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Distributive Policies and Economic Growth: An Optimal Taxation Approach

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Abstract

In an infinite-horizon, endogenous growth model a capital income cum investment subsidy tax is considered to investigate if distribution of income towards the non-accumulated factor of production (labour) retards growth and if capital income taxes are bad instruments to finance investment subsidies. The paper identifies conditions under which the tax scheme is better for growth than other distorting tax schemes. In the model a 'left-wing' (pro-labour) government acts growth maximizing and distributing income towards labour raises growth. A 'right-wing' (pro-capital) government's preferred policy is not growth maximizing under the tax scheme, but may generate higher growth than its optimal, growth maximizing policy under other tax schemes.

Keywords: Growth, Distribution, Investment Subsidies, Capital Income Taxes

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1 Introduction

Many theoretical models on growth and distributive policies suggest that increasing taxes for redistribution slows down growth. See e.g. Alesina and Rodrik (1994), Bertola (1993) and many others. This conclusion is reached by the following line of argument: Some taxes are optimal for the private incentive to invest and so optimal for the accumulated factor of production and growth. In comparison, a government that redistributes resources towards the non-accumulated factor of production levies higher taxes, causing lower steady state growth. The result crucially depends on the tax arrangement. Clearly, any policy that subsidizes investment is good for growth as is usually noted by the same authors. That raises the question how the subsidies are financed and what their distributional consequences are. Often *capital income taxation* is ruled out as a means of subsidy financing as it would *defeat the purpose of enhancing growth.* As an example see Bertola (1993), p. 1192. He does, however, analyze the effect of consumption taxes on growth and distribution.

In this paper and within an infinite-horizon, endogenous growth framework it is questioned whether (1) high capital income taxes are always bad for growth, (2) the optimal policies of the accumulated factor of production are necessarily growth maximizing, (3) shifting political power to the non-accumulated factor of production causes higher taxes and lower growth and (4) capital income taxation really does defeat the purpose of enhancing growth.[1]

In the model a *capital income cum investment subsidy tax* scheme

[1] A negative answer to the first question is e.g. provided by Uhlig and Yanagawa (1996) in a finite-horizon OLG growth model. More generally, and for infinite-horizon OLG growth models, Bertola (1996) shows that shifting income towards the non-accumulated factor of production may raise growth. However, he concludes that redistributing disposable income from accumulated to non-accumulated factors of production *necessarily* decreases the level and/or growth rate of income in *infinite-horizon* growth models. (Cf. Bertola (1998), p. 27; (1996), p. 1552.) But, for instance, Rehme (1995) or, in a different context, Pelloni and Waldmann (1997) show that the redistribution argument may also apply within 'standard', *infinite-horizon* growth models. Hence, the answer to the question above may be negative in more general settings.

(CICIST) is compared with a *wealth tax* scheme (WT). The tax schemes are meant to represent two broad classes of tax arrangements that may distort the investors' incentive to accumulate and both serve as metaphors for redistributive mechanisms. For the same assumption with respect to WT and examples of what other redistributive mechanisms WT may capture see Alesina and Rodrik (1991), (1994). In these papers policies that maximize growth are optimal for the accumulated factor of production. Thus, WT may also serve as a metaphor for models leading to that result. In contrast, CICIST is a metaphor for any redistributive, perhaps accumulation reducing policy that, in addition, subsidizes investment. Thus, CICIST is supposed to reflect the fact that in one way or another some governments subsidize investment more than others.[2]

In this paper and as is common, the non-accumulated factor of production is identified with labour and the accumulated factor is identified with capital. The model uses an important equivalence result due to Diamond and Mirrlees (1971) by assuming that the tax rate on capital income and for investment subsidies is equal. Thus, the tax scheme is tantamount to a tax on the consumption of the capital owners.[3] Suppose the government provides public inputs to production as in Barro (1990). Higher capital income taxes may then be good for the pre-tax return on capital if the public inputs positively affect production. Coupled with

[2]A society's choice of tax scheme depends on many diverse things such as history, politics, institutions etc. Although the paper argues that tax schemes play a great role in any analysis of the relationship between taxes and growth, an answer to the question why societies prefer one scheme to another one has to remain outside of this paper's analysis.

[3]In terms of implementability there are important differences, however. As a consumption tax the government would tax the capital owners' consumption so that a government representing their interests would not necessarily want to use it. On the other hand a pro-labour government may wish to use it. For both governments it would be difficult to determine whether a homogeneous consumption good was bought by a capital owner or a worker. So viewing the tax arrangement as a consumption tax raises various difficulties which do not arise when implemented as a capital income cum investment subsidy scheme. The conditions for the optimality of uniform commodity taxation have e.g. been analyzed by Sandmo (1976), or Besley and Jewitt (1995). For recent contributions on the relationship between consumption and (capital) income taxes see, for instance, Krusell, P., V. Quadrini and J. V. Ríos-Rull (1996) or Judd (1999).

an investment subsidy, a 'right-wing' (entirely pro-capitalists') government may want to use the tax scheme. Analogous reasoning holds for a 'left-wing' (entirely pro-workers') government.

The uniform tax rate may be justified as follows: Given everything else a 'right-wing' government would wish to set very low capital income taxes. But higher tax rates may be called for if public services raise the return on capital, and if the government wants to subsidize investment. So setting similar tax rates on capital income and for investment subsidies is a reasonable choice for a 'right-wing' government. Similarly, a 'left-wing' government would wish to set a high tax rate for redistributive reasons. But that hinders investment, and is bad for the growth of wages. Thus, a left-wing government would have to strike a balance between financing investment subsidies and redistribution. Hence, setting similar tax rates is also a reasonable choice for a 'left-wing' government. With these justifications and for simplicity a *uniform* tax rate on capital income and for investment subsidies is assumed.

In the paper the governments may want to expropriate the capital stock and run the economy more efficiently themselves. As that is rather unrealistic, it is ruled out and each government respects the right of private property.

It is shown that for optimizing capital owners the distorting effect of capital income taxes is removed by the investment subsidy and the positive effect of public inputs to production. The capital income tax only has a negative effect on the capital owners' consumption level. In the economy's market equilibrium steady state growth depends only on the *pre-tax* return to capital.

In the paper the equilibrium relationship between pre-tax factor incomes and taxes as well as growth and taxes is strictly positive. However, the capital income component of the share of disposable income in total income decreases with taxes while that for the wage component remains fixed. Thus, higher taxes redistribute relatively more income to labour and raise growth. Furthermore, growth is maximized, when the capitalists are taxed maximally. It is shown that CICIST allows for higher growth than WT if - under either tax scheme - a government targets the

same ratio of public inputs in production to the capital stock or if the capital owners are sufficiently impatient and a government targets the same ratio of tax revenues to tax base. Consequently, CICIST appears conducive to high growth.

In contrast to most optimal growth models impatience is *not* necessarily bad for growth in the model and there is a concave relationship between the growth and the time preference rate. As the tax scheme operates like a consumption tax, more impatience raises the capital owners' instantaneous level of consumption and with it the tax revenues, the government channels into production which in turn raise the return on capital and growth. Furthermore, it is shown that the more impatient the capital owners are, the lower the capital income tax rate must be to maintain a given growth rate.

In a public policy analysis it is investigated what tax rates a welfare maximizing government would choose.[4] In the model a time-consistent policy with non-zero capital income taxes is optimal. Furthermore, the optimal 'right-wing' tax policy does not maximize growth under CICIST. As capital income taxes reduce the investors' instantaneous consumption the 'right-wing' government chooses a tax rate that represents the optimal trade-off between generating high income through raising enough tax revenues in order to raise the return on capital on the one hand while reducing consumption on the other hand.

