POLICY ANNOUNCEMENTS AND WELFARE*

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Abstract

In this paper we show that the announcement of information can be detrimental to welfare. We consider an economy in which agents face idiosyncratic and aggregate risk. The policy maker learns about the aggregate shock before it directly impacts on the allocation, and can decide to announce that information early. Agents engage in risk-sharing contracts consistent with voluntary participation incentives. By early announcements the policy maker distorts agents' insurance possibilities, thereby increasing the variance of the optimal consumption allocation and worsening welfare ex-ante. As a particular application, we consider the problem of a monetary authority, which has the option of announcing shifts in the inflation target early. In this economy, monetary policy has real effects captured by a cash-in advance constraint. A fraction of firms need to set prices one period in advance, so that a late announcement of inflation target shifts results in welfare-reducing distortions of relative prices, if no idiosyncratic risk is present. However, with idiosyncratic risk of households – modeled as employment opportunities – we show that it may be better for the central bank to remain secretive.

JEL classification: D81, D86, E21, E52, E65.
Keywords: Social value of information, policy announcements, monetary policy, transparency

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1 Introduction

Should policy makers reveal what their future policies will be? The Economist (2004) replied *It's not always good to talk*, reflecting the literature on the social value of information that stresses coordination motives built upon Keynes's famous beauty contest (Morris and Shin (2002), and more recently Angeletos and Pavan (2007)). In that literature better public information may be undesirable due to the inefficient weight agents assign to public information relative to private information. In this paper we show that more precise public information on risks that are common to all agents may be detrimental to welfare even in the absence of a signal-extraction problem.

We consider an environment with idiosyncratic and aggregate risks. The idiosyncratic risks are insurable through a risk-sharing arrangement with voluntary participation. The arrangement is sustainable if in any period after knowing their idiosyncratic state, households choose not to walk away from the agreement. The lack of commitment creates a tension for high income households between current consumption and future benefits of the contract.

Information plays a crucial role in the trade-off between future insurance and current incentives. We introduce a public signal announced by a policy maker on the future aggregate state. The signal is common to all agents, and does not resolve households' idiosyncratic uncertainty.

As our main result we formally show that less precise public information about the aggregate future state is preferable over perfect public information. The mechanism is the following. The amount of the consumption good that the high income agents are willing to give up in the current period reflects future benefits of the contract relative to the outside option. In particular, if the signal indicates that the future state is likely to be one in which the benefits of the contract are relatively large, then the agents are willing to give up a larger share of current period consumption goods for these future benefits of the contract. Similarly, if the signal informs that the future state is likely to be one in which risk sharing benefits are low, then the high income agents are less likely to share their good fortune. Therefore, if the public signal is informative, the optimal consumption allocation spreads out to account for all possible realizations of the signal. On the other hand, when the signal is completely uninformative, the risk-sharing agreement is only contingent on idiosyncratic characteristics and aggregate resources. Comparing informative and completely uninformative signals, if all voluntary participation constraints for high income households are binding, high income agents obtain the same expected utility over realizations of the signal under perfect information as the utility under uninformative signals. For risk-averse households this implies that high income agents under perfect information consume more on average than under imperfect information on the future aggregate state. Correspondingly, from the resource constraint it follows that low income households are better off under imperfect information. Therefore, ex-ante risk averse agents prefer uninformative policy announcements.

As an application, we embed this mechanism into a monetary economy, in which households are subject to a cash-in-advance constraint, and in which a fraction of firms need to set prices one period in advance. Households face idiosyncratically fluctuating employment opportunities. In order to smooth these fluctuations, households engage in risk-sharing contracts with voluntary participation. The monetary authority is assumed to pursue a stochastically fluctuating inflation target. The target is known to the monetary authority one period in advance, and it may choose to release that information with certain precision. The precision of information affects the economy in two ways. First and more conventionally, more precise announcements allow sticky-price firms to set their prices correctly, thereby resulting in less price distortions and a better allocation of resources. Second - and this is the new effect here - more precise announcements distort risk sharing and thereby worsens the contractual insurance possibilities ex-ante. Furthermore, we show that the level of patience needed to sustain perfect risk sharing is strictly increasing in the precision of the public signal.

The question of the social value of information has been extensively studied in the literature. Morris and Shin (2002) show that better public information may be undesirable in the presence of private information if coordination of agents is driven by strategic complementarities of their actions. Angeletos and Pavan (2007) draw a general conclusion that the social value of information can be either way if the first best is different from the equilibrium under perfect information. In the environments analyzed in that literature, however, if public information is the only source of information, an increase in precision is always beneficial. Cuikerman and Meltzer (1986) show the undesirability of better information when the monetary authority has a different objective than the agents.

Our study is closely related to the literature on the relationship between risk sharing and information. Hirshleifer (1971) was among the first to point out that perfect information makes risk averse agents ex-ante worse off if this leads to an evaporation of risks that can be shared in a competitive equilibrium. Schlee (2001) shows under which general conditions better public information about tradable risks is Pareto inferior. In contrast, we consider the welfare effects of better public information about the non-tradable aggregate risks under incomplete markets.

The role of voluntary participation is emphasized by Kocherlakota (1996) in an environment without commitment. In this environment Kocherlakota explains the empirically observed positive correlation between income and consumption. Thomas and Worrall (1988) were among the first to study history-dependent contracts with risk-averse workers lacking commitment. The properties of stationary contracts in comparison to the first best are characterized by Coate and Ravallion (1993). Attanasio and Rios-Rull (2000) argue that in village economies where agreements are not enforceable, public insurance may crowd out private insurance arrangements. In comparison to that literature, we highlight the welfare effects of policy announcements on which agents form expectations about future states of the economy.

The remainder of the paper is organized as follows. In the next section we start with a simple two-period example to highlight the basic voluntary risk-sharing mechanism involved, and state out main result in that simple environment. In the third section we set up a model that integrates the mechanism into a monetary production economy with infinite horizon. In the following section we state the main results for that application. The last section concludes.

2 Two-period model

We set up a simple example that captures the interaction of individual incentives and the precision of public signals on aggregate risks. Assuming that participation in a risk-sharing agreement is voluntary we show that risk averse agents prefer completely uninformative public signals on the aggregate risk over perfectly informative signals.

Consider a two period pure exchange economy with two agents. In each period with equal probability one agent is endowed with a high income y^h and the other is endowed with low income y^l. Aggregate endowments can be further affected by government policy. In the second period the government can either tax away all the goods (type-b policy) or imposes no tax (type-g policy).

The preferences of both agents are given by

$$E[u(c_1) + \beta u(c_2)]$$

where c_1 and c_2 is consumption in the first and in the second period respectively, $\beta > 0$ is a discount factor, and u is a period utility function, which is assumed to be increasing and strictly concave.

If agents are able to commit, an optimal risk-sharing arrangement on which the agents may agree at date zero is perfect risk sharing.[1] The commitment requirement is crucial. After observing current endowments an agent with a high income may have an incentive to deviate from the perfect risk-sharing agreement, making such agreement unsustainable.

To capture this idea we analyze risk-sharing possibilities under two-sided lack of commitment by introducing voluntary participation constraints. These constraints characterize the trade off between first period consumption and the value of risk sharing provided by the contract in the second period. We require a limited sort of commitment, and consider agreements sustainable from a period 1 perspective.[2] An agreement is sustainable from a period 1 perspective if after observing the first period endowments each agent at least weakly prefers to stay in the agreement than to defect into autarky.

We confront two environments different in information precision about the future

[1] Throughout the section we focus on the socially optimal agreement with equal Pareto weights.

[2] If we do not require this limited sort of commitment, agents always consume their endowments in the second period.

policy. In the environment of perfect information agents know the second period government policy when they decide in the first period whether to sustain the risk-sharing agreement or deviate to autarky.

The voluntary participation constraints if information is perfect are given by

$$u(c^h_{1g}) + \beta\frac{1}{2}\left(u(c^{hh}_{2g}) + u(c^{hl}_{2g})\right) \geq u(y^h) + \beta\frac{1}{2}\left(u(y^h) + u(y^l)\right) \tag{1}$$

$$u(c^h_{1b}) + \beta u(0) \geq u(y^h) + \beta u(0) \tag{2}$$

$$u(c^l_{1g}) + \beta\frac{1}{2}\left(u(c^{lh}_{2g}) + u(c^{ll}_{2g})\right) \geq u(y^h) + \beta\frac{1}{2}\left(u(y^h) + u(y^l)\right) \tag{3}$$

$$u(c^l_{1b}) + \beta u(0) \geq u(y^l) + \beta u(0), \tag{4}$$

where c^i_{1k} is period-1 consumption of an agent with y^i first period endowment under k-type government policy, and c^{ij}_{2k} is period-2 consumption of an agent with y^i endowment in the first period and y^j endowment in the second period. The first two constraints are relevant for the agent with high first period income and the latter describe the agent with low first period income. The left hand side of each constraint constitutes utility of staying in the contract, and the right hand side is the outside option of living in autarky.

