerjee (2000), Bacchetta and van Wincoop (1998), Boldrin and Levine (2001), Dooley (2000), and Meng and Velasco (1999) consider the endogeneity of investors' behavior as being a determinant part in the modeling of the relation between volatility and growth.

Given the exogeneity of investors' behavior, the model chosen is not necessarily an international financial crises model, it does not explain why crisis occur and consequently does not present a timing theory for these events nor an indication of what are the warning signals for such an event. It can nevertheless contribute to the analysis of potential negative correlations between volatility and growth. It focuses on the volatility of the private sector investments fitting the pattern of international capital flows in the last decade. In particular, the model fits the case of a small open economy where domestic investors can only buy national projects because of resources limitation, like in a developing country for example, and where at the same time there are large overseas investments which flow into the economy financing domestic projects[3]. This is a realistic assumption fitting the fact that Northern countries might be willing

[3]"Capital flows to Asian countries have been sometimes massive, sometimes exceeding 10 percent of GDP", Ito (1998).

to lend to emerging countries (and did indeed lend funds) as they are well capitalized and tend to have fewer investment opportunities with high rates of return, as most such opportunities are exploited as they emerge[4].

Another key assumption of the model is about imperfect domestic financial markets: domestic entrepreneurs can not diversify their portfolio and thus can not reduce the risk associated with the "fickleness". If they could, the whole model would not be able to present negative effects due to foreign investors. This assumption implies a domestic economy which is either rather small, has a financial system not well developed or disposes of a relatively low endowment of capital which again is a plausible hypothesis in the case of a developing country. As Aghion, Bacchetta and Banerjee (2000) suggest in their paper, "at very high levels of financial development most firms' investment is not constrained by cash flow so shocks to cash flow are irrelevant. On the other hand, at very low levels of financial development, firms cannot borrow very much in any case and therefore their response to cash-flows shocks will be rather muted - extra cash means more investments but only a little more."[5] In this case we shall also think of an intermediate level of financial development. Agénor and Aizeman (1998), Aizeman (1997) and Chang and Velasco (2001), Chinn and Kletzer (2000) explain a negative impact of capital liberalization on growth with imperfect domestic financial sectors while Levine and Zervos (1998), Rajan and Zingales (1998) and King and Levine (1993a,b) confirm empirically how important a good financial system is for growth. In contrast, Scott and Uhlig (1999) model assumes that capital markets are imperfect but does not bases it analysis on this. In other words the mechanism through which foreign investors negatively affect growth is not determined by information asymmetries or problems of agency in financial intermediation but by the rising risk premium associated with higher volatility and the relative discouraging effect on entrepreneurial activity.

As it emerges from these few remarks on the characteristics of the model, it is clear that its main objective is to determine the macroeconomic implications of fickle

[4]Hermalin and Rose, (1999) argue that large capital flows are motivated by the combination of the following factors: large differences in capital abundance, desire for systemic risk diversification, and an inefficient domestic financial structure

[5]Aghion, Bacchetta and Banerjee (2000), pg 4.

investors without considering the causes behind the volatility. The model is therefore necessarily a partial equilibrium one.

The model is a discrete time overlapping generation model where subjects live two periods and can choose if to become entrepreneurs and invest in new projects or to remain simple workforce when "old". The dynamics is given by the number of projects carried out and the growth effect these have. International capital flows contribute at the financing of projects inserting an element of volatility in the model. The steady state analysis shows a mean-variance trade-off with respect to the presence of fickle investors. More overseas capital means more capital available for investments and thus, following the conventional wisdom, higher growth. On the other side though, an increase of capital inflows is accompanied by higher volatility and a higher risk of a reversal of the flows; this in contrast has a negative impact on growth. Correlating growth with different grades of capital inflows and relative volatility (measured by the standard deviation) Uhlig and Scott (1999) obtain a hump shaped growth curve implying that after reaching a certain level of optimal investments, the economy cannot deal with the volatility associated with these inflows and growth declines. The model thus provides support for the introduction of capital controls or maybe of a Tobin-type tax.

Aim of my analysis is to verify these results in a dynamic framework. Does the dynamic analysis of this model have the same policy implication as in the steady state? Is the modeling of investment volatility represent a determinant part of the analysis? That is, do results change if capital inflows are modeled as a ARCH process or is the model able to replicate its conclusions independently from the shape volatility takes? The analysis will entail standard impulse-response analysis to verify the dynamic behavior of the single variables in response to shocks to the capital inflows and to the number of projects. The ultimate aim of my analysis is to derive some policy implications on how developing countries' governments can optimize the process of capital market integration avoiding or minimizing the risks arising from unstable capital flows.

The plan of the thesis is as follows. Chapter 2 presents some stylized facts and briefly discusses the literature on international capital mobility and financial/currency crises in emerging market economies. Chapter 3 presents the model in detail. Chapter 4 gives the results of impulse-response analysis and simulations. Chapter 5 sketches some policy implications. Chapter 6 summarizes and concludes. All the derivations and the MATLAB programs are contained in the Appendix.

2 Facts and Literature

Over the last two decades, several emerging economies have liberalized their financial markets eliminating various current and capital controls and restrictions. Some developing countries introduced economic reform including financial sectors. Others allowed direct investment into various industries. This capital liberalization process together with low interest rates in advanced countries prompted many investors to search high yield opportunities in emerging countries (Ito, 1998). As a result the nineties witnessed a surge of capital flows towards emerging market economies and a change in the nature of capital flows to emerging market economies. "Net private capital flows to emerging markets increased seven-fold from 1990 to 1996. In 1990, total capital flows to emerging markets (developing countries and transition economies) were about 30 billion dollars.[...] In 1993, the total amount of capital flows was up to 160 billion dollars [...]."[6]. Moreover, during the nineties, instead of direct lending to developing countries, portfolio flows and foreign direct investment (FDI) became the dominant source of capital inflows towards emerging market economies. Governments also began to rely more on issuing debt securities than on foreign commercial bank loans (Bacchetta & van Wincoop, 1998). Typically now, most commercial bank lending to emerging market economies goes directly to the private sector, often channeled through banks and other financial institutions.

The idea that capital market integration increases economic performance and brings long-term benefits by fostering financial development is conventional wisdom. Indeed, capital market liberalization is an important component of the Washington consensus. There are several channels through which external investors positively influence the domestic economy's growth rate. First, financial openness increases the

[6] Ito (1998), pg.4

supply of capital at the disposal of an economy thus allowing for more portfolio risk diversification and consumption smoothing through borrowing and lending (Mishkin, 1998). Second, countries with a more open capital account, in theory, have the ability to finance a larger current account deficit and thus increase the volume of foreign savings. If increases in foreign savings are not matched by a complete crowding out of domestic savings, aggregate savings will be higher allowing for higher investment and thus, faster growth (Edwards, 2001). Third, increased access to the domestic financial system by foreign banks raises the efficiency of the banking system in their intermediation function between borrowers and savers. This lowers the cost of investment (Mishkin, 1998). Finally, foreign direct investments[7] are believed to be less likely to reverse the direction of flown and contribute to the industrialization process of the host country increasing the productivity and the efficiency of the real sector through technology, foreign management, know how, and intangible assets transfers (Edison, Klein, Ricci & Sløk, 2002).[8]

In support of these ideas, there is now a burgeoning theoretical and empirical literature that assesses the positive impact of capital market liberalization and growth. For instance, King and Levine (1993a,b), Levine and Zervos (1996), Rajan and Zingales (1997), Bekeart and Harvey (1998, 2000), Obstfeld (1996), and Ito (1998) all find a positive relation between the development of financial market integration and economic growth. Bartolini and Drazen (1997) describe free capital mobility as a possible signal for the government to enhance the credibility of a broader reform plan and assign it a positive effect. Frenkel, Nickel, Schmidt, and Stadtman (2001) suggest that capital controls exert negative effects on growth because they induce a dampening effect on investment activity. They nevertheless add to this conclusion the remark that opening capital markets can also involve some negative effects due to increased exchange rate volatility. Quinn's (1997) empirical estimates suggest that the change in capital account liberalization has a strongly significant effect on the growth in real

[7]Direct investment is usually defined as a purchase of more than 10 percent in equities of a particular company. In comparison with bank credit/deposits or bonds, they are considered to be more difficult and costly to be withdrawn. See Lipsey (2001) for an analysis of Foreign Direct Investments in the last three financial crisis.

[8]see also Ito (1998) for a regression of Asian countries' growth rates on FDI flows into the region.

GDP per capita. His results are nevertheless a bit unclear as he does not distinguish between financial openness and a broader measure of openness. Bekeart, Harvey, and Lundblad (2001) examine the impact of stock market liberalization on economic growth and find that financial liberalization leads to a 1 percent increase in annual per capita GDP. Barro (2001) fails to find a persisting adverse influence of currency and banking crises on economic growth.

There is then a range of studies which detect a positive correlation between capital account liberalization and growth but evidenciate that these positive effects vary considerably among countries (Chanda, 2001, and Edison, Klein, Ricci & Sløk, 2002) suggesting that there might be different factors influencing the macroeconomic impact of international capital flows[9]. Edwards (2001) finds that the positive relationship between capital account openness and productivity performance only manifests itself after the country in question has reached a certain degree of development. Klein and Olivei (1999) find a positive effect of capital liberalization on growth among industrial countries, but they do not find evidence that capital account liberalization promotes growth in non-industrial countries.

In contrast, the incidence of financial crisis in Asia and Russia, and the repeated recurrence of financial crises in Latin America supports the notion that financial openness increases financial fragility and the related risk of incurring in a currency and/or financial crisis with relative output losses, especially in developing countries. Some academics raised the question of the optimal quantity of capital flows arguing that too high level of overseas investments can actually hamper growth.

International financial interactions magnify the importance of two pervasive domestic financial markets phenomena: asymmetric information and enforcement risk. To the extent that capital account liberalization takes place in a country with a weak banking system and a poor prudential supervision system, capital inflows can incentivate excessive risk-taking activities undermining the solidity of the whole fi-

[9]Interesting Chanda (2001) suggests that the impact of foreign investments may vary with the level of ethnic and linguistic heterogeneity in the society considered a proxy for the number of interest groups. In general other researchers point more on financial and banking systems' efficiency as well as legal systems and level of industrialization to explain differences in growth due to capital inflows. On this more common position see Rossi (1999)

nancial system (Kraay, 1998). Beyond these microeconomic issues of information and enforcement, international "[...] capital flows are associated with two additional macroeconomic risks that are essentially absent in the domestic context. The first is sovereign risk; governments can choose to default on their international context. The second is the risk that international capital flows create macroeconomic instability through monetary spillovers."[10] The advantages and positive aspects of capital inflows are completely lost if the quantity of foreign investments becomes larger than the current account deficits, putting appreciation pressure on the currency in an environment where the emerging country is trying to keep a stable exchange rate to the dollar (Kahn and Reinhard, 1995). Without wanting to provide an explanation for currency crises and adventure into the vast literature on the topic, the main lesson drawn from the last financial crises is that too large capital inflows contribute to increase a developing economy's fragility. This is due to a well-known economic principle, the Mundell's celebrated "Incompatible Trinity": a small open economy cannot have free capital flow, a fixed exchange rate, and independent monetary policy at the same time.

As net capital flows into a country, domestic international reserves grow and correspondingly also the country's money supply is going to grow. The result is that monetary authorities lose the control over an important determinant of macroeconomic stability, i.e. money supply. Loosening of monetary policy can fuel inflation, bubbles in asset prices, especially stock, bond, and real estate prices[11]. Intervention of the authorities to sterilize reserves movements might actually lead to an increase of capital flows due to increases of the domestic interest rate. If the country chose to fix its exchange rate, it gives up its ability to conduct monetary policy for purely domestic reasons and as a result it bears higher risks of business-cycle fluctuations (Feldstein, 2002). In sum, the microeconomic benefits of capital inflows listed at the beginning of the Chapter come at the cost of the increased risk of domestic fluctuations.

Regarding this issue, i.e. channels through which capital inflows can hamper growth, I will refer to the main characteristics of the model in the next chapter.

[10]Hermalin, B.E. and Rose, A. (1999), pg.1

[11]Hermalin and Rose, (1999), and Ito, (1998)

The literature presented on the interaction between capital mobility and growth is just meant to provide a framework, or better, an overview of the current discussion in order to be able to place the model analyzed. Scott and Uhlig (1999) neglect the negative aspects of capital mobility presented above (in particular they do not consider at all the tensions between capital flows and policy authorities) and focus on the effects investments, and in particular their volatility, have on the entrepreneurs' decisions. The risk represented by the productivity of the projects financed with foreign resources and the rate of return on the funds invested in consideration of the obligation to repay the international borrowing is also not considered in the model. As explained in the next chapter, in the model discussed here, overseas investments are not allowed to finance consumption, excluding in this way another possible way of increasing domestic fragility. The mechanism through which outside investors adversely affect the Scott and Uhlig (1999) economy is given by the capital inflows' volatility and the increase in risk premium this requires.

The literature considering negative correlation between capital account liberalization and growth is (like in the case of a positive correlation) quite abundant. Meng and Velasco (1999) provide a theoretical model explaining why de-regulating the capital account may be destabilizing. In particular, foreign capital is believed to harm emerging market economies through its volatility. Aizenman (1995) and Agénor and Aizenman (1998) outline models where opening the economy to unrestricted inflows of capital may lead to a welfare reduction. Aghion, Bacchetta and Banerjee (2000) analyze the role of financial factors for small open economies and deduce from their model that full capital account liberalization can destabilize economies which are at an intermediate level of financial development.

Empirically, Easterly (2001) finds that the growth rate of countries adopting the Washington consensus was zero per cent during 1980-98. Likewise, Krugman (1993) is sceptical about the benefits of capital market liberalization while Mathieson and Rojaz-Suarez (1992) suggest that open capital account may actually undermine structural reform programs. Grilli and Milesi-Ferretti (1995) find no support for the hypothesis that capital account liberalization promotes growth. Finally, Rodrik (1998), and Kraay (1998) find an insignificant relationship between capital market liberalization and growth. Chari and Henry (2002) can not find a supportive evidence for an increase of allocative efficiency following capital liberalization. Indeed, Ramey and Ramey (1995) document a negative relationship between growth and volatility. In general there has been an increasing skepticism about the positive effects of foreign capital inflows.

The lack of a clear result from these studies is due partly to the complexity associated with measuring the actual amount of capital flows versus what is legally declared as the amount of capital mobility by countries (Edwards, 2001) and to the variety of indexes used for measuring the degree of openness of a country's capital market[12]. Nevertheless, a wider theoretical support may help empirical studies providing clearer hypotheses to test.

This chapter did avoid a review of the literature about financial crises and investors behavior as it might be misleading. For models on rational investors behavior I limit myself to refer to Agénor, Bhandari and Flood (1992) for a review of the first generation models pioneered by Krugman (1979) which interpret speculative attacks as the natural and anticipated demise of an inconsistent policy regime. The second generation models a la Obstfeld (1996), reviewed by Eichengreen, Rose and Wyplosz (1996), explain speculative attacks in terms of the fundamentals identified in the first generation models, but the fundamentals are themselves sensitive to shifts in private expectations about the future.

3 The Model

Scott ad Uhlig (1999) present an overlapping generations model in discrete time where agents live two periods. In the first period of their lives, agents work providing one unit of labor and save their entire wage earning. At the end of this period they decide if they want to remain workers or to become entrepreneurs and start new projects.

If they remain workers they supply ν efficiency units of labor. If $\nu < 1$ the old workers are less productive than the young ones and if $\nu > 1$ the contrary is true and

[12]For a review of the indeces used in the empirical literature see Edison , Klein, Ricci, & Sløk, 2002

the old workers enjoy higher productivity due to accrued experience. Old and young workers are perfect substitutes.

The share of population which become entrepreneurs in period t is given by the fraction $0 \leq e_t \leq 1$.

Therefore in every period t, the total amount of efficiency units of labor n_t is given by the young workers (i.e. $= 1$) plus the old workers:

$$n_t = 1 + \nu(1 - e_{t-1})$$

The key choice variable that determines long run growth is the fraction e_t: this part of the population improves overall productivity γ_t by the parameter ψ.

$$\gamma_{t+1} = \gamma_t(1 + \psi e_t) \tag{1}$$

The economy grows due to the new ideas or new technologies, represented by the parameter ψ, introduced by the new entrepreneurs:

$$\frac{\gamma_{t+1}}{\gamma_t} = 1 + \psi e_t$$

The total number of projects in operation at time t, q_t is given by

$$q_t = (1 - \delta)q_{t-1} + e_{t-1}$$

where δ denotes the depreciation rate of the projects.

The output of each project i is:

$$y_{t,i} = \gamma_t n_{t,i}^{\alpha}$$

The only production costs of the projects are labor costs. Labor is paid a wage w_t per efficiency unit. In every period t each project maximizes dividends - output minus costs - hiring the optimal amount of $n_{t,i}$ units of labor.

$$d_{t,i} = \max_{n_{t,i}} \gamma_t n_{t,i}^{\alpha} - w_t n_{t,i}$$

Total output is given by the continuum of projects present in the economy at time t:

$$y_t = \int_0^{q_t} y_{t,i} \mathbf{d}i$$

Assuming that all the firms are identical, the aggregate production function is given by a usual Cobb-Douglas with constant returns to scale

$$y_t = \gamma_t q_t^{1-\alpha} n_t^{\alpha}$$

where the factors are paid their marginal product

$$w_t n_t = \alpha y_t$$

$$d_t q_t = (1-\alpha) y_t$$

As already mentioned in chapter 2 the subjects in this model can only buy domestic projects and, in the second period, are not supposed to save either. The international dimension is given by the fact that projects are bought partly also from outside investors. Therefore, Scott and Uhlig (1999) assume imperfect international financial markets which is highly plausible in the light of the stylized facts of the last financial crises[13]. The model could therefore represent a developing country with a poorly developed financial system or with a shortage of national capital which receives relatively large capital inflows from industrialized countries. As already seen these flows can be motivated by low interest rate in Northern countries, the desire of industrialized countries to diversify their portfolio or imperfect domestic financial markets (Hermalin & Rose, 1999).

The valuation of the projects is given by the price ex-dividend per project p_t times the number of projects which are taking place in time t, q_t.

$z_t w_t$ are the total resources invested in projects such that

$$z_t w_t = p_t q_t$$

where $(z_t - 1)w_t$ are the overseas funds. It is assumed $z_t \in (0, \infty)$ where z_t is random but stationary and reflects the impact of volatile investors. If there are no

[13]See Mishkin (1998) for a review of the sequence of events in the Mexican and East Asia crises.

international investors then $z_t \equiv 1$ while if the amount of funds provided by overseas inflows is equal the domestic investments $z_t = 2$. Given the assumption made about the domestic financial market the case $z_t < 1$, where investors as a group are selling short, is not considered.

In my first set of simulations I will model z_t as a simple autoregressive process, and will then compare the results with simulation performed using a ARCH-process to describe z_t. Being z_t a financial variable the second option might seem more appropriate . Nevertheless given the nonlinearities of the model in itself it might be the case that the shape of z_t does not play a big role in determining the results.

As usual the return earned in time $t+1$, R_{t+1} is given by the dividends plus the change in the assets' price from time t to time $t+1$, obviously net of the fraction δ of dead projects.

$$R_{t+1} = (1-\delta)\frac{d_{t+1} + p_{t+1}}{p_t}$$

Agents decide whether to become entrepreneurs or to remain workers on the basis of the expected consumption $E_t[c^{(e)}_{t+1}]$ or $E_t[c^{(w)}_{t+1}]$. $c^{(e)}_{t+1}$ is the consumption of the entrepreneurs and is given by the return on the wage earnings gained in time t and subsequently invested, the dividends and the price of the projects at the end of the second period.

$$c^{(e)}_{t+1} = R_{t+1}w_t + d_{t+1} + p_{t+1}$$

From the definition of the return earned in $t+1$ we know that $d_{t+1} + p_{t+1} = p_t \frac{R_{t+1}}{(1-\delta)}$, therefore the entrepreneurs consumption can be rewritten as

$$c^{(e)}_{t+1} = R_{t+1}\Big(w_t + \frac{p_t}{1-\delta}\Big)$$

The experienced worker consumption is given just by return on the wage in time t and the wage in the next period $t+1$, times their efficiency units provided

$$c^{(w)}_{t+i} = R_{t+1}w_t + \nu w_{t+1}$$

where w_{t+1} is known at time t.

The arbitrage condition necessary for the agents to be indifferent between becoming entrepreneurs or supplying ν units of efficiency labor as experienced workers is defined by the equivalence of the expected respective utilities $u(c)$.

$$E_t[u(c_{t+1}^{(e)})] \equiv E_t[u(c_{t+1}^{(w)})] \tag{2}$$

As entrepreneurs take a higher risk than workers they need an incentive, a reward to do so. This is given by a positive risk premium π_t[14] times the wage of next period w_{t+1}. The reward for being an entrepreneur is therefore proportional to w_{t+1} which is considered a measure of wealth in the second period.

$$E_t[c_{t+1}^{(e)}] \equiv E_t[c_{t+1}^{(w)}] + \pi_t w_{t+1} \tag{3}$$

Assumption 1

$$\frac{1}{\alpha} + \frac{1}{\delta} + \frac{\pi_t}{\delta\nu} > 1$$

The assumption is not so stringent: as $0 \leq \alpha \leq 1$ and $0 \leq \delta \leq 1$ the first two terms are clearly bigger than 1. For the last term to be bigger than 1 it is necessary that the risk premium is bigger than the product of the fraction of dying projects and the productivity of the older workers. In other words the risk premium should be big enough to counterbalance the disincentive to become entrepreneur given by an increase in productivity of experienced workers and a high fraction of unsuccessful projects. Given the first two terms, the condition of the last term is clearly not so stringent. It will become clearer later, looking at the expression for $\bar{e}$, that this assumption is necessary in order to have a positive share of subjects becoming entrepreneur ($e_t > 0$)

The dynamics of this model are determined by the equations defining the number of projects occurring in period $t+1$, q_{t+1} and the share of entrepreneurs e_t:

$$q_{t+1} = \frac{1}{\nu + \alpha\pi_t}\Big((1-\alpha)(1+\nu) + \alpha E_t[z_{t+1}] + \nu(1-\delta)(1-\alpha)q_t\Big) \tag{4}$$

[14]The derivation of the positivity of the risk premium is shown in the Appendix.

$$e_t = \frac{1}{\nu + \alpha\pi_t}\Big((1-\alpha)(1+\nu) + \alpha E_t[z_{t+1}] - \alpha(\nu+\pi_t)(1-\delta)q_t\Big) \tag{5}$$

The dynamics of the model's endogenous state variable q is determined by a first-order difference equation given a process for π_t. The autoregressive coefficient for q_t is $\frac{\nu(1-\alpha)(1-\delta)}{\nu+\pi_t\alpha}$, implying that the number of projects will decline if the fraction of surviving projects $(1-\delta)$ and the profitability of projects $(1-\alpha)$ are decreasing and if the risk premium π_t increases. In equation (5), the share of entrepreneurs in time t depends on the existing projects q_t and on the expected capital inflows in next period $E_t[z_{t+1}]$, given a process for π_t. The entrepreneurial decision depends therefore on the expectations that agentshave about the price of projects in the following period. A further analysis of the dynamics of the model is done in the Section 4.

3.1 The steady state

Scott and Uhlig (1999) make the simplifying assumption that in the steady state the entrepreneurial risk premium $\bar{\pi}$ is a constant independent of the state of the economy or its parameters. All the other variables are calculated as usual in the steady state, i.e. $e_t = e_{t+1} = \bar{e}$. Considering that in the steady state $\bar{q} = \frac{\bar{e}}{\delta}$, eq.(4) leads to the following share of entrepreneurs in the economy:

$$\bar{e} = \frac{1+\nu}{\nu}\frac{\frac{1}{\alpha} + \frac{1}{1+\nu}E[z] - 1}{\frac{1}{\alpha} + \frac{1}{\delta} + \frac{\bar{\pi}}{\delta\nu} - 1} \tag{6}$$

The steady state growth is determined by the share of subjects in the economy which choose to invest in innovative activities becoming entrepreneur:

$$\bar{g} = \frac{y_{t+1}}{y_t} = 1 + \psi\bar{e} \tag{7}$$

In eq.(6) the meaning of assumption 1 becomes clear: it needs to hold to have a fraction of population becoming entrepreneur. If the risk premium is too small respect to the experience premium ν agents will just prefer not to get involved with a riskier activity and will remain workers. If nevertheless the risk premium is big enough an increase in the fraction of dying projects leads to a decrease in the number of projects and therefore to an increase of the return on them and an increase in entrepreneurs.

Eq.(6) also reflects the fact that if expected overseas investments ($E[z]$) increase, putting more capital at disposal of the economy and raising the price of projects, the subjects in the economy will have a higher incentive to invest in risky activities. Further, if the share of labor α increases, the incentive to become entrepreneur decreases unless the outside investments are big enough to counterbalance this effect and lead actually to an increase in e and thus in growth.

Therefore, Scott and Uhlig (1999) identify in the share of entrepreneurs the main channel for their "mean effect", i.e. the larger the amount of foreign investments, the larger the share of entrepreneurs, and thus the steeper the growth path of the economy. This effect is opposite to the "variance" effect, given by the volatility foreign investments can display, measured by σ_z^2.

3.2 The variance effect

Given a constant relative risk aversion utility function in the form

$$u(c) = \frac{c^{1-\eta} - 1}{1 - \eta}$$

where η is the marginal risk aversion of the subjects, Scott and Uhlig (1999) derive an expression for the risk premium π_t such that

$$\pi_t = \frac{\eta}{2} \frac{\sigma_{t,c^e}^2 - \sigma_{t,c^w}^2}{w_{t+i} E_t[c_{t+1}^w]} \tag{8}$$

where σ_{t,c^e}^2 and σ_{t,c^w}^2 are the variances of respectively $c_{t+1}^{(e)}$ and $c_{t+1}^{(w)}$ conditional on information up to date t. In the Appendix it is shown that the condition for the risk premium to be positive ($\sigma_{t,c^e}^2 > \sigma_{t,c^w}^2$) is satisfied and that π_t is bounded. Eq.(8) can be expressed as

$$\pi_t = \eta \frac{(1-\delta)(\frac{q_t}{z_t}) + 0.5}{\Big((\frac{q_t}{z_t})(\frac{1-\alpha}{\alpha}(1 + \nu(1 - e_t)) + E_t[z_{t+1}]) + \nu q_{t+1}\Big) q_{t+1} (1-\delta)^2} \sigma_{t,z}^2 \tag{9}$$

The variance effect influences the economy through its effect on the risk premium π_t as can be seen from equation (9). For higher levels of volatility of foreign investments $\sigma_{t,z}^2$, a higher reward for being entrepreneur is requested and therefore a smaller proportion of population (see eq.(5)) will invest in new projects and growth will be lower[15]. The same is true if the relative risk aversion η increases: a more risk averse subject will need a higher reward for not choosing the safe and quiet life of an experienced worker. Eq.(9) is an implicit equation in π_t as the risk premium depends on the number of projects q_{t+1} and on the share of entrepreneurs in the economy e_t and these in turn depend on the risk premium. Following Scott and Uhlig (1999) it is possible to combine the three eqs.(9),(5) and (4) to obtain an explicit quadric expression of π_t dependent only on the endogenous state variable q_t and on the exogenous state variables z_t, $E_t[z_{t+1}]$ and $\sigma^2_{t,z}$. As a negative risk premium is not economically meaningful, the system of equations can actually be solved picking always the positive solution[16].

3.3 Results in steady state

Summarizing the model, there is a mean effect which increases growth and a variance effect which acts in the opposite direction, this is the *mean-variance trade-off* result in Scott and Uhlig (1999). The variance effect is given by the variance of z_t, $\sigma^2_{t,z}$. The greater the variance of foreign investment, the more risk-averse agents will be scared away from entrepreneurship into the safer dependent employment. z_t represents therefore a random variable. To be able to better represent this variable and model the mean-variance trade-off, Scott and Uhlig propose a transformation of it. Knowing that the analysis focuses on the case where $z > 1$, z can be considered as a scaled version of a random variable X.

$$z - 1 = \lambda X, \text{ where } E[X] = 1 \text{ and } Var[X] = \xi^2$$

therefore $E[z] = \lambda + 1$, $Var[z] = \lambda^2 \xi^2$ and$\sigma_z = \lambda\xi$. Thank to this transformation Scott and Uhlig are able to illustrate the mean and the variance effect depending on the

[15] Like for σ^2_{t,c^e} and σ^2_{t,c^w}, also $\sigma^2_{t,z}$ is defined as the variance of z_{t+1}, condition on information up to and including date t

[16] Combining eqs.(9), (5) and (4) it also possible to obtain a third-degree polynomial expression for $\bar{q}$. This is shown in the Appendix A. Nevertheless this equation is not used to derive the steady state results as the authors prefer to reiterate eq.(9) in steady state.

