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Solow residuals without capital stocks

Michael C. Burda [a,b,c,*], Battista Severgnini [d]

[a] Humboldt University Berlin, Spandauer Straße 1, D-10178 Berlin, Germany
[b] CEPR, United Kingdom
[c] IZA, Germany
[d] Copenhagen Business School, Porcelænshaven 16 A, DK-2000 Frederiksberg, Denmark

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ABSTRACT

We use synthetic data generated by a prototypical stochastic growth model to assess the accuracy of the Solow residual (Solow, 1957) as a measure of total factor productivity (TFP) growth when the capital stock in use is measured with error. We propose two alternative measurements based on current investment expenditures: one eliminates the capital stock by direct substitution, while the other employs generalized differences of detrended data and the Malmquist index. In short samples, these measures can exhibit consistently lower root mean squared errors than the Solow–Törnqvist counterpart. Capital measurement problems are particularly severe for economies still far from their steady state. This drawback of the Solow residual is thus most acute in applications in which its accuracy is most highly valued. As an application, we compute and compare TFP growth measures for developing countries in the Heston–Summers dataset.

1. Introduction

For more than fifty years, the Solow decomposition has served as the standard measurement of total factor productivity (TFP) growth in economics and management.[1] Its considerable popularity derives from the absence of restrictive assumptions regarding the production technology, statistical model or econometric specification.[2] In his seminal paper, Solow (1957) used the decomposition to demonstrate the limits of accounting for economic growth with changes in observable inputs. The Solow residual has been important for research on the sources of long-run growth and economic development as well as business cycle fluctuations.[3] According to the Social Sciences Citation Index, the Solow paper has been referenced more than 1800 times since its publication.[4]

Despite its unchallenged preeminence, the precision of the Solow residual as a measurement tool has yet to be systematically evaluated. This is because the "true" evolution of total factor productivity is fundamentally unknown. Yet there are several reasons to suspect the quality of both microeconomic and macroeconomic TFP measurements. First, the capital stock is unobservable in practice and is estimated as a function of past investment expenditures plus an estimate of an unknown initial condition. Uncertainty surrounding that initial condition, the mismeasurement of investment expenditures, as well as the depreciation, obsolescence and decommissioning of capital in subsequent periods can imply significant measurement error. Second, as many scholars of productivity analysis have stressed, the Solow residual is based on an assumption of full efficiency, but in fact represents a mix of changes in total factor productivity and efficiency of factor utilization.[5] Intertemporal variation in capacity utilization can bias an unadjusted calculation of the Solow residual as a measure of total factor productivity (Burnside et al., 1993, 1995). Because the perpetual inventory method (PIM) is the backbone of capital measurement for the OECD and

* Corresponding author. Tel.: +49 30 2093 5638; fax: +49 30 2093 5696.
E-mail addresses: burda@wiwi.hu-berlin.de (M.C. Burda), bs.eco@cbs.dk (B. Severgnini).

[1] See, for example, Jorgenson and Griliches (1967), Kuznets (1971), Denison (1972), Maddison (1992), Hulten (1992), O'Mahony and van Ark (2003).

[2] See Griliches (1996).

[3] See the references in Hulten et al. (2001).

[4] Source: Social Sciences Citation Index, October 2013.

[5] For an interpretation of the Solow residual as the difference between TFP growth and efficiency, see Mohnen and ten Raa (2002). Macroeconomists have also studied this issue; see for example Summers (1986), Burnside et al. (1993), and King and Rebelo (1999).

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national accounting agencies in practice (Pritchett, 2000; O'Mahony and Timmer, 2009; Schreyer, 2009; McGrattan and Prescott, 2010), capital mismeasurement continues to pose a problem for growth accounting, especially for developing and transition countries[6] and when new types of capital are studied (e.g. research and development (R&D), information and communication technology (ICT), intangible and public capital).

In this paper, we evaluate the error of the Solow growth accounting measure using quantitative macroeconomic theory. A prototypical stochastic growth model serves as a laboratory for studying the implications of constructing capital stocks under conditions often encountered in developmental applications, i.e. with relatively short series of investment expenditures and an arbitrary initial condition. Using artificial data generated by that model, we show that measurement problems can be severe for developing or transition economies. This drawback of the Solow residual is thus most acute in applications in which its accuracy is most highly valued.