The resource feasibility constraints are

$$c^h_{1g} + c^l_{1g} = c^h_{1b} + c^l_{1b} = c^{hh}_{2g} + c^{ll}_{2g} = c^{hl}_{2g} + c^{lh}_{2g} = y^h + y^l.$$

The second environment is set to represent completely imperfect information. In the first period after observing their current endowments – without knowing the government policy in the second period – agents decide about their participation in the risk-sharing agreement. Correspondingly, the voluntary participation constraints read

$$u(c_1^h) + \beta\frac{1}{4}\left(u(c_{2g}^{hh}) + u(c_{2g}^{hl}) + 2u(0)\right) \geq u(y^h) + \beta\frac{1}{4}\left(u(y^h) + u(y^l) + 2u(0)\right) \tag{5}$$

$$u(c_1^l) + \beta\frac{1}{4}\left(u(c_{2g}^{lh}) + u(c_{2g}^{ll}) + 2u(0)\right) \geq u(y^l) + \beta\frac{1}{4}\left(u(y^h) + u(y^l) + 2u(0)\right), \tag{6}$$

where c_1^i is period-1 consumption of agent with y^i first period endowment, and resource feasibility requires

$$c_1^h + c_1^l = c_{2g}^{hh} + c_{2g}^{ll} = c_{2g}^{hl} + c_{2g}^{lh} = y^h + y^l.$$

Our goal is to highlight that additional aggregate information is harmful for the optimal insurance of idiosyncratic risks under voluntary participation.

Theorem 1 *Under completely imperfect information social welfare is strictly higher than under perfect information about future government policies.*

Proof. One can distinguish three cases depending on which participation constraints are binding. If under perfect information both participation constraints are binding for the high endowment agent then it follows immediately from the maximization problem that

$$c_{1g}^h = c_{2g}^{hh} = c_{2g}^{hl}$$

and similarly if under imperfect information the participation constraint for the high endowment agent is binding then

$$c_1^h = c_{2g}^{hh} = c_{2g}^{hl}$$

We thus compare the environments in terms of the first period allocations.

From (1), (2) and (5) we get that

$$\left(\frac{1}{2}+\frac{\beta}{2}\right)u(c_{1g}^h)+\frac{1}{2}u(c_{1b}^h)=\left(1+\frac{\beta}{2}\right)u(c_1^h), \tag{7}$$

i.e. the agent with high first period endowment obtains the same expected utility in both environments. From (7) strict concavity implies that

$$\left(\frac{1}{2}+\frac{\beta}{2}\right)c_{1g}^h+\frac{1}{2}c_{1b}^h>\left(1+\frac{\beta}{2}\right)c_1^h. \tag{8}$$

For for the expected utility of the agent with a low income in the first period under perfect and imperfect information it follows

$$\begin{aligned}&\left(\frac{1}{2}+\frac{\beta}{2}\right)u(c_{1g}^l)+\frac{1}{2}u(c_{1b}^l)<\left(1+\frac{\beta}{2}\right)u\left(\frac{1+\beta}{2+\beta}c_{1g}^l+\frac{1}{2+\beta}c_{1b}^l\right)\\&=\left(1+\frac{\beta}{2}\right)u\left(y-\frac{1+\beta}{2+\beta}c_{1g}^h-\frac{1}{2+\beta}c_{1b}^h\right)<\left(1+\frac{\beta}{2}\right)u(y-c_1^h)=\left(1+\frac{\beta}{2}\right)u(c_1^l),\end{aligned} \tag{9}$$

where the first inequality is due to strict concavity and the second one is implied by (8). Thus, the agent with low endowment is strictly better off under completely imperfect information. Adding up (7) and (9) we get that imperfect information is preferable for this case.

To complete the proof, if the participation constraints in the environment of imperfect information are not binding, then the optimal allocation in the environment is perfect risk sharing. This outcome is preferable to the one under perfect information where the first best is not incentive compatible because the participation constraints for a high tax future policy (2) and (4) always hold with equality. If the participation constraint

(1) is not binding but (5) does bind, imperfect information is still preferable. It can be seen that as agents become more patient the first period allocation for perfect information can not be improved upon, but under imperfect information the agents' utility is increasing towards the first best. ■

In the next section we embed this mechanism into a richer environment with a monetary authority which announces a signal on its future inflation target. In that application we extend the simple example in several dimensions. First, we abstract from any commitment and extend time horizon to infinite. Second, we allow for continuity in information precision, which affects agents' decisions twice. On the one hand it influences households' optimal risk-sharing possibilities under voluntary participation, and on the other hand it plays a role for the optimal pricing decisions of monopolistic competitive firms.

3 Environment

In this section we integrate the voluntary risk-sharing mechanism into a monetary production economy. We proceed in two steps. First, we present the environment and describe the equilibrium for given risk-sharing transfers among households. Second, we set up the social planner's problem to determine the optimal pure insurance transfers under voluntary participation.

We consider a production economy with a continuum of households of measure one and a single perishable consumption good.

Households are identical ex-ante. Household's preferences over the stream of consumption are given by

$$E\left[\sum_{t=0}^{\infty}\beta^t u(c_t^i)\right] \tag{10}$$

where c_t^i is consumption of household i in period t, $0 < \beta < 1$ is the time discount factor, and u is the period utility function. We assume the period utility function to be twice-differentiable, increasing, and strictly concave.

Each household consists of two members: a shopper and a worker. Each period the worker earns idiosyncratic income, inelastically supplying one unit of labor to one of the two production sectors, and the shopper buys consumption goods. Money is the only means for facilitating transactions and transferring wealth across periods. The period budget constraint of household i is

$$M_t^i + p_t c_t^i = M_{t-1}^i + p_t w_t^f + d_t^i + p_t \tau_t^i, \tag{11}$$

where M_t^i are nominal money holdings at the end of period t, d_t^i are nominal profits distributed to the household, τ_t^i are real transfers prescribed by a risk-sharing contract, w_t^f is the real wage in production sector f, and p_t is the aggregate price level.

Shopper and worker are separated within a period by different activities. A worker earns money and gives it to the shopper for purchasing consumption goods in the next period. A shopper is required to exchange all the money for consumption goods[3]

$$p_t x_t^i = M_{t-1}^i, \tag{12}$$

[3]Alternatively, the cash-in-advance constraint can be stated with inequality and restrictions on the set of risk-sharing transfers are imposed (for a sufficiently large lower bound on inflation) such that the cash-in-advance constraint is binding in any equilibrium.

where $x_t^i = c_t^i - \tau_t^i$ is the amount of the consumption good directly bought in the market.

The production part of the economy is represented by final good firms and intermediate good firms. In each period there are two final good firms, which both produce the identical consumption good by aggregating over differentiated intermediate goods, specific to that final good firm, according to

$$y_t^f = \left(\int_0^1 (y_t^{fj})^{1-\rho} dj \right)^{1/(1-\rho)}, \tag{13}$$

where y_t^f is the amount of the consumption good produced by final good firm f, y_t^{fj} is an intermediate good produced by differentiated good firm fj, and ρ is the inverse of the elasticity of substitution between differentiated goods.

For each final good firm f, intermediate goods are produced by a continuum of differentiated good firms. The production technology of the differentiated good firms is given by

$$y_t^{fj} = a_t^f l_t^{fj}, \tag{14}$$

where l_t^{fj} is the labor input. The productivity of the differentiated firms a_t^f is the same for each final good firm, but different across the final firms. A final good firm and the corresponding intermediate good firms constitute a production sector.

Acting under perfect competition, final good firms minimize costs by choosing the factor demand for each intermediate good to satisfy aggregate demand. The cost minimization problem is

$$\min \int p_t^{fj} y_t^{fj} dj \tag{15}$$

subject to the technology constraint (13), where the final firm f chooses the input of intermediate goods y_t^{fj} taking prices of the goods p_t^{fj} as given.

The intermediate good producers act under monopolistic competition. A measure λ of monopolistically competitive firms maximize profits subject to the actual demand of their product. The rest of the firms need to preset prices one period in advance. These non-flexible price firms maximize expected profits based on the public signal on future inflation.

The profit maximization problem of the flexible price monopolistically competitive firms is

$$\max(p_t^{fj} y_t^{fj} - p_t w_t^f l_t^{fj}) \tag{16}$$

given nominal sector wages and the demand of the final good firm, and subject to the production technology (14). Similarly, price presetting firms maximize expected profits

$$\max E_{t-1}[p_t^{fj} y_t^{fj} - p_t w_t^f l_t^{fj} | s_{t-1}] \tag{17}$$

by setting period t price p_t^{fj} in period $t-1$, given a public signal s_{t-1} on period t inflation.

In each period, each worker is randomly assigned either to be employed in the sector of high productivity a^h, or to work for firms with low productivity a^l. After selling the final goods to the shoppers, labor income and profits of the monopolistically competitive firms are equally distributed among workers of that sector.

Money is issued by the monetary authority that follows a stochastic inflation target. The stochastic properties of the inflation target process are known to all agents in the economy. In addition, the monetary authority knows the inflation target one period in advance, and provides a public signal on the inflation target with certain precision. The exogenous process for the inflation target is given by an i.i.d process with two states of equal probability: high inflation state π_h and low inflation state π_l.[4]

The inflation process coincides with the target by appropriate money injections. Seigniorage is spent for government purposes.[5] The government budget constraint is

$$p_t g_t = M_t - M_{t-1}, \tag{18}$$

where g_t denotes real government expenditures, and M_t is the aggregate money supply. The exogenous process for the inflation target is given by an i.i.d process with two states of equal probability: high inflation state π_h and low inflation state π_l. Similarly, the public signal on next period inflation takes two values, a high realization s_h and a low realization s_l. The precision of the public signal is given by $\kappa \equiv \text{Prob}[\pi_j|s_j]$, with $1/2 \leq \kappa \leq 1$.