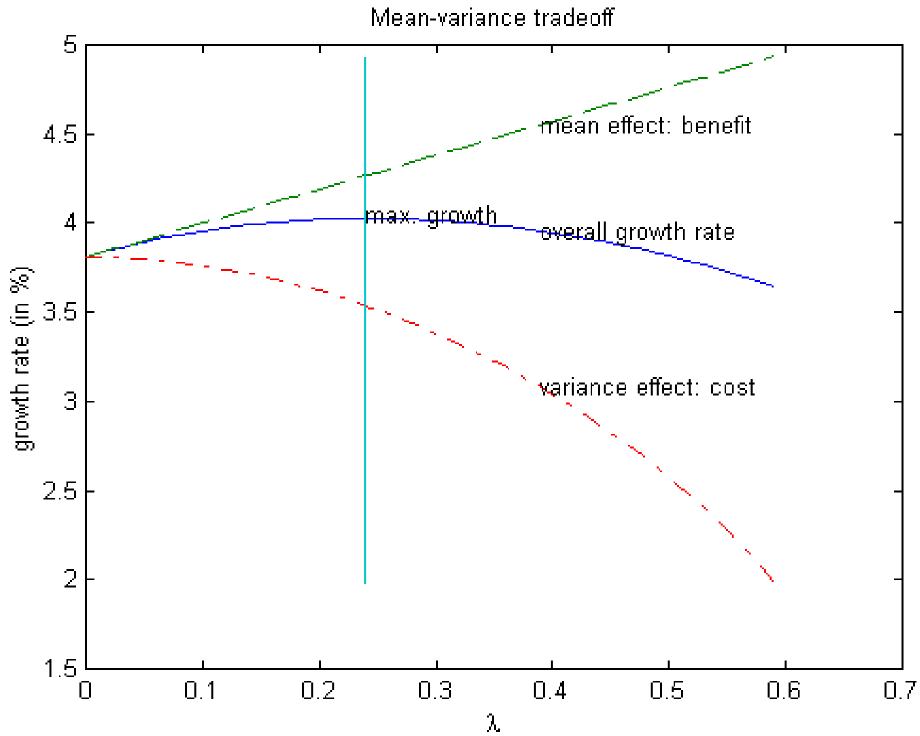


Figure 1: THE MEAN-VARIANCE TRADE-OFF

amount of foreign capital inflows $\lambda = z - 1$[17]. Alternatively, λ could be interpreted as a measure of the capital market openness. The growth effect of an increase in outside financing $z-1$ is therefore a linear increasing function of λ, whereas the costs associated with an increase of $z - 1$ are an increasing quadratic function of λ. Given increasing levels of λ, the growth function of the small economy described by the model is therefore hump-shaped illustrated in Fig.(1)[18].

Fig.(1) has been obtained for a "baseline" parameterization, using $\nu = 1$, $\alpha = 2/3$, $\delta = 0.1$, $\psi = 0.2$ and $\eta = 5$, assuming $\xi = 1$ and solving for the entrepreneurial risk premium at the steady state. Clearly for the mean effect benefit curve, one has $\sigma_z = 0$, while for the variance effect curve $\bar{z} = 1$, the overall growth rate curve combines both effects. To better illustrate this tradeoff, I report in the following the numerical results provided in Scott and Uhlig (1999). In Table (1) the inflows levels $\bar{z}$ and the volatility levels σ_z are varied, while in Table (2), the effects of the relative risk aversion η as well

[17]As the expected value of X is equal to 1, $\bar{z} - 1 = \lambda$.

[18]The graph is obtained from Scott and Uhlig (1999)

	$\bar{z} = 1.00$		$\bar{z} = 1.10$		$\bar{z} = 1.20$	
	$\bar{\gamma}$	$\bar{e}$	$\bar{\gamma}$	$\bar{e}$	$\bar{\gamma}$	$\bar{e}$
	π	σ_R	π	σ_R	π	σ_R
$\sigma_z = 0.00$	3.81	19.05	4.00	20.00	4.19	20.95
	0.00	0.00	0.00	0.00	0.00	0.00
$\sigma_z = 0.10$	3.76	18.81	3.96	19.78	4.15	20.74
	1.31	10.38	1.18	9.45	1.07	8.68
$\sigma_z = 0.20$	3.62	18.10	3.82	19.11	4.02	20.10
	5.48	20.72	4.92	18.88	4.44	17.34
$\sigma_z = 0.30$	3.38	16.91	3.59	17.97	3.81	19.03
	13.29	31.01	11.83	28.25	10.61	25.95
$\sigma_z = 0.40$	3.04	15.19	3.27	16.36	3.50	17.50
	26.64	41.22	23.38	37.55	20.72	34.50

Note: all the numbers are in percent.

Table 1: MEAN-VARIANCE EFFECT IN THE STEADY STATE: VARYING $\bar{z}$ AND σ_z

as of the volatility-to-mean ratio ξ[19] are verified. Table (1) illustrates the trade-off quite clearly: for any given $\bar{z}$ an increase of σ_z leads to a decrease in the growth rate $\bar{\gamma}$ and of the share of entrepreneurs $\bar{e}$ and correspondingly to an increase of the risk premium π and of the volatility of return σ_R. Whereas for any given level of volatility σ_z, an increase in external funds $\bar{z}$ leads to increasing growth rates $\bar{\gamma}$ and share of entrepreneurs $\bar{e}$ and a decrease of the risk premium π and of the return volatility σ_R.

Table (2) investigates the issue of the optimal size of foreign investments. Growth $\bar{\gamma}$, entrepreneurs share $\bar{e}$, risk premium π, and the return volatility σ_R decrease for increasing volatility-to-mean ratio ξ, given a certain level of marginal risk aversion η. [20]. Similarly, growth $\bar{\gamma}$, entrepreneurs share $\bar{e}$, risk premium π, and the return volatility σ_R decrease for increasing marginal risk aversion η, given a certain level of volatility-

[19]As $\sigma_z = \xi(\bar{z} - 1)$, ξ can be interpreted as the volatility-to-mean ratio: $\xi = \frac{\sigma_z}{\bar{z}-1}$

[20]π and σ_R decrease as well because even if the volatility-to-mean ratio ξ increases, in absolute terms σ_z is decreasing. As seen in Table (1) this leads to a decrease of both the variables.

	$\eta = 1.00$		$\eta = 2.00$		$\eta = 3.00$	
	$\bar{z}$	σ_z	$\bar{z}$	σ_z	$\bar{z}$	σ_z
	$\bar{\gamma}$	$\bar{e}$	$\bar{\gamma}$	$\bar{e}$	$\bar{\gamma}$	$\bar{e}$
	π	σ_R	π	σ_R	π	σ_R
$\xi = 1.00$	4.21	3.21	1.83	0.83	1.46	0.46
	6.02	30.10	4.48	22.41	4.20	21.02
	68.26	80.87	21.35	47.42	12.02	32.83
$\xi = 2.00$	1.32	0.63	1.14	0.28	1.09	0.18
	4.09	20.45	3.94	19.69	3.89	19.47
	8.32	50.11	3.71	25.69	2.35	17.16
$\xi = 5.00$	1.04	0.21	1.02	0.11	1.01	0.07
	3.85	19.24	3.83	19.15	3.82	19.11
	1.11	20.93	0.56	10.68	0.38	7.17

Note: all the numbers are in percent.

Table 2: Mean-variance effect in the steady state: varying η and ξ

to-mean ratio ξ. From these results, it is clear that even for small volatility-to-mean ratios ξ and marginal risk aversion η, the volatility effect can be quite relevant.

3.4 A question of approximation

Eq.(8) has been obtained by approximating the expected utility of the two subjects with a second order Taylor expansion and then compare them. The Taylor expansion is a method of local approximation of a function $f(x)$ around a point x_0[21] The

[21] I briefly recall here the formula to facilitate the following analysis.

$$f(x) \approx f(x_0) + f'(x_0)(x - x_0) + f''(x_0)\frac{(x - x_0)^2}{2!} + ... + f^n(x_0)\frac{(x - x_0)^n}{n!}$$

See Judd (1999) for more details.

approximation for $E_t\Big[\frac{(c^e_{t+1})^{1-\eta}-1}{1-\eta}\Big]$ around the point $E_t[c^w_{t+1}]$ is

$$E_t\Big[\frac{(c^e_{t+1})^{1-\eta}-1}{1-\eta}\Big] = \frac{(E_t[c^w_{t+1}])^{1-\eta}-1}{1-\eta}+\pi_t w_{t+1}(E_t[c^w_{t+1}])^{-\eta}-\frac{\eta}{2}(E_t[c^w_{t+1}])^{-\eta-1}(\sigma^2_{t,c^e}+\pi_t^2 w^2_{t+1}) \quad (10)$$

Scott and Uhlig (1999) drop the term $\pi_t^2 w^2_{t+1}$, most probably in consideration of the fact that π_t is a small value, always smaller than 1, and that, after all, it is about approximation. Without dropping this term eq.(8) will obviously be quadratic and more precisely of the form[22]

$$\tilde{\pi}_t^2 - \Big(\frac{2}{\eta}\frac{E_t[c^w_{t+1}]}{w_{t+1}}\Big)\tilde{\pi}_t + \frac{(\sigma^2_{t,c^e}-\sigma^2_{t,c^w})}{w^2_{t+1}} \quad (11)$$

Substituting out for $E_t[c^w_{t+1}]$ and for σ^2_{t,c^e} and σ^2_{t,c^w} and crunching a bit on it delivers

$$\tilde{\pi}_t^2-\Big(\frac{2}{\eta}\Big[\Big(\frac{(1-\alpha)}{\alpha}\frac{1+\nu(1-e_t)}{q_{t+1}}+\frac{E_t[z_{t+1}]}{q_{t+1}}\Big)\frac{q_t}{z_t}+\nu\Big]\Big)\tilde{\pi}_t+\frac{\sigma^2_z}{q^2_{t+1}}\Big(\frac{1+2\frac{q_t}{z_t}(1-\delta)}{(1-\delta)^2}\Big)=0 \quad (12)$$

This is again, as eq.(9) an implicit equation in $\tilde{\pi}_t$ as q_{t+1} and e_t depend on their turn on $\tilde{\pi}_t$, substituting eqs.(5) and (4) in this equation yields a huge second degree polynomial in $\tilde{\pi}_t$. Before trying to do that and be confronted with this mathematical challenge, I want to verify if using the values for q_{t+1}, q_t and e_t derived by the Scott and Uhlig (1999) formula, I can find values for $\tilde{\pi}_t$ close to π_t and therefore justify the neglect ion of the term $\pi_t^2 w^2_{t+1}$ in eq(10). The results of this exercise for different values of λ and keeping z_t constant over time, are shown in table(3).

$\tilde{\pi}_{t,2}$ can clearly be neglected as it is not economically meaningful; considering the number in percentage, a risk premium of over 100% would clearly prevent any subject from becoming entrepreneur. $\tilde{\pi}_t$ on the contrary is relatively close to π_t for low levels of λ.

On the basis of these results, the approximation procedure of Scott and Uhlig (1999) can be justified for low values of λ, it becomes more imprecise for higher levels of λ.

Not satisfied yet, I try to derive an explicit second order polynomial in $\tilde{\pi}_t$, substituting for q_{t+1} and e_t. The polynomial can be written in the characteristic form

$$A_t\tilde{\pi}_t^2 + B_t\tilde{\pi}_t + C_t = 0$$

[22] For the derivations, see the Appendix.

λ	$\tilde{\pi}_{t,1}$	$\tilde{\pi}_{t,2}$	π_t
0.1	0.0118	1.1155	0.0118
0.2	0.0442	1.0542	0.0435
0.3	0.0954	0.9786	0.0916
0.4	0.1683	0.8847	0.1542
0.5	0.1683	0.8847	0.2305
0.59	0.2771	0.7578	0.2988

Table 3: COMPARISON OF RESULTS $\tilde{\pi}_{t,1/2}$ AND π_t

where

$$A_t = 1 + \frac{\alpha^2\sigma_z^2 N_t}{D_t^2} - \frac{2}{\eta}\frac{(1-\alpha)}{D_t}\frac{q_t}{z_t}\Big((1+\nu) + \alpha\nu(1-\alpha)q_t + \frac{\alpha}{(1-\alpha)}E_t[z_{t+1}]\Big)$$

$$B_t = \frac{2\nu\alpha\sigma_z^2 N_t}{D_t^2} - \frac{2}{\eta}\nu - \frac{2}{\eta}\frac{\nu}{D_t}\frac{q_t}{z_t}(1-\alpha)\Big(\frac{(1+\nu)}{\alpha} - \frac{(1-\alpha)}{\alpha}(1+\nu) + \nu(1-\delta)q_t + \alpha E_t[z_{t+1}]\Big)$$

$$C_t = \frac{\nu^2\sigma_z^2 N_t}{D_t^2}$$

and

$$N_t = \Big(\frac{1 + 2\frac{q_t}{z_t}(1-\delta)}{(1-\delta)^2}\Big)$$

$$D_t = (1+\nu)(1-\alpha) + \nu(1-\alpha)(1-\delta)q_t + \alpha E_t[z_{t+1}]$$

Luckily, all the efforts are rewarded by a nice result: using the same procedure of Scott and Uhlig (1999) and reiterating in the steady state on this formula to obtain the solutions, I obtain results quite close to the original ones for $\tilde{\bar{\pi}}_1$ as shown in Table(4). The choice between the two roots is not so clearcut for high values of λ[23]. The values of the two solutions seem to get closer.

In conclusion, dropping the term $\pi_t^2 w_{t+1}^2$ saves actually a lot of troubles and on the basis of the results presented in Tables(3) and (4) can to a certain extent be justified. As it was not aim of this thesis to explore the vast field of approximation technics and the missing term was discovered quite late in the compiling of the thesis, the question

[23]In Table(4) as well as in Table(3) a higher λ means a higher $\bar{z}$ and a correspondingly higher σ_z.

λ	$\tilde{\bar{\pi}}_1$	$\tilde{\bar{\pi}}_2$	$\bar{\pi}$
0.1	0.0120	1.0959	0.0118
0.2	0.0469	0.9864	0.0444
0.3	0.1102	0.8291	0.0957
0.4	0.2390	0.7999	0.1666
0.5	0.4360	0.4350	0.2605
0.59	0.3739	0.3742	0.3572

Table 4: COMPARISON OF RESULTS $\tilde{\bar{\pi}}_{1/2}$ AND $\bar{\pi}$

has not been investigated any further and the dynamic analysis has been carried out on the Scott and Uhlig (1999) model[24].

4 THE DYNAMICS

The dynamics of the model are determined by the three equations (5),(4) and (9). The endogenous state variable is q_t, while e_t and π_t are the other endogenous variables and z_t and $\sigma_{t,z}$ are the exogenous variables which describe the impact of foreign investors. The implicit equation for π_t (9) can be made explicit plugging eqs.(5) and (4) into it and obtaining a quadratic polynomial dependent only on the endogenous state variable q_t and the exogenous variables z_{t+1} and $\sigma^2_{t,z}$ in the form

$$A_t\pi_t^2 + B_t\pi_t + C_t = 0$$

For values of $\sigma^2_{t,z}$ not too large this polynomial has just one economically meaningful solution. A negative risk premium is indeed not acceptable as a reward for the choice of a higher risk. The solution will thus always be

$$\pi_t^{(1,2)} = \frac{-1}{2A_t}(B_t - \sqrt{B_t^2 - 4A_tC_t})$$

[24]As approximation technics to overcome this problem one can think of numerical integration methods as presented in Judd (1999), chapter 7.

Therefore the dynamics of the model can be summarized as a system of three equations

$$q_{t+1} = f(q_t, \pi_t, z_{t+1})$$
$$\pi_t = f(q_t, z_{t+1}, \sigma^2_{t,z})$$
$$e_t = f(q_t, \pi_t, z_{t+1})$$

plus a process for the exogenous variable z_{t+1} which determines also $\sigma^2_{t,z}$. Recalling the scaled version of $z_t = \lambda X + 1$ introduced in the previous chapter, the process to be modeled is x_t while λ can be seen as a filter representing the outside capital flows. For the impulse-response analysis x_t follows an AR(1) process with a persistence parameter $\rho = 0.95$

$$x_t = \rho x_{t-1} + \epsilon_t$$

where ϵ_t is a random variable $\epsilon \in (0,1)$ with distribution $\epsilon_t \sim N(0,1)$. The shock has been obtained setting first $x_t = 0$ for $\forall t$ and $\epsilon_t = 0$ $\forall t$ except for $t = 1$. The endogenous variables are set at the steady state: $q_1 = \bar{q}$, $e_1 = \bar{e}$, $\pi_1 = \bar{\pi}$.

Using the scaled version of the variable has the implication that for low levels of λ, the shock will clearly be relatively smaller than for higher levels of foreign investments $z - 1 = \lambda$. This fact is reflected in the reaction of the variables shown in Fig. (2) and (3). In these Figures the variables are shown in levels to have an overview of how these variables react for different levels of λ. For low λ, say $\lambda = 0.1$ the reaction can hardly be noticed. The shock will just produce a jump in z from $z_0 = 1$ to $z_1 = 1.1$; the reactions to this can be hardly captured by the following graphs.

As it was expected, an increase of foreign capital inflows leads to increases in the share of entrepreneurs, in the number of projects, in the growth rate, in the price of projects, in the wages. The effect on the number of projects q_t is the most long lasting one. This is explained by its autoregressive coefficient $\frac{\nu(1-\alpha)(1-\delta)}{\nu+\pi_t\alpha}$ which makes the shock effect quite persistent in particular for small values of π_t. The risk premium π_t is clearly affected both by the mean through z_{t+1} and the variance effect through $\sigma^2_{t,z}$. We can see that at first the shock leads to a fall of the risk premium. Intuitively, as more capital flows in, more subjects will be entrepreneurs and more projects will be bought, as more capital is at disposal in the economy, the choice of becoming entrepreneur will be appealing also at lower risk premium. As the mean effect of the shock fades away though, the volatility effect becomes more relevant and leads to an increase in the risk premium. π_t will stabilize at higher levels.

From Fig.(3) it is clear that an increase of the agents' willingness to be entrepreneurs causes a corresponding drop in the number of workers. An increased number of projects and a smaller population of workers lead to wage increases. Correspondingly a higher demand for projects drives their price up[25]. The rate of return first increases following the increase in the project's price, and then falls due to a decrease in the dividends. As more projects are implemented, it is likely that the dividends they create will fall. The share of output used to compensate capital is going to be divided by a higher number of projects.

To have a better insight of the dynamic and be able to compare them, the variables have been transformed into logarithmic deviations from the steady state: for every variable[26]

$$\hat{e}_t = loge_t - log\bar{e}$$

The values for the steady state variables for different λ's have been obtained by Scott and Uhlig (1999). The deviations are shown for different λ in Fig.(4), (5), (6), and (7)[27]. The dynamics described above is replicated in the second set of graphs. It is though interesting to compare the amplitude and the timing of their changes. For instance, it is clear that the most reactive variable is e_t, jumping immediately to a high level and coming back to its steady state values correspondingly fast. n_t follows, of course in the opposite direction, one period later, but in a much smoother way, while, as already seen, q_t increases but less sharply and is more persistent. The persistence effect is therefore determined by the interplay of e_t and q_t: the number

[25] w_t is defined as share of output y_t, i.e. $w_t = \frac{\alpha y_t}{n_t}$. The variable has not been detrended, this is why it keeps growing. The same is true for d_t and p_t, respectively $d_t = \frac{(1-\alpha)y_t}{q_t}$ and $p_t = \frac{z_t w_t}{q_t}$. d_t is not reported in Figg. (2) and (3) as it is the exact complement of w_t, (output is shared between capital revenues - dividends d_t - and labor - wages w_t. d_t is nevertheless reported in the next figures, which represent the impulse response in percentage deviations from the steady state.

[26] For a detailed discussion on dynamic analysis see Uhlig(1997).

[27] For the variables e_t, q_t, n_t, π_t and the growth rate g_t the graphs has been plotted in a bigger scale to be better able to see the relevant movements of the variables.

of entrepreneurs e_t rises sharply immediately, creating new projects, q_t, these on turn will increase and perish much later in time and will thus have a lasting effect on growth.

The risk premium reacts almost imperceptibly to the mean effect of the shock for low levels of λ while it increases in the variance effect once z_t goes back to 1. This is due to the way the shock has been modeled; while the amount of foreign investments can be shocked and be brought from 1 to $1 + \lambda$ and back to 1, the variance of the process z_t keeps being constant through out the time and is equal to $\sigma^2_{t,z} = \lambda^2 \xi^2$. As it was assumed that $Var[X] = \xi^2 = 1$, the volatility of the process is constant and equal to λ^2 once the foreign investors enter the market.

The growth rate in dynamics does not depend only on e_t but is determined by e_t, n_t and q_t.

$$g_t = (1 + \psi e_t)(\frac{n_{t+1}}{n_t})^{\alpha}(\frac{q_{t+1}}{q_t})^{1-\alpha} - 1 \tag{13}$$

As n_t is also a function of e_t, it is obvious that the growth rate amplifies the reactions of e_t, it therefore first drops following the fall in n_t and then increases more than all other variables replicating in a bigger scale the behavior of e_t. Both e_t and q_t are inverse functions of π_t, i.e. if π_t increases they decrease.

Thus, also the growth rate depends on π_t. Where the variance effect is not counterbalanced by a strong mean effect, in other words for low λ's, the growth rate, e_t, and q_t, will be negatively influenced by an increase in π_t and will stabilize at lower level than the initial steady state. For high λ's the contrary will happen as they will stabilize at the same or at a higher level. This result - lower growth for lower λ and higher growth for higher λ - can look at odds with the results in the steady state shown in Fig.(1) but actually is not.

This can be better understood considering the volatility-to-mean ratio $\xi = \frac{\sigma_z}{z-1}$. In the steady state this is clearly constant and $\bar{z} - 1 = \lambda$ whereas in the dynamics, it changes over time as z_t is shocked: $z_t - 1$ peaks at the value λ in the first period and then declines. Consequently keeping $\sigma_{t,z}$ constant, ξ will increase as z goes back to its original value 1. This means that the volatility effect will be stronger over the time as we have noticed from the rising behavior of π_t, but it also means that for lower λ the mean effect spurred by the initial shock is not strong enough to counterbalance the volatility effect.

This effect is indeed not noticed in the next section where the state variable has been shocked keeping the volatility and the amount of foreign investments constant.

Growth rate stabilizes around its new steady state relatively fast and in five to six periods the positive effect of a shock in the amount of capital flowing into the economy is exhausted. Obviously, all the deviations are wider for larger λs.

The variables in Fig.(7) behave as expected: dividends and wages are just complementary, one drops and the other rises. The price rises as the demand for projects increases. The rate of return, being a function of the price and of the dividends, first raises following the effect on prices and then drops following the effect on dividends.