To deal with capital stock measurement error, we propose two alternative measurements of TFP growth. Both eliminate capital stocks from the Solow calculation, while introducing their own, different sources of errors. The first, based on direct substitution, requires an estimate of the user cost of capital, but is relatively robust for economies far from their steady state paths. The second, based on generalized first differences of national accounts data, requires an estimate of an initial condition for TFP growth and is more appropriate for economies close to their steady state. To implement the latter approach, we improve on the choice of starting value by exploiting the properties of the Malmquist index. Next, we use our synthetic data to evaluate the impact of these competing errors in a horse race. In short series, our measures outperform the traditional Solow residual and reduce the root mean squared by as much as one third. Depending on the application, our alternative measurements can be seen as either complements to or substitutes for the conventional Solow–Törnqvist approach.

The rest of paper is organized as follows. In Section 2, we review the Solow residual as a measure of TFP growth and the relationship between the Solow decomposition and the capital measurement problem. Section 3 employs a prototypical DSGE model – the stochastic growth model with variable capacity utilization – as a laboratory for evaluating the quality of the Solow residual as TFP growth measure. In Section 4, we propose two alternative TFP growth measurements and Section 5 reports comparative quantitative evaluations (a “horse race”) under varying assumptions concerning data available to the analyst. In Section 6, we construct and compare TFP growth measures for developing economies in the Penn World Tables database, for which good estimates of capital stocks are generally unavailable. Section 7 concludes.

2. The Solow residual and the capital measurement problem

2.1. The Solow residual after a half-century: a brief review

Solow (1957) considered a standard neoclassical production function $Y_t = F(A_t, K_t, N_t)$ expressing output (Y_t) in period t as a constant returns function of a homogeneous physical capital stock (K_t), employment (N_t) and the level of total factor productivity (A_t). He defined TFP growth as $\frac{\dot{Y}_t}{Y_t} - \alpha_t \frac{\dot{K}_t}{K_t} - (1-\alpha_t)\frac{\dot{N}_t}{N_t}$, the difference of the observable growth rate of output and a weighted average of the growth of the two inputs, where α_t and $1 - \alpha_t$ are local output elasticities of capital and labor; a dot denotes the time derivative (e.g.$\dot{A} = dA/dt$). In practice, the Solow decomposition generally measures TFP growth (α_t) in discrete time as (Barro, 1999; Barro and Sala-i-Martin, 2003):

$$a_t = \frac{\Delta Y_t}{Y_{t-1}} - \alpha \frac{\Delta K_t}{K_{t-1}} - (1-\alpha)\frac{\Delta N_t}{N_{t-1}} \tag{1}$$

where K_t denotes capital at the beginning of period t. When factor markets are competitive, output elasticities of capital and labor correspond to aggregate factor income shares, which are constant in the case of the Cobb–Douglas production function; for most technologies which allow for factor substitution, Eq. (1) gives a reasonable first-order approximation.[7]

Yet the Solow residual itself is hardly free of measurement error; Abramovitz (1956) called it a “measure of our ignorance”.[8] Denison (1972) and others extended the TFP measurement paradigm to a larger set of production factors, and continued to find that the residual is the most significant factor driving output growth. Since Christensen et al. (1973), it has become commonplace to employ the so-called Törnqvist index specification:

$$a_t^{ST} = \Delta \ln Y_t - \overline{\alpha}_{t-1} \Delta \ln K_t - (1-\overline{\alpha}_{t-1})\Delta \ln N_t \tag{2}$$

where $\overline{\alpha}_{t-1} = \frac{\alpha_{t-1}+\alpha_t}{2}$ (see Törnqvist (1936)). This formulation reduces measurement error and is exact if the production function is translog (Diewert, 1976). Denison (1962) and Hall and Jones (1999) have employed the Solow approximation across space to measure total factor productivity relative to a benchmark economy.

Measurement error can arise for reasons besides the specification of the production function. While output and employment are directly observable and readily quantifiable, capital measures rely on a number of assumptions, many of which lay at the center of the famous capital controversy between Joan Robinson and Paul Samuelson. Our paper lends more credence to the position taken by Robinson, albeit for reasons more nuanced than those she adduced (see Robinson (1953)).