Interestingly, the capital owners always prefer a wealth tax scheme under which they act growth maximizing. As growth may be higher under this paper's tax scheme, the preferred choice of the capital owners implies that they value the direct tax effect on their consumption level

[4]Thus, the paper contributes to discussions on optimal capital income taxation. In that literature some models argue in favour of zero long-run taxes on capital income as, for example, in Eaton (1981), Hamilton (1987), Judd (1985), Chamley (1985), (1986) or Lucas (1990), whereas others imply non-zero taxes as in e.g. Jones, Manuelli and Rossi (1993), (1997) or Uhlig and Yanagawa (1996). Analyzing the relationship between expenditure (consumption) and capital income taxes Atkinson and Sadmo (1980) identify conditions under which zero capital income taxes may yield first best allocations. On the same relationship, Judd (1999) shows that under more general conditions the optimal capital income tax should be zero on average. However, the generality of his result has been challenged by Lansing (1999).

higher than the intertemporal effect of having higher income and consumption in the future. The paper identifies conditions under which the capitalists' preferred tax policies generate higher growth under this paper's scheme than under the wealth tax scheme. Hence, the accumulated factor of production may not choose the growth maximizing tax base and may therefore not act growth maximizing in comparison to a tax scheme where it would actually maximize growth.

Furthermore, if the social planner uses an income cum investment subsidy tax arrangement, placing *more social weight* on the *welfare of the non-accumulated factor of production* (workers) *raises the optimal tax rate on the income of the accumulated factor of production* (capital) and through this *the growth rate*. Hence, it may not be optimal for high growth to shift all political power to the accumulated factor of production. The result is in direct contrast to what is shown in many models.[5]

The 'right-wing' government acts like a growth maximizer under the wealth tax scheme. With this paper's tax scheme a 'left-wing' government acts like a growth maximizer. Thus, a switch in tax bases may induce an important switch in optimal policies. Hence, one may observe an economy with a government that represents only the interests of the non-accumulated factor of production (labour) to distribute income to that factor *and* have higher growth than an economy represented by a government solely concerned about the welfare of the accumulated factor of production (capital).

The paper is organized as follows: Section 2 presents the model set-up, and derives the market equilibrium. Section 2.1 provides a public policy analysis and compares optimal tax policies. Section 3 draws some conclusions.

[5]See e.g. Alesina and Rodrik (1994). A result similar to this paper's is obtained in Bertola (1993), but notice that in comparison to a wealth tax scheme taxation of capital income *does not* defeat the purpose of enhancing growth in this model.

2 The Model

There are two types of many identical, price-taking, infinitely lived individuals who are all equally impatient. The capital owners (k) own capital equally and no labour, and the workers (W) own labour equally, but no capital.[6] Both groups derive logarithmic utility from the consumption of a homogeneous, malleable good. There are many firms which are owned by the capitalists. Aggregate output is produced by (raw) labour and capital according to a Barro (1990) production technology

$$Y_t = A\, K_t^{\alpha}\, G_t^{1-\alpha}\, L_t^{1-\alpha} \quad , \quad 0 < \alpha < 1 \tag{1}$$

where Y_t is total output, K_t is the real capital stock[7], G_t are total public inputs to production and A is a constant efficiency index, which depends on cultural, institutional and technological development. At each point in time (raw) labour is inelastically supplied and the total labour endowment equals unity, $L_t = 1$. For simplicity the paper abstracts from problems arising from the depreciation of the capital stock.

The Public Sector. At each point in time the government taxes the capital income of and grants an investment tax subsidy to the capital

[6] This assumption uses a short-cut of a result in Bertola (1993). He has shown in an endogenous growth model that for utility maximizing, infinitely lived agents who do not own initial capital, it is not optimal to save/invest out of wage income along a balanced growth path. Similarly, it is not optimal to work for those who only own capital initially. Thus, the model set-up is reminiscent of Kaldor (1956), where different proportions of profits and wages are saved. However, in Kaldorian models growth determines factor share incomes, whereas in endogenous growth models the direction is rather from factor shares to growth. Furthermore, the logic of the model would not change if instead one introduced a representative household who derives wage as well as capital income and makes investment decisions, and the government represented 'economic classes' within that household.

[7] Alternatively, one may assume that K_t is broad capital and that human and physical capital are strict complements. For a justification of the latter approach see, for instance, Mankiw, Romer and Weil (1992), p. 416. Both assumptions would allow one to concentrate on the distributional conflict between the accumulated and the non-accumulated factor of production, without changing the paper's qualitative results.

owning households at the rate θ_t. The tax arrangement amounts to a tax on the capitalist owners' consumption, which is implemented as a capital income cum investment subsidy tax (CICST).[8] The government runs a balanced budget at each point in time and uses the tax revenues to provide public services that feed back into production

$$G_t = \theta_t[r_t K_t - \dot{K}_t] \tag{2}$$

where the RHS denotes tax revenues net of investment subsidies. By assumption it is impossible to tax all capital income, that is, $\theta_t \in [0, 1-\epsilon]$ where ϵ is small.[9] Letting $\gamma \equiv \frac{\dot{K}_t}{K_t}$ notice that $\frac{G_t}{K_t}$ is constant over time when $\dot{r}_t = \dot{\gamma}$ and $\dot{\theta}_t = 0$.

The Private Sector. The *firms* operate in a perfectly competitive environment, maximize profits, and take G_t as given. The capital owners rent capital to and demand shares of the firms, which are collateralized one-to-one by capital. The markets for assets, capital and labour clear at each point in time so that the firms face a path of uniform, market clearing rental rates for capital and labour. Given perfect competition the firms rent capital and hire labour in spot markets in each period. The price of output Y_t serves as numéraire and is set equal to 1 at each date, implying that the price of capital K_t in terms of overall consumption stays at unity. Given constant returns to capital and labour, factor payments

[8] Thus, a *Ramsey Tax Problem* is contemplated. See Diamond and Mirrlees (1971) or Atkinson and Stiglitz (1980), chpt. 12. In order to see the equivalence let $q = 1 + t_c$ where q is the price consumption goods command in terms of producer prices normalized to be one and fixed. The government taxes consumption at rate t_c. Let Y^k denote the capital owners' pre-tax income minus pre-tax investment. Then a consumption tax is equivalent to an income cum investment tax if the capital owners' budget constraints satisfy $(1 + t_c)C^k = Y^k \Leftrightarrow C^k = (1 - \theta)Y^k$ which is true if $t_c = \frac{\theta}{1-\theta}$.

[9] A small ϵ captures that the upper bound on tax rates, consistent with no tax-induced expropriation, may still be large, that is, it may be close to, but it is less than one. For ease of calculations it is often assumed that $\epsilon \to 0$ when the effects of maximal taxation are analyzed. Then the reader should bear in mind that maximal taxation in this market economy model with private property is not meant to be the same as outright expropriation.

exhaust output so that profit maximization implies

$$r_t = \frac{\partial Y_t}{\partial K_t} = \alpha A \left(\frac{G_t}{K_t}\right)^{1-\alpha} \quad \text{and} \quad w_t = \frac{\partial Y_t}{\partial L_t} = (1-\alpha) A \left(\frac{G_t}{K_t}\right)^{1-\alpha} K_t \quad (3)$$

where $L_t = 1, \forall t$. If $\frac{G_t}{K_t}$ is constant over time, the marginal product of capital is constant, whereas the wages would grow with the capital stock. Note that the ratio of public inputs to the capital stock has a positive bearing on the pre-tax return on capital and (initial) wages.

The *workers* derive a utility stream from consuming their entire wages. Their intertemporal utility is given by $\int_0^\infty \ln C_t^W \, e^{-\rho t} dt$ where $C_t^W = w_t$. They do not invest and are not taxed by assumption.

