Master Thesis:

Fiscal Policy in An Estimated Dynamic Stochastic General Equilibrium Model

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Abstract

In this paper we follow closely with "An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area" by Frank Smets and Raf Wouters and supplement their model with consumption tax rate, capital tax rate and labor tax rate and lump sum tax. We solve the extended model, loglinearize it and implement it into Toolkit and Dynare. Then it is estimated with Bayesian techniques using observed marcoeconomic time series. Estimated parameters and standard deviations of shocks from our extended model using the same dataset used by Smets and Wouters (2003) agree with those values published in Smets and Wouters (2003). But after adding two more time series into observed data and detrending them by HP filter, the estimation results vary from those of Smets and Wouters (2003). The differences may come from more observation or different detrending tools.

Contents

1 Introduction

Frank Smets and Raf Wouters developed in 2003 an empirical stochastic dynamic general equilibrium (DSGE) model with sticky prices and wages for Euro area. The Smets and Wouters (2003) model incorporates various a multitude of features, such as habit formation, costs of capital adjustment, capacity utilization cost, sticky prices and wages following the Calvo mechanism. Ten structural shocks are also introduced to analyse the volatility of the business cycle developments in the euro area economy. The model is estimated based on quarterly data of the seven euro area macroeconomic variables, i.e., real GDP, consumption, investment, employment, real wages, inflation and the nominal short-term interest rate. Using Bayesian estimation and validation techniques, the estimated model shows its advantage over more standard, unrestricted time series models, such as vector auto regressions.

This paper seeks to investigate the effects of fiscal policy shocks in a New-Keynesian DSGE model. We supplement the euro area model by Smets and Wouters (2003) with consumption tax, labor tax, capital tax, and lump sum tax. The model features nominal price rigidities and wage rigidities as well real frictions such as adjustment costs in investment and habit information in consumption. We also estimate the model using Bayesian estimation techniques to match the observed macroeconomic series including real GDP, consumption, investment, inflation, short-term interest rate, wages, employment, government transfer and government expenditure.

The paper is organized as follows: section two gives a brief overview of literature. Section three describes our extended DSGE model following closely with the model designed by Smets and Wouters (2003) but adding fiscal policy features. Section four gives details on how to solve the model and how to loglinearize the equations, particularly the wage equation and inflation equation. Then we explain the implementation of the loglinearized the model into Toolkit. Afterwards the impulse response pictures to thirteen structural shocks are presented and analysed. Furthermore, in Section six we implement and estimate in Dynare three versions of model and data. Estimation parameters and standard deviations of shocks from our extended model using the same dataset used by Smets and Wouters (2003) agree with those values published in Smets and Wouters (2003). But after adding two more time series into observed data and detrending them by HP filter, the estimation results vary from those of Smets and Wouters (2003). Concluding remarks are stated in Section seven.

2 Literature

Recent years has witnessed the development of dynamic stochastic general equilibrium (DSGE) models with microeconomic foundations. These micro-founded DSGE models make evaluating consequences of macroeconomic policies easier as the utility of the households can be taken as a measure of welfare. The DSGE model in Smets and Wouters (2003) provides a good example of such a micro-founded DSGE model for monetary policy analysis.

Smets and Wouters (2003) incorporate a multitude of rich elements from previous literature. The model shares a lot of common features with Christiano, Eichenbaum, and Evans (2001). For example, the cost of changing the utilisation rate is expressed in terms of consumption goods as in CEE (2001). Besides, the cost of adjusting the capital stock is modelled as a function of the change in investment, instead of the level of investment as is commonly done.

Some features of the Smets and Wouters (2003) model are based on other resources. The sticky nominal prices and wages in the model are set as in Erceg, Henderson and Levin (2000) using a Calvo mechanism. Following Fuhrer (2000) and McCallum and Nelson (1999), an external habit formation variable is used to introduce the persistence in the consumption process observed in empirical data.

Besides, in order to introduce taxes and government transfer into the model, we follow Trabandt and Uhlig (2006) to set taxes to follow a first order autoregressive process.

3 The Model

In this section we present the model that we derive and estimate later. Our model is an extended version of the New-Keynesian DSGE model of the euro area developed by Smets and Wouters(2003).There are 4 types of agents in this model, households, firms and a fiscal authority and a monetary authority. Households maximize their sum of discounted utility over an infinite time horizon. The utility of households depends positively on the consumption of goods, relative to a time varying external habit variable. The households provide a differentiated type of labor and so has a monopoly power. They are the price setters in the labor market. Firms produce differentiated goods using labor inputs and capital inputs. The prices of goods are decided following the Calvo settings.

3.1 The Households

There is a continuum of households indexed by $\tau \in [0,1]$. Each household maximises its total discounted utility over an infinite time horizon:

$$E_0 \sum \beta^t U_t^\tau \tag{3.1}$$

The discount facotr is β and the utility function is specified as:

$$U_t^\tau = \epsilon_t^b \left\{ \frac{1}{1-\sigma_c} \left(C_t^\tau - Ht\right)^{1-\sigma_c} - \frac{\epsilon_t^L}{1+\sigma_l} \left(l_t^\tau\right)^{1+\sigma_l} \right\} \tag{3.2}$$

Utility of each household is positively related to consumption C_t and negatively related to labor supply $l_t.H_t$ is an exteral habit variable, which is a proportional to aggregate past consumption: $H_t = hC_{t-1}$. σ_c is the inverse of the intertemporal substitution elasticity. σ_l is the inverse of the elasticity of work effort to the real wage. ϵ_t^b is a preference shock and ϵ_t^L is a labour supply shock. Both of them follow a first order autoregressive process with an indepently identically distributed normal error, i.e. $\epsilon_t^b = \rho_b \epsilon_{t-1}^b + \eta_t^b$ and $\epsilon_t^L = \rho_L \epsilon_{t-1}^L + \eta_t^L$.

Each household is also subject to an intertemporal buget constrait given as:

$$
\begin{aligned}
0 &= (1+\tau_t^c)C_t^\tau + I_t^\tau - (1-\tau_t^n)W_t l_t^\tau - A_t^\tau - Div_t^\tau + TR_t \\
&\quad -(1-\tau_t^k)(r_t^k z_t^\tau k_{t-1}^\tau - \psi(Z_t^\tau)k_{t-1}^\tau) - \frac{B_{t-1}}{P_t} + b_t \frac{B_t}{P_t} \qquad (3.3)
\end{aligned}
$$

Where τ_t^n represents the labor tax rate levied on labor income. τ_t^c indicates the consumption tax rate. And τ_t^k is the capital tax rate on income from renting out capital. TR_t represents the lump sum tax. DIV_t are dividends from the intermediate firms.

Z_t^τ here is the utilization rate of capital. $\psi(Z_t^\tau)$ represents the cost of capital utilization. Following Christiano, Eichenbaum, and Evans (2001), the steady state of capital utilization rate is set as 1, and at steady state the cost of capital utilization is set as 0 ($\bar{z} = 1$ and $\psi(1) = 0$).

Each household offers a differentiated type of labor. Thus households have a monopoly power on labor supply and they are the price setters in the labor market. Following Calvo (1983), households can only reoptimize their wage with probability $1-\xi_w$ when they receive a random signal to change their wage. The rest ξ_w households can not reoptimize their wage and can only adjust their wage according to the following scheme:

$$
W_t^\tau = (\frac{P_{t-1}}{P_{t-2}})^{\gamma_w} W_{t-1}^\tau \qquad (3.4)
$$

Where γ_w is the degree of indexation to past inflation.

3.2 Technologies and Firms

In the economy two types of firms exist. Final-good firms produce final good for consumption and investment by the households. The final-good sector is perfectly competitive. Intermediate-good firms have monopoly power and provide differentiated intermediate good j ($j \in [0,1]$) for the final-good firms.

3.2.1 Final-good Firms

Final goods are produced using the intermediate goods by the following technology:

$$Y_t = [\int (y_t^j)^{1/(1+\lambda_{p,t})} dj]^{1+\lambda_{p,t}} \tag{3.5}$$

Where y_t^j represents the amount of intermediate good j used in the production of final goods. $\lambda_{p,t}$ determines the markup in the goods market. $\lambda_{p,t}$ follows the process: $\lambda_{p,t} = \lambda_p + \eta_t^p$. η_t^p is a cost push shock and is i.i.d. normal.

Final-good firms minimize their cost and thus:

$$y_t^j = Y_t(\frac{P_t^j}{P_t})^{-(1+\lambda_{p,t})/\lambda_{p,t}} \tag{3.6}$$

Where P_t^j is the price of intermediate goods while P_t is the price of final goods. As the final good sector is perfectly competitive, the intermediate-good price and the final-good price have the following relation:

$$P_t = [\int (p_t^j)^{-1/\lambda_{p,t}} dj]^{-\lambda_{p,t}} \tag{3.7}$$

3.2.2 Intermediate-good firms

Each intermediate good j is produced by a single firm using the following technology:

$$y_t^j = \epsilon_t^a \tilde{K}_{j,t}^{\alpha} L_{j,t}^{1-\alpha} - \Phi \tag{3.8}$$

Here ϵ_t^a denotes the technology shock. ϵ_t^a follows a first order autoregressive process, i.e., $\epsilon_t^a = \rho_t^a \epsilon_{t-1}^a + \eta_t^a$. $\tilde{K}_{j,t}$ is the effectively utilized capital stock given by $\tilde{K}_{j,t} = z_t K_{j,t-1}$. $L_{j,t}$ denotes the input of labor and Φ is the fixed cost.

The marginal cost of the intermediate firms is given by:

$$mc_t = \frac{1}{\epsilon_t^a} r_t^{k\alpha} W_t^{1-\alpha} (\alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)} \tag{3.9}$$

So at time period t, the nominal profit of firm j is given by:

$$\pi_t^j = (P_t^j - mc_t) Y_t (\frac{P_t^j}{P_t})^{-(1+\lambda_{p,t})/\lambda_{p,t}} - mc_t \Phi \tag{3.10}$$

4 Model Analysis

4.1 Consumption, Investment and Capital Accumulation

Each household maximizes its sum of discounted utilities over an infinite horizon subject to the budget constraint:

$$
\begin{aligned}
L \quad = \quad & E_t \sum_{t=0}^{\infty} \beta^t \epsilon_t^b [\frac{1}{1-\sigma_c} \left(C_t^\tau - Ht\right)^{1-\sigma_c} - \frac{\epsilon_t^L}{1+\sigma_l} \left(l_t^\tau\right)^{1+\sigma_l}] \\
& -\lambda_t \beta^t [(1+\tau_t^c) C_t^\tau - (1-\tau_t^n) W_t l_t^\tau - A_t^\tau - Div_t^\tau + TR_t + I_t^\tau \\
& -(1-\tau_t^k)(r_t^k z_t^\tau k_{t-1}^\tau - \psi(Z_t^\tau) k_{t-1}^\tau) - \frac{B_{t-1}}{P_t} + b_t \frac{B_t}{P_t}] \\
& -\mu_t \beta^t [k_t - (1-\delta) k_{t-1} - I_t (1 - S(\epsilon_t^I I_t / I_{t-1}))]
\end{aligned} \tag{4.1}
$$

The first order condition with respect to consumption gives:

$$\lambda_t = \frac{1}{1+\tau_t^c} (C_t^\tau - Ht)^{-\sigma_c} \epsilon_t^b \tag{4.2}$$

Where λ_t is the marginal utility of consumption.

The first order condition with respect to real capital is:

$$Q_t = Et[\beta \frac{\lambda_{t+1}}{\lambda_t} [(1-\delta) Q_{t+1} + (1-\tau_t^k)(r_{t+1}^k Z_{t+1}^\tau - \psi(Z_{t+1}^\tau))]] \tag{4.3}$$

Where

$$Q_t \equiv \mu_t / \lambda_t \tag{4.4}$$

The first order condition with respect to investment results in:

$$1 = Et[Q_t - Q_t [S(\frac{\epsilon_t^I I_t}{I_{t-1}}) + S'(\frac{\epsilon_t^I I_t}{I_{t-1}}) \frac{\epsilon_t^I I_t}{I_{t-1}}] + \beta Q_{t+1} \frac{\lambda_{t+1}}{\lambda_t} S'(\frac{\epsilon_{t+1}^I I_{t+1}}{I_t}) \frac{\epsilon_{t+1}^I I_{t+1}}{I_t}] \tag{4.5}$$

Where S(.) is the adjustment function of changes in investment. The steady states of S(.) and S'(.) are set as 0 following CEE (2001). ϵ_t^I denotes a shock to investment cost function. It follows a first order autoregressive process with a i.i.d. normal error term: $\epsilon_t^I = \rho^I \epsilon_{t-1}^I + \eta_t^I$.

The first order condition with respect to the utilization rate of capital gives:

$$r_t^k = \psi'(Z_t^\tau) \tag{4.6}$$

4.2 Wage Setting

The households who can reoptimize their wage choose the level of nominal wage to maximise their objective function subject to the intertemporal budget constraint and the labor demand equation:

$$L = E_t \sum_{i=0}^{\infty} \beta^i \xi_w^i U_{t+i} - \lambda_{t+i}[... - (1-\tau_{t+i}^n) w_{t+i}^\tau l_{t+i}^\tau / P_{t+i} - A_{t+i}^\tau + I_{t+i}^\tau + ...] \tag{4.7}$$

Meanwhile, the first derivative of labor with respect to nominal wage is:

$$\begin{aligned} \frac{\partial l_{t+i}^\tau}{\partial w_t^\tau} &= \frac{\partial (\frac{w_{t+i}^\tau}{W_{t+i}})^{-\frac{1+\lambda_{w,t+i}}{\lambda_{w,t+i}}} L_{t+i}}{\partial w_t^\tau} \\ &= -\frac{1+\lambda_{w,t+i}}{\lambda_{w,t+i}} \frac{l_{t+i}^\tau}{w_t^\tau} \end{aligned} \tag{4.8}$$

$$\tag{4.9}$$

So the resulting first order condition with respect to nominal wage is as follows:

$$E_t \sum_{i=0}^{\infty} \beta^i \xi_w^i (1-\tau_{t+i}^n) \frac{w_t^\tau}{P_{t+i}} \frac{l_{t+i}^\tau U^C t+i}{1+\lambda_{w,t+i}} (P_{t+i-1}/P_{t-1})^{\gamma_w} = E_t \sum_{i=0}^{\infty} \beta^i \xi_w^i l_{t+i}^\tau U_{t+i}^l \tag{4.10}$$

The law of motion of the aggregate wage index is:

$$(W_t)^{-1/\lambda_{w,t}} = \xi_w (W_{t-1}(\frac{P_{t-1}}{P_{t-2}})^{\gamma_w})^{-1/\lambda w,t} + (1-\xi_w)(w_t^h)^{-1/\lambda w,t} \tag{4.11}$$

4.3 Price setting

Each intermediate firm j has market power in the market of its own good. Each firm chooses its price level to maximize the sum of expected profits. The discount rate is the $\beta\rho_t$, which is consistent with the pricing kernel for the households: $\rho_t = \frac{\lambda_{t+k}}{\lambda_t}\frac{1}{P_{t+k}}$.

Following Calvo (1983), firms can only change their their prices when they receive a random price-change signal. Each firm has a probabiity $1-\xi_p$ to reoptimize its price at a certain period. Firms that can not reoptimize their price only adjust according to indexed inflation of previous period. Firms who receive a price-change signal reoptimize their price at time to maximize thier sum of expected profits. The resulting first order condition is as follows:

$$Et\sum_{i=0}^{\infty}\beta^i\xi_p^i\lambda_{t+i}y_{t+i}^j\frac{\tilde{P}_t^j}{P_t}(\frac{(P_{t+i-1}/P_{t-1})^{\gamma_p}}{P_{t+i}/P_t}) = Et\sum_{i=0}^{\infty}\beta^i\xi_p^i(1+\lambda_{p,t+i})mc_{t+i} \tag{4.12}$$

The law of motion of the aggregate price index is:

$$(P_t)^{-1/\lambda_{p,t}} = \xi_p(P_{t-1}(\frac{P_{t-1}}{P_{t-2}})^{\gamma_p})^{-1/\lambda p,t} + (1-\xi_p)(\tilde{p}_t^j)^{-1/\lambda p,t} \tag{4.13}$$

Cost minimization of firms also results in the following equalization of marginal cost:

$$\frac{W_t L_{j,t}}{r_t^k \tilde{K}_{j,t}} = \frac{1-\alpha}{\alpha} \tag{4.14}$$

The above equation shows that the capital labor ratio are identical for every intermediate firm j.

