

OPTIMAL INTEREST RATE STABILIZATION IN A BASIC STICKY-PRICE MODEL^{*}

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Abstract

This paper studies optimal monetary policy with the nominal interest rate as the single policy instrument in an economy, where firms set prices in a staggered way without indexation and real money balances contribute separately to households' utility. The optimal deterministic steady state under commitment is the Friedman rule – even if the importance assigned to the utility of money is small relative to consumption and leisure. We approximate the model around the optimal steady state as the long-run policy target. Optimal monetary policy is characterized by stabilization of the nominal interest rate instead of inflation stabilization as the predominant principle.

JEL classification: E32, E52, E58.

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1 Introduction

What is the primary aim of optimal monetary policy? In the existing literature there are two major views that deliver opposite recommendations for the optimal conduct of monetary policy in the short and in the long run. The first branch goes back to Friedman (1969) and evaluates monetary policy in the long run with fully flexible prices and under perfect competition. In order to equate the private opportunity costs for holding money to the zero social costs to produce it, the nominal interest rate should be zero. The other view considers optimal monetary policy in the short run in the presence of nominal rigidities and imperfect competition (e.g. Woodford, 2003a, ch.6-8; Benigno and Woodford, 2005; Khan et al., 2003; Schmitt-Grohé and Uribe, 2004, 2005). A key feature of this literature is that the authors consider small fluctuations around the (almost) zero inflation steady state, implying that optimal policy nearly completely offsets the distortions due to price dispersion – even in the presence of a monetary friction. The predominant principle is inflation stabilization, while the nominal interest rate should adjust relatively freely to support this principle (Woodford, 2003a).

In this paper we revisit the issue of optimal monetary policy in a sticky price model in the presence of a transaction friction. The foremost contribution is to challenge the conventional view that the Friedman rule loses out to the goal of price stability once price stickiness is introduced. We show that the widely used money-in-the utility function model (MIU) implies that Friedman’s rule is optimal even when large amounts of price stickiness are present. This is in contrast to the key message of papers such as Woodford (2003a), Khan, King and Wolman (2003) and Schmitt-Grohé and Uribe (2004, 2005) and others. Second, we find that the primary aim of optimal policy in the short run is to stabilize the nominal interest rate instead of inflation.

Our analysis is set in a dynamic stochastic general equilibrium model with imperfect competition and Calvo’s staggered price setting (1983) without indexation. A transaction

friction is introduced via the textbook money-in-the-utility-function approach (Sidrauski, 1967; Woodford, 2003a; Walsh, 2003) with consumption and real money balances entering in a separable way. We abstract from interactions between fiscal and monetary policy by assuming that the government has access to lump-sum taxes. Since we assume an output subsidy that offsets the steady state distortion created by monopolistic competition, the policy maker faces two distortions: price dispersion due to staggered price setting calls for an optimal inflation of zero, implying costs of money holdings. However, the monetary distortion can only be offset by setting the nominal interest rate to zero.

We choose the long-run target of monetary policy to be the welfare-maximizing deterministic steady state. Remarkably, we find that even for very low values for the weight of money in the utility function relative to consumption and leisure, it is optimal to fully offset the monetary distortion and to allow for a small degree of price dispersion. I.e. the Friedman rule is optimal even in the presence of Calvo-style staggered price setting. This result holds for wide a range of parameter values including low weights for real money balances in the utility function. To understand this finding, note that the welfare cost of price dispersion arising from long-run deflation required by the Friedman rule is small relative to the loss from a positive nominal interest rate. While the welfare loss due to price dispersion hinges primarily on the frequency of price adjustment, the utility losses of a positive interest rate crucially depends on the sensitivity of money demand to the nominal interest rate. In an MIU framework, the latter increases strongly as interest rates fall. Thereby, the taxation of money holdings via a positive interest rate becomes suboptimal.

We linearize the model around the optimal steady state and derive a quadratic approximation to the utility of the representative household. This welfare based loss function serves as the central bank's objective, and it depends on three arguments: the unconditional variances of inflation, the output gap, and on the variance of the nominal interest rate. While the weight for the variation in the output gap relative to inflation depends ex-

clusively on structural parameters unrelated to policy, the relative weight for interest rate variability also hinges on steady state values that are under control of policy in the long run. Remarkably, the preference to stabilize fluctuations in the nominal interest rate increases as optimal steady state inflation moves towards Friedman's rule of deflation. This increase is primarily driven by the rise in the interest elasticity of money demand. Correspondingly, the importance to account for monetary frictions depends upon the steady state chosen for approximation: The long-run optimal policy is key for optimal policy reactions in the short run. Since we approximate our model around a steady state implied by the Friedman rule, the primary goal of optimal monetary policy is to stabilize variations in the interest rate rather than in inflation. Given the high weight attached to interest rate stabilization, optimal monetary policy requires abstaining from fluctuations in the nominal interest rate. Instead, the nominal interest rate is literally fixed in response to various kinds of disturbances.

We show that choosing a long-run deflation target according to the Friedman rule does not generally undermine the central banks ability to stabilize the welfare relevant fluctuations around that target. On the contrary, the welfare loss arising from fluctuations around the Friedman steady state can be lower than the loss arising from fluctuations around the zero inflation steady state. Overall, we find support for the Friedman rule even in case of a reasonable amount of nominal rigidity due to staggered price setting a la Calvo: The Friedman rule yields higher steady state utility and can also improve welfare effects of fluctuations around the steady state compared to price stability.

We address the issue of the zero bound constraint on the nominal interest rate in the following way. First, we impose that the gross nominal interest rate exceeds unity in the deterministic steady state by a small amount. This assumption does not exclude the possibility of an occasionally binding lower bound constraint in response to shocks. Second, we approximate the probability of hitting the lower bound. We find that it is minor. To

be more precise the standard deviation of the nominal interest rate under optimal policy is so small relative to the buffer between the steady state nominal rate and unity, that the likelihood of a binding lower bound is low.

Related Literature

We now turn to the related literature. Most closely related to our paper is the work by Woodford (2003a, Chapter 6-7; Woodford, 2003b) and Schmitt-Grohé and Uribe (2005). Woodford also studies optimal monetary policy in a money-in-the-utility function framework with staggered price setting. In contrast to our analysis, the model is log-linearized around the zero inflation steady state. This approximation point then implies very different dynamics for the nominal interest rate. In his analysis, the nominal interest rate reacts rather sharply to shocks while the optimal path of inflation is relatively smooth over the cycle (see Woodford, 2003a: 504). Our contribution is to show that the optimal policy prescriptions differ substantially once one takes into account the interactions between long run and short run optimal policy.

Schmitt-Grohé and Uribe (2005) and Khan et al. (2003) also analyze optimal monetary policy with nominal rigidities and a monetary friction. These papers adopt a transaction technology approach to introducing money into the model. While Khan (2003) use a different time dependent pricing model than we do, the economic environment of Schmitt-Grohé and Uribe (2005) is more similar to our framework. They analyze a medium scale model with staggered price setting a la Calvo and various additional distortions. They find that the central bank should aim at price stability and stabilization of inflation as the main principle. The difference between their key finding and our results is explained as follows. The money-in-the-utility function approach we employ has different implications for money demand at low interest rates compared to the transactions technology in Schmitt-Grohé and Uribe. The MIU framework implies that the interest-elasticity of

money demand increases by large amounts as the nominal interest rate approaches the lower bound. Correspondingly, welfare costs of taxing money balances with positive interest rates in the long run and varying the nominal interest rate in the short run increase substantially. This is not the case for their transaction cost technology. Our contribution is to show that both the degree of price dispersion, as well as the sensitivity of money demand with respect to nominal interest rates at low levels, are decisive for the conduct of optimal policy.

We are not the first in showing that the Friedman rule can be optimal in economies with sticky prices. Adão, Correia, and Teles (2003) prove that the Friedman rule is optimal in an economy with imperfect competition, a cash in advance constraint, and prices that are set one period in advance. In contrast, our analysis assumes sticky prices à la Calvo, which implies that there are costs to deflation due to the dispersion of relative prices. Closer to our work is King and Wolman (1996). They show that setting the nominal interest rate at a minuscule amount above zero maximizes steady state welfare in a model with Calvo pricing and a transaction cost technology. We obtain the same result in an MIU framework. While King and Wolman (1996) focus on a static analysis of optimal policy our main contribution is dynamic: We derive the guiding principles for the optimal conduct of policy in the short run that follow of choosing the Friedman rule as long run target.

Methodologically, this paper differs from Khan et al. (2003) and Schmitt-Grohé (2005) by working with the linear-quadratic framework, rather than with the time invariant Ramsey approach. By showing that the weight on nominal interest stabilization in the loss function depends on the steady state values under control of the central bank, this approach helps to point out intuitively how long run optimal policy and short run stabilization policies are interrelated. In addition, the guiding principle of optimal monetary policy is directly transparent in the size of the relative weights to stabilize the nominal interest rate, inflation, and the output gap.

The remainder of this paper proceeds as follows: in section 2 we set up the model. In section 3 we compute the optimal steady state under commitment and derive a quadratic approximation of the utility of the representative household. In section 4 we derive the optimal monetary policy responses in the short run for 2 policy regimes: the first one has Friedman's Rule, and the other one has zero inflation as its long-run target. The last section concludes.