An *equilibrium with incomplete markets* is an allocation $\{c_t^i, x_t^i, M_t^i, d_t^i, y_t^f, y_t^{fj}, M_t, g_t\}$ and a price system $\{p_t, p_t^{fj}, w_t^f\}$ such that given exogenous processes for the inflation target $\{\pi_t\}$, the public signal $\{s_t\}$, and assignments of households to production sectors $\{a_t^i\}$, the risk-sharing contract transfers $\{\tau_t^i\}$, and initial conditions for the distribution of nominal money balances $\{M_{-1}^i\}$, initial price setting of non-flexible price firms $\{p_0^{fj}\}$, and initial aggregate price level normalization $p_{-1} = 1$

(i) for each household i an allocation $\{c_t^i, x_t^i, M_t^i\}$ maximizes household's utility (10) subject to the budget constraint (11) and the cash-in-advance constraint (12), given prices $\{p_t, w_t^f\}$ and profits $\{d_t^i\}$,

(ii) for each production sector f the production allocation $\{y_t^f, y_t^{fj}\}$, prices $\{p_t, p_t^{fj}, w_t^f\}$ and profits $\{d_t^i\}$ solve the cost minimization problem of the final good firms (15),

[4] The inflation process and productivity are assumed to be non-degenerate $\pi_l < \pi_h$ and $a^l < a^h$.

[5] Alternatively, when seigniorage is equally distributed back to households our main results stay valid.

and the profit maximization problems of the differentiated good firms (16) and (17),

(iii) monetary injections are consistent with the inflation target

$$p_t = \pi_t p_{t-1},$$

(iv) the government budget constraint is fulfilled, and

(v) markets are clear

$$\int c_t^i di + g_t = \int y_t^f df, \quad \int M_t^i di = M_t, \quad \int l_t^{fj} dj = \frac{1}{2}.$$

In the following we assume that the low realization of the inflation target is large enough to satisfy the resource feasibility with non-negative government expenditures.

The main element of our model is households' risk-sharing contract under voluntary participation. Without risk-sharing transfers the consumption allocation that results from the rational expectation equilibrium is not efficient from an ex-ante perspective due to market incompleteness which prevents households from optimal borrowing and lending. However, the efficient use of a complete set of securities requires commitment or enforceability of the arrangements. In the absence of commitment the consumption allocation can still be improved by risk-sharing transfers consistent with voluntary participation incentives. We set up a social planner problem to determine the optimal transfers under voluntary participation in the equilibrium with incomplete markets.

Voluntary participation in social insurance provided by the risk-sharing transfers means that in each period households may decline the offered risk-sharing contract. In such a case they live in an economy with no transfers, consuming only the goods bought directly in the market.

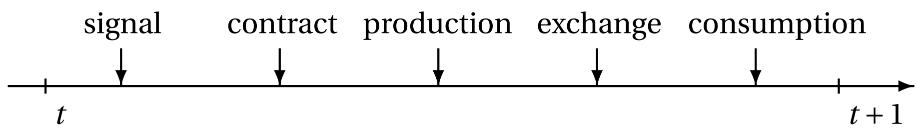

Figure 1: Timing of events

The timing of events is illustrated in Figure 1. In each period, first, agents obtain a public signal on next period's inflation target, and become aware of the current period inflation target.[6] Second, households decide on sustaining a risk-sharing contract that prescribes transfers $\{\tau_t^i\}$. Third, workers inelastically supply their labor into the production process. Fourth, market exchange takes place. Flexible price monopolistic firms set price for the current period, shoppers receive consumption goods in exchange for money balances held from the previous period, workers receive wages and shares of profits and the government collects seigniorage from money injections. Fifth, among shoppers an exchange according to the risk-sharing contract takes place. Finally, members of each household meet together, consume, money balances are passed from worker to shopper for next period consumption purchases, and non-flexible price firms preset prices for the next period based on the public signal on the future inflation target.

Formally, the risk-sharing contract is built upon the consumption allocation $\{x_t^i\}$ of the incomplete market equilibrium with no transfers as the outside option. Let the individual public state at time t be $h_t^i = (x_t^i, x_t^{-i}, s_t)$, where s_t is the public signal about inflation in period $t+1$. A consumption allocation $\{c_t^i\}$ is *sustainable* if there exist trans-

[6]An alternative timing of events that leads to exactly the same results and does not require the awareness of current period inflation includes shoppers' trading first, followed by the risk sharing contract decision, and workers' realization of income.

fers $\{\tau_t^i\}$ such that

(i) the consumption allocation $\{c_t^i\}$ solves the rational expectation equilibrium with the transfers $\tau_t^i(h_t^i)$

(ii) for each household i and state h_t^i, the consumption allocation $\{c_t^i\}$ is weakly preferable to the outside option $\{x_t^i\}$, which solves the rational expectation equilibrium with no transfers

$$E\left[\sum_{j=0}^{\infty} \beta^{t+j} u(c_{t+j}^i)|h_t^i\right] \geq E\left[\sum_{j=0}^{\infty} \beta^{t+j} u(x_{t+j}^i)|h_t^i\right] \tag{19}$$

(iii) and the transfers $\{\tau_t^i\}$ are resource-feasible

$$\int \tau_t^i(h_t^i) di = 0. \tag{20}$$

The key element of the information set in period t is the public signal on inflation provided by the monetary authority. The signal helps to resolve inflation uncertainty for the agents.

We define the *socially optimal contract under voluntary participation* as a consumption allocation $\{c_t^i\}$ that provides the highest expected utility among the set of sustainable allocations.[7]

It is natural to compare the optimal contract under voluntary participation to an optimal contract under commitment. We define the *benchmark allocation* as a consump-

[7] We restrict our analysis to pure insurance arrangements as emphasized by Kimball (1988), Coate and Ravallion (1993), and Ligon et al. (2002). This precludes a lending element in the risk-sharing arrangements. A household that receives a transfer may be willing to "pay back" the donor by accepting a less favorable transfer agreement in the future. This in turn may induce a higher transfer from the donor today and may result in better risk sharing.

tion allocation that provides the highest expected utility among the set of equilibrium consumption allocations for resource-feasible transfers $\{\tau_t^i\}$.

4 Results under flexible prices

In this section we focus on the effect of announcements about future monetary policy on the outcome of the socially optimal risk-sharing contract under voluntary participation. To highlight this effect we abstain from the effect of public signals on optimal pricing decisions of presetting firms. We avoid the pricing friction on the firm side by assuming that all intermediate firms are flexible price firms.

As our main result we show that better precision is undesirable because it harms individual risk-sharing possibilities. In addition, we show that under more informative signals perfect risk sharing requires a higher degree of patience to be supported as a sustainable allocation.

In the absence of any price presetting firms, and due to constant labor supply, the income of household i earned in period t depends only on the productivity f of the sector where the worker is employed. The income consists of labor income and profits and is given in real terms by $w_t^f + d_t^i / p_t = a^f$. Correspondingly, the disposable income before risk-sharing transfers is

$$x_t^i(\pi_j) = a^f / \pi_j,$$

when inflation in period t is $p_t / p_{t-1} = \pi_j$. In the equilibrium with incomplete markets the risk-sharing transfers directly affect the consumption allocation

$$c_t^i(\pi_j, s_k) = a^f / \pi_j + \tau(a^f, \pi_j, s_k),$$

for a period t signal indicating an inflation rate π_k in the next period.

As an initial point of our analysis we show that the optimal contract exists and is unique by employing the theorem of the maximum. For two productivity states, two inflation states, and two signals on next period's inflation rate, the sustainable allocations $\{c^i(\pi_j, s_k)\}_{i,j,k=\{h,l\}}$ fulfill the participation constraints (19), which for a high inflation signal s_h are written as

$$u(c^i(\pi_j, s_h)) + \left(\beta\kappa + \frac{\beta^2}{1-\beta}\frac{1}{2}\right) V_{rs}(\pi_h) + \left(\beta(1-\kappa) + \frac{\beta^2}{1-\beta}\frac{1}{2}\right) V_{rs}(\pi_l) \geq \\ u(x^i(\pi_j)) + \left(\beta\kappa + \frac{\beta^2}{1-\beta}\frac{1}{2}\right) V_{at}(\pi_h) + \left(\beta(1-\kappa) + \frac{\beta^2}{1-\beta}\frac{1}{2}\right) V_{at}(\pi_l), \quad (21)$$

and for a low inflation signal s_l are given by

$$u(c^i(\pi_j, s_l)) + \left(\beta(1-\kappa) + \frac{\beta^2}{1-\beta}\frac{1}{2}\right) V_{rs}(\pi_h) + \left(\beta\kappa + \frac{\beta^2}{1-\beta}\frac{1}{2}\right) V_{rs}(\pi_l) \geq \\ u(x^i(\pi_j)) + \left(\beta(1-\kappa) + \frac{\beta^2}{1-\beta}\frac{1}{2}\right) V_{at}(\pi_h) + \left(\beta\kappa + \frac{\beta^2}{1-\beta}\frac{1}{2}\right) V_{at}(\pi_l), \quad (22)$$

for each productivity realization i and current inflation state j, where the value of the contract in j-inflation state is

$$V_{rs}(\pi_j) = \sum_{i,k} \frac{1}{4} u(c^i(\pi_j, s_k)),$$

and the value of the equilibrium allocation without transfers in inflation state j is

$$V_{at}(\pi_j) = \sum_{i} \frac{1}{2} u(x^i(\pi_j)).$$

Due to strict concavity of the utility function, the set of allocations that satisfy the participation constraints is convex. The implications for the optimal contract are stated formally in the next lemma.

Lemma 1 *The socially optimal contract exists and is unique. The contract and the social welfare are continuous functions in the precision of the public signal.*

Proof. For any precision of the public signal, the set of sustainable allocations is nonempty and compact-valued. The outside option allocation is always in the set of sustainable allocations, and the restrictions imposed by the participation constrains and consumption feasibility define a bounded and closed set. Furthermore, it can be shown that the sustainable set is a continuous correspondence of the signal precision. Given that the objective function is continuous, by the Theorem of the Maximum (Berge, 1963) there exists a solution to the optimal contract problem for any public signal precision, and the highest expected utility is continuous in signal precision.