At each period the *capital owners* choose how much of their income to consume or invest, and they take the paths of r_t and θ_t as given. Their instantaneous budget constraint is given by $C_t^k = (1-\theta_t)[r_t K_t - \dot{K}_t]$ so that consumption depends on after-tax capital income minus after-tax investment.[10]

Rearranging the capital owners solve

$$\max_{C_t^k} \int_0^\infty \ln C_t^k \, e^{-\rho t} dt \quad (4)$$

$$s.t. \qquad \dot{k} = r_t K_t - \tfrac{C_t^k}{1-\theta_t} \quad (5)$$

$$k(0) = \text{given}, \quad k(\infty) = \text{free}. \quad (6)$$

The present value Hamiltonian for this problem is $\mathcal{H} = \ln C_t^k + \mu_t (r_t K_t - \frac{C_t^k}{1-\theta_t})$ where μ_t denotes the current value shadow price of one more unit of investment at date t. The necessary FOCs for the maximization of $\mathcal{H}(\cdot)$ are given by equations (5), (6) and

$$\frac{1}{C_t^k} = \frac{\mu_t}{1-\theta_t} \quad (7)$$

[10] Notice that the budget constraint suggests yet another implementation of the consumption tax. As $C_t^k = (1-\theta_t) r_t K_t - \dot{K}_t + \theta_t \dot{K}_t$, the term $\theta_t \dot{K}_t$ may be interpreted as a special form of politically determined capital depreciation allowance which is directly and positively related to the amount invested.

$$\dot{\mu}_t = \mu_t \rho - r_t \mu_t \tag{8}$$

$$\lim_{t \to \infty} K_t \mu_t e^{-\rho t} = 0. \tag{9}$$

where the transversality condition (9) ensures that the present value of the capital stock approaches zero asymptotically. In Appendix A it is shown that in the optimum

$$\gamma \equiv \frac{\dot{K}_t}{K_t} = r_t - \rho. \tag{10}$$

Thus, in the model the growth rate of wealth γ depends on the *pre-tax* return on capital, because the capital owners have perfect foresight and know that they receive an investment subsidy. In the optimum the distorting effect of capital income taxation is exactly offset by the accumulation inducing effect of the investment subsidy. The distorting effect is, however, present in the capital owners' instantaneous level of consumption, which follows the rule $C_t^k = (1 - \theta_t)\rho K_t$. More impatience causes the capital owners to value current consumption more than future consumption, which makes them consume more per units of capital. Furthermore, an increase in θ_t reduces the capital owners' instantaneous consumption per units of capital at each date t. But then consumption grows at $\gamma_{C_t^k} \equiv \frac{\dot{C}_t^k}{C_t^k} = \gamma - \frac{\dot{\theta}_t}{1-\theta_t}$ which does not equal γ in general. However, when θ_t is constant, then $\gamma = \gamma_c$. In that case $\frac{G_t}{K_t}$ is constant, so that $\dot{\gamma} = \dot{r}_t = 0$. This will be the case in equilibrium as shown below.

Market Equilibrium. For arbitrary paths of θ_t the economy's resource constraint is satisfied at each date if $I_t = Y_t - C_t - G_t$ where $C_t = C_t^W + C_t^k$. Private sector optimality requires $\gamma = \frac{\dot{K}_t}{K_t} = r_t - \rho$ so that in equilibrium the government's budget constraint becomes

$$G_t = \theta_t(r_t K_t - \dot{K}_t) = \theta_t(r_t K_t - \gamma_t K_t) = \theta_t \rho K_t. \tag{11}$$

Hence, for arbitrary paths of θ_t the equilibrium factor rewards in (3) equal

$$r_t = \alpha A[\theta_t \rho]^{1-\alpha} \quad \text{and} \quad w_t \equiv \eta(\theta_t) K_t = (1-\alpha) A[\theta_t \rho]^{1-\alpha} K_t. \tag{12}$$

Notice that r_t and η_t are continuous, increasing and concave in θ_t. Thus, higher tax rates raise the return on capital and (initial) wages. Surprisingly the marginal products depend on preference parameters. But that is, of course, due to the fact that in a model with productive government inputs and a tax scheme that operates like a consumption tax the marginal products should clearly depend on preference parameters. Also, the return on capital is higher the more *impatient* the investors are. More impatience makes the capital owners consume more per units of capital, which increases the tax revenues that are channelled into production only to raise the return on capital and growth.[11]

From the production function one verifies for given G_t that $Y_t = r_t K_t + w_t$, since $L_t = 1$. Thus, the economy's resource constraint is satisfied if

$$I_t = \dot{K}_t = r_t K_t + w_t - C_t^k - C_t^W - \theta_t \rho K_t. \tag{13}$$

But private sector optimality entails $\dot{K}_t = \gamma K_t = (r_t - \rho) K_t$, $C_t^k = (1-\theta_t)\rho K_t$ and $C_t^W = \eta(\theta_t) K_t$ so that the resource constraint is met and the economy is in equilibrium at each point in time. However, over time and for arbitrary paths of θ_t the economic aggregates may grow at different rates. Growth will only be balanced when the tax policies are constant over time.

To put more structure on the tax policies consider a government that wants to maximize the growth of any aggregate variable featuring in the overall resource constraint. In Appendix B the following is shown:

[11] In Appendix E it is shown that for constant policies and the more general case of iso-elastic utility with preference for consumption smoothing the return on capital is also increasing in ρ, that is, if the intertemporal elasticity of substitution of consumption between different dates varies between zero and one, more impatience raises the steady state rate of return on capital.

Proposition 1 *At each point in time the growth rates of output, wage or capital income, the capital owners' or workers' consumption, capital or government expenditure are each maximized by the constant policy which taxes capital income at its maximum rate,* $\theta_t = 1 - \epsilon$.

Clearly, growth maximizing policies are bad for the capital owners as they reduce their consumption to a level close to zero. Hence, for the capital owners it has very different welfare implications whether the government maximizes the level or the growth of their consumption. Another, perhaps more surprising implication is that the paper's *capital income tax* scheme calls for maximal taxation of the reproducible factor of production, *if* the objective is to maximize growth of the aggregates above, which are often considered in the literature.

Next, consider disposable income $Y_t^d = [(r_t - \theta_t\rho)K_t] + [\eta_t K_t]$ as given from (13). Expressed in terms of total income (=output) and for constant policies one gets $\frac{Y_t^d}{Y_t} = [\alpha - \frac{(\theta_t\rho)^\alpha}{A}] + [1 - \alpha]$ which decreases in θ_t. Thus, higher taxes reduce overall disposable income in terms of total income. This reduction is completely due to the capital income component $\zeta(\theta) \equiv [\alpha - \frac{(\theta_t\rho)^\alpha}{A}]$, because the wage component remains constant at $1 - \alpha$. Thus and in relative terms, higher taxes redistribute income towards labour, but they also raise balanced growth, as γ is strictly increasing in taxes.[12]

Comparison to a Wealth Tax Market Equilibrium. Let technology, preferences etc. be as in this paper with the only difference that the government taxes *wealth*. In such a framework Alesina and Rodrik show that a constant tax policy is optimal for the governments they consider. Thus, assume that governments pursue constant tax policies.[13] The capitalists' dynamic budget constraint is then given by $C_t^k = (r - \tau)k_t - \dot{k}_t$ where τ is the tax rate, levied on the capital owners' wealth. Solving a

[12] Below and for WT one verifies that $\zeta(\tau)$ is also decreasing, but growth is concave in τ. Furthermore, any government attaching social weight to the non-accumulated factor of production will choose $\tau > \hat{\tau}$ in which case there is a positive relation between $\zeta(\tau)$ and growth. See Lemma 2 below.

[13] Dropping time subscripts indicates constant policies from now on.

problem analogous to the one presented above implies $\gamma(\tau) = r(\tau) - \tau - \rho$, where $r(\tau) = \alpha A \left(\frac{G_t}{K_t}\right)^{1-\alpha}$, $G_t = \tau K_t$ and growth is concave in τ. Denote $\gamma(\theta)$ and $\gamma(\tau)$ as the growth rate under CICIST or WT, respectively. Suppose the government maintains the same ratio of G_t to K_t for all t under either tax scheme. Then

Proposition 2 *If the government maintains the same ratio of G_t to K_t under either tax scheme, $\frac{G_t(\theta)}{K_t} = \frac{G_t(\tau)}{K_t} > 0$ for all t, then the pre-tax returns on capital are equal, $r(\theta) = r(\tau)$, but growth is higher under the capital income tax cum investment subsidy tax than under the wealth tax scheme, $\gamma(\theta) > \gamma(\tau)$.*

Thus, a government may fare better in terms of growth with CICIST. Notice that the result would be qualitatively the same if the government fixed $\frac{G_t}{Y_t}$ instead.