2.2. The capital measurement problem

The capital stock poses a particular problem in growth accounting because it is not measured or observed directly, but is constructed by statistical agencies using time series of investment expenditures and ancillary information. At some level, capital stocks always represent the forward integration of the “Goldsmith equation” (Goldsmith, 1955)

$$K_{t+1} = (1-\delta_t)K_t + I_t, \quad t = 0, 1, \ldots \tag{3}$$

from some initial condition K_0, given sequences of periodic investment expenditures $\{I_t\}$ and depreciation rates $\{\delta_t\}$:

$$K_{t+1} = \left[\prod_{i=0}^{t}(1-\delta_{t-i})\right]K_0 + \sum_{i=0}^{t}\left[\prod_{j=0}^{i}\left(1-\delta_{t-j}\right)\right]I_{t-i}. \tag{4}$$

The capital stock available for production tomorrow is the weighted sum of an initial capital value, K_0, and subsequent investment expenditures up to the present, with weights corresponding to their respective

[6] Several authors have explored total factor productivity in developing countries assuming different measurements of capital. See, for example, Dadkhah and Zahedi (1986) and Young (1995b).

[7] We offer only a cursory survey of growth accounting methods here, which were anticipated by Tinbergen (1942) and pioneered by Solow (1957) and Denison (1962). For more detailed reviews of the Solow decomposition, see Diewert and Nakamura (2003, 2007) and ten Raa and Shestalova (2011).

[8] Solow himself wrote:

> *“[L]et me be explicit that I would not try to justify what follow by calling on fancy theorems on aggregation and index numbers. Either this kind of aggregate economics appeals or it doesn't.[…] If it does, one can draw some useful conclusions from the results.”* Solow (1957: 312).

undepreciated components. If the depreciation rate is constant and equal to δ, Eq. (4) collapses to an expression found in, e.g., Hulten (1990)[9]

$$K_{t+1} = (1-\delta)^{t+1}K_0 + \sum_{i=0}^{t}(1-\delta)^{i+1}I_{t-i} \tag{5}$$

Eq. (5) shows that mismeasurement of the initial capital stock casts a long shadow on the construction of the Solow residual. The problem can only be solved by pushing the initial condition sufficiently far back into the past; yet with the exception of a few countries,[10] long time series for investment are unavailable. The perpetual inventory method of constructing capital series was thus criticized by Ward (1976) and Mayes and Young (1994), who proposed alternatives based on estimation methods.[11]

Even today, capital stock estimation relies heavily on PIM and the implied error remains a widely-recognized problem in growth accounting as well as productivity measurement.[12] Employing long time series for the US, Gollop and Jorgenson (1980) equate capital at time $t = 0$ to investment in that period. The US Bureau of Economic Analysis (Reinsdorf and Cover (2005) and Sliker (2007)) set $K_0 = \left(\frac{1+g^I}{\delta+g^I}\right)I_0$, which is consistent with a steady state in which capital grows at rate g^I and is depreciated at rate δ.[13] Caselli (2005) confirms that capital measurement error induced by the initial guess is most severe for the poorest countries. Rather than employing the standard steady-state condition $K_0 = \frac{I_0}{(g^I+\delta)}$ (e.g. Kohli (1982)), he estimates initial conditions for capital stocks of the poorest countries using[14]:

$$K_0 = K_0^*\left(\frac{Y_0}{Y_0^*}\right)^{\frac{1}{\alpha}}\left(\frac{N_0^*}{N_0}\right)^{\frac{1-\alpha}{\alpha}} \tag{6}$$

where the star refers to values from a benchmark economy (here, the United States). The precision of Caselli's innovative approach will depend, among other things, on the distance of the benchmark economy from its steady state. In addition, Eq. (6) assumes that total factor productivity levels are identical to those in the US in the base year, which is inconsistent with the findings of Hall and Jones (1999). Most importantly, benchmark estimates of the US capital stocks are also likely subject to significant measurement error.

