

Why Run a Million Regressions? Endogenous Policy and Cross-Country Growth Empirics

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Abstract

This paper analyzes the link between growth and public policy when the latter depends on economically important fundamentals. When policy is endogenous the measured effects of policy on growth will generally be biased. Using a widely quoted theoretical model, the *signs* of the biases are derived. It is shown that the usually reported effects on growth of tax rate variables related to GDP, the ratio of public investment to total investment and the ratio of redistributive transfers to GDP are generally biased downwards. Based on these signed biases the paper provides a theory-based "lens" through which some existing empirical results can be discussed that seem puzzling from a theoretical viewpoint. Based on OECD data, empirical evidence appears to support the theoretical predictions.

KEYWORDS: Growth, Endogenous Public Policy, Cross-Sectional Models

JEL classification: O4, D3, C2

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1 Introduction

This paper analyzes some empirical findings that appear to be at odds with theoretical predictions as regards the effects of policy on growth. In this context, almost all researchers acknowledge that their empirical work may be riddled by endogeneity problems. This paper concentrates on exactly these problems. More precisely, it attributes the discrepancy of results to the fact that policy is economically endogenous and that treating it as exogenous provides one with a misleading picture of the relationship between fiscal policy and growth.

For instance, many authors have investigated the effects of taxation on long-run growth. Although employing similar (theoretical) frameworks, their conclusions differ widely. See, for example, King and Rebelo (1990), Lucas (1990), Rebelo (1991), Jones, Manuelli, and Rossi (1993), Pecorino (1993), Pecorino (1994), or Stokey and Rebelo (1995).

The impact of productive government spending has, for instance, been analyzed by Aschauer (1989), Barro (1990) or Barro and Sala-i-Martin (1992). According to their theoretical results productive government spending should play a significant role in influencing the private return on capital and through that the long-run growth rate.

The link between (re-)distribution and growth has e.g. been analyzed by Bertola (1993), Alesina and Rodrik (1994), Persson and Tabellini (1994), or Perotti (1996). These studies often find that theoretically redistribution of resources from the accumulated towards the non-accumulated factor of production should be expected to affect growth negatively.

To test these theoretical predictions a large number of contributions has proceeded by taking averages of their data over time and run simple cross-country OLS regressions over these averages.¹ For that reason the paper concentrates on simple statistics based on time averages of variables, bearing in mind that simple statistics may also be relevant for more sophisticated methods.

For instance, Koester and Kormendi (1989), Barro (1991), Barro (1997),

¹In the paper 'simple cross-country OLS regression' is meant to reflect that procedure of handling the time series dimension of the data. Of course, 'simple' does not mean simplistic, since the availability of data may not allow for another or a 'better' method of analysis. More recently some authors have advocated the use of *dynamic* panel data methods to pay explicit attention to the time series dimension. See e.g. Caselli, Esquivel, and Lefort (1996). But the latter methods seem to have their own problems as e.g. argued by Barro (1997), p. 37, Temple (1999), p. 132, and e.g. analyzed by Banerjee, Marcellino, and Osbat (2000).

Levine and Renelt (1992), Easterly and Rebelo (1993a), or Sala-i-Martin (1997) have analyzed the effects of fiscal policy on growth. Most of them find that tax rates or other, tax financed fiscal variables have a negative, but - when controlling for initial income - insignificant effect on growth. That would cast doubt on Barro (1990) or Barro and Sala-i-Martin (1992) who show that some of these variables should have a significant and (often) positive effect.

In this paper it is assumed that public policy takes account of fundamental economic variables that may or may not be included in growth regressions. The assumption of endogenous policy implies that the OLS estimates of the effect of public policy variables on growth are generally *biased*, implying that correct statistical inferences would not really be possible.²

Based on a simplified version of the widely quoted model of Alesina and Rodrik (1994), theoretical correlations between public policy and fundamental economic variables are derived to *sign* the biases. That, in turn, allows for interpretations of some empirical findings on the link between policy and growth that seem puzzling from a theoretical viewpoint.

The theory would predict that the estimates for the effects on growth of tax rate variables related to the tax base are generally biased upwards and so overestimated. Thus, any reported negative effect of taxes on growth is understated, if measured by these variables.³

However, in the macroeconomics literature tax rates are usually measured by variables related to GDP.⁴ It is shown that the estimates of the effect on growth of tax rate variables related to GDP are generally biased downwards. Thus, any reported negative (positive) effect of tax rates on growth would be overstated (understated), if measured by those variables. The systematic underestimation implies that the theoretical prediction that high tax rates negatively affect growth may be inherently untestable. Hence, it matters what variables one uses for tax

²Most of the contributions mentioned in this paper do not rigorously focus on the bias problem when policy is endogenous. Interesting econometric analyses that may provide solutions to a bias problem are e.g. provided by Olley and Pakes (1996) and Levinsohn and Petrin (2003). However, these authors focus on the link between unobservables and regressors when estimating production functions, not on the link between policy and growth.

³Effect is not meant to be causal. In this paper effect means that some underlying economic fundamental influences policy which in turn bears on growth. Then the 'true' effect of policy on growth is spurious really, but can be picked up by simple linear operationalizations of the model.

⁴Common variables are 'the ratio of tax revenues to GDP' for average tax rates, or 'the ratio of the change in tax revenues to the change in GDP' for marginal tax rates.

rates in growth regressions.

If public policy is assumed to be exogenous, the estimated coefficients of the effect on growth of the ratio of productive public expenditure to total investment or to GDP are biased downwards under all policies considered. If public policy is assumed to be endogenous, the point estimates of the effect on growth of the ratio of public investment to total investment are biased downwards.

That has interesting implications for any hypothesis that claims that the breakdown of total investment between public and private investment does not materially affect growth. It also casts doubt on the hypothesis that a typical country comes close to the quantity of public investment that maximizes growth.

Under the assumption of endogenous policy, the estimated coefficients of the effect on growth of the ratio of redistributive transfers to GDP are generically biased downwards. That renders the hypothesis that redistribution is bad for growth untestable for the following reason: The prediction of the theoretical model is that redistributive transfers are bad for growth. However, in the model an increase in efficiency makes an optimizing, redistributing government grant less transfers to the non-accumulated factor of production. This last effect is ignored in cross-country growth regressions when one assumes that public policy is exogenous.

For theoretical reasons many researchers expect a negative coefficient for the effect of redistributive transfers on growth. However, many people find positive coefficients (for example, for the effect of social security contributions on growth, see Sala-i-Martin (1996)). As any downward bias of the estimated coefficients may be as large as minus infinity, a reported negative coefficient cannot corroborate the hypothesis that redistribution is bad for growth. On the other hand, any downward bias is perfectly consistent with many empirical findings and the alternative hypothesis that redistribution is not bad for growth.

Next, empirical evidence for a sample of OECD countries is presented. I find tentative support for the theoretical predictions for a selection of policy variables analyzed in the paper. In particular, I find that the biases in simple growth regressions due to the omission of an indicator of aggregate efficiency do have the directions predicted by the theory.

The main insight of the paper's analysis is that the disentanglement of the interplay of economic fundamentals and policy on the one hand and policy and

growth on the other is useful and may provide one with new interpretations of existing results.

The paper is organized as follows: Section 2 presents the theoretical model and analyzes endogenous policy. The next section uses theoretical correlations derived from the model to analyze the effect of tax, public investment and redistributive transfer variables on growth. Section 5 provides concluding remarks.

2 Theory

Consider a private ownership economy that is populated by two types of price-taking, infinitely lived individuals who are all equally patient. One group of agents, the capitalists (k), owns wealth equally and does not work. The other group of agents, the workers (W), owns (raw) labour equally, but no capital.⁵ Population is stationary and each group of agents derives logarithmic utility from the consumption of a homogeneous, malleable good. Aggregate output is produced according to

$$Y_t = A K_t^\alpha G_t^{1-\alpha} L_t^{1-\alpha} \quad , \quad 0 < \alpha < 1 \quad (1)$$

where Y_t denotes aggregate output, K_t is the real capital stock, L_t is labour supplied, and G_t are public inputs to production. See, for instance, Barro (1990).⁶ Capital is broadly defined and includes human capital.⁷ The index A reflects the economy's state of technology. It depends on cultural, institutional and technological development and captures long-run exogenous factors that play a role in the production process. Labour is inelastically supplied and normalized so that the total labour endowment equals unity, $L_t = 1$. The model abstracts from the depreciation of capital so that output and factor returns are really defined in net terms.

The government taxes wealth at the constant rate τ , redistributes a constant

⁵The assumption may be justified by various arguments, especially for the long run. See e.g. Kaldor (1956), Pasinetti (1962), Schlicht (1975), Bourguignon (1981), or Bertola (1993).

⁶I also follow Alesina and Rodrik (1994) and Barro and Sala-i-Martin (1995), ch. 4, who use this representation of a production technology. Of course, the results are sensitive to the choice of this function. As it has often been used, however, I follow work based on this setup.

⁷This eliminates a separate treatment of how human capital is accumulated. By assumption the economies are perfectly competitive and the return on human capital services equals that of physical capital services.

share λ of its tax revenues to the workers and runs a balanced budget:

$$\tau K_t = G_t + \lambda \tau K_t.$$

The LHS depicts the tax revenues and the RHS public expenditures. The workers receive $\lambda \tau K_t$ as transfers and G_t is spent on public inputs to production. Thus, we follow Alesina and Rodrik (1994) and consider a wealth tax scheme that represents a broad class of distributive tax arrangements, which distort the investors' incentive to accumulate.⁸

There are many identical, profit-maximizing firms which operate in a perfectly competitive environment. They are owned by the capital owners who rent capital to and demand shares of the firms. The shares are collateralized one-to-one by capital. The markets for assets and capital are assumed to clear at each point in time. The firms take G_t as given and rent capital and labour in spot markets in each period. The price of output y_t serves as numéraire and is set equal to one. Profit maximization entails that firms pay each factor of production its marginal product,

$$r = \alpha A[(1 - \lambda)\tau]^{1-\alpha} \quad (2)$$

$$w_t \equiv \eta(\tau, \lambda)K_t = (1 - \alpha)A[(1 - \lambda)\tau]^{1-\alpha} K_t. \quad (3)$$

Because of the productive role of government services policy has a bearing on the marginal products. The return on capital is constant over time while the wages grow with the capital stock. Notice that more redistribution lowers r and η , while higher taxes raise them.

The workers derive utility from consuming their entire income. They do not invest and are not taxed. Their intertemporal welfare is given by

$$\int_0^\infty \ln C_t^W e^{-\rho t} dt \quad \text{where} \quad C_t^W = \eta(\tau, \lambda)K_t + \lambda \tau K_t. \quad (4)$$

⁸For arguments what other redistributive mechanisms this tax scheme metaphorically also captures see Alesina and Rodrik's paper. They also show that the optimal policies are constant over time and, thus, time-consistent. For convenience constancy of policy is assumed from the beginning in this paper. Notice that we contemplate a distortionary tax scheme. Nondistortionary consumption taxes may have different effects. But as many studies analyze the effects of distortionary taxes on accumulation we concentrate on these, as they are an important phenomenon in the real world.

The capitalists choose how much to consume or invest, and they have perfect foresight about the prices and tax rates, which they take as *given*. They maximize their intertemporal utility according to

$$\max_{C_t^k} \int_0^\infty \ln C_t^k e^{-\rho t} dt \quad (5)$$

$$s.t. \quad \dot{K}_t = (r - \tau)K_t - C_t^k \quad (6)$$

$$K(0) = \bar{K}_0, \quad K(\infty) = \text{free}, \quad (7)$$

where equation (6) is the capitalists' dynamic budget constraint which depends on their after-tax income $(r - \tau)K_t$. In Appendix A it is shown that their consumption and wealth optimally grow at

$$\gamma \equiv \frac{\dot{C}_t^k}{C_t^k} = \frac{\dot{K}_t}{K_t} = (r - \tau) - \rho \quad (8)$$

which is increasing in the after-tax return on capital and constant over time.