4.4 Equilibrium conditions

The goods market is in equilibrium when the total output equals the consumption, investment, capital utilization costs and government expenditure:

$$Y_t = C_t + I_t + G_t + \psi(Z_t)K_{t-1} \tag{4.15}$$

The equilibrium in the government sector is fulfilled when the government expenditure and transfer equal tax revenues:

$$G_t = \tau_t^c C_t + \tau_t^k (r_t^k Z_t K_{t-1} - \psi(Z_t)K_{t-1}) + \tau_t^n W_t L_t - F_t \tag{4.16}$$

The equlibrium in the capital market is fulfilled when the capital demand of intermediate firms equals capital supply offered by households. The equlibrium in the labor market is fulfilled when the labor demand by firms equals labor supply provided by households at the wage level they have chosen.

4.5 Log-linearizing the model

In this subsection we loglinearize the equations around their steady state. The hat sign above a variable represents its log deviation from its steady state.

4.5.1 How to log-linearize the inflation equation

In order to log-linearize the aggregate price index equation, we first consider a simple case:

$$A_t = B_t^{C_t} \tag{4.17}$$

Taylor approximation gives:

$$\hat{A}_t = \frac{\partial A}{\partial B}\frac{\bar{B}}{\bar{A}}\hat{B}_t + \frac{\partial A}{\partial C}\frac{\bar{C}}{\bar{A}}\hat{C}_t \tag{4.18}$$

$$\hat{A}_t = \bar{C}\bar{B}^{\bar{C}-1}\frac{\bar{B}}{\bar{A}}\hat{B}_t + (\ln \bar{B})\bar{B}^{\bar{C}}\hat{C}_t \tag{4.19}$$

$$\hat{A}_t = \bar{C}\hat{B}_t + (\ln \bar{B})\bar{C}\hat{C}_t \tag{4.20}$$

Applying the above results to the aggregate price index equation we reach:

$$\hat{P}_t = \xi_p(\hat{P}_{t-1} + \gamma_p \hat{\pi}_{t-1}) + (1-\xi_p)\ \hat{\tilde{P}}_t^j \tag{4.21}$$

Next, we log linearize Calvo pricing equation as follows:

$$Et\sum_{i=0}^{\infty}\beta^i\xi_p^i(\hat{\lambda'}_{p,t+i} + \hat{mc}_{t+i} + \hat{P}_{t+i} - \gamma_p\hat{P}_{t+i-1}) = Et\sum_{i=0}^{\infty}\beta^i\xi_p^i\hat{\tilde{P}}_t^j - \gamma_p\hat{P}_{t-1} \tag{4.22}$$

$$Et\sum_{i=0}^{\infty}\beta^i\xi_p^i(\hat{\lambda'}_{p,t+i} + \hat{mc}_{t+i} + \hat{P}_{t+i} - \gamma_p\hat{P}_{t+i-1}) = \frac{1}{1-\beta\xi_p}\hat{\tilde{P}}_t^j - \gamma_p\hat{P}_{t-1} \tag{4.23}$$

For time period t+1, we have a likewise equation:

$$Et\sum_{i=0}^{\infty}\beta^i\xi_p^i(\hat{\lambda'}_{p,t+i+1} + \hat{mc}_{t+i+1} + \hat{P}_{t+i+1} - \gamma_p\hat{P}_{t+i}) = \frac{1}{1-\beta\xi_p}\hat{\tilde{P}}_{t+1}^j - \gamma_p\hat{P}_t \tag{4.24}$$

Combining equations for time period t and t+1, we reach:

$$\hat{\lambda'}_{p,t} + \hat{mc}_t + \hat{P}_t - \gamma_p\hat{P}_{t-1} = \frac{1}{1-\beta\xi_p}[\hat{\tilde{P}}_t^j - \gamma_p\hat{P}_{t-1} - \beta\xi_p^i(\hat{\tilde{P}}_{t+1}^j - \gamma_p\hat{P}_t)] \tag{4.25}$$

Plugging the loglinearized aggregate price index equation into the above equation we get:

$$\hat{\lambda'}_{p,t} + \hat{mc}_t = \frac{1}{(1-\beta\xi_p)}\frac{1}{(1-\xi_p)}[-\beta\xi_p\pi_{t+1} - \gamma_p\xi_p\pi_{t-1} + \xi_p(1+\beta\gamma_p)\pi_t] \tag{4.26}$$

Plugging in the marginal cost equation and then rearranging terms we reach the inflation equation:

$$\widehat{\pi}_t = \frac{\beta}{1+\beta\gamma_p} E_t \widehat{\pi}_{t+1} + \frac{\gamma_p}{1+\beta\gamma_p} \widehat{\pi}_{t-1} + \frac{1}{1+\beta\gamma_p} \frac{(1-\beta\xi_p)1-\xi_p)}{\xi_p} [\alpha \widehat{r}_t^k + (1-\alpha)\widehat{w}_t - \widehat{\epsilon}_t^a + \eta_t^p] \tag{4.27}$$

4.5.2 How to log-linearize the wage equation

From the calvo wage equation we have:

$$Et \sum_{i=0}^{\infty} \beta^i \xi_w^i (\hat{\tau}_{t+i}^n + \hat{U}_{t+i}^{\tau,l} - \hat{U}_{t+i}^c + \hat{P}_{t+i} - \gamma_w \hat{P}_{t+i-1}) = \frac{1}{(1-\beta\xi_w)} (\hat{w^\tau}_t - \gamma_w \hat{P}_{t-1}) \tag{4.28}$$

From utility function we derive the marginal utility of consumption and marginal disutility of labor for each household h:

$$\hat{U}_t^c = \hat{\epsilon}_t^b - \frac{\sigma_c}{1-h} (\hat{C}_t - h\hat{C}_{t-1}) \tag{4.29}$$

$$\hat{U}_t^{\tau,l} = \hat{\epsilon}_t^b + \hat{\epsilon}_t^l + \sigma_l \hat{L}_t^h \tag{4.30}$$

From the labor demand equation we know:

$$\hat{L}_t^\tau = -\frac{1+\lambda_w}{\lambda_w} (\hat{w^\tau}_t - \hat{W}_t) + \hat{L}_t \tag{4.31}$$

Thus

$$\hat{U}_t^{\tau,l} = \hat{\epsilon}_t^b + \hat{\epsilon}_t^l + \sigma_l \frac{1+\lambda_w}{\lambda_w} (\hat{W}_t - \hat{w^\tau}_t) + \sigma_l \hat{L}_t \tag{4.32}$$

We know that $\hat{w^\tau}_{t+i} = (\frac{P_{t+i-1}}{P_{t-1}})^{\gamma_w} \hat{w^\tau}_t$. Besides, we define $k_w = 1 + \sigma_l \frac{1+\lambda_w}{\lambda_w}$ to simplify the coefficients:

$$\begin{aligned} \frac{k_w}{(1-\beta\xi_w)} (\hat{w^\tau}_t - \gamma_w \hat{P}_{t-1}) \quad = \quad & Et \sum_{i=0}^{\infty} \beta^i \xi_w^i (\hat{\tau}_{t+i}^n + \epsilon_{t+i}^l + \sigma_l L_{t+i} + \frac{\sigma_c}{1-h} (\hat{C}_{t+i} - h\hat{C}_{t+i-1}) \\ & + (k_w - 1) W_{t+i} + \hat{P}_{t+i} - k_w \gamma_w \hat{P}_{t+i-1}) \end{aligned} \tag{4.33}$$

Similarly we have:

$$\begin{aligned}\frac{k_w}{(1-\beta\xi_w)}(\hat{w^\tau}_{t+1}-\gamma_w\hat{P}_t) &= Et\sum_{i=0}^{\infty}\beta^i\xi_w^i(\hat{\tau}_{t+i+1}^n+\epsilon_{t+i+1}^l+\sigma_l L_{t+i+1}+\frac{\sigma_c}{1-h}(\hat{C}_{t+i+1}-h\hat{C}_{t+i}) \\ &\quad +(k_w-1)W_{t+i+1}+\hat{P}_{t+i+1}-k_w\gamma_w\hat{P}_{t+i}) \qquad (4.34)\end{aligned}$$

Combining the above two equations we reach:

$$\begin{aligned}\hat{w^h}_t-\beta\xi_w\hat{w^\tau}_{t+1} &= \frac{1-\beta\xi_w}{k_w}(\hat{\tau}_t^n+\epsilon_t^l+\sigma_l L_t+\frac{\sigma_c}{1-h}(\hat{C}_{t+1}-h\hat{C}_t) \\ &\quad \frac{k_w-1}{k_w}(1-\beta\xi_w)\hat{W}_t+\frac{(1-\beta\xi_w)}{k_w}\hat{P}_t-\beta\xi_w\gamma_w\hat{\pi}_t \qquad (4.35)\end{aligned}$$

The loglinearized wage index equation are derived in the same way as the price index equation:

$$\hat{W}_t=\xi_w(\hat{W}_{t-1}+\gamma_w\hat{\pi}_{t-1})+(1-\xi_w)\hat{w^\tau}_t \qquad (4.36)$$

We use ς to denote real wage, where $\hat{\varsigma}_t=\hat{W}_t-\hat{P}_t$. So the above equation can be expressed in terms of real wage:

$$\hat{\varsigma}_t+\hat{P}_t=\xi_w(\hat{\varsigma}_{t-1}+\hat{P}_{t-1}+\gamma_w\hat{\pi}_{t-1})+(1-\xi_w)\hat{w^\tau}_t \qquad (4.37)$$

Rearranging the terms gives:

$$\hat{w^\tau}_t=\frac{1}{(1-\xi_w)}[\hat{\varsigma}_t-\xi_w\hat{\varsigma}_{t-1}+\hat{P}_t-\xi_w\hat{P}_{t-1}-\xi_w\gamma_w\hat{\pi}_{t-1}] \qquad (4.38)$$

For $\hat{w^\tau}_{t+1}$ we have:

$$\hat{w^\tau}_{t+1}=\frac{1}{(1-\xi_w)}[\hat{\varsigma}_{t+1}-\xi_w\hat{\varsigma}_t+\hat{P}_{t+1}-\xi_w\hat{P}_t-\xi_w\gamma_w\hat{\pi}_t] \qquad (4.39)$$

Plugging the above equation into the loglinearized aggregate wage index equation and then rearranging terms, we reach the wage equation:

$$\begin{aligned}\widehat{\varsigma}_t &= \frac{\beta}{1+\beta}E_t\widehat{\varsigma}_{t+1}+\frac{1}{1+\beta}\widehat{\varsigma}_{t-1}+\frac{\beta}{1+\beta}E_t\widehat{\pi}_{t+1}-\frac{1+\beta\gamma_w}{1+\beta}\widehat{\pi}_t+\frac{\gamma_w}{1+\beta}\widehat{\pi}_{t-1}\\ &\quad +\frac{1}{1+\beta}\frac{(1-\beta\xi_w)(1-\xi_w)}{(1+\frac{(1+\lambda_w)\sigma_L}{\lambda_w})\xi_w}[-\widehat{\varsigma}_t+\sigma_L\widehat{L}_t+\frac{\sigma_c}{1-h}(\widehat{C}_t-h\widehat{C}_{t-1})+\widehat{\epsilon}_t^L+\hat{\tau}_t^n]\end{aligned} \tag{4.40}$$

4.5.3 The log-linearized model

$$\hat{\lambda}_t = \hat{\epsilon}_t^b - \frac{1}{1-h}\sigma_c[\hat{C}_t - h\hat{C}_{t-1}] + \hat{\tau}_t^c \tag{4.41}$$

$$\hat{Z}_t = \psi \hat{r}_t^k \tag{4.42}$$

Consumption equation is given as:

$$\widehat{C}_t = \frac{h}{1+h}\widehat{C}_{t-1} + \frac{1}{1+h}E_t\widehat{C}_{t+1} - \frac{1-h}{(1+h)\sigma_c}(\widehat{R}_t - E_t\widehat{\pi}_{t+1} + \widehat{\epsilon}_t^b - \widehat{\epsilon}_{t+1}^b + \hat{\tau}_t^c - \hat{\tau}_{t+1}^c) \tag{4.43}$$

Q equation is given as:

$$\widehat{Q}_t = -(\widehat{R}_t - E_t\widehat{\pi}_{t+1}) + \frac{1-\delta}{1-\delta+\overline{r}^k}E_t\widehat{Q}_{t+1} + \frac{\overline{r}^k}{1-\delta+\overline{r}^k}E_t\widehat{r}_{t+1}^k + \eta_t^Q - (1-\beta+\beta\delta)\hat{\tau}_t^k \tag{4.44}$$

The investment equation is given by:

$$\widehat{I}_t = \frac{1}{1+\beta}\widehat{I}_{t-1} + \frac{\beta}{1+\beta}E_t\widehat{I}_{t+1} + \frac{\varphi}{1+\beta}\widehat{Q}_t + \widehat{\epsilon}_t^I \tag{4.45}$$

The capital accumulation equation is given by:

$$\widehat{K}_t = (1-\delta)\widehat{K}_{t-1} + \delta\widehat{I}_{t-1} \tag{4.46}$$

The labor demand equation comes from the equalisation of marginal costs of labor and capital:

$$\widehat{L}_t = -\widehat{\varsigma}_t + (1+\psi)\widehat{r}_t^k + \widehat{K}_{t-1} \tag{4.47}$$

Market equilibrium is given as:

$$\hat{Y}_t = \phi\hat{\epsilon}_t^a + \phi\alpha\psi\hat{r}_t^k + \phi\alpha K_{t-1} + \phi(1-\alpha)L_t \tag{4.48}$$

$$\hat{Y}_t = \frac{\bar{C}}{\bar{Y}}\hat{C}_t + \frac{\bar{I}}{\bar{Y}}\hat{I}_t + \hat{G}_t \tag{4.49}$$

Government fulfills:

$$\hat{G}_t = \bar{Y}\bar{C}\hat{\tau}_t^c + \bar{Y}\bar{W}\bar{L}\hat{\tau}_t^n + \bar{Y}\bar{K}\bar{r}^k\hat{\tau}_t^k - \bar{Y}\bar{TR}\hat{TR}_t \tag{4.50}$$

The inflation equation is given by:

$$\widehat{\pi}_t = \frac{\beta}{1+\beta\gamma_p} E_t \widehat{\pi}_{t+1} + \frac{\gamma_p}{1+\beta\gamma_p} \widehat{\pi}_{t-1} + \frac{1}{1+\beta\gamma_p} \frac{(1-\beta\xi_p)1-\xi_p)}{\xi_p} [\alpha \widehat{r}_t^k + (1-\alpha)\widehat{w}_t - \widehat{\epsilon}_t^a + \eta_t^p] \tag{4.51}$$

The wage equation is given as:

$$\begin{aligned} \widehat{\varsigma}_t \quad = \quad & \frac{\beta}{1+\beta} E_t \widehat{\varsigma}_{t+1} + \frac{1}{1+\beta} \widehat{\varsigma}_{t-1} + \frac{\beta}{1+\beta} E_t \widehat{\pi}_{t+1} - \frac{1+\beta\gamma_w}{1+\beta} \widehat{\pi}_t + \frac{\gamma_w}{1+\beta} \widehat{\pi}_{t-1} \\ & + \frac{1}{1+\beta} \frac{(1-\beta\xi_w)(1-\xi_w)}{(1+\frac{(1+\lambda_w)\sigma_L}{\lambda_w})\xi_w} [-\widehat{\varsigma}_t + \sigma_L \widehat{L}_t + \frac{\sigma_c}{1-h}(\widehat{C}_t - h\widehat{C}_{t-1}) + \widehat{\epsilon}_t^L + \hat{\tau}_t^n + \eta_t^w] \end{aligned} \tag{4.52}$$

The monetary policy reaction function is given by:

$$\begin{aligned} \widehat{R}_t \quad = \quad & \rho \widehat{R}_{t-1} + (1-\rho) \left\{ \bar{\pi}_t + r_\pi (\widehat{\pi}_{t-1} - \bar{\pi}_t) + r_Y (\widehat{Y}_t - \widehat{Y}_t^P) \right\} \\ & + r_{\Delta\pi} (\widehat{\pi}_t - \widehat{\pi}_{t-1}) + r_{\Delta Y} \left[\widehat{Y}_t - \widehat{Y}_t^P - (\widehat{Y}_{t-1} - \widehat{Y}_{t-1}^P) \right] + \eta_t^R \end{aligned} \tag{4.53}$$

5 Implementation in Toolkit

In order to calibrate the model, we have to create two systems based on the loglinearized equations from 4.43 to 4.53. The first system is a flexible system where there is no price stickiness, wage stickiness or three cost-push shocks. The other one is the sticky system where prices and wages are set following a Calvo mechanism. We use the potential output produced in the flexible system to calculate the output gap in the Taylor rule. In each system, there are 9 endogenous variables and 2 state variables. The 9 endogenous variables are capital, consumption, investment, inflation, wages, output, interest rate, government transfer and real capital stock. The 2 state variables are labour and return on capital.