2.3. Measurement error, depreciation and capital utilization

The initial condition problem noted by Caselli (2005) applies a fortiori to a more general setting in which the initial value of capital is measured with error, if depreciation is stochastic, or is unobservable. Suppose that the elements of the sequence of depreciation rates δ_t move around some constant value δ. It is possible to rewrite Eq. (6) as:

$$K_{t+1} = (1-\delta)^{t+1}K_0 + \sum_{i=0}^{t}(1-\delta)^{i+1}I_{t-i} + \left[\prod_{i=0}^{t}\frac{(1-\delta_{t-i})}{(1-\delta)}-1\right](1-\delta)^{t+1}K_0 + \sum_{i=0}^{t}\left[\prod_{j=0}^{i}\frac{\left(1-\delta_{t-j}\right)}{(1-\delta)}-1\right](1-\delta)^{t+i}I_{t-i} \tag{7}$$

Eq. (7) expresses the true capital stock in $t+1$ as the sum of three components: 1) an initial capital stock, net of assumed depreciation at some constant rate δ, plus the contribution of investment $\{I_s\}_{s=0}^{t}$, also expressed net of depreciation at rate δ; 2) mismeasurement of the initial condition's contribution due to fluctuation of depreciation about the assumed constant value; and 3) mismeasurement of the contribution of all investment expenditures from period 0 to t. Each of these three components represents a potential source of measurement error. The first component contains errors involving the initial valuation of the capital stock. For the most part, the second and third components are unobservable. Ignored in most estimates of capital, they represent a potentially significant source of mismeasurement which would contaminate a Solow residual calculation.

The interaction between the depreciation of capital and capacity utilization is also important for macroeconomic modeling. Time-varying depreciation rates imply changing relative weights of old and new investment in the construction of the capital stock. In dynamic stochastic general equilibrium models, the depreciation rate is generally assumed constant, despite empirical evidence to the contrary (see Burnside et al. (1995) and Corrado and Mattey (1997)).[15]

3. Capital Measurement and the Solow Residual: A Quantitative Assessment

3.1. The Stochastic Growth Model as a Laboratory

A central innovation of this paper is an evaluation of TFP growth measurement using synthetic data generated by a known, prototypical model of economic growth and fluctuations. To this end, we employ the standard, neoclassical framework (King and Rebelo, 1999), in which the first and second welfare theorems hold and markets are complete, to allow for variable capacity utilization, following Greenwood et al. (1988), Burnside et al. (1995), and Wen (1998). Employing this well-understood model as a laboratory, we assess quantitatively the limitations of the Solow residual measurement. In this section, we briefly describe the model and the data which it generates. Details can be found in Appendix A.

The model represents fluctuations and economic growth as purposeful responses to the evolution of total factor productivity. A representative household supplies capital services and labor to firms, which produce output using a constant returns, Cobb–Douglas production technology. The household plans consumption, investment, capacity utilization and labor supply in order to maximize expected discounted utility. Consumption and leisure enter utility in a conventional time-separable and concave fashion, whereas fluctuations in labor are less costly than those in consumption. The household can choose to utilize their accumulated capital more intensively, at the cost of increasingly

[9] From the perspective of measurement theory, four general problems arise with using capital stock data estimated by statistical agencies (see Diewert and Nakamura (2007) for more a detailed discussion of these issues). First, the construction of capital stocks presumes an accurate measurement of the initial condition K_0. The shorter the series under consideration, the more likely such measurement error will affect the precision of the Solow residual. Second, it is difficult to distinguish utilized capital at any point in time from that which is idle. Solow (1957) also argued that the appropriate measurement should be of "*capital in use, not capital in place*". Third, depreciation is unobservable. For some sectors and some types of capital, it is difficult if not impossible to apply an appropriate depreciation rate; this is especially true of the retail sector. Fourth, many intangible input stocks such as cumulated research and development effort and advertising goodwill are not included in measured capital.

[10] For example, Denmark and the United States publish investment data dating from 1832 and 1901 respectively; most industrialized economies only report data since the 1960s or afterwards.

[11] Schreyer (2001) suggests comparing initial capital estimates with five different benchmarks: 1) population census data, which take into account different types of dwellings; 2) fire insurance records; 3) company accounts; 4) administrative property records, which provides values of residential and commercial buildings at current market prices; and 5) company share valuation.

[12] See, for example, the recent OECD manual on measuring capital (Schreyer (2009)), the US Bureau of Economic Analysis (http://www.bea.gov/national/pdf/NIPAhandbookch1-4.pdf) and its methodological appendix observe that initial conditions can affect capital measurement if time series are short.

[13] Griliches (1980) used $K_0 = \rho\frac{I_0}{Y_0}$ as an initial condition for measuring R&D capital stocks, where ρ is a parameter to be estimated.

[14] In his original formulation, Caselli (2005) considers an extended production function with human capital.