In addition, the set of sustainable allocations is convex-valued due to the concavity of the utility function, and the objective function is strictly concave. By the Maximum Theorem under Convexity the optimal contract is unique and continuous in signal precision. ■

Since households are ex-ante the same, the benchmark allocation is perfect risk sharing $c_t^i = (x_t^h + x_t^l)/2$ for all households i. This allocation may not be sustainable due to the additional restrictions which are brought up by the lack of commitment inherent in voluntary participation. Notably, among the restrictions only participation constraints of high productivity agents can be binding for the optimal contract. Except if the only sustainable allocation is the no-transfer equilibrium, low productivity households always gain of staying in the contract relative to their outside option because the optimal risk-sharing contract prescribes transfers from high productivity households.

Though voluntary participation imposes additional restrictions to the optimal contract, this does not mean that the benchmark allocation is never attainable. Indeed, perfect risk sharing may still be the socially optimal contract if the discount factor β is high enough. This result, commonly known as the Folk Theorem is established in the following lemma.

Lemma 2 *If perfect risk sharing is the socially optimal contract for discount factor $\bar{\beta}$ and for any signal precision, then for any $\beta \geq \bar{\beta}$ the optimal contract is perfect risk sharing.*

Proof. Perfect risk sharing provides the highest ex-ante utility. In the participation constraints a higher β increases the future value of perfect risk sharing relative to the allocation in the equilibrium without transfers, leaving the current incentives to deviate unaffected. Therefore, if the participation constraints are not binding for $\bar{\beta}$, they are not binding for any $\beta \geq \bar{\beta}$. ■

Furthermore, we can characterize the role of signal precision on the sustainability of perfect risk sharing by the following proposition. We show that the level of patience that is needed to sustain perfect risk sharing $\bar{\beta}(\kappa)$ increases in precision of the signal.

Proposition 1 *Let $\bar{\beta}(\kappa)$ be the cut-point such that for each $\beta \geq \bar{\beta}(\kappa)$ perfect risk sharing is the socially optimal contract. The cut-point $\bar{\beta}(\kappa)$ is increasing in the precision of the public signal.*

Proof is provided in Appendix A.1.

This result is driven by the gain the optimal contract offers relative to the equilibrium in the absence of transfers for different inflation rates. This gain can be higher either under low or under high inflation. Under low inflation, aggregate resources are higher, which tends to scale up the value of the contract, the value of the outside option, and the gain of the contract relative to the allocation of the no-transfer equilibrium. We refer to this effect as the wealth effect. On the other hand, under high inflation the lower resources may lead to the higher benefits of risk sharing relative to the outside option if the curvature of the utility function is high. We name this effect the risk aversion effect.

If the wealth effect and the risk aversion effect do not cancel out, the cut-off value is strictly increasing in precision. In this case, among the participation constraints for high productivity households there is one which imposes the tightest restriction. Moreover, this constraint always provides a stronger restriction under informative signals than under uninformative signals. Suppose that the wealth effect dominates, which means that the relative gain of the optimal contract is lower under high inflation. While the current period loss of staying in the contract is independent of signal precision, under the high next period inflation signal the expected future gain for high productivity agents is lower for informative signals than for uninformative signals. Therefore, the level of patience needed to sustain the perfect risk sharing allocation is higher under an informative signal.

On the lower end of sustainable contract, if the level of patience is relatively low, the set of sustainable allocations may shrink to one point, the equilibrium allocation in the absence of transfers. As a characteristic of the optimal contract relative to the outside option, we point out that the value of the arrangement can not be lower than the value of the allocation in the no-transfer equilibrium for any inflation state.

Lemma 3 *The socially optimal contract satisfies* $V_{rs}(\pi_j) - V_{at}(\pi_j) \geq 0$ *for all inflation states* π_j.

Proof is provided in Appendix A.2.

In analogy to Lemma 2, if the equilibrium with no transfers is the only sustainable allocation for a certain level of patience then the socially optimal allocation is again the outside option if households are even less patient.

Lemma 4 *If for a certain discount factor $\underline{\beta}$ the equilibrium allocation in the absence of transfers is the socially optimal contract for any signal precision, then for any $\beta \le \underline{\beta}$ the socially optimal contract is the equilibrium allocation in the absence of transfers.*

Proof. Assume that for some $\beta \le \underline{\beta}$ there exists an optimal contract different from the equilibrium allocation with no transfers. The contract allocation is sustainable. By Lemma 3, the value of this contract is at least as good as the value of defecting into the outside option. Then for $\underline{\beta}$ the allocation is also sustainable since the value of the contract other than the outside option gets an even higher weight in the participation constraints. This contradicts that for $\underline{\beta}$ the optimal contract is the no-transfer equilibrium allocation. ■

If the optimal contract prescribes either perfect risk sharing or the allocation in the absence of transfers, the influence of information precision is limited to households' perception of given allocations in participation constraints.

A number of studies indicate that more realistic is the case when risk sharing is neither perfect nor absent but partial.[8] This case is analyzed below. We show that the transfers prescribed by the contract are a function of precision, and the signal can shape the resulting consumption allocation significantly. As our main novel result we show that precision in public signals harms social welfare.

[8]See e.g. Townsend (1994) or more recently Ligon et al. (2002).

If perfect risk sharing is not sustainable, a number of participation constraints of high productivity agents are binding. Which constraints are binding depends on the current gain from deviation and the future value of the contract. We focus on the case when all constraints are binding and state below sufficient conditions for such case to apply.

Lemma 5 *If all participation constraints for high productivity agents are violated under perfect risk-sharing contract then all the constraints are binding under the optimal contract.*

Proof is provided in Appendix A.3.

Binding participation constraints imply that perfect risk sharing is not optimal, however on the other hand, the optimal contract may be given by another extreme, which is outside option. In the following lemma we provide conditions under which there exists a socially optimal contract different from the consumption allocation in the absence of transfers. In particular, we consider a situation when the signal is uninformative.

Lemma 6 *Consider the case of an uninformative public signal. If all participation constraints for high productivity agents are binding for the optimal contract and*

$$\frac{1}{2}\left(\frac{u'(x^l(\pi_h))}{u'(x^h(\pi_h))}+\frac{u'(x^l(\pi_l))}{u'(x^h(\pi_l))}\right)>\frac{2-\beta}{\beta}$$

then the socially optimal contract is not the consumption allocation of the equilibrium in the absence of transfers.

The proof is provided in Appendix A.4.

As our main result, we provide conditions for social welfare to be decreasing in the precision of the public signal. If the optimal contract is either perfect risk sharing or outside option at then the signal precision does not directly affect the contract and social welfare. Lemmas 5 and 6 provide sufficient conditions for an socially optimal contract other than perfect risk sharing or the outside option. Given this, we show that the social welfare is decreasing in signal precision in a neighborhood of the perfectly uninformative signal.

Theorem 2 *If for any precision of the public signal all participation constraints for high productivity agents are binding for the optimal contract and the equilibrium allocation in the absence of transfers is not the only sustainable contract, then there exists a neighborhood of an uninformative signal in which social welfare is decreasing in the precision of the public signal.*

The proof can be found in Appendix A.5.

The negative influence of more precise signals on social welfare can be illustrated as follows. Assume that under an uninformative signal the wealth effect dominates the risk aversion effect, i.e. the optimal contract provides higher value relative to the equilibrium allocation without transfers under low inflation than under high inflation. Suppose that the realized signal indicates that the next period inflation is more likely to be high. The signal reveals that the future value of the contract is lower, which is an unfavorable outcome for all households. Therefore the high productivity agents require higher current period consumption for any current inflation rate. On the contrary, under the low inflation signal, which indicates the brighter future, the high productivity agents can be satisfied with lower current period consumption. Adding up, the consumption prescribed by the optimal contract diverges under different signals. From an ex-ante perspective the signal precision increases the consumption cross-variance, therefore risk aversive agents prefer less informative signals.

The negative value of information does not depend on whether the wealth effect or the risk aversion effect is dominant. If the risk aversion effect were dominating, the high productivity agents would require lower current period consumption following a high inflation signal, and would demand higher current period consumption following a low signal. Nonetheless, from an ex-ante perspective such divergences is still welfare decreasing for risk-averse agents.

The effect of signal's precision on social welfare due to risk sharing is of second order. Up to first order the change in consumption for high and low inflation signals is exactly opposite. In addition, there is a second order effect of the signal precision on consumption, which is positive for all high productivity agents, and negative for all low productivity agents. The effect moves the consumption of heterogeneous households further apart, and leads to a decrease in ex-ante utility of risk-averse households.[9]

While we prove in Theorem 2 that the social value of information is negative in a neighborhood of an uninformative signal, we have numerical evidence that indicates that this result holds globally. In our numerical example we consider CRRA-preferences and calibrate the inflation process to match variance and mean of the U.S. postwar consumer price index.[10]

We characterize how the precision of public signals affects optimal insurance under voluntary participation when prices are flexible. If the optimal contract is partial risk sharing, the precision of the signal effectively influences the distribution of con-

[9]The importance of second order terms in the model's solution for welfare analysis is highlighted by Kim and Kim (2003) and Woodford (2003).

[10]The results of this exercise are available on request.

sumption in the risk-sharing arrangement. We show that higher precision in signals is socially undesirable because this increases the variance of consumption across states. In addition, we find that the level of patience needed to sustain the perfect risk sharing allocation is strictly increasing in the precision of the signal. While the social value of information under flexible prices is negative, more precision can be welfare improving if not all prices are perfectly flexible.