2 The model

We consider an economy that consists of a continuum of infinitely lived households indexed with $j \in [0, 1]$. It is assumed that households have identical initial asset endowments and identical preferences. Household j acts as a monopolistic supplier of labor services l_j . Lower (upper) case letters denote real (nominal) variables. At the beginning of period t , households' financial wealth comprises money $M_{j,t-1}$, a portfolio of state contingent claims on other households yielding a (random) payment $Z_{j,t}$, and one period nominally non-state contingent government bonds $B_{j,t-1}$ carried over from the previous period. Assuming complete financial markets let $q_{t,t+1}$ denote the period t price of one unit of currency in a particular state of period $t + 1$ normalized by the probability of occurrence of that state, conditional on the information available in period t . Then, the price of a random payoff Z_{t+1} in period $t + 1$ is given by $E_t[q_{t,t+1}Z_{j,t+1}]$. The budget constraint of the representative household reads

$$M_{j,t} + B_{j,t} + E_t[q_{t,t+1}Z_{j,t+1}] + P_t c_{j,t} \leq R_{t-1}B_{j,t-1} + M_{j,t-1} + Z_{j,t} + P_t w_{j,t} l_{j,t} + \int_0^1 D_{j,i,t} di - P_t T_t, \quad (1)$$

where c_t denotes a Dixit-Stiglitz aggregate of consumption with elasticity of substitution θ , P_t the aggregate price level, $w_{j,t}$ the real wage rate for labor services $l_{j,t}$ of type j , T_t a lump-

sum tax, R_t the gross nominal interest rate on government bonds, and D_{it} dividends of monopolistically competitive firms. Further, households have to fulfill the no-Ponzi game condition, $\lim_{i \rightarrow \infty} E_t q_{t,t+i} (M_{jt+i} + B_{jt+i} + Z_{jt+1+i}) \geq 0$. The objective of the representative household is

$$E_{t_0} \sum_{t=t_0}^{\infty} \beta^t \{u(c_{jt}, \zeta_t) - v(l_{jt}) + z(M_{jt}/P_t)\}, \quad \beta \in (0, 1), \quad (2)$$

where β denotes the subjective discount factor and $M_{jt}/P_t = m_{jt}$ end-of-period real money balances. Note that our specification of utility is consistent with recent findings by Andrés, López-Salido and Vallés (2006) for the Euro area and by Ireland (2004) for the US. They estimate the role of money for the business cycle and find that preferences are separable between consumption and real money balances.

We assume that households' utility can be affected by a disturbance term ζ_t with mean 1 that can alter the utility of consumption. To avoid additional complexities, we set $u_{c\zeta} = u_c$ at the deterministic steady state. For each value of ζ , the instantaneous utility function is assumed to be non-decreasing in consumption and real balances, decreasing in labor time, strictly concave, twice continuously differentiable, and to fulfill the Inada conditions.

Households are wage-setters supplying differentiated types of labor l_j which are transformed into aggregate labor l_t with $l_t^{(\epsilon_t-1)/\epsilon_t} = \int_0^1 l_{jt}^{(\epsilon_t-1)/\epsilon_t} dj$. We assume that the elasticity of substitution between different types of labor, $\epsilon_t > 1$, varies exogenously over time. The time variation in this markup parameter introduces a so called cost-push shock into the model that gives rise to a stabilization problem for the central bank. Cost minimization implies that the demand for differentiated labor services l_{jt} , is given by $l_{jt} = (w_{jt}/w_t)^{-\epsilon_t} l_t$, where the aggregate real wage rate w_t is given by $w_t^{1-\epsilon_t} = \int_0^1 w_{jt}^{1-\epsilon_t} dj$. Maximizing (2) subject to (1) and the no-Ponzi game condition for given initial values $M_{t_0-1} > 0$, Z_0 , B_{t_0-1} , and $R_{t_0-1} \geq 0$ leads to the following first order conditions for consumption, money, the

real wage rate for labor type j , government bonds, and contingent claims:

$$\lambda_{jt} = u_c(c_{jt}, \zeta_t), \quad v_l(l_{jt}) = w_{jt} \lambda_{jt} / \mu_t^w, \quad (3)$$

$$\lambda_{jt} - z_m(m_{jt}) = \beta E_t \frac{\lambda_{jt+1}}{\pi_{jt+1}}, \quad q_{t,t+1} = \frac{\beta \lambda_{jt+1}}{\pi_{t+1} \lambda_{jt}}, \quad \lambda_{jt} = \beta R_t E_t \frac{\lambda_{jt+1}}{\pi_{t+1}} \quad (4)$$

where λ_{jt} denotes a Lagrange multiplier, π_t the inflation rate $\pi_t = P_t/P_{t-1}$, and $\mu_t^w = \epsilon_t/(\epsilon_t - 1)$ the stochastic wage mark-up with mean $\bar{\mu}^w > 1$. The first order condition for contingent claims holds for each state in period $t + 1$, and determines the price of one unit of currency for a particular state at time $t + 1$ normalized by the conditional probability of occurrence of that state in units of currency in period t . The absence of arbitrage opportunities between government bonds and contingent claims requires $R_t = 1/E_t q_{t,t+1}$. The optimum is further characterized by the budget constraint (1) holding with equality and by the transversality condition $\lim_{i \rightarrow \infty} E_t \beta^i \lambda_{jt+i} (M_{jt+i} + B_{jt+i} + Z_{jt+i+1}) / P_{jt+i} = 0$.

The final consumption good Y_t is an aggregate of differentiated goods produced by monopolistically competitive firms indexed with $i \in [0, 1]$ and defined as $y_t^{\frac{\theta-1}{\theta}} = \int_0^1 y_{it}^{\frac{\theta-1}{\theta}} di$, with $\theta > 1$. Let P_{it} and P_t denote the price of good i set by firm i and the price index for the final good. The demand for each differentiated good is $y_{it}^d = (P_{it}/P_t)^{-\theta} y_t$, with $P_t^{1-\theta} = \int_0^1 P_{it}^{1-\theta} di$. A firm i produces good y_i using a technology that is linear in the labor bundle $l_{it} = [\int_0^1 l_{jit}^{(\epsilon_t-1)/\epsilon_t} dj]^{\epsilon_t/(\epsilon_t-1)}$: $y_{it} = a_t l_{it}$, where $l_t = \int_0^1 l_{it} di$ and a_t is a productivity shock with mean 1. Labor demand satisfies: $mc_{it} = w_t/a_t$, where $mc_{it} = mc_t$ denotes real marginal cost independent of the quantity that is produced by the firm.

We allow for a nominal rigidity in form of a staggered price setting as developed by Calvo (1983). Each period firms may reset their prices with the probability $1 - \alpha$ independently of the time elapsed since the last price setting. The fraction $\alpha \in [0, 1]$ of firms are assumed to keep their previous period's prices, $P_{it} = P_{it-1}$, i.e. indexation is absent.

Firms are assumed to maximize their market value, which equals the expected sum of discounted dividends $E_t \sum_{T=t}^{\infty} q_{t,T} D_{iT}$, where $D_{iT} \equiv P_{iT} y_{iT} (1 - \tau) - P_t m c_t y_{iT}$ and we used that firms also have access to contingent claims. Here, τ denotes an exogenous sales tax introduced to offset the inefficiency of steady state output due to markup pricing (Rotemberg and Woodford, 1999). In each period a measure $1 - \alpha$ of randomly selected firms set new prices \tilde{P}_{it} as the solution to $\max_{\tilde{P}_{it}} E_t \sum_{T=t}^{\infty} \alpha^{T-t} q_{t,T} (\tilde{P}_{iT} y_{iT} (1 - \tau) - P_T m c_T y_{iT})$, s.t. $y_{iT} = (\tilde{P}_{it})^{-\theta} P_T^{\theta} y_T$. The first order condition for the price of re-optimizing producers is given by

$$\frac{\tilde{P}_{it}}{P_t} = \frac{\theta}{\theta - 1} \frac{F_t}{K_t}, \quad (5)$$

where K_t and F_t are defined by the following expressions:

$$F_t = E_t \sum_{T=t}^{\infty} (\alpha \beta)^{T-t} u_c(c_T, \zeta_T^{(1)}) y_T \left(\frac{P_T}{P_t} \right)^{\theta} m c_T \quad (6)$$

and

$$K_t = E_t \sum_{T=t}^{\infty} (\alpha \beta)^{T-t} u_c(c_T, \zeta_T^{(1)}) (1 - \tau) y_T \left(\frac{P_T}{P_t} \right)^{\theta-1}. \quad (7)$$

Aggregate output is given by $y_t = a_t l_t / \Delta_t$, where $\Delta_t = \int_0^1 (P_{it} / P_t)^{-\theta} di \geq 1$ and thus $\Delta_t = (1 - \alpha)(\tilde{P}_t / P_t)^{-\theta} + \alpha \pi_t^{\theta} \Delta_{t-1}$. The dispersion measure Δ_t captures the welfare decreasing effects of staggered price setting. If prices are flexible, $\alpha = 0$, then the first order condition for the optimal price of the differentiated good reads: $m c_t = (1 - \tau)^{\frac{\theta-1}{\theta}}$.

The public sector consists of a fiscal and a monetary authority. The central bank as the monetary authority is assumed to control the short-term interest rate R_t as the single instrument. The fiscal authority issues risk-free one period bonds, has to finance exogenous government expenditures $P_t G_t$, receives lump-sum taxes from households, transfers from the monetary authority, and tax-income from an exogenous given constant sales tax τ , such that the consolidated budget constraint reads: $R_{t-1} B_{t-1} + M_{t-1} + P_t G_t = M_t + B_t +$

$P_t T_t + \int_0^1 P_{it} y_{it} \tau di$. The exogenous government expenditures G_t evolve around a mean \bar{G} , which is restricted to be a constant fraction of output, $\bar{G} = \bar{y}(1 - sc)$. We assume that tax policy guarantees government solvency, i.e., ensures $\lim_{i \rightarrow \infty} (M_{t+i} + B_{t+i}) \prod_{v=1}^i R_{t+v}^{-1} = 0$. Due to the existence of the lump-sum tax, we consider only the demand effect of government expenditures and focus exclusively on optimal monetary policy.

We collect the exogenous disturbances in the vector $\xi_t = [\zeta_t, a_t, G_t, \mu_t^w]$. It is assumed that the percentage deviation of each of the elements of the vector from their means evolve according to autonomous AR(1)-processes with autocorrelation coefficients $\rho_\zeta, \rho_a, \rho_G, \rho_\mu \in [0, 1)$. The innovations are assumed to be i.i.d..