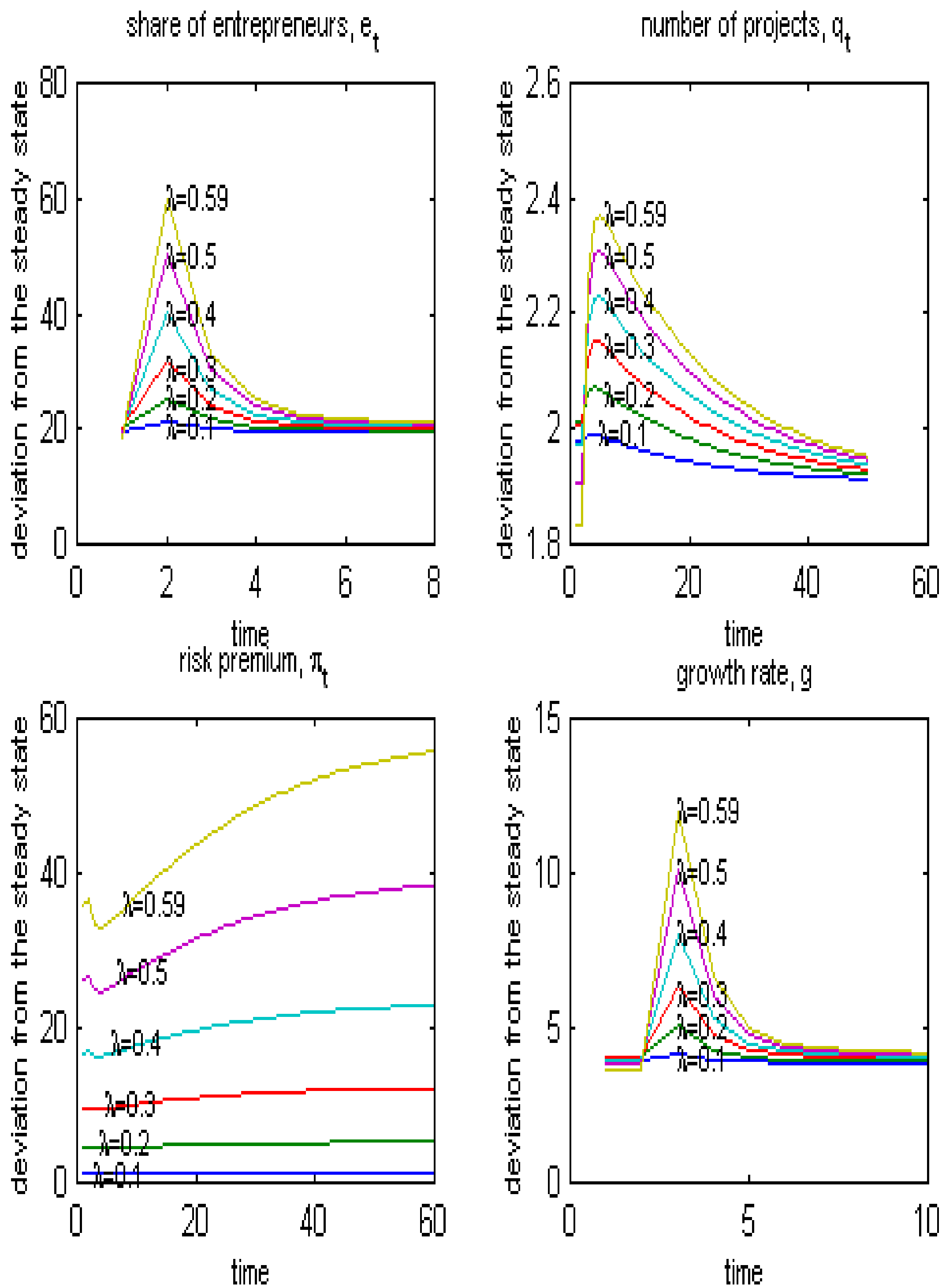


Figure 2: IMPULSE RESPONSE TO A SHOCK IN THE FOREIGN INVESTMENTS. LEVELS

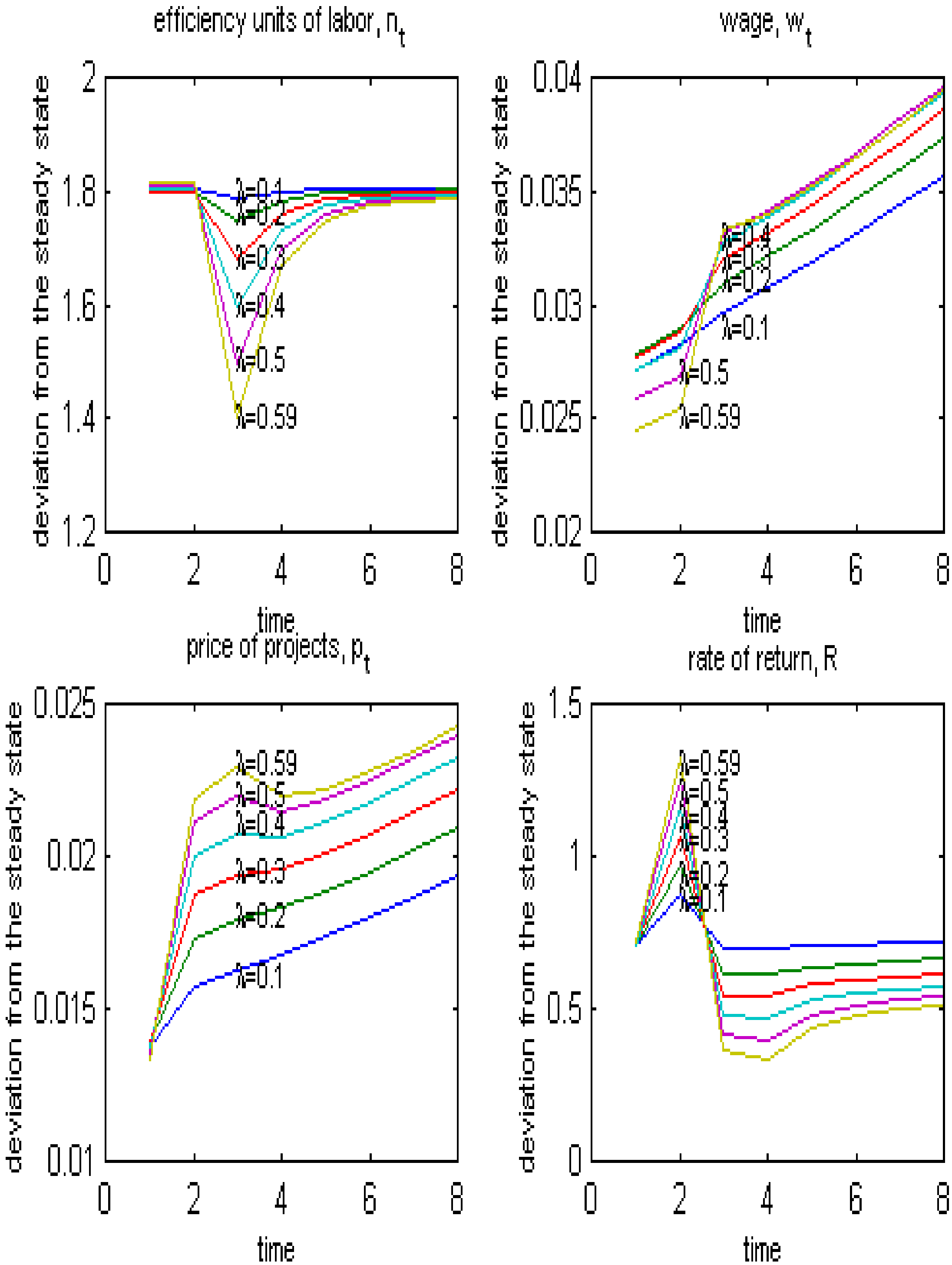


Figure 3: Impulse response to a shock in the foreign investments. Levels

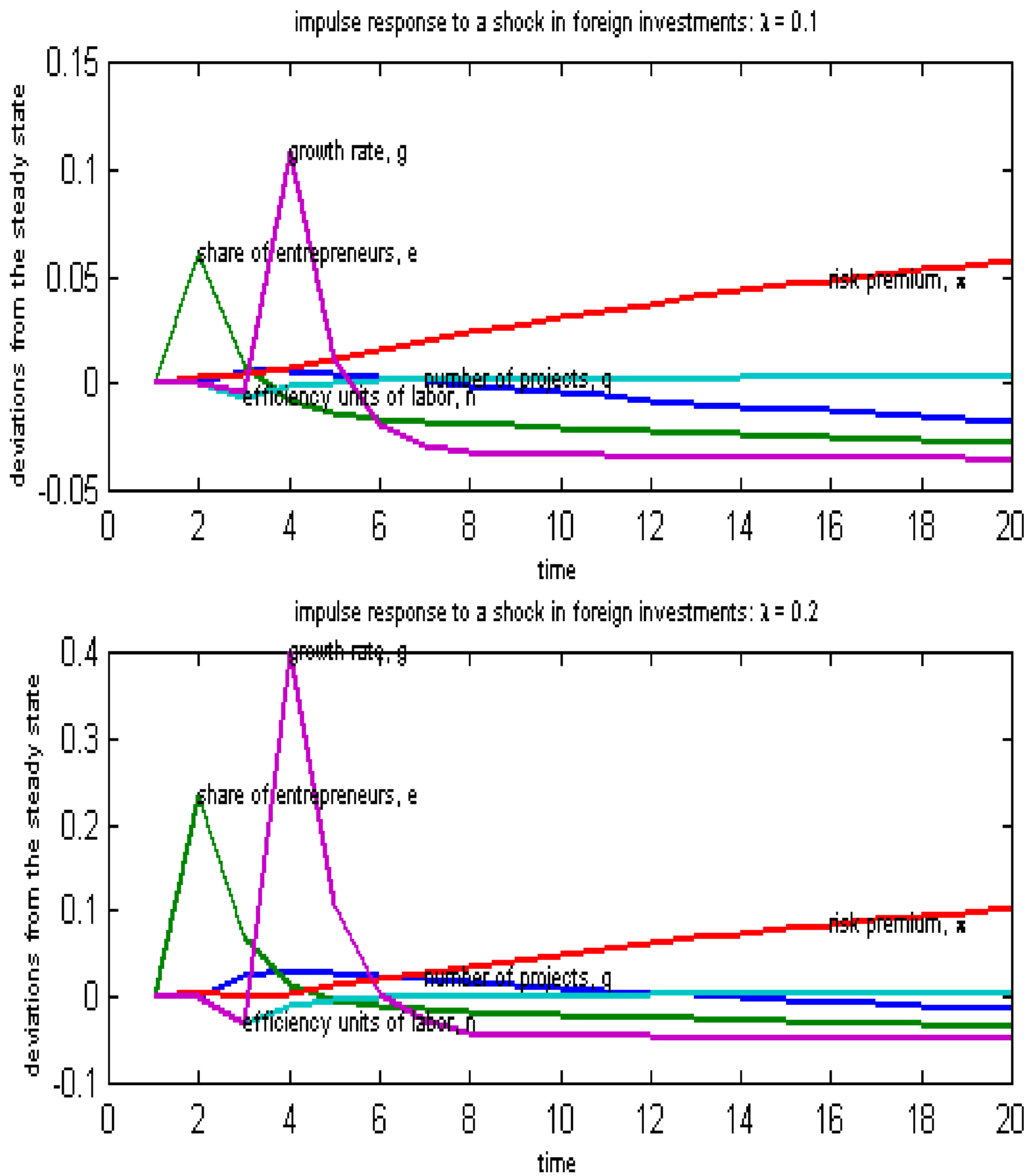


Figure 4: IMPULSE RESPONSE TO A SHOCK IN THE FOREIGN INVESTMENTS: $\lambda = 0.1$ AND $\lambda = 0.2$

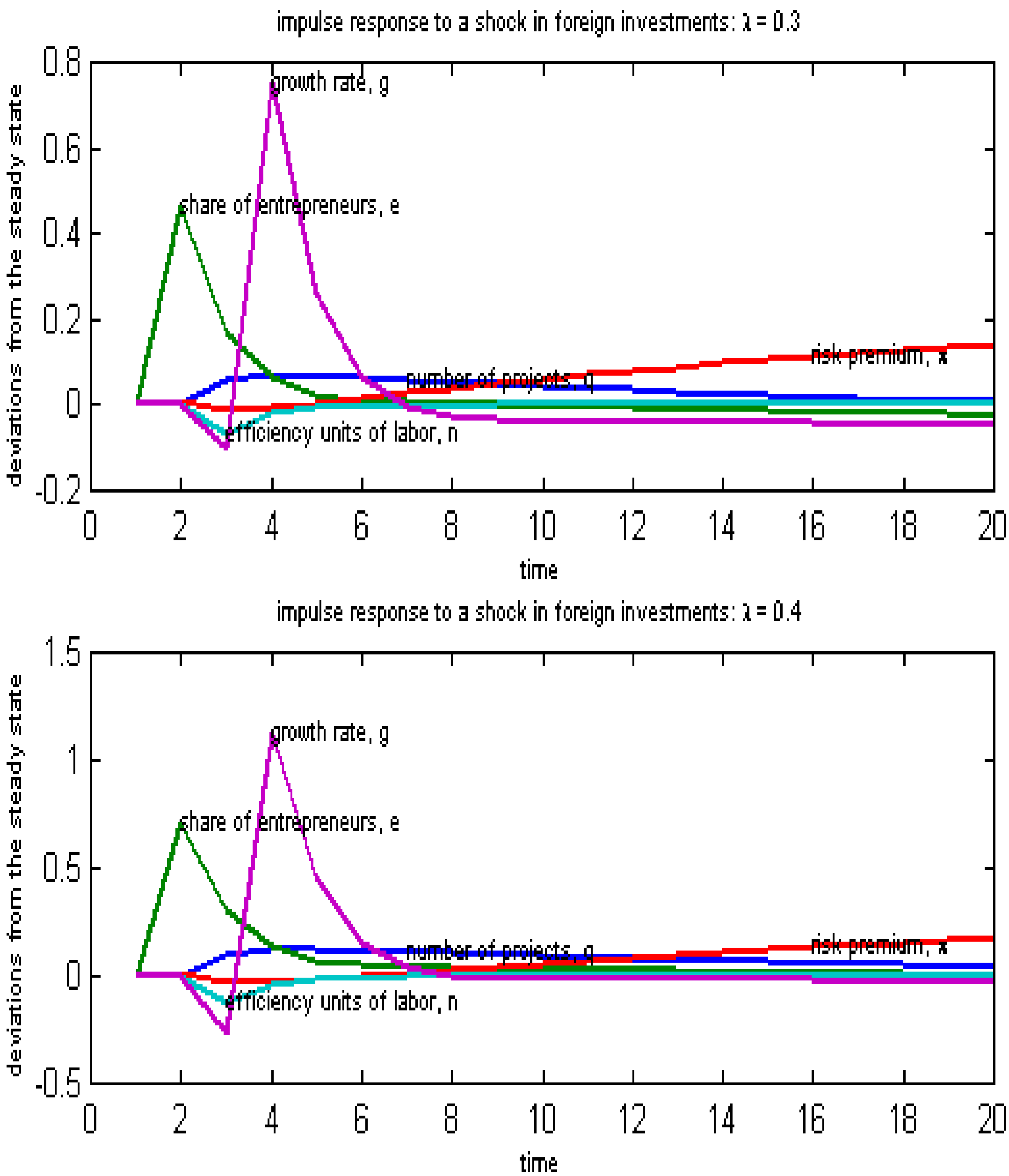


Figure 5: Impulse response to a shock in the foreign investments: $\lambda = 0.3$ and $\lambda = 0.4$

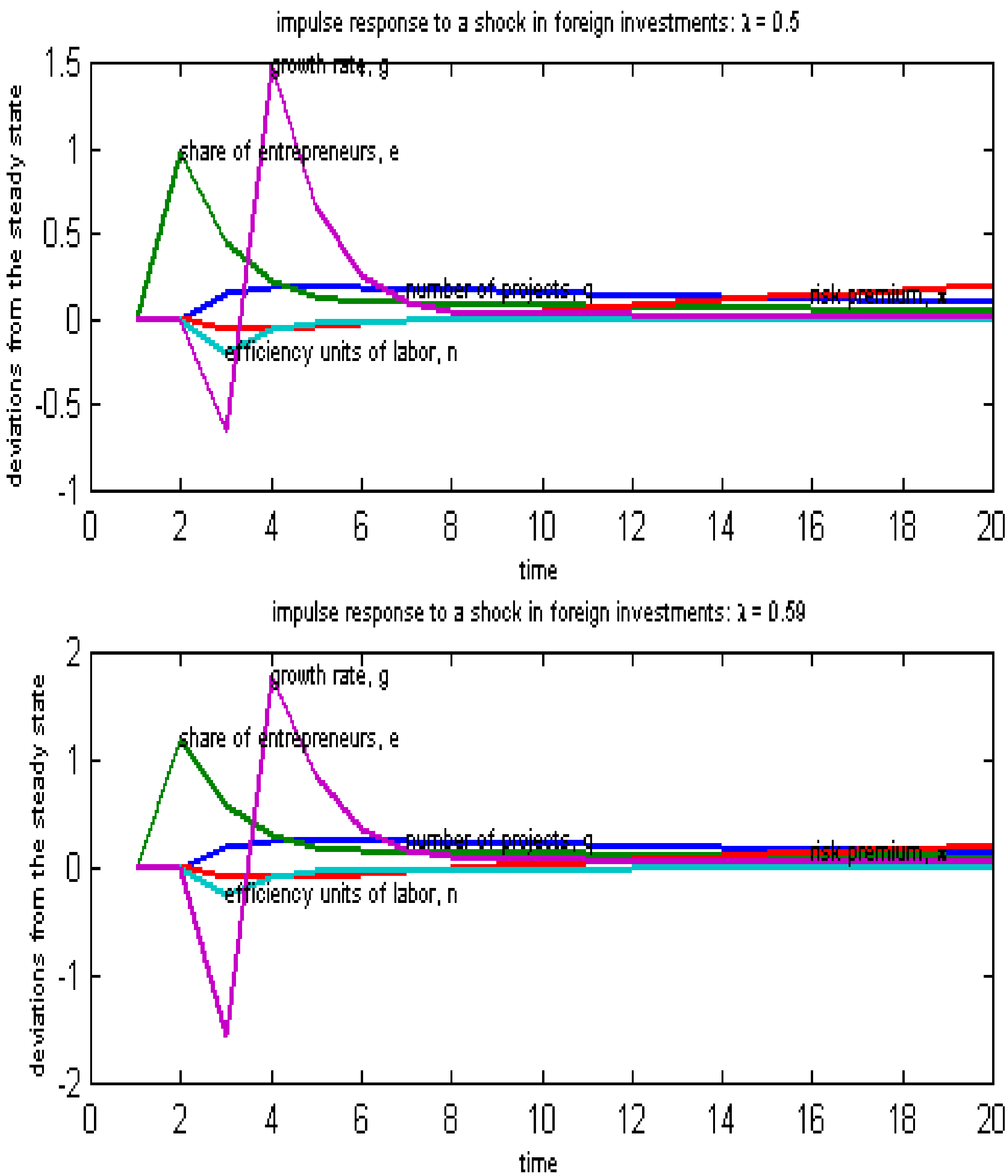


Figure 6: Impulse response to a shock in the foreign investments: $\lambda = 0.5$ and $\lambda = 0.59$

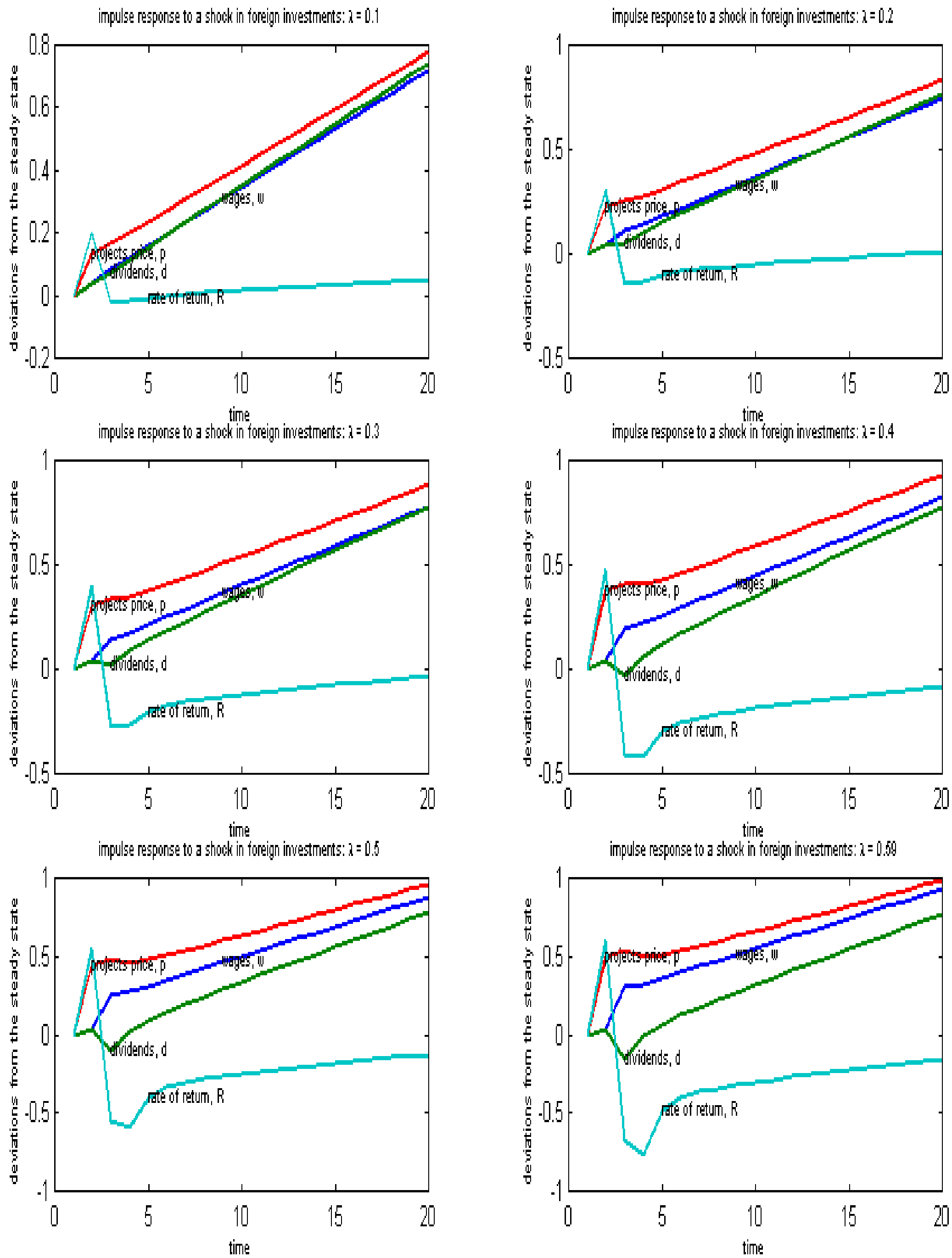


Figure 7: Impulse response to a shock in the foreign investments

4.1 A shock to the endogenous state variable q_t

In this section I shocked negatively the number of projects q_t keeping the amount of foreign investments λ and their volatility $\sigma^2_{t,z}$ constant over time. The results are shown in Figg.(8) and (9) for different values of λ.

In opposition to the previous section, where the effect on the whole system of a positive shock on the exogenous variable z_t was illustrated, I choose now to implement a negative shock. Per assumption the model does not foresee short selling of projects which implies that I am considering only cases where $z_t \geq 1$. Implementing a negative shock to the number of projects is therefore a way to circumvent this restriction[28].

This event could be represented by an occasion where capital flows are politically "frozen" and therefore foreign investors are not allowed or willing to retrieve their funds but the number of projects implemented has been reduced drastically due to some political uncertainties or dramatic events, a flood or drought, which have led domestic and foreign investor to stop their activity temporarily. This could be the case of war but also the full unexpected exploitation of domestic natural resources or the government sudden decision to abandon a sector of production in economies still centrally controlled of the sort of the Asian economies[29].

The behavior of the variables is roughly the same for different values of λ, the scale of them just changes. For a low number of projects in time t there will be more subjects willing to become entrepreneurs in time $t+1$ expecting a higher productivity on the new projects. This reaction was expected to be like this, recalling eq.(5), q_t enters in the expression with a negative sign, therefore a drop in the state variable creates a positive jump in e_t. The worker population will accordingly shrink, the wage raise and correspondingly the dividends fall.

As subjects are keen on buying new projects, their price raises following the reac-

[28] A negative shock of 100% means that $q_1 = 0$ or in other words $-1 = log q_t - log \bar{q}$. Since calculating $log 0$ obviously determines a problem in the program, the percentage deviations for the relative graphs have been calculated as: $\frac{q_t}{\bar{q}} - 1$. The graphs should not be influenced by this procedure.

[29] As in the model it is not considered the role of the exchange rate, I do not want to motivate this as a worsening of the terms of trade as it might have been the case for the Asian crisis. Ito (1998)

tion of e_t. Prices raise more than the dividends fall and therefore the rate of return, R_t raises sharply and only after its peak in the second period, is pulled down by the falling dividends[30].

The growth rate is not highly affected by this shock; on the one hand there is the negative effect on q_t, but on the other there is the cumulated - through γ_t and through n_t - positive effect of e_t which brings the growth rate back in line after a short fall.

π_t initially grows driven by the increase in the number of entrepreneurs and then goes back to its steady state. For higher λ it stabilizes at a lower level than the initial steady state. This can be explained by the behavior of e_t and q_t: as the number of projects is low, the share of agents willing to take risk is larger and higher amount of foreign capitals will be needed for new projects. This can be seen in the behavior of both the share of entrepreneurs and the number of projects, both stabilizing at a higher level than the original one for high levels of λ. In this case therefore, the mean effect is dominant even for high levels of funds. This result is quite interesting as it could mean that the mean-tradeoff is conditional on the initial conditions of the economy. If the economy is below its production potential, a higher amount of international investments could be completely absorbed in the production activity and the volatility effect might be overbalanced by the mean effect.

[30]The rate of return is basically determined by two ratios: $\frac{d_{t+1}}{p_t}$ and $\frac{p_{t+1}}{p_t}$. If p_t is raising sharply from period t to period $t+1$, the second ratio will be predominant and will actually lead the rate of return to rise even sharper.

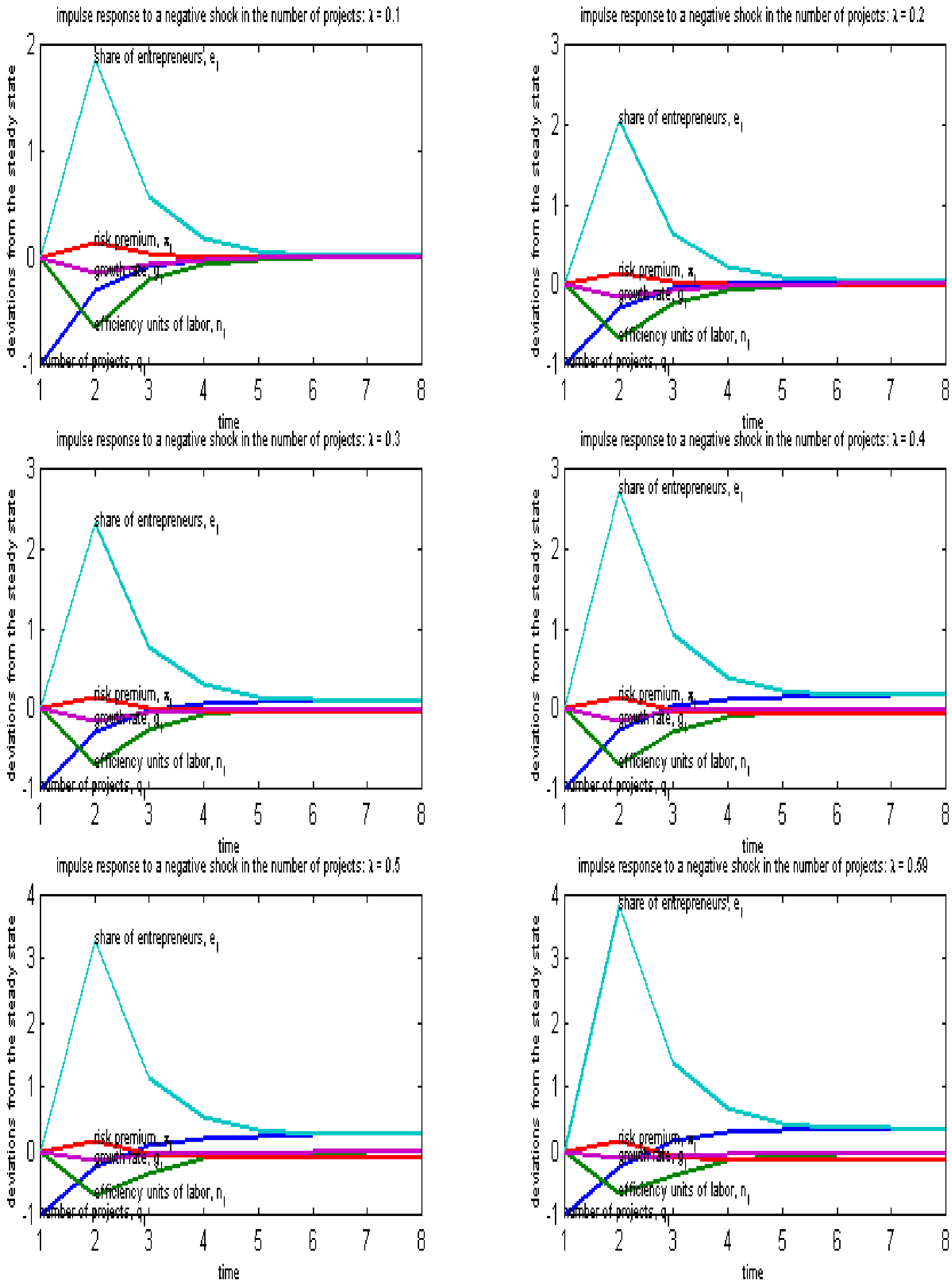


Figure 8: IMPULSE RESPONSE TO A NEGATIVE SHOCK IN THE NUMBER OF PROJECTS q_t

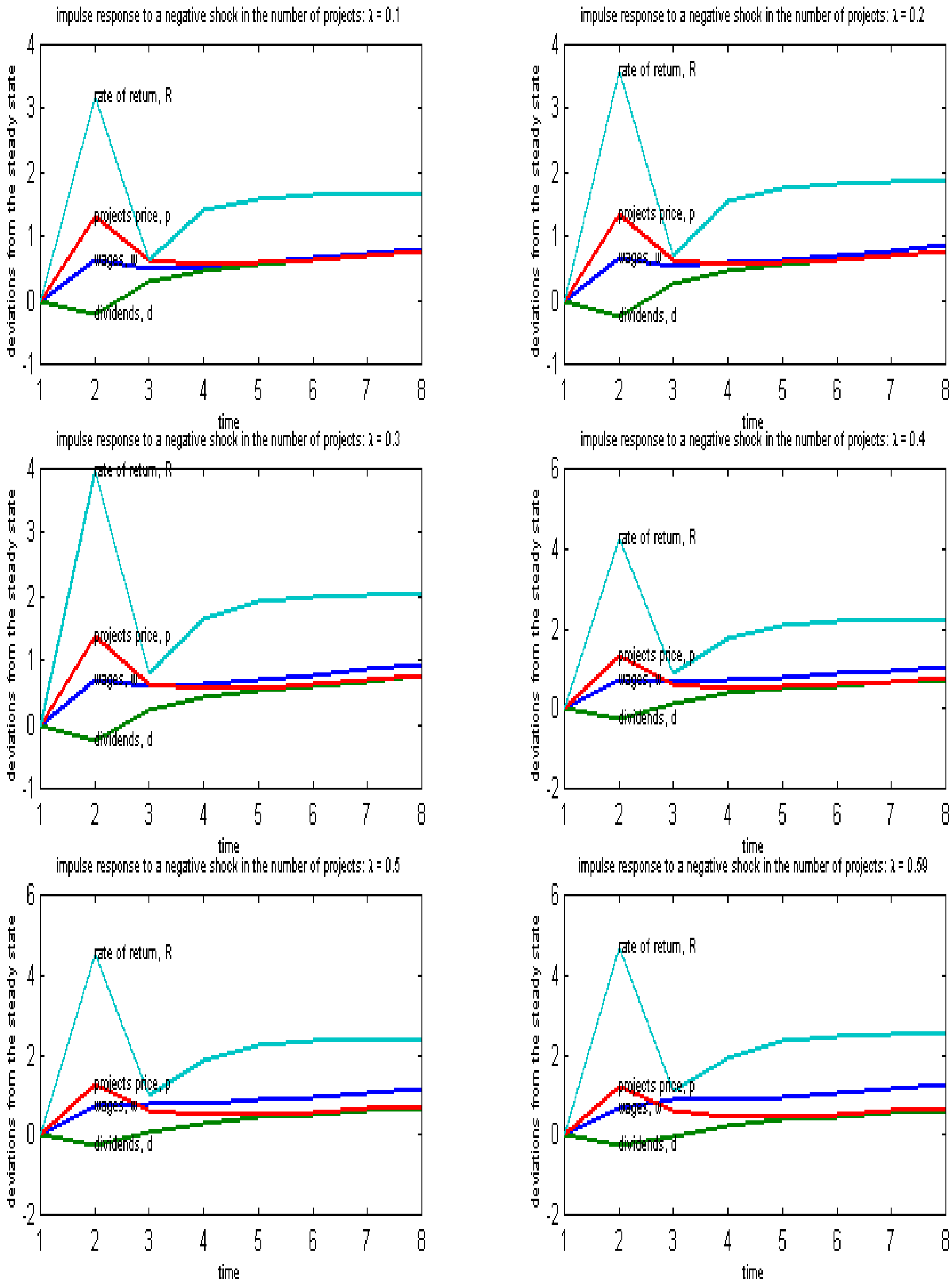


Figure 9: Impulse response to a negative shock in the number of projects q_t

4.2 The mean-variance trade-off: simulations

In this section I will try to reproduce the mean-variace trade-off in the dynamics. To do this the amount of foreign investments has been modeled in a way that the condition $E[X] = 1$ is met and the variance can be determined and changed easily. The variable x_t will therefore follow the autoregressive process

$$x_t = c + \rho x_{t-1} + \sqrt{(1-\rho^2)}\epsilon_t$$

where ϵ_t is a random variable with distribution $\epsilon_t \sim N(0,1)$, $\rho = 0.95$ and $c = 0.05$.

As the unconditional mean of such a process is given by

$$E[x_t] = E[c] + E[\rho x_{t-1}] + E[\sqrt{(1-\rho^2)}\epsilon_t]$$

$$E[X] = \frac{c}{1-\rho}$$

(as $E[\epsilon] = 0$)

choosing $c = 1 - \rho$ allows us to meet the condition on the expectation of X.