[15] See the OECD manual (Schreyer, 2009) on capital stock estimation for more details on the measurement of depreciation.

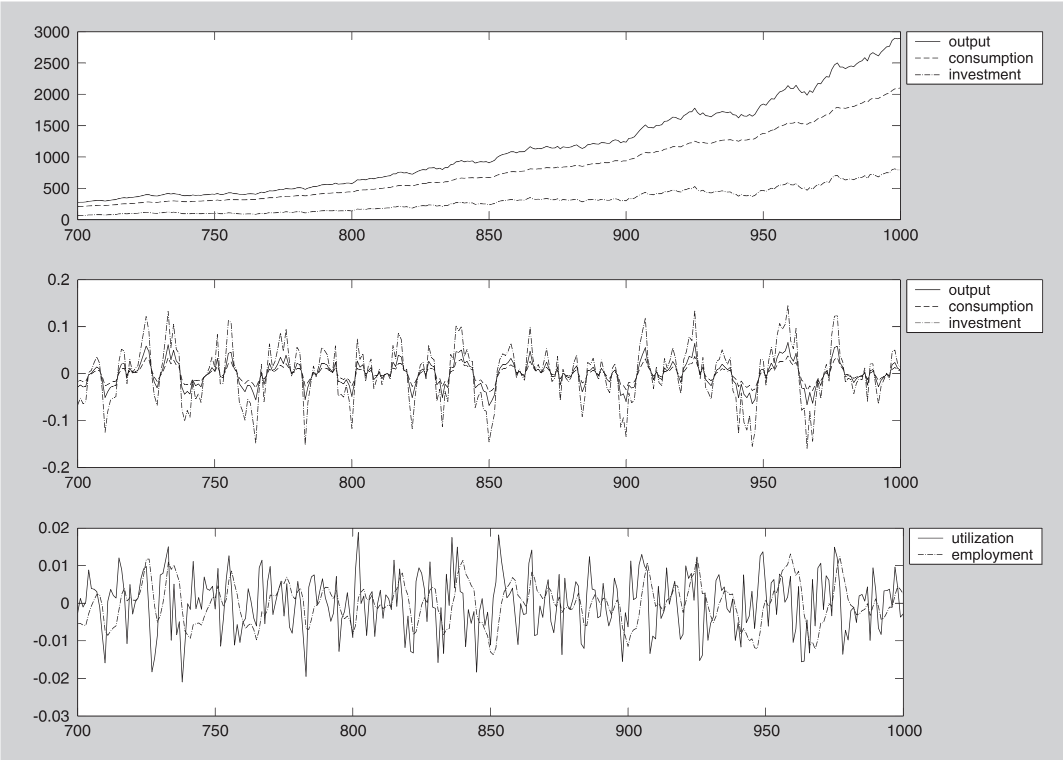

Fig. 1. A typical time series realization for the mature economy in levels and in H–P detrended form, periods 700–1000.

higher depreciation. The logarithm of total factor productivity evolves as a linear time trend plus a stationary AR(1) process.

3.2. Construction of the data sets

The model was calibrated to the US economy with standard parameter values described in Appendix A. Each realization (simulation) of the artificial economy consists of sequences of 1200 quarterly observations of output $\{Y_t\}$, total factor productivity $\{A_t\}$, true capital stock $\{K_t\}$, employment $\{N_t\}$, consumption $\{C_t\}$, investment $\{I_t\}$, capacity utilization $\{U_t\}$, rental price of capital $\{\kappa_t\}$, and the wage $\{\omega_t\}$. The initial condition for TFP (A_0) was drawn from a normal distribution with zero mean and unit standard deviation and the capital stock in period zero (K_0) is set to its deterministic steady-state value; the model is allowed to run 100 quarters before samples were drawn. For each realization, data were generated for both "mature" and "transition" economies. A mature economy is drawn from a realization starting in period 700, while a transition economy consists of the same realization as a mature economy until period 699, after which the capital stock is reduced to half its original value. The economy's equilibrium is then re-computed with this lower initial capital stock from period 700 to 1200. In Fig. 1 we display a representative time series realization of the mature economy in original and H–P detrended form with detrending parameter set at 1600.

The model economy's properties are summarized in Table 1 and compared with moments of the Hansen's (1985) stochastic growth model as well as US data reported by Stock and Watson (1999) and Dejong and Dave (2007). Our benchmark model is thus capable of replicating key features of the US economy.