The return on capital depends on the rate of time preference in the model (*Ramsey result*) and so it is interesting to know under what other conditions the ratio result holds. Suppose a government would target the same ratio of tax revenues to tax base under either tax scheme.[14] That would imply $\theta = \tau$. But then $\gamma(\theta) > \gamma(\tau)$ if

$$\begin{aligned} \alpha A (\theta\rho)^{1-\alpha} - \rho &> \alpha A (\tau)^{1-\alpha} - \tau - \rho \\ \rho &> \left[1 - \frac{\tau^{\alpha}}{\alpha A}\right]^{\frac{1}{1-\alpha}}. \end{aligned}$$

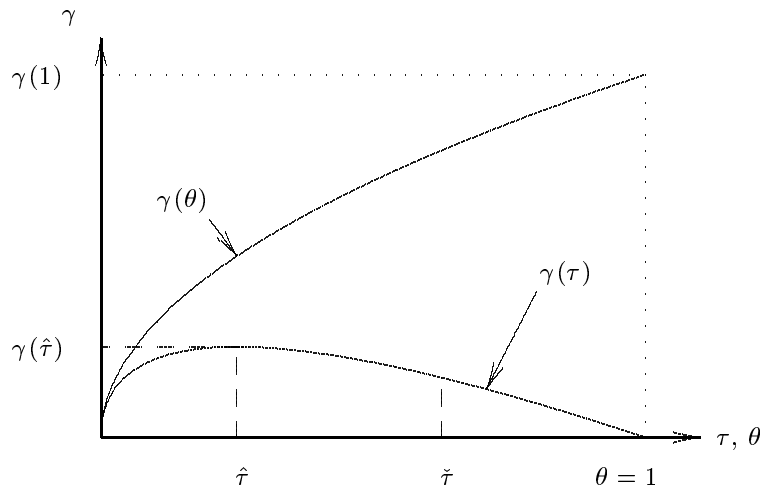
The growth maximizing wealth tax rate is $\tau = [\alpha(1-\alpha)A]^{\frac{1}{\alpha}} \equiv \hat{\tau}$. If $\theta = \hat{\tau}$ then the condition amounts to $\rho > \alpha^{\frac{1}{1-\alpha}}$.

Proposition 3 *If the government targets the same ratio of tax revenues to tax base under either tax scheme, then $\theta = \tau$. Furthermore, if the agents are sufficiently impatient, $\rho > \alpha^{\frac{1}{1-\alpha}}$, and $\theta = \hat{\tau}$, where $\hat{\tau}$ maximizes $\gamma(\tau)$, then $\gamma(\theta)_{|\theta=\hat{\tau}} > \gamma(\tau)_{|\tau=\hat{\tau}}$ so that growth would be higher under CICIST than under WT.*

[14] Recall that under CICIST the tax base is $TB = rK_t - \dot{K}_t$ and tax revenues are $\theta_t TB$.

Figure 1 below visualizes the result for the case where the conditions for the proposition hold.

Figure 1: γ as a function of θ or τ



With sufficient impatience CICIST generates higher growth than WT. The model's tax scheme neither generates an inverted U-shaped (Barro (1990) or Alesina and Rodrik (1994)) nor a U-shaped relationship (Persson and Tabellini (1994)) between growth and taxes. Instead, it is strictly positive. The reason is that in terms of growth the positive effect of granting investment subsidies outweighs the negative effect of levying taxes on capital income. In contrast to most optimal growth models (e.g. Cass (1965) or Koopmans (1965)) impatience (higher ρ) is *not* necessarily bad for growth in this model. For given tax policies growth is first increasing and then decreasing in the rate of time preference ρ, and maximized if

$$[\alpha(1-\alpha)A]\,\theta = (\theta\rho)^{\alpha} \quad \Leftrightarrow \quad \rho^{*} = \theta^{\frac{1}{\alpha}} \left(\frac{\hat{\tau}}{\theta}\right).$$

Thus, for given policy and when $\rho < \rho^{*}$, growth could be higher if the agents discounted future utility more. In that situation an increase in impatience would raise growth. Furthermore, given that γ is concave in ρ, it is possible that two economies have identical technologies, pursue the same policies and exhibit the same growth performance, although

the agents in one economy are more impatient than in another economy. Notice that a higher θ requires a higher ρ^*.

Proposition 4 *Assume that the tax policies are given. Under the wealth tax scheme growth, $\gamma(\tau)$, is maximized if ρ is very small. Under the capital income cum investment subsidy tax scheme growth, $\gamma(\theta)$, is concave in ρ and maximized if the rate of time preference equals ρ^*.*

The model's tax arrangement is equivalent to a tax on the capital owners' consumption which depends positively on their rate of time preference. Thus, if the capital owners are more impatient, they will choose higher consumption (per units of capital) and that raises the tax revenues available to the government. The revenues in turn may be used to provide productive services, thereby raising the return on capital and growth .

Note that $\gamma(\hat{\tau}) = \frac{\alpha}{1-\alpha}\ \hat{\tau} - \rho$ and $\hat{\tau} = [\alpha(1-\alpha)A]^{\frac{1}{\alpha}}$. Thus, for $\gamma(\theta) = \gamma(\hat{\tau})$

$$\alpha A\left[\theta\rho\right]^{1-\alpha} = \frac{\alpha\ \hat{\tau}}{1-\alpha} \quad \text{, that is} \quad \theta = \left[\frac{\hat{\tau}}{(1-\alpha)A}\right]^{\frac{1}{1-\alpha}}\ \frac{1}{\rho}.$$

Lemma 1 *If $\theta = \left[\frac{\hat{\tau}}{(1-\alpha)A}\right]^{\frac{1}{1-\alpha}}\ \frac{1}{\rho}$, then $\gamma(\theta) = \gamma(\hat{\tau})$.*

Thus, there is an interesting trade-off between the tax rate and the time preference rate for given growth. The more impatient the capital owners are, the lower the taxes have to be for maintaining a given growth rate. Again that is due to the fact that more impatient capital owners consume more, but also generate higher tax revenues used for productive services.

2.1 Public Policy Analysis

The government cares about the workers or the capital owners. Respecting the right of private property, it chooses taxes to maximize the welfare

function

$$W = (1-\beta)\ V^r(C_t^k) + \beta\ V^l(C_t^W) \tag{14}$$

where V^r, V^l are the intertemporal utility indices of the capital owners and workers, respectively. The parameter $\beta \in [0,1]$ represents the welfare weight attached to the two groups. The constancy of β is justified by interpreting it as reflecting the political and socio-economic institutions in the economy. Then the fact that governments alternate in office is less of an issue since institutional features are usually constant for long periods of time. If $\beta = 1(0)$, the government is 'left-wing' ('right-wing') and cares about the workers (capital owners) only.

Alesina and Rodrik show that the optimal policies maximizing (14) under WT are constant and characterized by the following:

Lemma 2 (Alesina and Rodrik) *Under the wealth tax scheme the optimal policies which maximize $W(\tau)$ are such that*

1. *the growth rate, $\gamma(\tau)$, is inversely related to the social weight attached to welfare of the workers, β.*

2. *the policy, which is optimal for the capital owners, i.e. when $\beta = 0$, is given by $\tau = \hat{\tau}$, maximizes growth and depends on technological parameters only.*

Thus, under WT a government placing more weight on the welfare of the non-accumulated factor of production chooses a higher than the growth maximizing ('right-wing') tax rate.