We take most of the parameter values in the Mode column of Table 1 of Smets and Wouters(2003) for our calibration. We set the first order autoregressive coefficients of consumption tax rate, capital tax rate and labor tax rate as 0.9. As for the standard errors of the tax rates, we set them all equal 0.16. Besides, we set the percentage of government transfer to GDP as 0.2. The following table gives a detailed description of the parameters.

Parameters	Value	Description
β	0.99	discount factor
τ	0.025	depreciation rate of capital
α	0.3	capital output ratio
ψ	$1/0.169$	inverse of the elasticity of the capital utilization cost function
γ_p	0.469	degree of partial indexation of price
γ_w	0.763	degree of partial indexation of wage
λ_w	0.5	mark up in wage setting
ty	0.01	percentage of government transfer to GDP
ξ_f^p	0.00000001	Calvo price stickiness in the flexible system
ξ_f^w	0.00000001	Calvo wage stickiness in the flexible system
ξ_s^p	0.908	Calvo price stickiness in the sticky system
ξ_s^w	0.737	Calvo wage stickiness in the sticky system
σ_L	2.4	inverse of elasticity of work effort
σ_c	1.353	coefficient of the relative risk aversion of the household
h	0.573	habit portion of past consumption
ϕ	1.408	share of fixed cost in production plus 1
φ	$1/6.771$	inverse of investment adjustment cost
$\overline{r}_k$	$1/\beta - 1 + \tau$	steady state return on capital
k_y	8.8	capital output ratio
inv_y	0.22	share of investment to GDP
c_y	0.6	share of consumption to GDP
g_y	$1 - c_y - inv_y$	historical average share of government expenditure in GDP
r_π^Δ	0.14	inflation growth coefficient
r_y	0.099	output gap coefficient

Parameter	Value	Description
r_y^{Δ}	0.159	output gap growth coefficient
ρ	0.961	autoregressive parameter on lagged interest rate
r_π	1.684	inflation coefficient
ρ_{ϵ_L}	0.889	autoregressive parameter of labour supply shock
ρ_{ϵ_a}	0.823	autoregressive parameter of productivity shock
ρ_{ϵ_b}	0.855	autoregressive parameter of preference shock
ρ_G	0.949	autoregressive parameter of government expenditure shock
$\rho_{\overline{\pi}}$	0.924	autoregressive parameter of inflation objective shock
ρ_{ϵ_i}	0.927	autoregressive parameter of investment shock
ρ_{ϵ_r}	0	autoregressive parameter of interest rate shock,IID
ρ_{λ_w}	0	autoregressive parameter of wage markup,IID
ρ_{tc}	0.9	autoregressive parameter of consumption tax,IID
ρ_{tk}	0.9	autoregressive parameter of capital tax,IID
ρ_{tl}	0.9	autoregressive parameter of labor tax,IID
ρ_q	0	autoregressive parameter of return on equity,IID
ρ_{λ_p}	0	autoregressive parameter of price mark-up shock,IID
σ_{ϵ_L}	3.52	stand error of labour supply shock
σ_{ϵ_a}	0.598	stand error of productivity shock
σ_{ϵ_b}	0.336	stand error of preference shock
σ_G	0.325	stand error of government expenditure shock
$\sigma_{\overline{\pi}}$	0.017	stand error inflation objective shock
σ_{ϵ_r}	0.081	stand error of interest rate shock
σ_{ϵ_i}	0.085	stand error of investment shock
σ_{λ_p}	0.16	stand error of mark-up shock
σ_{λ_w}	0.289	stand error of wage mark-up shock
σ_{ϵ_q}	0.604	stand error of equity premium shock
σ_{tc}	0.16	stand error of consumption tax shock
σ_{tk}	0.16	stand error of capital tax shock
σ_{tl}	0.16	stand error of labor tax shock

Besides, we create another 4 indices for parameters in order to reduce the size of matrices. They are as follows:

Parameter	Value	Description
$index1f$	$\frac{(1-\beta\xi_w^f)(1-\xi_w^f)}{(1+\beta)(1+\frac{(1+\lambda_w)\sigma_L}{\lambda_w})\xi_w^f}$	an index in the wage equation in the flexible system
$index1s$	$\frac{(1-\beta\xi_w^s)(1-\xi_w^s)}{(1+\beta)(1+\frac{(1+\lambda_w)\sigma_L}{\lambda_w})\xi_w^s}$	an index in the wage equation in the sticky system
$index2f$	$\frac{(1-\beta\xi_p^f)(1-\xi_p^f)}{(1+\beta)(1+\frac{(1+\lambda_p)\sigma_L}{\lambda_p})\xi_p^f}$	an index in the inflation equation in the flexible system
$index2s$	$\frac{(1-\beta\xi_p^s)(1-\xi_p^s)}{(1+\beta)(1+\frac{(1+\lambda_p)\sigma_L}{\lambda_p})\xi_p^s}$	an index in the inflation equation in the sticky system

5.1 Rescaling the shock size

In the Toolkit, shocks are standardised to be 1 per cent. But the size of shocks in Smets and Woutes (2003) is set to equal one standard deviation. Therefore, we have to rescale the shock size in matrices DD and MM. We create a matrix which is a diagonal matrix with the standard deviation of each shock on the diagonal. Multiplying DD and MM with this matrix gives us the same shock size as those of Smets and Wouters (2003).

5.2 Smets and Wouters JEEA (2003) V.S. Dynare Codes

People may have observed that the equations in our implementation are not the same as those in Smets and Wouters (2003). This is because we have discovered one mistake and several differences in the process of our implementation. First, we think the sign of the labour supply shock in the wage equation should be positive instead of negative, because common sense would suggest that increase in labour supply will lead to decrease in real wage. We have also guessed that the size of shocks is somewhat different, because the impulse responses produced by the Toolkit show the same shapes but different magnitude.

We have communicated with the authors of the original paper about the above findings. Raf Wouters confirmed that they did have the wrong sign before the labour supply shock, and that they rescaled the shocks to make them more robust to changes in other parameters. Therefore, we have also modified our equations in Toolkit so as to benchmark with those equations in Dynare written by Smets and Wouters. The following table gives a list of shocks that have been rescaled.

Equation Number as in Smets and Wouters (2003)	Our modifications
28	take out ϵ^b_{t+1}
29	take out ϵ^I_{t+1}
29	ϵ^b_t rescaled to equal 1
35a	ϵ^G_t rescaled to equal 1
32	η^p_t rescaled to equal 1
33	η^w_t rescaled to equal 1

6 Impulse Response Analysis

In this section we present graphs of the impulse reponses of marcoeconomic variables to various shocks using our estimated model. Compared with results from Smets and Wouters (2003) Dynare codes, our impulse reponses of shocks excluding tax rate shocks are exactly the same. The scenarios of three more tax rate shocks look reasonable. We explain below the impulse responses to each shock in detail.

Figure 1 shows the responses of output, consumption, inflation, interest rate, real wage, return on capital, labor, equity premium and investment following a technology shock. We see that output, consumption and investment rise after the improvement of technology. But the return on capital, interest rate and labor input falls. The fall in the labor input is due to the decrease in marginal cost after a rise in technology. This is in consistent with the estimated results in the United States as presented in Gali (1999).

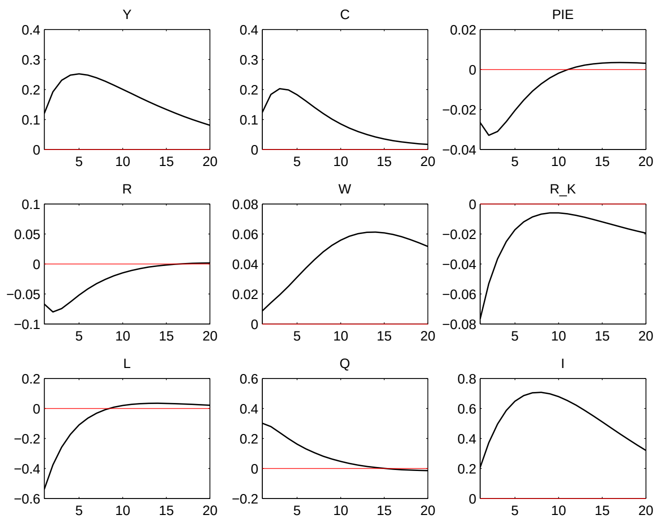

Figure 1: Technology Shock

Figure 2 shows the effects of a preference shock. After a positive change in preference, consumption and output grow significant. Meanwhile, investment declines sharply, with a lowest point around -1.2. The fall in investment is due to the crowding out effect on investment. The increased demand results in a rise in more labor input. The effect on inflation and return on capital are relatively slight.

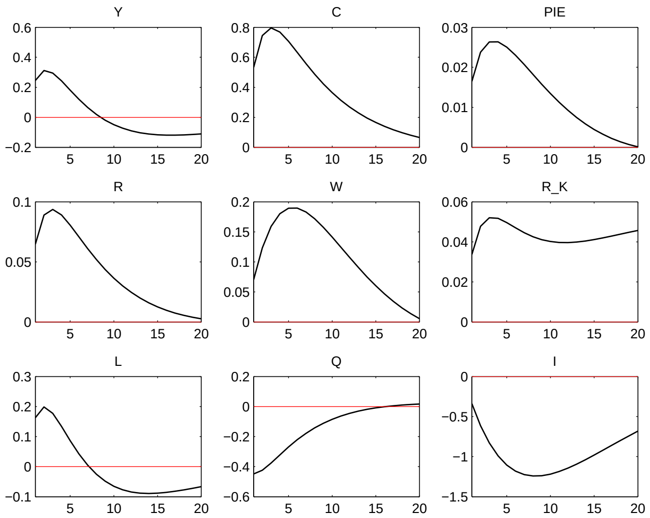

Figure 2: Preference Shock

Impulse responses to a government expenditure shock is depicted in Figure 3. We can see strong crowding out effects on private consumption and investment following an increase in government expenditure. Both of them decrease significant after the shock while rental rate on capital and real wage increase slightly.

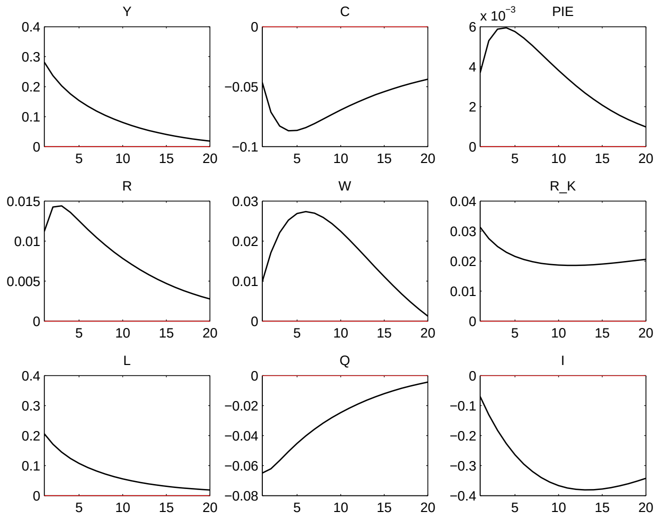

Figure 3: Government Expenditure Shock

Figure 4 presents the effects of an positive investment shock on the economy. Output and labor input increase significantly after a rise in investment. While consumption first falls and then recovers.

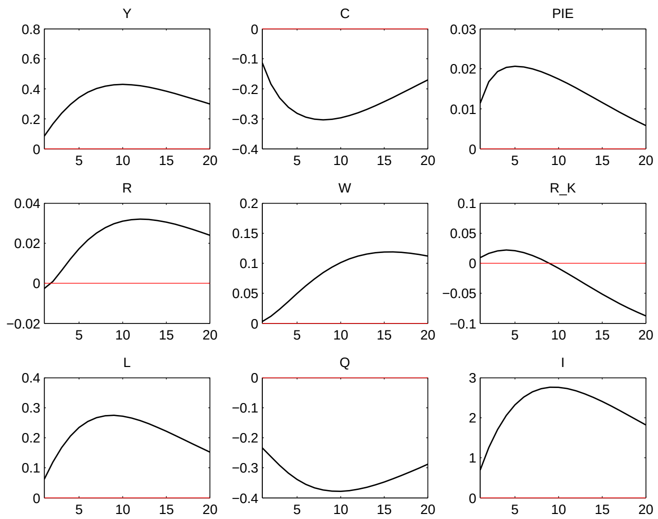

Figure 4: Investment Shock

Figure 5 depicts the scenario of a labor supply shock. Output, consumption, investment increase significantly after a rise in labor supply. This is similar to the case of a positive technology shock. But here the real wage level drops in contrast to a slight increase in real wage in the case of a technology shock. Inflation rate also falls here.

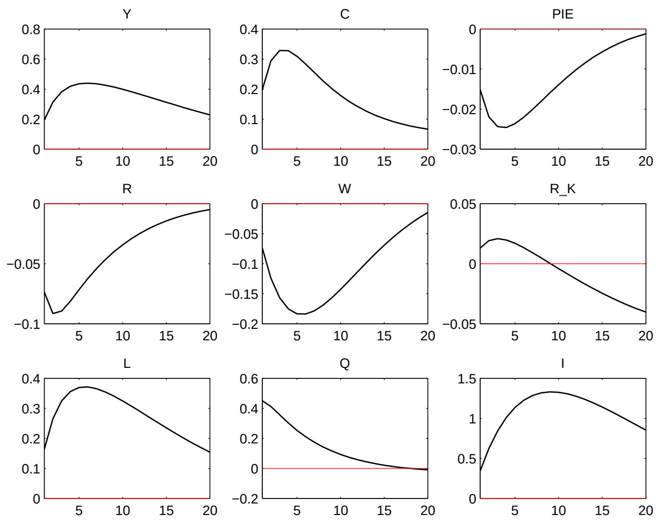

Figure 5: Labor Supply Shock

Figure 6 and 7 plot the effects of the two monetary policy shocks. Figure 6 presents the scenario of a positive shock to the inflation objective while Figure 7 presents the case of an increase in nominal interest rate. After rising the inflation objective, output, consumption and inflation all increase slightly. Nominal interest rate also start increasing after the shock. The responses in Figure 7 look quite different. In Figure 7, output, consumption and investment all drop with a hump shape. Real wage also falls in line with the stylized facts.

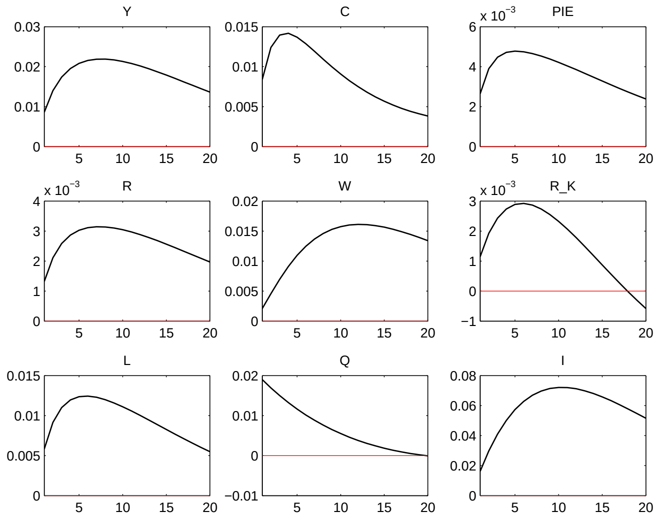

Figure 6: Inflation Objective Shock

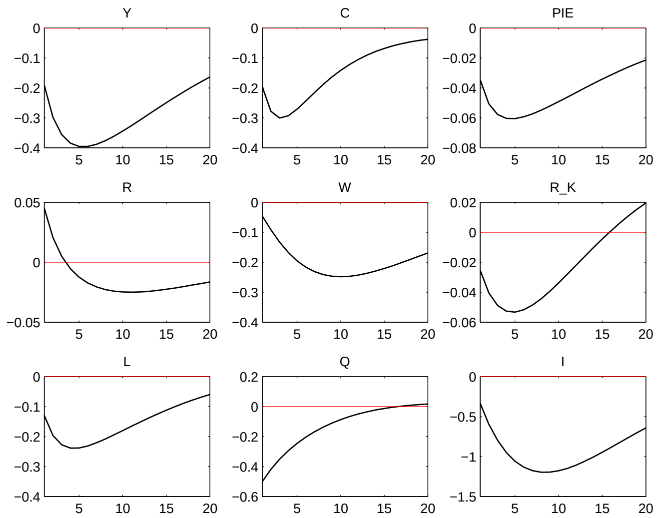

Figure 7: Interest rate Shock

Figure 8, 9 and 10 shows how the economy responds to the three cost push shocks. The effects of these three shocks are quantitatively smaller compared with the previous shocks.