The recursive equilibrium is defined as follows:

Definition 1 *Given initial values, $M_{t_0-1} > 0$, $P_{t_0-1} > 0$ and $\Delta_{t_0-1} \geq 1$, a monetary policy and a ricardian fiscal policy $T_t \forall t \geq t_0$, a sales tax τ , a rational expectations equilibrium (REE) for $R_t \geq 1$, is a set of sequences $\{y_t, c_t, l_t, mc_t, w_t, \Delta_t, P_t, \tilde{P}_t, m_t, R_t\}_{t=t_0}^\infty$ satisfying the firms' first order condition $mc_t = w_t/a_t$, (5) with $\tilde{P}_{it} = \tilde{P}_t$, and $P_t^{1-\theta} = \alpha P_{t-1}^{1-\theta} + (1-\alpha)\tilde{P}_t^{1-\theta}$, the households' first order conditions $u_c(y_t - G_t, \zeta_t) w_t = v_l(l_t) \mu_t^w$, $u_c(y_t - G_t, \zeta_t)/P_t = \beta R_t E_t u_c(y_{t+1} - G_{t+1}, \zeta_{t+1})/P_{t+1}$, $z_m(m_t) = u_c(y_t - G_t, \zeta_t)(R_t - 1)/R_t$, the aggregate resource constraint $y_t = a_t l_t / \Delta_t$, where $\Delta_t = (1-\alpha)(\tilde{P}_t/P_t)^{-\theta} + \alpha(P_t/P_{t-1})^\theta \Delta_{t-1}$, clearing of the goods market $c_t + G_t = y_t$ and the transversality condition, for $\{\xi_t\}_{t=t_0}^\infty$.*

3 The linear-quadratic optimal policy problem

In a first step, we compute the optimal deterministic steady state of the economy as the one that maximizes steady state utility. This steady state is our point of expansion for the log-linear approximation of the model's equilibrium conditions as well as for the derivation of the purely quadratic welfare measure. As we will see, long run and short run op-

timal policy are closely interrelated. Throughout we assume that the steady state is rendered efficient by an appropriate setting of the tax rate.

3.1 The optimal steady state

Our approach to optimal policy in the long run is to maximize steady state utility. Wolman (1999) shows that this criterion gives slightly different prescriptions to optimal policy in the long-run than the time invariant Ramsey concept, but these differences are quantitatively very small. Both approaches differ in our case due to the presence of forward-looking equations and one endogenous state variable, namely price dispersion.

The nonlinear optimization problem for the central bank is to choose steady state values for output, price dispersion, the denominator (K) and the numerator (F), the nominal interest rate and inflation to maximize steady state utility of the representative household

$$\max \mathcal{L} = u(y - G, \zeta) - v(\Delta y / a) + z(m(R, y - G, \zeta)), \quad (8)$$

subject to the firms' optimal pricing condition, the recursive formulation of the functions K and F , the evolution of the dispersion measures and the euler equation:

$$\rho(\pi)^{\frac{1}{1-\theta}} K = \frac{\theta}{\theta - 1} F \quad (9)$$

$$K = u_c(y - G, \zeta)(1 - \tau)y + \beta \alpha K \pi^{\theta-1} \quad (10)$$

$$F = v_l(y \Delta / a) y \mu^w + \alpha \beta F \pi^\theta \quad (11)$$

$$\Delta = (1 - \alpha) \rho(\pi)^{\frac{\theta}{\theta-1}} + \alpha \Delta \pi^\theta \quad (12)$$

and

$$\pi = \beta R, \quad (13)$$

with $\rho(\pi) \equiv (1 - \alpha\pi^{\theta-1})(1 - \alpha)^{-1}$.¹

To simplify the analysis and to solve for the optimal steady numerically, we assume that households' utility is given by the usual CRRA specification:

$$\frac{c^{1-\sigma_c}}{1-\sigma_c} - a_2 \frac{l^{1+\omega}}{1+\omega} + a_1 \frac{m^{1-\sigma_m}}{1-\sigma_m}, \quad (14)$$

σ_c , σ_m positive and ω non-negative. Here, $a_1 \geq 0$ denotes the weight for the utility stemming from real money balances relative to the utility of consumption and a_2 the corresponding relative weight for the disutility of labor.² We assume that the zero bound on the nominal interest rate is not binding in expectations. This is equivalent to assuming that inflation in the deterministic steady state is at least $\pi \geq \beta + \epsilon$ for a small parameter $\epsilon > 0$. The reason for this assumption is twofold. Economically, the resulting buffer allows the central bank to adjust its instrument downward in response to a shock (at least by a small amount). Technically, the CRRA preferences do not display a satiation point for real money balances at a finite level. However, by imposing a lower bound on the steady state nominal interest, real money balances are still bounded – even if inflation equals $\beta + \epsilon$ (and $R - 1 = \beta^{-1}\epsilon$). Then all first and second partial derivatives of utility with respect to c and m exhibit well defined finite limiting values (e.g. $z_{mm} < 0$) as c, m approach their corresponding finite values at the ϵ lower bound, c_ϵ, m_ϵ . In particular, this implies that the interest elasticity of money demand, $\eta_R(R_\epsilon) = z_m(m_\epsilon)[m_\epsilon z_{mm}(m_\epsilon)(R_\epsilon - 1)]^{-1}$ is well-defined and finite since $R_\epsilon - 1 = \beta^{-1}\epsilon$ is a small positive scalar.

σ_c	σ_m	ω	β	a_1	a_2	$sc = \bar{c}/\bar{y}$	$\bar{\mu}^w$	θ	α	ϵ
2	2.5	0.5	0.99	1/99	25	0.8	7/6	6	0.66	0.0001

Table 1: Baseline calibration

¹To simplify the notation, steady state values in the following denoted without a time subscript.

²The first order conditions and the constraints of the Ramsey problem in the deterministic steady state for the assumed CRRA preferences can be found in appendix 6.1.

In our baseline calibration we set $\theta = 6$ and $\alpha = 0.66$, where the latter can be found for example in Walsh (2005) or Woodford (2003a). The parameter a_2 is set such that agents work 1/3 of their available time in the steady state.

We calibrate the money demand block of our model to be in line with the existing literature and U.S. times series data. In particular, we set the annual interest semi-elasticity of money demand, $\partial \log m / \partial R = -[R(R-1)\sigma_m]^{-1}$ equal to -4.47 at an annual interest rate of $R = 1.083$. This is in line with Lucas (2000) and Woodford (2003a). In calibrating this elasticity we have assumed an average annual inflation rate of 4 per cent together with a real interest rate of 4.3 per cent such that $R = 1.083$. It then follows that $\sigma_m = 2.5$. Note that the semi-elasticity and the elasticity of money demand, $\eta_R(R) \equiv [(R-1)\sigma_m]^{-1} > 0$, increases (in absolute terms) as interest rates decrease. We assume a degree of relative risk aversion $\sigma_c = 2$. This implies an output elasticity of money demand $\sigma_c / (s_c \sigma_m) = 1$. Furthermore, we set the parameter $a_1 = 1/99$ such that at a nominal interest rate of $R = 1.083$ the annual ratio of M1 over nominal GDP equals 0.2. This value is consistent with postwar U.S. data and similar to the one used by Schmitt-Grohé and Uribe (2004, 2005).

Then the following numerical result for the ϵ steady state holds:

Result 1 (Optimal Steady State) *If $a_1 \geq 1/3513$ and the other parameters are given by the baseline calibration, optimal inflation in the deterministic steady state π is $\beta + \epsilon = 0.9901$. The associated optimal price dispersion $\bar{\Delta}$ is 1.0014, while the optimal nominal interest rate \bar{R} is $1.0001 > 1$.*

Under the baseline calibration, we find that the optimal steady-state value for inflation is the lower bound, $\pi = \beta + \epsilon$, i.e. it involves deflation. Correspondingly, the nominal interest rate is almost zero.

Since a_1 is an unobserved preference parameter, it is difficult to assess whether the critical value $a_1 = 1/3513$ implies a large or small role for money in the utility function.

However, the annual steady state ratio of $M1$ over nominal GDP implied by this critical value is 0.048. Hence, even if the importance of money in transactions - as measured by this ratio - falls by 76% from its baseline value of 0.2, the Friedman rule would still be optimal. Therefore, the Friedman rule is optimal in our model even when money provides a very small flow of utility.

Why does the Friedman rule turn out to be optimal even when the importance of real money balances in the utility function is very low? Optimal monetary policy seeks to minimize two distortions created by price dispersion and the transaction friction, since the monopolistic distortion is eliminated in the steady state by an output subsidy.³ Price dispersion calls for an inflation rate of zero, while the monetary friction requires deflation. Correspondingly, we expect our optimal gross inflation rate to be found between β and unity. First, while studies such as Kiley (2002) and Ascari (2004) have shown that relatively small amounts of trend inflation are associated with relatively large welfare costs under Calvo pricing, this is not the case for long run deflation. Figure 4 in the appendix shows that the price dispersion arising from long run deflation is relatively small. The second reason for the optimality of Friedman's rule is an adaption of a general principle of optimal taxation in public finance. Since the interest rate acts like a tax on money holdings, it should be low due to the fact that money demand is elastic with respect to interest under price stability.

While the choice for ϵ is arbitrary, our results are not very sensitive to the magnitude of ϵ (see Figure 1). The graph plots optimal annual inflation against the degree of price dispersion α . Remarkably, our threshold levels for the optimality of Friedman's rule differ substantially from the results obtained by Schmitt-Grohé and Uribe (2005, Figure 1).

³The output subsidy of $\tau = 1 - (1 - \alpha\beta\pi^{\theta-1})\mu^w\theta\rho(\pi)^{1/(\theta-1)}[(1 - \alpha\beta\pi^\theta)(\theta - 1)]^{-1} < 0$ depends on steady state deflation. However, this feature does not favor the Friedman Rule in the steady state. If we were to apply the subsidy under zero-inflation, $\tau = 1 - \mu^w\theta/(\theta - 1)$, the Friedman Rule would be optimal for even smaller relative weights of money in the utility function. The reason is as follows. First, note that steady state output is lower when the subsidy does not depend on trend deflation. Note further that the utility loss that households suffer due to a positive steady state price dispersion is weighted with the steady state output.