Equilibrium. In steady state the economy is characterized by *balanced growth* at the rate γ , which is first increasing and then decreasing in τ for given λ . Growth is maximized when $\tau = [\alpha(1 - \alpha)A]^\frac{1}{\alpha} \equiv \hat{\tau}$ and $\lambda = 0$. If taxes higher than $\hat{\tau}$ are levied, growth is traded off against redistribution when $\lambda > 0$ and $\tilde{\tau} > \hat{\tau}$.

Furthermore, $r - \tau = \tau(\alpha A[(1 - \lambda)\tau]^{-\alpha} - 1)$ so that for *given* policy an increase in efficiency A or in the share of capital α raises growth.

Policy. The intertemporal welfare of an entirely pro-capital, V^r , resp. entirely pro-labour government, V^l , is given by

$$V^r(C_t^k) = \frac{\ln(\rho K_0)}{\rho} + \frac{\gamma}{\rho^2} \quad \text{and} \quad V^l(C_t^W) = \frac{\ln[(\eta(\tau, \lambda) + \lambda\tau)K_0]}{\rho} + \frac{\gamma}{\rho^2}. \quad (9)$$

See Appendix B. The governments respect the right of private property and maximize the welfare of their clientele under the condition $\lambda \geq 0$.

The optimal pro-labour policy is derived in Appendix C and is given by

If $\rho \geq [(1 - \alpha)A]^\frac{1}{\alpha}$ then:

$$\tau = \rho, \quad \lambda = 1 - \frac{[(1 - \alpha)A]^{\frac{1}{\alpha}}}{\rho}. \quad (10)$$

$$\text{If } \rho < [(1 - \alpha)A]^{\frac{1}{\alpha}} \text{ then:} \\ \tau[1 - \alpha(1 - \alpha)A\tau^{-\alpha}] = \rho(1 - \alpha), \quad \lambda = 0. \quad (11)$$

Denote the optimal pro-labour tax rate by $\check{\tau}$ and notice that $\check{\tau} > \hat{\tau}$ for $\rho > 0$.⁹ Furthermore, for a wide range of parameter values there is no direct redistribution under the optimal pro-labour policy. In particular, if the agents are sufficiently patient or the economy is very efficient, the owners of the non-accumulated factor of production (workers) prefer to have higher growth instead of direct redistribution. This is because high growth may be better for their income stream and so long-run welfare than direct (unproductive) transfers. In that way the model distinguishes between redistributing and non-redistributing (pro-labour) governments.

In contrast, the pro-capital government chooses $\tau = \hat{\tau}$, does not redistribute and acts growth maximizing in this model by granting the maximum after-tax return on capital to the capital owners.

All optimal policies depend on A , α , and ρ . Thus, policy is endogenous in the paper. The rate of time preference will not be considered any further because it is mostly considered a soft variable, which is very hard to measure. For the other variables I find the following

Table 1: Growth and Policy Effects

	PC		PL $_{\lambda=0}$		PL $_{\lambda \geq 0}$		
	$\hat{\tau}$	$\hat{\gamma}$	$\check{\tau}$	$\check{\gamma}$	$\check{\tau}$	λ	$\check{\gamma}$
A	+	+	+	+	0	-	+
α	?	+	?	+	0	?	+

PC - pro-capital, PL - pro-labour
Sign: (+) - positive, (-) - negative, (?) - ambiguous

See Appendix D. Thus, an increase in efficiency raises growth, does not imply

⁹If the expression in bracket in (11) were zero, then $\check{\tau} = \hat{\tau}$. This would hold if $\rho \rightarrow 0$. Thus, unless the workers are extremely patient, they will not choose the growth maximizing tax rate $\hat{\tau}$ in this model.

lower tax rates but calls for lower redistribution under all policies considered. This is due to the externality of public inputs. Higher growth requires more tax revenues for productive services channelled into production in order to raise the return on capital and so growth.

In turn, a higher share of capital also raises growth under all policies, but that effect is ambiguous as regards taxes and redistribution. This is interesting because it means that shifting relatively more factor income towards capital may call for higher or lower taxes in order to generate a higher after-tax income for capital and higher growth. For instance, Alesina and Rodrik (1994), fn. 7, cite theoretical support for the prediction that an increase in α raises redistributive pressure leading to higher taxes. Thus, higher taxes may even be optimal for a government that is only concerned about the welfare of the owners of the accumulated factor of production.

3 Growth Empirics

According to the theory the empirical, long-run relationship between growth and policy for a country i is of the form

$$\gamma_i = f(\tau_i(\alpha_i, A_i), \lambda_i(\alpha_i, A_i), \alpha_i, A_i) = f(\alpha_i, A_i) \quad (12)$$

where $f(\cdot)$ is a non-linear function of α_i and A_i which are assumed to be country-specific, that is, independent and thus uncorrelated across countries.¹⁰

An analysis of exogenous, once-and-for-all changes in A_i is similar in spirit to models with exogenous technological change, which is commonly thought to be unobservable.¹¹ Clearly, countries differ widely in the level of development, but reliable data and concepts capturing the essence of development are not readily available. Thus, A_i is taken to be unobservable, implying that information on A_i

¹⁰Under the assumption of exogenous policy $\gamma_i = g(\tau_i, \lambda_i, \alpha_i, A_i)$ where τ_i and λ_i are independent of the other variables included in $g(\cdot)$. Notice that $f(\cdot)$ and $g(\cdot)$ may be observationally equivalent when particular combinations of α_i and A_i lead to the same growth rate under either assumption. Thus, assume that empirical and theoretical researchers agree that the Data Generating Mechanism (DGP) is given by the joint probability distribution $D(\gamma, \tau, \lambda, \alpha, A)$, which is expressed in terms of steady state variables and, thus, ignores any time dependence. That reflects the procedure to take time-averages of data which are considered of interest.

¹¹The discussion about the *Solow-Residual* reflects these difficulties. See, for instance, Barro and Sala-i-Martin (1995), chpt. 10.4.

would be contained in the disturbance term and would not feature separately in the regressions. See e.g. Mankiw, Romer, and Weil (1992). Then the second-best, but operationally viable, linear model would be

$$\gamma_i = \beta_0 + \beta_1\alpha_i + \beta_2\tau_i(\alpha_i, A_i) + \beta_3\lambda_i(\alpha_i, A_i) + v_i \quad (13)$$

where $v_i = v_i(A_i, \epsilon_i)$ is a country-specific disturbance term which depends on A_i and ϵ_i . The latter is assumed to be uncorrelated with A_i as well as with each of the regressors, and $E(\epsilon_i) = 0$.

If that model were estimated by OLS, multicollinearity and the omission of a relevant variable would be a problem. As the error term correlates with the regressors or any transformation thereof depending on α or A , the conditions of *weak exogeneity* in the sense of Engle, Hendry, and Richard (1983) are violated.¹²

A standard justification for treating policy as exogenous is a randomization argument. For example, Barro (1989a) argues that in a large sample public policies may be treated as randomly generated. That comes close to saying that policies are exogenous. But in light of this paper's analysis the argument would not hold. Even if all countries had different governments with different welfare functions so that policies looked randomly chosen, the model predicts that *all* policies would be influenced by fundamental economic variables included or not included in the regressions. The paper concentrates on exactly that problem.

Taxes. Most studies investigating the relation between taxes and growth use

$$av_1 = \frac{TAR}{TAB}, \quad m_1 = \frac{dTAR}{dTAB}, \quad av_2 = \frac{TAR}{GDP}, \quad m_2 = \frac{dTAR}{dGDP}$$

as regressor variables where TAR denotes the total tax revenues in a country, and TAB denotes the tax base. Usually, av_1 and av_2 represent *average tax rates*, whereas m_1 and m_2 are often used as proxies for *marginal tax rates*.¹³

¹²Thus, reported t -statistics will not report the true significance levels and statistical inferences are not really possible. However, here the focus is on the problem caused by assuming that policy is exogenous.

¹³All these variables are problematic representations for aggregate relationships between fiscal policy and growth. For instance, the 'marginal tax rate' m_2 is commonly obtained by regressing tax revenues on GDP, where the resulting regression coefficient is interpreted as the 'marginal tax rate'. See, for instance, Koester and Kormendi (1989) or Easterly and Rebelo (1993b). That raises the question whether GDP really is the relevant tax base.

From the model τ is equal to both av_1 and m_1 , which are related to the *tax base*.¹⁴ Suppose $\gamma_i = \delta_0^1 + \delta_1^1 \tau_i + u_i^1$ is run and it is assumed that policy is exogenous. In that case the OLS estimator of δ_1^1 , call it d_1^1 , would be unbiased.¹⁵ However, assuming that policy is exogenous when in fact it is endogenous yields biased estimates, because $d_1^1 = \frac{\sum_i (\tau_i - \bar{\tau}) \gamma_i}{\sum_i (\tau_i - \bar{\tau})^2}$ and substituting in the 'true' model (13) yields $d_1^1 = \beta_2 + \beta_1 \sum_i c_i \alpha_i + \beta_3 \sum_i c_i \lambda_i + \sum_i c_i v_i$ where $c_i = \frac{(\tau_i - \bar{\tau})}{\sum_i (\tau_i - \bar{\tau})^2}$ and $v_i = v_i(A_i, \epsilon_i)$. Taking expectations under the assumption that policy is in fact endogenous one obtains¹⁶

$$E(d_1^1) = \beta_2 + \beta_1 E \left[\sum_i c_i \alpha_i \right] + \beta_3 E \left[\sum_i c_i \lambda_i \right] + E \left[\sum_i c_i v_i \right].$$

Hence, d_1^1 is generally biased because the sum of the expectations of the expressions on the RHS is non-zero.

The bias is due to three factors. First, the expression $\sum_i c_i v_i$ is positive for all non-redistributing governments since $cov(\tau_i, A_i) > 0$ from Table 1. Second, $cov(\tau_i, \lambda_i) \geq 0$, which is positive if governments redistribute. Third, if $cov(\tau_i, \alpha_i) \geq 0$, the bias is clearly positive.

Proposition 1 *Let av_1 or m_1 have the same properties as τ . Simple cross-country OLS regressions of the growth rate on τ , av_1 or m_1 assuming that policy is exogenous, when in fact it is endogenous, yields generically biased estimates. In many cases and definitely if $cov(\tau, \alpha) \geq 0$, the bias would be positive.*

Thus, the coefficients measuring the effect of these variables on growth are *overestimated* so that either any reported negative effect is understated or any reported positive effect is overstated. In the theoretical literature higher taxes are usually shown to affect growth negatively. Hence, if one found a negative effect for regressors like τ , av_1 or m_1 one could be 'sure' that the effect is indeed

¹⁴The equality appears problematic as one would often expect that marginal tax rates (m) are not lower and often higher than average tax rates (av). One should bear in mind, however, that for economies with flat-rate tax systems, which are often analyzed and sometimes advocated to be adopted in the theoretical literature, $av = m$ would certainly hold. Thus, a discussion of the effect of such tax variables on growth in simple OLS cross-country growth regressions may be worth for that and for theoretical reasons.

¹⁵The superscript 1 indicates that the regression is run on variables related to the tax base.

¹⁶Under exogenous policy the expectation would be conditional on $(\tau_i, \lambda_i, A_i, \alpha_i)$, since all researchers agree on the 'true' DGP by assumption. Under endogenous policy the expectation is only conditional on (A_i, α_i) , of course.

negative. Even a reported positive effect would not invalidate the prediction that the 'true' effect is really negative. Hence, Proposition 1 provides a negative result for hypothesis testing, but it offers an important 'positive' link to empirical research in that it provides an argument that any theoretically derived negative relationship between taxes and growth is 'true', *if* the regressors are τ , av_1 or m_1 and have the theoretical model's properties.

However, Proposition 1 rests on strong assumptions. The determination of a country's tax base most relevant for growth is extremely difficult. Usually governments raise all sorts of taxes on different tax bases. Furthermore, countries do not necessarily use the same tax bases and for some tax bases data are difficult to obtain. For that reason the vast majority of cross-country growth studies use av_2 or m_2 as regressors.