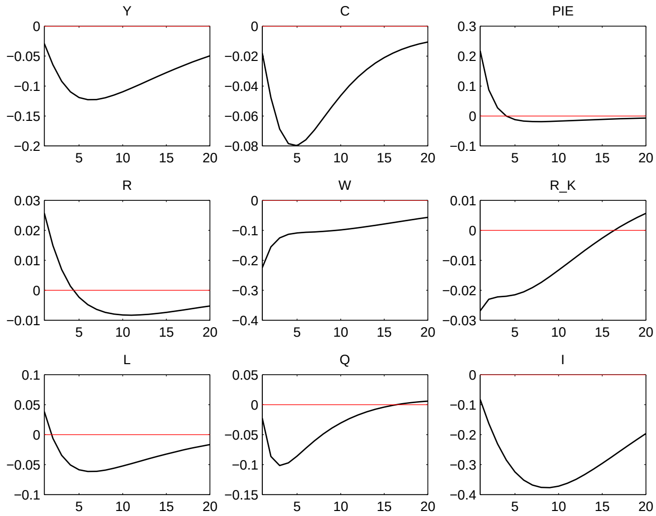

Figure 8: Price Mark-up Shock

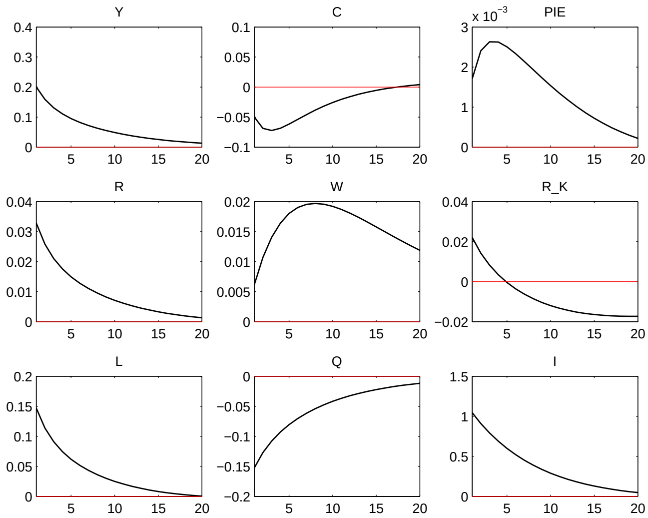

Figure 9: Equity Premium Shock

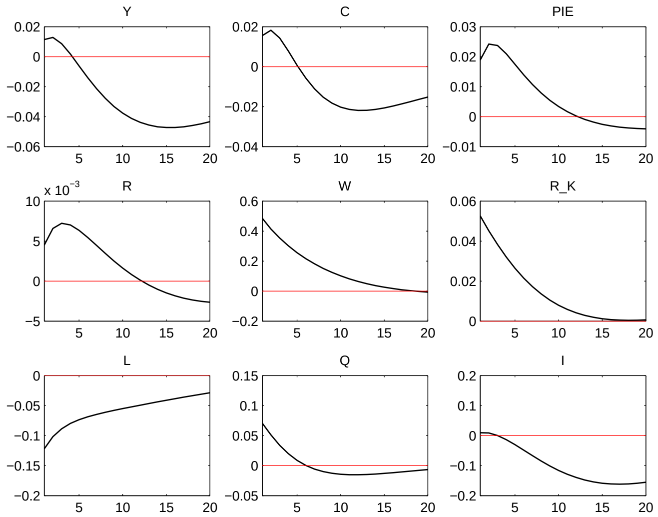

Figure 10: Wage Mark-up Shock

Figure 11, 12, and 13 plots of the effects of three fiscal policy shocks. Figure 11 shows the impulse responses to a rise in consumption tax rate. Consumption drops as consumption gets more expensive, but in the long term output increases because households are more willing to supply labor. In contrast, in Figure12 we see that a rise in capital tax rate make output decrease. Households are less motivated to save and invest when a higher tax rate on capital return is levied. Figure 12 depicts the effects of a rise in labor tax rate. Gross wage increases but households are less motivated to supply labor. Output, consumption and investment first increase for a short while and then keep declining.

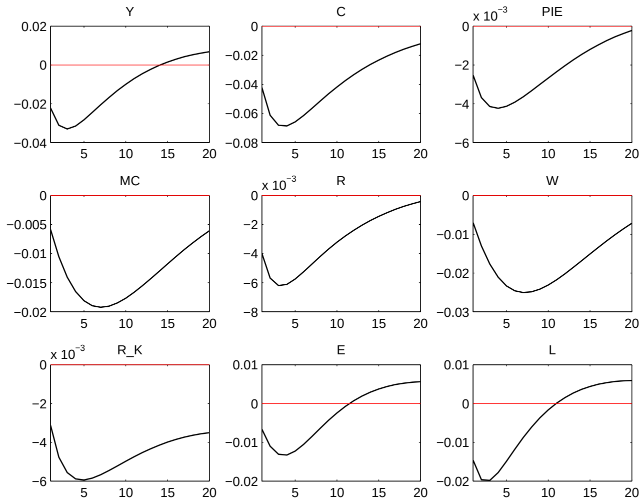

Figure 11: Consumption Tax Shock

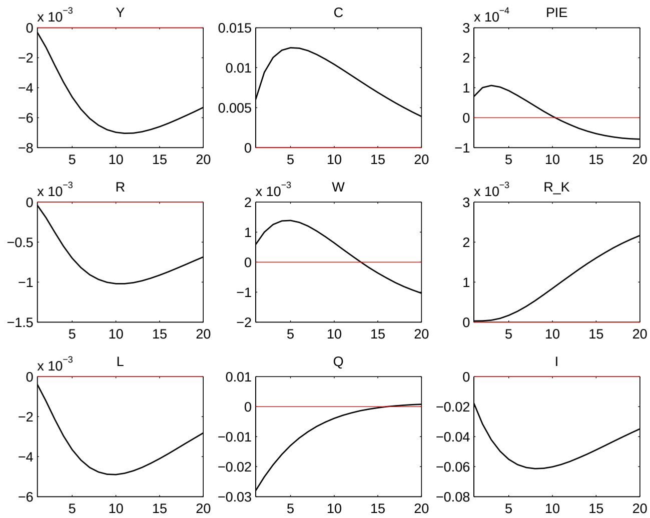

Figure 12: Capital Tax Shock

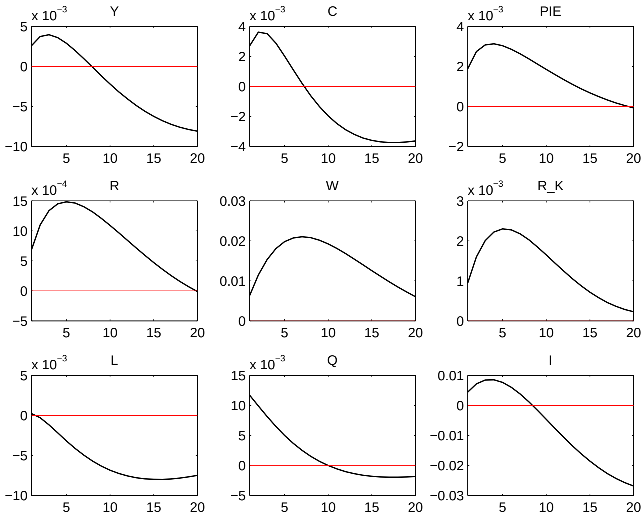

Figure 13: Labor Tax Shock

7 Estimation in Dynare

Following Smets and Wouters (2003), we apply Bayesian approach to estimate the parameters and standard deviations of shocks in our model. We first give a short introduction about how Bayesian method works. We also motivate Bayesian estimation by explaining its advantages. Next, we present estimation results for three combinations of model and data. The first version we estimate the original Smets and Wouters (2003) model with the dataset they provided. The second version we estimate our extended model including taxes and transfer using the same dataset Smets and Wouters (2003) use. At last, we add 2 more observed time series, i.e., government spending and government transfer into the data, detrended them with HP filter, and then use the detrended data to estimate the extended model. We have found that estimation results from the first and second version are almost the same as the results published in Table 1 in Smets and Wouters (2003). But the estimated parameters and standard deviations look different in the third version. The differences may come from different detrending methods or more observations included in the dataset of the third version.

7.1 Basic Mechanics of Bayesian Estimation

In this section we introduce some basics of Bayesian estimation. Bayesian estimation is an alternative approach to the classical sampling theory. In a Bayesian framework, probability of an event is based on a degree of an individual's believe in how likely or unlikely the event is to happen. This belief is subjective because different persons have different expectation about probabilities that the same event is to occur. Besides, parameters are considered as random variables that has a probability distribution, while in the classical sampling theory parameters are set to have a fixed value so that no probability distributions can be assigned to them.

The Bayesian probability distribution is derived from a prior distribution, which is knowledge existing before observing any sample information, combined with sample information. The derived distribution from priors and samples is called a posterior distribution. This procedure is known as Bayes' theorem.

The first step of the Bayesian approach is to specify a density function for priors, for example: $g(\theta)$. Where θ is a vector of parameters in which we are interested. g(.) is the probability density function for priors. It could be a normal, gamma, shifted gamma, inverse gamma, beta, generalized beta, or uniform function.

Second, given θ, a conditional density function $f(y|\theta)$ describes all the sample information about θ, where y is a vector of sample observations. Defining h as the joint density function for y and θ, we can write:

$$h(\theta, y) = f(y|\theta)g(\theta) = g(y|\theta)f(y) \tag{7.1}$$

Rearranging the terms in the above equation gives the expression known as Bayes' theorem:

$$g(y|\theta) = \frac{f(y|\theta)}{f(y)}g(\theta) \tag{7.2}$$

$g(y|\theta)$ is the posterior density function that we are interested in. We see that it summarizes all the available information about θ from both the prior distribution $g(\theta)$and information from the sample observation $\frac{f(y|\theta)}{f(y)}$. To express the idea in words, we have:

Posterior information = sample information * prior information

Bayesian approach is becoming popular for estimating macroeconomic models. The following advantages of using Bayesian method are reasons for its increasing popularity: First, compared with GMM estimation that is based on particular equilibrium conditions, Bayesian approach fit to estimate a complete system of solved DSGE model. Second, Bayesian approach takes into account prior information so that the posterior distributions will not peak at strange points. Besides, information from prior distributions also help the identification of parameters. Finally, using Bayesian estimation researchers can compare competing models based on fit. The posterior distributions can be used to tell which model fits the data most.

7.2 Estimation Results

In this section we presents the estimation results in Dynare. We have 3 versions of estimation. The first version we estimate the original Smets and Wouters (2003) model with the dataset they provided. In Figure 14, we see that the estimated parameters and standard deviations of shocks have almost the same values as those published in Table 1 in Smets and Wouters JEEA(2003). But there are still slight differences scaled around 0.01 between our results and published results. We think the differences are due to discrepancies between the model from the published JEEA version and the model presented in the Dynare codes of Smets and Wouters. As stated in Section 5.2, Smets and Wouters rescaled some shocks in their Dynare codes.

Figure 15 presents the estimation results of our extended model including taxes and transfers using the same dataset as in Smets and Wouters Dynare codes including 7 time series. Comparing Figure 14 and Figure 15, We observe that adding tax rates and transfer into the model has a very small effect on the estimated parameters and standard deviations of shocks. The differences between the first order autoregressive coefficients in Figure 14 and those in Figure 15 are around 0.002. The estimated Calvo price, Calvo wage and Calvo employment parameters are also almost the same in the two different estimations. The estimated standard deviations of shocks are not very different either. As for the three more tax rate shocks, their first order autoregressive coefficients are estimated to be around 0.92 while their standard deviations are estimated to be around 0.046.

Figure 16 has a quite different story from Figure 14 and 15. Here we have estimated our extended model using our own HP-filtered data including 9 observed time series. We have collected all 9 time series from AWM dataset from Fagan and Henry(2001), which is also the date source of Smets and Wouter(2003). The 9 time series are output per capita, consumption per capita, investment per capita, government consumption per capita, government transfer per capita, gross nominal interest rate, inflation rate, employment rate, and wage per capita. They were logged and then detrended by HP filter in Eviews 5. In the program, the priors of standard deviations of shocks are modified in line with our data.

The differences between Figure 16 and Figure 14 should not come from the extension of the DSGE model, but from two different datasets. Compared with the one from Smets and Wouters, the dataset we use include 2 more time series, i.e., government expenditure and government transfer. Second, the method we used to detrend the data is HP filter while Smets and Wouters used Kalmann filter. More observation and different detrending tool result in changes in estimated mode values. If there were more time, it would be interesting to try detrending our data with Kalmann filter and then compare the results again.

```
RESULTS FROM POSTERIOR MAXIMIZATION
parameters
           prior mean     mode    s.d. t-stat prior pstdev

      rho_a   0.850   0.7995  0.0614 13.0279 beta  0.1000
     rho_pb   0.850   0.9136  0.0837 10.9172 beta  0.1000
      rho_b   0.850   0.8494  0.0360 23.6025 beta  0.1000
      rho_g   0.850   0.9485  0.0291 32.6456 beta  0.1000
      rho_l   0.850   0.8748  0.0682 12.8184 beta  0.1000
      rho_i   0.850   0.9400  0.0264 35.5566 beta  0.1000
      phi_i   4.000   7.1620  1.0954  6.5384 norm  1.5000
      sig_c   1.000   1.3137  0.2667  4.9252 norm  0.3750
        hab   0.700   0.5764  0.0707  8.1568 beta  0.1000
       xi_w   0.750   0.7454  0.0498 14.9802 beta  0.0500
      sig_l   2.000   2.2154  0.5916  3.7448 norm  0.7500
       xi_p   0.750   0.9082  0.0111 81.5360 beta  0.0500
       xi_e   0.500   0.6058  0.0473 12.8172 beta  0.1500
    gamma_w   0.750   0.7647  0.1854  4.1239 beta  0.1500
    gamma_p   0.750   0.4358  0.1012  4.3062 beta  0.1500
      czcap   0.200   0.2351  0.0679  3.4625 norm  0.0750
      phi_y   1.450   1.4221  0.1094 12.9986 norm  0.1250
      r_pie   1.700   1.6938  0.1000 16.9425 norm  0.1000
      r_dpi   0.300   0.1138  0.0510  2.2308 norm  0.1000
        rho   0.800   0.9743  0.0129 75.3252 beta  0.1000
        r_y   0.125   0.1136  0.0441  2.5772 norm  0.0500
       r_dy   0.063   0.1497  0.0297  5.0324 norm  0.0500
standard deviation of shocks
           prior mean     mode    s.d. t-stat prior pstdev

        E_A   0.400   0.6108  0.1121  5.4482 invg  2.0000
  E_PIE_BAR   0.020   0.0092  0.0038  2.4501 invg  10.0000
        E_B   0.200   0.3338  0.0976  3.4214 invg  2.0000
        E_G   0.300   0.3224  0.0254 12.7184 invg  2.0000
        E_L   1.000   3.8411  1.3353  2.8766 invg  2.0000
        E_I   0.100   0.0534  0.0160  3.3353 invg  2.0000
      ETA_R   0.100   0.0625  0.0278  2.2450 invg  2.0000
      ETA_Q   0.400   0.6140  0.0605 10.1442 invg  2.0000
      ETA_P   0.150   0.1580  0.0151 10.4493 invg  2.0000
      ETA_W   0.250   0.2867  0.0264 10.8713 invg  2.0000

Log data density [Laplace approximation] is -276.953607.
```