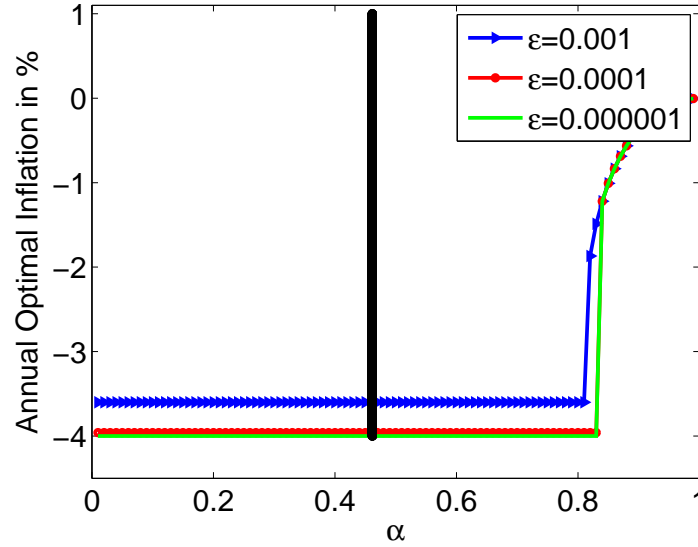


Figure 1: Optimal annual inflation and Calvo parameter α . The vertical line denotes the critical value in Schmitt-Grohe and Uribe (2005) for which the Friedman rule ceases to be optimal.

While the Friedman rule in our model is optimal until the degree of price dispersion is below 0.81, Schmitt-Grohé and Uribe find a considerably lower breaking point of approximately 0.46 (see the vertical line in Figure 1), since the welfare costs of positive interest rates are lower in their transaction costs specification. To be more precise the MIU framework unlike theirs implies that the interest-elasticity of money demand increases by large amounts as the nominal interest rate approaches the lower bound.⁴

Which parameters influence the lower bound on a_1 , i.e. the minimum weight for money in the utility function that renders the Friedman rule optimal? Put differently, which structural features work in favor for the Friedman rule and when does price dispersion become the main focus of monetary policy? To gain intuition for this question, we compare the outcomes of the Friedman rule and a zero inflation policy and derive an analytical expression of the threshold for which the former dominates the latter policy.

⁴While not uncontroversial this property is not special to our MIU formulation. It can be obtained in non-separable MIU specifications as well as in MIU models with a satiation point for real money balances and in transactions cost models.

Proposition 1 (Friedman's Rule and Zero Inflation) *Assume that preferences are of the separable CRRA type and logarithmic, $\sigma_m = \sigma_c = 1$, and $a_2 = 1$. Then the Friedman Rule steady state, $\pi_{FR} = \beta + \epsilon$, yields higher utility than the zero inflation steady state, $\pi_{ZERO} = 1$, if and only if*

$$a_1 > \underline{a_1} \equiv \frac{\frac{\Delta_{FR}-1}{(1+\omega)sc} + \frac{\omega}{1+\omega} \ln[\Delta_{FR}]}{\ln[R_{FR}\eta_{R,FR}(R_{ZERO}\eta_{R,ZERO})^{-1}] - \omega/(1+\omega) \ln[\Delta_{FR}]}$$

with Δ_{FR} as the price dispersion associated with $\pi = \beta + \epsilon$ and $R_{FR}\eta_{R,FR}(R_{ZERO}\eta_{R,ZERO})^{-1} = (1 - \beta)(1 + \beta^{-1}\epsilon)/\beta^{-1}\epsilon$.

Proof see appendix 6.2.

$R_{ZERO} = \beta^{-1}$ and $R_{FR} = 1 + \beta^{-1}\epsilon$ denote the gross nominal interest rate under zero inflation and Friedman's rule. Evidently, the Friedman rule performs better than a zero inflation regime, when the degree of price dispersion associated with the Friedman rule, Δ_{FR} is small. But at least equally important is the sensitivity of money demand with respect to interest rates under Friedman's rule, $\eta_{R,FR}$, compared to the corresponding elasticity if zero inflation applies, $\eta_{R,ZERO}$. If these elasticities differ substantially, the amount and utility of real money balances in both regimes differs too. As will become clear below, this elasticity heavily influences the possible welfare losses due to positive interest rates. Furthermore, a large fraction of private consumption, sc , favors the Friedman rule. The intuition is as follows. Consider a value for a_1 such that the Friedman rule delivers the same steady state welfare as the zero inflation policy. If the fraction of government expenditures decreases, people have to work less since less output has to be produced. Due to price dispersion, people work more under the Friedman Rule, such that their marginal disutility of labor is always higher than under the zero inflation regime. Correspondingly, a one percent decrease in labor in both regimes leads to relatively larger utility gains in the Friedman Rule regime.

In the following subsection we consider optimal monetary policy in the short run, as-

suming the baseline calibration, such that $\beta + \epsilon$ is the optimal inflation rate.

3.2 Approximating the model around the optimal steady state

The model is log-linearized around the optimal deterministic steady state $\pi = \beta + \epsilon < 1$, i.e. under trend deflation and closely follows the approximation around trend inflation (Ascari, 2004). The rational expectations equilibrium for the log-linear-approximate model is then a set of sequences $\{\hat{y}_t, \hat{\pi}_t, \hat{m}_t, \hat{R}_t, \hat{F}_t\}_{t=t_0}^{\infty}$ consistent with the following set of equilibrium conditions⁵

$$\sigma(E_t \hat{y}_{t+1} - \hat{y}_t + g_t - g_{t+1}) = \hat{R}_t - \hat{\pi}_{t+1}, \quad (15)$$

$$\hat{m}_t = \frac{\sigma}{\sigma_m}(\hat{y}_t - g_t) - \eta_{R,FR} \hat{R}_t, \quad (16)$$

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa^*(\omega + \sigma)(\hat{y}_t - \hat{y}_t^z) + \frac{\kappa^*(\bar{\pi} - 1)}{1 - \alpha\beta\pi^\theta}[(\sigma - 1)\hat{y}_t + \hat{F}_t] \quad (17)$$

$$\hat{F}_t = (1 - \alpha\beta\pi^\theta)[(1 + \omega)\hat{y}_t + \hat{\mu}_t^w - (1 + \omega)\hat{a}_t] + \alpha\beta\pi^\theta E_t(\theta\hat{\pi}_{t+1} + \hat{F}_{t+1}), \quad (18)$$

where $\eta_{R,FR} = [\sigma_m(R_{FR} - 1)]^{-1}$, $sc = c/y$, $\sigma_c = -u_{cc}c/u_c > 0$, $\sigma = \sigma_c sc^{-1}$, $\omega = v_{ll}l/v_l > 0$, $g_t = (G_t - G)/y + \sigma^{-1}\hat{\zeta}_t$, $\kappa^* = (1 - \alpha\pi^{\theta-1})(1 - \beta\alpha\pi^\theta)/(\alpha\pi^\theta)$, disturbances are collected in $\hat{y}_t^z = ((1 + \omega)\hat{a}_t + \sigma g_t - \hat{\mu}_t^w)/(\omega + \sigma)$, $\sigma_m = -z_{mm}(\bar{m})\bar{m}/z_m(\bar{m}) > 0$, the transversality condition, for a monetary policy, a sequence $\{\hat{\xi}_t\}_{t=t_0}^{\infty}$, and given initial values M_{t_0-1} and P_{t_0-1} . Further \hat{z}_t denotes the percent deviation of a generic variable z_t from its steady state value z . In addition we assume that the bounds on the fluctuations of the shock vector $\|\log \xi_t\|$ are sufficiently tight, such that ξ_t remains in the neighborhood of its steady state value.

⁵The derivation of the aggregate supply curve can be found in our working paper version.

3.3 A quadratic policy objective

In this section we derive a purely quadratic welfare measure for the utility of the average household as the relevant objective for optimal monetary policy in the short run.

We assume that the welfare-relevant objective is the expected and discounted average utility level of all households, which is given by

$$U_{t_0} \equiv E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \{u(c_t, \zeta_t) - \int_0^1 v(l_{jt}) dj + z(M_t/P_t)\}. \quad (19)$$

Our aim is to derive a quadratic loss function that yields an accurate second order approximation of the average utility of all households. We seek to evaluate the approximated level of utility by using the log-linearized conditions (15)-(18) describing the competitive equilibrium – that is, we set up the familiar linear-quadratic optimal policy problem. A correct welfare ranking of alternative policies requires a second-order approximation of utility that involves no linear terms – at least in expectations (see Woodford, 2003a, ch.6).

The existence of a non-zero linear term in the utility approximation crucially relies on the distortions of the steady state output relative to the efficient output level as consequences of price and wage-setting power, distortionary taxation and trend deflation that are represented in ϕ :

$$1 - \phi = \rho(\pi)^{\frac{1}{1-\theta}} (1 - \tau) \frac{\theta - 1}{\mu^{\omega\theta}} \frac{1 - \alpha\beta\pi^\theta}{1 - \alpha\beta\pi^{\theta-1}} = \frac{v_l}{u_c}. \quad (20)$$

If this inefficiency gap is zero or only of first order in ϕ , the linear term in the second order approximation vanishes. Following Rotemberg and Woodford (1997) we assume that the sales tax plays a role of an output subsidy that offsets exactly the steady state output distortion. Since we assume separability between consumption and real money balances, this implies that real balance effects do not contribute to this inefficiency measure.

As Carlstrom and Fuerst (2004) point out, the inclusion of money demand fundamentally changes optimal monetary policy responses even in case if one assumes – as we do – real balances do not effect the dynamic evolution of inflation and output in the competitive equilibrium. The reason is that variations in the nominal interest rate contribute to the relevant distortions the policy maker seeks to stabilize. As we will show below, the relative weight of variations in the interest rate that enters the welfare measure is substantially increased if we approximate around the optimal steady state. In the following proposition we derive a quadratic Taylor-series approximation to (19).