But the choice of regressors may affect the underlying structure of the estimated and the statistical model. Going from τ or av_2 to m_2 often entails a non-linear reparametrization of an original estimated model or a different model altogether.¹⁷ For instance, the choice of av_2 as a regressor variable means that $\gamma = f^2(\frac{\tau K_t}{Y_t}, \alpha_i, A_i, \rho_i) = f^2(\alpha_i, A_i, \rho_i)$ which is usually not the same as $f(\cdot)$ in (12). Instead of trying to find a reparametrization of the original model assume all policy variables react to the fundamental variables as in Table 1 and $f^2(av_2, \cdot)$ can be linearized as $\gamma_i = \beta_0^2 + \beta_1^2 \alpha_i + \beta_2^2 av_{2i} + v_i^2(A_i, \epsilon_i)$. The same assumptions and arguments apply for m_2 and $f^2(m_2, \cdot)$.

From the theoretical model it is not difficult to verify that¹⁸

$$av_2 \equiv \frac{\tau K_t}{Y_t} = \frac{\tau^\alpha}{A(1-\lambda)^{1-\alpha}} \quad \text{and} \quad m_2 \equiv \frac{d(\tau K_t)}{dY_t} = \frac{av_2}{1-\alpha}. \quad (14)$$

Thus, $m_2 > av_2$ which is a property most researcher would expect. For instance, Koester and Kormendi (1989), p. 370/1, find that marginal tax rates m_2 are significantly different from average tax rates av_2 and that '*across countries marginal*

¹⁷It should be borne in mind that the theoretical model is supposed to capture essential properties of the relation between taxes and growth. Thus, it is perfectly valid and common scientific procedure to reduce a problem to a simple model and use some, but not necessarily the closest estimable and statistical models to test the theoretical predictions. On this point see, for instance, Spanos (1986), chpts. 1.2 and 26.

¹⁸Recall $Y_t = AK_t^\alpha G_t^{1-\alpha}$ and $G_t = (1-\lambda)\tau k_t$. Then $av_2 = \frac{\tau K_t}{Y_t} = \frac{\tau K_t}{AK_t^\alpha ((1-\lambda)\tau K_t)^{1-\alpha}}$ which reduces to the expression in (14). Furthermore, $\frac{dY_t}{d(\tau K_t)} = (1-\alpha)AK_t^\alpha (1-\lambda)^{1-\alpha} (\tau K_t)^{-\alpha}$. Taking the inverse yields $\frac{d\tau K_t}{dY_t} = \frac{\tau^\alpha}{(1-\alpha)A(1-\lambda)^{1-\alpha}} = \frac{av_2}{1-\alpha}$.

tax rates average about one-and-one-half to two times average tax rates’.

Notice that even under the assumption of exogenous policy the coefficients on av_2 and m_2 would be biased downwards. If policy is endogenous, then $av_2(\hat{\tau}) = \alpha(1 - \alpha)$, $m_2(\hat{\tau}) = \alpha$ and so $\frac{dav_2(\hat{\tau})}{dA} = \frac{dm_2(\hat{\tau})}{dA} = 0$. For non-redistributing policies $\frac{dav_2}{dA} = \frac{\tau^\alpha}{A^2} \left[\frac{\alpha A}{\tau} \frac{d\tau}{dA} - 1 \right]$ and $\frac{dm_2}{dA} = \left(\frac{1}{1-\alpha} \right) \frac{dav_2}{dA}$. Furthermore, $\frac{\alpha A}{\tau} \frac{d\tau}{dA} < 1$ if

$$\begin{aligned} \alpha^2(1 - \alpha)A(\tau^\alpha - \alpha(1 - \alpha)^2A)^{-1} &< 1 \\ \alpha(1 - \alpha)^2A + \alpha^2(1 - \alpha)A &< \tau^\alpha \\ \hat{\tau} &< \tau \end{aligned} \tag{15}$$

which is true for $\check{\tau} > \hat{\tau}$. Thus, $\frac{dav_2(\check{\tau})}{dA}$ and $\frac{dm_2(\check{\tau})}{dA}$ are negative. For a redistributing government $av_2 = \frac{\rho}{A[(1-\alpha)A]^{\frac{1-\alpha}{\alpha}}}$ which decreases in A . By a similar argument one verifies that m_2 decreases in A under that policy. Hence, $cov(av_{2i}, A_i)$ and $cov(m_{2i}, A_i)$ are non-positive.

To keep matters simple assume that the covariance of av_{2i} (m_{2i}) and α_i is non-positive.¹⁹ Then av_2 co-varies with A or α in the same way as m_2 does.

Suppose one assumes exogenous policy, although it is not, and runs the regression $\gamma_i = \delta_0^2 + \delta_1^2 av_{2i} + u_i^2$ where the superscript 2 indicates that model $f^2(\cdot)$ is being analyzed. Proceeding as above reveals that d_1^2 is generically biased downwards, that is, it is biased down unless all governments act growth maximizing and $cov(av_{2i}, \alpha_i) = 0$. Hence, any measured negative effect of av_2 on γ may often be overstated. Similar arguments hold for m_2 .

Proposition 2 *Assume policy variables react to fundamental economic variables as in Table 1, the DGPs for $f(\cdot)$ and $f^2(\cdot)$ are the same, and $cov(av_{2i}, \alpha_i)$ and $cov(m_{2i}, \alpha_i)$ are non-positive. A simple cross-country OLS regression of γ on av_2 or m_2 yields generically biased estimates. Unless all governments in the sample pursue growth maximizing policies, the estimates are biased downwards.*

The downward bias may be explained as follows: Tax rates higher than the growth maximizing ones lead to relatively lower growth. That explains why many authors find a negative point estimate for δ_1^2 . For instance, Koester and Kormendi

¹⁹For instance, for *given* tax policies of all non-redistributing governments an increase in α reduces av_2 . Thus, the results below are conditional on that or that $cov(av_{2i}, A_i)$ ($cov(m_{2i}, A_i)$) is more important than $cov(av_{2i}, \alpha_i)$ ($cov(m_{2i}, \alpha_i)$). The condition is testable and any results derived from it are therefore falsifiable.

(1989) report $d_1^1 = -0.074$ (-2.18) for a simple regression of growth on average tax rates.²⁰ (All t-statistics are shown in parentheses.) However, in the model the bad direct effect of high taxes is compensated by the good effect of taxes channelled into production via public services. That good effect is larger when A is higher and it is ignored when assuming that policy is exogenous.

For instance, Levine and Renelt (1992), Engen and Skinner (1992), or Sala-i-Martin (1997) find that controlling for initial income y_0 renders their estimated effects of taxes on growth insignificant. However, putting initial income in the regression first and then adding a tax variable often renders the estimated coefficient of the effect of initial income on growth insignificant. For instance, Easterly and Rebelo (1993a) find in their growth regressions that seven(!) out of thirteen tax variables render the estimated coefficients for the effect of initial income on growth insignificant. This possibility is often ignored, although it provides a rationale for not including *either* tax measures *or* initial income in growth regressions. Thus, from a statistical viewpoint alone there is no reason why authors such Levine and Renelt (1992) or Sala-i-Martin (1997) conclude that fiscal variables are mostly non-robust regressors when fiscal variables are highly correlated with initial income.

A downward bias of d_1^2 plays an important role in explaining why one is likely to obtain insignificant coefficients in a regression $\gamma_i = \xi_0^2 + \xi_1^2 av_{2i} + \xi_2^2 y_{0i} + w_i^2$. Elementary econometrics tells one that the OLS estimator of ξ_1^2 is given by

$$\hat{\xi}_1^2 = \frac{d_1^2 - \hat{\xi}_{13}^2 \hat{\xi}_{32}^2}{1 - \hat{\xi}_{23}^2 \hat{\xi}_{32}^2}$$

where d_1^2 reflects the slope of the simple regression of γ_i on av_2 , $\hat{\xi}_{13}^2$ the slope of the regression of γ_i on y_{0i} , $\hat{\xi}_{23}^2$ the slope of the regression of av_2 on y_{0i} and $\hat{\xi}_{32}^2$ the slope of the regression of y_{0i} on av_2 . E.g., Koester and Kormendi report

$$\begin{aligned} d_1^2 &= -0.074 \quad (-2.18) & , & \quad \hat{\xi}_{13}^2 = -0.053 \quad (-3.52), \\ \hat{\xi}_{23}^2 &= +0.293 \quad (+6.32) & , & \quad \hat{\xi}_{32}^2 = -0.005 \quad (-0.11), \end{aligned}$$

from which the imputed value of $\hat{\xi}_{32}^2$ is 1.3389. It is instructive to see that including initial income reduces the point estimate for the effect of taxes on growth (d_1^2)

²⁰The reported coefficient on m_2 was -0.025 (-1.87).

vs. $\hat{\xi}_1^2$) by a factor of 14.8 and makes it statistically insignificant.²¹

Suppose $\hat{\xi}_{13}^2$, $\hat{\xi}_{23}^2$ and $\hat{\xi}_{32}^2$ were unbiased estimates. By Proposition 2 any negative value of d_1^2 is overstated so that the true δ_1^2 would be less negative than the value reported. If one fixes the other estimates one gets

$$\hat{\xi}_1^2 = \frac{d_1^2 - (-0.053) (+1.3389)}{1 - (+0.293) (+1.3389)} = (d_1^2 + 0.0710) \times 1.6455.$$

Underestimation means that a negative estimate d_1^2 would overstate any true (negative) δ_1^2 , which raises the problem of how big the bias is. Suppose $\delta_1^2 = d_1^2 \times x$ where $x \in (-\infty, 1)$ and that the unknown 'true' value is non-positive, $\delta_1^2 \leq 0$.

Table 2: Bias Effects on $\hat{\xi}_1^2$ when $d_1^2 = -0.074$

x	0.99	0.95	0.9	0.85	0.80	0.5	0.10
$\hat{\xi}_1^2$	-0.004	+0.001	+0.007	+0.013	+0.019	+0.056	+0.104

Thus, a five-percent bias in d_1^2 raises $\hat{\xi}_1^2$ from -0.005 to $+0.001$, that is, it pushes the point estimate of $\hat{\xi}_1^2$ towards zero. That is interesting, because many researchers control for initial income first and then add policy variables to their growth regressions. Most of them find that variables such as av_2 take on values close to zero. Given the bias in d_1^2 the 'true' point estimate of $\hat{\xi}_1^2$ is likely to be positive and maybe statistically significant. Most authors argue that they find insignificant effects for variables such as av_2 because of the high correlation between initial income and av_2 , that is, high $\hat{\xi}_{23}^2$ or $\hat{\xi}_{32}^2$.

However, a $\hat{\xi}_1^2$ which is close to zero may mean that the 'true' ξ_1^2 is positive and that biases in d_1^2 push $\hat{\xi}_1^2$ down towards zero. Hence, one cannot exclude the possibility that a higher 'ratio of tax revenues to GDP' may actually have a positive effect on growth.

By the same token one cannot exclude the possibility that marginal tax rates, measured by m_2 , positively affect growth. For example, Perotti (1996) is puzzled to find positive and significant coefficients for the effect of marginal tax rates on growth in his 2SLS regressions.

²¹Notice that taxes and initial income are positively and comparatively strongly correlated ($\hat{\xi}_{23}^2$), indicating that *Wagner's Law* holds which asserts a positive relation between the size of the government and per capita income. Furthermore, initial income is negatively related to growth, which is often interpreted as indicating conditional convergence in the growth process, that is, countries with lower initial income tend to have higher subsequent growth.

A negative bias in the coefficients for av_2 or m_2 implies that the theoretical prediction that higher taxes negatively affect growth is inherently untestable. Any reported value of the effect of tax variables such as av_2 or m_2 on growth is always *underestimated*, that is, a reported negative effect would be overstated and a reported positive effect would be understated. As a reported negative effect may well be positive in 'reality' then, the theoretical prediction cannot be corroborated when using av_2 or m_2 and the paper's theoretical model captures general properties of the effect of taxes and policy on growth.

Hence, it matters a great deal which tax variables one includes in growth regressions.