The unconditional variance is

$$Var[x_t] = Var[c] + \rho^2 Var[x_{t-1}] + (1-\rho^2)Var[\epsilon_t]$$

$$\sigma_x^2 = \frac{1-\rho^2}{1-\rho^2}\sigma_\epsilon^2 = \sigma_\epsilon^2 = 1$$

This simple process allows us to determine and change the variance of x_t just by changing the coefficient in front of ϵ[31].

This will be useful to test how the model reacts to a change in the parameters in particular of ξ which, as seen in the steady state represent the standard deviation of X but also the volatility-to-mean ratio.

To make sure that the mean and the variance will effectively be the unconditional ones, I will choose a long time period.

[31] For more on time series see Hamilton(1994)

For $t = 1, 2, 3...T$ with $T > 10000$, choosing the first value x_0 out of a distribution $N \sim (1, 0)$ and not considering the first 100 periods to allow the process to stabilize around its mean, to make sure that the process will eventually have a mean and a variance equal to the unconditional ones given above[32].

As we can see from Fig.(10), the mean-variance trade-off seems not to hold in the dynamic analysis. Growth does not display a hump-hill shaped surface, on the contrary, it seems to keep rising for higher levels of λ. The mean effect seems actually to be quite strong casting some doubts on the validity of the Scott and Uhlig (1999) result.

Nevertheless if we examine the graph displayed in Fig.(10) from another perspective like in Fig.(11) one can recognize that the first impression is not totally correct. For the lower blips of the growth series it is actually recognizable a fall of growth for higher values of λ. Where the mean effect given by $\lambda x_t + 1$ is limited as x_t is below its average, the variance effect does matter and leads to a falling growth rate, whereas if x_t is above its mean, the mean effect will be preponderant and the economy will display increasing growth rates.

This is easily explained considering the volatility-to-mean ratio ξ. In the steady state $x_t = \bar{x} = 1$, therefore ξ is just the ratio of $\frac{\sigma_z}{\bar{z}-1}$ where it was clear that $\sigma_z = \lambda\sigma_x$ and $\bar{z} - 1 = \lambda$. It is obvious that in the steady state as long as $\sigma_x = \xi$, the volatility-to-mean ratio is always equal to 1. If this condition holds than the growth rate is hump shaped.

In the simulation, clearly the process x_t will be randomly above and below its mean, and consequently ξ will change from period to period[33].

This was true also in the steady state, the hump shaped curve in Fig.(1) can easily change shape if the parameters are changed. For $\xi > 1$ it just looks like a falling curve and the opposite is true for $\xi < 1$ as one can see from Fig.(12).

The result of Scott of Uhlig (1999) can therefore be confirmed by the dynamic analysis with a bigger emphasis on the volatility-to-mean ratio. This ratio determines

[32]For a better reading of the graph, though, only 100 periods have been considered.

[33]In this section therefore I prefer to distinguish between volatility-to-mean ratio and standard deviation of x_t and indicate with ξ the first one and with σ_x the second one as they are no more bounded to be coincident concepts.

if the capital inflows will have a beneficial rather than a disruptive effect on the economy. In the case where capital flows consist prevalently of FDI, which are considered very little volatile, the amount of these funds, our λ, should not, according to this model, hamper growth.

As a consequence, the specification of the model would need a more precise definition of investments[34].

Scott and Uhlig (1999) structure capital flows as relatively productive ones; on the one hand capital flows into projects and would let one think of Direct Investments, while on the other hand volatility is supposed to increase proportionally to the amount of these flows regardless of the kind of investments done.

The model therefore offers an interesting insight of the channels through which foreign investments affect growth: on the one side there is the positive mean effect through the share of entrepreneurs, on the other there is the dampening variance effect trough the risk premium. The fundamental question is which one will be preponderant, and as a consequence the fundamental policy question will be not much on how much to liberalize capital markets but which flows present a too high volatility-to-mean ratio and need therefore to be controlled.

In the last decade, flows to developing countries have been so large that the consideration of the mean effect in comparison to the volatility has most probably been given more weight. It would have been quite difficult to imagine that such huge amount of capital could reverse their destination at once. The main problem seems therefore to be the ability to foresee and evaluate the volatility correctly.

In the following, this result has been verified against different parameters, in particular the volatility-to-mean ratio and the marginal aversion to risk η. It is difficult to notice any difference due to the change in η, the real difference is determined by the changing σ_x. The range of variation of the series is of course larger and the falling growth effect for the periods where the shock is below its mean is more evident.

This is even more evident if I use an ARCH(1) process for x_t as in Figg(16) and

[34]For an empirical study of the mean-variance effect see Liew, C. (2002) Master thesis. In her thesis she finds that the mean-volatility effect is significant only on certain capital flows, in particular debt flows -portfolio bond investment and bank- and trade- related bonds.

(17). In Fig(17) is even recognizable for some periods a maximum growth peak around values of λ between 0.3 and 0.4.

The ARCH process has been chosen in the consideration that this model is able to better reproduce financial data series with high volatility periods alternated to low volatility times[35].

The process has been modeled as follows

$$x_t = c + \rho x_{t-1} + \epsilon_t$$

where

$$\epsilon_t = \nu_t \sqrt{\alpha_0 + \alpha_1 \epsilon_{t-1}^2}$$

ν_t is a white noise process such that $E[\nu_t] = 0$ and $\sigma_\nu^2 = 1$, ν_t and ϵ_{t-1} are independent of each other, and α_0 and α_1 are constants such that $\alpha_0 > 0$ and $0 < \alpha_1 < 1$.

The parameters have been chosen such to display unconditional mean

$$E[X] = \frac{c}{1-\rho} = 1$$

and unconditional variance

$$Var[x_t] = \Big(\frac{\alpha_0}{1-\alpha_1}\Big)\Big(\frac{1}{1-\rho^2}\Big) = 25$$

or, in other words, $\sigma_x = 5$.

Comparing Fig(15), where the initial shock process for $\eta = 5$ and $\eta = 2$ given a volatility of $\sigma_x = 5$ has been plotted, with the ARCH process for the same values of η, depicted in Fig(17), it seems clear that the modeling of the stochastic process does not determine a significant difference. The main result here is the ability of the model to produce lasting cycles in the economy's growth rate. λ seem to have a magnifying effect on these cycles.

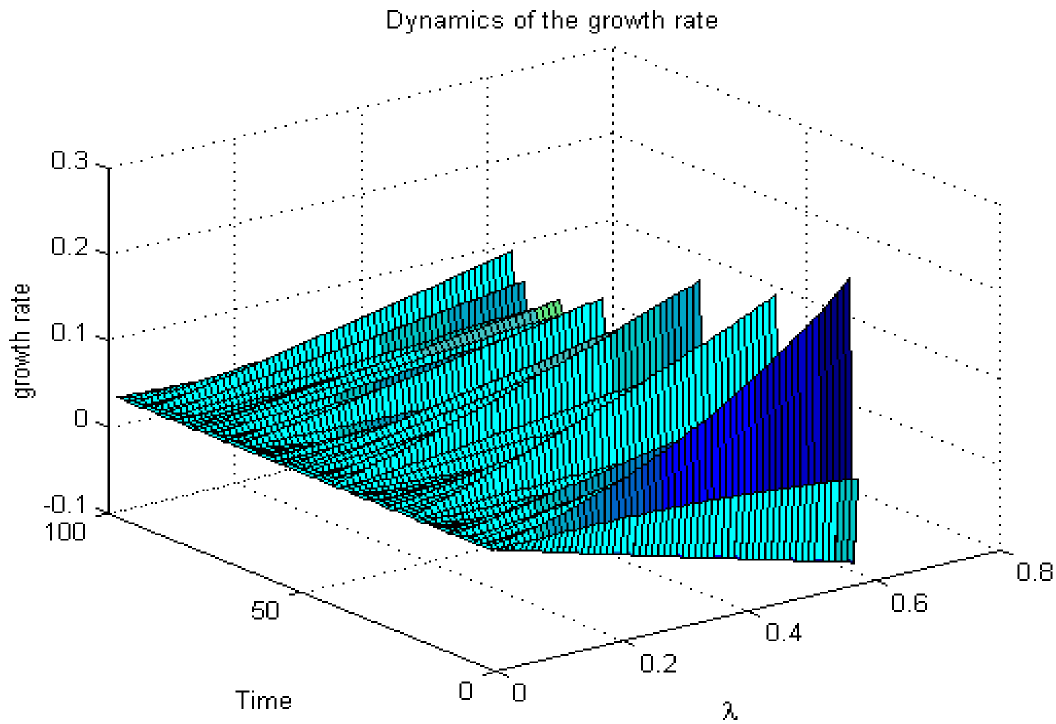


Figure 10: SIMULATION OF THE GROWTH RATE

[35] For more details on ARCH and GARCH processes see Enders (1994).

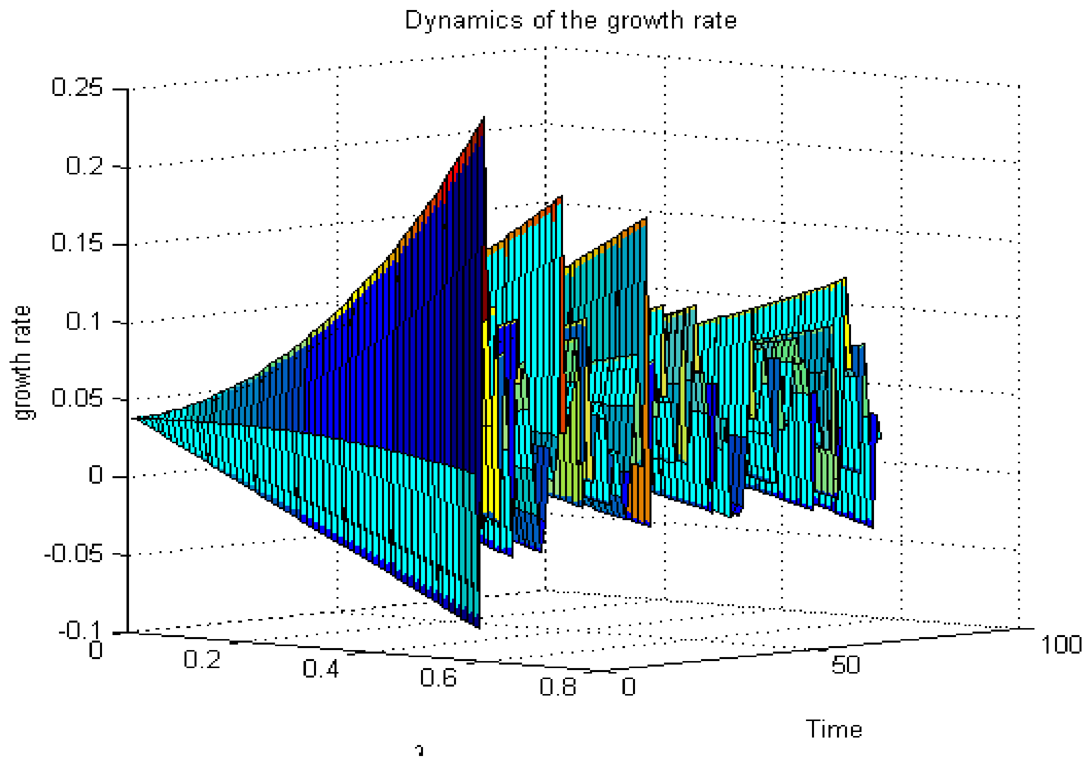


Figure 11: Simulation of the growth rate, from a different perspective

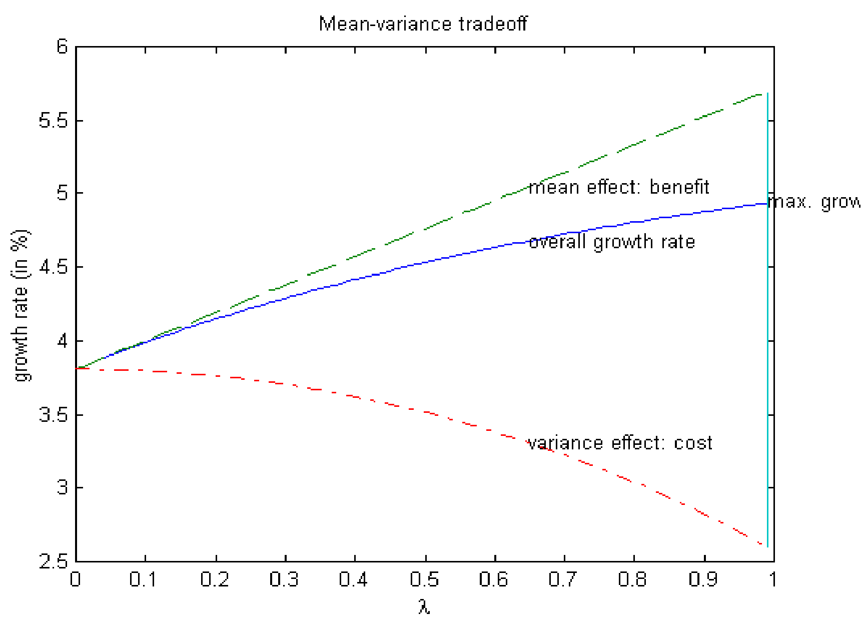


Figure 12: Growth rate in steady state for $\xi = 0.5$

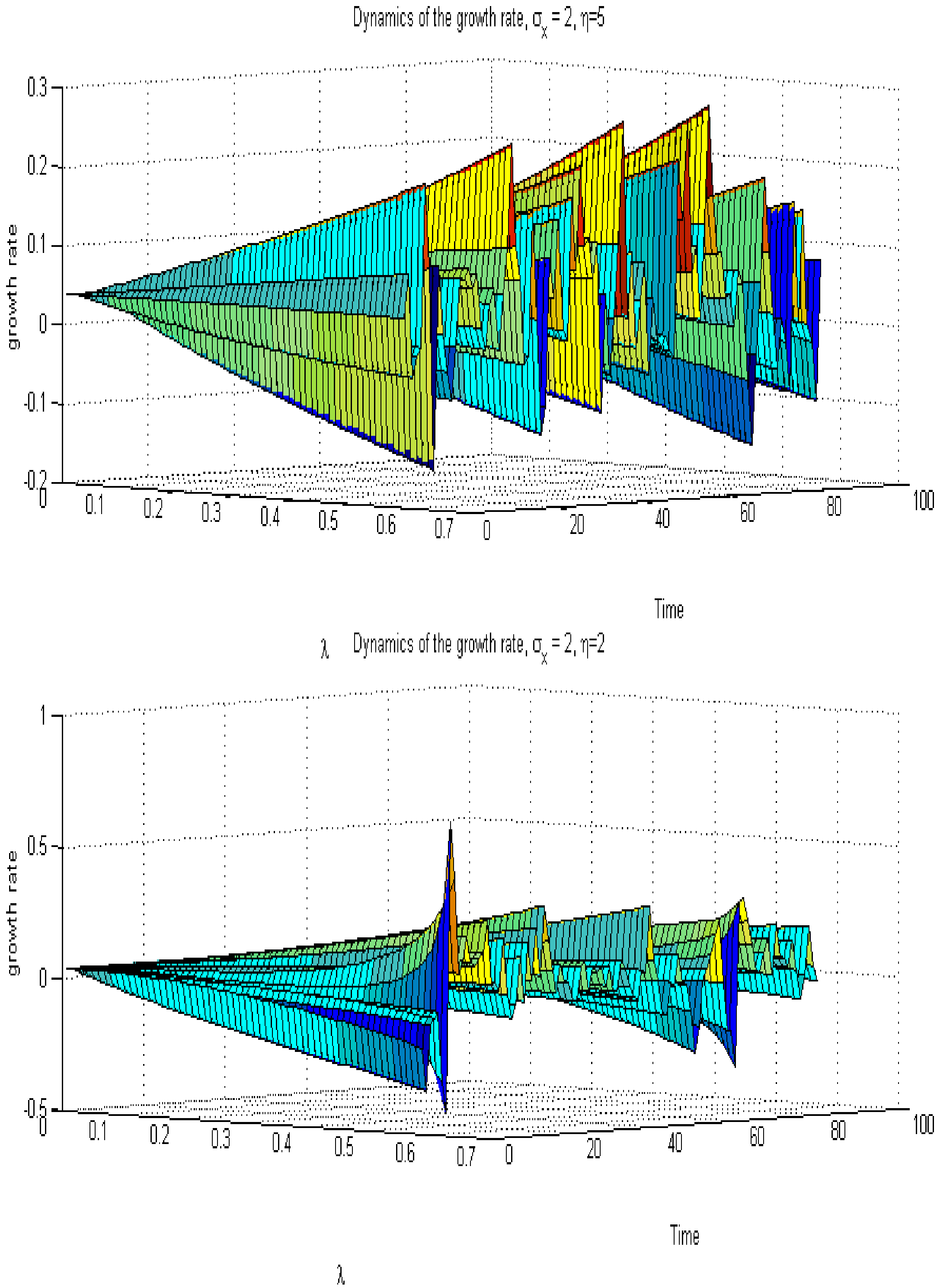
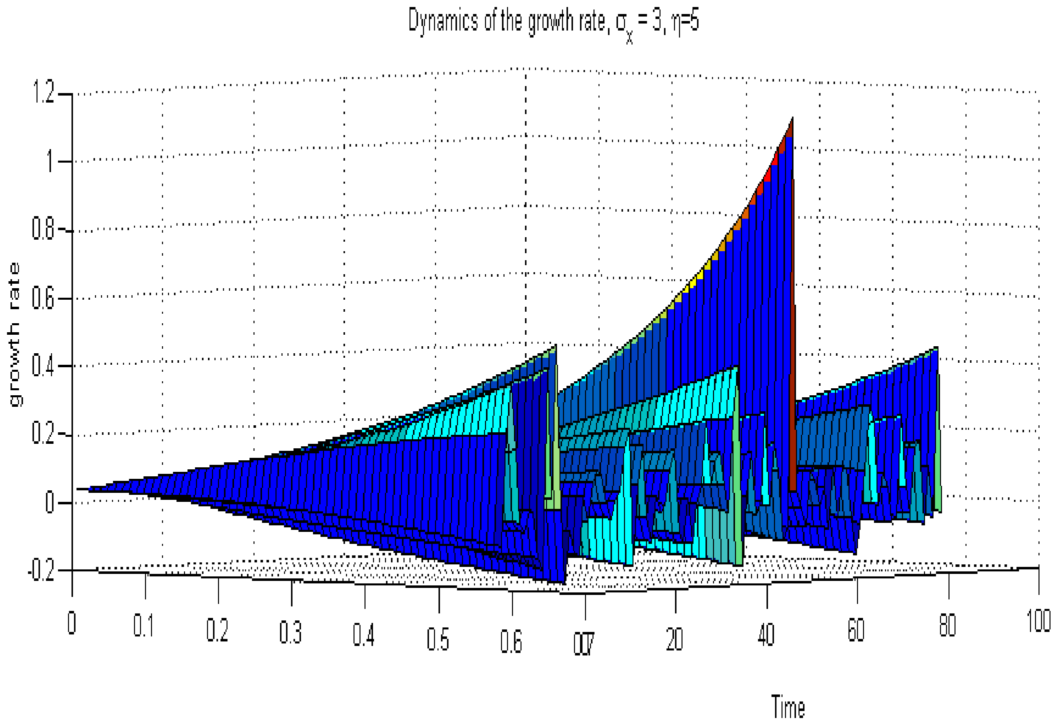


Figure 13: Simulations of the growth rate, $\sigma_x = 2$, $\eta = 5$ and $\eta = 2$



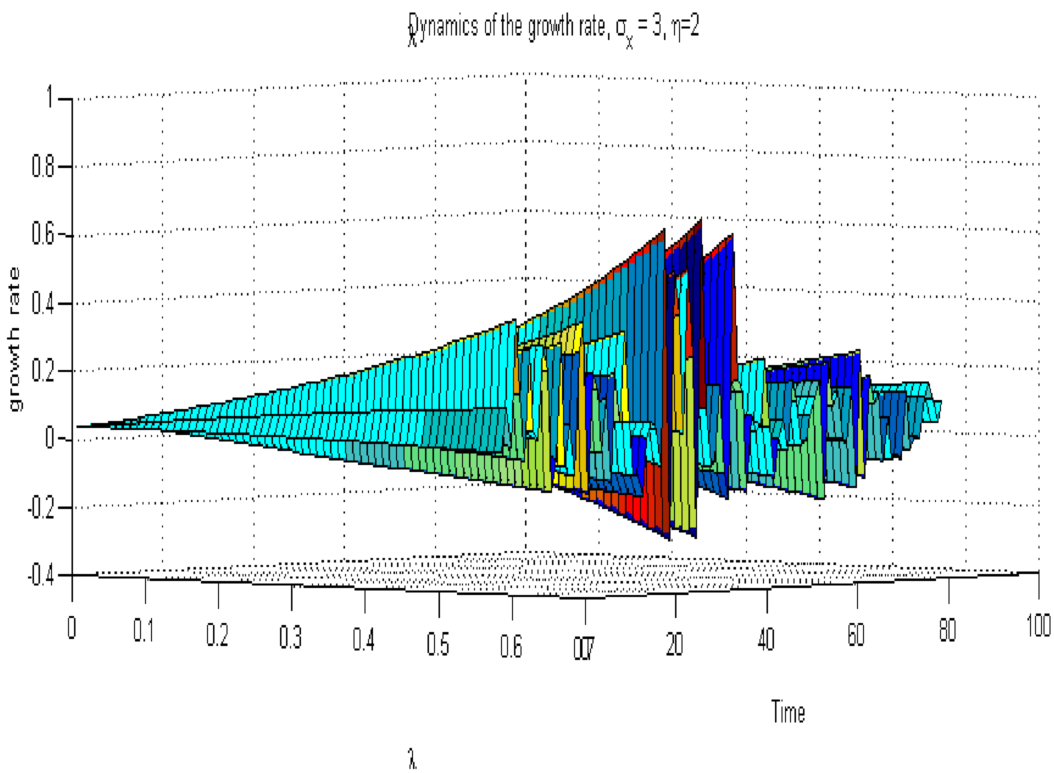


Figure 14: SIMULATIONS OF THE GROWTH RATE, $\sigma_x = 3$, $\eta = 5$ AND $\eta = 2$

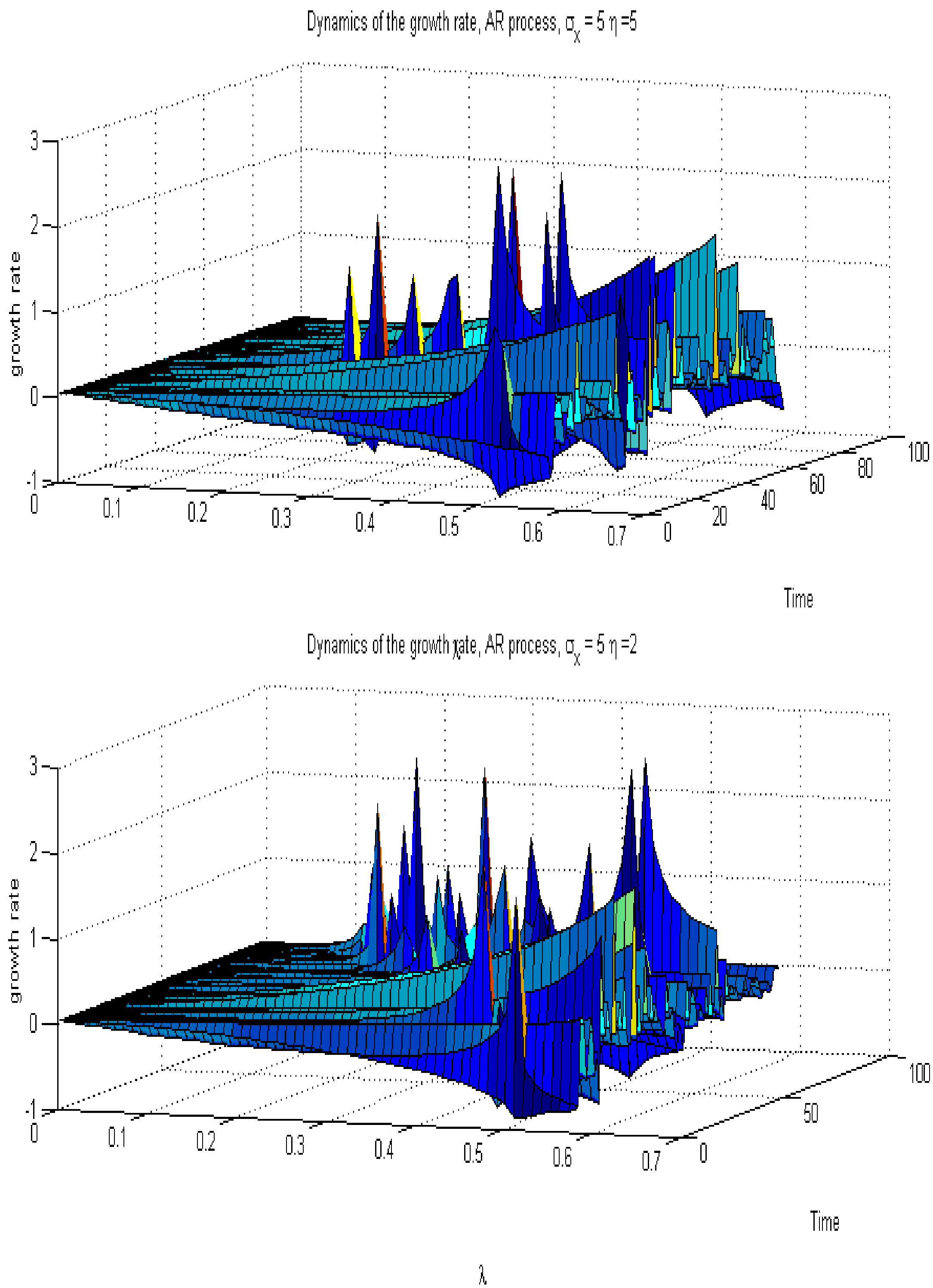


Figure 15: SIMULATIONS OF THE GROWTH RATE, $\sigma_x = 5$, $\eta = 5$ AND $\eta = 2$

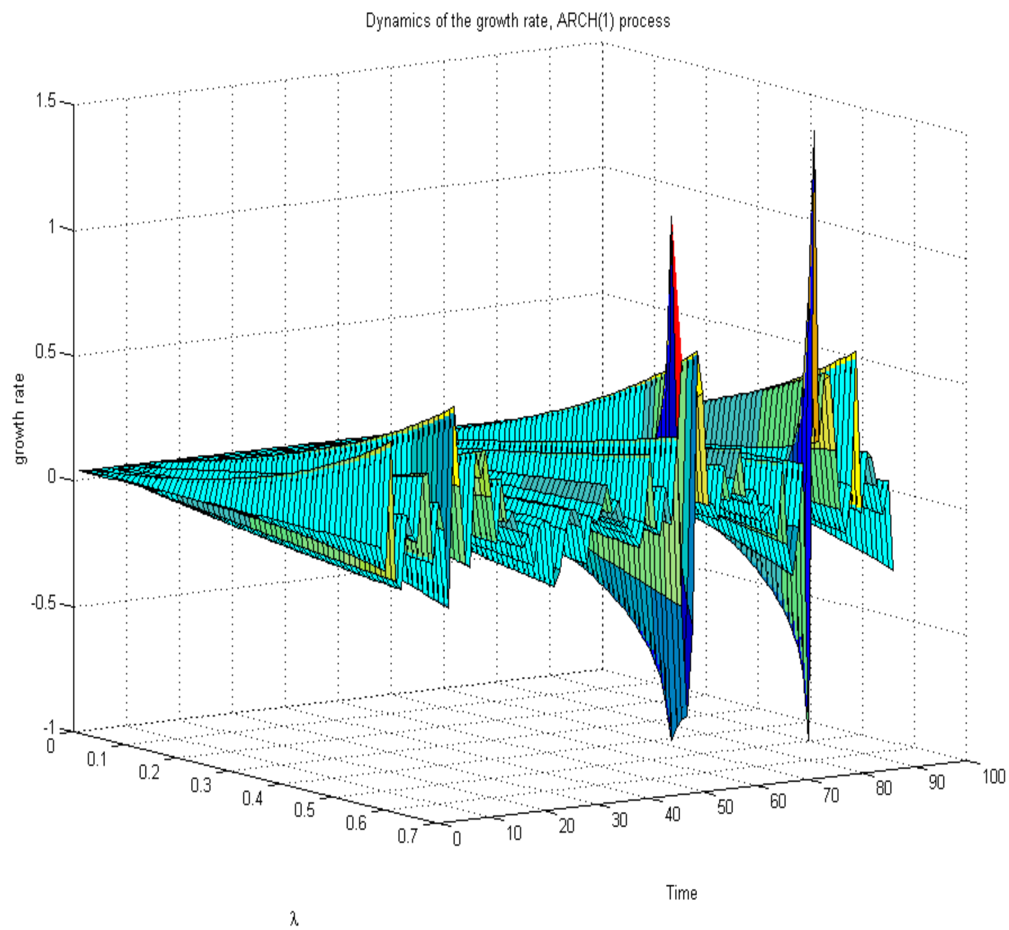


Figure 16: Simulations of the growth rate, ARCH(1)