Proposition 2 (Quadratic Approximation to Utility) *If the fluctuations in y_t around y , R_t around R , ξ_t around ξ , π_t around π are small enough, π and Δ are close enough to 1, and if the steady state distortions ϕ vanish due to the existence of an appropriate subsidy τ , the utility of the average household can be approximated by:*

$$U_{t_0} = -\Omega E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} [\lambda_x (\hat{y}_t - \hat{y}_t^*)^2 + \hat{\pi}_t^2 + \lambda_R \hat{R}_t^2] + t.i.s.p. + \mathcal{O}(\|\hat{\xi}_t, \varsigma\|^3), \quad (21)$$

where *t.i.s.p.* indicate terms independent of stabilization policy, $\kappa = (1 - \alpha)(1 - \alpha\beta)(\omega + \sigma)/\alpha$, $\Omega = \frac{u_c y \theta (\omega + \sigma)}{2\kappa}$,

$$\lambda_x = \frac{\kappa}{\theta}, \quad (22)$$

$$\lambda_R = \frac{\eta_{R,FR} \lambda_x}{\nu(\omega + \sigma)}, \quad (23)$$

and

$$\hat{y}_t^* = \frac{\sigma g_t + (1 + \omega) \hat{a}_t}{\omega + \sigma}, \quad (24)$$

where $\nu = y/m > 0$ and $\eta_{R,FR}$ is the interest elasticity of money demand at the Friedman rule steady state.

Proof see appendix 6.3.

Under the conditions given in proposition 2, the relative weights of inflation, output gap and the nominal interest rates correspond to the results in Woodford (2003a). Our analysis differs from Woodford (2003a), because the steady state values relate to the lower bound and no longer to price stability as in his analysis.

Remarkably, only the weight to stabilize fluctuations in the nominal interest rate depends on steady state values, ν and $\eta_{R,FR}$. Since we approximate our model around the deterministic steady state consistent with the Friedman Rule, the value for the former is small and the value for the latter is large, implying a high preference to stabilize variations in the opportunity costs to hold money. To set up the optimal policy problem, we need to rewrite the relevant constraints, i.e. the Euler-equation, the law of motion for \hat{F}_t and the aggregate supply curve in terms of the welfare-relevant output gap, $x_t = \hat{y}_t - \hat{y}_t^*$:

$$\hat{R}_t = \hat{\pi}_{t+1} + \sigma(E_t x_{t+1} - x_t) + n_t, \quad (25)$$

$$\hat{F}_t = (1 - \alpha\beta\bar{\pi}^\theta)(1 + \omega)x_t + u_t + \alpha\beta\bar{\pi}^\theta E_t(\theta\hat{\pi}_{t+1} + \hat{F}_{t+1}) \quad (26)$$

and

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \eta_4 x_t + \frac{\kappa^*(\bar{\pi} - 1)}{1 - \alpha\beta\bar{\pi}^\theta} \hat{F}_t + s_t. \quad (27)$$

Here, n_t , u_t , s_t denote linear combinations of the elements of $\hat{\xi}_t$ and η_4 is a constant, which are defined in appendix 6.4. Note, that the money demand condition does not enter the set of relevant constraints of the policy problem. Nevertheless it influences the optimal decision via the quadratic loss function, in which it plays an important role in determining the relative weight of interest rate variations.

4 Optimal monetary policy in the short run

Our approach to optimal policy in the short run is the timeless perspective. At $t = t_0$ the central bank respects prior commitments made in the infinite past (Woodford, 2003a). Hence, the associated optimality conditions will be time invariant which marks the difference to a standard commitment approach. In particular, the optimality conditions in the initial period do not differ from those in later periods. We showed that the optimal policy in the long run is to follow the Friedman rule. In this section we consider the implications for optimal policy in the short run, if deflation – instead of zero inflation – is chosen as the optimal long run target. In particular, we consider the optimal reaction to various kinds of disturbances and evaluate the resulting stabilization loss of both regimes.

4.1 Optimal response to shocks

Our impulse responses analysis distinguishes two cases. In the first case, our set of equilibrium conditions is log-linearized around the optimal steady state in which the inflation rate is equal to $\beta + \epsilon$. In the second case, we follow the conventional procedure and approximate around a steady state of zero inflation. The choice of a point of expansion for the log-linearization affects both the loss function and equilibrium conditions. Log-linearizing around the Friedman rule increases the relative weight on the stabilization of the nominal interest rate and affects the coefficients in the Phillips curve.

When we log-linearize around the optimal steady state corresponding to the Friedman rule, we find that the central bank essentially keeps the nominal interest rate fixed in response to any of the shocks present in our model. Consider first the optimal response to a technology shock displayed in Figure 2. A Taylor expansion around zero inflation suggests that the central bank should lower the annualized nominal rate by roughly 12 basis points and then gradually return to the steady state. However, linearization around the Friedman

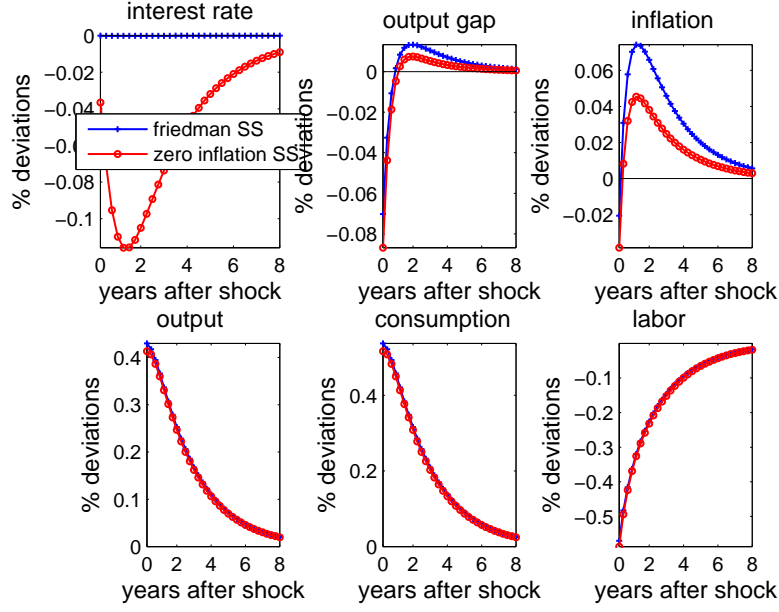


Figure 2: Responses to technology shock

rule implies that the nominal rate is literally fixed. In line with this finding, the approximation around the Friedman rule implies more volatile response of inflation and the output gap than what is suggested by linearization around the zero inflation steady state. A stronger stabilization of the nominal interest rate necessarily implies that the other arguments in the loss function can only be stabilized less.

Impulse responses to the other shocks deliver a similar message: Linearization around the Friedman steady state implies that the nominal interest rate is literally fixed. To understand this, note that the interest elasticity of money demand, $[\sigma_m(R-1)]^{-1}$ becomes very large as R approaches its lower bound. For our baseline calibration this elasticity is roughly -4000 at $R = 1 + \beta^{-1}\epsilon$. Despite the fact that the marginal utility of real balances is close to zero, this large elasticity explains why the central bank wishes to hold the nominal rate constant under the Friedman rule.

When deriving the quadratic policy objective we need to assume that price dispersion in the steady state was small. Does this assumption hold in our model? Note from Fig-

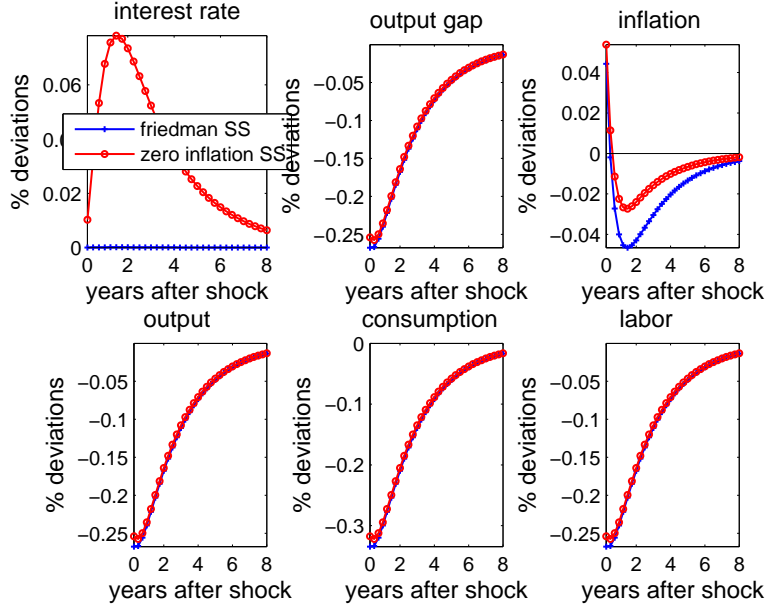


Figure 3: Responses to wage markup shock

ure 4 in the appendix that the dispersion measure is lower for deflation than for inflation. Hence, the condition is more likely to be fulfilled when the model is approximated around a deflationary steady state.⁶ To further reassure the reader, we compare the impulse response functions from the linear-quadratic approach to those obtained from linearizing the first order conditions of the non-linear time invariant Ramsey problem (43)-(48), as well as the constraints (49)-(53) and log-linearizes them around the optimal steady state (see appendix 6.5). The results of this experiment are displayed in Figure 5 in the appendix. The impulse responses are remarkably similar indicating the accuracy of our linear quadratic approach.⁷

⁶This depends crucially on the absence of strategic complementarities in price setting (Levin, Lopez-Salido and Yun, 2006).

⁷To induce the system of first-order conditions of the Ramsey planner to have the same steady state as the one chosen for our expansion point of the linear-quadratic problem, we have to add a constant to the first-order condition of the Ramsey planner for the nominal interest rate that is non-zero. This constant picks up the steady state slack that arises because the Friedman rule steady state constitutes a corner solution. The constant plays no further role for the dynamics.