Public Investment. Sometimes growth is regressed on the ratio of government investment to GDP or total (private and public) investment to test how public investment such as changes in infrastructure affect growth. Theoretically, a significant and often positive relationship is expected. See, for instance, Aschauer (1989), Barro (1990), Barro and Sala-i-Martin (1992), or Barro and Sala-i-Martin (1995), chpt. 4.4. Surprisingly, in many empirical studies such as Barro (1989b), (1991), or Barro and Sala-i-Martin (1995) chpt. 12.3 the effect of these variables on growth turns out to be statistically insignificant.

In this paper long-run total investment is given by $I_t = \gamma K_t$,

$$\mathfrak{p} \equiv \frac{G_t}{Y_t} = \frac{1}{A} \left(\frac{G_t}{K_t} \right)^\alpha = \frac{((1-\lambda)\tau)^\alpha}{A} \quad \text{and} \quad \mathfrak{q} \equiv \frac{G_t}{\gamma K_t} = \frac{(1-\lambda)\tau}{\gamma}. \quad (16)$$

Barro (1991) uses public expenditure data to test his theoretical predictions which are based on G_t arguing that it is a proxy for the change in the stock of public capital. That is a problematic assumption, possible implications of which he discusses. For consistency with empirical results, which are often based on that, assume that G_t is indeed a good proxy for public investment, that is, a proxy for the change in the stock of public capital for which data are difficult to obtain. Furthermore, in line with the previous discussion on taxes and for simplicity assume that the correlation between these ratios and α_i is less important than that between the ratios and A_i . Thus, all results below are conditional on that assumption.

Interestingly, for *given* tax policies both \mathfrak{p} and \mathfrak{q} are decreasing in A or α .

Thus, the coefficients measuring the effect of these variables on growth would be biased downwards even under the assumption of exogenous policy and when A and α would not be included in the regression .

Lemma 1 *For given tax policies (exogenous policy), simple cross-country OLS regressions of the growth rate on \mathbf{p} or \mathbf{q} produce coefficients that are biased downwards, when omitting A and/or α .*

For endogenous policy the ratio of public investment to total investment (\mathbf{q}) is discussed first. To this end let $b \equiv (1 - \lambda)\tau$. Then $\frac{db}{dA} = -\frac{d\lambda}{dA} \tau + \frac{d\tau}{dA} (1 - \lambda)$ which is positive for all policies considered. I want to show that $\frac{dq}{dA} < 0$.

$$\text{sgn} \left(\frac{dq}{dA} \right) = \frac{db}{dA} \gamma - \frac{d\gamma}{dA} b = \frac{db}{dA} (r - \tau - \rho) - \left(\frac{dr}{dA} - \frac{d\tau}{dA} \right) b = \mathbf{q}_1 + \mathbf{q}_2$$

where $\mathbf{q}_1 \equiv \left(\frac{db}{dA} r - \frac{dr}{dA} b \right)$ and $\mathbf{q}_2 \equiv \frac{db}{dA} (-\tau - \rho) + \frac{d\tau}{dA} b$. Clearly $\mathbf{q}_2 < 0$, since

$$\begin{aligned} \mathbf{q}_2 &= \left(-\frac{d\lambda}{dA} \tau + \frac{d\tau}{dA} (1 - \lambda) \right) (-\tau - \rho) + \frac{d\tau}{dA} (1 - \lambda)\tau \\ &= \frac{d\lambda}{dA} \tau(\tau + \rho) - \frac{d\tau}{dA} (1 - \lambda)\rho < 0 \end{aligned}$$

with $\frac{d\lambda}{dA} \leq 0$ and $\frac{d\tau}{dA} \geq 0$ and at least one of the derivatives being non-zero for all policies considered. For \mathbf{q}_1 one obtains

$$\begin{aligned} \mathbf{q}_1 &= \frac{db}{dA} r - \left(r_A + r_b \frac{db}{dA} \right) b = (r - r_b b) \frac{db}{dA} - r_A b \\ &= \alpha^2 A b^{1-\alpha} \frac{db}{dA} - r_A b = \alpha b^{1-\alpha} \left[\alpha A \frac{db}{dA} - b \right] \end{aligned}$$

which is non-positive for all non-redistributing policies since for $\lambda = 0$ one gets $\alpha A \frac{d\tau}{dA} \leq \tau$ as has been shown in (15). For a redistributing government $b = [(1 - \alpha)A]^{\frac{1}{\alpha}}$ and $\frac{db}{dA} = \frac{b}{\alpha A}$ so that $\mathbf{q}_1 = 0$. Hence, $\mathbf{q}_1 + \mathbf{q}_2 < 0$ and so $\frac{dq}{dA} < 0$.

The ratio of government investment to GDP is $\mathbf{p} = \frac{b^\alpha}{A}$ so that

$$\text{sgn} \left(\frac{d\mathbf{p}}{dA} \right) = \alpha A b^{\alpha-1} \frac{db}{dA} - b^\alpha = b^\alpha \left[\frac{db}{dA} \frac{\alpha A}{b} - 1 \right]$$

which is zero for growth maximizing and redistributing policies. For non-redistributing policies it is negative.

Proposition 3 *Assume policy variables react to fundamental economic variables as in Table 1, the DGP is the same as that for $f(\cdot)$ and γ is regressed on \mathbf{q} or \mathbf{p} . If $\text{cov}(\mathbf{q}_i, \alpha_i)$ and $\text{cov}(\mathbf{p}_i, \alpha_i)$ are non-positive, then in simple cross-country OLS regressions*

1. *the estimated coefficients measuring the effect of \mathbf{q} on growth are biased downwards under all policies considered.*
2. *the estimated coefficients measuring the effect of \mathbf{p} on growth are biased downwards if all governments pursue non-redistributing, pro-labour policies. The estimated coefficients may be unbiased if all governments pursue growth maximizing, or redistributing policies and \mathbf{p}_i is uncorrelated with α_i .*

Barro (1991), Table IV, reports that the estimated coefficient for \mathbf{q} was 0.014 (0.636) and that for \mathbf{p} was 0.13 (1.3) for regressions that included many variables. When also including the significantly positive variable i/y the estimated coefficient for \mathbf{p} became negative and was -0.015 (-0.126).

Barro (1990), p. S124, interprets the estimated coefficient for \mathbf{q} as corroboration of his hypothesis that *'the typical country comes close to the quantity of public investment that maximizes the growth rate.'* As violations of the standard assumptions for hypothesis testing make such an argument invalid, Proposition 3 provides one reason why such a hypothesis may rest on shaky ground. As any measured positive point estimate of the effect of \mathbf{q} on growth may be *underestimated*, it is likely that the ratio of public to total investment positively affects growth, *even if* all governments pursue growth maximizing policies, and although the estimate is statistically insignificant. This is because by Lemma 1 the coefficients on \mathbf{q} would be biased downwards, when, for instance, omitting A . Thus, one can hypothesize that the "true" coefficient is really positive.

For a sample with heterogeneous policies and a "true" estimate that is likely to be significantly positive, a hypothesis compatible with the proposition should be that, given everything else, a typical country that increases public investment is likely to increase the growth rate, no matter what policy is pursued.²² Given

²²Notice that a significantly positive coefficient does not entail that it should be growth maximizing to push \mathbf{q} up to 1 when tax base or collection constraints curtail such choices. Those and other provisos are meant by 'given everything else'.

the downward biases for the measured effects of q on growth one may be led to conclude that an increase in q may raise growth.

Barro interprets the statistically insignificant, but negative coefficient on p , when the variable i/y is controlled for, as an indication that '*there is no separate effect on growth from the breakdown of total investment between private and public components*'. That suggests the hypothesis that an optimizing public sector invests according to the same criteria as the private sector does. For instance, the public sector may wish to maximize the social return on its investment, just as private sector investors wish to maximize the private return on their investments. Barro implicitly assumes that maximization of the social return on public investment also maximizes the growth rate.

First of all, it is worth noting that under the assumption of exogenous policy the estimated coefficients for the effect of p on growth are biased downwards (see Lemma 1). Under the assumption of endogenous policy the estimated coefficients for the effect of p would be biased downwards if the sample contained non-redistributing, pro-labour governments.

Secondly, under the assumption of endogenous policy the negative point estimate suggests that the breakdown matters. A reported value of -0.015 for the effect of p on growth means that holding total investment constant and raising public investment with a compensating cut in private investment reduces growth. Thus, a negative point estimate in that regression implies that a typical country's policy which crowds out private (steady state) investment is bad for growth. That implication may be a valid result (unbiased estimates) for samples where, according to Proposition 3, all countries are led by growth maximizing, or redistributing, pro-labour policies and $cov(p, \alpha_i) = 0$. But given unbiasedness and statistical insignificance, it follows that other than growth maximizing policies would also lead to no crowding out of public and private investment. That, in turn, implies that a statistically insignificant value does not mean that governments have maximized the social return on public investment. Thus, other welfare objectives are compatible with a statistically insignificant coefficient.

Barro and others view their empirical results as "ongoing research" and are aware of the econometric problems mentioned above. However, they does not investigate the implications of endogenous policy in detail. Here it follows that under endogenous policy the estimated coefficients for the effect on growth of the

ratio of public investment to total investment are generally biased downwards. The estimated coefficients for the effect on growth of the ratio of public investment to GDP may be unbiased if that ratio is uncorrelated with the share of capital and other than non-redistributing, pro-labour policies are pursued. Otherwise, there is a downward bias in the estimated coefficients.

Interestingly, if one assumes exogenous policy, then by Lemma 1 the coefficients measuring the effect of that variable on growth are definitely biased downwards due to omitting important variables. Hence, reported statistically insignificant negative point estimates for that coefficient in simple growth regressions may be either biased downwards and so the 'true' coefficient may really be positive or they are unbiased, but provide no corroboration of the hypothesis that all countries are lead by growth maximizing policies.

Redistributive Transfers. Some researchers test the effect of redistributive transfers such as social benefits, social security contributions etc. on growth. Transfers of that kind are political instruments to correct for socially unwanted pre-tax income inequality. In that context growth is often regressed on the ratio of redistributive transfers to GDP, which - in this model - is given by²³

$$\mathfrak{b} \equiv \frac{\lambda\tau K_t}{Y_t} = \frac{\lambda\tau K_t}{AK_t^\alpha G_t^{1-\alpha}} = \frac{\tau K_t \left(\frac{G_t}{K_t}\right)^\alpha}{AG_t} = \frac{\lambda((1-\lambda)\tau)^\alpha}{A(1-\lambda)}$$

where $\tau = \rho$ from (10), $G_t = (1-\lambda)\tau K_t$ and by assumption some governments in the sample choose $\lambda > 0$ so that data on redistribution are actually available. Then a regression of γ on \mathfrak{b} produces biased estimates for the following reasoning:

Firstly, for *given* (i.e. exogenous) policy and $cov(\mathfrak{b}_i, \alpha_i) \leq 0$, the estimated effect of \mathfrak{b} on growth are biased downwards. Secondly, under the assumption of endogenous policy Table 1 implies that $\frac{d\mathfrak{b}}{d\alpha}$ and so $cov(\mathfrak{b}_i, \alpha_i)$ are ambiguous in sign. Then $\frac{d\mathfrak{b}}{dA} = \mathfrak{b}_\lambda \frac{d\lambda}{dA} + \lambda_A < 0$ since $\lambda_A < 0$, $\mathfrak{b}_\lambda > 0$, $\frac{d\lambda}{dA} < 0$ and $\tau = \rho$ which is independent of A , suggesting that simple OLS estimates for the effect of \mathfrak{b}_i on γ_i are biased downwards, that is, if $cov(\mathfrak{b}_i, \alpha_i) \leq 0$ in the sample or the covariance is less important than that of $cov(\mathfrak{b}_i, A_i)$ the estimated effect of transfers on growth are biased downwards.

²³Table 1 implies that a regression of the growth rate on the ratio of transfers to tax revenues, that is, λ would also yield coefficients that are biased downwards.