For CICIST it is now shown that a government maximizing $W(\theta_t)$ chooses a constant policy. Thus, let the government solve

$$\max_{\theta_t} \int_0^\infty \left((1-\beta)\ln C_t^k + \beta \ln C_t^W\right) e^{-\rho t} dt \tag{15}$$

$$s.t. \qquad C_t^k = (1-\theta_t)\rho K_t \tag{16}$$

$$C_t^W = \eta(\theta_t) K_t \tag{17}$$

$$\dot{K}_t = \gamma_t K_t, \tag{18}$$

plus the private sector optimality and the equilibrium conditions, which feature in γ_t, r_t and η_t. The last equation implies $K_t = K_0 e^{\int_0^t \gamma_s ds}$. Substitution of the constraints into the objective function (15) implies

$$\int_0^\infty \left((1-\beta) \ln \left((1-\theta_t) \rho \, K_0 e^{\int_0^t \gamma_s ds} \right) + \beta \ln \left(\eta(\theta_t) \, K_0 e^{\int_0^t \gamma_s ds} \right) \right) e^{-\rho t} dt.$$

Simplification and collecting terms yields

$$\int_0^\infty \left((1-\beta) \ln \left((1-\theta_t) \rho K_0 \right) + \int_0^t \gamma_s ds + \beta \ln \left(\eta(\theta_t) \, K_0 \right) \right) e^{-\rho t} dt.$$

Integrating by parts establishes

$$\int_0^\infty \left(\int_0^t \gamma_s ds \right) e^{-\rho t} dt = \lim_{t \to \infty} \left[-\frac{\int_0^t \gamma_s ds}{\rho e^{\rho t}} \right]_0^t + \frac{1}{\rho} \int_0^\infty \gamma_t e^{-\rho t} dt.$$

For the limit expression notice that

$$\lim_{t \to \infty} \left[-\frac{\int_0^t \gamma_s ds}{\rho e^{\rho t}} \right]_0^t = \lim_{t \to \infty} \left[-\frac{\int_0^t \gamma_s ds}{\rho e^{\rho t}} + \frac{\int_0^0 \gamma_s ds}{\rho e^{\rho 0}} \right] = \lim_{t \to \infty} \left[-\frac{\int_0^t \gamma_s ds}{\rho e^{\rho t}} \right]$$

and $\int_0^t \gamma(\theta(s)) ds \leq \int_0^t \gamma(\theta_s = 1) ds = t\gamma(1)$. Under the assumption that $\theta(t)$ is continuous in t and by the concavity of $\gamma(\theta)$ the limit expression exists. (Thus, there are no jumps or points of discontinuity.) Furthermore, γ_s is bounded as $\gamma_s \in [\gamma(\theta = 0), \gamma(\theta = 1)]$ and γ is increasing in θ. By l'Hôpital's rule

$$\lim_{t \to \infty} \left[-\frac{\int_0^t \gamma_s ds}{\rho e^{\rho t}} \right] = \lim_{t \to \infty} \left[-\frac{\gamma_t}{\rho^2 e^{\rho t}} \right] = 0.$$

Hence, $\int_0^\infty \left(\int_0^t \gamma_s ds \right) e^{-\rho t} dt = \frac{1}{\rho} \int_0^\infty \gamma_t e^{-\rho t} dt$. But then the government's objective function reduces to

$$W(\theta_t) = \int_0^\infty \left((1-\beta) \ln \left((1-\theta_t) \rho \right) + \frac{\gamma(\theta_t)}{\rho} + \beta \ln \left(\eta(\theta_t) \right) + \ln K_0 \right) e^{-\rho t} dt.$$

The integral has the structure $\int_0^\infty F(\theta_t, \dot{\theta}_t, \theta)dt$ and for its maximization the necessary Euler equation $F_{\theta_t} - \frac{dF_{\dot{\theta}_t}}{dt} = 0$ is given by $-\frac{1-\beta}{1-\theta_t} + \frac{1}{\rho}\frac{d\gamma}{dr_t}\frac{dr_t}{d\theta_t} + \beta\frac{d\eta_t}{d\theta_t}\frac{1}{\eta_t} = 0$. Evaluating the derivatives yields

$$-\frac{(1-\beta)}{1-\theta_t} + \alpha(1-\alpha)A[\theta_t\rho]^{-\alpha} + \beta\left(\frac{1-\alpha}{\theta_t}\right) = 0. \tag{19}$$

But the θ_t solving this equation depends on constants only so that a time-invariant policy is optimal. Clearly, $\theta_t = 0$ does not solve the equation. Thus, the economy is characterized by steady state, balanced growth at the rate γ.

Lemma 3 *The government's optimal policy is time-invariant so that the economy exhibits steady state, balanced growth. Zero taxation of capital income is never optimal under the capital income cum investment subsidy tax scheme.*

For constant policies one verifies that

$$W(\theta) = (1-\beta)\,\frac{\ln[(1-\theta)\rho K_0]}{\rho} + \beta\,\frac{\ln[\eta\, K_0]}{\rho} + \frac{\gamma}{\rho^2}. \tag{20}$$

Notice that the optimal tax rate $\tilde{\theta} = f(\alpha, A, \rho, \beta)$ is *unique*, because $W(\theta)$ is concave as $\gamma_{\theta\theta} < 0$ so that $W(\theta)$ is a sum of concave functions.[15]

From equation (19) the $\tilde{\theta}$ chosen by a right-wing ($\beta = 0$) government satisfies

$$\theta = \frac{\hat{\tau}\ (1-\theta)^{\frac{1}{\alpha}}}{\rho} \tag{21}$$

where $\hat{\tau}$ is the growth maximizing wealth tax rate. Let $\tilde{\theta^r}$ denote the solution to the equation above. It is obvious that $\theta = 1$ ($\epsilon \to 0$) does not solve the equation and that $\tilde{\theta^r}$ is decreasing in the time preference rate.

[15] To see this notice that $W_{\theta\theta} = -\frac{1-\beta}{\rho(1-\theta)^2} - \frac{(1-\alpha)\beta}{\rho\theta^2} + \frac{\gamma_{\theta\theta}}{\rho^2} < 0, \forall\ \theta \in [0,1]$.

Proposition 5 *A right-wing ($\beta = 0$) government does not maximize growth and its optimal tax rate is determined by* $\tilde{\theta}^r = \dfrac{\hat{\tau}\ (1-\tilde{\theta}^r)^{\frac{1}{\alpha}}}{\rho} < 1$ *and decreases in the rate of time preference, ρ.*

The intuition for $\tilde{\theta}^r < 1$ is not difficult to understand. On the one hand the right-wing government wants to set a high tax rate, as that is good for the capital owners' income level and growth, which positively affects the capitalists' utility. On the other hand higher taxes reduce their consumption, which negatively affects their utility. $\tilde{\theta}^r$ represents the optimal trade-off for this problem.

Whereas under the WT the optimal right-wing policy is independent of preferences, $\tilde{\theta}^r$ pays attention to technology *and* the intertemporal preferences of the capital owners. Surprisingly, it is optimal for patient capital owners to be taxed more heavily in the model. The reason is that patient capital owners consume too little, generating not enough tax revenues for productive government inputs in production. To compensate for that the right-wing government chooses higher taxes to obtain the growth rate which is optimal for the capital owners' welfare.