4.2 Welfare analysis

In this subsection we compare the welfare implications of the two policy regimes – the long run deflation target according to the Friedman rule vs. zero inflation as the long run target. Using (21) a second-order accurate approximation to the utility of the average household is given by:

$$U_{t_0} = E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} U_t \approx \frac{1}{1-\beta} \bar{U} - \Omega E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \lambda_x (\hat{y}_t - \hat{y}_t^*)^2 + \hat{\pi}_t^2 + \lambda_R \hat{R}_t^2. \quad (28)$$

The first part, the discounted steady state utility, is shown to be higher if the Friedman rule is optimal. The second part, the stabilization loss, that relates to the optimal policy reaction in the short run, is not necessarily lower under the Friedman rule regime than under zero inflation. Which of those two parts dominates depends on the calibration of the model, e.g. increasing the variances of the innovations amplifies the welfare loss due to short run fluctuations. In line with the spirit of the timeless perspective, we do not compute welfare conditional on a particular initial state vector at time t_0 . Our short run stabilization loss is given by the discounted and weighted sum of unconditional variances:

$$SL = -\frac{1}{1-\beta} \Omega \{var(\hat{\pi}) + \lambda_x var(x) + \lambda_R var(\hat{R})\} = -\frac{1}{1-\beta} \Omega L, \quad (29)$$

Here L is proportional to the unconditional expectation of period utility. In table 2 below we list the relative loss differences under the two policy regimes for a range of relative weights for the utility of real money balances given our baseline calibration for other parameters. For this purpose we calibrate the stochastic shock processes to match the standard deviations of real private consumption and government spending of U.S. data during the post-Volcker period.⁸ All exogenous processes are assumed to be autocorre-

⁸The quarterly data is logged and detrended via the Hodrick-Prescott filter with a smoothing parameter of 10,000. The obtained standard deviation of private consumption is 0.0123, for government expenditures

lated with coefficient 0.9. We have chosen a standard deviation of the innovations to the taste shock of 0.0001, for the markup shock 0.00015, for the government spending shock 0.0075 and for the technology shock 0.0096.

a_1	λ_R^{ZERO}	λ_R^{FR}	$bcc^{ZERO} - bcc^{FR}$	dU_{CE}	σ_{FR}	σ_{ZERO}
1/20	2.3426	1472	0.0017%	1.3617%	317	41
1/50	1.6238	1020	0.0011%	0.8959%	220	31
1/99	1.2355	777	0.0005%	0.6423%	167	25
1/150	1.0463	658	0.0002%	0.5182%	142	23
1/189	0.9539	600	0.00%	0.4574%	129	21
1/250	0.8530	536	-0.0002%	0.3909%	116	20
1/500	0.6464	406	-0.0008%	0.2544%	88	17
1/1000	0.4899	308	-0.0014%	0.1506%	66	15

Table 2: Welfare Analysis: $\epsilon = 0.0001$

The results in Table 2 shows that the larger the preference parameter a_1 the larger is the weight for interest rate stabilization in the loss function, λ_R . Here, λ_R^{ZERO} denotes the weight when the model is approximated around the zero inflation and λ_R^{FR} denotes the weight for the approximation around the Friedman rule steady state.⁹ The fourth column displays the difference in stabilization loss or business-cycle costs under both regimes. To be more precise it depicts how much steady state consumption agents are willing to give up permanently to compensate for short run fluctuations in the regime with zero inflation as long run target, bcc^{ZERO} , relative to the Friedman rule regime, bcc^{FR} . Evidently, this difference is small, e.g. 0.0005% under the baseline calibration with $a_1 = 1/99$. The resulting stabilization loss, when approximating around the Friedman rule steady state is superior to the stabilization loss around zero inflation if a_1 is large enough.

The (technical) intuition for this is a trade off effect between predictability and possible welfare losses in the neighborhood of the steady state of each regime. If the Friedman

we obtain 0.0172.

⁹Table 3 in the appendix gives the corresponding results for $\epsilon = .000001$, i.e. if the assumed lower bound is closer to the zero bound.

rule is the expansion point, then the reduced form involves 4 jump variables, \hat{R}_t , x_t , $\hat{\pi}_t$ and \hat{F}_t , as well as 3 endogenous state variables, the multipliers on the relevant constraints, (25)-(27). If zero inflation is chosen as the approximation point, the reduced form does not involve \hat{F}_t and exhibits only the two multipliers associated with the aggregate supply curve and the euler equation as endogenous state variables. On the one hand, the fundamental solution in the Friedman regime is characterized by an additional history dependent variable. This tends to increase prediction power by reducing the forecast error variances of inflation, output gap and the nominal interest rate.¹⁰ On the other hand, however, possible welfare losses in the neighborhood of the zero inflation steady state are lower, steady state utility is "flatter" around $\pi = 1$ (see Figure 6 in the appendix). If the relative weight of real money balances decreases, the additional state variable loses prediction power, while possible welfare losses around the zero inflation steady state decrease.

While there is a cut-off value in terms of stabilization loss, overall utility composed of steady state utility minus stabilization loss, is higher under the Friedman rule than under zero inflation (dU_{CE}). The third but last column of Table 2 depicts this overall difference in utility expressed in steady state consumption equivalents of the Friedman rule steady state. Under the baseline calibration ($a_1 = 1/99$) agents are willing to give up permanently 0.64% of their consumption in the Friedman rule steady state until they are indifferent between the Friedman rule and the zero inflation regime.

We address the issue of the lower bound approximately in a way proposed originally by Rotemberg and Woodford (1997) and more recently by Schmitt-Grohé and Uribe (2005). First, we compute the optimal steady state under the assumption that the steady state nominal interest rate is at least slightly positive, $R - 1 = \beta^{-1}\epsilon > 0$ in the Friedman rule regime, and $R = \beta^{-1} > 0$ under zero inflation. However, this does not exclude an occa-

¹⁰E.g. Woodford (2003b) or Walsh (2003b) find that history-dependence can be beneficial for social welfare in forward-looking models. Note however, that in our case the models are not structurally nested, since in the Friedman regime more jump variables must be pinned down.

sionally binding zero bound. The entries σ_{FR} and σ_{ZERO} shed light on how likely it is that the lower bound on the nominal interest rate binds if the economy fluctuates around the Friedman rule ϵ steady state or around price stability. We calculate the standard deviation of the nominal interest rate under the optimal policy implied by both policy regimes. The term σ_{FR} then expresses the size of the interval from $R = 1.0001$ to the lower bound $R = 1$ in terms of this standard deviation. The entry σ_{ZERO} does the same, but now the approximation is computed around a zero inflation steady state. Hence, larger values for σ_{FR} or for σ_{ZERO} imply that the lower bound is less likely to be binding. Note that our results imply a low probability that the nominal interest rate hits the lower bound, i.e. $R_t = 1$. Even for a small relative weight of real money balances, $a_1 = 1/1000$, the resulting standard deviation for the nominal interest rate is small relative to ϵ . A symmetric confidence interval around $R = 1.0001$ of up to 66 standard deviations could be constructed until the lower bound is included. If we decrease ϵ , i.e. if the assumed lower bound moves closer to zero, the corresponding number of standard deviations increases to 418 (see table 3 in the appendix). This implies that the effect to stabilize the nominal interest even more (higher relative weight λ_R^{FR}) dominates the effect of the smaller distance to the zero bound. Correspondingly, if zero inflation is chosen as the expansion point, the probability to hit the lower bound is even higher (see the last column). We stress that our attempt to approximating the probability of lower bound violations ignores certain feedback channels.¹¹ Nevertheless, computing the variance of the nominal interest rate is one way to gauge the severity of the lower bound constraint in linear models.

¹¹Recent work by Adam and Billi (2006) or Eggertsson and Woodford (2003) explicitly accounts for the non-linear lower bound constraint and shows how the possibility of a binding constraint affects agents' decisions. They find, that the lower bound constraint may amplify the adverse effects of shocks and trigger a stronger response of monetary policy.

5 Conclusion

We study optimal monetary policy in an economy without capital, where firms set prices in a staggered way without indexation and real money balances are assumed to provide utility. Accounting for a sizeable degree of nominal rigidity, the optimal deterministic steady state that maximizes steady state utility is to follow the Friedman rule, even if the importance assigned to the utility of money is small relative to consumption and leisure.

We approximate the model around this optimal steady state as the long-run policy target and derive a second order approximation to households' utility. Optimal interest rate policy is shown to abstain from reacting sharply to changes in the state of the economy. Instead of stabilizing inflation, the primary goal of the central bank is to stabilize fluctuations in the nominal interest rate.

We stress that our model is not about direct and quantitative advice on optimal monetary policy. It is too stylized for this purpose. The foremost contribution of this paper is to challenge the conventional view that the Friedman rule loses out to the goal of price stability once price stickiness is introduced. We show that a widely used money-in-the-utility function model implies that the Friedman rule is optimal even when large amounts of price stickiness are present. When the economy fluctuates around the Friedman rule steady state, central bankers should keep the nominal interest stable over the business cycle. This result is explained by the large interest elasticity of money demand that obtains in our MIU model when the nominal rate is close to zero. There is little empirical evidence on the behavior of money demand in the major industrialized countries for very low interest rates. This is unfortunate as the interest elasticity at low interest rates is a key difference between our MIU framework and the transactions technology employed in other papers that come to different policy prescriptions. Therefore, future research on optimal monetary policy in sticky price models benefits from a better understanding of money demand. Recent work by Ireland (2007) contributes to this issue and points towards a change in U.S.

money demand at low rates in the post 1980 period.