Proposition 4 *If $\text{cov}(b_i, \alpha_i) \leq 0$ and some countries in the sample are led by redistributing governments the model predicts that in simple cross-country OLS regressions the estimates measuring the effect on growth of the ratio of redistributive transfers to GDP are biased downwards.*

Hence, any reported bad effect of redistributive transfers is overstated and any reported good effect is understated. The downward bias would even be present if one assumed that policy is exogenous and when omitting A . Many theoretical models show that redistributive transfers are bad for growth. The reason for overstating any measured negative effect of redistributive transfers on growth in this model is due to the prediction that countries with more efficient economies and redistributing governments choose to redistribute *less* resources per units of taxes collected.

As Proposition 4 does not say anything on the magnitude of the bias, the latter may be as large as minus infinity. That means, the hypothesis that redistributive transfers are bad for growth cannot be validated by any reported negative coefficient on transfers in simple OLS growth regressions. Thus, for simple growth regressions Proposition 4 would render the hypothesis generically untestable.

In fact, these arguments may suggest the opposite hypothesis, namely that redistributive transfers affect growth in a positive way. For instance, Perotti (1994) is surprised to find a significantly positive coefficient on redistributive transfers in his regressions of investment on policy variables. He quotes other studies that have found a positive relation between transfers and growth. (See ftn. 8 of his paper.) Sala-i-Martin (1996) states that it is surprising that among the three components of public spending - public investment, public consumption and public transfers - the only one that seems to be positively related to growth is the redistributive transfer variable. For instance, social security contributions are often found to affect growth in a positive way.²⁴

²⁴A positive coefficient might suggest that a different theoretical model is needed. In that context, Rehme (2004) shows, however, that a nonlinear model featuring a negative effect of redistribution on growth under the assumption of exogenous policy, might actually imply a theoretically expected "true" positive coefficient in a linear empirical model, when the latter is viewed as a linear approximation and policy is endogenous. Thus, a positive estimate might be compatible with a model featuring a negative "effect" of redistribution on growth when policy is endogenous.

4 Empirical Evidence

In this section empirical checks are provided for the theoretical predictions of the previous sections. The following empirical evidence should be viewed as suggestive only, because reliable data on tax rates and redistribution are in general not easy to obtain for a large set of countries.²⁵ Therefore, I concentrate on the subset of OECD countries for which data are more readily available.²⁶

In terms of methodology I will only focus on estimation issues in the empirical exercises that follow. We will not address issues of statistical significance, which rely on estimation anyway. For a justification of such an approach see, for example, McCloskey (1985) and McCloskey and Ziliak (1996).

Given that one could contemplate many different policy variables related to the theoretical analysis, I will restrict the empirical checks below by focussing on three key variables analyzed before. They are an indicator for average tax rates, one for redistribution and one for public investment in terms of GDP. Furthermore, I will concentrate on efficiency A as the prime mover of policy and growth. Thus, we fix the share of capital α and the time preference rate at some level for the the analysis below.

4.1 The Data

Countries differ widely in the level of development, but reliable data capturing the *level* of development are not easily available. In order to calculate the model's state of technology, A , I have proceeded as follows: Following Hall and Jones (1999) I have used a *levels accounting* framework based on a production function approach. Under the assumptions made the aggregate production function is given by $Y = AG^{1-\alpha}K^\alpha L^{1-\alpha}$ where $L = 1$ by assumption.²⁷ From this the level

²⁵For a detailed description of the data and the methods used in this paper confer <http://www.tu-darmstadt.de/~rehme/endopol/data-million.htm>. The sample is presented at the end of the paper.

²⁶An advantage of this, though, is that they form a homogeneous group so that one does not need to control for regional disparities by means of dummy variables as has so often been done in cross-country research.

²⁷Thus, we ignore level effects due to the size of the labour force. A slight modification of the model would lead to the same procedure as is shown in Rehme (2004).

of technology can be calculated as

$$A = \left(\frac{G}{Y}\right)^{\alpha-1} \left(\frac{K}{Y}\right)^{-\alpha}.$$

Using data from Barro and Lee (1994), Summers and Heston (1991) and the OECD, I have calculated long-run averages for G/Y and K/Y for the period 1970-2000.²⁸ The data for $G/Y \equiv \mathfrak{p}$ are captured by the variable *INVPUB* and denote public investment in terms of GDP.

Following Mankiw, Romer, and Weil (1992) I have set the share of broad capital α equal to 0.67. The resulting variable measuring multi-factor productivity is called GA . The cross-country productivity differences are expressed relative to the United States for which the value is set at unity.

Redistribution is measured following Milanovic (2000). Based on household data from the Luxembourg Income Study (LIS) redistribution is calculated from Gini indices for the distribution of households' factor income.²⁹ These they are compared to the Gini indices for the distribution of disposable income. The difference between them is then related to the Gini coefficient for factor incomes.³⁰ Milanovic provides data for 24, mostly OECD countries with a total of 79 observations. As there are not enough observation for the initial year 1970, I take averages for these differences in Gini coefficients in terms of the Gini coefficient

²⁸Notice that we have assumed that G is the *flow* of public inputs in production. Barro and Lee (1994) provide data for G/Y by their variable *INVPUB* which measures the ratio of nominal public domestic investment (fixed capital formation) to nominal GDP. For the period 1970-90 K/Y was calculated from Summers and Heston (1991), PWT (Mark 5.6), who provide data of capital per worker (KAPW) and real GDP per worker. For the period 1990-2000 I have used OECD data for government fixed capital formation and real GDP. Notice that according to the model $K/Y = A^{-1} \left(\frac{K}{G}\right)^{1-\alpha} = \tau^{\alpha-1}/A$ which is constant, no matter whether capital is broadly defined or not. This is what I assume to be the case here. Thus, even though capital may only include physical capital in KAPW, it would still be a proxy for the models capital-output ratio. See the online appendix for more details.

²⁹For further, more detailed accounts of why moving from the factor income distribution to the distribution of personal incomes makes sense in our model framework see Rehme (2000) and Rehme (2006).

³⁰Factor income is defined as pre-transfer *and* pre-tax income, and includes wages income from self-employment, income from ownership of physical and financial capital and gifts. Factor income also includes public pensions. Gross income, in turn, equals factor income *plus* social insurance transfers, which includes sick pay, disability pay, social retirement benefits, child or family allowances, maternity pay, military or veterans benefits and near-cash benefits. Gross income *minus* mandatory employee contributions *minus* income tax equals disposable income. See the Luxembourg Income Study for the variable definitions at <http://www.lisproject.org/techdoc/summary.pdf>.

for factor incomes over the sample period 1970-2000. The resulting variable is called $RRED$ and is taken to proxy the model's λ . Thus, $RRED = \frac{Gini^{FI} - Gini^{DI}}{Gini^{FI}}$ where FI denotes factor income and DI represents disposable income.

To proxy for the average tax rate we concentrate on the indicator for tax revenues in terms of GDP, av_2 . To this end I have used data from the OECD Revenue Statistics 1965-2001, OECD (2002). For these years the ratio of tax revenue to GDP is provided for 30 countries. Here we focus on the time average tax rate for the period 1970-2000 and call that variable $TAX7090$.

Finally, long-run growth rates for the period 1970-2000, called GR , and the logarithm of initial income in 1970, $LN70$, were calculated using the Penn World Table (Mark 6.1) provided by Heston, Summers, and Aten (2002).

The following table provides information on the descriptive statistics for the resulting sample of 29 countries.

Table 2: Descriptive Statistics

Variable	Mean	Std. Dev.	Minimum	Maximum	No. of Obs.
GA	0.713	0.166	0.440	1.010	29
GR	0.023	0.010	0.010	0.060	29
$TAX7090$	0.345	0.077	0.170	0.482	29
$RRED$	0.337	0.096	0.160	0.510	21
$INVPUB$	0.061	0.048	0.025	0.194	29
$LN70$	9.243	0.452	7.930	9.920	27

For the variables of interest, av_2 , λ , G/Y , and A , this means that the average tax-revenues-to-GDP ratio over the sample period, $TAX7090$, is about 34 percent with a standard deviation of 7.7 percentage points. Redistribution $RRED$ is such that government intervention by means of taxes and transfers amounts to 34 percent of the pre-tax inequality. Public Investment, $INVPUB$, is 6 percent on average. Finally, in the sample the average country features a level of productivity, GA , that reaches roughly 71 percent of the level that pertains to the United States of America.³¹

³¹Thus, the data confirm findings that show that the United States is among the most productive countries in the world. See e.g. Treffer (1995) and many others. In the sample countries like France (0.82), Germany (0.75), Japan (0.52) or the United Kingdom (0.90) feature lower values for multi-factor productivity - after controlling for public inputs in production. In that sense the U.S. is an exception. Interestingly, Iceland features a value greater than unity, suggesting that it is more productive than the U.S. Of course, values of A lower than unity also indicate that public inputs in production and the other factor inputs contribute more to explanations of output in these countries than in the U.S. That is because A can also be

The next table presents the pairwise correlations between the variables.³²

Table 3: Pairwise Correlations

	<i>GA</i>	<i>GR</i>	<i>TAX7090</i>	<i>RRED</i>	<i>INVPUB</i>	<i>LN70</i>
<i>GR</i>	0.108 (29)	1				
<i>TAX7090</i>	-0.158 (29)	-0.192 (29)	1			
<i>RRED</i>	-0.412 (21)	-0.060 (21)	0.666 (17)	1		
<i>INVPUB</i>	-0.611 (29)	-0.151 (29)	0.074 (23)	0.596 (21)	1	
<i>LN70</i>	0.133 (27)	-0.589 (27)	0.356 (23)	-0.245 (19)	-0.430 (27)	1

Number of observations in brackets.

For the variables of interest the theoretically derived signs for the correlations are generally borne out by the data. For example, the correlation between *TAX7090* and *GA* is negative (-0.158), that between *RRED* and *GA* is negative (-0.412), and that between *INVPUB* and *GA* is negative (-0.611). Furthermore, the correlations show a positive (simple, uncontrolled) relationship between taxes, *TAX7090*, and redistribution, *RRED*. In addition, the correlations between initial income and *TAX7090*, *RRED* and *GA* all show the expected signs.

Therefore, the signs of the pairwise correlations seem to support the model's theoretical predictions.

4.2 Regressions and Biases

We now look at the effects of the biases as predicted by the theory. One simple way to check this, is to compare estimated coefficients in growth regressions with policy variables when an index of efficiency is included or not.³³ This will provide additional information whether the direction of the biases is as predicted.

taken as an indicator of our "ignorance" about other, exogenous factors bearing on aggregate production. Thus, the percentage of exogenous, "unexplained" factors describing aggregate output is larger in the U.S. and Iceland than, for example, in Germany or Japan.

³²These pairwise correlations use as much information from the data as possible. Thus, we get correlations for a maximum of 29 countries and a minimum of 19 countries for the correlation between tax rates and redistribution. This is due to missing observations.

³³Again we will only concentrate on the estimated coefficients and not on their standard errors or significance levels.

Thus, consider the simple regression³⁴

$$\text{Model 1: } GR = \beta_0^1 + \beta_1^1 X + \epsilon^1$$

where X is $TAX7090$, $RRED$, or $INVPUB$. The estimated coefficient for the policy variable X , $\hat{\beta}_1^1$, can then be compared to the the estimate $\hat{\beta}_1^2$ in

$$\text{Model 2: } GR = \beta_0^2 + \beta_1^2 X + \beta_2^2 GA + \epsilon^2.$$

The difference between $\hat{\beta}_1^1$ and $\hat{\beta}_1^2$ may then be taken to indicate whether the bias due to an omitted variable in Model 1 has the direction predicted by the theory.

As a robustness check we will also look at the conventional growth regressions that also include initial income as a regressor. Thus, we compare the estimates of β_1^3 and β_1^4 of

$$\text{Model 3: } GR = \beta_0^3 + \beta_1^3 X + \beta_2^3 LNY70 + \epsilon^3$$

$$\text{Model 4: } GR = \beta_0^4 + \beta_1^4 X + \beta_2^4 LNY70 + \beta_3^4 GA\epsilon^4.$$

The resulting estimated coefficients for these regressions are reported below.