3.3. Measuring measurement error of the Solow residual

The data generated by the artificial economy can be used to evaluate the precision of the Solow residual. In what follows, we sketch this procedure and provide a first evaluation of its accuracy. The basis of comparison is the average root mean squared error (RMSE) with respect to true TFP growth, computed over 100 independent samples of either 50 or 200 periods, starting in period 700, for realizations of both the mature and the transition economy.[16] The Solow residual measure is calculated as a Törnqvist index described in Eq. (2).[17] The true capital stock is never observable to the analyst; instead, PIM is applied to investment data and an initial capital stock is estimated using methods described above. In the baseline scenario A, the analyst observes neither the rate of capacity utilization nor the true depreciation rate. Alternatively, the analyst observes the utilization rate only (Scenario B) or both the utilization and the true depreciation rate (Scenario C). In (B) and (C) a modified Solow residual calculation is used to exploit information on capacity utilization.[18] When depreciation is not observed directly, a quarterly rate of 0.015 is employed. The parameter g^I, which is used for the BEA estimate of the initial capital stock, is computed as the average growth rate of observed investment data. Scenario D, which mimics the context of developing economies, repeats the exercise in Scenario A for data sampled annually as sums over the year (for flows) or first-quarter values (for stocks).

The results reported in Table 2 show that even under these ideal conditions, the initial condition of the capital stock is a significant source of error for the Solow residual. In the A scenario with 50 observations, the RMSE is 0.90 with a standard error of 0.10. (If the analyst could in fact observe the true capital stock, the RMSE would decline to 0.63.) Without the capital stock but with access to data on capital utilization (Scenario B) the RMSE falls to 0.64. Additional information on (time-varying) depreciation (Scenario C) is not useful, nor is the gain significant from more data observations (moving from 50 to 200). As

[16] $RMSE = [MEAN(a_t^{ST} - a_t)^2]^{0.5}$, where a_t is the true rate of total factor productivity growth.

[17] Note that for Cobb–Douglas production technology and competitive factor markets, factor shares and output elasticities are constant, so the Törnqvist index and lagged factor share versions are equivalent.

[18] $a_t^{ST} = \frac{\Delta Y_t}{Y_{t-1}} - \overline{\alpha}\left(\frac{\Delta K_t}{K_{t-1}} + \frac{\Delta U_t}{U_{t-1}}\right) - (1-\overline{\alpha})\frac{\Delta N_t}{N_{t-1}}$.

Table 1
Comparative statistical properties of the model economy.

Series	Model economy (200 quarters)	Hansen (1985)		US Data	US Data
		Divisible labor model	Indivisible labor model	1953Q1–1996Q4 (Stock and Watson (1999))	1948Q1–2004Q4 (Dejong and Dave (2007))
Cross-correlations with output					
Consumption	0.99	0.89	0.87	0.90	–
Investment	1.00	0.99	0.99	0.89	–
Employment	0.48	0.98	0.98	0.89	–
Productivity	1.00	0.98	0.87	0.77	–
Std. dev. normalized by std. dev. of output					
Consumption	0.52	0.46	0.29	0.76	0.46
Investment	2.08	2.38	3.24	2.99	4.23
Employment	0.62	0.34	0.77	1.56	1.05

expected, average RMSE declines with sample size. At a sample length of 50 years (200 quarters), the annualized root mean squared error converges to about 0.64. Our results are strikingly different for the transition economy. First, the RMSE with 50 observations in the A-Scenario rises to 3.27, more than three times as large as the mature economy case. As the sample size increases to 200, the RMSE falls to 1.82 but remains significantly above the mature economy level; all scenarios are still characterized by significant measurement error for small samples and especially in transition economies. For annual data (Scenario D), the ST measures convey a comparable impression of systematic imprecision relative to the "true" value when the capital stock is observed correctly (3.23).

Our results show that the Solow residual can be subject to considerable measurement error. In scenario A, about 40% of this error in the smaller dataset is due to the estimated initial capital stock, while the rest is due to unobservable depreciation and capacity utilization. Measurement error in K_0 will be significant when 1) the depreciation rate is low and 2) the time series under consideration is short. For conventional rates of depreciation, errors in estimating the initial condition can have long-lasting effects on estimated capital stocks. In the following two sections, we propose two capital stock-free alternatives to the Solow residual.