6 Appendix

6.1 Constraints and CRRA preferences in the steady state

Suppose that the utility function is of the CRRA form. Given an output subsidy that renders the steady state efficient, constraints (9)-(13), and the money demand equation can be combined to solve for Δ , y , c , l , R and m in terms of inflation.

$$\Delta = \frac{(1 - \alpha)\rho^{\frac{\theta}{\theta-1}}}{1 - \alpha\pi^\theta}, \quad (30)$$

with $\rho = (1 - \alpha\pi^{\theta-1})/(1 - \alpha)$,

$$y = \left[\frac{1}{a_2 \Delta^\omega s c^{\sigma_c}} \right]^{\frac{1}{\sigma_c + \omega}}, \quad (31)$$

$$c = y s c, \quad (32)$$

$$l = y \Delta \quad (33)$$

$$R = \frac{\pi}{\beta}, \quad (34)$$

$$m = [R/(R - 1) y^{\sigma_c} a_1 s c^{\sigma_c}]^{1/\sigma_m}. \quad (35)$$

6.2 Proof proposition 1

Consider first the steady state utility if the inflation rate is zero. Correspondingly, gross inflation and price dispersion are 1, such that $y_{ZERO} = l_{ZERO}$. Using (31)-(35), one can compute $y_{ZERO} = 1/s c^{1/(1+\omega)} = l_{ZERO}$, $c_{ZERO} = s c^{\omega/(1+\omega)}$ and $m_{ZERO} = R_{ZERO} \eta_{R,ZERO} a_1 s c^{\omega/(1+\omega)}$. Then the period steady state utility of the average household is given by

$$u_{ZERO} = (1 + a_1) \frac{\omega}{1 + \omega} \ln(s c) - \frac{1}{(1 + \omega) s c} - a_1 \ln(1 - \beta) + a_1 \ln(a_1). \quad (36)$$

If $\pi = \beta + \epsilon$, then price dispersion is $\Delta_{FR} > 1$, and output equals $y_{FR} = 1/(sc\Delta_{FR}^\omega)^{1/(1+\omega)} < y_{ZERO}$, while $l_{FR} = (\Delta_{FR}/sc)^{1/(1+\omega)} > l_{ZERO}$. Consumption and real money balances are then given by $c_{FR} = (sc/\Delta_{FR})^{\omega/(1+\omega)} < c_{ZERO}$ and $m_{FR} = R_{FR}\eta_{R,FR}a_1(sc/\Delta_{FR})^{\omega/(1+\omega)} > m_{ZERO}$. In this case, the period steady state utility is

$$u_{FR} = (1 + a_1) \frac{\omega}{1 + \omega} \ln(sc) - \frac{\omega}{1 + \omega} (1 + a_1) \ln(\Delta_{FR}) - \frac{\Delta_{FR}}{(1 + \omega)sc} + a_1 \ln\left(\frac{1 + \beta^{-1}\epsilon}{\beta^{-1}\epsilon}\right) + a_1 \ln(a_1). \quad (37)$$

Comparing (36) and (37), the Friedman rule yields higher utility as long as $a_1 > \underline{a}_1$. ■

6.3 Proof proposition 2

The period utility function of the average household in equilibrium is given by:

$$\int_0^1 [u(y_t - G_t, \zeta_t) - v(l_{jt}) + z(m_t)] dj = u(y_t - G_t) + z(m_t) - \int_0^1 v(l_{jt}) dj.$$

To derive (21) we need to impose that, in the optimal steady state, real money balances are sufficiently close to being satiated (see Woodford, 2003a, Assumption 6.1), the price dispersion associated with optimal inflation is sufficiently small, as well as that optimal inflation is close enough to one.

The first summands can be approximated to second order by:

$$u(y_t - G_t, \zeta_t) = u_c y [\hat{y}_t + \frac{(1 - \sigma)}{2} \hat{y}_t^2 + \sigma g_t \hat{y}_t] + t.i.s.p. + \mathcal{O}(\|\hat{\xi}_t, \hat{y}_t\|^3), \quad (38)$$

where we used that $(x_t - x) = x(\hat{x}_t + 0.5\hat{x}_t^2) + \mathcal{O}(\|\hat{x}_t\|^3)$, t.i.s.p denotes terms independent of stabilization policy, $u_{c\zeta} = u_c$, $\zeta = 1$, $\sigma = \sigma_c sc^{-1}$, $\hat{G}_t = (G_t - G)/y$, and that $g_t = \hat{G}_t + \sigma^{-1}\hat{\zeta}_t$.

The utility of real money balances can be approximated by:

$$z(m) = u_c y \left[\frac{z_m m}{u_c y} \hat{m}_t + 0.5 \frac{z_m m}{u_c y} (1 - \sigma_m) \hat{m}_t^2 \right] + t.i.s.p. + \mathcal{O}(\|\hat{m}_t\|^3).$$

We treat $(R - 1)/R$ as an expansion parameter, implying that $z_m/u_c - 0 = (R - 1)/R - 0$ is at least of first order. Since we expand our model at a point near the zero bound, this means that the marginal utility of real money balances is close to zero.

Applying a first-order approximation to the money demand equation, and using that the coefficients $\sigma/\sigma_m = u_{cc}scy(R - 1)/(Rz_{mm}m)$ and $s_m = z_m m/(u_c y)$ are of first order in $(R - 1)/R$, we can approximate $z(m_t)$ as:

$$z(m_t) = -u_c y \frac{1}{2\sigma_m(R - 1)v} \hat{R}_t^2 + t.i.s.p. + \mathcal{O}(\|\hat{\xi}_t, \hat{y}_t, \hat{R}_t\|^3), \quad (39)$$

where we assumed that $(R - 1)/R - 0$ is of second order, implying that the linear term drops out in the quadratic approximation.

The third part of households' period utility can be approximated by:

$$v(l_t) = v_y y [\hat{y}_t + \frac{1 + \omega}{2} \hat{y}_t^2 - (1 + \omega) \hat{a}_t \hat{y}_t + \hat{\Delta}_t] + t.i.s.p. + \mathcal{O}(\|\hat{\xi}_t, \hat{y}_t, \hat{\Delta}_t^{0.5}, \varsigma\|^3), \quad (40)$$

with $v_y y = v_\Delta \Delta = v_l l$. This approximation is based on the assumption that

$$\int_0^1 (P_i/P)^{-\theta} di = (\check{P}/P)^{-\theta} - 1 = \mathcal{O}(\|\varsigma\|^3) \quad (41)$$

. Here \check{P} denotes the average long-term individual price and we collect in ς the distortions of the relative price due to price dispersion in the optimal steady state.¹² It follows that

$$\Delta_t - 1 = \frac{\theta}{2} var_i \ln(P_{it}) + \mathcal{O}(\|\hat{p}_{it}, \hat{\xi}_t, \varsigma\|^3),$$

and correspondingly $\hat{\Delta}_t$ are of second order. Connecting (38), (39) and (40) by the relation-

¹²This assumption depends on the absence of strategic complementarities in price setting (Yun, Levin, Lopez-Salido and Yun, 2006). Then, price dispersion in the steady state is lower for deflation than for inflation (see Figure 4.)

ship $v_y/u_c = amc/\mu^w = (1 - \phi)$, with

$$\phi = 1 - \rho(\pi)^{\frac{1}{1-\theta}} (1 - \tau) \frac{\theta - 1}{\mu^w \theta} \frac{1 - \alpha \beta \pi^\theta}{1 - \alpha \beta \pi^{\theta-1}},$$

results in:

$$\begin{aligned} U(c_t, l_t, m_t) = & -u_c y [-\phi \hat{y}_t + \frac{\sigma + \omega - \phi(1 + \omega)}{2} \hat{y}_t^2 - \hat{y}_t (\sigma g_t + (1 - \phi)(1 + \omega) \hat{a}_t) \\ & + (1 - \phi) \frac{\theta}{2} \text{var}_i \ln(P_{it}) + \frac{1}{2\sigma_m(R - 1)v} \hat{R}_t^2] + t.i.s.p. + \mathcal{O}(\|\hat{\xi}_t, \varsigma\|^3). \end{aligned}$$

Using the sales tax as a sales subsidy by setting

$$1 - \tau = [\rho(\pi)^{\frac{1}{1-\theta}} \frac{\theta - 1}{\mu^w \theta} \frac{1 - \alpha \beta \pi^\theta}{1 - \alpha \beta \pi^{\theta-1}}]^{-1},$$

the linear term in the welfare approximation above vanishes:

$$U(t) = -\frac{u_c y}{2} [(\sigma + \omega)(\hat{y}_t - \hat{y}_t^*)^2 + \theta \text{var}_i \ln(P_{it}) + \frac{1}{\sigma_m(R - 1)v} \hat{R}_t^2] + t.i.s.p. + \mathcal{O}(\|\hat{\xi}_t, \varsigma\|^3). \quad (42)$$

The variance of $\ln(P_t(i))$ is given by

$$\text{var}_i(\ln P_{it}) = \alpha \text{var}_i \ln(P_{it-1}) + \frac{\alpha}{1 - \alpha} \hat{\pi}_t^2 + t.i.s.p. + \mathcal{O}(\|\hat{\xi}_t, \varsigma^{2/3}\|^3),$$

where we assumed that $\ln(\pi) = 0 + \mathcal{O}(\|\varsigma\|^2)$. Iterating the equation above forward starting from any $\text{var}_i \ln(P_{it_0-1})$ in the period before policy applies, for $t \geq t_0$ results in :

$$\text{var}_i \ln(P_{it}) = \sum_{s=t_0}^t \alpha^{t-s} \frac{\alpha}{1 - \alpha} \hat{\pi}_s^2 + t.i.s.p. + \mathcal{O}(\|\hat{\xi}_t, \varsigma^{2/3}\|^3),$$

where we used that the initial price dispersion $\text{var}_i \ln(P_{it_0-1})$ is $t.i.s.p.$

Discounting, summing up and substituting for $\text{var}_i \ln(P_{it})$ in (42) results in (21) in

proposition 2. ■

6.4 Definition of the disturbances n_t , u_t and s_t

The exogenous fluctuations are defines as:

$$n_t = \eta_2 E_t(\hat{a}_{t+1} - \hat{a}_t) - \eta_1 \sigma E_t(g_{t+1} - g_t),$$

$$u_t = \eta_3 \hat{a}_t + (1 - \alpha \beta \bar{\pi}^\theta) \hat{\mu}_t^w + \eta_3 \sigma g_t$$

and

$$s_t = \eta_5 \hat{a}_t + \eta_6 \sigma g_t + \kappa^* \hat{\mu}_t^w.$$

The constants η_i , $i = 1, ..6$ are defined as:

$$\eta_1 = \frac{\omega}{\omega + \sigma},$$

$$\eta_2 = \frac{\sigma(1 + \omega)}{\omega + \sigma},$$

$$\eta_3 = (1 - \alpha \beta \bar{\pi}^\theta)(1 + \omega) \frac{1 - \sigma}{\omega + \sigma},$$

$$\eta_4 = \kappa^* [\omega + \sigma + \frac{(1 - \bar{\pi})(1 - \sigma)}{1 - \alpha \beta \bar{\pi}^\theta}],$$

$$\eta_5 = \frac{\eta_4(1 + \omega)}{\omega + \sigma} - \kappa^*(1 + \omega)$$

and

$$\eta_6 = \frac{\eta_4}{\omega + \sigma} + \kappa^* (\frac{1 - \bar{\pi}}{1 - \alpha \beta \bar{\pi}^\theta} - 1).$$