Table 4: Regression coefficients

Variable	Model 1 $\hat{\beta}_1^1$	Model 2 $\hat{\beta}_1^2$	Model 3 $\hat{\beta}_1^3$	Model 4 $\hat{\beta}_1^4$
$TAX7090$	-0.0245	-0.0229	0.0054	0.0078
$RRED$	-0.0047	0.0020	0.0082	0.0111
$INVPUB$	-0.0305	-0.0275	-0.0696	-0.0749

These coefficients suggest the following: The variable $TAX7090$ is taken to proxy the model's av_2 , that is, total tax revenues in terms of GDP. The theory predicts that the "effect" of av_2 on growth is biased downwards, when ignoring the effect of A . (Proposition 2.) The estimated coefficients lend support to that result. This is because when including GA in the regressions the estimated coefficients on $TAX7090$ are higher than those when excluding GA . For instance, the estimated coefficient for Model 1 is -0.0245 , which is lower than that for

³⁴Of course, one often includes more than one policy variable in growth regressions. For simplicity, we concentrate on the single variable case here. An analysis focussing on multivariate policy bias problems is provided in Rehme (2004), which yields complementary results.

Model 2 with -0.0229 and GA included in that regression. The same holds when comparing Models 3 and 4 for $TAX7090$.³⁵

$RRED$ proxies for λ . According to the theory the effect of λ on growth is biased downwards. (Proposition 4.) The estimated coefficients seem to support that claim. The estimated coefficient for Model 1 is -0.0047 , which is lower than that for Model 2 with -0.0020 and GA included in the regression. The same holds when comparing Models 3 and 4 for $RRED$.

For our proxy of $\mathfrak{p} \equiv \frac{G_t}{Y_t}$, called $INVPUB$, it is predicted that its measured effect on growth should be biased downwards. (Proposition 3.) The comparison of Models 1 and 2 support this claim, because the estimated coefficient for Model 1 is -0.0305 which is lower than that for Model 2 with -0.0275 and GA included in the regression. However, comparing Models 3 and 4 would not support the theoretical prediction.³⁶

From this I conclude that in the majority of simple checks, based on empirical evidence, the theoretical predictions are borne out by this paper's data.

5 Conclusion

Within a common theoretical framework it is shown how optimizing governments take account of fundamental economic variables when making their decisions so that public policy is economically endogenous and has interesting effects on long-run growth. This exercise provides a theory-grounded "lens" through which many existing empirical results on the link between policy and growth may be interpreted. Several findings of the paper are noteworthy.

First, under certain conditions the policies optimal for the accumulated factor of production (growth maximization) may also be pursued if a government has other welfare objectives. An increase in technological efficiency generally raises optimal taxes and growth under the policies considered.

³⁵Notice that we get a sign reversal moving from regressions excluding initial income to those including initial income. This replicates such findings as, for instance, found by Easterly and Rebelo (1993a).

³⁶Notice that Models 3 and 4 do not tally with the theory. The theory is based on a simple endogenous growth model where initial income should not necessarily feature in the regression. In that sense Models 3 and 4 are of auxiliary nature for the theory of this paper, although they have been used very extensively in the empirical growth literature.

Second, when policy is endogenous the estimated coefficients on policy variables are generically biased. The direction of the biases are deduced from the theoretical model, providing a useful lens which permits us to formulate explanations for certain, sometimes puzzling point estimates found in the existing empirical literature.

Third, it is shown that the effects on growth of some widely used variables such as tax rate variables related to GDP, the ratio of public investment to total investment and the ratio of redistributive transfers to GDP are generally biased downwards.

For instance, for the latter this implies the following: Under the assumption that policy is economically endogenous, the effect of redistributive transfer variables on growth are generally underestimated so that the hypothesis that redistribution is bad for growth may not be testable. The downward bias is, however, perfectly consistent with empirical findings in the literature which find a positive association between redistributive transfers and growth. It may also represent corroboration of the hypothesis that redistribution is not bad for growth.

Finally, using data for a sample of OECD countries, I find suggestive evidence for the theoretical results derived in this paper.

The purpose of the paper is to contribute to steps in the direction of the disentanglement of the interplay of long-run economic fundamentals - like social fabrics, technology, history, institutions, or geography - and policy on the one hand, and policy and growth on the other. Such a disentanglement may provide interesting avenues for further theoretical and empirical research on this problem.

A The capitalists' optimum

The current value Hamiltonian for the problem (5) - (7) is given by

$$H = \ln C_t^k + \mu_t((r - \tau)K_t - C_t^k). \quad (\text{A1})$$

The necessary first order conditions for its maximization are given by (6), (7) and

$$\frac{1}{C_t^k} - \mu_t = 0 \quad (\text{A2a})$$

$$\dot{\mu}_t = \mu_t \rho - \mu_t (r - \tau) \quad (\text{A2b})$$

$$\lim_{t \rightarrow \infty} K_t \mu_t e^{-\rho t} = 0, \quad (\text{A2c})$$

where the positive co-state variable μ_t represents the instantaneous shadow price of one more unit of investment at date t .

From (A2a) and (A2b) consumption grows at $\gamma \equiv \frac{\dot{C}_t^k}{C_t^k} = R - \rho$ where $R \equiv (r - \tau)$ and constant. Thus, $C_t = C_0 e^{(R-\rho)t}$ where C_0 remains to be determined. Substituting for C_t in (6) implies $\dot{K}_t = R K_t - C_0 e^{(R-\rho)t}$ which is a first order, linear differential equation in K_t and solved as follows

$$\begin{aligned} \dot{K}_t - R K_t &= -C_0 e^{\gamma t} \\ e^{-Rt} (\dot{K}_t - R K_t) &= -e^{-Rt} C_0 e^{\gamma t} \\ \int e^{-Rt} (\dot{K}_t - R K_t) dt &= - \int C_0 e^{-\rho t} dt. \end{aligned}$$

The last equation is an exact differential equation with integrating factor e^{-Rt} . The LHS is solved by $K_t e^{-Rt} + b_0$ and the RHS is solved by $\frac{C_0}{\rho} e^{-\rho t} + b_1$, where b_0, b_1 are arbitrary constants. Thus, $K_t = \frac{C_0}{\rho} e^{(R-\rho)t} + b e^{Rt}$ where $b = b_1 - b_0$. Substituting this into the transversality condition implies

$$\frac{1}{C_0} \lim_{t \rightarrow \infty} \left(\frac{C_0}{\rho} e^{(R-\rho)t} + b e^{Rt} \right) e^{-Rt} = \lim_{t \rightarrow \infty} \left(\frac{1}{\rho} e^{-\rho t} + \frac{b}{C_0} \right) = 0$$

which holds if the arbitrary constant b is set equal to zero. Then $K_t = \frac{C_0}{\rho} e^{\gamma t} \Rightarrow \gamma_k = \gamma = R - \rho$ so that consumption and wealth grow at the same constant rate in the optimum. Furthermore, the optimal level of consumption at each date is given by $C_t = \rho K_t$.

B Welfare

The agents' welfare is $V(C^j) = \int_0^t \ln C_t^j e^{-\rho t}$ where $j = k, W$. Let $t \rightarrow \infty$ and use integration by parts. For this define $v_2 = \ln C_t^j$, and $dv_1 = e^{-\rho t} dt$. Recall $dv_2 = \frac{\dot{C}_t^j}{C_t^j} = \gamma$ and constant. Then $v_1 = -\frac{1}{\rho} e^{-\rho t}$ so that

$$\begin{aligned} \int_0^\infty \ln C_t^j e^{-\rho t} dt &= \frac{1}{\rho} \left[-\ln C_t^j e^{-\rho t} \right]_0^\infty + \frac{1}{\rho} \int_0^\infty \gamma e^{-\rho t} dt \\ &= \frac{\ln C_0^j}{\rho} - \frac{1}{\rho^2} [\gamma e^{-\rho t}]_0^\infty \end{aligned}$$

where $C_0^k = \rho K_0$ and $C_0^W = (\eta + \lambda\tau)K_0$. Evaluation at the particular limits yields the expressions in (9).

C Optimal Policies

The government solves: $\max_{\tau, \lambda} (1 - \beta) V^r + \beta V^l$ s.t. $\lambda \geq 0$ where β is the social weight attached to the workers' welfare. The FOCs are

$$\beta \frac{\eta_\tau + \lambda}{(\eta + \lambda\tau)\rho} + \frac{\gamma_\tau}{\rho^2} = 0 \quad , \quad \lambda \left(\beta \frac{\eta_\lambda + \tau}{(\eta + \lambda\tau)\rho} + \frac{\gamma_\lambda}{\rho^2} \right) = 0.$$

Notice that γ_τ must be negative for the first equation to hold, so in the optimum $\tau > \hat{\tau}$. Concentrating on an interior solution for λ , simplifying, rearranging and division of the resulting two equations by one another yields

$$\frac{\eta_\tau + \lambda}{\eta_\lambda + \tau} = \frac{\gamma_\tau}{\gamma_\lambda}. \quad (\text{C1})$$

Then $\gamma_\lambda = r_\lambda$ and $\gamma_\tau = r_\tau - 1$ imply $(\eta_\tau + \lambda)r_\lambda = (\eta_\lambda + \tau)(r_\tau - 1)$ which upon multiplying out becomes $\eta_\tau r_\lambda + \lambda r_\lambda = r_\tau \eta_\lambda + r_\tau \tau - \eta_\lambda - \tau$. Notice $r_\lambda \eta_\tau = r_\tau \eta_\lambda$ and $\eta = \frac{1-\alpha}{\alpha} r$. Then $\lambda r_\lambda = r_\tau \tau - \frac{1-\alpha}{\alpha} r_\lambda - \tau$ and so

$$\left(\lambda + \frac{1-\alpha}{\alpha} \right) r_\lambda = \tau r_\tau - \tau \Leftrightarrow \left(\lambda + \frac{1-\alpha}{\alpha} \right) = \frac{\tau r_\tau}{r_\lambda} - \frac{\tau}{r_\lambda}.$$

Recall $r_\tau = \alpha E(1 - \lambda)$, $r_\lambda = \alpha E(-\tau)$ where $E = (1 - \alpha)A[(1 - \lambda)\tau]^{-\alpha}$. Thus, $\frac{\tau r_\tau}{r_\lambda} = -\frac{\tau \alpha E(1-\lambda)}{\alpha E \tau} = -(1 - \lambda)$ and $\lambda + (1 - \lambda) + \frac{1-\alpha}{\alpha} = -\frac{\tau}{r_\lambda} \Leftrightarrow \frac{r_\lambda}{\alpha} = -\tau$ and so

$$\tau = \frac{[(1 - \alpha)A]^\frac{1}{\alpha}}{1 - \lambda}. \quad (\text{C2})$$

Notice that for this τ we have $E = 1$. For the first order condition for τ we note that $\eta = (1 - \alpha)A[(1 - \lambda)\tau]^{1-\alpha} = E[(1 - \lambda)\tau] = [(1 - \alpha)A]^\frac{1}{\alpha}$. Furthermore, $\eta_\tau = (1 - \alpha)(1 - \lambda)$, $r_\tau = \alpha(1 - \lambda)$. Eqn. (C2) implies $\lambda = 1 - \frac{[(1-\alpha)A]^\frac{1}{\alpha}}{\tau}$ so that

$$\eta + \lambda\tau = [(1 - \alpha)A]^\frac{1}{\alpha} + \tau \left(1 - \frac{[(1 - \alpha)A]^\frac{1}{\alpha}}{\tau} \right) = \tau.$$

Then the first order condition for τ becomes

$$\beta \frac{\eta_\tau + \lambda}{(\eta + \lambda\tau)} = -\frac{\gamma_\tau}{\rho} \Leftrightarrow \frac{\eta_\tau + \lambda}{\tau} = -\frac{\gamma_\tau}{\beta\rho} \Leftrightarrow \frac{\eta_\tau + \lambda}{\gamma_\tau} = -\frac{\tau}{\beta\rho}.$$

But from above $\frac{\eta_\tau + \lambda}{\gamma_\tau} = \frac{(1-\alpha)(1-\lambda)+\lambda}{\alpha(1-\lambda)-1} = -1$ so that $\tau = \beta\rho$. Thus,

$$\tau = \beta\rho \quad \text{and} \quad \lambda = 1 - \frac{[(1 - \alpha)A]^\frac{1}{\alpha}}{\beta\rho}. \quad (\text{C3})$$

which is equation (10) when $\beta = 1$. Recall that these equations hold for $\lambda \geq 0$, thus for $\beta\rho \geq [(1 - \alpha)A]^\frac{1}{\alpha}$.