5 Results under imperfectly flexible prices

In the previous section we abstracted from any pricing friction to show the negative social value of precise information. To capture positive effects of better information we introduce a positive fraction of intermediate good producers that preset their prices one period in advance (Woodford, 2003). We show that more precise information leads to an increase in aggregate resources such that the social value of information is positive.

Solving the cost minimization problem of the perfectly competitive final good firms (15) we get the demand for each of the variety goods

$$y_t^{fj} = \left(\frac{p_t^{fj}}{p_t}\right)^{-1/\rho} y_t^f, \tag{23}$$

where the aggregate price level is defined by

$$p_t = p_t^f \equiv \left(\int_0^1 (p_t^{fj})^{1-1/\rho} dj\right)^{1/(1-1/\rho)}. \tag{24}$$

Using the production technology (14), the final good firm demand (23), and integrating over all monopolistically competitive firms in the sector, production per worker in sector f is given by

$$y_t^f = \frac{a^f}{\int \left(\frac{p_t^{fj}}{p_t} \right)^{-1/\rho} dj}, \tag{25}$$

with $\int \left(\frac{p_t^{fj}}{p_t} \right)^{-1/\rho} dj \geq 1$ by Jensens' inequality. The highest level of production is achievable if all differentiated good firms set the same price, $p_t^{fj} = p_t$, which is the case if all firms are flexible in their pricing decision.

The pricing decision of monopolistically competitive firms is divided in two groups. A share λ of firms of each type set price according to actual demand (26). The other $(1 - \lambda)$ firms preset prices a period ahead based on the public signal on inflation by solving the expected profit maximization problem (17). For the flexible price monopolistically competitive firms, solving the profit maximization problem (16) we get

$$p_t^{fj} = \mu \frac{w_t^f}{a^f} p_t, \tag{26}$$

where $\mu = 1/(1-\rho)$ is a fixed mark-up above real marginal costs.

When there is a positive measure of presetting firms, aggregate resources are not longer determined by productivity alone. Instead, current period production depends in addition on the accuracy of pricing decisions of firms that had to set their prices in the previous period. As an intermediate step of our analysis to establish the positive social value of public information, we show that aggregate resources are increasing in the precision of public signal.

Proposition 2 *There exists a neighborhood of perfectly flexible prices, in which expected aggregate resources are strictly increasing in the precision of the public signal.*

The proof is provided in Appendix A.6.

The intuition for this result is the following. The less precise the signal is the larger is the inflation prediction error by presetting firms. As a result, the prices set by presetting firms differ more from those set by flexible price firms. The resulting dispersion in relative prices of differentiated goods diminishes resources available for consumption, as it can be seen from (25).

In turn, the expected utility of households is also increasing in precision, unless households are too risk averse. We state this result in the following Proposition.

Proposition 3 *Consider the cases of perfect risk sharing or outside option being the socially optimal contract for any public signal precision. Assume that preferences are characterized by a relative risk aversion of less or equal then 2. Then there exists a neighborhood of perfectly flexible prices in which social welfare is increasing in the precision of the public signal.*

The proof is provided in Appendix A.7.

This result provides sufficient conditions for better information to be socially valued. By assuming that either perfect risk sharing or outside option is the socially optimal contract for any precision, we exclusively consider the pricing mechanism.

There are two effects on welfare if signal's precision increases. When the signal gets more precise firms that preset price put a larger weight on it, which results in larger spread in output. Risk-aversion of 2 or less is sufficient for this effect not to be welfare decreasing. On the other hand, the probability of the lowest outcome is decreasing if precision increases, and this always increases welfare. Therefore, the assumption on risk-aversion may be too restrictive.

Proposition 3 indicates that the positive effect of information on social welfare is guaranteed to be valid only in a neighborhood of perfectly flexible prices. However, in a numerical example with CRRA-preferences, and inflation process calibrated to match the U.S. postwar consumer price index, we can show that the positive value of information is likely to be a global phenomenon.[11]

It is worth to remind that the welfare effect of public announcements is present as long as the inflation process is stochastic. If the central bank follows constant inflation policy, perfectly communicating it to the public, than signal precision does not anymore distort agents' risk-sharing possibilities. While neutral under flexible prices, the social value of information is advantageous under imperfectly flexible prices since aggregate resources increase in the accuracy of policy announcements. In that sense our contribution is in fact pro and not con transparency.[12]

6 Conclusion

In this paper we study the welfare effects of policy announcements. As our novel result we highlight that more precise public information on risks, which are common to all agents may harm welfare by limiting voluntary risk-sharing opportunities. Technically, the optimal allocation under informative signals exhibits a higher cross-variance of consumption than under uninformative signals. We illustrate this effect in a two period model.

We embed the risk-sharing mechanism into a production economy, in which monetary policy has real effects captured by a cash-in-advance constraint and a fraction of firms sets prices in advance. The monetary authority announces a public signal on fu-

[11]The results of this exercise are available on request. Currently, we are working on a numerical example that integrates both effects of public information precision – the risk sharing and the resource effect.

[12]We borrowed this expression from Svensson (2006).

ture inflation.

First, we characterize the optimal stationary contract under voluntary participation in the absence of firms that preset prices. We find that the level of patience needed to sustain the first best allocation is increasing in the precision of the public signal. Then we show that in the optimal contract all participation constraints are binding for high productivity households if all the constraints were violated under the first best allocation. As our main result, we prove that there is a neighborhood around the optimal allocation under the perfectly uninformative signal in which the social welfare is strictly decreasing in the precision of the public signal. The risk-sharing effect on welfare is of second order.

Second, we analyze the case when a positive fraction of monopolistically competitive firms sets price one period in advance. Alleviating the negative effect on households' risk-sharing possibilities, a more precise signal reduces the prediction error of firms that set prices in advance, and aggregate resources increase. The negative effect of better precision dominates if households are sufficiently risk averse and the distribution of idiosyncratic income is dispersed. However, if perfect risk sharing or the equilibrium allocation without transfers is the socially optimal contract for any precision, the social value of better information is positive.

A Appendix

A.1 Proof of Proposition 1

The cut-point for β is characterized by participation constraints that become binding. Among the participation constraints only constraints for a high productivity agent can be binding, which limits consideration to four cases.

Consider a case such that

$$u(\bar{x}(\pi_l)) - \frac{1}{2}(u(x^h(\pi_l)) + u(x^l(\pi_l))) \le u(\bar{x}(\pi_h)) - \frac{1}{2}(u(x^h(\pi_h)) + u(x^l(\pi_h))) \tag{27}$$

$$u(x^h(\pi_l)) - u(\bar{x}(\pi_l)) \le u(x^h(\pi_h)) - u(\bar{x}(\pi_h)) \tag{28}$$

where $\bar{x}(\pi_j) \equiv (x^h(\pi_j)) + u(x^l(\pi_j))/2$. The first inequality (27) states that for the perfect risk-sharing contract provides higher value in comparison to the outside option under high inflation $V_{rs}(\pi_l) - V_{at}(\pi_l) \le V_{rs}(\pi_h) - V_{at}(\pi_h)$, where the value of the perfect risk-sharing contract is $V_{rs}(\pi_j) = u(\bar{x}(\pi_j))$. The second inequality (28) implies that the current period deviation for a high ability agent is more beneficial in the high inflation state. Therefore, for any precision of the signal, the participation constraint of high productivity agents under high current inflation that receive a low future inflation signal is the one that imposes the tightest restriction. This constraint is

$$\begin{aligned} u(\bar{x}(\pi_h)) - u(x^h(\pi_h)) + \bar{\beta}\kappa(V_{rs}(\pi_l) - V_{at}(\pi_l)) \\ + \bar{\beta}(1-\kappa)(V_{rs}(\pi_h) - V_{at}(\pi_h)) + \frac{(\bar{\beta})^2}{1-\bar{\beta}}(V_{rs} - V_{at}) = 0 \end{aligned} \tag{29}$$

where $V_{rs} = (u(\bar{x}(\pi_h)) + u(\bar{x}(\pi_l)))/2$.

There exists a unique positive solution to (29) due to $u(x^h(\pi_h)) - u(\bar{x}(\pi_h)) > 0$; the solution can be shown to satisfy $0 < \bar{\beta}(\kappa) < 1$.

Differentiating (29) we get

$$\frac{d\bar{\beta}}{d\kappa} = \frac{\bar{\beta}(1-\bar{\beta})(V_{rs}(\pi_h) - V_{at}(\pi_h) - V_{rs}(\pi_l) + V_{at}(\pi_l))}{u(x^h(\pi_h)) - u(\bar{x}(\pi_h)) + dV(\kappa) + 2\bar{\beta}(dV(1/2) - dV(\kappa))} \ge 0.$$

where

$$dV(\kappa) \equiv \kappa(V_{rs}(\pi_l) - V_{at}(\pi_l)) + (1-\kappa)(V_{rs}(\pi_h) - V_{at}(\pi_h)) \geq 0.$$

The other cases are similar to the case considered above.