It is an interesting question whether the capital owners are better off under this paper's tax scheme or under WT. In appendix C it is shown that the capital owners' welfare under WT is given by $V^r(\tau) = \frac{\ln[\rho K_0]}{\rho} + \frac{\gamma(\tau)}{\rho^2}$. Under CICIST the highest welfare to be obtained by the capital owners is $V^r(\tilde{\theta}^r)$ and under WT it is $V^r(\hat{\tau})$. The model implies $V^r(\theta^r) < V^r(\hat{\tau})$ because $V^r(\theta) < V^r(\tau)$ holds if

$$\frac{\ln[(1-\theta)\rho K_0]}{\rho} - \frac{\ln[\rho K_0]}{\rho} + \frac{B}{\rho} < 0 \quad \Leftrightarrow \quad (1-\theta)e^B < 1$$

where $B \equiv \frac{\gamma(\theta)-\gamma(\tau)}{\rho}$. Notice that $e^B = (1+\frac{B}{1!}+\frac{B^2}{2!}+...)$. Then a sufficient condition for the last inequality to hold is $(1+B) < \frac{1}{1-\theta}$, that is, $B < \frac{\theta}{1-\theta}$. Thus, $V(\theta) < V(\tau)$ if

$$\gamma(\theta) - \gamma(\tau) < \frac{\theta\rho}{1-\theta} \quad \Leftrightarrow \quad \alpha A(\theta\rho)^{1-\alpha} - \alpha A(\tau)^{1-\alpha} + \tau < \frac{\theta\rho}{1-\theta}.$$

Evaluate the last inequality at $\hat{\tau}$ and $\tilde{\theta}^r$, which are two numbers, note that $\tilde{\theta}^r\rho = \hat{\tau}(1-\tilde{\theta}^r)^{\frac{1}{\alpha}}$ from equation (21) and substitute for $\tilde{\theta}^r\rho$ above to obtain

$$\alpha A\hat{\tau}^{1-\alpha}(1-\tilde{\theta}^r)^{\frac{1-\alpha}{\alpha}} - \alpha A\hat{\tau}^{1-\alpha} + \hat{\tau} < \frac{\hat{\tau}(1-\tilde{\theta}^r)^{\frac{1}{\alpha}}}{1-\tilde{\theta}^r}$$

Divide by $\hat{\tau}$, note $\hat{\tau}^{\alpha} = \alpha(1-\alpha)A$, and simplify to get

$$\frac{(1-\tilde{\theta}^r)^{\frac{1}{\alpha}-1}}{1-\alpha} - \frac{1}{1-\alpha} + 1 < (1-\tilde{\theta}^r)^{\frac{1}{\alpha}-1} \quad \Leftrightarrow \quad (1-\tilde{\theta}^r)^{\frac{1}{\alpha}} < 1-\tilde{\theta}^r$$

which is true so that indeed $V^r(\hat{\tau}) > V^r(\tilde{\theta}^r)$. Hence,

Proposition 6 *The capital owners' optimal policies under either tax scheme imply that they would prefer the wealth tax scheme to the capital income cum investment subsidy tax scheme.*

The result may not look very surprising if one recalls that CICIST works like a tax on the capital owners' consumption reducing their utility. However, the growth rate may be higher under the paper's capital income tax scheme. Thus, the result establishes that the capital owners value the direct effect on their consumption level higher than the intertemporal effect of having higher income and so higher consumption in the future.

The right-wing government represents the accumulated factor of production and acts growth maximizing under WT, but does not do so under CICIST. However, for a wide range of parameter values the optimal policy for the capital owners under this model's tax scheme generates higher growth. Thus, even though a right-wing government does not maximize growth under CICIST its policy may generate a higher growth rate than under WT. If $\gamma(\theta) > \gamma(\hat{\tau})$, then $\theta > \left[\frac{\hat{\tau}}{(1-\alpha)A}\right]^{\frac{1}{1-\alpha}} \frac{1}{\rho}$ by Lemma 1. Substitution of $\tilde{\theta}^r = \frac{\hat{\tau}(1-\tilde{\theta}^r)^{\frac{1}{\alpha}}}{\rho}$ yields

$$\hat{\tau}(1-\tilde{\theta}^r)^{\frac{1}{\alpha}} > \left[\frac{\hat{\tau}}{(1-\alpha)A}\right]^{\frac{1}{1-\alpha}} \quad \Leftrightarrow \quad (1-\tilde{\theta}^r)^{\frac{1}{\alpha}} > \hat{\tau}^{\frac{\alpha}{1-\alpha}}\left[\frac{1}{(1-\alpha)A}\right]^{\frac{1}{1-\alpha}},$$

that is, $(1-\tilde{\theta^r}) > \alpha^{\frac{\alpha}{1-\alpha}}$. Whether this inequality holds is not easily analyzed analytically, but the following table presents a numerical simulation showing that there exist parameter values for which $\gamma(\hat{\tau}) < \gamma(\tilde{\theta^r})$.

Numerical Simulation for $A = 1$							
	ρ	α	$\hat{\tau}$	$\tilde{\theta^r}$	$\gamma(\hat{\tau})$	$\gamma(\tilde{\theta^r})$	Δ
1.	0.01	0.25	0.001	0.086	-0.010	-0.009	+
2.	0.01	0.50	0.063	0.672	0.053	0.032	-
3.	0.01	0.75	0.107	0.851	0.312	0.218	-
4.	0.05	0.25	0.001	0.023	-0.050	-0.049	+
5.	0.05	0.50	0.063	0.420	0.013	0.023	+
6.	0.05	0.75	0.107	0.611	0.272	0.263	-
7.	0.10	0.25	0.001	0.012	-0.100	-0.099	+
8.	0.10	0.50	0.063	0.303	-0.038	- 0.013	+
9.	0.10	0.75	0.107	0.466	0.222	0.248	+

where $\Delta = sgn\left(\gamma(\tilde{\theta^r}) - \gamma(\hat{\tau})\right)$.[16] From the table and for given α an increase in ρ causes the right-wing policy to generate higher growth under CICIST than under WT. A similar conclusion can be reached for given ρ and increases in α.

Proposition 7 *$\exists\ \alpha, A$ and ρ such that $\gamma(\hat{\tau}) < \gamma(\tilde{\theta^r})$ so that the preferred policy of the accumulated factor of production may generate higher growth under the capital income cum subsidy tax than under the wealth tax scheme.*

The proposition casts doubt on models that identify growth maximizing and optimal policies of the owners of the accumulated factor of

[16] How the simulation was carried out is explained in Appendix D.

production. In this model the owners of the accumulated factor of production prefer a wealth tax scheme (Proposition 6) and a government representing their interests acts growth maximizing under that scheme. But that choice is not growth maximizing in comparison to a tax scheme that the accumulated factor owners would not choose and under which their optimal policy is not growth maximizing, but may still generate higher growth than under the accumulated factor owners' preferred (wealth) tax scheme (Proposition 7). Hence, the model provides an example that the owners of the accumulated factor of production do not always choose a growth maximizing tax base.

Next, it is shown that an increase in β, that is, an increase in the weight attached to the welfare of the non-accumulated factor of production (workers) increases the optimal θ. If $\beta > 0$ then $\tilde{\theta}$ solves (19) so that $W(\tilde{\theta}(\beta), \beta)_\theta = 0$. Totally differentiate with respect to β to obtain[17]

$$W_{\theta\theta} \frac{\partial \tilde{\theta}}{\partial \beta} + W_{\theta\beta} = 0.$$

Concavity of $W(\theta)$ entails $W_{\theta\theta} < 0$. Notice that from (20)

$$W_{\theta\beta} = \frac{1}{(1-\theta)\rho} + \frac{1-\alpha}{\theta\rho} > 0$$

[17]For a similar proof in a different context see Mirrlees (1986).

which implies

$$\frac{\partial \tilde{\theta}}{\partial \beta} = -\frac{W(\tilde{\theta})_{\theta\beta}}{W(\tilde{\theta})_{\theta\theta}} > 0$$

so that any optimal $\tilde{\theta}$ is increasing in β. But an increase in β also raises the growth rate since $\gamma_\theta > 0$. Thus,

Proposition 8 *An increase in β raises $\tilde{\theta}$ and $\gamma(\tilde{\theta})$.*

This is an important result and in direct contrast to Lemma 2. If the social planner uses the CICIST arrangement, placing *more weight* (higher β) on the welfare of the *non-accumulated factor of production* (workers) *raises the optimal tax rate on the income of the accumulated factor of production* (capital) and through this *the growth rate*. Hence, under CICIST it is not optimal for high growth to shift all political power to the accumulated factor of production.[18]

From (20) one readily verifies $W_\theta(\beta = 1) > 0$ for a left-wing government.

Proposition 9 *A left-wing government sets $\theta^l = 1 - \epsilon$ and maximizes growth.*

Under WT a right-wing government acts like a growth maximizer in the optimum. In contrast, under CICIST a left-wing government acts

[18] A similar result is obtained in Bertola (1993), but notice that taxation of capital income *does not* defeat the purpose of enhancing growth in this model.

like a growth maximizer. Thus, a switch from WT to CICIST induces an important switch in optimal policies. In particular, it makes a right and left-wing government switch roles in terms of who maximizes growth.