4. TFP growth measurement without capital stocks: two alternatives

4.1. Direct substitution (DS)

The first strategy for estimating TFP relies on direct substitution. Differentiation of the production function $Y_t = F(A_t, U_t, K_t, N_t)$ with respect to time, insertion of the capital transition equation $\dot{K}_t = I_t - \delta_t K_t$ and rearrangement yields

$$\frac{\dot{Y}_t}{Y_t} = \frac{AF_A}{Y_t}\dot{A}_t + F_K\frac{I_t}{Y_t} + \alpha_t\left(\frac{\dot{U}_t}{U_t} - \delta_t\right) + (1-\alpha_t)\frac{\dot{N}_t}{N_t}, \tag{8}$$

where as before α_t is the local elasticity of output with respect to the capital input. In an economy with competitive factor markets, the marginal product of capital F_K equates κ_t, the user cost of capital in t. This equation is adapted to discrete time to obtain the DS measure of TFP growth, a_t^{DS}:

$$a_t^{DS} = \frac{\Delta Y_t}{Y_{t-1}} - \kappa_{t-1}\frac{I_{t-1}}{Y_{t-1}} + \alpha_{t-1}\left(\delta_{t-1} - \frac{\Delta U_t}{U_{t-1}}\right) - (1-\alpha_{t-1})\frac{\Delta N_t}{N_{t-1}}. \tag{9}$$

The substitution eliminates the capital stock from the TFP calculation.

In a world in which all variables are perfectly observed, the DS and ST measures are identical. Any advantage of the DS derives from higher precision, in a root mean squared error sense, of measuring the current investment rate and the user cost versus the total productive capital stock. The DS approach will be a better measurement of TFP growth to the extent that 1) the capital stock is unobservable or poorly measured; 2) capital depreciation is unobservable or poorly measured and varies over time; 3) the last gross increment to the capital stock is more likely to be completely utilized than older capital.[19] The DS measure implies an imputed contribution of capital to growth equal to $\frac{\Delta Y_t}{Y_{t-1}} - a_t^{DS} - (1-\alpha_{t-1})\frac{\Delta N_t}{N_{t-1}} - \alpha_{t-1}\frac{\Delta U_t}{U_{t-1}}$.

4.2. Generalized differences of deviations from the steady state (GD)

If an economy or sector is close to its steady state, it may be more appropriate to measure total factor productivity growth as deviations from some long-run deterministic trend path estimated using trend regression, moving averages or the Hodrick–Prescott filter (Hodrick and Prescott (1997)). Consider a balanced growth steady state in which all observable variables are growing at rate g. If $\hat{X}_t$ denotes the deviation of X_t around a steady state value $\overline{X}_t$, then the discrete-time production function and the Goldsmith Eq. (3) can be approximated as

$$\hat{Y}_t = \hat{A}_t + \alpha\left(\hat{K}_t + \hat{U}_t\right) + (1-\alpha)\hat{N}_t \tag{10}$$

and

$$\hat{K}_t = \frac{(1-\delta)}{(1+g)}\hat{K}_{t-1} + \iota\hat{I}_{t-1}, \tag{11}$$

respectively, where $\iota = \frac{\overline{(I/K)}}{(1+g)}$, and the capital elasticity $\alpha \equiv \frac{F_K K}{Y_t}$ and depreciation rate δ are constant, following standard steady state restrictions on grand ratios emphasized by King et al. (1988). Multiplying both sides of Eq. (10) by $\left(1 - \frac{(1-\delta)}{(1+g)}L\right)$ and substituting Eq. (11), we can express TFP growth in generalized differences as

$$\left(1 - \frac{(1-\delta)}{(1+g)}L\right)a_t^{GD} = \left(1 - \frac{(1-\delta)}{(1+g)}L\right)\hat{Y}_t - \iota\alpha\hat{I}_{t-1} - \left(1 - \frac{(1-\delta)}{(1+g)}L\right)\alpha\hat{U}_t - \left(1 - \frac{(1-\delta)}{(1+g)}L\right)(1-\alpha)\hat{N}_t. \tag{12}$$

The generalized differences eliminate the capital stock completely from the computation. Given an initial condition, $\hat{a}_0^{GD}$, the sequence $\left\{\hat{a}_t^{GD}\right\}$ may be recovered for $t = 1,\ldots,T$ using

$$\hat{a}_t^{GD} = \left(\frac{1-\delta}{1+g}\right)^t \hat{a}_0^{GD} + \sum_{i=0}^{t-1}\left(\frac{1-\delta}{1+g}\right)^i\left[\hat{Y}_{t-i} - \alpha\left(\iota\hat{I}_{t-1-i} + \hat{U}_{t-i}\right) - (1-\alpha)\hat{N}_{t-i}\right]. \tag{13}$$

[19] While the possibility of heterogenous utilization of capital (due to vintage) is not considered explicitly in the model, examples of it abound in reality, such as electrical power generation or transportation.