A.2 Proof of Lemma 3

Let $\{c^i(h_t)\}$ be the optimal contract. By contradiction if there exists j such that $V_{rs}(\pi_j) - V_{at}(\pi_j) < 0$ then there exists a signal k such that $c^h(\pi_j, s_k) > x^h(\pi_j, s_k)$ (otherwise if for all signals $c^h(\pi_j, s_k) \geq x^h(\pi_j, s_k)$ and respectively by resource constraints $c^l(\pi_j, s_k) \leq x^l(\pi_j, s_k)$ then it would be a contradiction of $V_{rs}(\pi_j) - V_{at}(\pi_j) < 0$ due to concavity of the utility function). If the participation constraint for the high productivity agent under j inflation and k signal holds with equality then the future value of the contract is lower than the outside option value, and taking into account that for the low productivity agent from the resource constraint $c^l(\pi_j, s_k) < x^l(\pi_j, s_k)$ the participation constraint for the low productivity agent is violated. Therefore, the considered participation constraint for the high productivity agent can only hold with inequality. Then, consider a consumption allocation $\{\tilde{c}^i(h_t)\}$ given by

$$\tilde{c}^h(\pi_j, s_k) = c^h(\pi_j, s_k) - \varepsilon, \quad \tilde{c}^l(\pi_j, s_k) = c^l(\pi_j, s_k) + \varepsilon, \quad \tilde{c}^i(h^i_t) = c^i(h^i_t) \text{ otherwise}$$

There exists $\varepsilon > 0$ such that the consumption allocation $\{\tilde{c}(h^i_t)\}$ is sustainable, and by concavity it provides higher utility then the allocation $\{c(h^i_t)\}$, which contradicts that $\{c(h^i_t)\}$ is the socially optimal contract.

A.3 Proof of Lemma 5

First, we show that for any public state h_t^i the optimal consumption allocation satisfies

$$c^h(h_t^i) > \bar{x}(h_t^i) > c^l(h_t^i).$$

As an example consider the participation constraint for households of high productivity in the previous period under currently high inflation that receive a high signal on future inflation

$$u(c^h(\pi_h, s_h)) + \beta(\kappa V_{rs}(\pi_h) + (1-\kappa)V_{rs}(\pi_l)) + \frac{\beta^2}{1-\beta}V_{rs} \geq$$
$$u(x^h(\pi_h)) + \beta(\kappa V_{at}(\pi_h) + (1-\kappa)V_{at}(\pi_l)) + \frac{\beta^2}{1-\beta}V_{at}.$$

where the unconditional expected value of the contract is

$$V_{rs} \equiv \frac{V_{rs}(\pi_h) + V_{rs}(\pi_l)}{2}$$

and the values of outside option is

$$V_{at} \equiv \frac{V_{at}(\pi_h) + V_{at}(\pi_l)}{2}$$

Under perfect risk sharing the participation constraints are

$$u(\bar{x}(\pi_h)) + \beta(\kappa u(\bar{x}(\pi_h)) + (1-\kappa)u(\bar{x}(\pi_l))) + \frac{\beta^2}{1-\beta}V_{prs} <$$
$$u(x^h(\pi_h)) + \beta(\kappa V_{at}(\pi_h) + (1-\kappa)V_{at}(\pi_l)) + \frac{\beta^2}{1-\beta}V_{at}$$

where the value of perfect risk-sharing contract

$$V_{prs} \equiv \frac{u(\bar{x}(\pi_h)) + u(\bar{x}(\pi_l))}{2}.$$

Combining the corresponding pairs of constraints we get

$$u(c^h(\pi_h, s_h)) + \beta(\kappa V_{rs}(\pi_h) + (1-\kappa) V_{rs}(\pi_l)) + \frac{\beta^2}{1-\beta} V_{rs} >$$
$$u(\bar{x}(\pi_h)) + \beta(\kappa u(\bar{x}(\pi_h)) + (1-\kappa) u(\bar{x}(\pi_l))) + \frac{\beta^2}{1-\beta} V_{prs}$$

Taking into account that

$$V_{rs}(\pi_h) \le u(\bar{x}(\pi_h)) \quad V_{rs}(\pi_l) \le u(\bar{x}(\pi_l)) \quad V_{rs} \le V_{prs},$$

we get

$$u(c^h(\pi_h, s_h)) > u(\bar{x}(\pi_h))$$

or, combining with resource feasibility

$$c^h(\pi_h, s_h) > \bar{x}(\pi_h) > c^l(\pi_h, s_h).$$

Similarly we can show the same inequalities for the other public states.

Second, by contradiction, assume that there is one participation constraint for high productivity agents that is not binding. The Lagrangian of the optimal contract problem can be written as

$$\mathscr{L} = (1 + \sum_{h_t^i \in \tilde{H}} \lambda(h_t^i))(u(c^h(\pi_j, s_k)) + u(c^l(\pi_j, s_k)))$$

$$+ \mu(\pi_j, s_k)(c^h(\pi_j, s_k) + c^l(\pi_j, s_k) - 2\bar{x}(\pi_j)) + \xi(c(\tilde{H})))$$

where (π_j, s_k) is the state for which the participation constraint is not binding, $\tilde{H}$ is the set of all possible states, excluding (π_j, s_k), $\lambda(h_t^i)$ are the normalized Lagrange multipliers for the participation constraints, and $\mu(h_t^i)$ are the Lagrange multipliers for resource constraints. The Lagrange multiplier for the participation constraint for state (π_j, s_k) is zero and is explicitly excluded from the summation.

Solving the optimal contract problem we get

$$c^h(\pi_j, s_k) = c^l(\pi_j, s_k) = \bar{x}(\pi_j)$$

for the non-binding state, which contradicts the partial risk-sharing condition stated above.

A.4 Proof of Lemma 6

The optimal contract under uninformative signals can be written as fixed point problem in terms of the contract value. If signals are uninformative, the number of participation constraints of high productivity households reduces to two. If the value of the contract is w the two participation constrains, which are assumed to hold with equality can be written as

$$u(c^h(\pi_h, w)) + \frac{\beta}{1-\beta} w = u(x^h(\pi_h)) + \frac{\beta}{1-\beta} V_{at}$$

$$u(c^h(\pi_l, w)) + \frac{\beta}{1-\beta} w = u(x^h(\pi_l)) + \frac{\beta}{1-\beta} V_{at},$$

and resources are given by

$$c^h(\pi_h, w) + c^l(\pi_h, w) = x^h(\pi_h) + x^l(\pi_h)$$

$$c^h(\pi_l, w) + c^l(\pi_l, w) = x^h(\pi_l) + x^l(\pi_l).$$

The participation constraints imply that the optimal contract problem reduces to the solution of the following fixed point problem $w = V_{rs}(w)$, where

$$V_{rs}(w) \equiv \frac{1}{4}\Big(u(c^h(\pi_h, w)) + u(c^l(\pi_h, w) + u(c^h(\pi_l, w) + u(c^l(\pi_l, w)\Big).$$

The existence of a unique non-autarkic socially optimal contract is proved by showing that $V_{rs}(w)$ is a monotonically increasing concave function. Requiring a slope greater than unity at the outside option allocation guarantees the unique existence of a non-autarkic socially optimal contract. From the participation constraints and resource constraints it follows that $V_{rs}(w)$ is strictly increasing[13]

$$V'_{rs}(w) = \frac{1}{4}\frac{\beta}{1-\beta}\left(-2 + \frac{u'(c^l(\pi_h))}{u'(c^h(\pi_h))} + \frac{u'(c^l(\pi_l))}{u'(c^h(\pi_l))}\right) > 0,$$

since perfect risk sharing is not sustainable per assumption. Concavity of $V_{rs}(w)$ is implied by

$$\frac{d}{dw}\left(\frac{u'(c^l(\pi_h, w))}{u'(c^h(\pi_h, w))}\right) = \frac{\beta}{1-\beta}\frac{1}{\big(u'(c^h(\pi_h))\big)^2}\left(u''(c^l(\pi_h)) + u''(c^h(\pi_h))\frac{u'(c^l(\pi_h))}{u'(c^h(\pi_h))}\right) < 0.$$

[13]To simplify notation we suppress in the following the contingency of the allocation on w.

Next, the solution to the optimal contracting problem is given by $w = V_{rs}(w)$. Due to concavity of $V_{rs}(w)$ there are at most two solution for the fixed point problem. By problem construction, one solution is V_{at}. Note that the derivative of $V_{rs}(w)$ at V_{at} is higher than at any partial risk-sharing allocation. The second solution preferable to outside option exists if the derivative of $f(V_{rs})$ at V_{at} is greater than 1, that is

$$\frac{1}{2}\left(\frac{u'(x^l(\pi_h))}{u'(x^h(\pi_h))}+\frac{u'(x^l(\pi_l))}{u'(x^h(\pi_l))}\right)\geq\frac{2-\beta}{\beta}.$$

In summary, the solution at the socially contract is characterized by $V'_{rs}(w) \leq 1$ or

$$\frac{1}{2}\left(\frac{u'(c^l(\pi_h))}{u'(c^h(\pi_h))}+\frac{u'(c^l(\pi_l))}{u'(c^h(\pi_l))}\right)\leq\frac{2-\beta}{\beta}.$$

A.5 Proof of Theorem 2

First we compute a quadratic approximation in signal precision to the optimal contract around the optimal allocation for uninformative signals. Second we show that this implies that social welfare is decreasing in precision in the neighborhood of $\kappa = 1/2$ if the optimal contract is not the outside option.