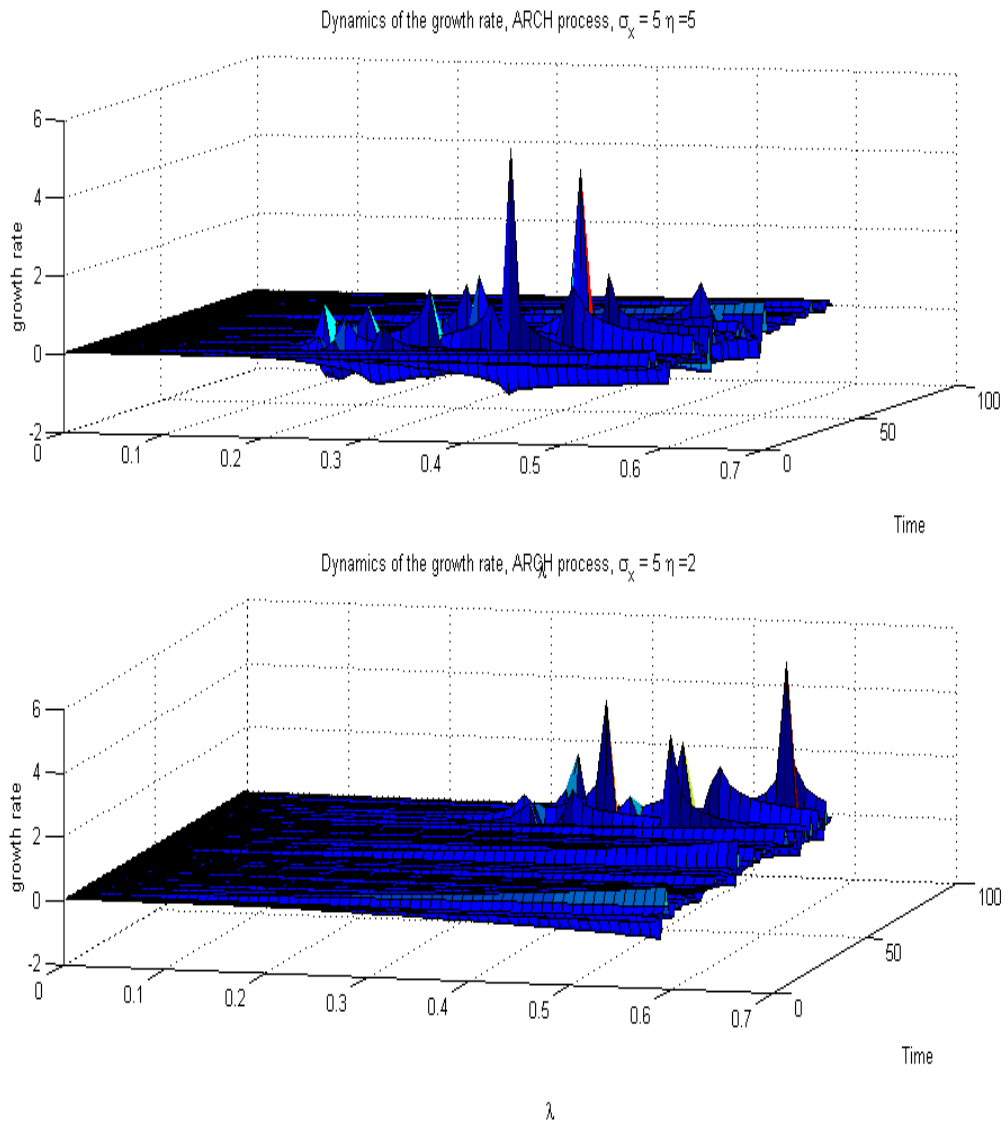


Figure 17: Simulations of the growth rate, ARCH(1), $\eta = 5$ and $\eta = 2$

5 Discussion

The dynamic analysis confirms the existence of a mean-variance trade-off pointing out the importance of a correct evaluation of capital flows' volatility. In fact, the results show that the risk associated with capital liberalization is not so much determined by the quantity of capital flows as argued by some authors (Kahn and Reinhard, 1995) as by the volatility-to-mean ratio. In this context it is clear that the channel through which Scott and Uhlig (1999) determine a negative impact of foreign investors on growth presupposes an intermediate level of financial development but it is not explained by it.

This adds a new dimension to the discussion on capital market liberalization. The model outlined does not refer to expectations, foreign currency debt, information asymmetries, agency problems, nominal prices rigidities, net worth balances, domestic liquidity, the role of monetary institutions or irrational investors behavior. The mechanism explained relies only on the discouraging effect a higher risk premium has on investment activities. It is therefore a purely private channel. As the key variable in the model is the volatility-to-mean ratio, this analysis would suggest the necessity of improving the ability to control volatility without discouraging capital flows.

The Scott and Uhlig (1999) paper contributes to the current discussion on controls on capital inflows giving a hint on how these have to be implemented. Neglecting the current discussion about their effectivity, capital controls may, in theory, be a good device to allow emerging market economies to improve their volatility-to-mean ratio and benefit the most from international investments[36]. Specifically, this would imply a discriminating policy towards capital flows: productive investments which can be retrieved from a country only with difficulty like in the case of Foreign Direct Investments should be encouraged while speculative kind of flows should be discouraged. However this policy is to be pursued with the highest transparency in order to avoid investors' uncertainties about the government type. In fact, international investors seem to be the most courted, finicky, unstable and irrational category of subjects on the scene of economics. Investment conditions should be appealing, offering not only

[36] See Edwards(1999) and Rossi (1999) for a discussion on the inefficiency of capital controls.

a good rate of return but also: a stable and transparent political environment, low taxation, broad information on the domestic market, a solid banking and financial system and stringent supervision mechanisms.

In other words, offering the right investment conditions, will attract the desired productive and low-volatility international flows. The best way of reaching an effective capital control policy it therefore a policy which leads to an improvement of the domestic financial system and consequently to a decrease in the need of capital controls. In fact, to keep a good access to capital markets is a fundamental factor in avoiding crises (Krugman, 1999).

A better financial system will in turn have certainly an influence on the level of the risk premium as presented in the model. As the role of financial systems is mainly that of processing informations and provide a contact between potential lenders and entrepreneurs, more transparency and less uncertainty will clearly lead to a decrease in the risk premium required by risk averse investors. In this context, the risk premium presented in the model could be interpreted as the price of the information asymmetries present in the market. As many authors suggested, the main objective of governments in this field should be to ensure the stability, transparency, and well functioning of the banking and more in general of the financial intermediation sector. International institutions such as the Bank for International Settlements and the International Accounting Standards Committee developed a set of guidelines to strengthen regulatory, supervisory, and accounting practices exactly following this logic. Further, the International Monetary Fund and the World Bank have stepped up surveillance of the financial sector in individual countries and urged developing economies, in particular, to adopt these guidelines (Rossi 1999).

Alternatively to capital inflows controls, some authors have argued in favor of a Tobin, or transactions tax on capital flows. The transaction tax is supposed to deter the most volatile short term capital flows: "a half percent tax translates into an annual rate of 4% on a three months' round trip into a foreign money market, more for shorter round trips. It is this effect that creates room for differences in domestic interest rates, allowing national monetary policies to respond to domestic macroeconomic needs. The same tax would be a smaller deterrent to slower round trips. It would be a negligible consideration in long-term portfolio or direct investments in other economies."[37] Unfortunately, to be effective, this kind of tax needs to be universal and uniform; applying it only in certain countries or with different rates across jurisdictions would make it distortionary. Therefore as much as it can be an appealing idea, this tax is not a feasible option in the near future.

6 Summary and Concluding Remarks

The last decade witnessed several financial crises in emerging market economies. These events gave raise to a lively debate on the role of international capital flows. The well established idea that foreign investments foster growth has been questioned and the structure of financial markets in developing countries has been scrutinized. International institutions like the IMF and the World Bank reconsidered their position on unconditional capital account liberalization. The current discussion on the topic motivated me to approach it from a dynamic point of view.

This thesis analyzed the dynamic behavior of Scott and Uhlig (1999) fickle investors model. In their paper, presented in chapter 3, the authors outline a mechanism through which foreign investors can negatively affect long-run growth. This mechanism presupposes the existence of fickle investors without endogenising their behavior. In the same way, the authors make the assumption of imperfect financial market but do not rely on this hypothesis to explain their mean-to-volatility trade-off result. An increase in capital flows leads to an increase in the number of entrepreneurs and in the number of projects. At the same time the increase volatility associated with the capital flows raises the risk premium required by the subjects to embark into new productive activities rather than safely remain workforce. The combination of the two effects allows the determination of the maximizing amount of capital flows.

In chapter 4 I presented standard impulse-response analysis to a shock in foreign investments and to a negative shock in the number of projects. Further, simulations of the growth rate for different stochastic processes and parameters have been shown. The dynamic behavior of the model confirms the existence of a mean-volatility trade-

[37] Eichengreen, Tobin, and Wyplosz (1995), pg 165

off. Yet the determination of the optimal amount of foreign investments is not so straightforward. The key variable which determines the impact of overseas capital on growth is the volatility in rapport of the amount of capital. This implies that highly stable flows have a predominant positive "mean" effect and lead to an increase in growth. A shock in the financial conditions produces lasting real effects through the number of projects and can induce cycles. The amount of capital inflows, as measured by λ in the model, amplifies the cycles produced by the exogenous stochastic variable.

A different modeling of the stochastic process does not reveal major changes in the results. Also a different risk aversion does not.

The results support a more prundential approach towards capital market liberalizations for countries at an intermediate level of financial development. Governements should aim at encouraging stable capital inflows on the one hand and limit the more volatile kind of investments on the other. These policies should be implemented in a transparent way and be coupled with a reform of the domestic banking and financial sector. A solid and well supervised finacial sector is the best policy to attract productive investments and foster growth.

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A Appendix - the Scott and Uhlig (1999) model

This appendix is more detailed and extensive than the one in Scott and Uhlig (1999) as some of the derivations are quite laborious to carry out and it is maybe of help to be able to see all the relevant passages.

To derive the aggregate production function first maximize the dividends function of the single project over the units of labor:

$$d_{t,i} = \max_{n_{t,i}} \gamma_t n_{t,1}^{\alpha} - w_t n_{t,i}$$

$$\frac{\partial d_{t,i}}{\partial n_{t,i}} = \gamma_t \alpha n_{t,i}^{\alpha-1} - w_t = 0$$

multiply both sides of the equation by $n_{t,i}$

$$\gamma_t \alpha n_{t,i}^{\alpha} = w_t n_{t,i}$$

as the individual project's output $y_{t,i}$ is $\gamma_t n_{t,i}^{\alpha}$ the last equation becomes

$$\alpha y_{t,i} = w_t n_{t,i}$$

If labor is paid its marginal product, then $w_t = \alpha$ and therefore

$$\alpha \int_0^{q_t} y_{t,i} \mathbf{d} i = w_t \int_0^{q_t} n_{t,i}$$

and thus

$$\alpha y_t = w_t n_t$$

and by symmetry

$$(1 - \alpha) y_t = d_t q_t$$

A.1 Derivation of the number of entrepreneurs, e_t and of the number of projects, q_t

The arbitrage equation is

$$E_t[c^{(e)}_{t+1}] \equiv E_t[c^{(w)}_{t+1} + \pi_t w_{t+1}]$$

but w_{t+1} is known at time t as it is π_t, therefore $\pi_t w_{t+1}$ is given at date t and one can rewrite the arbitrage equation as

$$E_t[c^{(e)}_{t+1}] \equiv E_t[c^{(w)}_{t+1}] + \pi_t w_{t+1}$$

Now, inserting the relative definitions for the consumption of the entrepreneurs and of the workers the equation becomes

$$E_t[R_{t+1}w_t + d_{t+1} + p_{t+1}] = E_t[R_{t+1}w_t + \nu w_{t+1}] + \pi_t w_{t+1}$$

As expectations are additive and can be divided in different brackets, the term $E_t[R_{t+1}w_t]$ falls out. d_{t+1} and w_{t+1} are known at time t therefore one obtains

$$d_{t+1} + E_t[p_{t+1}] = (\nu + \pi_t)w_{t+1}$$

multiply both sides by the term $\frac{n_{t+1}}{d_{t+1}}$

$$n_{t+1} + \frac{n_{t+1}}{d_{t+1}}E_t[p_{t+1}] = (\nu + \pi_t)\frac{w_{t+1}n_{t+1}}{d_{t+1}}$$

But $w_t n_t = \alpha y_t$ and $d_t q_t = (1-\alpha)y_t$, and consequently $n_{t+1} = \frac{\alpha y_{t+1}}{w_{t+1}}$ and $d_{t+1} = \frac{(1-\alpha)y_{t+1}}{q_{t+1}}$ thus one can use the relation

$$\frac{n_{t+1}}{d_{t+1}} = \frac{\frac{\alpha y_{t+i}}{w_{t+1}}}{\frac{(1-\alpha)y_{t+1}}{q_{t+1}}} = \frac{\alpha}{1-\alpha}\frac{q_{t+1}}{w_{t+1}}$$

back in the arbitrage equation

$$n_{t+1} + \frac{\alpha}{1-\alpha}\frac{q_{t+1}}{w_{t+1}}E_t[p_{t+1}] = (\nu + \pi_t)\frac{\alpha}{1-\alpha}q_{t+1}$$

But $z_t w_t = p_t q_t$ and $z_{t+1}w_{t+1} = p_{t+1}q_{t+1}$ and therefore $p_{t+1} = \frac{z_{t+1}w_{t+1}}{q_{t+1}}$

$$n_{t+1} + \frac{\alpha}{1-\alpha}\frac{q_{t+1}}{w_{t+1}}E_t\Big[\frac{z_{t+1}w_{t+1}}{q_{t+1}}\Big] = (\nu+\pi_t)\frac{\alpha}{1-\alpha}q_{t+1}$$

Taking expectations on both sides and assuming $E_t\Big[Cov\Big(z_{t+1}, \frac{w_{t+1}}{q_{t+1}}\Big)\Big] = 0$ one obtains

$$E_t\Big[n_{t+1} + \frac{\alpha}{1-\alpha}\frac{q_{t+1}}{w_{t+1}}E_t[z_{t+1}]E_t\Big[\frac{w_{t+1}}{q_{t+1}}\Big]\Big] = (\nu+\pi_t)\frac{\alpha}{1-\alpha}E_t[q_{t+1}]$$

$$E_t\Big[n_{t+1} + \frac{\alpha}{1-\alpha}E_t[z_{t+1}]\Big] = (\nu+\pi_t)\frac{\alpha}{1-\alpha}E_t[q_{t+1}]$$

The expectation of an expectation is just an expectation

$$E_t\Big[n_{t+1} + \frac{\alpha}{1-\alpha}z_{t+1}\Big] = (\nu+\pi_t)\frac{\alpha}{1-\alpha}E_t[q_{t+1}]$$

But $q_t = (1-\delta)q_{t-1} + e_{t-1}$, therefore $q_{t+1} = (1-\delta)q_t + e_t$
and $n_t = 1 + \nu(1-e_{t-1})$, therefore $n_{t+1} = 1 + \nu(1-e_t)$
and n_{t+1} is known at time t, therefore $E_t[n_{t+1}] = n_{t+1}$
inserting these relations in the original equation one obtains

$$1 + \nu(1-e_t) + \frac{\alpha}{1-\alpha}E_t[z_{t+1}] = (\nu+\pi_t)\frac{\alpha}{1-\alpha}E_t[(1-\delta)q_t + e_t] \tag{14}$$

solving for e_t

$$1 + \nu - \nu e_t + \frac{\alpha}{1-\alpha}E_t[z_{t+1}] = (\nu+\pi_t)\frac{\alpha}{1-\alpha}(1-\delta)q_t + (\nu+\pi_t)\frac{\alpha}{1-\alpha}e_t$$

$$1 + \nu + \frac{\alpha}{1-\alpha}E_t[z_{t+1}] - (\nu+\pi_t)\frac{\alpha}{1-\alpha}(1-\delta)q_t = \nu e_t + (\nu+\pi_t)\frac{\alpha}{1-\alpha}e_t$$

$$1 + \nu + \frac{\alpha}{1-\alpha}E_t[z_{t+1}] - (\nu+\pi_t)\frac{\alpha}{1-\alpha}(1-\delta)q_t = e_t\Big(\frac{\nu+\alpha\pi_t}{1-\alpha}\Big)$$

the number of entrepreneurs is therefore given by

$$e_t = \frac{1}{\nu + \alpha\pi_t}\Big((1-\alpha)(1+\nu) + \alpha E_t[z_{t+1}] - \alpha(\nu+\pi_t)(1-\delta)q_t\Big)$$

Knowing that $q_{t+1} = (1-\delta)q_t + e_t$ and therefore $e_t = q_{t+1} - (1-\delta)q_t$ one can derive the equation for q_{t+1} substituting this for e_t

$$q_{t+1} - (1-\delta)q_t = \frac{1}{\nu + \alpha\pi_t}\Big((1-\alpha)(1+\nu) + \alpha E_t[z_{t+1}] - \alpha(\nu+\pi_t)(1-\delta)q_t\Big)$$

$$q_{t+1} = \frac{1}{\nu + \alpha\pi_t}\Big((1-\alpha)(1+\nu) + \alpha E_t[z_{t+1}] + \nu(1-\delta)(1-\alpha)q_t\Big)$$

A.2 Derivation of the steady state

To derive the steady state growth rate, let just first consider the production function

$$y_t = \gamma_t q_t^{1-\alpha} n_t^{\alpha}$$

and the growth rate of this economy given by

$$\frac{y_{t+1}}{y_t} = \Big(\frac{\gamma_{t+1}}{\gamma_t}\Big)\Big(\frac{q_{t+1}}{q_t}\Big)^{1-\alpha}\Big(\frac{n_{t+1}}{n_t}\Big)^{\alpha}$$

now, in the steady state, $e_{t+1} = e_t = \bar{e}$

$$\bar{q} = (1-\delta)\bar{q} + \bar{e}$$

and therefore in the steady state

$$\bar{q} = \frac{\bar{e}}{\delta}$$

$$\frac{n_{t+1}}{n_t} = \frac{1+\nu(1-\delta\bar{q})}{1+\nu(1-\delta\bar{q})} \Rightarrow = 1$$

$$\frac{q_{t+1}}{q_t} = (1-\delta) + \frac{\bar{e}}{\bar{q}}$$

$$\frac{q_{t+1}}{q_t} = (1-\delta) + \frac{\delta\bar{q}}{\bar{q}} \Rightarrow = 1$$

$$\frac{y_{t+1}}{y_t} = \Big(\frac{\gamma_{t+1}}{\gamma_t}\Big)(1)^{1-\alpha}(1)^{\alpha}$$

$$\frac{\gamma_{t+1}}{\gamma_t} = 1 + \psi\bar{e}$$

therefore the steady state growth rate is

$$\bar{g} = \frac{y_{t+1}}{y_t} = 1 + \psi\bar{e}$$

To solve for the steady-state growth path, Scott and Uhlig (1999) do the simplifying approximation that in the steady state the risk premium is a constant $\bar{\pi}$ "independent from the state of the economy or its parameter"[38]

Therefore equation (5) can be written as

$$\bar{e} = \frac{1}{\nu + \alpha\bar{\pi}}\Big((1-\alpha)(1+\nu) + \alpha E[z] - \alpha(\nu + \pi_t)(1-\delta)\frac{\bar{e}}{\delta}\Big)$$

$$\bar{e}\Big(\frac{(\nu + \bar{\pi}\alpha)\delta + (\nu + \bar{\pi})\alpha(1-\delta)}{(\nu + \bar{\pi}\alpha)\delta}\Big) = \frac{1}{\nu + \alpha\bar{\pi}}((1-\alpha)(1+\nu) + \alpha E[z])$$

$$\bar{e} = \frac{(1+\nu)(1-\alpha) + \alpha E[z]}{\nu + \frac{\nu\alpha}{\delta} + \frac{\bar{\pi}\alpha}{\delta} - \nu\alpha}$$

The expression for the steady state growth path of agents becoming entrepreneurs is

$$\bar{e} = \frac{1+\nu}{\nu}\frac{\frac{1}{\alpha} + \frac{1}{1+\nu}E[z] - 1}{\frac{1}{\alpha} + \frac{1}{\delta} + \frac{\bar{\pi}}{\delta\nu} - 1}$$

Looking at the denominator of this equation clarifies the assumption made by Scott and Uhlig (1999)

$$\frac{1}{\alpha} + \frac{1}{\delta} + \frac{\bar{\pi}}{\delta\nu} > 1$$

If this were not the case, then the denominator would be negative and therefore the steady state entrepreneurs fraction were also negative which is nonsense in the model.

[38]Scott and Uhlig, 1999, pg. 1352

A.3 Derivation of the risk premium, proof that it is positive and bounded

The utility function is of the form

$$u(c) = \frac{c^{1-\eta} - 1}{1-\eta}$$

From before

$$c_{t+1}^{(e)} = E_t[c_{t+1}^{(w)} + \pi_t w_{t+1}] + \varepsilon_{t+1,c^e}$$

and

$$c_{t+i}^{(w)} = E_t[c_{t+1}^{(w)}] + \varepsilon_{t+1,c^w}$$

Expected utility for an entrepreneur will simply mean plugging in $c_{t+1}^{(e)}$ into $u(c)$ above and taking a second order Taylor approximation of $E_t[u(c_{t+1}^{(e)})]$ and $E_t[u(c_{t+1}^{(w)})]$.

$$E_t\Big[\frac{(c_{t+1}^e)^{1-\eta} - 1}{1-\eta}\Big] = \frac{(E_t[c_{t+1}^w])^{1-\eta} - 1}{1-\eta} + \pi_t w_{t+1}(E_t[c_{t+1}^w])^{-\eta} - \frac{\eta}{2}(E_t[c_{t+1}^w])^{-\eta-1}\sigma_{t,c^e}^2$$

Likewise

$$E_t\Big[\frac{(c_{t+1}^w)^{1-\eta} - 1}{1-\eta}\Big] = \frac{(E_t[c_{t+1}^w])^{1-\eta} - 1}{1-\eta} - \frac{\eta}{2}(E_t[c_{t+1}^w])^{-\eta-1}\sigma_{t,c^w}^2$$

where $\sigma_{t,c^{(e)}}^2 = E_t[\epsilon_{t+1,c^{(e)}}^2]$ and $\sigma_{t,c^{(w)}}^2 = E_t[\epsilon_{t+1,c^{(w)}}^2]$

From the first equation derive an expression for $\frac{(E_t[c_{t+1}^w])^{1-\eta} - 1}{1-\eta}$

$$E_t\Big[\frac{(c_{t+1}^e)^{1-\eta} - 1}{1-\eta}\Big] - \pi_t w_{t+1}(E_t[c_{t+1}^w])^{-\eta} + \frac{\eta}{2}(E_t[c_{t+1}^w])^{-\eta-1}\sigma_{t,c^e}^2 = \frac{(E_t[c_{t+1}^w])^{1-\eta} - 1}{1-\eta}$$

and equate to the second expression

$$E_t\Big[\frac{(c_{t+1}^e)^{1-\eta} - 1}{1-\eta}\Big] - \pi_t w_{t+1}(E_t[c_{t+1}^w])^{-\eta} + \frac{\eta}{2}(E_t[c_{t+1}^w])^{-\eta-1}\sigma_{t,c^e}^2 =$$

$$= E_t\Big[\frac{(c_{t+1}^w)^{1-\eta} - 1}{1-\eta}\Big] + \frac{\eta}{2}(E_t[c_{t+1}^w])^{-\eta-1}\sigma_{t,c^w}^2$$

so, one obtains

$$E_t\Big[\frac{(c^e_{t+1})^{1-\eta}-1}{1-\eta}\Big]-E_t\Big[\frac{(c^w_{t+1})^{1-\eta}-1}{1-\eta}\Big] = \pi_t w_{t+1}(E_t[c^w_{t+1}])^{-\eta}+\frac{\eta}{2}(E_t[c^w_{t+1}])^{-\eta-1}(\sigma^2_{t,c^w}-\sigma^2_{t,c^e})$$

but from the arbitrage condition $E_t[u(c^e_{t+1})] = E_t[u(c^w_{t+1})]$ the left hand side results being zero

$$0 = \pi_t w_{t+1}(E_t[c^w_{t+1}])^{-\eta} + \frac{\eta}{2}(E_t[c^w_{t+1}])^{-\eta-1}(\sigma^2_{t,c^w} - \sigma^2_{t,c^e})$$

solving for π_t, one obtains

$$\pi_t = \frac{\eta}{2}\frac{\sigma^2_{t,c^e} - \sigma^2_{t,c^w}}{w_{t+i}E_t[c^w_{t+1}]}$$

Proof that the risk premium is positive

To prove that π_t is positive it has to be shown that $\sigma^2_{t,c^e} > \sigma^2_{t,c^w}$

$$c^{(e)}_{t+1} = R_{t+1}\Big(w_t + \frac{p_t}{1-\delta}\Big)$$

$$E_t[c^{(e)}_{t+1}] = E_t[R_{t+1}]\Big(w_t + \frac{p_t}{1-\delta}\Big)$$

$$c^{(e)}_{t+1} - E_t[c^{(e)}_{t+1}] = \Big(w_t + \frac{p_t}{1-\delta}\Big)(R_{t+1} - E_t[R_{t+1}])$$

Let $c^{(e)}_{t+1} - E_t[c^{(e)}_{t+1}] = \varepsilon_{t+1,c^e}$ and $R_{t+1} - E_t[R_{t+1}] = \varepsilon_{t+1,R}$, thus

$$\varepsilon_{t+1,c^e} = \Big(w_t + \frac{p_t}{1-\delta}\Big)\varepsilon_{t+1,R}$$

and from before

$$c^{(w)}_{t+i} = R_{t+1}w_t + \nu w_{t+1}$$

$$E_t[c^{(w)}_{t+i}] = E_t[R_{t+1}]w_t + \nu w_{t+1}$$

again, subtracting the expectations from the first equation and let $c^{(w)}_{t+i} - E_t[c^{(w)}_{t+i}] = \varepsilon_{t+1,c^w}$ one obtains

$$\varepsilon_{t+1,c^w} = \varepsilon_{t+1,R} w_t$$

then I need to demonstrate that $(\varepsilon_{t+1,c^e})^2 > (\varepsilon_{t+1,c^w})^2$

$$\Big(w_t + \frac{p_t}{1-\delta}\Big)^2 \varepsilon_{t+1,R}^2 > \varepsilon_{t+1,R}^2 w_t^2$$

$$\Big(\frac{p_t}{1-\delta}\Big)^2 + 2\frac{w_t p_t}{1-\delta} > 0$$

which is always true as $\delta \in [0,1)$

A.4 Derivation of the equation for π_t in the form given in the text

Remembering that

$$R_{t+1} = (1-\delta)\frac{d_{t+1} + p_{t+1}}{p_t}$$

and plugging

$$p_{t+1} = \frac{z_{t+1} w_{t+1}}{p_t q_{t+1}}$$

into the equation for R_{t+1} and as usual deriving $\varepsilon_{t+1,z} = z_{t+1} - E_t[z_{t+1}]$

$$\varepsilon_{t+1,R} = \Big(\frac{w_{t+1}}{p_t q_{t+1}}\Big)\varepsilon_{t+1,z}$$

knowing that $Var(cx) = c^2 Var(x)$ then

$$\sigma_{t,c^e}^2 = \Big(w_t + \frac{p_t}{1-\delta}\Big)^2 \sigma_R^2$$

and

$$\sigma_{t,c^w}^2 = w_t^2 \sigma_R^2$$

and

$$\sigma_R^2 = \Big(\frac{w_{t+1}}{p_t q_{t+1}}\Big)^2 \sigma_z^2$$

plugging these into the expression for π_t

$$\pi_t = \frac{\eta\Big(\Big(w_t + \frac{p_t}{1-\delta}\Big)^2 - w_t^2\Big)\sigma_R^2}{2w_{t+1}E_t[c_{t+1}^w]}$$

$$\pi_t = \frac{\eta\Big(\frac{p_t^2}{(1-\delta)^2} + 2\frac{w_t p_t}{1-\delta}\Big)\frac{w_{t+1}^2}{p_t^2 q_{t+1}^2}\sigma_z^2}{2w_{t+1}E_t[c_{t+1}^w]}$$

$$\pi_t = \eta\frac{w_{t+1}}{E_t[c_{t+1}^w]}\frac{1}{2}\Big(\frac{1}{(1-\delta)^2 q_{t+1}^2} + \frac{2w_t}{(1-\delta)p_t q_{t+1}^2}\Big)\sigma_z^2$$

Now, $z_t w_t = p_t q_t$, therefore $\frac{w_t}{p_t} = \frac{q_t}{z_t}$

$$\pi_t = \eta\frac{w_{t+1}}{E_t[c_{t+1}^w]}\frac{0.5 + (q_t/z_t)(1-\delta)}{(1-\delta)^2 q_{t+1}^2}\sigma_z^2$$

From the production function $\frac{(1-\alpha)y_{t+1}}{\alpha y_{t+1}} = \frac{d_{t+1}q_{t+1}}{w_{t+1}n_{t+1}}$ therefore

$$\frac{1-\alpha}{\alpha}\frac{w_{t+1}n_{t+1}}{d_{t+1}} = q_{t+1}$$

Moreover since $E_t[c_{t+1}^e] = E_t[R_{t+1}](w_t + p_t/(1-\delta))$ and $E_t[c_{t+1}^e] = E_t[c_{t+1}^w] + \pi_t w_{t+1}$ and $E_t[c_{t+1}^w] = E_t[R_{t+1}]w_t + \nu w_{t+1}$

$$E_t[R_{t+1}]\Big(w_t + \frac{p_t}{1-\delta}\Big) = E_t[R_{t+1}]w_t + \nu w_{t+1} + \pi_t w_{t+1}$$

solve for $E_t[R_{t+1}]$ and plug it back into the expression for $E_t[c_{t+1}^w]$

$$E_t[c_{t+1}^w] = w_{t+1}\Big(\frac{1-\delta}{p_t}w_t(\nu + \pi_t) + \nu\Big)$$

plugging these results into the expression for π_t and knowing that $w_t = \frac{p_t q_t}{z_t}$ one obtains

$$\pi_t = \eta\frac{(1-\delta)(q_t/z_t) + 0.5}{((1-\delta)(q_t/z_t)(\nu + \pi_t) + \nu)q_{t+1}^2(1-\delta)^2}\sigma_{t,z}^2$$

simplyfing a bit further one gets

$$\pi_t = \eta \frac{(1-\delta)(q_t/z_t) + 0.5}{((1-\delta)(q_t/z_t)(\nu+\pi_t)((1-\delta)q_t + e_t) + \nu q_{t+1})q_{t+1}(1-\delta)^2} \sigma^2_{t,z}$$

recalling the equation () in the appendix, the denominator of the last expression can be changed to obtain

$$\pi_t = \eta \frac{(1-\delta)(\frac{q_t}{z_t}) + 0.5}{\left((\frac{q_t}{z_t})(\frac{1-\alpha}{\alpha}(1+\nu(1-e_t)) + E_t[z_{t+1}]) + \nu q_{t+1}\right)q_{t+1}(1-\delta)^2} \sigma^2_{t,z}$$

Proof that the risk premium is bounded

If both the numerator and the denominator of eq.(9) are multiplied by z_t

$$\pi_t = \eta \frac{(1-\delta)q_t + 0.5 z_t}{\left(q_t(\frac{1-\alpha}{\alpha}(1+\nu(1-e_t)) + E_t[z_{t+1}]) + \nu q_{t+1} z_t\right)q_{t+1}(1-\delta)^2} \sigma^2_{t,z}$$

and then look what happens if $z_t \to 0$, one obtains

$$\pi_t \to \eta \frac{\alpha}{(1-\delta)(1-\alpha)q_{t+1}(1+\nu(1-e_t))} \sigma^2_{t,z}$$

For $z_t \to \infty$, eq.(9) yields

$$\pi_t = \eta \frac{1}{(1-\delta)^2 \nu q^2_{t+1}} \sigma^2_{t,z}$$

Therefore for q_{t+1} strictly bounded from below $q_{t+1} > \underline{q}$ and for a $\sigma^2_{t,z}$ bounded from above, π_t is also bounded.