6.5 The optimal policy problem from a timeless perspective

Under the timeless perspective the first order necessary conditions with respect to y_t , Δ_t , K_t , F_t , R_t and π_t for all $t \geq t_0$ are given by:

$$u_c(t) - \Delta_t v_l(t)/a_t + z_m(t)m_c(t) + \lambda_{2t}(1-\tau)[u_{cc}(t)y_t + u_c(t)] + \lambda_{3t}\mu_t^w[v_{ll}(t)\Delta_t y_t/a_t + v_l(t)] - \lambda_{5t}u_{cc}(t) + \lambda_{5t-1}\frac{u_{cc}(t)R_{t-1}}{\pi_t} \doteq 0 \quad (43)$$

$$-y_t v_l(t)/a_t + \lambda_{3t}v_{ll}(t)y_t^2\mu_t^w + \lambda_{4t} - \lambda_{4t+1}\beta\alpha\pi_{t+1}^\theta \doteq 0 \quad (44)$$

$$-\lambda_{1t}\rho(t)^{\frac{1}{1-\theta}} - [\lambda_{2t} - \alpha\pi_t^{\theta-1}\lambda_{2t-1}] \doteq 0 \quad (45)$$

$$\frac{\theta}{\theta-1}\lambda_{1t} - [\lambda_{3t} - \alpha\pi_t^\theta\lambda_{3t-1}] \doteq 0 \quad (46)$$

$$z_m(t)m_R(t) + \lambda_{5t}\beta\frac{u_c(t+1)}{\pi_{t+1}} \leq 0, \quad (47)$$

and

$$-\lambda_{1t}K_t\frac{\alpha}{1-\alpha}\pi_t^{\theta-2}\rho(t)^{\frac{\theta}{1-\theta}} + \lambda_{2t-1}\alpha K_t(\theta-1)\pi_t^{\theta-2} + \lambda_{3t-1}\alpha F_t\theta\pi_t^{\theta-1} + \lambda_{4t}[\theta\alpha\pi_t^{\theta-2}\rho(t)^{\frac{1}{\theta-1}} - \alpha\theta\pi_t^{\theta-1}\Delta_{t-1}] - \lambda_{5t-1}R_{t-1}\frac{u_c(t)}{\pi_t^2} \doteq 0. \quad (48)$$

Note that λ_{2t_0-1} , λ_{3t_0} and λ_{5t_0-1} are the multiplier requiring initial commitment. The multipliers $\lambda_{1t} - \lambda_{5t}$ are associated with the following constraints:

$$\rho(\pi_t)^{\frac{1}{1-\theta}}K_t = \frac{\theta}{\theta-1}F_t \quad (49)$$

$$K_t = u_c(y_t - G_t, \zeta_t)(1-\tau)y_t + \beta\alpha E_t K_{t+1}\pi_{t+1}^{\theta-1} \quad (50)$$

$$F_t = v_l(y_t\Delta_t/a_t)y_t\mu_t^w + \alpha\beta E_t F_{t+1}\pi_{t+1}^\theta \quad (51)$$

$$\Delta_t = (1-\alpha)\rho(\pi_t)^{\frac{\theta}{\theta-1}} + \alpha\Delta_{t-1}\pi_t^\theta \quad (52)$$

and

$$u_c(y_t - G_t, \zeta_t) = \beta R_t E_t \frac{u_c(y_{t+1} - G_{t+1}, \zeta_{t+1})}{\pi_{t+1}}, \quad (53)$$

with $\rho(\pi_t) \equiv (1 - \alpha \pi_t^{\theta-1})(1 - \alpha)^{-1}$ for $R_t \geq 1$.

Additional figures and tables

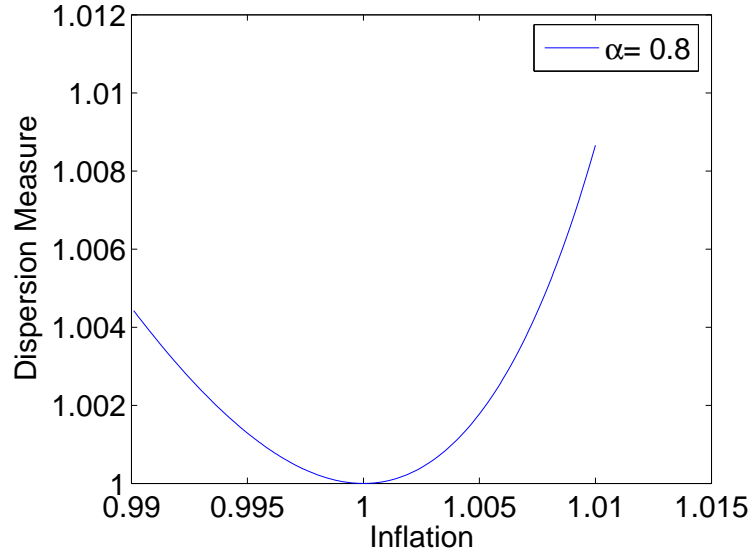


Figure 4: Steady state price dispersion as a function of inflation.

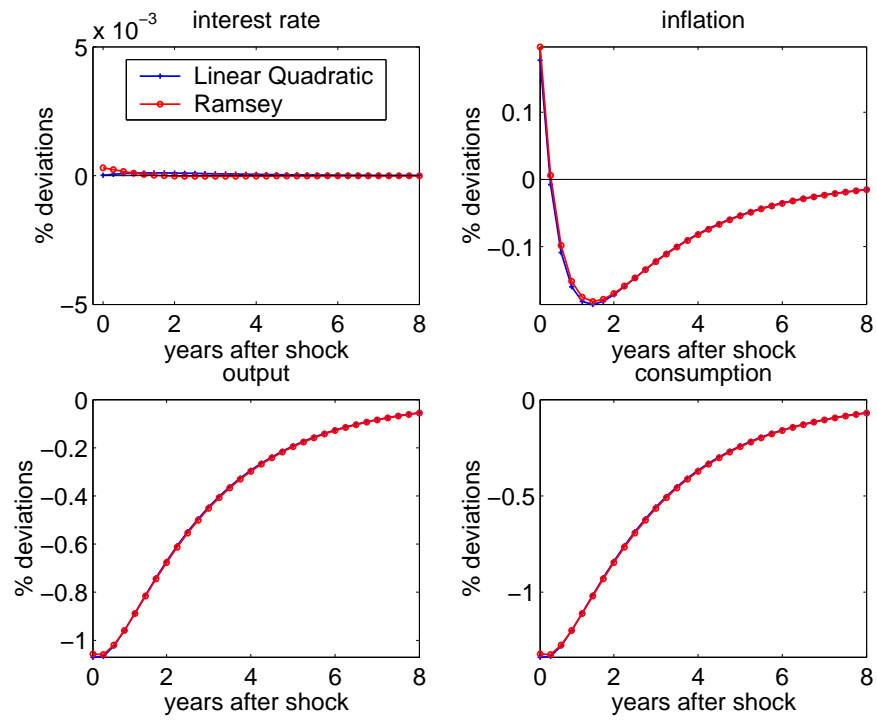


Figure 5: Impulse responses to a wage markup shock under optimal policy computed by the linear quadratic approximation and by the time invariant Ramsey approach.

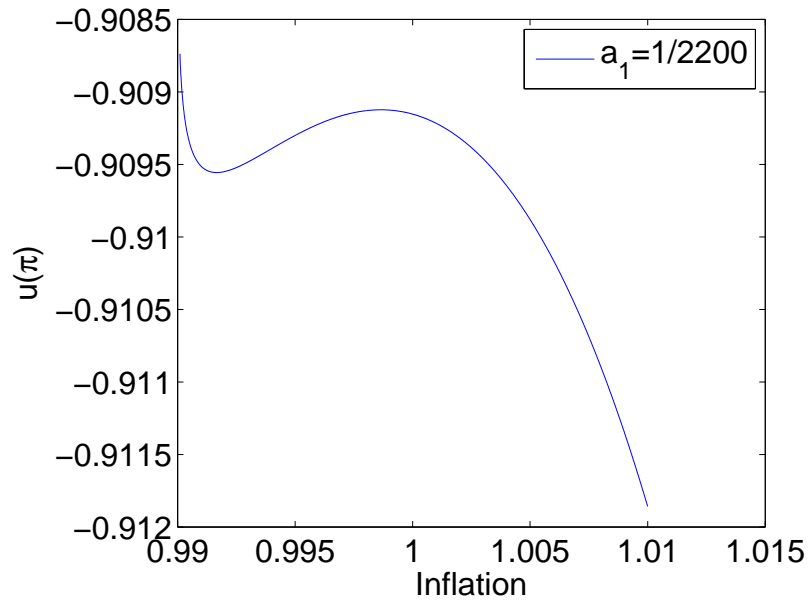


Figure 6: Welfare and Inflation in the steady state

a_1	λ_R^{ZERO}	λ_R^{FR}	$bcc^{ZERO} - bcc^{FR}$	dU_{CE}	$\sigma(\text{FR})$	$\sigma(ZERO)$
1/20	2.3426	928920	0.0017%	1.4543%	1999	41
1/50	1.6238	643870	0.0011%	0.9598%	1386	31
1/99	1.2355	488930	0.0005%	0.6904%	1054	25
1/150	1.0463	414910	0.0002%	0.5585	893	23
1/189	0.9539	378270	0.00%	0.4940%	814	21
1/250	0.8530	338230	-0.0002%	0.4233%	728	20
1/500	0.6464	256330	-0.0009%	0.2783%	552	17
1/1000	0.4899	19460	-0.0015%	0.1680%	418	15

Table 3: Welfare Analysis: $\epsilon = 0.000001$

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