Suppose $\lambda = 0$, then the first order condition becomes

$$\frac{\eta_\tau}{\eta} = -\frac{r_\tau - 1}{\beta\rho} \Leftrightarrow \frac{(1 - \alpha)E}{\tau E} = -\frac{\alpha E - 1}{\beta\rho} \Leftrightarrow (1 - \alpha)\beta\rho = \tau - \alpha\tau E$$

so that the solution with $\lambda = 0$ is given by

$$(1 - \alpha)\beta\rho = \tau [1 - \alpha(1 - \alpha)A\tau^{-\alpha}] \quad (\text{C4})$$

which holds only if $\beta\rho < [(1 - \alpha)A]^\frac{1}{\alpha}$. For $\beta = 1$ this is equation (11) in the text.

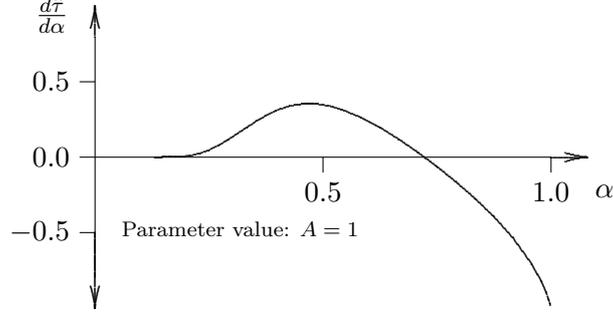
If $\beta = 0$, then $\tau = \hat{\tau}$. Thus, the pro-capital policy maximizes growth.

D Reactions under Endogenous Policy

Pro-Capital. $\hat{\gamma} = \frac{\alpha \hat{\tau}}{1 - \alpha} - \rho$ and $\hat{\tau} = [\alpha(1 - \alpha)A]^\frac{1}{\alpha}$. Clearly, $\frac{d\hat{\tau}}{dA} > 0$ and $\frac{d\hat{\gamma}}{dA} > 0$. Furthermore,

$$\begin{aligned} \frac{d\hat{\tau}}{d\alpha} &= \frac{[\alpha(1 - \alpha)A]^\frac{1}{\alpha} - 1}{\alpha} (1 - 2\alpha)A - \frac{[\alpha(1 - \alpha)A]^\frac{1}{\alpha} \ln[\alpha(1 - \alpha)A]}{\alpha^2} \\ &= \frac{\hat{\tau} (1 - 2\alpha)}{\alpha^2(1 - \alpha)} - \frac{\hat{\tau} \ln \hat{\tau}}{\alpha} \end{aligned} \quad (\text{D1})$$

which is not easy to evaluate. However, the following plot establishes that $\frac{d\hat{\tau}}{d\alpha} \geq 0$ for a particular level of A .



Thus, for two different values of the share of capital one may have the same $\hat{\tau}$. As $\frac{d\hat{\tau}}{d\alpha} \geq 0$ the sign of $\frac{d\hat{\gamma}}{d\alpha}$ is not clear. For the calculation of $\frac{d\hat{\gamma}}{d\alpha}$ rearrange to get $(\hat{\gamma} + \rho) = \frac{\alpha \hat{\tau}}{1-\alpha}$. Then

$$\begin{aligned} \ln(\hat{\gamma} + \rho) &= \ln \alpha - \ln(1 - \alpha) + \ln \hat{\tau} \\ &= \ln \alpha - \ln(1 - \alpha) + \frac{\ln(\alpha(1 - \alpha)A)}{\alpha} \\ &= \left(\frac{\alpha + 1}{\alpha}\right) \ln \alpha + \left(\frac{1 - \alpha}{\alpha}\right) \ln(1 - \alpha) + \left(\frac{1}{\alpha}\right) \ln A. \end{aligned}$$

For the effect of a change in α on this expression one gets

$$\begin{aligned} \frac{d \ln(\hat{\gamma} + \rho)}{d\alpha} &= -\left(\frac{1}{\alpha^2}\right) \ln \alpha + \left(\frac{\alpha + 1}{\alpha^2}\right) - \left(\frac{1}{\alpha^2}\right) \ln(1 - \alpha) - \frac{1}{\alpha} - \left(\frac{1}{\alpha^2}\right) \ln A \\ &= \frac{1}{\alpha} \left[\frac{1}{\alpha} - \left(\frac{1}{\alpha}\right) \ln(\alpha(1 - \alpha)A) \right] = \frac{1}{\alpha} \left[\frac{1}{\alpha} - \ln \hat{\tau} \right]. \end{aligned}$$

As $\hat{\tau} < 1$ the expression is positive. Then $\frac{d \ln(\hat{\gamma} + \rho)}{d\alpha} > 0$ which implies $\frac{d\hat{\gamma}}{d\alpha} > 0$.

Redistributing, Pro-Labour. By equation (10) $\hat{\tau} = \rho$ so that $\frac{d\hat{\tau}}{d\alpha} = 0 = \frac{d\hat{\tau}}{dA}$. As $\lambda = 1 - \frac{[(1-\alpha)A]^{\frac{1}{\alpha}}}{\rho}$ it follows that $\frac{d\lambda}{dA} < 0$. Next, let $c \equiv (1 - \alpha)A$, then

$$\frac{d\lambda}{d\alpha} = -\frac{1}{\rho} \left[-\frac{Ac^{\frac{1-\alpha}{\alpha}}}{\alpha} - \frac{c^{\frac{1}{\alpha}} \ln c}{\alpha^2} \right] = \frac{c^{\frac{1}{\alpha}}}{\alpha\rho} \left[\frac{1}{1-\alpha} + \frac{\ln(1-\alpha) + \ln A}{\alpha} \right].$$

Suppose A is low (e.g. $A < e^{-\frac{\alpha}{1-\alpha}}$), then $\frac{d\lambda}{d\alpha} < 0$. Next, suppose $c \approx 1$, then $\frac{d\lambda}{d\alpha} > 0$. Thus, the sign of $\frac{d\lambda}{d\alpha}$ is ambiguous.

From equation (10) $r = \alpha A[(1 - \lambda)\tau]^{1-\alpha} = \frac{\alpha}{1-\alpha} [(1 - \alpha)A]^{\frac{1}{\alpha}}$ so that $\frac{d\hat{\gamma}}{dA} > 0$ under

that policy. Furthermore, rearrangement yields

$$\ln(\gamma + 2\rho) = \ln \alpha + \frac{1 - \alpha}{\alpha} \ln(1 - \alpha) + \frac{1}{\alpha} \ln A.$$

Taking the derivative yields

$$\frac{d \ln(\gamma + 2\rho)}{d\alpha} = \frac{1}{\alpha} - \frac{1}{\alpha^2} \ln[(1 - \alpha)A] - \frac{(1 - \alpha)}{\alpha(1 - \alpha)}$$

which is positive since $(1 - \alpha)A = (1 - \lambda)\tau < 1$ in (10). Hence, $\frac{d\check{\gamma}}{d\alpha} > 0$.

Non-Redistributing, Pro-Labour. For $\lambda = 0$ the optimal tax rate $\check{\tau}$ solves equation (11), that is, $z = \frac{\tau}{1 - \alpha} - \alpha A \tau^{1 - \alpha} - \rho = 0$. The partial derivatives of z are given by

$$z_\tau = \frac{1}{1 - \alpha} - (1 - \alpha)\alpha A \tau^{-\alpha} \quad \text{and} \quad z_\alpha = \frac{\tau}{(1 - \alpha)^2} - A \tau^{1 - \alpha} + \alpha A \tau^{1 - \alpha} \ln \tau$$

with z_τ being positive for all $\tau > \hat{\tau}$. As $z_A < 0$ it follows that $\frac{d\check{\tau}}{dA} = -\frac{z_A}{z_\tau} > 0$ under that policy. Furthermore,

$$\begin{aligned} \frac{d\tau}{d\alpha} = -\frac{z_\alpha}{z_\tau} &= -\frac{\frac{\tau}{(1 - \alpha)} - (1 - \alpha)A \tau^{1 - \alpha} + \alpha(1 - \alpha)A \tau^{1 - \alpha} \ln \tau}{1 - (1 - \alpha)^2 \alpha A \tau^{-\alpha}} \\ &= \frac{\tau \left[-\frac{1}{(1 - \alpha)} + (1 - \alpha)A \tau^{-\alpha} - \alpha(1 - \alpha)A \tau^{-\alpha} \ln \tau \right]}{1 - (1 - \alpha)^2 \alpha A \tau^{-\alpha}} \end{aligned}$$

where the first term in z_α is positive, but the sum of the other two terms is negative. However, $\check{\tau} \in \left((\alpha(1 - \alpha)A)^{\frac{1}{\alpha}}, ((1 - \alpha)A)^{\frac{1}{\alpha}} \right)$. Suppose $\check{\tau} \rightarrow (\alpha(1 - \alpha)A)^{\frac{1}{\alpha}}$ and $A = 1$. Then the $\frac{d\tau}{d\alpha}$ reduces to

$$\frac{\hat{\tau} (1 - 2\alpha)}{\alpha^2 (1 - \alpha)} - \frac{\hat{\tau} \ln \hat{\tau}}{\alpha}$$

which is the same expression as that for $\frac{d\hat{\tau}}{d\alpha}$ in (D1). Hence, $\frac{d\check{\tau}}{d\alpha} \geq 0$.

For the growth rate one finds $\frac{d\gamma}{dA} = r_A + (r_\tau - 1) \frac{d\tau}{dA} > 0$ if

$$\begin{aligned} \alpha \tau^{1 - \alpha} &> (1 - \alpha(1 - \alpha)A \tau^{-\alpha}) \left[\alpha(1 - \alpha)\tau (\tau^\alpha - \alpha(1 - \alpha)^2 A)^{-1} \right] \\ \tau^\alpha - \alpha^2 (1 - \alpha)^2 A &> (1 - \alpha)\tau^\alpha - \alpha^2 (1 - \alpha)^2 A \end{aligned}$$

which is equivalent to $1 > 1 - \alpha$ and true. Thus, $\frac{d\tilde{\gamma}}{dA} > 0$ if $\lambda = 0$ in (11). Furthermore,

$$\begin{aligned}\frac{d\tilde{\gamma}}{d\alpha} &= A\tau^{1-\alpha} - \frac{d\tau}{d\alpha} - \alpha A [\tau^{1-\alpha} \ln \tau] + \alpha(1-\alpha)A\tau^{-\alpha} \frac{d\tau}{d\alpha} \\ &= A\tau^{1-\alpha} - \alpha A [\tau^{1-\alpha} \ln \tau] - [1 - \alpha(1-\alpha)A\tau^{-\alpha}] \frac{d\tau}{d\alpha}.\end{aligned}$$

I want to show that $\frac{d\tilde{\gamma}}{d\alpha} > 0$ for any $\tilde{\tau} \in \left((\alpha(1-\alpha)A)^{\frac{1}{\alpha}}, ((1-\alpha)A)^{\frac{1}{\alpha}} \right)$. For that it suffices to show that $\frac{d\tau}{d\alpha} < A\tau^{1-\alpha}$, since $-\alpha A [\tau^{1-\alpha} \ln \tau]$ is non-negative. For the rest of the proof it is convenient to represent the solution space $\tilde{\tau}$ in the form

$$\tilde{\tau} = x((1-\alpha)A)^{\frac{1}{\alpha}} \quad \text{where } x \in \left(\alpha^{\frac{1}{\alpha}}, 1 \right) \Leftrightarrow \tilde{\tau} \in \left((\alpha(1-\alpha)A)^{\frac{1}{\alpha}}, ((1-\alpha)A)^{\frac{1}{\alpha}} \right).$$

Higher x implies a higher optimal $\tilde{\tau}$. I want to show that $\frac{d\tau}{d\alpha} < A\tau^{1-\alpha}$, that is,

$$\begin{aligned}\frac{\frac{\tau}{(1-\alpha)} - (1-\alpha)A\tau^{1-\alpha} + \alpha(1-\alpha)A\tau^{1-\alpha} \ln \tau}{1 - (1-\alpha)^2\alpha A\tau^{-\alpha}} &< A\tau^{1-\alpha} \\ \frac{\tau^\alpha}{A(1-\alpha)^2} - 1 + \alpha \ln \tau &< \frac{1}{1-\alpha} - \alpha(1-\alpha)A\tau^{-\alpha}.\end{aligned}$$

Substituting $\tilde{\tau}$ for τ yields

$$\begin{aligned}\frac{x^\alpha}{1-\alpha} - 1 + \alpha \ln \tilde{\tau} &< \frac{1}{1-\alpha} - \frac{\alpha}{x^\alpha} \\ \frac{\alpha}{x^\alpha} - 1 + \alpha \ln \tilde{\tau} &< \frac{1-x^\alpha}{1-\alpha}\end{aligned}$$

and holds since $\alpha \ln \tilde{\tau}$ is unambiguously negative, $\frac{\alpha}{x^\alpha} < 1$ and $x^\alpha > 1$ for all $x \in \left(\alpha^{\frac{1}{\alpha}}, 1 \right)$. Hence, $\frac{d\tilde{\gamma}}{d\alpha} > 0$.