Figure 14: Estimation Results of the SW Model with SW Data

RESULTS FROM POSTERIOR MAXIMIZATION

parameters

	prior mean	mode	s.d.	t-stat	prior	pstdev
rho_tc	0.850	0.9143	0.0827	11.0516	beta	0.1000
rho_tk	0.850	0.9241	0.0899	10.2736	beta	0.1000
rho_tl	0.850	0.9240	0.0980	9.4273	beta	0.1000
rho_g	0.850	0.9490	0.0293	32.3825	beta	0.1000
rho_a	0.850	0.7993	0.0614	13.0231	beta	0.1000
rho_pb	0.850	0.9155	0.0851	10.7530	beta	0.1000
rho_b	0.850	0.8501	0.0360	23.5881	beta	0.1000
rho_l	0.850	0.8727	0.0689	12.6631	beta	0.1000
rho_i	0.850	0.9399	0.0265	35.4261	beta	0.1000
phi_i	4.000	7.1647	1.0958	6.5381	norm	1.5000
sig_c	1.000	1.3169	0.2664	4.9433	norm	0.3750
hab	0.700	0.5755	0.0707	8.1402	beta	0.1000
xi_w	0.750	0.7471	0.0500	14.9517	beta	0.0500
sig_l	2.000	2.2145	0.5918	3.7418	norm	0.7500
xi_p	0.750	0.9081	0.0112	81.2757	beta	0.0500
xi_e	0.500	0.6057	0.0473	12.8133	beta	0.1500
gamma_w	0.750	0.7684	0.1848	4.1577	beta	0.1500
gamma_p	0.750	0.4351	0.1013	4.2954	beta	0.1500
czcap	0.200	0.2353	0.0679	3.4643	norm	0.0750
phi_y	1.450	1.4219	0.1094	12.9965	norm	0.1250
r_pie	1.700	1.6939	0.1000	16.9435	norm	0.1000
r_dpi	0.300	0.1129	0.0508	2.2245	norm	0.1000
rho	0.800	0.9748	0.0128	76.3902	beta	0.1000
r_y	0.125	0.1141	0.0440	2.5948	norm	0.0500
r_dy	0.063	0.1491	0.0298	4.9950	norm	0.0500

standard deviation of shocks

	prior mean	mode	s.d.	t-stat	prior	pstdev
E_A	0.400	0.6109	0.1121	5.4482	invg	2.0000
E_PIE_BAR	0.020	0.0092	0.0038	2.4501	invg	10.0000
E_B	0.200	0.3323	0.0977	3.4012	invg	2.0000
E_L	1.000	3.8750	1.3389	2.8941	invg	2.0000
E_I	0.100	0.0537	0.0161	3.3404	invg	2.0000
ETA_R	0.100	0.0616	0.0277	2.2190	invg	2.0000
ETA_Q	0.400	0.6138	0.0606	10.1289	invg	2.0000
ETA_P	0.150	0.1579	0.0151	10.4499	invg	2.0000
ETA_W	0.250	0.2870	0.0264	10.8749	invg	2.0000
E_TC	0.100	0.0471	0.0201	2.3436	invg	2.0000
E_TK	0.100	0.0461	0.0188	2.4490	invg	2.0000
E_TL	0.100	0.0462	0.0189	2.4433	invg	2.0000
E_G	0.300	0.3209	0.0255	12.6048	invg	2.0000

Log data density [Laplace approximation] is -277.862506.

Figure 15: Estimation Results of the Tax SW Model with SW data

```
RESULTS FROM POSTERIOR MAXIMIZATION
parameters
           prior mean     mode    s.d. t-stat prior pstdev

      rho_tc   0.850   0.9090  0.0864 10.5202 beta  0.1000
      rho_tk   0.850   0.9189  0.0847 10.8519 beta  0.1000
      rho_tl   0.850   0.8352  0.0573 14.5851 beta  0.1000
       rho_g   0.850   0.7735  0.0465 16.6530 beta  0.1000
       rho_a   0.850   0.4622  0.0819  5.6447 beta  0.1000
      rho_pb   0.850   0.9901  0.0087 113.2312 beta  0.1000
       rho_b   0.850   0.2851  0.0883  3.2272 beta  0.1000
       rho_l   0.850   0.9208  0.0843 10.9229 beta  0.1000
       rho_i   0.850   0.5061  0.0997  5.0777 beta  0.1000
       phi_i   4.000   7.6676  1.0534  7.2792 norm  1.5000
       sig_c   1.000   1.5760  0.3077  5.1212 norm  0.3750
         hab   0.700   0.8164  0.0490 16.6554 beta  0.1000
        xi_w   0.750   0.7404  0.0351 21.0887 beta  0.0500
       sig_l   2.000   1.8604  0.8207  2.2667 norm  0.7500
        xi_p   0.750   0.7962  0.0262 30.3479 beta  0.0500
        xi_e   0.500   0.7467  0.0225 33.2132 beta  0.1500
     gamma_w   0.750   0.8696  0.0654 13.3025 beta  0.1500
     gamma_p   0.750   0.9493  0.0441 21.5472 beta  0.1500
       czcap   0.200   0.2321  0.0705  3.2922 norm  0.0750
       phi_y   1.450   1.5036  0.1114 13.4956 norm  0.1250
       r_pie   1.700   1.5685  0.1040 15.0799 norm  0.1000
       r_dpi   0.300   0.3801  0.0864  4.3986 norm  0.1000
         rho   0.800   0.7702  0.0285 26.9864 beta  0.1000
         r_y   0.125   0.1880  0.0448  4.1917 norm  0.0500
        r_dy   0.063   0.1941  0.0476  4.0781 norm  0.0500
standard deviation of shocks
           prior mean     mode    s.d. t-stat prior pstdev

         E_A   0.002   0.0085  0.0016  5.1393 invg  2.0000
   E_PIE_BAR   0.007   0.0052  0.0009  6.0705 invg  10.0000
         E_B   0.005   0.0314  0.0086  3.6547 invg  2.0000
         E_L   0.001   0.0005  0.0001  3.5282 invg  2.0000
         E_I   0.006   0.0016  0.0003  4.8009 invg  2.0000
       ETA_R   0.005   0.0071  0.0005 14.5358 invg  2.0000
       ETA_Q   0.010   0.0062  0.0005 11.3593 invg  2.0000
       ETA_P   0.008   0.0020  0.0001 14.6382 invg  2.0000
       ETA_W   0.005   0.0027  0.0002 15.1517 invg  2.0000
        E_TC   0.001   0.0004  0.0001  3.7217 invg  2.0000
        E_TK   0.001   0.0005  0.0001  3.5733 invg  2.0000
        E_TL   0.001   0.0043  0.0003 16.9165 invg  2.0000
         E_G   0.003   0.0031  0.0002 17.2707 invg  2.0000

Log data density [Laplace approximation] is 3822.596877.
```


Figure 16: Estimation Results of the Tax SW Model with HP data

8 Concluding Remarks

In this paper, we have extended the DSGE model by Smets and Wouter (2003) to add in fiscal policy features. The extended model not only features sticky price, sticky wage, capital utilization rate, but also introduces a full set of structural shocks including shocks to tax rates. The extended model is calibrated using parameter values from Smets and Wouters (2003). We also estimate the model using Bayesian techniques.

The effects of structural shocks on macroeconomic variables are shown in the impulse response analysis. Overall, the impulse response graphs to shocks except tax rate shocks from our extended model look the same as those produced by Smets and Wouters (2003) codes. The labor supply shock and the preference shock contribute to a large fraction of fluctuation in output, consumption, and investment. The three cost push shocks have a comparably small effect on the economy. The effects of three tax shocks are also quantitatively minor.

Second, the empirical estimation using Bayesian approach yields interesting results. We have tried three different combinations of model and data. The first version estimates the original Smets and Wouters (2003) model using their own data. The estimated parameters and standard deviations of shocks look almost the same as those published in their paper. The second version estimates the extended model using the data from Smets and Wouters. The estimation results remain very similar to the first version. Then we add two more time series, government transfer and government expenditure into the observed data and detrend them with HP filter. The estimation results using the new data for the extended model differ from what we have reached in the first two versions. The estimated price stickiness is less significant while the estimated employment stickiness is more considerable.

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10 Appendix List

Appendix A: Dynare Codes for the Extended Model Using HP-filtered Data

Appendix B: HP-detrended Time Series Used for Dynare Estimation

Appendix C: Prior and Posterior Distributions from Estimating the Extended Model with HP-detrended Data

Appendix D: Matrices in Toolkit Codes

```
var MC E EF R_KF QF CF IF YF LF PIEF WF RF R_K Q C I Y L PIE W R EE_A PIE_BAR EE_B EE_G EE_TC EE_TL EE_TK TR EE_L EE_I KF K one;

varexo E_A E_B E_TC E_TL E_TK E_G E_L E_I ETA_R E_PIE_BAR ETA_Q ETA_P ETA_W  ;

parameters xi_e lambda_w alpha czcap beta phi_i tau sig_c hab ccs cinvs ty phi_y gamma_w xi_w gamma_p xi_p sig_l r_dpi r_pie r_d↙
y r_y rho rho_a rho_pb rho_b rho_tc rho_tl rho_tk rho_g rho_l rho_i  ;
alpha=.30;
beta=.99;
tau=0.025;
ccs=0.6;
cinvs=.22;  //% alpha*(tau+ctrend)/R_K   R_K=ctrend/beta-1+tau
ty=0.01;
lambda_w = 0.5;
phi_i= 6.771;
sig_c=   1.353;
hab=    0.573;
xi_w=   0.737;
sig_l=    2.400;
xi_p=   0.908;
xi_e= 0.599;
gamma_w=    0.763;
gamma_p=    0.469;
czcap=    0.169;
phi_y=    1.408;
r_pie=     1.684;
r_dpi=    0.14;
rho=      0.961;
r_y=      0.099;
r_dy=     0.159;
rho_a=    0.823;
rho_b=    0.855;
rho_l=   0.889;
rho_i=   0.927;
rho_pb=  0.924;
rho_tc= 0.9;
rho_tl= 0.9;
rho_tk= 0.9;
rho_g= 0.949;


model(linear);
          CF = (1/(1+hab))*(CF(1)+hab*CF(-1))-((1-hab)/((1+hab)*sig_c))*(RF-PIEF(1)-EE_B) ;
          0 =  alpha*R_KF+(1-alpha)*WF -EE_A ;
```

Figure 17: Appendix A: Dynare Codes-1

```
PIEF = 0*one;
IF = (1/(1+beta))* ((  IF(-1) + beta*(IF(1)))+(1/phi_i)*QF)+0*ETA_Q+EE_I ;
QF = -(RF-PIEF(1))+(1-beta*(1-tau))*((0+czcap)/czcap)*R_KF(1)+beta*(1-tau)*QF(1) +0*EE_I ;
KF =  (1-tau)*KF(-1)+tau*IF(-1) ;
YF = ccs*CF+cinvs*IF +EE_G  ;
YF = 1*phi_y*( alpha*KF+alpha*(1/czcap)*R_KF+(1-alpha)*LF+EE_A ) ;
WF = (sig_c/(1-hab))*(CF-hab*CF(-1)) + sig_l*LF - EE_L ;
LF = R_KF*((1+czcap)/czcap)-WF+KF ;
EF = EF(-1)+EF(1)-EF+(LF-EF)*((1-xi_e)*(1-xi_e*beta)/(xi_e));

C = (hab/(1+hab))*C(-1)+(1/(1+hab))*C(1)-((1-hab)/((1+hab)*sig_c))*(R-PIE(1)-EE_B +EE_TC-EE_TC(1)) ;
I = (1/(1+beta))* ((  I(-1) + beta*(I(1)))+(1/phi_i)*Q )+1*ETA_Q+1*EE_I ;
Q = -(R-PIE(1))+(1-beta*(1-tau))*((0+czcap)/czcap)*R_K(1)+beta*(1-tau)*Q(1)-(1-beta+ beta*tau)* EE_TK +EE_I*0+0*ETA_Q ↙
;
K =  (1-tau)*K(-1)+tau*I(-1) ;
Y = (ccs*C+cinvs*I)+ ccs*EE_TC +EE_G;
TR = (1/ty)*(cinvs*(1/tau)*(1/beta-1+tau)*EE_TK +(1-alpha)/alpha*cinvs*(1/tau)*(1/beta-1+tau)*EE_TL-EE_G);
Y = phi_y*( alpha*K+alpha*(1/czcap)*R_K+(1-alpha)*L ) +phi_y*EE_A  ;
PIE = (1/(1+beta*gamma_p))*
      (
      (beta)*(PIE(1)) +(gamma_p)*(PIE(-1))
      +((1-xi_p)*(1-beta*xi_p)/(xi_p))*(MC)
      )  + ETA_P ;

MC = alpha*R_K+(1-alpha)*W -EE_A;
W =  (1/(1+beta))*(beta*W(+1)+W(-1))
      +(beta/(1+beta))*(PIE(+1))
      -((1+beta*gamma_w)/(1+beta))*(PIE)
      +(gamma_w/(1+beta))*(PIE(-1))
      -(1/(1+beta))*(((1-beta*xi_w)*(1-xi_w))/(((1+(((1+lambda_w)*sig_l)/(lambda_w))))*xi_w))*(W-sig_l*L-(sig_c/(1-hab↙
))*(C-hab*C(-1)) +EE_L -EE_TL)
      +ETA_W;
L = R_K*((1+czcap)/czcap)-W+K ;
R = r_dpi*(PIE-PIE(-1))
    +(1-rho)*(r_pie*(PIE(-1)-PIE_BAR)+r_y*(Y-YF))
    +r_dy*(Y-YF-(Y(-1)-YF(-1)))
    +rho*(R(-1)-PIE_BAR)
    +PIE_BAR
    +ETA_R;
E = E(-1)+E(1)-E+(L-E)*((1-xi_e)*(1-xi_e*beta)/(xi_e));


EE_A = (rho_a)*EE_A(-1)  + E_A;
```

Figure 18: Appendix A: Dynare Codes-2

```
          PIE_BAR = rho_pb*PIE_BAR(-1)+ E_PIE_BAR ;
          EE_B = rho_b*EE_B(-1) + E_B ;
          EE_G = rho_g*EE_G(-1) + E_G ;
          EE_L = rho_l*EE_L(-1) + E_L ;
          EE_I = rho_i*EE_I(-1) + E_I ;
          EE_TC = rho_tc* EE_TC(-1) + E_TC;
          EE_TK = rho_tk* EE_TK(-1) + E_TK;
          EE_TL = rho_tl* EE_TL(-1) + E_TL;
          one = 0*one(-1) ;
end;


shocks;
var E_A; stderr 0.598;
var E_B; stderr 0.336;
var E_I; stderr 0.085;
var E_L; stderr 3.520;
var ETA_P; stderr 0.160;
var ETA_W; stderr 0.289;
var ETA_R; stderr 0.081;
var ETA_Q; stderr 0.604;
var E_PIE_BAR; stderr 0.017;
var E_TC; stderr 0.16;
var E_TL; stderr 0.16;
var E_TK; stderr 0.16;
var E_G; stderr 0.325;
end;

//stoch_simul(irf=20) Y C PIE R W R_K L Q I K ;

// stoch_simul generates what kind of standard errors for the shocks ?

steady(solve_algo=0);
//check;
//stoch_simul(periods=200,irf=20,simul_seed=3) Y C PIE MC R W R_K E L I ;

//datatomfile('ddd',[]);

// new syntax

estimated_params;
// PARAM NAME, INITVAL, LB, UB, PRIOR_SHAPE, PRIOR_P1, PRIOR_P2, PRIOR_P3, PRIOR_P4, JSCALE
// PRIOR_SHAPE: BETA_PDF, GAMMA_PDF, NORMAL_PDF, INV_GAMMA_PDF
```