Derivation of an explicit quadratic equation in π_t

Inserting equations (5) and (4) in the equation for π_t (9) one obtains an enormous expression which can be reduced to an explicit quadratic equation in π_t as given in Scott and Uhlig (1999). The expression can be reduced into humans dimensions using the following abbreviations:[39]

$$D_t = (1+\nu)(1-\alpha) + \nu(1-\alpha)(1-\delta)q_t + \alpha E_t[z_{t+1}]$$

$$F_t = \eta((1-\delta)(q_t/z_t) + 0.5)\sigma^2_{t,z}$$

[39] also in this case I use Scott and Uhlig notation.

then

$$q_{t+1} = \frac{1}{\nu + \pi_t \alpha} D_t$$

first insert q_{t+1}

$$\pi_t \Big[(1-\delta)^2 \frac{D_t}{\nu + \pi_t \alpha} \Big(\frac{q_t}{z_t} \Big(\frac{1-\alpha}{\alpha} (1 + \nu(1 - e_t)) + E_t[z_{t+1}] \Big) + \frac{\nu D_t}{\nu + \pi_t \alpha} \Big) \Big] = F_t$$

and then e_t

$$\pi_t \Big[(1-\delta)^2 \frac{D_t}{\nu + \pi_t \alpha} \Big(\frac{q_t}{z_t} \Big(\frac{1-\alpha}{\alpha} (1 + \nu - \frac{\nu}{\nu + \pi_t \alpha}$$

$$\Big((1-\alpha)(1+\nu) + \alpha E_t[z_{t+1}] - \alpha(\nu + \pi_t)(1-\delta) q_t \Big) + E_t[z_{t+1}] \Big]$$

$$= F_t(\nu + \pi_t \alpha) - \frac{\nu \pi_t D_t^2 (1-\delta)^2}{\nu + \pi_t \alpha}$$

As the algebra involved here is not too difficult I avoid reporting all the passages, and just report the final equation as given in Scott and Uhlig (1999).

$$A_t \pi_t^2 + B_t \pi_t + C_t = 0$$

where

$$A_t = (1-\delta)^2 (1-\alpha) D_t \frac{q_t}{z_t} \Big(\nu(1-\delta) q_t + 1 + \nu + \frac{E_t[z_{t+1}]}{1-\alpha} \Big) - \alpha^2 F_t$$

$$B_t = (1-\delta)^2 D_t \Big(\nu \frac{1-\alpha}{\alpha} \frac{q_t}{z_t} \Big(\alpha \nu (1-\delta) q_t - (1+\nu)(1-\alpha)$$

$$-\alpha E_t[z_{t+1}] + 1 + \nu + \frac{\alpha}{1-\alpha} E_t[z_{t+1}] \Big) + \nu D_t \Big) - 2\alpha \nu F_t$$

$$C_t = -\nu^2 F_t$$

The solutions is then given by the characteristic equation:

$$\pi_t^{(1,2)} = \frac{-1}{2A_t} (B_t \pm \sqrt{B_t^2 - 4 A_t C_t})$$

A.5 Derivation of the steady state number of projects $\bar{q}$

Considering eq.(6) and substituting for $\bar{e} = \delta\bar{q}$ one obtains an equation for $\bar{q}$ which look like the following:

$$\bar{q} = \frac{\chi_1}{\chi_2 + \bar{\pi}} \tag{15}$$

where

$$\chi_1 = \frac{1+\nu}{\alpha} + E[z] - (1+\nu)$$

and

$$\chi_2 = \frac{\nu\delta}{\alpha} + \nu(1-\delta)$$

Equation (9) in the steady state can be rewritten as:

$$\bar{\pi} = \frac{\chi_3 + \chi_4\bar{q}}{\bar{q}(\chi_5 + \chi_6\bar{q} + \chi_7\bar{q}^2)} \tag{16}$$

where

$$\chi_3 = 0.5\sigma_{t,z}^2\eta$$

$$\chi_4 = \frac{\sigma_{t,z}^2\eta(1-\delta)}{\bar{z}}$$

$$\chi_5 = 0$$

$$\chi_6 = \frac{1}{(1-\delta)^2}\Big(\frac{(1-\alpha)(1+\nu)}{\alpha\bar{z}} + E[z] + \nu\Big)$$

$$\chi_7 = \frac{1}{(1-\delta)^2}\Big(\frac{1-\alpha}{\alpha}\delta\nu\Big)$$

Multiplying equations (15) and (16) for the respective denominators leads to the two equations

$$\bar{\pi}\bar{q} = \chi_1 - \chi_2\bar{q}$$

$$\bar{\pi}\bar{q}(\chi_5 + \chi_6\bar{q} + \chi_7\bar{q}^2) = \chi_3 + \chi_4\bar{q}$$

Substituting the first equation into the second one obtains a third-order polynomial in $\bar{q}$[40]

$$\chi_1\chi_5 - \chi_3 + (\chi_1\chi_6 - \chi_2\chi_5 - \chi_4)\bar{q} + (\chi_1\chi_7 - \chi_2\chi_6)\bar{q}^2 - \chi_2\chi_7\bar{q}^3 = 0$$

A.6 A question of approximation

It has been shown in chapter 3.4 that the correct Taylor expansion for the expected utility of the entrepreneurs is:

$$E_t\Big[\frac{(c^e_{t+1})^{1-\eta} - 1}{1-\eta}\Big] = \frac{(E_t[c^w_{t+1}])^{1-\eta} - 1}{1-\eta} + \pi_t w_{t+1}(E_t[c^w_{t+1}])^{-\eta} - \frac{\eta}{2}(E_t[c^w_{t+1}])^{-\eta-1}(\sigma^2_{t,c^e} + \pi^2_t w^2_{t+1})$$

while the expression for the expected utility of the workers is:

$$E_t\Big[\frac{(c^w_{t+1})^{1-\eta} - 1}{1-\eta}\Big] = \frac{(E_t[c^w_{t+1}])^{1-\eta} - 1}{1-\eta} - \frac{\eta}{2}(E_t[c^w_{t+1}])^{-\eta-1}\sigma^2_{t,c^w}$$

Considering that deriving π_t not dropping the term $\pi_t w_{t+1}$ will lead to different results I call the new risk premium $\tilde{\pi}_t$ to distinguish it from the old one. Subtracting the second equation from the first one yields:

$$0 = \tilde{\pi}_t w_{t+1} E_t[c^w_{t+1}]^{-\eta} - \frac{\eta}{2} E_t[c^w_{t+1}]^{-\eta-1}(\sigma^2_{t,c^e} + \tilde{\pi}^2_t w^2_{t+1} - \sigma^2_{t,c^w})$$

solving for $\tilde{\pi}_t$

[40] Scott and Uhlig (1999) have a couple of small typing errors in this equation (for example: why to include χ_5 in the equation is if this is zero?). This nevertheless does not affect their results as this equation is not used to derive the steady state values used in their analysis. They prefer to use the second order equation for $\bar{\pi}$ knowing that only one solution is significant (the positive one) and to reiterate the calculations until the result does converge to the steady state.

$$\tilde{\pi}_t^2 w_{t+1}^2 (\frac{\eta}{2} E_t[c_{t+1}^w]^{-\eta-1}) = \tilde{\pi}_t w_{t+1} E_t[c_{t+1}^w]^{-\eta} - \frac{\eta}{2} E_t[c_{t+1}^w]^{-\eta-1}(\sigma_{t,c^e}^2 - \sigma_{t,c^w}^2)$$

$$\tilde{\pi}_t^2 - \Big(\frac{2}{\eta}\frac{E_t[c_{t+1}^w]}{w_{t+1}}\Big)\tilde{\pi}_t + \frac{(\sigma_{t,c^e}^2 - \sigma_{t,c^w}^2)}{w_{t+1}^2} \tag{17}$$

Where $E_t[c_{t+1}^w] = E_t[R_{t+1}w_t + \nu w_{t+1}]$ and $\sigma_{t,c^e}^2 = (w_t + \frac{p_t}{(1-\delta)})^2 \sigma_{t,R}^2$, $\sigma_{t,c^w}^2 = w_t^2 \sigma_{t,R}^2$ where $\sigma_{t,R}^2 = \Big(\frac{w_{t+1}^2}{p_t q_{t+1}}\Big)\sigma_z^2$

therefore

$$\tilde{\pi}_t^2 - \Big(\frac{2}{\eta}\Big[\frac{E_t[R_{t+1}]w_t}{w_{t+1}} + \nu\Big]\Big)\tilde{\pi}_t + \frac{1}{w_{t+1}^2}\Big(\frac{w_{t+1}^2}{p_t^2 q_{t+1}^2}\sigma_z^2\Big[\Big(w_t + \frac{p_t}{(1-\delta)}\Big)^2 - w_t^2\Big]\Big) = 0$$

$$\tilde{\pi}_t^2 - \Big(\frac{2}{\eta}\Big[\frac{E_t[R_{t+1}]w_t}{w_{t+1}} + \nu\Big]\Big)\tilde{\pi}_t + \frac{\sigma_z^2}{p_t^2 q_{t+1}^2}\Big(\frac{p_t^2}{(1-\delta)^2} + \frac{2w_t p_t}{(1-\delta)}\Big) = 0$$

recalling that $w_t/p_t = q_t/z_t$

$$\tilde{\pi}_t^2 - \Big(\frac{2}{\eta}\Big[\frac{E_t[R_{t+1}]w_t}{w_{t+1}} + \nu\Big]\Big)\tilde{\pi}_t + \frac{\sigma_z^2}{q_{t+1}^2}\Big(\frac{1 + 2\frac{q_t}{z_t}(1-\delta)}{(1-\delta)^2}\Big) = 0 \tag{18}$$

substituting in for $E_t[R_{t+1}] = \frac{E_t[d_{t+1}+p_{t+1}]}{p_t}$ and knowing that $w_t/p_t = q_t/z_t$

$$\tilde{\pi}_t^2 - \Big(\frac{2}{\eta}\Big[\frac{E_t[d_{t+1}+p_{t+1}]}{w_{t+1}}\frac{q_t}{z_t} + \nu\Big]\Big)\tilde{\pi}_t + \frac{\sigma_z^2}{q_{t+1}^2}\Big(\frac{1 + 2\frac{q_t}{z_t}(1-\delta)}{(1-\delta)^2}\Big) = 0$$

knowing that $d_{t+1} = \frac{(1-\alpha)y_{t+1}}{q_{t+1}}$ and $p_{t+1} = \frac{E_t[z_{t+1}]w_{t+1}}{q_{t+1}}$

$$\tilde{\pi}_t^2 - \Big(\frac{2}{\eta}\Big[\Big(\frac{(1-\alpha)}{\alpha}\frac{n_{t+1}}{q_{t+1}} + \frac{E_t[z_{t+1}]}{q_{t+1}}\Big)\frac{q_t}{z_t} + \nu\Big]\Big)\tilde{\pi}_t + \frac{\sigma_z^2}{q_{t+1}^2}\Big(\frac{1 + 2\frac{q_t}{z_t}(1-\delta)}{(1-\delta)^2}\Big) = 0$$

Derivation of an explicit equation for $\tilde{\pi}_t$

Using the same definition for D_t as on the section on the original π_t, q_{t+1} can be defined as $q_{t+1} = \frac{D_t}{\nu + \tilde{\pi}_t \alpha}$ substituting one obtains

$$\tilde{\pi}_t^2 - \Big(\frac{2}{\eta}\Big(\frac{(\nu + \tilde{\pi}_t\alpha)(1-\alpha)}{\alpha D_t}\Big((1 + \nu - \nu e_t) + \alpha E_t[z_{t+1}]\Big)\frac{q_t}{z_t} + \nu\Big)\Big)\tilde{\pi}_t$$

$$+\frac{(\nu+\tilde{\pi}_t\alpha)^2\sigma_z^2}{D_t^2}\Big(\frac{1+2\frac{q_t}{z_t}(1-\delta)}{(1-\delta)^2}\Big)=0$$

substituting for e_t yields a second order polynomial which can be rewritten as:

$$A_t\tilde{\pi}_t^2+B_t\tilde{\pi}_t+C_t=0$$

where

$$A_t=1+\frac{\alpha^2\sigma_z^2N_t}{D_t^2}-\frac{2}{\eta}\frac{(1-\alpha)}{D_t}\frac{q_t}{z_t}\Big((1+\nu)+\alpha\nu(1-\alpha)q_t+\frac{\alpha}{(1-\alpha)}E_t[z_{t+1}]\Big)$$

$$B_t=\frac{2\nu\alpha\sigma_z^2N_t}{D_t^2}-\frac{2}{\eta}\nu-\frac{2}{\eta}\frac{\nu}{D_t}\frac{q_t}{z_t}(1-\alpha)\Big(\frac{(1+\nu)}{\alpha}-\frac{(1-\alpha)}{\alpha}(1+\nu)+\nu(1-\delta)q_t+\alpha E_t[z_{t+1}]\Big)$$

$$C_t=\frac{\nu^2\sigma_z^2N_t}{D_t^2}$$

and

$$N_t=\Big(\frac{1+2\frac{q_t}{z_t}(1-\delta)}{(1-\delta)^2}\Big)$$

$$D_t=(1+\nu)(1-\alpha)+\nu(1-\alpha)(1-\delta)q_t+\alpha E_t[z_{t+1}]$$

A.7 The Scott and Uhlig (1999) MATLAB programs

The following programs were provided by Scott and Uhlig (1999)

```
% PARAMS.M sets some parameters

% Global parameters

nu = 1;

alpha = 2/3;

z_bar = 1;

delta = 0.1;
```

psi = 0.2;

sig_x = 1;

eta = 5;

sig_z = 0;

pi_bar = 0;

chi = 0;

xi = 1;

% Some initialization for some routines

max_lambda = 1;

min_lambda = 0;

ind_lam = 0;

min_q = 0;

max_q = 1/delta;

warning = 0;

==

% mean_var.m calculates some things for the mean-variance model

```
% mean_var.m calculates some things for the mean-variance model

DISP_PI = 0;

DISP_WARNING = 1;

DISP_CRITICAL = 0;

D_t = (1+nu)*(1-alpha)+nu*(1-alpha)*(1-delta)*q_t + alpha*(expec_z + chi);

F_t = eta*((1-delta)*q_t/z_t + 0.5)*sig2_z;
```

```
A_t = (1-delta)^2*(1-alpha)*D_t*(q_t/z_t)*( nu*(1-delta)*q_t+1+nu+...

      expec_z/(1-alpha) ) - alpha^2 * F_t;

B_t = (1 - delta)^2 * D_t * (nu*((1-alpha)/alpha) *(q_t/z_t) * ...

   (alpha*nu*(1-delta)*q_t - (1+nu)*(1-alpha)-alpha*(expec_z + chi) ...

   + 1 + nu + (alpha/(1-alpha))*expec_z ) + nu*D_t) - 2*alpha*nu*F_t;

C_t = - nu^2 * F_t;

warning = (B_t^2 - 4*A_t*C_t < 0);

if warning & DISP_WARNING,

   disp('MEAN_VAR.M: WARNING!  Complex-valued risk premium!');

end;
if abs(A_t / B_t) < .000001,

   pi_t1 = - C_t / B_t;

   pi_t2 = pi_t1;

else

   pi_t1 = (-1.0/(2*A_t)) * (B_t - sqrt( B_t^2 - 4*A_t*C_t) );

   pi_t2 = (-1.0/(2*A_t)) * (B_t + sqrt( B_t^2 - 4*A_t*C_t) );

end;

warning = warning | ( (pi_t1 < 0) & (pi_t2 < 0) );

critical = (abs(1-alpha) < .000001) | (abs(alpha) < .000001) |(abs(A_t) < .000001) | (abs(z_t) < .000001);

if DISP_PI | (DISP_CRITICAL & critical),

   disp(sprintf('pi_t1 = %6.3f, pi_t2 = %6.3f, A_t = %6.3f',pi_t1,pi_t2,A_t));

end;

==============================================================================

% dyn.m calculates the dynamics, starting from some specified
```

```
% q_t, and using z_bar

DO_PLOTDYN = 1;

HORIZON = 10;

% DYN_CHOICE = 0;

% disp('To get anything interesting, change DYN_CHOICE in dyn.m here!!');
z_arr = zeros(1,HORIZON) + z_bar;

sig_arr = zeros(1,HORIZON) + sig_z;

q_t = q_bar;
if DYN_CHOICE == 1;

   q_t = q_t*1.1;

elseif DYN_CHOICE == 2,

   z_arr(2) = z_arr(2) + 0.1;

elseif DYN_CHOICE == 3,

   z_arr = z_arr + 0.1;

   z_arr(1) = z_bar;

elseif DYN_CHOICE == 4,

   sig_arr = zeros(1,HORIZON) + sig_z;

   sig_arr(2) = sig_arr(2) + .1;

elseif DYN_CHOICE == 5,

   sig_arr = sig_arr + .1;

   sig_arr(1) = sig_z;

else

end;

q_arr = zeros(1,HORIZON);

e_arr = zeros(1,HORIZON);
```

```
pi_arr = zeros(1,HORIZON);

mx_hor = 1;

no_further = 0;

for j_hor = 1 : HORIZON,

   q_arr(j_hor) = q_t;

   z_t = z_arr(j_hor);

   sig2_z = (sig_arr(j_hor))^2;

   calc_pi;

   % pi_t1 = pi_bar;

   if (~warning) & (~no_further),

      pi_arr(j_hor) = pi_t1;

      e_nxt = (1/(nu+pi_t1*alpha)) * ...

         ((1+nu)*(1-alpha)-(nu+pi_t1)*alpha*(1-delta)*q_t + alpha*z_arr(j_hor));

      e_nxt = max(min(1,e_nxt),0);

      e_arr(j_hor) = e_nxt;

      q_t = (1 - delta)*q_t + e_nxt;

      mx_hor = j_hor;

   else

      no_further = 1;

      % disp(sprintf('j_hor=%5.0f:imaginary number!',j_hor));

   end;

end;

if DO_PLOTDYN,

   hndl = plot(1:mx_hor,100*e_arr(1:mx_hor));
```

```
   xlabel('Time');

   ylabel('Entrepreneurs (in %)');

if DYN_CHOICE == 1;

   title('Initial q_t 10 percent above steady state');

elseif DYN_CHOICE == 2,

   title('Temporary increase of z by 0.1');

elseif DYN_CHOICE == 3,

   title('Permanent increase of z by 0.1');

elseif DYN_CHOICE == 4,

   title('Temporary increase of \sigma_z by 0.1');

elseif DYN_CHOICE == 5,

   title('Permanent increase of \sigma_z by 0.1');

else

   title('Dynamics');

end;

set(hndl,'LineWidth',20);

ph=get(hndl,'Parent');

set(ph,'FontSize',18);

th=get(ph,'Title');set(th,'FontSize',18);

xh=get(ph,'XLabel');set(xh,'FontSize',18);

yh=get(ph,'YLabel');set(yh,'FontSize',18);

end;

====================================================================

% findmax finds the maximum lambda for given xi by
```

```
% calling hump twice.
min_lambda = 0;

max_lambda = 5;

hump;

[g,ind_lam] = max(gro_tab);

if ind_lam == max(size(gro_tab)),

   min_lambda = lam_tab(max(1,ind_lam - 2));

   max_lambda = lam_tab(ind_lam);

elseif ind_lam == 1,

   min_lambda = lam_tab(ind_lam);

   max_lambda = lam_tab(min(ind_lam+2,max(size(lam_tab))));

else

   min_lambda = lam_tab(ind_lam-1);

   max_lambda = lam_tab(ind_lam+1);

end;

hump;

[g,ind_lam] = max(gro_tab);

z_bar = lam_tab(ind_lam) + 1;

sig_z = lam_tab(ind_lam) * xi;

iterate;

====================================================================

% HUMP calculates the growth rate as a function of

% an increase in the external funds, which linearly increases

% its standard deviations.  There will typically be a hump

% shaped relationship because the benefits ("mean effect") are
```

```
% linear in the mean external funds, whereas the costs ("variance

% effect") are quadratic in the level of funds: pi is proportional

% to the variance, not the standard deviation of the fickle funds.
DO_MAX = 0;

DO_PLOTHUMP = 0;
if DO_MAX,

   if xi > 1,

      max_lambda = 1/(xi-1); %cutoff at std.dev. = funds

   end;

end;

lam_tab = min_lambda + (0 : .01 : 1)*(max_lambda-min_lambda);

sz_tab = max(size(lam_tab));

gro_tab = lam_tab * 0;

mx_j = 1;

no_further = 0;

for j_lam = 1 : sz_tab,

   z_bar = lam_tab(j_lam) + 1;

   sig_z = lam_tab(j_lam) * xi;

   iterate;

   if (~warning) & (~no_further),

      gro_tab(j_lam) = 100*(gamma_bar - 1);

      mx_j = j_lam;

   else

      no_further = 1;

      % disp(sprintf('j_lam=%5.0f:imaginary number!',j_lam));
```

```
   end;

end;

======================================================================

% iterate iterates on the risk premium to calculate the

% steady state growth rate.
DISP_STEPS = 0;

DISP_ITER = 0;
nn = 5;

steady;

j = 0;

pi_bar = 0;

pi_t1 = pi_bar;

if DISP_STEPS,

   disp(sprintf('j=%3.0f, pi_bar = %6.3f, q_bar = %6.3f, growth rate

= %6.3f',j,pi_bar,q_bar,100*(gamma_bar - 1)));

end;

for j = 1 : nn,

   pi_old = pi_bar;

   calc_pi;

   pi_bar = real(pi_t1);

   steady;

   if DISP_STEPS,

      disp(sprintf('j=%3.0f, pi_bar = %6.3f, q_bar = %6.3f, growth rate

= %6.3f',j,pi_bar,q_bar,100*(gamma_bar - 1)));

   end;
```

```
end;

warning = warning | (abs(pi_old - pi_bar) > 0.01);
if DISP_ITER,

   disp(sprintf('j=%3.0f, pi_bar = %6.3f, growth rate = %6.3f',j,pi_bar,100*(gamma_bar - 1)));

end;

======================================================================

% q_func plots the mapping q -> pi -> steadystate q
% you need to specify min_q and max_q
q_tab = min_q + (0 : .001 : 1)*max_q;

pi_bar_tab = 0*q_tab;

q_bar_tab = 0*q_tab;

e_nxt_tab = 0*q_tab;

for q_ind = 1 : max(size(q_tab)),

   q_t = q_tab(q_ind);

   calc_pi;

   pi_bar = real(pi_t1);

   e_nxt = (1/(nu+pi_bar*alpha)) * ...

      ((1+nu)*(1-alpha)-(nu+pi_bar)*alpha*(1-delta)*q_t + alpha*z_bar);

   e_nxt_tab(q_ind) = max(min(1,e_nxt),0);

   steady;

   pi_bar_tab(q_ind) = pi_bar;

   q_bar_tab(q_ind) = q_bar;

end;
%plot(q_tab,q_bar_tab,q_tab,q_tab,'--');
scl=100*delta;

plot(scl*q_tab,scl*q_bar_tab,scl*q_tab,scl*q_tab,'--',...
```

```
   scl*q_tab,100*e_nxt_tab,'-.',scl*q_tab,100 + 0*q_tab);

xlabel('e-bar in percent');

ylabel('e-bar in percent');

======================================================================

% STEADY.M calculates the steady state, given pi_bar and chi



DISP_STEADY = 0;

DISP_CRITICAL = 0;
sig2_z = sig_z^2;

denominator = 1/alpha + 1/delta + pi_bar/(delta*nu) - 1;

numerator = 1/alpha + (z_bar + chi)/(1+nu) - 1;

if (abs(denominator/numerator) < (1+nu)/nu),

   e_bar = 1;
else
   e_bar = ((1+nu)/nu) * numerator/denominator;

end;

e_bar = max(0,min(e_bar,1));

q_bar = e_bar/delta;

n_bar = 1 + nu*(1-e_bar);

gamma_bar = 1 + psi * e_bar;

rho_bar = (alpha/(1-alpha)) * (1/(z_bar*n_bar));

sig_R = 100*gamma_bar*sig_z/z_bar;
q_t = q_bar;

e_t = e_bar;

z_t = z_bar;

expec_z = z_bar;
critical = (abs(denominator) < .000001) | (abs(z_bar) < .000001);
```

```
if DISP_STEADY | (DISP_CRITICAL & critical),

   disp(sprintf('e_bar = %6.2f',e_bar));

   disp(sprintf('q_bar = %6.2f',q_bar));

   disp(sprintf('gamma_bar = %6.2f',gamma_bar));

   disp(sprintf('rho_bar = %6.2f',rho_bar));

   disp(sprintf('z_bar = %6.2f, denominator = %6.2f, pi_bar = %6.2f',z_bar,denominator,pi_bar));

end;

=======================================================================

% table1 creates a table of results, varying z_bar and sig_z

% You need to call params.m first.
z_arr = [1, 1.1, 1.2 ];

sig_arr = [ 0, .1, .2, .3, .4 ];
row_tab = sig_arr;

col_tab = z_arr;
DO_TAB1 = 1;

DO_TAB2 = 0;
tabulate;

========================================================================

% TABLE2 calculates the point of maximal growth for various

% values of xi, which is the "fickle standard deviation per

% unit of outside investment".  It also varies nu
nu_arr = [1,2,3];

xi_arr = [0.5, 1, 2, 5];

DO_TAB2 = 1;

DO_TAB1 = 0;
col_tab = nu_arr;

row_tab = xi_arr;
```

```
rows = max(size(row_tab));
cols = max(size(col_tab));
res_l0 = zeros(1,2*cols);
res_l1 = zeros(1,2*cols);
res_l2 = zeros(1,2*cols);
if DO_TAB2,
   disp('Rows: eta, Columns: xi');
end;
disp('Boxes:');
disp('z_bar, sig_z');
disp('growth-rate, e-bar');
disp('premium pi , sig_R');
disp(' ');
disp(['       ',sprintf('& %5.2f &       ',col_tab),' \\ \hline']);
for i_row = 1 : rows,
   for j_col = 1 : cols,
     if DO_TAB2,
         eta = col_tab(j_col);
         xi = row_tab(i_row);
         findmax;
      end;
      gro_rate = 100*(gamma_bar - 1);
      res_l0(2*j_col-1) = z_bar;
      res_l0(2*j_col) = sig_z;
```

```
      res_l1(2*j_col-1) = gro_rate;

      res_l1(2*j_col) = 100*e_bar;

      res_l2(2*j_col-1) = 100*pi_bar;

      res_l2(2*j_col) = sig_R;

   end;

   disp([sprintf('%5.2f  ',row_tab(i_row)),sprintf('& %5.2f ',res_l0),' \\']);

   disp([sprintf('       ',row_tab(i_row)),sprintf('& %5.2f ',res_l1),' \\']);

   disp(['       ',sprintf('& %5.2f ',res_l2),' \\ \hline']);

end;

============================================================================

% tradeoff.m plots the mean-variance tradeoff, using params.m
params;

hump;


pi_bar = 0;
DO_MAX = 0;
if DO_MAX,

   if xi > 1,

      max_lambda = 1/(xi-1); %cutoff at std.dev. = funds

   end;

end;
lam_tab = min_lambda + (0 : .01 : 1)*(max_lambda-min_lambda);

sz_tab = max(size(lam_tab));

gro_tab = lam_tab * 0;

gro_mean = 0 * gro_tab;

gro_var = 0 * gro_tab;
```

```
mx_j = 1;

no_further = 0;

for j_lam = 1 : sz_tab,

   z_bar = lam_tab(j_lam) + 1;

   sig_z = lam_tab(j_lam) * xi;

   iterate;

   if (~warning) & (~no_further),

      gro_tab(j_lam) = 100*(gamma_bar - 1);

      mx_j = j_lam;

   else

      no_further = 1;

      % disp(sprintf('j_lam=%5.0f:imaginary number!',j_lam));

   end;

   z_bar = lam_tab(j_lam) + 1;

   sig_z = 0;

   pi_bar = 0;

   steady;

   gro_mean(j_lam) = 100*(gamma_bar - 1);

   z_bar = 1;

   sig_z = lam_tab(j_lam) * xi;

   iterate;

   if (~warning) & (~no_further),

      gro_var(j_lam) = 100*(gamma_bar - 1);

      mx_j = j_lam;
```

```
   else

      no_further = 1;

      % disp(sprintf('j_lam=%5.0f:imaginary number!',j_lam));

   end;

end;
mx_j = mx_j - 1;
lam_tab = lam_tab(1:mx_j);

gro_tab = real(gro_tab(1:mx_j));

gro_mean = real(gro_mean(1:mx_j));

gro_var = real(gro_var(1:mx_j));

[gro_mx,ind] = max(gro_tab);

lam_mx = lam_tab(ind);

all_max = max([max(gro_tab),max(gro_mean),max(gro_var)]);

all_min = min([min(gro_tab),min(gro_mean),min(gro_var)]);

plot(lam_tab,gro_tab,lam_tab,gro_mean,'--',lam_tab,gro_var,'-.',...

   [lam_mx,lam_mx],[all_min,all_max],'-');

title('Mean-variance tradeoff');

xlabel('\lambda');

ylabel('growth rate (in %)');

lab = max(1,floor(mx_j*2/3));

text(lam_tab(lab),gro_tab(lab),'overall growth rate');

text(lam_tab(lab),gro_mean(lab),'mean effect: benefit');

text(lam_tab(lab),gro_var(lab),'variance effect: cost');

text(lam_mx,gro_tab(ind)+.02,'max. growth');

========================================================================
rows = max(size(row_tab));
```

```
cols = max(size(col_tab));
e_arr     = zeros(cols,rows);

g_arr     = zeros(cols,rows);

pi_arr    = zeros(cols,rows);

sigR_arr  = zeros(cols,rows);

res_l1 = zeros(1,2*cols);

res_l2 = zeros(1,2*cols);
if DO_TAB1,

   disp('Rows: z_bar, Columns: sig_z');

end;

disp('Boxes:');

disp('growth-rate, e-bar');

disp('premium pi , sig_R');

disp(' ');

disp(['       ',sprintf('& %5.2f &       ',col_tab),' \\ \hline']);

for i_row = 1 : rows,

   for j_col = 1 : cols,

     if DO_TAB1,

         z_bar = col_tab(j_col);

         sig_z = row_tab(i_row);

      end;

      iterate;

      gro_rate = 100*(gamma_bar - 1);

      e_arr(i_row,j_col) = 100*e_bar;

      g_arr(i_row,j_col) = gro_rate;
```

```
      pi_arr(i_row,j_col) = 100*pi_bar;

      e_arr(i_row,j_col) = sig_R;

      res_l1(2*j_col-1) = gro_rate;

      res_l1(2*j_col) = 100*e_bar;

      res_l2(2*j_col-1) = 100*pi_bar;

      res_l2(2*j_col) = sig_R;

   end;

   disp([sprintf('%5.2f  ',row_tab(i_row)),sprintf('& %5.2f ',res_l1),' \\']);

   disp(['       ',sprintf('& %5.2f ',res_l2),' \\ \hline']);

end;

======================================================================}
```