3 Conclusion

Three points that are often made in the theoretical literature on growth and distributive policies. First, increasing taxes for redistributive purposes slows down growth. Second, the optimal policies of the accumulated factor of production can be identified with growth maximization. Third, capital income taxation defeats the purpose of enhancing growth when used as a means to finance investment subsidies.

This paper challenges all three points by showing that maximal taxation of the accumulated factor of production may be growth maximizing, the non-accumulated factor of production may act growth maximizing, and capital income taxes are not necessarily bad instruments for investment subsidy financing.

The model analyzes a capital income cum investment subsidy tax scheme which operates like a tax on the capital owners' consumption. The paper argues that the implementability of the tax scheme can be justified for 'right-wing' and 'left-wing' governments. It is shown that for optimizing agents the investment subsidies remove the distorting effect of capital income taxation. In equilibrium growth depends only and

positively on the pre-tax return to capital. Impatience is not necessarily bad for growth in the model and it is growth maximizing to tax capital income maximally. The reason is that the tax scheme operates like a consumption tax. More impatience causes the capital owners to consume more, raising the government's tax revenues that are channelled into production as public inputs, thereby raising the return to capital and growth.

In a public policy analysis the optimal policies under the model's tax scheme are compared with those generated under a wealth tax scheme. The paper implies that a 'right-wing' government does not maximize growth under the model's tax scheme, although it does so under the wealth tax scheme which it prefers. But the capital owners' optimal ('right-wing') policy under the model's tax scheme may generate higher growth than their optimal, growth maximizing policy under the wealth tax scheme. Thus, the paper shows that the preferred policy of the accumulated factor of production is not always good for growth.

Furthermore, it is shown that placing *more weight* on the welfare of the *non-accumulated factor of production* (workers) leads the social planner to *raise* the optimal *tax rate* on the income of the accumulated factor of production (capital) and through this the *growth rate*. Hence, under this paper's tax arrangement it is not optimal for high growth to shift all political power to the accumulated factor of production.

In fact, a 'left-wing' government acts like a growth maximizer in

the model. Thus, a switch in tax schemes may induce an important switch in optimal policies. The results imply that due to differences in tax arrangements one may observe an economy with a government that represents only the interests of the non-accumulated factor of production (labour) to have higher growth than an economy represented by a government solely concerned about the accumulated factor of production (capital).

Several caveats apply. The set-up of the model has been highly aggregated. In reality workers own capital and capital owners supply labour. It would be desirable to know more about how exactly the government achieves targeting personal investment. These and other problems are left for further research.

A The capital owners' optimum

By equation (8) the shadow price evolves according to $\mu_t = \mu_0\, e^{-\int_0^t (r_s-\rho)ds}$ where μ_0 is a positive constant which equals $\frac{1-\theta_0}{C_0}$. Then the transversality condition (9) boils down to

$$\mu_0 \lim_{t\to\infty} e^{-\int_0^t (r_s-\rho)ds} K_t\, e^{-\rho t} = \mu_0 \lim_{t\to\infty} K_t\, e^{-\int_0^t r_s\, ds} = 0.$$

Let $D_t \equiv \frac{C_t^k}{1-\theta_t}$. Equations (7) and (8) imply that $\gamma_D \equiv \frac{\dot{D}_t}{D_t} = r_t - \rho$. Hence, actual consumption C_t^k grows at

$$\frac{\dot{C}_t^k}{C_t^k} = \frac{\dot{D}_t}{D_t} - \frac{\dot{\theta}_t}{1-\theta_t}. \tag{A1}$$

At any date D_t is described by $D_t = D_0\, e^{\int_0^t (r_s-\rho)ds}$ where D_0 remains to be determined. Substituting for D_t in (5) implies

$$\dot{K}_t = r_t K_t - D_0\, e^{\int_0^t (r_s-\rho)ds}$$

which is a first order, linear differential equation in K_t. It is solved as follows

$$\begin{aligned}
\dot{K}_t - r_t K_t &= -D_0\, e^{\int_0^t (r_s-\rho)ds} \\
e^{-\int_0^t r_s\, ds}\left(\dot{K}_t - r_t K_t\right) &= -e^{-\int_0^t r_s\, ds}\, D_0\, e^{\int_0^t (r_s-\rho)ds} \\
\int e^{-\int_0^t r_s\, ds}\left(\dot{K}_t - r_t K_t\right) dt &= -\int D_0\, e^{-\rho t} dt.
\end{aligned}$$

The last equation is an exact differential equation with integrating factor $e^{-\int_0^t r_s\,ds}$. The LHS is solved by $K_t\ e^{-\int_0^t r_s\,ds} + b_0$ and the RHS is solved by $\frac{D_0}{\rho}\ e^{-\rho t} + b_1$, where b_0, b_1 are arbitrary constants. Thus,

$$K_t = \frac{D_0}{\rho}\ e^{\int_0^t (r_s-\rho)\,ds} + b\ e^{\int_0^t r_s\,ds} \tag{A2}$$

where $b = b_1 - b_0$. Substituting this into the transversality condition implies

$$\frac{1}{D_0}\lim_{t\to\infty}\left(\frac{D_0}{\rho}\ e^{\int_0^t (r_s-\rho)\,ds} + b\ e^{\int_0^t r_s\,ds}\right)\ e^{-\int_0^t r_s\,ds} = \lim_{t\to\infty}\left(\frac{1}{\rho}\ e^{-\rho t} + \frac{b}{D_0}\right) = 0$$

which holds if the arbitrary constant b equals zero. Then equation (A2) becomes

$$K_t = \frac{D_0}{\rho}\ e^{\int_0^t (r_s-\rho)\,ds} \quad \Rightarrow \quad \gamma \equiv \frac{\dot{K}_t}{K_t} = \gamma_D = r_t - \rho$$

so that D_t and wealth K_t grow at the same rate in the optimum. Furthermore, in the optimum instantaneous consumption is determined by the rule $D_t = \rho K_t$ so that $C_t^k = (1-\theta_t)\rho K_t$. If θ_t is constant, then $\gamma_{C_t^k} \equiv \frac{\dot{C}_t^k}{C_t^k} = \frac{\dot{D}_t}{D_t} = \frac{\dot{K}_t}{K_t}$ so that consumption would grow at the same rate as wealth.

B Growth maximizing policies

Recall $\gamma \equiv \frac{\dot{K}_t}{K_t} = r_t - \rho$. For all other variables let γ_i denote the growth rate of variable i. Then one verifies that

$$\gamma_G = \gamma + \frac{\dot{\theta}_t}{\theta_t}, \quad \gamma_Y = \gamma_{rK} = \gamma_{\eta K} = \gamma_{C^W} = \gamma + \frac{(1-\alpha)\dot{\theta}_t}{\theta_t}, \quad \gamma_{C^k} = \gamma - \frac{\dot{\theta}_t}{1-\theta_t}.$$

Notice that the workers' consumption grows at the same rate as wage, capital or total income. All these growth rates are in general time dependent and have the structure $\gamma_i(\theta_t, \dot{\theta}_t, t)$. A government that wants to maximize the growth of these aggregates solves[19]

$$\max_{\theta_t} \gamma_i(\theta_t, \dot{\theta}_t, t) \quad s.t. \quad \theta_t \leq 1 - \epsilon \tag{B1}$$

Suppose $\theta_t < 1-\epsilon$ for all t. Then a necessary condition for a maximum is that the Euler equation $\frac{d\gamma_i}{d\theta_t} - d\left(\frac{d\gamma_i}{d\dot{\theta}}\right)/dt = 0$ holds. Notice that $\frac{d\gamma}{d\theta_t} > 0$. For $\gamma_Y, \gamma_G, \gamma_{C^W}$ the derivative expression $\frac{d\gamma_i}{d\theta_t} - d\left(\frac{d\gamma_i}{d\dot{\theta}}\right)/dt$ is given by

$$\left(\frac{d\gamma}{d\theta_t} - \frac{c_i\ \dot{\theta}_t}{\theta_t^2}\right) - \left(-\frac{c_i\ \dot{\theta}_t}{\theta_t^2}\right) = \frac{d\gamma}{d\theta_t}$$

where c_i is a positive constant. The derivative is positive so that the Euler equation is *not* satisfied. Furthermore, it does *not* hold for γ_{C^k}