Consider the second order approximation of the optimal consumption of high productivity household $c^h(\pi_j, s_k; \hat{\kappa})$ near $\kappa = 1/2$

$$c^h(\pi_j, s_k; \hat{\kappa}) = \bar{c}^h_j + \alpha_{jk}\hat{\kappa} + \frac{1}{2}\gamma_{jk}\hat{\kappa}^2 + \mathcal{O}(\hat{\kappa}^3),$$

where $\hat{\kappa} \equiv \kappa - 1/2$. Then the second order approximation of the period utility of the high productivity household is

$$u(c^h(\pi_j, s_k; \hat{\kappa})) = u(\bar{c}_j^h) + u'(\bar{c}_j^h)\alpha_{jk}\hat{\kappa} + \frac{1}{2}(u''(\bar{c}_j^h)(\alpha_{jk})^2 + u'(\bar{c}_j^h)\gamma_{jk})\hat{\kappa}^2 + \mathcal{O}(\hat{\kappa}^3)$$

From the resource constraint we write the optimal consumption of the low productivity agent as

$$c^l(\pi_j, s_k; \hat{\kappa}) = \bar{c}_j^l - \alpha_{jk}\hat{\kappa} - \frac{1}{2}\gamma_{jk}\hat{\kappa}^2 + \mathcal{O}(\hat{\kappa}^3),$$

and similarly the second order approximation of the utility $u(c^l(\pi_j, s_k; \hat{\kappa}))$ is

$$u(c^l(\pi_j, s_k; \hat{\kappa})) = u(\bar{c}_j^l) - u'(\bar{c}_j^l)\alpha_{jk}\hat{\kappa} + \frac{1}{2}(u''(\bar{c}_j^l)(\alpha_{jk})^2 - u'(\bar{c}_j^l)\gamma_{jk})\hat{\kappa}^2 + \mathcal{O}(\hat{\kappa}^3)$$

The expected utility from the contract in j-inflation state is

$$\begin{aligned} V_{rs}(\pi_j; \hat{\kappa}) = &\frac{1}{2}(u(\bar{c}_j^h) + u(\bar{c}_j^l)) + \frac{1}{4}(u'(\bar{c}_j^h) - u'(\bar{c}_j^l))(\alpha_{jh} + \alpha_{jl})\hat{\kappa} \\ &+ \frac{1}{8}((u''(\bar{c}_j^h) + u''(\bar{c}_j^l))((\alpha_{jh})^2 + (\alpha_{jl})^2) + (u'(\bar{c}_j^h) - u'(\bar{c}_j^l))(\gamma_{jh} + \gamma_{jl}))\hat{\kappa}^2 + \mathcal{O}(\hat{\kappa}^3) \end{aligned}$$

If we put the approximation into the participation constraints

$$\begin{aligned} u(c^h(\pi_j, s_h)) + \beta(V_{rs}(\pi_h) - V_{rs}(\pi_l))\hat{\kappa} + \frac{\beta}{1-\beta}V_{rs} = \\ u(x^h(\pi_j)) + \beta(V_{at}(\pi_h) - V_{at}(\pi_l))\hat{\kappa} + \frac{\beta}{1-\beta}V_{at} \end{aligned}$$

where the social welfare is $V_{rs} \equiv (V_{rs}(\pi_h) + V_{rs}(\pi_l))/2$, we get

$$u(\bar{c}_j^h) + u'(\bar{c}_j^h)\alpha_{jh}\hat{\kappa} + \frac{1}{2}(u''(\bar{c}_j^h)(\alpha_{jh})^2 + u'(\bar{c}_j^h)\gamma_{jh})\hat{\kappa}^2$$
$$+ \beta\left(\frac{1}{2}(u(\bar{c}_h^h) + u(\bar{c}_h^l) - \frac{1}{2}(u(\bar{c}_l^h) + u(\bar{c}_l^l))\right)\hat{\kappa}$$
$$+ \beta\left(\frac{1}{4}(u'(\bar{c}_h^h) - u'(\bar{c}_h^l))(\alpha_{hh} + \alpha_{hl}) - \frac{1}{4}(u'(\bar{c}_l^h) - u'(\bar{c}_l^l))(\alpha_{lh} + \alpha_{ll})\right)\hat{\kappa}^2$$
$$+ \frac{\beta}{1-\beta}V_{rs}(\hat{\kappa}) + \mathcal{O}(\hat{\kappa}^3) = u(x^h(\pi_j)) + \beta(V_{at}(\pi_h) - V_{at}(\pi_l))\hat{\kappa} + \frac{\beta}{1-\beta}V_{at}$$

and similarly for the the other states.

Combining the terms for the first power of $\hat{\kappa}$ we obtain for the high next period inflation signal

$$u'(\bar{c}_j^h)\alpha_{jh} + \beta\left(\frac{1}{2}(u(\bar{c}_h^h) + u(\bar{c}_h^l)) - \frac{1}{2}(u(\bar{c}_l^h) + u(\bar{c}_l^l))\right)$$
$$+ \frac{\beta}{1-\beta}\left(\frac{1}{8}(u'(\bar{c}_h^h) - u'(\bar{c}_h^l))(\alpha_{hh} + \alpha_{hl}) + \frac{1}{8}(u'(\bar{c}_l^h) - u'(\bar{c}_l^l))(\alpha_{lh} + \alpha_{ll})\right) =$$
$$\beta(V_{at}(\pi_h) - V_{at}(\pi_l))$$

and similarly for the low inflation signal

$$u'(\bar{c}_j^h)\alpha_{jl} + \beta\left(\frac{1}{2}(u(\bar{c}_l^h) + u(\bar{c}_l^l)) - \frac{1}{2}(u(\bar{c}_h^h) + u(\bar{c}_h^l))\right)$$
$$+ \frac{\beta}{1-\beta}\left(\frac{1}{8}(u'(\bar{c}_h^h) - u'(\bar{c}_h^l))(\alpha_{hh} + \alpha_{hl}) + \frac{1}{8}(u'(\bar{c}_l^h) - u'(\bar{c}_l^l))(\alpha_{lh} + \alpha_{ll})\right) =$$
$$\beta(V_{at}(\pi_l) - V_{at}(\pi_h))$$

Solving for $\{\alpha^{jk}\}$ we get

$$u'(\bar{c}_j^h)\alpha_{jh} = \beta((V_{at}(\pi_h) - V_{at}(\pi_l)) - (\bar{V}_{rs}(\pi_h) - \bar{V}_{rs}(\pi_l)))$$

$$u'(\bar{c}_j^h)\alpha_{jl} = \beta((V_{at}(\pi_l) - V_{at}(\pi_h)) - (\bar{V}_{rs}(\pi_l) - \bar{V}_{rs}(\pi_h)))$$

which satisfy

$$\alpha_{hh} + \alpha_{hl} = \alpha_{lh} + \alpha_{ll} = 0$$

Under partial risk sharing, social welfare is decreasing in precision if all γ_{jk} are positive which is established in the following.

First, combining the terms for the second power of $\hat{\kappa}$, and taking into account the first order solution we get

$$\frac{1}{2}(u''(\bar{c}_j^h)(\alpha_{jk})^2 + u'(\bar{c}_j^h)\gamma_{jk}) + \frac{\beta}{1-\beta}\frac{1}{2}\bar{V}_{rs}'' = 0$$

Taking again into account the symmetric first order effect $\alpha_{jh} = -\alpha_{jl}$ we obtain

$$\gamma_{hh} = \gamma_{hl} \quad \text{and} \quad \gamma_{lh} = \gamma_{ll},$$

which implies that the second order terms do not depend on realization of the public signal.

Expanding $\bar{V}''_{rs}$, the participation constraints for the second power of $\hat{\kappa}$ are

$$\begin{aligned}
&\frac{1}{2}(u''(\bar{c}_h^h)(\alpha_{hk})^2 + u'(\bar{c}_h^h)\gamma_{hk}) \\
&\qquad + \frac{\beta}{1-\beta}\frac{1}{8}((u''(\bar{c}_h^h) + u''(\bar{c}_h^l))(\alpha_{hk})^2 + (u'(\bar{c}_h^h) - u'(\bar{c}_h^l))\gamma_{hk} \\
&\qquad\qquad + (u''(\bar{c}_l^h) + u''(\bar{c}_l^l))(\alpha_{lk})^2 + (u'(\bar{c}_l^h) - u'(\bar{c}_l^l))\gamma_{lk}) = 0 \quad (30)
\end{aligned}$$

$$\begin{aligned}
&\frac{1}{2}(u''(\bar{c}_l^h)(\alpha_{lk})^2 + u'(\bar{c}_l^h)\gamma_{lk}) \\
&\qquad + \frac{\beta}{1-\beta}\frac{1}{8}((u''(\bar{c}_h^h) + u''(\bar{c}_h^l))(\alpha_{hk})^2 + (u'(\bar{c}_h^h) - u'(\bar{c}_h^l))\gamma_{hk} \\
&\qquad\qquad + (u''(\bar{c}_l^h) + u''(\bar{c}_l^l))(\alpha_{lk})^2 + (u'(\bar{c}_l^h) - u'(\bar{c}_l^l))\gamma_{lk}) = 0 \quad (31)
\end{aligned}$$

The determinant of the linear system (30)-(31) for two unknowns γ_{hk} and γ_{lk} is

$$\begin{aligned}
\Delta \equiv &\left[\frac{1}{2}u'(\bar{c}_h^h) + \frac{\beta}{1-\beta}\frac{1}{8}(u'(\bar{c}_h^h) - u'(\bar{c}_h^l))\right]\left[\frac{1}{2}u'(\bar{c}_l^h) + \frac{\beta}{1-\beta}\frac{1}{8}(u'(\bar{c}_l^h) - u'(\bar{c}_l^l))\right] \\
&- \left[\frac{\beta}{1-\beta}\frac{1}{8}(u'(\bar{c}_h^h) - u'(\bar{c}_h^l))\right]\left[\frac{\beta}{1-\beta}\frac{1}{8}(u'(\bar{c}_l^h) - u'(\bar{c}_l^l))\right] \\
&= \frac{1}{8}u'(\bar{c}_h^h)u'(\bar{c}_l^h)\frac{\beta}{1-\beta}\left[\frac{2-\beta}{\beta} - \frac{1}{2}\left(\frac{u'(\bar{c}_l^l)}{u'(\bar{c}_l^h)} + \frac{u'(\bar{c}_h^l)}{u'(\bar{c}_h^h)}\right)\right]
\end{aligned}$$

Note that the optimal allocation under perfectly uninformative signal indeed satisfies

$$\frac{1}{2}\left(\frac{u'(\bar{c}_l^l)}{u'(\bar{c}_l^h)} + \frac{u'(\bar{c}_h^l)}{u'(\bar{c}_h^h)}\right) \leq \frac{2-\beta}{\beta} \quad (32)$$

as provided in the proof of Lemma 6, and therefore $\Delta \geq 0$.