A.8 A question of approximation - the programs

```
%pitilde.m calculates pitilde given the old values for q_t and e_t

%it recall steadylam. For different values of lambda remember to

%change the variable lam and the index of the steady values (11 for lambda=0.1

%21 for lam =0.2 and so on...the max lambda is 0.59 -index 60)

params;

lam=0.1;

T=100;
e_ti=zeros(T+1,1);
q_ti=zeros(T+1,1);
pi_ti=zeros(T+1,1);
gamma_ti=zeros(T+1,1);
n_ti=zeros(T+1,1);
w_ti=zeros(T+1,1);
d_ti=zeros(T+1,1);
R_ti=zeros(T+1,1);
p_ti=zeros(T+1,1);
g_t=zeros(T+1,1);
```

```
M_t=zeros(T+1,1);
N_t=zeros(T+1,1);
y_t = zeros(T+1,1);
steadylam;
e_ti(1)=e_tab(11);
q_ti(1)=q_tab(11);
   pi_ti(1)=pi_tab(11);
   gamma_ti(1)=gamma_tab(11);
    %z_bar(n) = lam_tab((51)) + 1;
    n_ti(1) = n_tab(11);
   g_t(1) = gamma_tab(11);
    sig2_zi=(lam_tab(11))^2*(sig_x^2);
z_bar=lam +1;
for i=2:T+1
%y_t(1)=1;
q_ti(i)=(1/(nu+pi_ti(i-1)*alpha))*((1+nu)*(1-alpha)+nu*(1-alpha)*(1-delta)*q_ti(i-1) +alpha*z_bar);
e_ti(i-1)=(1/(nu+pi_ti(i-1)*alpha))*((1+nu)*(1-alpha)-(nu+pi_ti(i-1))*alpha*(1-delta)*q_ti(i-1) + alpha*z_bar);
D_t(i-1) = (1+nu)*(1-alpha)+nu*(1-alpha)*(1-delta)*q_ti(i-1) + alpha*z_bar;
F_t(i-1) = eta*((1-delta)*q_ti(i-1)/z_bar+ 0.5)*sig2_zi;

A_t(i-1) = (1-delta)^2*(1-alpha)*D_t(i-1)*(q_ti(i-1)/z_bar)*( nu*(1-delta)*q_ti(i-1)+1+nu+z_bar/(1-alpha) ) ...

   - alpha^2 * F_t(i-1);

B_t(i-1) = (1 - delta)^2 * D_t(i-1) * (nu*((1-alpha)/alpha) *(q_ti(i-1)/z_bar) * ...

   (alpha*nu*(1-delta)*q_ti(i-1) - (1+nu)*(1-alpha)-alpha*z_bar ...

   + 1 + nu + (alpha/(1-alpha))*z_bar ) + nu*D_t(i-1)) - 2*alpha*nu*F_t(i-1);

C_t(i-1) = - nu^2 * F_t(i-1);
pi_ti(i-1) = (-1.0/(2*A_t(i-1))) * (B_t(i-1) - sqrt( B_t(i-1)^2 - 4*A_t(i-1)*C_t(i-1)) );

gamma_ti(i) = gamma_ti(i-1)*(1+psi*e_ti(i-1));
n_ti(i) = 1 + nu*(1-e_ti(i-1));
M_t(i) = -(2/eta)*((((1-alpha)/alpha)*(n_ti(i)/q_ti(i))+(z_bar/q_ti(i)))*(q_ti(i-1)/z_bar)+nu);

N_t(i)= sig2_zi*((1+2*(1-delta)*(q_ti(i-1)/z_bar))/(((1-delta)^2)*(q_ti(i)^2)));

pi_tilde1(i) = - (1/2) * (M_t(i) - sqrt(M_t(i)^2 - 4*N_t(i)));

pi_tilde2(i) = - (1/2) * (M_t(i) + sqrt(M_t(i)^2 - 4*N_t(i)));
end
pi_tilde1 = real(pi_tilde1);

pi_tilde2 = real(pi_tilde2);

================================================================================
```

```
%steadylam.m produces steady state values for all the variables

%for different lambdas calculates mean - variance effect and aggregate effect

params;
if xi > 1,

      max_lambda = 1/(xi -1); %cutoff at std.dev. = funds

   end;

lam_tab = min_lambda + (0 : .01 : 1)*(max_lambda-min_lambda);

sz_tab = max(size(lam_tab));
e_tab = lam_tab * 0;
q_tab = 0 * e_tab;
pi_tab = 0 * e_tab;
n_tab= 0* e_tab;
w_tab=0*e_tab;
d_tab= 0* e_tab;
p_tab= 0* e_tab;
R_tab= 0* e_tab;
gamma_tab=0*e_tab;
e_mean = lam_tab * 0;
q_mean = 0 * e_tab;
pi_mean = 0 * e_tab;
n_mean= 0* e_tab;
gamma_mean=0*e_tab;
e_mean = lam_tab * 0;
q_mean = 0 * e_tab;
pi_mean = 0 * e_tab;
n_mean= 0* e_tab;
gamma_mean=0*e_tab;
mx_j=1;
no_further=0;
for j_lam = 1 : 60,
   sig_z = lam_tab(j_lam) * xi;
z_bar = lam_tab(j_lam) + 1;
iterate;
 if (~warning) & (~no_further),
   e_tab(j_lam) = e_bar;
   q_tab(j_lam) = q_bar;
   pi_tab(j_lam) = pi_bar;
   n_tab(j_lam) = n_bar;
   sig_z_tab(j_lam) = sig_z;
   gamma_tab(j_lam) = (gamma_bar - 1 );
      mx_j = j_lam;
```

```
   else
     no_further = 1;
end;
 z_bar = lam_tab(j_lam) + 1;
   sig_z = 0;
   pi_bar = 0;
   steady;
   e_mean(j_lam) = 100*e_bar;
    q_mean(j_lam) = 100*q_bar;
   pi_mean(j_lam) = 100*pi_bar;
   n_mean(j_lam) = n_bar;
   gamma_mean(j_lam) = 100*(gamma_bar - 1 );
   z_bar = 1;
   sig_z = lam_tab(j_lam) * xi;
   iterate;
   if (~warning) & (~no_further),
      e_var(j_lam) = 100*e_bar;
    q_var(j_lam) = 100*q_bar;
   pi_var(j_lam) = 100*pi_bar;
   n_var(j_lam) = n_bar;
      gamma_var(j_lam) = 100*(gamma_bar - 1);
      mx_j = j_lam;
   else
      no_further = 1;
      % disp(sprintf('j_lam=%5.0f:imaginary number!',j_lam));
   end;
end;
mx_j=mx_j - 1;
lam_tab = lam_tab(1:mx_j);
e_tab = real(e_tab(1:mx_j));
q_tab = real(q_tab(1:mx_j));
pi_tab = real(pi_tab(1:mx_j));
n_tab = real(n_tab(1:mx_j));
gamma_tab = real(gamma_tab(1:mx_j));
e_mean = real(e_mean(1:mx_j));
q_mean = real(q_mean(1:mx_j));
pi_mean = real(pi_mean(1:mx_j));
n_mean = real(n_mean(1:mx_j));
gamma_mean = real(gamma_mean(1:mx_j));
e_var = real(e_var(1:mx_j));
q_var = real(q_var(1:mx_j));
pi_var = real(pi_var(1:mx_j));
n_var = real(n_var(1:mx_j));
gamma_var = real(gamma_var(1:mx_j));
========================================================================
%pitilde2.m gives the new formula for pitilde in the ss
DISP_PI = 0;
DISP_WARNING = 0;
```

```
DISP_CRITICAL = 0;
D_t = (1+nu)*(1-alpha)+nu*(1-alpha)*(1-delta)*q_t + alpha*(expec_z + chi);
N_t= (1+2*(1-delta)*(q_t/z_t))/((1-delta)^2);
C_t = (sig2_z*(nu^2)*N_t)/(D_t^2);
A_t = 1 + (N_t*sig2_z*alpha^2)/D_t^2 -(2/eta)*(q_t/z_bar)*(alpha/D_t)*(((1-alpha)/alpha)*(1+nu)...
   + nu*(1-alpha)*(1-delta)*q_t+z_bar);
B_t = (sig2_z*2*nu*alpha*N_t)/D_t^2 - (2/eta)*nu - (2/eta)*(q_t/z_bar)*(nu/D_t)*((-(1-alpha)^2/alpha)*(1+nu)...
   +((1-alpha)/alpha)*(1+nu) + nu*(1-delta)*q_t*(1-alpha) + alpha*z_bar);
warning = (B_t^2 - 4*A_t*C_t < 0);
if warning & DISP_WARNING,
   disp('MEAN_VAR.M: WARNING!  Complex-valued risk premium!');
end;
if abs(A_t / B_t) < .000001,
   pi_t1 = - C_t / B_t;
   pi_t2 = pi_t1;
else
   pi_t1 = (-1.0/(2*A_t)) * (B_t - sqrt( B_t^2 - 4*A_t*C_t) );
   pi_t2 = (-1.0/(2*A_t)) * (B_t + sqrt( B_t^2 - 4*A_t*C_t) );
end;

warning = warning | ( (pi_t1 < 0) & (pi_t2 < 0) );
critical = (abs(1-alpha) < .000001) | (abs(alpha) < .000001) |(abs(A_t) < .000001) | (abs(z_t) < .000001);
if DISP_PI | (DISP_CRITICAL & critical),
   disp(sprintf('pi_t1 = %6.3f, pi_t2 = %6.3f, A_t = %6.3f',pi_t1,pi_t2,A_t));
end;
============================================================================

%iterate2.m uses the same principle of iterate but uses the new formula for pitilde
%insert different values for z_bar and sig_z
DISP_STEPS = 0;
DISP_ITER = 0;
nn = 5;
steady;
j = 0;
pi_bar = 0;
pi_t1 = pi_bar;
if DISP_STEPS,
   disp(sprintf('j=%3.0f, pi_bar = %6.3f, q_bar = %6.3f, growth rate = %6.3f',j,pi_bar,q_bar,100*(gamma_bar - 1)));
end;
for j = 1 : nn,
   pi_old = pi_bar;
   steadytilde;
   pi_bar = real(pi_t1);
   steady;
   if DISP_STEPS,
      disp(sprintf('j=%3.0f, pi_bar = %6.3f, q_bar = %6.3f, growth rate = %6.3f',j,pi_bar,q_bar,100*(gamma_bar - 1)));
   end;
end;
```

```
warning = warning | (abs(pi_old - pi_bar) > 0.01);
if DISP_ITER,
   disp(sprintf('j=%3.0f, pi_bar = %6.3f, growth rate = %6.3f',j,pi_bar,100*(gamma_bar - 1)));
end;
```

B Appendix - Derivations of the dynamic analysis

In the dynamic perspective the growth rate is given by:

$$g_t = \frac{y_{t+1}}{y_t} = (\frac{\gamma_{t+1}}{\gamma_t})(\frac{n_{t+1}}{n_t})^{\alpha}(\frac{q_{t+1}}{q_t})^{1-\alpha}$$

therefore

$$g_t = (1+\psi e_t)(\frac{n_{t+1}}{n_t})^{\alpha}(\frac{q_{t+1}}{q_t})^{1-\alpha} - 1$$

For the programming use the following equations for the second set variables (w_t, p_t, d_t, and R_t):

$$w_{t+1} = \alpha \frac{y_{t+1}}{n_{t+1}}$$

knowing that $y_{t+1} = \gamma_{t+1} q_{t+1}^{1-\alpha} n_{t+1}^{\alpha}$

$$w_{t+1} = \alpha \gamma_{t+i} \left(\frac{q_{t+1}}{n_{t+1}}\right)^{1-\alpha}$$

$$d_{t+1} = \frac{(1-\alpha) y_{t+1}}{q_{t+1}}$$

therefore, following the same logic as for eq(**??**), one obtains

$$d_{t+1} = (1-\alpha)\gamma_{t+1} \left(\frac{n_{t+1}}{q_{t+1}}\right)^{\alpha}$$

As $q_t p_t = z w_t$ the price equation is given by

$$E_t[p_{t+1}] = \frac{E_t[z_{t+1} w_{t+1}]}{q_{t+1}}$$

For R_{t+1} the expression is given in Scott and Uhlig (1999)