Figure 19: Appendix A: Dynare Codes-3

```
stderr E_A,0.0025,0.0001,1,INV_GAMMA_PDF,0.002,2;
stderr E_PIE_BAR,0.009,0.0001,1,INV_GAMMA_PDF,0.007,10;
stderr E_B,0.0057,0.0001,1,INV_GAMMA_PDF,0.005,2;
stderr E_L,0.0018,0.0001,1,INV_GAMMA_PDF,0.001,2;
stderr E_I,0.0085,0.0001,1,INV_GAMMA_PDF,0.006,2;
stderr ETA_R,0.0055,0.0001,1,INV_GAMMA_PDF,0.005,2;
stderr ETA_Q,0.019,0.0001,1,INV_GAMMA_PDF,0.01,2;
stderr ETA_P,0.0086,0.0001,1,INV_GAMMA_PDF,0.008,2;
stderr ETA_W,0.0057,0.0001,1,INV_GAMMA_PDF,0.005,2;
stderr E_TC,0.001,0.0001,1,INV_GAMMA_PDF,0.001,2;
stderr E_TK,0.001,0.0001,1,INV_GAMMA_PDF,0.001,2;
stderr E_TL,0.001,0.0001,1,INV_GAMMA_PDF,0.001,2;
stderr E_G,0.0043,0.0001,1,INV_GAMMA_PDF,0.003,2;
rho_tc,0.7,.1,.9999,BETA_PDF,0.85,0.1;
rho_tk,0.7,.1,.9999,BETA_PDF,0.85,0.1;
rho_tl,0.7,.1,.9999,BETA_PDF,0.85,0.1;
rho_g,.9502,.1,.9999,BETA_PDF,0.85,0.1;
rho_a,.9722,.1,.9999,BETA_PDF,0.85,0.1;
rho_pb,.85,.1,.999,BETA_PDF,0.85,0.1;
rho_b,.7647,.1,.99,BETA_PDF,0.85,0.1;
rho_l,.9542,.1,.9999,BETA_PDF,0.85,0.1;
rho_i,.6705,.1,.99,BETA_PDF,0.85,0.1;
phi_i,5.2083,1,15,NORMAL_PDF,4,1.5;
sig_c,0.9817,0.25,3,NORMAL_PDF,1,0.375;
hab,0.5612,0.3,0.95,BETA_PDF,0.7,0.1;
xi_w,0.7661,0.3,0.9,BETA_PDF,0.75,0.05;
sig_l,1.7526,0.5,5,NORMAL_PDF,2,0.75;
xi_p,0.8684,0.3,0.95,BETA_PDF,0.75,0.05;
xi_e,0.5724,0.1,0.95,BETA_PDF,0.5,0.15;
gamma_w,0.6202,0.1,0.99,BETA_PDF,0.75,0.15;
gamma_p,0.6638,0.1,0.99,BETA_PDF,0.75,0.15;
czcap,0.2516,0.01,2,NORMAL_PDF,0.2,0.075;
phi_y,1.3011,1.001,2,NORMAL_PDF,1.45,0.125;
r_pie,1.4616,1.2,2,NORMAL_PDF,1.7,0.1;
r_dpi,0.1144,0.01,0.5,NORMAL_PDF,0.3,0.1;
rho,0.8865,0.5,0.99,BETA_PDF,0.8,0.10;
r_y,0.0571,0.01,0.2,NORMAL_PDF,0.125,0.05;
r_dy,0.2228,0.05,0.5,NORMAL_PDF,0.0625,0.05;
end;

varobs Y C I E PIE W R TR EE_G;

estimation(datafile=taxswdata,nograph,first_obs=1, mh_replic=20000);
```

Figure 20: Appendix A: Dynare Codes-4

	C	E	EE_G	I	PIE	R	TR	W	Y
1970Q1	-0.010952	-0.002426	-0.010692	-0.056777	0.023609	0.012722	0.001627	-0.021993	-0.012454
1970Q2	-0.008366	-0.001287	-0.003247	-0.005887	0.023592	0.011946	0.005584	-0.013241	-0.002476
1970Q3	-0.007779	-0.001269	-0.000515	0.000889	0.017864	0.008159	0.006487	-0.004258	-0.001114
1970Q4	0.000398	-0.000526	0.004434	-0.011186	0.008468	0.004286	0.004692	0.008342	0.000592
1971Q1	-0.004227	-0.000219	-0.004061	-0.024904	0.0143	-0.003064	-0.015948	-0.000846	-0.011933
1971Q2	-0.001835	-0.000454	0.00112	-0.007903	0.004762	-0.009139	-0.014595	-0.006177	-0.010844
1971Q3	0.000481	0.000148	0.005112	-0.006159	-0.001541	-0.007157	-0.002677	0.002434	-0.00106
1971Q4	0.001314	-0.000672	0.001473	-0.000804	-0.00908	-0.010061	0.003778	0.002174	-0.003104
1972Q1	0.006766	-0.00129	0.00651	0.003408	-0.008497	-0.020085	0.007236	0.00861	0.000829
1972Q2	0.001504	-0.000924	0.001923	0.011837	-0.016402	-0.023939	0.004537	0.001843	-0.001458
1972Q3	0.008161	-0.000756	0.000276	0.018672	-0.024579	-0.024128	0.007245	0.001204	0.001559
1972Q4	0.006709	0.000432	-0.000897	0.035542	-0.02923	-0.010853	0.000421	-0.000719	0.005265
1973Q1	0.016503	0.002068	0.006183	0.050754	-0.021091	-0.008455	-0.00698	0.005622	0.014128
1973Q2	0.01109	0.000523	-0.005091	0.041936	-0.020861	0.005943	-0.014548	0.001521	0.011387
1973Q3	0.006584	0.003739	-0.012084	0.043955	-0.017863	0.020427	-0.01781	0.000237	0.016219
1973Q4	0.005958	0.004378	-0.007866	0.04315	-0.021093	0.027101	-0.02557	0.007819	0.017514
1974Q1	-0.002004	0.004872	-0.002517	0.037781	-0.012649	0.027329	-0.039657	0.002364	0.019413
1974Q2	0.000298	0.005587	-0.001529	0.01113	-0.003433	0.028537	-0.052741	0.010891	0.015269
1974Q3	-0.001556	0.005006	-0.003134	0.011379	0.00603	0.02827	-0.040203	-0.001396	0.014999
1974Q4	-0.021172	0.001101	-0.004503	-0.02065	0.013446	0.019823	-0.020541	-0.001931	-0.009662
1975Q1	-0.021892	-0.001273	-0.001397	-0.034382	0.01315	0.00243	0.002165	-0.010032	-0.024895
1975Q2	-0.012745	-0.00398	0.00501	-0.045348	0.011841	-0.014401	0.040218	-0.00732	-0.023152
1975Q3	-0.006958	-0.003312	0.01333	-0.040276	0.005857	-0.021271	0.058425	-0.003032	-0.019142
1975Q4	-0.003501	-0.005188	0.006803	-0.026891	0.000324	-0.019866	0.045092	0.000331	-0.017259
1976Q1	0.001363	-0.004138	0.003392	-0.027879	-0.005403	-0.016817	0.03786	0.000668	-0.007822
1976Q2	-0.0014	-0.003308	0.001949	-0.027922	0.009049	-0.003103	0.018296	-0.002	-0.00361
1976Q3	-0.002683	-0.001829	0.003039	-0.033043	0.010418	0.00314	0.013237	0.005542	-0.003014
1976Q4	0.000437	-0.002212	0.000868	-0.011218	0.009721	0.000814	0.021786	0.007662	0.006053
1977Q1	-0.005226	-0.001473	-0.005002	-0.001617	0.007127	0.008867	0.017864	0.002287	0.003296
1977Q2	-0.000493	-0.002019	-0.009774	-0.018582	0.011626	-4.90E-05	0.010245	0.004957	-0.002556
1977Q3	0.00232	-0.002574	-0.003165	-0.013459	0.00722	-0.004842	0.007664	0.005438	-0.005988
1977Q4	0.005671	-0.001477	0.004731	-0.003096	0.003633	-0.005156	0.012163	0.001606	9.95E-05
1978Q1	0.001786	-0.000217	0.002072	-0.002431	0.001438	-0.016687	0.009149	0.002404	0.000277
1978Q2	0.005409	-0.000209	0.001575	0.005246	-0.000246	-0.019869	0.010822	-0.003526	0.004947
1978Q3	0.005595	5.27E-05	0.004787	0.003465	-0.003462	-0.010208	0.005514	-0.002786	0.00286
1978Q4	0.010024	0.00082	0.005443	0.009603	-0.00883	-0.01262	0.007661	-0.003459	0.008324
1979Q1	0.010724	0.002422	0.005692	-0.007466	-0.012151	-0.028076	0.00458	0.001392	0.007514
1979Q2	0.021154	0.002401	0.003704	0.026611	-0.013705	-0.015118	0.009503	-0.003014	0.015175
1979Q3	0.009018	0.002703	-0.000203	0.028489	-0.010215	-0.002244	-0.001349	0.004676	0.011956
1979Q4	0.014486	0.005309	-0.002609	0.043424	-0.008825	0.010442	-0.010035	0.004747	0.016736
1980Q1	0.019765	0.006638	-0.000361	0.054395	-0.004922	0.008667	-0.017599	0.010055	0.022273
1980Q2	0.007859	0.006832	0.002254	0.036031	-0.00356	0.011117	-0.04094	0.002229	0.011639
1980Q3	0.010069	0.006162	0.00049	0.031659	-0.004876	0.000489	-0.044761	0.001557	0.005834
1980Q4	0.00636	0.006482	-0.002225	0.024326	-0.008205	0.002718	-0.03131	0.004203	0.002844
1981Q1	0.002441	0.006271	0.015337	0.015982	-0.009719	0.008267	-0.017851	-0.001673	0.000231
1981Q2	-0.002885	0.003347	-0.001012	0.015366	-0.003958	0.02545	-0.004686	0.003815	-0.001376
1981Q3	-0.002996	0.001584	-0.000917	0.009806	0.000723	0.030283	0.001184	0.007832	-0.002565
1981Q4	-0.002133	-0.000759	-0.005606	-0.012235	0.004814	0.023999	0.003647	0.007075	-0.004974
1982Q1	-0.000849	-0.001107	0.004525	-0.009741	0.005583	0.014674	0.007847	0.001549	-0.004331
1982Q2	-0.004883	-0.000267	-0.000328	-0.006891	0.010835	0.015838	0.008664	-0.000606	-0.004657
1982Q3	-0.012474	-0.001746	-0.001918	-0.014224	0.010491	0.007528	0.002498	-0.000593	-0.012738
1982Q4	-0.009194	-0.003449	-0.005611	-0.021047	0.010538	0.002171	-0.001189	-0.00035	-0.016534
1983Q1	-0.006029	-0.004729	0.000732	-0.011483	0.010939	-0.002757	0.001085	-0.004301	-0.013074
1983Q2	-0.008416	-0.004482	-0.002744	-0.009464	0.009506	-0.001394	0.002101	-0.001645	-0.009559
1983Q3	-0.011644	-0.003251	-0.003199	-0.004724	0.012203	0.003815	0.003316	0.00283	-0.009036

Figure 21: Appendix B: Observed Time Series -1

1983Q4	-0.00717	-0.003625	-0.002791	-0.01413	0.013164	0.005426	0.008065	-0.001803	-0.002704
1984Q1	-0.001531	-0.004404	-0.004952	-0.008328	0.012574	0.001002	0.014136	0.0018	0.002087
1984Q2	-0.007437	-0.003832	-0.003077	-0.032881	0.009783	-0.00192	0.003266	-0.009554	-0.005671
1984Q3	-0.010221	-0.003023	-0.004511	-0.022812	0.008924	-0.003673	0.006489	-0.007668	-0.000457
1984Q4	-0.013251	-0.003608	-0.001302	-0.018345	0.002652	-0.004016	0.005909	-0.000174	-0.000305
1985Q1	-0.009812	-0.004106	-0.001447	-0.034919	0.003622	-0.004236	0.001604	0.001747	-0.003435
1985Q2	-0.007437	-0.002538	0.000192	-0.023546	0.001834	-0.001255	0.005967	-0.000435	0.001781
1985Q3	-0.00401	-0.001837	0.001402	-0.009761	0.002908	-0.004898	0.010919	0.000867	0.004468
1985Q4	-0.00434	-0.001905	0.00307	-0.00706	0.003303	-0.009695	0.012209	-0.002042	0.002947
1986Q1	-0.008908	-0.002159	-0.001095	-0.026818	0.008016	-0.006991	0.005272	-0.006504	-0.008192
1986Q2	0.002572	-0.001234	0.001931	-0.009626	0.008494	-0.012256	0.020915	-0.007182	0.003885
1986Q3	0.00252	-0.000206	0.001761	-0.00576	0.006629	-0.013674	0.017922	-0.007318	0.001956
1986Q4	0.000144	-0.001312	-3.13E-05	-0.010317	0.002815	-0.011677	0.004001	-0.0104	-0.005151
1987Q1	-0.007097	-0.002905	0.001822	-0.039739	-0.001267	-0.008934	-0.017943	-0.009866	-0.018604
1987Q2	0.002754	-0.002273	0.005122	-0.018462	-0.003724	-0.005626	-0.017878	-0.006092	-0.008861
1987Q3	0.002162	-0.003423	0.003862	-0.009133	-0.008276	-0.004932	-0.019665	-0.002302	-0.007054
1987Q4	0.007205	-0.003029	0.006769	-0.006237	-0.007813	-0.005664	-0.01184	-0.001106	-0.001617
1988Q1	-0.001966	-0.003219	0.005323	-0.004636	-0.009532	-0.015027	-0.011146	-0.003554	-0.005624
1988Q2	-0.004706	-0.002708	0.00227	0.001137	-0.009709	-0.017401	-0.00813	-0.003748	-0.004586
1988Q3	0.0022	-0.002525	0.000722	0.005867	-0.011936	-0.011519	-0.004204	-0.002333	0.000436
1988Q4	0.00413	-0.000426	0.00529	0.013984	-0.010043	-0.007785	-0.004079	-0.006754	0.004205
1989Q1	0.005728	0.000953	-0.004072	0.027438	-0.009054	0.000766	-0.00295	-0.003157	0.007956
1989Q2	0.004017	0.001975	-0.003262	0.025735	-0.011522	0.001334	-0.005252	-0.004961	0.009541
1989Q3	0.00578	0.002404	-0.007544	0.019825	-0.012559	0.004043	-0.010965	-0.001228	0.005808
1989Q4	0.006275	0.004094	-0.016534	0.030653	-0.010093	0.011834	-0.007019	-0.005106	0.006426
1990Q1	0.006044	0.006735	-0.00893	0.049486	-0.00704	0.012348	0.00119	-0.001382	0.0116
1990Q2	0.004646	0.008049	-0.008952	0.034614	-0.003916	0.006148	0.000726	0.000559	0.00846
1990Q3	-0.000283	0.008455	-0.012159	0.029099	-0.005702	0.004855	0.00136	0.001939	0.009542
1990Q4	0.000624	0.009786	-0.014445	0.028687	-0.00492	0.009099	-0.004093	0.007174	0.007445
1991Q1	0.003014	0.009987	-0.018008	0.020747	-0.002301	0.009421	-0.013791	-0.001555	0.005937
1991Q2	0.005858	0.008069	-0.009538	0.018433	0.001177	0.003809	-0.021652	0.007324	0.003342
1991Q3	-0.001442	0.005034	-0.000737	0.014541	0.003322	0.004582	-0.025723	0.008398	-0.000802
1991Q4	0.009434	0.005082	0.006309	0.026938	0.007772	0.007264	-0.015051	0.009771	0.005625
1992Q1	0.011238	0.004387	0.006654	0.043095	0.005031	0.009234	0.001477	0.017442	0.016426
1992Q2	0.008476	0.003373	0.002198	0.027293	0.004418	0.011597	-0.002917	0.015686	0.004869
1992Q3	0.005368	0.001606	0.008081	0.00968	0.003197	0.021576	-3.75E-05	0.026225	0.000714
1992Q4	0.01652	0.00394	0.015148	0.007331	0.002753	0.019049	0.005656	0.021578	0.001519
1993Q1	-0.001893	0.001675	0.013481	-0.019703	0.006096	0.014326	0.005211	0.005215	-0.007148
1993Q2	-0.005632	-0.002319	0.014049	-0.036023	0.006386	0.001836	0.017549	0.004464	-0.010269
1993Q3	-0.006322	-0.00482	0.010241	-0.035698	0.004647	-0.004742	0.030699	-7.77E-05	-0.010416
1993Q4	-0.005448	-0.006919	0.007222	-0.047599	0.004571	-0.00883	0.029833	-0.004216	-0.012982
1994Q1	-0.008667	-0.007923	0.010086	-0.039211	0.002383	-0.011614	0.02964	-0.002212	-0.008232
1994Q2	-0.008862	-0.007525	0.005841	-0.024391	0.001603	-0.013357	0.027697	-0.003112	-0.005635
1994Q3	-0.005256	-0.005701	0.003038	-0.015967	0.001574	-0.010546	0.018926	-0.00416	-0.003129
1994Q4	-0.004356	-0.004085	0.006368	0.005765	0.003012	-0.006741	0.003514	-0.004854	0.000542
1995Q1	-0.001361	-0.002003	-0.007738	-0.008963	0.002786	0.000216	-0.00734	-0.006803	0.003952
1995Q2	0.005568	-0.000511	-0.003208	0.002996	0.004931	0.004673	-0.016382	-0.00626	0.005904
1995Q3	0.00067	-0.001039	-0.000106	-0.008077	0.006998	0.002961	-0.016065	-0.00962	0.003208
1995Q4	-0.003371	-0.001529	0.005822	-0.002157	0.0061	0.003568	-0.009308	-0.008544	0.000107
1996Q1	0.002053	-0.001981	-0.002895	-0.039906	0.005324	-0.002251	-0.009386	-0.00485	-0.004577
1996Q2	-0.001804	-0.002689	0.000825	0.001443	0.003673	-0.00479	0.007046	-0.010314	-0.002343
1996Q3	-0.004261	-0.003404	0.0038	-0.00974	0.003073	-0.003893	0.009446	-0.006107	-0.004124
1996Q4	-0.008439	-0.003888	0.001375	-0.011181	0.001434	-0.005838	0.006283	-0.004977	-0.007566
1997Q1	-0.009115	-0.00436	0.002852	-0.031504	0.001267	-0.005427	0.00146	-0.000929	-0.007628
1997Q2	-0.005209	-0.004716	0.003067	-0.017713	-0.000515	-0.004772	0.00063	-0.002084	-0.002478
1997Q3	-0.008025	-0.004513	-0.001103	-0.021055	-5.48E-05	-0.003194	-0.004135	-0.004	-0.00189