As a final step, note that the elements of the right hand side of the linear system (30)-(31) are positive. Thus, for the solution to the system to be positive it is sufficient to require

$$\frac{1}{2}u'(\bar{c}_l^h) + \frac{\beta}{1-\beta}\frac{1}{8}(u'(\bar{c}_l^h) - u'(\bar{c}_l^l)) \geq 0$$

$$\frac{\beta}{1-\beta}\frac{1}{8}(u'(\bar{c}_l^h) - u'(\bar{c}_l^l)) \leq 0,$$

as well as

$$\frac{1}{2}u'(\bar{c}_h^h) + \frac{\beta}{1-\beta}\frac{1}{8}(u'(\bar{c}_h^h) - u'(\bar{c}_h^l)) \geq 0$$

$$\frac{\beta}{1-\beta}\frac{1}{8}(u'(\bar{c}_h^h) - u'(\bar{c}_h^l)) \leq 0.$$

The requirements simplify to

$$\frac{u'(\bar{c}_l^h)}{u'(\bar{c}_l^l)} \geq \frac{\beta}{4-3\beta}$$

$$\frac{u'(\bar{c}_h^h)}{u'(\bar{c}_h^l)} \geq \frac{\beta}{4-3\beta},$$

which are weaker than the inequality (32) already shown above.

A.6 Proof of Proposition 2

Log-linear second order approximations around $p_t^{fj} = p_t$ of the integrands in (24) and (25) are given by

$$\left(\frac{p_t^{fj}}{p_t}\right)^{1-1/\rho} = 1 + \left(1 - \frac{1}{\rho}\right)\hat{z}_t^{fj} + \left(1 - \frac{1}{\rho}\right)^2 \frac{(\hat{z}_t^{fj})^2}{2} + \mathcal{O}(\|\hat{z}_t^{fj}\|^3) \tag{33}$$

$$\left(\frac{p_t^{fj}}{p_t}\right)^{-1/\rho} = 1 - \frac{1}{\rho}\hat{z}_t^{fj} + \left(\frac{1}{\rho}\right)^2 \frac{(\hat{z}_t^{fj})^2}{2} + \mathcal{O}(\|\hat{z}_t^{fj}\|^3) \tag{34}$$

where $\hat{z}_t^{fj} = \log p_t^{fj} - \log p_t$.

Substituting the approximation (33) into the identity for the aggregate price level (24) we get

$$\int \hat{z}_t^{fj} dj + \left(1 - \frac{1}{\rho}\right) \int \frac{(\hat{z}_t^{fj})^2}{2} dj = \mathcal{O}(\| \hat{z}_t^{fj} \|^3),$$

and combining the expression with the approximation (34) we can write the denominator of (25) as

$$\int \left(\frac{p_t^{fj}}{p_t}\right)^{-1/\rho} dj = 1 + \frac{1}{2\rho} var_j \log p_t^{fj} + \mathcal{O}(\| p_t^{fj} \|^3).$$

Next, we compute the cross-variance of prices $var_j \log p_t^{fj}$ from the optimal price setting. From (24) up to a first order approximation, it follows that the aggregate price can be written as $\log p_t = \lambda \log p_{1t}^f + (1-\lambda) p_{2t}^f$, where p_{1t}^f is the flexible firm price and p_{2t}^f is the price preset by nonflexible firms. Similarly, solving the profit maximization problem for price presetting firms, up to a first order approximation $\log p_{2t}^f = E_{t-1}\left[\log p_{1t}^f | s_{t-1}\right]$. Then, the prediction error can be written as $\pi_t - E_{t-1}[\pi_t | s_{t-1}] = \lambda(\log p_{1t} - \log p_{2t})$, and therefore $var_j \log p_t^j = \lambda(1-\lambda)(\log p_{1t} - \log p_{2t})^2 = \frac{1-\lambda}{\lambda}(\pi_t - E_{t-1}[\pi_t | s_{t-1}])^2$. This implies

$$\int \left(\frac{p_t^{fj}}{p_t}\right)^{-1/\rho} dj = 1 + \varphi(\pi_t - E_{t-1}[\pi_t | s_{t-1}])^2 + \mathcal{O}(\| p_t^{fj} \|^3),$$

with $\varphi = (1-\lambda)/(2\rho\lambda)$.

In case of two inflation states, applying the approximation to (25), expected per capita production in each sector is

$$E[y_t^f] = \kappa \left(\frac{a^f}{1 + \phi(1-\kappa)^2}\right) + (1-\kappa)\left(\frac{a^f}{1 + \phi\kappa^2}\right).$$

Differentiating with respect to signal precision gives

$$\frac{\partial E[y_t^f]}{\partial \kappa} = a^f \left(\frac{1}{1+\phi(1-\kappa)^2} - \frac{1}{1+\phi\kappa^2} \right) + \phi 2 a^f \kappa (1-\kappa) \left(\frac{1}{\left(1+\phi(1-\kappa)^2\right)^2} - \frac{1}{\left(1+\phi\kappa^2\right)^2} \right) \geq 0 \quad \forall \quad 1/2 \leq \kappa \leq 1,$$

where $\phi \equiv \frac{1}{2\rho}\frac{1-\lambda}{\lambda}(\pi_h - \pi_l)^2$. Since the resources increase with better signals in each sector, aggregate resources (in per capita terms and in total) are also an increasing function in signal precision.

A.7 Proof of Proposition 3

First, we consider of perfect risk sharing being the socially optimal contract, in which case each household consumes the deflated average income across sectors, $\bar{x} \equiv (x_t^h + x_t^l)/2$. Building on results derived in Proposition 2, up to the second order approximation average deflated income reads:

$$\bar{x} = \frac{\bar{a}}{1 + \frac{1-\lambda}{\lambda}(\pi_t - E_{t-1}[\pi_t | s_{t-1}])^2},$$

where $\bar{a} = (a^h + a^l)/2$. In case of two inflation states the welfare is given by the following expression

$$\begin{aligned} W &= E[u(\bar{x})] \\ &= \frac{\kappa}{2} u\left(\frac{\bar{a}}{1+\phi(1-\kappa)^2}\frac{1}{\pi_h}\right) + \frac{1-\kappa}{2} u\left(\frac{\bar{a}}{1+\phi\kappa^2}\frac{1}{\pi_h}\right) \\ &+ \frac{\kappa}{2} u\left(\frac{\bar{a}}{1+\phi(1-\kappa)^2}\frac{1}{\pi_l}\right) + \frac{1-\kappa}{2} u\left(\frac{\bar{a}}{1+\phi\kappa^2}\frac{1}{\pi_l}\right). \end{aligned}$$

Differentiating with respect to κ results in

$$\begin{aligned} \frac{\partial W}{\partial \kappa} = \frac{1}{2} &\left[u\left(\frac{\bar{a}}{1+\phi(1-\kappa)^2}\frac{1}{\pi_h}\right) - u\left(\frac{\bar{a}}{1+\phi\kappa^2}\frac{1}{\pi_h}\right)\right] \\ &+ \frac{1}{2}\left[u\left(\frac{\bar{a}}{1+\phi(1-\kappa)^2}\frac{1}{\pi_l}\right) - u\left(\frac{\bar{a}}{1+\phi\kappa^2}\frac{1}{\pi_l}\right)\right] \\ &+ \Psi(\kappa,\pi_h) + \Psi(\kappa,\pi_l) \end{aligned}$$

where

$$\begin{aligned} \Psi(\kappa,\pi_i) \equiv \frac{\kappa}{2} u'&\left(\frac{\bar{a}}{1+\phi(1-\kappa)^2}\frac{1}{\pi_i}\right) \frac{2\phi\bar{a}(1-\kappa)}{\left(1+\phi(1-\kappa)^2\right)^2}\frac{1}{\pi_i} \\ &- \frac{1-\kappa}{2} u'\left(\frac{\bar{a}}{1+\phi\kappa^2}\frac{1}{\pi_i}\right) \frac{2\phi\bar{a}\kappa}{\left(1+\phi\kappa^2\right)^2}\frac{1}{\pi_i}. \end{aligned}$$

While the first two terms in the derivative of welfare are non-negative for $\kappa \geq 1/2$, the signs for the latter two are ambiguous in general. Taking into account the assumption on risk aversion, $f(c) \equiv c^2 u'(c)$ is an increasing function, and it follows for $\Delta(\kappa, \pi_i)$ that

$$\Psi(\kappa,\pi_i) = \phi\kappa(1-\kappa)\frac{\pi_i}{\bar{a}}\left(f\left(\frac{\bar{a}}{1+\phi(1-\kappa)^2}\frac{1}{\pi_i}\right) - f\left(\frac{\bar{a}}{1+\phi\kappa^2}\frac{1}{\pi_i}\right)\right) \geq 0$$

$$\forall \quad 1/2 \leq \kappa \leq 1.$$

The case when outside option is the socially optimal contract is similar.

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