$$R_{t+1} = (1-\delta)\frac{d_{t+1} + p_{t+1}}{p_t}$$

B.1 The dynamics - the programs

```
% PAR2.M sets some different parameters
% Global parameters
nu = 1;
alpha = 2/3;
z_bar = 1;
```

```
delta = 0.1;
psi = 0.2;
%sig_x=1;
sig_x=5;
%sig_x=5;
eta = 2;
%eta = 2;
%eta = 1;
% Some initialization for some routines
max_lambda = 1;
min_lambda = 0;
ind_lam = 0;
min_q = 0;
max_q = 1/delta;
warning = 0;

====================================================================
%shock:m produces a shock for the impulse-response analysis
T=100;
rho=0.95;
c=1;
r=zeros(T+1,1);
x=zeros(T+1,1);
r(2)=1;
for i=2:T+1
   x(i) = rho*x(i-1)+r(i);
end
%plot(x)

====================================================================
%shock2.m produces a shock with determinable variance and mean = 1
T=11000;
rho=0.95;
c=1;
r=randn(T+1,1);
x=ones(T+1,1);
S=zeros(T+1,1);
Var=0;
x(1)=rand(1);
for i=2:T+1
      x(i) = 0.05 + rho*x(i-1)+ (sqrt(25*(1-rho^2)))*r(i);
   %x=x(100:T+1);
   %mu = sum(x)/T;
   %S(i)=(x(i)-mu)^2;
end
x=x(100:T+1,1);
%for l=1:10000
 %   mu = sum(l)/T;
```

```
  % S(l)=(x(l)-mu)^2;
   %end
   %mu
   %Var=sum(S)/(T-1);
%Var
%sig_x=sqrt(Var);
%sig_x
%plot(x)
===================================================================

%shock3.m produces and ARCH process
T=100;
rho=0.95;
r=randn(T+1,1);
x=zeros(T+1,1);
epsilon=zeros(T+1,1);
epsilon(1)=0;
x(1)=rand(1);
for i=2:T+1
   epsilon(i) = r(i)*sqrt(1+0.5897*(epsilon(i-1)^2));
   x(i) = 0.05 + rho*x(i-1)+ epsilon(i);
   %x=x(100:T+1);
   %mu = sum(x)/T;
   %S(i)=(x(i)-mu)^2;
end
=================================================================

%prova.m creates plots for the impulse response analysis one-by-one variable
params;
shock;
M=60;
e_ti=zeros(M+1,59);
q_ti=zeros(M+1,59);
pi_ti=zeros(M+1,59);
gamma_ti=zeros(M+1,59);
n_ti=zeros(M+1,59);
g_t=zeros(M+1,59);
sig2_zi=zeros(1,59);
z=ones(M+1,59);
%z_bar=ones(1,92);
D_t=zeros(M+1,59);
F_t=zeros(M+1,59);
A_t=zeros(M+1,59);
B_t=zeros(M+1,59);
C_t=zeros(M+1,59);
L_t=zeros(M+1,59);
CAP_t=zeros(M+1,59);
w_ti=zeros(M+1,59);
```

```
p_ti=zeros(M+1,59);
d_ti=zeros(M+1,59);
R_ti=zeros(M+1,59);
steadylam;
%biglam;
for n=1:59
for i=2:M+1
    z(i,n)=(x(i)*lam_tab(n))+1;
e_ti(1,n)=e_tab(n);
q_ti(1,n)=q_tab(n);
   pi_ti(1,n)=pi_tab(n);
   gamma_ti(1,n)=gamma_tab(n);
    %z_bar(n) = lam_tab((n)) + 1;
    n_ti(1,n) = n_tab(n);
    g_t(1,n) = gamma_tab(n);
    sig2_zi(n)=(lam_tab(n))^2*(sig_x^2);
          w_ti(1,n)= alpha*gamma_ti(1,n)*((q_ti(1,n)/n_ti(1,n))^(1-alpha));
   d_ti(1,n)= (1-alpha)*gamma_ti(1,n)*((n_ti(1,n)/q_ti(1,n))^alpha);
   p_ti(1,n)= (z(1,n)*w_ti(1,n))/q_ti(1,n);
     R_ti(1,n)= (1-delta)*((d_ti(1,n)+p_ti(1,n))/p_ti(1,n))-1;
z(1,n)=1;
L_t(1,n)=1;
CAP_t(1,n)=1;
q_ti(i,n)=(1/(nu+pi_ti(i-1,n)*alpha))*((1+nu)*(1-alpha)+nu*(1-alpha)*(1-delta)*q_ti(i-1,n) +alpha*z(i,n));
e_ti(i-1,n)=(1/(nu+pi_ti(i-1,n)*alpha))*((1+nu)*(1-alpha)-(nu+pi_ti(i-1,n))*alpha*(1-delta)*q_ti(i-1,n) + alpha*z(i,n));
D_t(i-1,n) = (1+nu)*(1-alpha)+nu*(1-alpha)*(1-delta)*q_ti(i-1,n) + alpha*z(i,n);
F_t(i-1,n) = eta*((1-delta)*q_ti(i-1,n)/z(i-1,n) + 0.5)*sig2_zi(n);
A_t(i-1,n) = (1-delta)^2*(1-alpha)*D_t(i-1,n)*(q_ti(i-1,n)/z(i-1,n))*( nu*(1-delta)*q_ti(i-1,n)+1+nu+z(i,n)/(1-alpha) ) ...
   - alpha^2 * F_t(i-1,n);
B_t(i-1,n) = (1 - delta)^2 * D_t(i-1,n) * (nu*((1-alpha)/alpha) *(q_ti(i-1,n)/z(i-1,n)) * ...
    (alpha*nu*(1-delta)*q_ti(i-1,n) - (1+nu)*(1-alpha)-alpha*z(i,n) ...
   + 1 + nu + (alpha/(1-alpha))*z(i,n) ) + nu*D_t(i-1,n)) - 2*alpha*nu*F_t(i-1,n);
C_t(i-1,n) = - nu^2 * F_t(i-1,n);
pi_ti(i-1,n) = (-1.0/(2*A_t(i-1,n))) * (B_t(i-1,n) - sqrt( B_t(i-1,n)^2 - 4*A_t(i-1,n)*C_t(i-1,n)) );
gamma_ti(i,n)= gamma_ti(i-1,n)*(1+ psi*e_ti(i-1,n));
n_ti(i,n) = 1 + nu*(1-e_ti(i-1,n));
L_t(i,n) = n_ti(i,n)/n_ti(i-1,n);
CAP_t(i,n) = q_ti(i,n)/q_ti(i-1,n);
g_t(i,n) = ((1 + psi*e_ti(i-1,n))*(L_t(i)^alpha)*(CAP_t(i)^(1-alpha)))-1;
g_t=real(g_t);
w_ti(i,n)=alpha*gamma_ti(i,n)*((q_ti(i,n)/n_ti(i,n))^(1-alpha));
d_ti(i,n) = (1-alpha)*gamma_ti(i,n)*((n_ti(i,n)/q_ti(i,n))^alpha);
p_ti(i,n) = (z(i,n)*w_ti(i,n))/q_ti(i,n);
R_ti(i,n) = (1-delta)*((d_ti(i,n)+p_ti(i,n))/p_ti(i-1,n))-1;
end
end
time=1:1:60;
time=time';
```

```
e_impresp=e_ti.*100;
q_impresp=q_ti;
g_impresp=g_t.*100;
pi_impresp=pi_ti.*100;
e_lam01=e_impresp(1:8,11);
e_lam02=e_impresp(1:8,21);
e_lam03=e_impresp(1:8,31);
e_lam04=e_impresp(1:8,41);
e_lam05=e_impresp(1:8,51);
e_lam06=e_impresp(1:8,59);
if VAR == 1
subplot(2,2,1)
plot(time(1:8),e_lam01,time(1:8),e_lam02,time(1:8),e_lam03,time(1:8),e_lam04,...
   time(1:8),e_lam05,time(1:8),e_lam06);
title('share of entrepreneurs, e_t','FontSize',8);
xlabel('time','FontSize',8);
ylabel('deviation from the steady state','FontSize',8);
text(2,20,'\lambda=0.1','FontSize',8);
text(2,25,'\lambda=0.2','FontSize',8);
text(2,30,'\lambda=0.3','FontSize',8);
text(2,40,'\lambda=0.4','FontSize',8);
text(2,50,'\lambda=0.5','FontSize',8);
text(2,60,'\lambda=0.59','FontSize',8);
subplot(2,2,2)
plot(time(1:50),q_impresp(1:50,11),time(1:50),q_impresp(1:50,21),time(1:50),q_impresp(1:50,31),time(1:50),q_impresp(1:50,41
  time(1:50),q_impresp(1:50,51),time(1:50),q_impresp(1:50,59));
title('number of projects, q_t','FontSize',8);
xlabel('time','FontSize',8);
ylabel('deviation from the steady state','FontSize',8);
text(5,1.99,'\lambda=0.1','FontSize',8);
text(6,2.06,'\lambda=0.2','FontSize',8);
text(6,2.14,'\lambda=0.3','FontSize',8);
text(6,2.22,'\lambda=0.4','FontSize',8);
text(6,2.3,'\lambda=0.5','FontSize',8);
text(6,2.37,'\lambda=0.59','FontSize',8);
subplot(2,2,3)
plot(time,pi_impresp(1:60,11),time,pi_impresp(1:60,21),time,pi_impresp(1:60,31),time,pi_impresp(1:60,41),...
  time,pi_impresp(1:60,51),time,pi_impresp(1:60,59));
title('risk premium, \pi_t','FontSize',8);
xlabel('time','FontSize',8);
ylabel('deviation from the steady state','FontSize',8);
text(3,1,'\lambda=0.1','FontSize',8);
text(4,5,'\lambda=0.2','FontSize',8);
text(5,10,'\lambda=0.3','FontSize',8);
text(6,18,'\lambda=0.4','FontSize',8);
text(7,27,'\lambda=0.5','FontSize',8);
text(8,36,'\lambda=0.59','FontSize',8);
subplot(2,2,4)
```

```
plot(time(1:10),g_impresp(1:10,11),time(1:10),g_impresp(1:10,21),time(1:10),g_impresp(1:10,31),time(1:10),...
   g_impresp(1:10,41),time(1:10),g_impresp(1:10,51),time(1:10),g_impresp(1:10,59));
title('growth rate, g','FontSize',8);
xlabel('time','FontSize',8);
ylabel('deviation from the steady state','FontSize',8);
text(3,4,'\lambda=0.1','FontSize',8);
text(3,5,'\lambda=0.2','FontSize',8);
text(3,6,'\lambda=0.3','FontSize',8);
text(3,8,'\lambda=0.4','FontSize',8);
text(3,10,'\lambda=0.5','FontSize',8);
text(3,12,'\lambda=0.59','FontSize',8);
else
   subplot(2,2,1)
   plot(time(1:8),n_ti(1:8,11),time(1:8),n_ti(1:8,21),time(1:8),n_ti(1:8,31),...
time(1:8),n_ti(1:8,41),...
   time(1:8),n_ti(1:8,51),time(1:8),n_ti(1:8,59));
   title('efficiency units of labor, n_t','FontSize',8);
xlabel('time','FontSize',8);
ylabel('deviation from the steady state','FontSize',8);
text(3,1.8,'\lambda=0.1','FontSize',8);
text(3,1.76,'\lambda=0.2','FontSize',8);
text(3,1.68,'\lambda=0.3','FontSize',8);
text(3,1.6,'\lambda=0.4','FontSize',8);
text(3,1.5,'\lambda=0.5','FontSize',8);
text(3,1.4,'\lambda=0.59','FontSize',8);
subplot(2,2,2)
plot(time(1:8),w_ti(1:8,11),time(1:8),w_ti(1:8,21),time(1:8),w_ti(1:8,31),...
time(1:8),w_ti(1:8,41),...
time(1:8),w_ti(1:8,51),time(1:8),w_ti(1:8,59));
title('wage, w_t','FontSize',8);
xlabel('time','FontSize',8);
ylabel('deviation from the steady state','FontSize',8);
text(3,0.029,'\lambda=0.1','FontSize',8);
text(3,0.031,'\lambda=0.2','FontSize',8);
text(3,0.032,'\lambda=0.3','FontSize',8);
text(3,0.033,'\lambda=0.4','FontSize',8);
text(2,0.027,'\lambda=0.5','FontSize',8);
text(2,0.025,'\lambda=0.59','FontSize',8);
subplot(2,2,3)
plot(time(1:8),p_ti(1:8,11),time(1:8),p_ti(1:8,21),time(1:8),p_ti(1:8,31),
   time(1:8),p_ti(1:8,41),time(1:8),p_ti(1:8,51),time(1:8),p_ti(1:8,59));
title('price of projects, p_t','FontSize',8);
xlabel('time','FontSize',8);
ylabel('deviation from the steady state','FontSize',8);
text(3,0.016,'\lambda=0.1','FontSize',8);
text(3,0.018,'\lambda=0.2','FontSize',8);
text(3,0.0195,'\lambda=0.3','FontSize',8);
text(3,0.021,'\lambda=0.4','FontSize',8);
```

```
text(3,0.022,'\lambda=0.5','FontSize',8);
text(3,0.023,'\lambda=0.59','FontSize',8);
subplot(2,2,4)
plot(time(1:8),R_ti(1:8,11),time(1:8),R_ti(1:8,21),time(1:8),R_ti(1:8,31),time(1:8),...
   R_ti(1:8,41),time(1:8),R_ti(1:8,51),time(1:8),R_ti(1:8,59));
title('rate of return, R','FontSize',8);
xlabel('time','FontSize',8);
ylabel('deviation from the steady state','FontSize',8);
text(2,0.85,'\lambda=0.1','FontSize',8);
text(2,0.94,'\lambda=0.2','FontSize',8);
text(2,1.05,'\lambda=0.3','FontSize',8);
text(2,1.12,'\lambda=0.4','FontSize',8);
text(2,1.21,'\lambda=0.5','FontSize',8);
text(2,1.3,'\lambda=0.59','FontSize',8);
end
%time=time*ones(1,60);
%lam_tab=ones(60,1)*lam_tab;
%mesh(time,lam_tab,pi_impresp(1:60,:));
%mesh(time,lam_tab,e_impresp(1:60,:));
%mesh(time,lam_tab,q_impresp(1:60,:));
=====================================================================
%impresp.m produces imp-resp for a shock in the stochastic process
params;
%par2;
shock;
M=100;
e_ti=zeros(M+1,1);
q_ti=zeros(M+1,1);
pi_ti=zeros(M+1,1);
gamma_ti=zeros(M+1,1);
n_ti=zeros(M+1,1);
w_ti=zeros(M+1,1);
d_ti=zeros(M+1,1);
p_ti=zeros(M+1,1);
R_ti = zeros(M+1,1);
Ca_t = zeros(M+1,1);
L_t = zeros(M+1,1);
g_t=zeros(M+1,1);
sig2_zi=zeros(1,1);
z=ones(M+1,1);
D_t=zeros(M+1,1);
F_t=zeros(M+1,1);
A_t=zeros(M+1,1);
B_t=zeros(M+1,1);
C_t=zeros(M+1,1);
e=zeros(M-1,1);
q=zeros(M-1,1);
w=zeros(M-1,1);
```

```
pi=zeros(M-1,1);
n=zeros(M-1,1);
q=zeros(M-1,1);
p=zeros(M-1,1);
R=zeros(M-1,1);
d=zeros(M-1,1);
g=zeros(M-1,1);
steadylam;
if LAMBDA == 1
   m=11
   end
   if LAMBDA == 2
      m=21
      end
   if LAMBDA == 3
         m=31
      end
      if LAMBDA == 4
            m=41
         end
          if LAMBDA ==5
               m=51
            end
             if LAMBDA == 6
                  m=59
                  end
for i=2:M+1
    z(i)=(x(i)*lam_tab(m))+1;
e_ti(1)=e_tab(m);
q_ti(1)=q_tab(m);
   pi_ti(1)=pi_tab(m);
   gamma_ti(1)=gamma_tab(m);
   g_t(1)=gamma_tab(m);
n_ti(1)= n_tab(m);
   sig2_zi(m)=(lam_tab(m))^2*(sig_x^2);
   w_ti(1)= alpha*gamma_ti(1)*((q_ti(1)/n_ti(1))^(1-alpha));
   d_ti(1)= (1-alpha)*gamma_ti(1)*((n_ti(1)/q_ti(1))^alpha);
   p_ti(1)= (z(1)*w_ti(1))/q_ti(1);
   R_ti(1)= (1-delta)*((d_ti(1)+p_ti(1))/p_ti(1))-1;
z(1)=1;
q_ti(i)=(1/(nu+pi_ti(i-1)*alpha))*((1+nu)*(1-alpha)+nu*(1-alpha)*(1-delta)*q_ti(i-1) +alpha*z(i));
e_ti(i-1)=(1/(nu+pi_ti(i-1)*alpha))*((1+nu)*(1-alpha)-(nu+pi_ti(i-1))*alpha*(1-delta)*q_ti(i-1) + alpha*z(i));
D_t(i-1) = (1+nu)*(1-alpha)+nu*(1-alpha)*(1-delta)*q_ti(i-1) + alpha*z(i);
F_t(i-1) = eta*((1-delta)*q_ti(i-1)/z(i-1) + 0.5)*sig2_zi(m);
A_t(i-1) = (1-delta)^2*(1-alpha)*D_t(i-1)*(q_ti(i-1)/z(i-1))*( nu*(1-delta)*q_ti(i-1)+1+nu+z(i)/(1-alpha) ) ...
   - alpha^2 * F_t(i-1);
B_t(i-1) = (1 - delta)^2 * D_t(i-1) * (nu*((1-alpha)/alpha) *(q_ti(i-1)/z(i-1)) * ...
   (alpha*nu*(1-delta)*q_ti(i-1) - (1+nu)*(1-alpha)-alpha*z(i) ...
```

```
   + 1 + nu + (alpha/(1-alpha))*z(i) ) + nu*D_t(i-1)) - 2*alpha*nu*F_t(i-1);
C_t(i-1) = - nu^2 * F_t(i-1);
pi_ti(i-1) = (-1.0/(2*A_t(i-1))) * (B_t(i-1) - sqrt( B_t(i-1)^2 - 4*A_t(i-1)*C_t(i-1)) );
gamma_ti(i)=gamma_ti(i-1)*(1+ psi*e_ti(i-1));
n_ti(i) = 1 + nu*(1-e_ti(i-1));
Ca_t(i)= (n_ti(i)/q_ti(i))^alpha;
L_t(i)=(q_ti(i)/n_ti(i))^(1-alpha);
w_ti(i)= ((q_ti(i)/n_ti(i))^(1-alpha))*gamma_ti(i)*alpha;
d_ti(i) = (1-alpha)*gamma_ti(i)*(n_ti(i)/q_ti(i))^alpha;
p_ti(i) = (z(i)*w_ti(i))/q_ti(i);
R_ti(i) = (1-delta)*((d_ti(i)+p_ti(i))/p_ti(i-1))-1;
g_t(i) = (1 + psi*e_ti(i-1))*((n_ti(i)/n_ti(i-1))^ alpha)...
   *((q_ti(i)/q_ti(i-1))^(1-alpha)) -1;
g_t=real(g_t);
end
for j=2:M-1
 q(j)= log(q_ti(j))-log(q_ti(1));
 e(j)= log(e_ti(j))-log(e_ti(1));
 pi(j)= log(pi_ti(j))-log(pi_ti(1));
 g(j)=log(g_t(j))-log(g_t(1));
n(j)=log(n_ti(j))-log(n_ti(1));
w(j)=log(w_ti(j))-log(w_ti(1));
d(j)=log(d_ti(j))-log(d_ti(1));
p(j)=log(p_ti(j))-log(p_ti(1));
R(j)=log(R_ti(j))-log(R_ti(1));
end
w=real(w);
d=real(d);
p=real(p);
R=real(R);
if VARSET == 1;
time=1:1:20;
H = plot(time,q(1:20,1),time,e(1:20,1),time,pi(1:20,1),...
   time,n(1:20,1),time,g(1:20,1))
xlabel('time','FontSize',8);
ylabel('deviations from the steady state','FontSize',8);
if LAMBDA == 1;
   title('impulse response to a shock in foreign investments: \lambda = 0.1','FontSize',8);
elseif LAMBDA == 2,
    title('impulse response to a shock in foreign investments: \lambda = 0.2','FontSize',8);
 elseif LAMBDA == 3,
    title('impulse response to a shock in foreign investments: \lambda = 0.3','FontSize',8);
 elseif LAMBDA == 4,
     title('impulse response to a shock in foreign investments: \lambda = 0.4','FontSize',8);
  elseif LAMBDA == 5,
     title('impulse response to a shock in foreign investments: \lambda = 0.5','FontSize',8);
  else
     title('impulse response to a shock in foreign investments: \lambda = 0.59','FontSize',8);
```

```
  end
  set(H,'LineWidth',2);
text(time(7),q(7),'number of projects, q','FontSize',8);
text(time(2),e(2),'share of entrepreneurs, e','FontSize',8);
text(time(16),pi(16),'risk premium, \pi','FontSize',8);
text(time(3),n(3),'efficiency units of labor, n','FontSize',8);
text(time(4),g(4),'growth rate, g','FontSize',8);
end
if VARSET == 2;
time=1:1:20;
HN = plot(time,w(1:20,1),time,d(1:20,1),time,p(1:20,1),...
   time,R(1:20,1))
xlabel('time','FontSize',7);
ylabel('deviations from the steady state','FontSize',7);
if LAMBDA == 1;
   title('impulse response to a shock in foreign investments: \lambda = 0.1','FontSize',7);
elseif LAMBDA == 2,
    title('impulse response to a shock in foreign investments: \lambda = 0.2','FontSize',7);
 elseif LAMBDA == 3,
    title('impulse response to a shock in foreign investments: \lambda = 0.3','FontSize',7);
 elseif LAMBDA == 4,
    title('impulse response to a shock in foreign investments: \lambda = 0.4','FontSize',7);
  elseif LAMBDA == 5,
    title('impulse response to a shock in foreign investments: \lambda = 0.5','FontSize',7);
  else
     title('impulse response to a shock in foreign investments: \lambda = 0.59','FontSize',7);
  end
  set(HN,'LineWidth',2);
text(time(9),w(9),'wages, w','FontSize',7);
text(time(3),d(3),'dividends, d','FontSize',7);
text(time(2),p(2),'projects price, p','FontSize',7);
text(time(5),R(5),'rate of return, R','FontSize',7);
end
========================================================================
%plotir.m plots impresp
LAMBDA = 1;
VARSET = 1;
impresp;
if PLOTCHOICE ==1
subplot(2,1,1)
LAMBDA = 1;
impresp;
subplot(2,1,2)
LAMBDA = 2;
impresp;
end
if PLOTCHOICE ==2
subplot(2,1,1)
```

```
LAMBDA = 3;
impresp;
subplot(2,1,2)
LAMBDA = 4;
impresp;
end
if PLOTCHOICE ==3
subplot(2,1,1)
LAMBDA = 5;
impresp;
subplot(2,1,2)
LAMBDA = 6;
impresp;
end
=======================================================================
%ploirvar2.m plot impresp for the second set of variables
LAMBDA = 1;
VARSET = 2;
impresp;
subplot(3,2,1)
LAMBDA = 1;
impresp;
subplot(3,2,2)
LAMBDA = 2;
impresp;
subplot(3,2,3)
LAMBDA = 3;
impresp;
subplot(3,2,4)
LAMBDA = 4;
impresp;
subplot(3,2,5)
LAMBDA = 5;
impresp;
subplot(3,2,6)
LAMBDA = 6;
impresp;
========================================================================
%qshock.m produces a shock in the state variable
params;
M=100;
sig_x=1;
e_ti=zeros(M+1,1);
q_ti=zeros(M+1,1);
pi_ti=zeros(M+1,1);
gamma_ti=zeros(M+1,1);
n_ti=zeros(M+1,1);
w_ti=zeros(M+1,1);
```

```
d_ti=zeros(M+1,1);
p_ti=zeros(M+1,1);
R_ti = zeros(M+1,1);
g_t=zeros(M+1,1);
sig2_zi=zeros(1,1);
z_bar=ones(1,1);
D_t=zeros(M+1,1);
F_t=zeros(M+1,1);
A_t=zeros(M+1,1);
B_t=zeros(M+1,1);
C_t=zeros(M+1,1);
e=zeros(M-1,1);
q=zeros(M-1,1);
pi=zeros(M-1,1);
n=zeros(M-1,1);
q=zeros(M-1,1);
p=zeros(M-1,1);
R=zeros(M-1,1);
d=zeros(M-1,1);
g=zeros(M-1,1);
steadylam;
if LAMBDA == 1
   m=11
   end
   if LAMBDA == 2
      m=21
      end
   if LAMBDA == 3
         m=31
      end
      if LAMBDA == 4
            m=41
         end
          if LAMBDA ==5
               m=51
            end
             if LAMBDA == 6
                  m=59
               end

z_bar = lam_tab(m) + 1;%constant shock
for i=2:M+1
    %z(i)=(x(i)*lam_tab(m))+1;
 e_ti(1)=e_tab(m);
q_steady= q_tab(m);
q_ti(1)= 0;
pi_ti(1)=pi_tab(m);
   gamma_ti(1)=gamma_tab(m);
```

```
   g_t(1)=gamma_ti(1)*100;
   sig2_zi=(lam_tab(m))^2*(sig_x^2);
  n_ti(1)=n_tab(m);
%w_ti(1) = alpha*gamma_ti(1)*((q_ti(1)/n_ti(1))^(1-alpha));
   w_steady= alpha*gamma_ti(1)*((q_steady/n_ti(1))^(1-alpha));
   w_ti(1) = w_steady;
   %d_ti(1) = (1-alpha)*gamma_ti(1)*((n_ti(1)/q_ti(1))^alpha);
 d_steady = (1-alpha)*gamma_ti(1)*((n_ti(1)/q_steady)^alpha);
   d_ti(1)= d_steady;
   %p_ti(1)= (z_bar*w_ti(1))/q_ti(1);
   p_steady= (z_bar*w_steady)/q_steady;
   p_ti(1) = p_steady;
  R_steady= (1-delta)*((d_steady+p_steady)/p_steady)-1;
  R_ti(1) = R_steady;
  %R_ti(1) = (1-delta)*((d_ti(1)+p_ti(1))/p_ti(1))-1;
q_ti(i)=(1/(nu+pi_ti(i-1)*alpha))*((1+nu)*(1-alpha)+nu*(1-alpha)*(1-delta)*q_ti(i-1) +alpha*z_bar);
e_ti(i-1)=(1/(nu+pi_ti(i-1)*alpha))*((1+nu)*(1-alpha)-(nu+pi_ti(i-1))*alpha*(1-delta)*q_ti(i-1) + alpha*z_bar);
D_t(i-1) = (1+nu)*(1-alpha)+nu*(1-alpha)*(1-delta)*q_ti(i-1) + alpha*z_bar;
F_t(i-1) = eta*((1-delta)*q_ti(i-1)/z_bar + 0.5)*sig2_zi;
A_t(i-1) = (1-delta)^2*(1-alpha)*D_t(i-1)*(q_ti(i-1)/z_bar)*( nu*(1-delta)*q_ti(i-1)+1+nu+z_bar/(1-alpha) ) ...
   - alpha^2 * F_t(i-1);
B_t(i-1) = (1 - delta)^2 * D_t(i-1) * (nu*((1-alpha)/alpha) *(q_ti(i-1)/z_bar) * ...
   (alpha*nu*(1-delta)*q_ti(i-1) - (1+nu)*(1-alpha)-alpha*z_bar ...
   + 1 + nu + (alpha/(1-alpha))*z_bar ) + nu*D_t(i-1)) - 2*alpha*nu*F_t(i-1);
C_t(i-1) = - nu^2 * F_t(i-1);
pi_ti(i-1) = (-1.0/(2*A_t(i-1))) * (B_t(i-1) - sqrt( B_t(i-1)^2 - 4*A_t(i-1)*C_t(i-1)) );
gamma_ti(i)=gamma_ti(i-1)*(1+ psi*e_ti(i-1));
n_ti(i) = 1 + nu*(1-e_ti(i-1));
w_ti(i)=alpha*gamma_ti(i)*(q_ti(i)/n_ti(i))^(1-alpha);
d_ti(i) = (1-alpha)*gamma_ti(i)*(n_ti(i)/q_ti(i))^alpha;
p_ti(i) = (z_bar*w_ti(i))/q_ti(i);
R_ti(i) = (1-delta)*((d_ti(i)+p_ti(i))/p_ti(i-1));
p_ti(i)=real(p_ti(i));
w_ti(i)=real(w_ti(i));
d_ti(i)=real(d_ti(i));
R_ti(i)=real(R_ti(i));
end
for j=1:M-1
   if q_ti(j) < 0
        q(j)= -((abs(q_ti(j))/q_steady)-1);
else
    q(j)=(q_ti(j)/q_steady)-1;
   end
      if e_ti(j) <0
   e(j)= -((abs(e_ti(j))/e_ti(1))-1);
else
  e(j)=(e_ti(j)/e_ti(1))-1;
end
```

```
   if pi_ti(j) < 0
 pi(j)= -((abs(pi_ti(j))/pi_ti(1))-1);
else
    pi(j)=(pi_ti(j)/pi_ti(1))-1;
   end
  if n_ti(j)<0
  n(j)= -((abs(n_ti(j))/n_ti(1))-1);
else
   n(j)=(n_ti(j)/n_ti(1))-1;
   end
   if w_ti(j)<0
   w(j)= -((abs(w_ti(j))/w_steady)-1);
else
    w(j)=(w_ti(j)/w_steady)-1;
   end
   if p_ti(j)<0
         p(j)= -((abs(p_ti(j))/p_steady)-1);
else
      p(j)=(p_ti(j)/p_steady)-1;
end
if R_ti(j)<0
    R(j)= -((abs(R_ti(j))/R_steady)-1);
   else
         R(j)=(R_ti(j)/R_steady)-1;
   end
 %  if g_t(j)<0
  %    g(j)= -((abs(g_t(j))/g_t(1))-1);
%else
 %  g(j)=(g_t(j)/g_t(1))-1;
%   end
   if d_ti(j)<0
    d(j)= -((abs(d_ti(j))/d_steady)-1);
else
     d(j)=(d_ti(j)/d_steady)-1;
   end
   if gamma_ti(j)<0
    gamma(j)= -((abs(gamma_ti(j))/gamma_ti(1))-1);
else
    gamma(j)=(gamma_ti(j)/gamma_ti(1))-1;
   end
end
for t=2:M-1
   g(t)=gamma(t)*(n(t)^alpha)*q(t)^(1-alpha);
   g(1)=gamma(1);
   O(t)=n(t)^alpha;
   M(t)=q(t)^(1-alpha);
   end
w=real(w);
```

```
d=real(d);
p=real(p);
R=real(R);
 q = real(q);
 e = real(e);
 pi = real(pi);
n = real(n);
g =real(g);
if VARSET == 1;
time=1:1:8;
H = plot(time,q(1:8,1),time,n(1:8,1),time,pi(1:8,1),time,e(1:8,1),time,g(1:8,1))
xlabel('time','FontSize',7);
ylabel('deviations from the steady state','FontSize',7);
if LAMBDA == 1;
   title('impulse response to a negative shock in the number of projects: \lambda = 0.1','FontSize',7);
elseif LAMBDA == 2,
    title('impulse response to a negative shock in the number of projects: \lambda = 0.2','FontSize',7);
 elseif LAMBDA == 3,
    title('impulse response to a negative shock in the number of projects: \lambda = 0.3','FontSize',7);
 elseif LAMBDA == 4,
     title('impulse response to a negative shock in the number of projects: \lambda = 0.4','FontSize',7);
  elseif LAMBDA == 5,
     title('impulse response to a negative shock in the number of projects: \lambda = 0.5','FontSize',7);
  else
     title('impulse response to a negative shock in the number of projects: \lambda = 0.59','FontSize',7);
  end
  set(H,'LineWidth',2);
text(time(1),q(1),'number of projects, q_t','FontSize',7);
text(time(2),n(2),'efficiency units of labor, n_t','FontSize',7);
  text(time(2),e(2),'share of entrepreneurs, e_t','FontSize',7);
text(time(2),pi(2),'risk premium, \pi_t','FontSize',7);
text(time(2),g(2),'growth rate, g_t','FontSize',7);
end
if VARSET == 2;
time=1:1:8;
HN = plot(time,w(1:8),time,d(1:8),time,p(1:8),time,R(1:8))
xlabel('time','FontSize',7);
ylabel('deviations from the steady state','FontSize',7);
if LAMBDA == 1;
   title('impulse response to a negative shock in the number of projects: \lambda = 0.1','FontSize',7);
elseif LAMBDA == 2,
    title('impulse response to a negative shock in the number of projects: \lambda = 0.2','FontSize',7);
 elseif LAMBDA == 3,
    title('impulse response to a negative shock in the number of projects: \lambda = 0.3','FontSize',7);
 elseif LAMBDA == 4,
     title('impulse response to a negative shock in the number of projects: \lambda = 0.4','FontSize',7);
  elseif LAMBDA == 5,
     title('impulse response to a negative shock in the number of projects: \lambda = 0.5','FontSize',7);
```

```
  else
     title('impulse response to a negative shock in the number of projects: \lambda = 0.59','FontSize',7);
  end
  set(HN,'LineWidth',2);
text(time(2),w(2),'wages, w','FontSize',7);
text(time(2),d(2),'dividends, d','FontSize',7);
text(time(2),p(2),'projects price, p','FontSize',7);
text(time(2),R(2),'rate of return, R','FontSize',7);
end
if VARSET == 3;
   time=1:1:8;
HN = plot(time,pi(1:8),time,g(1:8),time,e(1:8))
xlabel('time','FontSize',7);
ylabel('deviations from the steady state','FontSize',7);
if LAMBDA == 1;
   title('impulse response to a shock in foreign investments: \lambda = 0.1','FontSize',7);
elseif LAMBDA == 2,
    title('impulse response to a shock in foreign investments: \lambda = 0.2','FontSize',7);
elseif LAMBDA == 3,
    title('impulse response to a shock in foreign investments: \lambda = 0.3','FontSize',7);
 elseif LAMBDA == 4,
     title('impulse response to a shock in foreign investments: \lambda = 0.4','FontSize',7);
  elseif LAMBDA == 5,
     title('impulse response to a shock in foreign investments: \lambda = 0.5','FontSize',7);
  else
     title('impulse response to a shock in foreign investments: \lambda = 0.59','FontSize',7);
  end
  set(HN,'LineWidth',2);
  text(time(4),g(4),'growth rate, g_t','FontSize',7);
    text(time(2),e(2),'share of entrepreneurs, e_t','FontSize',7);
text(time(2),pi(2),'risk premium, \pi','FontSize',7);
end
%plot(time,n,time,w,time,d,time,p,time,R);
===========================================================================
%plotqs.m plots qshock
LAMBDA = 1;
VARSET = 2;
qshock;
subplot(3,2,1)
LAMBDA = 1;
qshock;
subplot(3,2,2)
LAMBDA = 2;
qshock;
subplot(3,2,3)
LAMBDA = 3;
qshock;
subplot(3,2,4)
```

```
LAMBDA = 4;
qshock;
subplot(3,2,5)
LAMBDA = 5;
qshock;
subplot(3,2,6)
LAMBDA = 6;
qshock;
==========================================================================
%threed.m produces 3D simulations of the growth rate
par2;
%params;
shock3;
if sig_x > 1,
      max_lambda = 1/(sig_x -1); %cutoff at std.dev. = funds
   end;
lam_tab = min_lambda + (0 : .1 : 1)*(max_lambda-min_lambda);
sz_tab = max(size(lam_tab));
sig_z = zeros(1,59);
z_bar = zeros(1,59);
G=100;
q_ti=zeros(G+1,59);
e_ti=zeros(G+1,59);
D_t=zeros(G+1,59);
F_t=zeros(G+1,59);
A_t=zeros(G+1,59);
B_t=zeros(G+1,59);
C_t=zeros(G+1,59);
pi_ti=zeros(G+1,59);
n_ti=zeros(G+1,59);
g_t=zeros(G+1,59);
gamma_ti= zeros(G+1,59);
y_ti=zeros(G+1,59);
z=ones(G+1,59);
mx_j=1;
steadylam;
for n=1:59
for i=2:100
   z(i,n)=(x(i)*lam_tab(n))+1;
 e_ti(1,n)=e_tab(n);
q_ti(1,n)=q_tab(n);
   pi_ti(1,n)=pi_tab(n);
   gamma_ti(1,n)=gamma_tab(n);
   n_ti(1,n)=n_tab(n);
   sig2_zi(n)=(lam_tab(n))^2*(sig_x^2);
z(1,n)=1;
y_ti(1,n)=gamma_ti(1,n)*(q_ti(1,n)^(1-alpha))*n_ti(1,n)^alpha;
q_ti(i,n)=(1/(nu+pi_ti(i-1,n)*alpha))*((1+nu)*(1-alpha)+nu*(1-alpha)*(1-delta)*q_ti(i-1,n) +alpha*z(i,n));
```

```
e_ti(i-1,n)=(1/(nu+pi_ti(i-1,n)*alpha))*((1+nu)*(1-alpha)-(nu+pi_ti(i-1,n))*alpha*(1-delta)*q_ti(i-1,n) + alpha*z(i,n));
D_t(i-1,n) = (1+nu)*(1-alpha)+nu*(1-alpha)*(1-delta)*q_ti(i-1,n) + alpha*z(i,n);
F_t(i-1,n) = eta*((1-delta)*q_ti(i-1,n)/z(i-1,n) + 0.5)*sig2_zi(n);
A_t(i-1,n) = (1-delta)^2*(1-alpha)*D_t(i-1,n)*(q_ti(i-1,n)/z(i-1,n))*( nu*(1-delta)*q_ti(i-1,n)+1+nu+z(i,n)/(1-alpha) ) ...
   - alpha^2 * F_t(i-1,n);
B_t(i-1,n) = (1 - delta)^2 * D_t(i-1,n) * (nu*((1-alpha)/alpha) *(q_ti(i-1,n)/z(i-1,n)) * ...
    (alpha*nu*(1-delta)*q_ti(i-1,n) - (1+nu)*(1-alpha)-alpha*z(i,n) ...
   + 1 + nu + (alpha/(1-alpha))*z(i,n) ) + nu*D_t(i-1,n)) - 2*alpha*nu*F_t(i-1,n);
C_t(i-1,n) = - nu^2 * F_t(i-1,n);
pi_ti(i-1,n) = (-1.0/(2*A_t(i-1,n))) * (B_t(i-1,n) - sqrt( B_t(i-1,n)^2 - 4*A_t(i-1,n)*C_t(i-1,n)) );
n_ti(i,n) = 1 + nu*(1-e_ti(i-1,n));
gamma_ti(i,n) = gamma_ti(i-1,n)*(1+psi*e_ti(i-1,n));
y_ti(i-1,n) = gamma_ti(i-1,n)*(q_ti(i-1)^(1-alpha))*(n_ti(i-1,n)^alpha);
g_t(i,n) = (1 + psi*e_ti(i-1,n))*((q_ti(i,n)/q_ti(i-1,n))^(1- alpha))...
  *((n_ti(i,n)/n_ti(i-1,n))^alpha)-1;
g_t=real(g_t);
end
end
%for s=2:99
%  g_t(s,n) = log(y_ti(s,n))-log(y_ti(s-1,n));
%end
lam_tab=lam_tab';
lam_tab=lam_tab*ones(1,98);
time=1:1:98;
time=time';
time=time*ones(1,59);
time=time';
g_t=g_t';
subplot(2,1,2)
surf(lam_tab,time,g_t(1:59,2:99));
title('Dynamics of the growth rate, ARCH process, \sigma_x = 5 \eta =2');
xlabel('\lambda');
ylabel('Time');
zlabel('growth rate');
```

Statement of Copyright

Hereby I state that I compiled this Master thesis alone with help of the sources and programs (PcTex, Matlab) referred to in the text.

Claudia Trentini

Berlin, February 25, 2003