[19] For a discussion of inequality constraints in calculus of variation problems see e.g. Kamien and Schwartz (1991), section 14, or Chiang (1992), chpt. 6.

because

$$\left(\frac{d\gamma}{d\theta_t} - \frac{\dot{\theta}_t}{(1-\theta_t)^2}\right) - \left(-\frac{\dot{\theta}_t}{(1-\theta_t)^2}\right) = \frac{d\gamma}{d\theta_t}$$

is also positive. But as $\frac{d\gamma}{d\theta_t} > 0$, growth at any point in time is highest when $\theta_t = 1 - \epsilon$. Hence, at each date growth of $K_t, Y_t, C_t^k, C_t^W, \eta_t K_t, r_t K_t$ or G_t is maximized by the time invariant policy $\theta_t = 1 - \epsilon$ for all t.

C Welfare measures

In the steady state, balanced growth equilibrium $\gamma(\theta)$ and $\gamma(\tau)$ are constant. The workers' and capital owners' intertemporal welfare is given by $\int_0^t \ln C_t^j \; e^{-\rho t}$ where $j = k, W$. Let $t \to \infty$ and use integration by parts. For this define $v_2 = \ln C_t^j$, $dv_1 = e^{-\rho t} dt$. Recall that $C_t^k = (1-\theta)\rho K_t$ and $C_t^W = \eta K_t$ under CICIST. Then $dv_2 = \frac{\dot{C}^j}{C^j} = \gamma(\theta)$ and constant in steady state, and $v_1 = -\frac{1}{\rho} \; e^{-\rho t}$ so that

$$\begin{aligned}
\int_0^\infty \ln C_t^j \; e^{-\rho t} \; dt &= \frac{1}{\rho}\left[-\; \ln C_t^j \; e^{-\rho t}\right]_0^\infty + \frac{1}{\rho}\int_0^\infty \gamma \; e^{-\rho t} \; dt \\
&= \frac{\ln C_0^j}{\rho} - \frac{1}{\rho^2}\left[\gamma \; e^{-\rho t}\right]_0^\infty .
\end{aligned}$$

Evaluation at the particular limits and substitution for C_0^j yields the expressions of V^r, V^l in (20). Under WT the capital owners' budget constraint is $C^k = (r - \tau)K_t - \dot{K}_t$ so that instantaneous consumption

in steady state is given by $C^k = \rho K_t$. Proceeding as above with $\gamma(\tau)$ instead of $\gamma(\theta)$ gives the expression for $V^r(\tau)$ in the text.

D Numerical Simulation Procedure

I have defined the following variables in Mathematica

```
t   := (a*(1 - a)*A)^a^(-1)
gt  := -rho - t + a*A*t^(1 - a)
gth := -rho + a*A*(rho*th)^(1 - a)
c   := FindRoot[ths*rho - t*(1 - ths)^(1/a) == 0, {ths, 0}]
tst :=(1 - th) - a^(a/(1 - a))
```

where $th = \theta$, $t = \hat{\tau}$, $gt = \gamma(\hat{\tau})$ and $gth = \gamma(\theta)$. Setting $A = 1$ and for given values of α and ρ I have calculated $ths = \tilde{\theta}^r$, set $ths = th$ and calculated gt and gth, recording the values in the table. I have checked, but not recorded, the results with calculating tst.

E Iso-elastic utility

Suppose the capital owners have the instantaneous utility function

$$U(C_t^k) = \frac{C_t^{1-\nu} - 1}{1 - \nu} \quad , \quad \nu > 1 \tag{E1}$$

where the constant $\sigma = \frac{1}{\nu}$ represents the elasticity of intertemporal substitution. If $\nu \to 1$, $U(\cdot)$ reduces to logarithmic utility. A high ν implies a low elasticity intertemporal substitution, low σ. This means that the capital owners like to smooth consumption. In contrast, a high elasticity of substitution implies that the investors are indifferent to the timing of consumption. In that case the agents may defer consumption for a long time while investing in order to consume a large amount at a future date. By restricting $\nu > 1$ such behaviour is ruled out, implying $\sigma \in (0, 1)$.[20] Notice that a high time preference rate ρ implies that the investors *value* future consumption less than current consumption.

Assume tax policy is constant and the capital owners solve a problem similar to the one in the text under the dynamic budget constraint $C_t^k = (1-\theta)\left[rK_t - \dot{K}_t\right]$. It is not difficult to verify (see also e.g. Barro and Sala–i–Martin (1995), chpt. 2.1.2) that the steady state, balanced growth rate in a market equilibrium with arbitrary and constant tax rates is given by

$$\gamma = \frac{r-\rho}{\nu} = \sigma(r-\rho). \tag{E2}$$

[20] Notice that steady growth and a constant interest rate are consistent with many (ν, ρ) pairs of (unobservable) preference parameters. However, Hall (1988) infers from aggregate variability in the growth rates of consumption and interest rates that σ is much lower than unity in reality. For a similar argument see Bertola (1996), ftn. 6. Thus, for the argument the paper wants to make it may suffice to show that the model generalizes to all functions with $\nu > 1$.

Then the optimal level of consumption is determined by

$$C_t^k = (1-\theta)[r-\gamma]K_t = (1-\theta)\left[(1-\sigma)r + \sigma\rho\right]K_t$$

where $K_t = K_0 e^{\gamma t}$. Thus, σ and ρ have an effect on both the level and growth of the capital owners' consumption. For *given* θ and r an increase in ρ (more impatience) or a decrease in σ (more consumption smoothing) lower the growth rate and raise the fraction $\frac{C_t^k}{K_t}$, that is, the capital owners' steady state consumption per units of capital. Thus, more impatience or consumption smoothing make the capitalists *less willing to save* for *given* taxes and *given* $\frac{G_t}{K_t}$ and r.

From the balanced budget condition (2) one gets

$$b \equiv \frac{G_t}{K_t} = \theta\,(r-\gamma) = \theta(r - \sigma(r-\rho)).$$

In equilibrium $r = \alpha A\,(b)^{1-\alpha}$ so that b is implicitly defined by

$$b = \theta\left[(1-\sigma)\alpha A b^{1-\alpha} + \sigma\rho\right] \quad \Leftrightarrow \quad b^{\alpha} = \theta\left[(1-\sigma)\alpha A + \sigma\rho\, b^{\alpha-1}\right].$$

As $(\alpha b^{\alpha-1})\dot{b} = (\theta\sigma\rho(1-\alpha)b^{\alpha-2})\dot{b}$ is in general only satisfied if $\dot{b} = 0$, the fraction $\frac{G_t}{K_t}$ must be constant. Define $x \equiv b^{\alpha} - \theta\left[(1-\sigma)\alpha A + \sigma\rho\, b^{\alpha-1}\right]$ then $x_b = \alpha b^{\alpha-1} - \theta\sigma\rho(\alpha-1)b^{\alpha-2} > 0$ for all $b \in (0,1)$. One also verifies that $x_\rho < 0$ so that $\frac{db}{d\rho} > 0$ and hence $\frac{dr}{d\rho} > 0$. Also, $x_\sigma = \theta\left(\alpha A - \rho b^{\alpha-1}\right) \gtreqless 0$ depending on ρ, α and A for given θ. Thus, the effect

of an increase in σ on b and r is generally ambiguous. However, for sufficiently large ρ it is positive.

Hence, for iso-elastic utility with preference for consumption smoothing, $\sigma \in (0, 1)$, higher ρ or an increase in σ when ρ is sufficiently large would increase steady state growth.

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