References

- ALESINA, A., AND D. RODRIK (1994): "Distributive Politics and Economic Growth," *Quarterly Journal of Economics*, 109, 465–490.
- ASCHAUER, D. A. (1989): "Is Public Expenditure Productive?," *Journal of Monetary Economics*, 23, 177–200.
- BANERJEE, A., M. MARCELLINO, AND C. OSBAT (2000): "Some Cautions on the Use of Panel Methods for Integrated Series of Macro-Economic Data," Working Paper 170, IGIER, Bocconi University, Milano.
- BARRO, R. J. (1989a): "A Cross-Country Study of Growth, Saving, and Government," Working Paper 2855, NBER.
- (1989b): "Economic Growth in a Cross Section of Countries," Working Paper 3120, NBER.
- (1990): "Government Spending in a Simple Model of Endogenous Growth," *Journal of Political Economy*, 98, S103–S125.
- (1991): "Economic Growth in a Cross Section of Countries," *Quarterly Journal of Economics*, 106, 407–444.
- (1997): *Determinants of Economic Growth: A Cross-Country Empirical Study*. MIT Press, Cambridge, Mass.
- BARRO, R. J., AND J.-W. LEE (1994): "Sources of Economic Growth," *Carnegie Rochester Conference Series on Public Policy*, 40, 1–46.
- BARRO, R. J., AND X. SALA-I-MARTIN (1992): "Public Finance in Models of Economic Growth," *Review of Economic Studies*, 59, 645–661.
- (1995): *Economic Growth*. McGraw-Hill, New York.
- BERTOLA, G. (1993): "Factor Shares and Savings in Endogenous Growth," *American Economic Review*, 83, 1184–1198.
- BOURGUIGNON, F. (1981): "Pareto Superiority of Unegalitarian Equilibria in Stiglitz' Model of Wealth Distribution with Convex Saving Function," *Econometrica*, 49, 1469–1475.
- CASELLI, F., G. ESQUIVEL, AND F. LEFORT (1996): "Reopening the Convergence Debate: A New Look at Cross-Country Growth Empirics," *Journal of Economic Growth*, 1, 363–389.
- EASTERLY, W., AND S. REBELO (1993a): "Fiscal policy and economic growth. An empirical investigation," *Journal of Monetary Economics*, 32, 417–458.
- (1993b): "Marginal Income Tax Rates and Economic Growth in Developing Countries," *European Economic Review*, 37, 409–417.
- ENGEN, E. M., AND J. SKINNER (1992): "Fiscal Policy and Economic Growth," Working Paper 4223, NBER.
- ENGLE, R. F., D. F. HENDRY, AND J.-F. RICHARD (1983): "Exogeneity," *Econometrica*, 51, 277–304.
- HALL, R. E., AND C. I. JONES (1999): "Why Do Some Countries Produce So Much More Output Per Worker Than Others," *Quarterly Journal of Economics*, 114, 83–116.
- HESTON, A., R. SUMMERS, AND B. ATEN (2002): "Penn World Table Version 6.1," Discussion Paper October, Center for International Comparisons at the University of Pennsylvania.

- JONES, L. E., R. E. MANUELLI, AND P. E. ROSSI (1993): "Optimal Taxation in Models of Endogenous Growth," *Journal of Political Economy*, 101, 485–517.
- KALDOR, N. (1956): "Alternative Theories of Income Distribution," *Review of Economic Studies*, 48(5), 83–100.
- KING, R. G., AND S. REBELO (1990): "Public Policy and Economic Growth: Developing Neoclassical Implications," *Journal of Political Economy*, 98, S126–S150.
- KOESTER, R. B., AND R. C. KORMENDI (1989): "Taxation, Aggregate Activity and Economic Growth: Cross-Country Evidence on Supply-Side Hypotheses," *Economic Inquiry*, 27, 367–386.
- LEVINE, R., AND D. RENELT (1992): "A Sensitivity Analysis of Cross-Country Growth Regressions," *American Economic Review*, 82, 942–963.
- LEVINSOHN, J., AND A. PETRIN (2003): "Estimating Production Functions Using Inputs to Control for Unobservables," *Review of Economic Studies*, 70, 317–341.
- LUCAS, R. E. (1990): "Supply-Side Economics: An Analytical Review," *Oxford Economic Papers*, 42, 293–316.
- MANKIW, N. G., D. ROMER, AND D. N. WEIL (1992): "A Contribution to the Empirics of Economic Growth," *Quarterly Journal of Economics*, 107, 407–437.
- MCCLOSKEY, D. N. (1985): "The Loss Function Has Been Misaid: The Rhetoric of Significance Tests," *American Economic Review*, 75, 201–205.
- MCCLOSKEY, D. N., AND S. T. ZILIAK (1996): "The Standard Error of Regressions," *Journal of Economic Literature*, 34, 97–114.
- MILANOVIC, B. (2000): "The Median Voter Hypothesis, Income Inequality and Income Redistribution: An Empirical Test with the Required Data," Working Paper 256, Luxembourg Income Study, Syracuse, New York.
- OECD (2002): "Revenue Statistics 1965-2001," online data, OECD, Paris, available at <http://www.oecdwash.org/DATA/online.htm>.
- OLLEY, G. S., AND A. PAKES (1996): "The Dynamics of Productivity in the Telecommunications Equipment Industry," *Econometrica*, 64, 1263–1297.
- PASINETTI, L. (1962): "Rate of Profit and Income Distribution in Relation to the Rate of Economic Growth," *Review of Economic Studies*, 29, 267–279.
- PECORINO, P. (1993): "Tax structure and growth in a model with human capital," *Journal of Public Economics*, 52, 251–271.
- (1994): "The Growth Rate Effects of Tax Reform," *Oxford Economic Papers*, 46, 492–501.
- PEROTTI, R. (1994): "Income Distribution and Investment," *European Economic Review*, 38, 827–835.
- (1996): "Growth, Income Distribution, and Democracy: What the Data Say," *Journal of Economic Growth*, 1, 149–187.
- PERSSON, T., AND G. TABELLINI (1994): "Is Inequality Harmful for Growth?," *American Economic Review*, 84, 600–621.
- REBELO, S. (1991): "Long-Run Policy Analysis and Long-Run Growth," *Journal of Political Economy*, 99, 500–521.

- REHME, G. (2000): "Economic Growth and (Re-)Distributive Policies: A Comparative Dynamic Analysis," Working Paper ECO 00/13, European University Institute, Florence, Italy.
- (2004): "Endogenous Policy and Cross-Country Growth Empirics," mimeo, TU Darmstadt.
- (2006): "Endogenous (Re-)Distributive Policies and Economic Growth: A Comparative Static Analysis," mimeo, TU Darmstadt.
- SALA-I-MARTIN, X. (1996): "A Positive Theory of Social Security," *Journal of Economic Growth*, 1, 277–304.
- (1997): "I Just Ran Two Million Regressions," *American Economic Review, Papers and Proceedings*, 87, 178–188.
- SCHLICHT, E. (1975): "A Neoclassical Theory of Wealth Distribution," *Jahrbücher für Nationalökonomie und Statistik*, 189, 78–96.
- SPANOS, A. (1986): *Statistical Foundations of Econometric Modelling*. Cambridge University Press, Cambridge, UK.
- STOKEY, N. L., AND S. REBELO (1995): "Growth Effects of Flat-Rate Taxes," *Journal of Political Economy*, 103, 519–550.
- SUMMERS, R., AND A. HESTON (1991): "The Penn World Table (Mark 5): An Expanded Set of International Comparisons, 1950-1988," *Quarterly Journal of Economics*, 106, 327–368.
- TEMPLE, J. (1999): "The New Growth Evidence," *Journal of Economic Literature*, 37, 112–156.
- TREFLER, D. (1995): "The Case of Missing Trade and Other Mysteries," *American Economic Review*, 85, 1029–1046.

Data Appendix

Table 5: The Sample

	<i>TAX70</i>	<i>RRED</i>	<i>GA</i>	<i>LN70</i>	<i>INVPUB</i>	<i>GR</i>
Australia	0.225	0.28	0.614	9.600	0.064	0.018
Austria	0.346	n.a.	0.769	9.320	0.036	0.025
Belgium	0.345	0.49	0.849	9.400	0.025	0.023
Canada	0.308	0.25	0.846	9.560	0.029	0.021
Czech Republic	n.a.	0.50	0.489	n.a.	0.193	0.017
Denmark	0.392	0.41	0.758	9.670	0.029	0.017
Finland	0.319	0.37	0.619	9.330	0.033	0.025
France	0.341	0.29	0.816	9.410	0.031	0.020
Germany	0.323	0.33	0.747	9.420	0.030	0.021
Greece	0.224	n.a.	0.612	9.020	0.053	0.019
Hungary	n.a.	0.42	0.492	8.590	0.194	0.022
Iceland	0.269	n.a.	1.009	9.290	0.043	0.028
Ireland	0.288	0.32	0.833	8.890	0.037	0.043
Italy	0.261	0.27	0.905	9.320	0.027	0.022
Japan	0.200	n.a.	0.519	9.340	0.078	0.026
Korea	n.a.	n.a.	0.650	7.930	0.060	0.058
Luxembourg	0.249	0.34	0.615	9.620	0.049	0.036
Mexico	n.a.	n.a.	0.811	8.610	0.060	0.016
Netherlands	0.358	0.30	0.771	9.490	0.044	0.020
New Zealand	0.268	n.a.	0.639	9.520	0.061	0.011
Norway	0.345	0.37	0.445	9.320	0.074	0.030
Poland	n.a.	0.30	0.554	8.420	0.054	0.024
Portugal	0.194	n.a.	0.759	8.750	0.083	0.031
Slovak Republic	n.a.	0.51	0.490	n.a.	0.192	0.012
Spain	0.163	0.25	0.970	9.110	0.028	0.023
Sweden	0.387	0.43	0.708	9.600	0.041	0.016
Switzerland	0.225	0.16	0.497	9.920	0.042	0.009
United Kingdom	0.370	0.28	0.896	9.400	0.048	0.020
United States	0.277	0.20	1.000	9.710	0.026	0.023
Mean	0.290	0.34	0.713	9.243	0.061	0.023
SD	0.066	0.10	0.166	0.452	0.048	0.010

The growth rates for the Czech and Slovak Republic were calculated using data for former Czechoslovakia. The details on how the data were obtained can be found at <http://www.tu-darmstadt.de/~rehme/endopol/data-million.htm>.

Table 6: Plots of Selected Policy Variables and Growth Against Efficiency