Figure 22: Appendix B:Observed Time Series -2

1997Q4	-0.005037	-0.00428	-0.00744	-0.009017	0.000172	-0.000726	-0.00278	-0.002521	0.002039
1998Q1	-0.004085	-0.003641	-0.003618	-0.000464	0.00029	-0.001687	-0.001627	-0.007986	0.001921
1998Q2	-0.006123	-0.003333	-0.005484	-0.008111	0.001053	-0.001879	-0.004867	-0.005395	-0.002597
1998Q3	-0.004473	-0.003008	-0.007765	-0.001077	-0.000241	-0.001927	-0.006688	-0.007381	-0.005227
1998Q4	-0.001809	-0.002427	-0.008404	-0.005045	-0.000608	-0.004161	-0.010172	-0.000459	-0.009723
1999Q1	0.00257	-0.00112	-0.002588	0.011038	-0.003775	-0.008425	-0.00349	-0.003089	-0.006685
1999Q2	0.001701	8.61E-05	-0.004997	0.013874	-0.004192	-0.012158	-0.001385	0.002571	-0.00489
1999Q3	0.003749	0.001531	-0.003275	0.022266	-0.00657	-0.010933	-3.30E-05	0.00625	0.000376
1999Q4	0.007763	0.003103	-0.003605	0.025905	-0.009074	-0.003338	0.002881	0.010771	0.006741
2000Q1	0.010604	0.004413	0.003287	0.039053	-0.008484	-0.001728	-0.002578	0.011188	0.011253
2000Q2	0.013481	0.006519	-0.000822	0.035056	-0.008331	0.005663	-0.001421	0.009047	0.013297
2000Q3	0.008682	0.007027	-0.000951	0.036843	-0.007833	0.010565	-0.004437	0.007336	0.011245
2000Q4	0.00468	0.007746	-0.004409	0.031722	-0.009456	0.013746	-0.000207	0.006729	0.011336
2001Q1	0.011588	0.00855	9.57E-05	0.034849	-0.006165	0.011622	0.000299	0.006954	0.015662
2001Q2	0.011479	0.00812	-0.00126	0.025038	-0.003441	0.010773	-0.001975	0.002847	0.011685
2001Q3	0.007709	0.007154	-0.003217	0.010388	-0.002307	0.008373	-0.002969	0.001181	0.006767
2001Q4	0.001437	0.00568	0.001819	0.000709	0.000595	0.001086	-0.00377	7.91E-05	0.002388
2002Q1	-0.002771	0.004294	-0.00243	-0.005292	0.002381	0.001047	-0.004089	-0.002475	-0.000926
2002Q2	-0.003492	0.002301	0.003884	-0.021233	0.001105	0.002684	0.000278	-0.000832	-0.00023
2002Q3	-0.00079	0.000458	0.00346	-0.018574	0.003544	0.002724	0.000759	0.000261	-0.00028
2002Q4	-0.000582	-0.00065	0.005342	-0.013852	0.004273	0.001232	0.007976	-0.001213	-0.002475
2003Q1	-0.003501	-0.001915	0.001253	-0.014306	0.000737	-0.001948	0.009655	0.000186	-0.004297
2003Q2	-0.004037	-0.002906	0.002683	-0.017784	0.0021	-0.004133	0.004847	-0.001979	-0.009379
2003Q3	-0.004097	-0.003043	0.003731	-0.017639	0.005386	-0.005434	-0.00431	-0.005582	-0.008276
2003Q4	-0.006527	-0.004051	0.003874	-0.015609	0.003351	-0.004411	0.004738	-0.004172	-0.008338
2004Q1	-0.003383	-0.004302	-0.001002	-0.01146	0.001244	-0.004349	0.003878	0.001925	-0.002731
2004Q2	-0.004708	-0.003969	-0.001478	-0.011519	0.002724	-0.003271	-0.00458	-0.00101	-0.002621
2004Q3	-0.005858	-0.003741	-0.0006	-0.010209	0.001073	-0.002066	8.69E-05	-0.002689	-0.003771
2004Q4	-0.000534	-0.002708	-0.0034	-0.008028	0.000813	-0.000747	-0.004416	-0.000229	-0.004444
2005Q1	-0.002975	-0.002306	-0.005313	-0.00517	-0.000383	-0.00014	0.000839	-0.000313	-0.002725
2005Q2	-0.001264	-0.001078	-0.001593	0.004786	-0.001625	0.000532	0.000919	-0.001227	-0.000648
2005Q3	0.004619	0.001236	0.003759	0.015739	-0.001606	0.001413	0.005469	-0.001471	0.004472
2005Q4	0.002663	0.002821	0.002357	0.016587	0.00027	0.004317	-0.008098	0.000841	0.005028

Figure 23: Appendix B:Observed Time Series -3

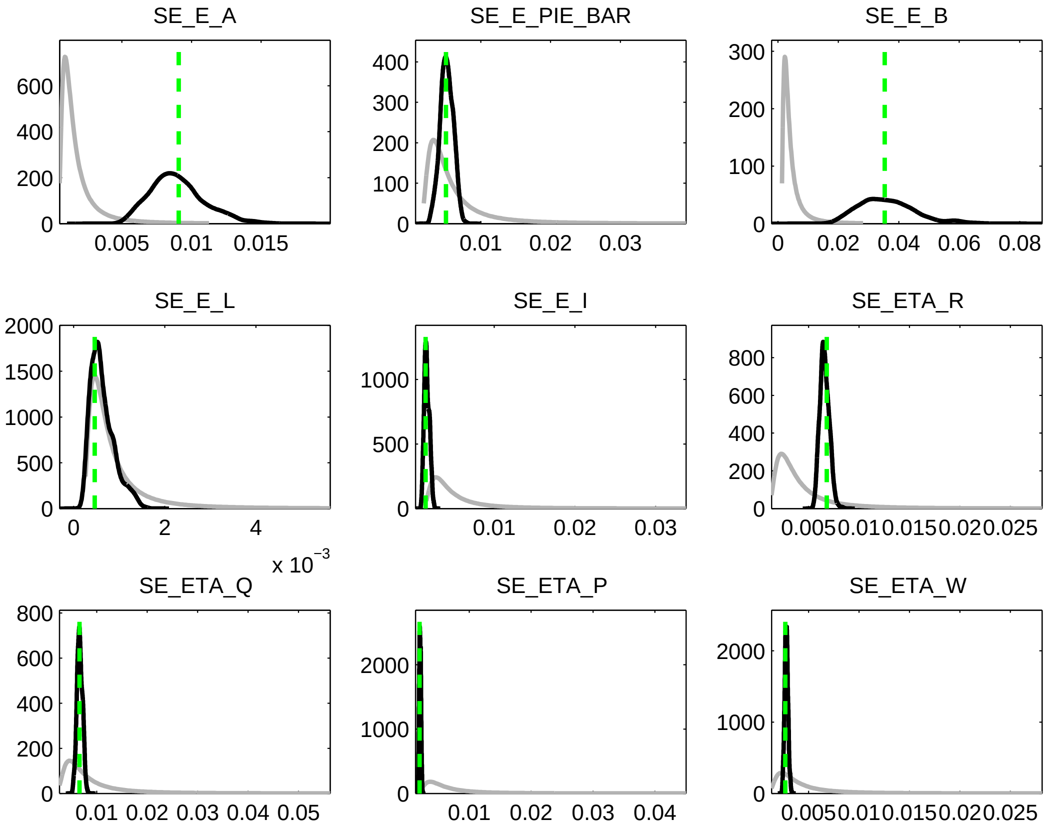

Figure 24: Appendix C: Prior and Posterior Distributions -1

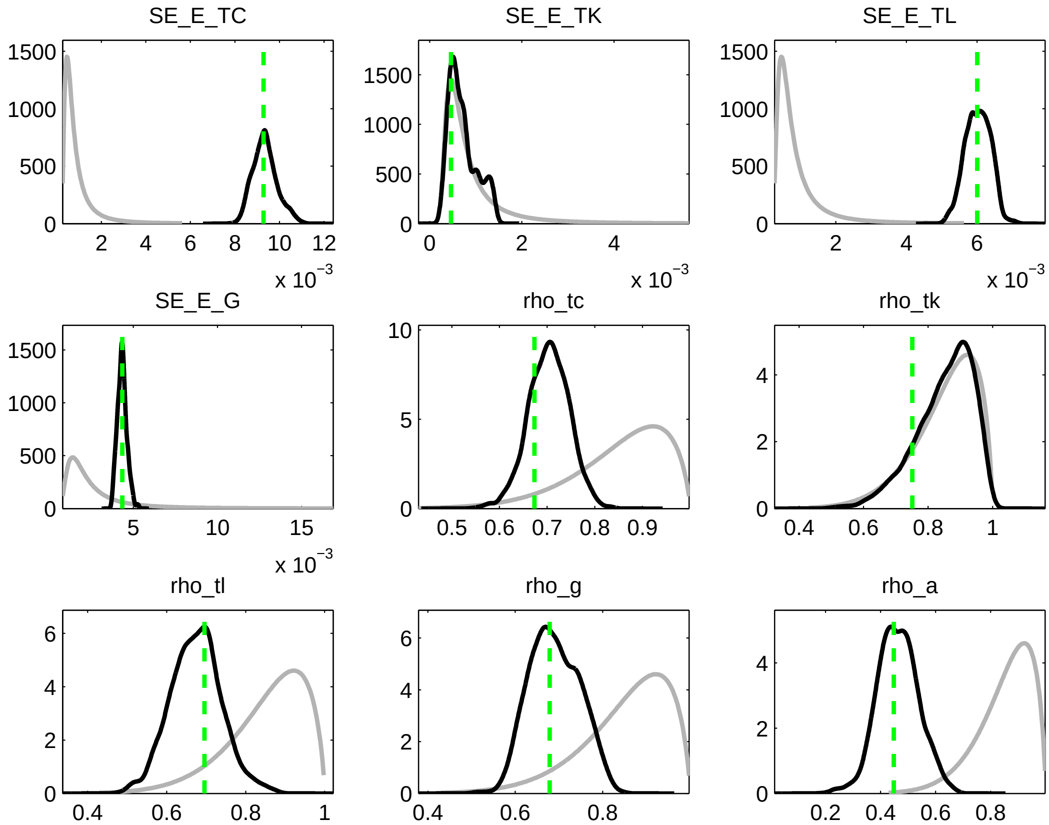

Figure 25: Appendix C: Prior and Posterior Distributions -2

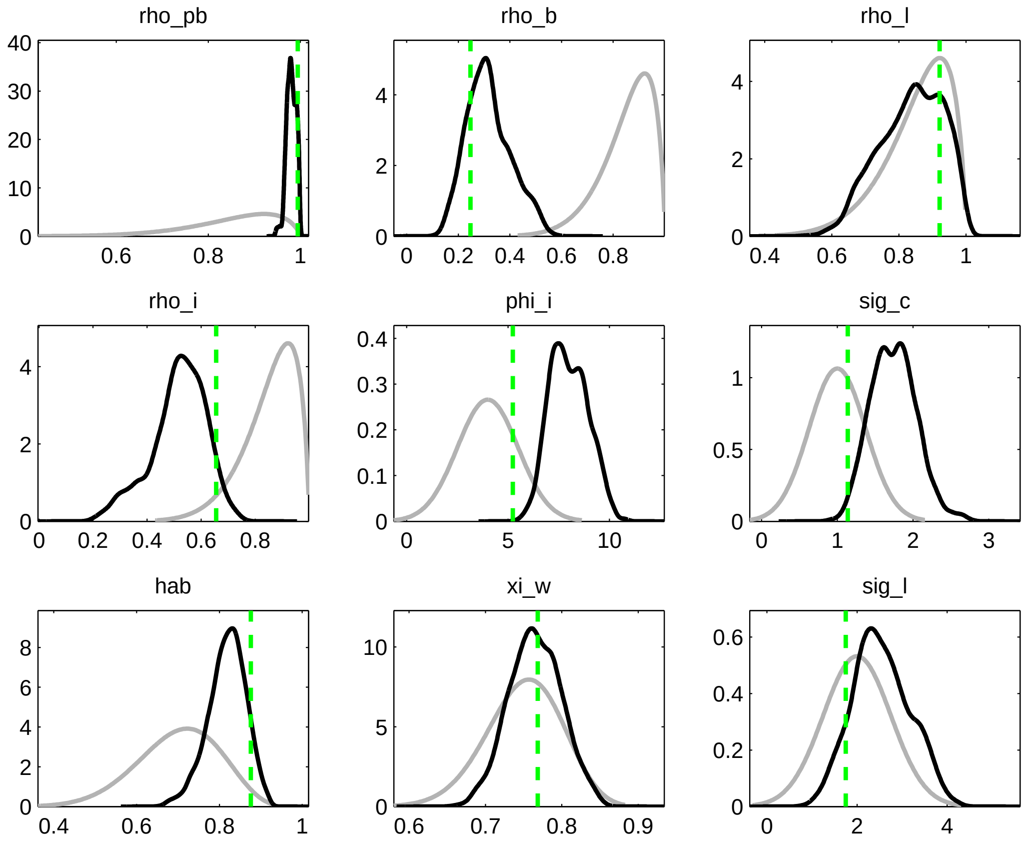

Figure 26: Appendix C: Prior and Posterior Distributions -3

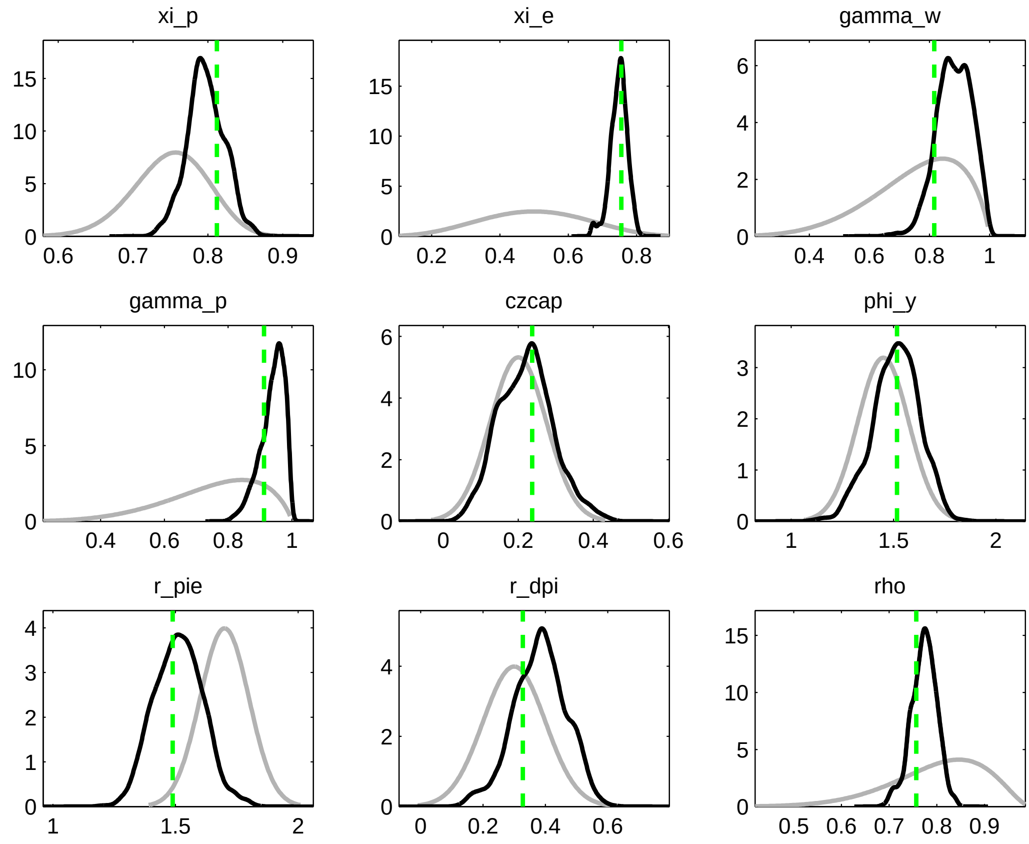

Figure 27: Appendix C: Prior and Posterior Distributions -4

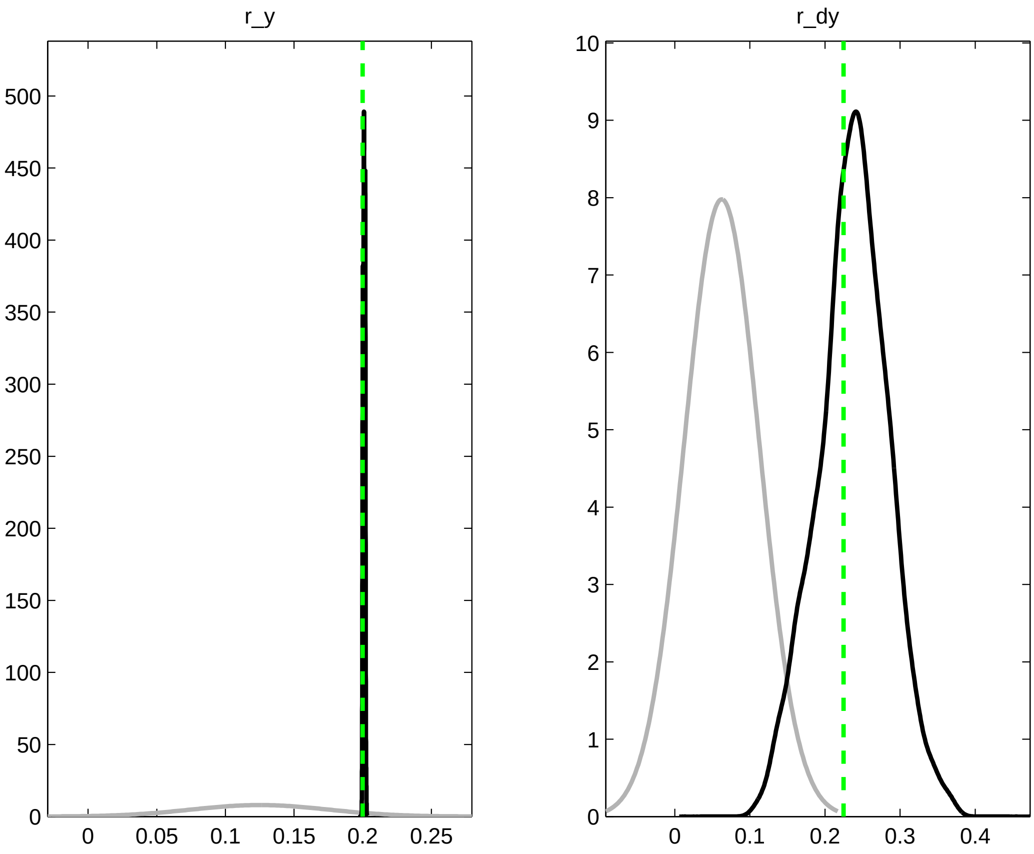

Figure 28: Appendix C: Prior and Posterior Distributions -5

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        0,  0,  0,  0,  0,  0,                   0,  0, -1,      0,        thau,    0,      0,  0,                   0,  0,  0
        0,  0,  0,  0,  0,  0,                   0,  0,  1,      0,          0,     0,     -1,  0,                   0,  0,  0
        0,  0,  0,  0,  0,  0,                   0,  0,  0,      c_y,    thau*k_y,  0,      0,  -1,                  0,  0,  0
        0,  0,  0,  0,  0,  0,                   0,  0,  alfa,   0,          0,     0,      0,  -1/fc,               0,  0,  0
        0,  0,  0,  0,  0,  -(1-rho)*r_y-r_d_y,  0,  0,  0,      0,          0,     r_d_pai, 0,  (1-rho)*r_y+r_d_y,  -1,  0,  0
        0,  0,  0,  0,  0,  0,                   0,  0,  0,      0,          0,     0,      0,  0,                   0,  0,  -ty];

BB = [ 1-thau,   0,    0,   0,                        0,   0,   0,   0,              0,  0,  0,  0,  0,  0,  0,  0,   0
         0,      0,    0,   0,                        0,   0,   0,   0,              0,  0,  0,  0,  0,  0,  0,  0,   0
         0,      0,    0,   0,                        0,   0,   0,   0,              0,  0,  0,  0,  0,  0,  0,  0,   0
         0,     0,    0,   0,                         0,   0,   0,   0 ,             0,  0,  0,  0,  0,  0,  0,  0,   0
         0,      0,    0,  (1-rho)*r_pai-r_d_pai,     0,   0,   rho, 0 ,             0,  0,  0,  0,  0,  0,  0,  0,   0

        0,  0,  0,  0,  0,  0,                  0,  0,  1-thau,   0,    0,   0,                        0,   0,       0,   0, 0
        0,  0,  0,  0,  0,  0,                  0,  0,  0,        0,    0,   0,                        0,   0,       0,   0, 0
        0,  0,  0,  0,  0,  0,                  0,  0,  0,        0,    0,   0,                        0,   0,       0,   0, 0
        0,  0,  0,  0,  0,  0,                  0,  0,  0,        0,    0,   0,                        0,   0,       0,   0, 0
        0,  0,  0,  0,  0,  r_d_y,              0,  0,  0,        0,    0,  (1-rho)*r_pai-r_d_pai,     0,  -r_d_y,  rho, 0, 0
        0,  0,  0,  0,  0,  0,                  0,  0,  0,        0,    0,   0,                        0,   0,       0,   0, 0  ];

CC = [  0,         0,        0, 0
       -1,       1+psi,      0, 0
        0,         0,        0, 0
      1-alfa,   alfa*psi,    0, 0
        0,         0,        0, 0

        0, 0,       0,         0
        0, 0,      -1,       1+psi,
        0, 0,       0,         0
        0, 0,    1-alfa,   alfa*psi,
        0, 0,       0,         0
        0, 0,       0,         0  ];

DD = [ 0,   0,   0,  0,                0,              0,   0,   0,   0,  0,    0, 0, 0
       0,   0,   0,  0,                0,              0,   0,   0,   0,  0     0, 0, 0
       0,   0,   0,  1,                0,              0,   0,   0,   0,  0     0, 0, 0
       0,   1,   0,  0,                0,              0,   0,   0,   0,  0     0, 0, 0
       0,   0,   0,  0,          (1-rho)*(1-r_pai),    1,   0,   0,   0,  0     0, 0, 0

      0,   0,   0,    0,                       0,   0,   0,   0,   0,  0     0,   0,          0
      0,   0,   0,    0,                       0,   0,   0,   0,   0,  0     0,   0,          0
      0,   0,   0,    1,                       0,   0,   0,   0,   0,  0     0,   0,          0
      0,   1,   0,    0,                       0,   0,   0,   0,   0,  0     0,   0,          0
      0,   0,   0,    0,        (1-rho)*(1-r_pai), 1,   0,   0,   0,  0     0,   0,          0
      0,   0,   0,    1,                        0,   0,   0,   0,   0,  0    c_y, inv_y/thau*r_bar_k, (1-alfa)/alfa/thau*inv_y*r_bar_k ]  ...
      * scale_shocks;
```

Figure 29: Appendix D: Toolkit Matrices-1

```
FF = [ 0,   1/(1+h),        0,        (1-h)/((1+h)*sigma_c),       0,           0,   0,        0,               0,  0,  0,  0,  0,  0,  0,  0,  0
       0,     0,       beta/(1+beta),             0,               0,           0,   0,        0,               0,  0,  0,  0,  0,  0,  0,  0,  0
       0,     0,            0,                    1,               0            0,   0,  (1-thau)*beta,         0,  0,  0,  0,  0,  0,  0,  0,  0
       0,     0,            0,          beta/(1+beta*gamma_p),     0,           0,   0,        0,               0,  0,  0,  0,  0,  0,  0,  0,  0
       0,     0,            0,          beta/(1+beta),         beta/(1+beta)    0,   0,        0,               0,  0,  0,  0,  0,  0,  0,  0,  0

       0,  0,  0,  0,  0,  0,  0,  0,                0,   1/(1+h),       0,        (1-h)/((1+h)*sigma_c),       0,           0,   0,        0,  0
       0,  0,  0,  0,  0,  0,  0,  0,                0,     0,     beta/(1+beta),            0,                 0,           0,   0,        0,  0
       0,  0,  0,  0,  0,  0,  0,  0,                0,     0,           0,                  1,                 0            0,   0,  (1-thau)*beta,  0
       0,  0,  0,  0,  0,  0,  0,  0,                0,     0,           0,          beta/(1+beta*gamma_p),     0,           0,   0,        0,  0
       0,  0,  0,  0,  0,  0,  0,  0,                0,     0,           0,          beta/(1+beta),         beta/(1+beta)    0,   0,        0,  0];

GG = [ 0,                    -1,      0,               0,              0,                 0,   (h-1)/((1+h)*sigma_c),          0,    0,  0,  0,  0,  0,  0,  0,  0,  0
       0,                     0,     -1,               0,              0,                 0,              0,       phai/(1+beta),    0,  0,  0,  0,  0,  0,  0,  0,  0
       0,                     0,      0,               0,              0,                 0,             -1,                 -1,    0,  0,  0,  0,  0,  0,  0,  0,  0
       0,                     0,      0,              -1,         index2f*(1-alfa),       0,              0,                 0,     0,  0,  0,  0,  0,  0,  0,  0,  0
       0,   index1f*sigma_c/(1-h),     0,    -(1+beta*gamma_w)/(1+beta),  -1-index1f,        0,              0,                 0,     0,  0,  0,  0,  0,  0,  0,  0,  0

        0,  0,  0,  0,  0,  0,  0,  0,  0,       -1,                  0,               0,              0,                        0,   (h-1)/((1+h)*sigma_c),          0,  0
        0,  0,  0,  0,  0,  0,  0,  0,  0,        0,                 -1,               0,              0,                        0,              0,       phai/(1+beta),  0
        0,  0,  0,  0,  0,  0,  0,  0,  0,        0,                  0,              -1,              0,                        0,             -1,                 -1,  0
        0,  0,  0,  0,  0,  0,  0,  0,  0,        0,                  0,               0,         index2s*(1-alfa),              0,              0,                  0,  0
        0,  0,  0,  0,  0,  0,  0,  0,  0,   index1s*sigma_c/(1-h),    0,    -(1+beta*gamma_w)/(1+beta),  -1-index1s,           0,              0,                  0,  0];

HH = [ 0,          h/(1+h),            0,                   0,                      0,          0,    0,    0,              0,  0,  0,  0,  0,  0,  0,  0,  0
       0,          0,                  1/(1+beta),          0,                      0,          0,    0,    0,              0,  0,  0,  0,  0,  0,  0,  0,  0
       0,          0,                  0,                   0,                      0,          0,    0,    0,              0,  0,  0,  0,  0,  0,  0,  0,  0
       0,          0,                  0,        gamma_p/(1+beta*gamma_p),          0,          0,    0,    0,              0,  0,  0,  0,  0,  0,  0,  0,  0
       0,   -index1f*sigma_c*h/(1-h),  0,        gamma_w/(1+beta),            1/(1+beta),       0,    0,    0,              0,  0,  0,  0,  0,  0,  0,  0,  0

         0,  0,  0,  0,  0,  0,  0,  0,             0,                h/(1+h),               0,                0,                     0,          0,    0,    0,  0
         0,  0,  0,  0,  0,  0,  0,  0,             0,                   0,                  1/(1+beta),       0,                     0,          0,    0,    0,  0
         0,  0,  0,  0,  0,  0,  0,  0,             0,                   0,                  0,                0,                     0,          0,    0,    0,  0
         0,  0,  0,  0,  0,  0,  0,  0,             0,                   0,                  0,        gamma_p/(1+beta*gamma_p),      0,          0,    0,    0,  0
         0,  0,  0,  0,  0,  0,  0,  0,             0,        -index1s*sigma_c*h/(1-h),      0,           gamma_w/(1+beta),       1/(1+beta),     0,    0,    0,  0];

JJ = [ 0,          0,              0,  0
       0,          0,              0,  0
       0,    r_bar_k*beta,         0,  0
       0,          0,              0,  0
       0,          0,              0,  0

        0,  0,        0,          0
        0,  0,        0,          0
        0,  0,        0,    r_bar_k*beta
        0,  0,        0,          0
        0,  0,        0,          0];

KK = [ 0,             0,                                    0,  0
       0,             0,                                    0,  0
       0,             0,                                    0,  0
       0,             index2f*alfa,                         0,  0
    sigma_L*index1f,    0,                                   0,  0

       0,  0,        0,                                    0
       0,  0,        0,                                    0
       0,  0,        0,                                    0
       0,  0,        0,                               index2s*alfa
       0,  0,        sigma_L*index1s,                      0];
```

Figure 30: Appendix D: Toolkit Matrices-2

```
LL = [ 0,   0,          0,        0,          0,    0,     0,             0,   0,   0,              0,    0,    0
       0,   0,          0,        0,          0,    0,     0,             0,   0,   0,              0,    0,    0
       0,   0,          0,        0,          0,    0,     0,             0,   0,   0,              0,    0,    0
       0,   0,          0,        0,          0,    0,     0,             0,   0,   0,              0,    0,    0
       0,   0,          0,        0,          0,    0,     0,             0,   0,   0,              0,    0,    0

       0,   0,          0,         0,       0,    0,     0,             0,   0,   0,             (1-h)/((1+h)*sigma_c),    0,    0
       0,   0,          0,         0,       0,    0,     0,             0,   0,   0,              0,    0,    0
       0,   0,          0,         0,       0,    0,     0,             0,   0,   0,              0,    0,    0
       0,   0,          0,         0,       0,    0,     0,             0,   0,   0,              0,    0,    0
       0,   0,          0,         0,       0,    0,     0,             0,   0,   0,              0,    0,    0]...
  * scale_shocks;

MM = [ 0,          0,          (1-h)/((1+h)*sigma_c),       0,  0,0,       0,  0, 0,  0,              0,    0,    0
       0,          0,           0,                          0,  0,0,       1,  0, 0,  1,              0,    0,    0
       0,          0,           0,                          0,  0,0,       0,  0  0,  0,              0,    0,    0
       0,          -index2f,    0,                          0,  0,0,       0,  0, 0,  0,              0,    0,    0
       -index1f,   0,           0,                          0   0,0,       0,  0  0,  0,              0,    0,    0


       0,          0,          (1-h)/((1+h)*sigma_c),       0,  0, 0,     0,  0,  0, 0,              -(1-h)/((1+h)*sigma_c),    0,    0
       0,          0,           0,                          0,  0, 0,     1,  0,  0, 1,              0,                         0,    0
       0,          0,           0,                          0,  0, 0,     0,  0,  0, 0,              0,       -(1-beta+beta*thau),    0
       0,    -index2s,          0,                          0,  0, 0,     0,  1,  0, 0,              0,                         0,    0
    -index1s,      0,           0,                          0   0, 0,     0,  0,  1, 0,              0,                         0,    index1s]...
  * scale_shocks;

NN= [ rho_epsilon_L,     0,          0,             0,       0,            0,              0,              0,             0,             0,    0,           0,          0
         0,        rho_epsilon_a,    0,             0,       0,            0,              0,              0,             0,             0,    0,           0,          0
         0,            0,        rho_epsilon_b,     0,       0,            0,              0,              0,             0,             0,    0,           0,          0
         0,            0,          0,             rho_G,     0,            0,              0,              0,             0,             0,    0,           0,          0
         0,            0,          0,             0,       rho_pai_bar,  0,              0,              0,             0,             0,    0,           0,          0
         0,            0,          0,             0,       0,            rho_epsilon_r,  0,              0,             0,             0,    0,           0,          0
         0,            0,          0,             0,       0,            0,              rho_epsilon_i,  0,             0,             0,    0,           0,          0
         0,            0,          0,             0,       0,            0,              0,              rho_lambda_p,  0,             0,    0,           0,          0
         0,            0,          0,             0,       0,            0,              0,              0,             rho_lambda_w,  0,    0,           0,          0
         0,            0,          0,             0,       0,            0,              0,              0,             0,             rho_q, 0,           0,          0
         0,            0,          0,             0,       0,            0,              0,              0,             0,             0,    rho_tau_c,   0,          0
         0,            0,          0,             0,       0,            0,              0,              0,             0,             0,    0,           rho_tau_k,  0
         0,            0,          0,             0,       0,            0,              0,              0,             0,             0,    0,           0,          rho_tau_l];
```

Figure 31: Appendix D: Toolkit Matrices-3