Table 2
A horse race: RMSEs (% per period) of traditional Solow–Törnqvist TFP growth for mature and transition economies (100 realizations, standard errors in parentheses).

	A (Quarterly)		B (Quarterly)		C (Quarterly)		D (Annual)	
	T = 50	T = 200	T = 50	T = 200	T = 50	T = 200	T = 12	T = 50
Mature economy	0.90 (0.10)	0.90 (0.06)	0.64 (0.09)	0.64 (0.05)	0.64 (0.09)	0.64 (0.05)	3.68 (0.87)	3.67 (0.40)
	A		B		C		D	
	T = 50	T = 200	T = 50	T = 200	T = 50	T = 200	T = 12	T = 50
Transition economy	3.27 (0.23)	1.82 (0.11)	2.34 (0.18)	1.31 (0.08)	2.31 (0.17)	1.31 (0.08)	4.11 (0.68)	3.93 (0.35)

A: Analyst observes quarterly data $\{Y_t, N_t, I_t, \omega_t\}$.
B: Analyst observes quarterly data $\{Y_t, N_t, I_t, \omega_t, U_t\}$.
C: Analyst observes quarterly data $\{Y_t, N_t, I_t, \omega_t, U_t, \delta_t\}$.
D: Analyst observes annual time-aggregated data from Scenario A.
Note: a^{ST} is computed using the BEA estimate of K_0.

From the sequence $\left\{\hat{a}_t^{GD}\right\}_{t=1}^{T}$ it is straightforward to recover the TFP growth measure $\{a_t^{GD}\}$, given an estimate of the initial condition, $\hat{a}_0^{GD}$, and using the approximation $a_t^{GD} \approx ln\left(\frac{A_t}{A_{t-1}}\right)$.[20]

Our estimate, which is based on the Malmquist index, is given by $\hat{a}_0^{GD} = ln\left(A_0/\overline{A}_0\right)$ and is described in detail in Appendix B. It equals the geometric mean of labor productivity growth and output growth in the first period. Capital's implied contribution to growth is given by $\frac{\Delta Y_t}{Y_{t-1}} - a_t^{GD} - (1-\alpha_{t-1})\frac{\Delta N_t}{N_{t-1}} - \alpha_{t-1}\frac{\Delta U_t}{U_{t-1}}$.

4.3. The need for numerical evaluation

The central difference between the two alternatives to the Solow residual is the point around which the approximation is taken. The DS approach employs the levels of factor inputs in the previous period and is appropriate when the economy is far from its steady state. In the GD approach, the point of approximation is a balanced growth path along which the capital elasticity, s_K, the growth rate g, and the grand ratio I/K are constant. The advantages and disadvantages of each measurement will depend on the application at hand.

While both measurements eliminate capital from the TFP measurement, they introduce other forms of measurement error. The DS method replaces the capital stock with a more accurately measured gross investment flow and a depreciation rate which is likely to be time-varying but possibly unobservable in practice. The capital rental price κ_t can be obtained from independent sources or economic theory, but is also measured with error. Similarly, the GD procedure accentuates the marginal contribution of new capital but substitutes another form of measurement error (TFP growth in the initial period). Given that the GD method necessarily assumes a constant rate of depreciation, it will tend to do worse when the depreciation rate is in fact endogenous and procyclical. It should also perform poorly for economies or sectors which are far from their steady states. On the other hand, it is likely to be more appropriate for business cycle applications involving developed countries. To determine which measurement error is greater, we must turn to simulation methods.

5. A horse race of TFP growth measurements

5.1. Preliminaries

To evaluate and compare our alternative TFP growth measurements with the standard Solow–Törnqvist residual, we generate capital stock series using PIM and either 1) the BEA approach (Reinsdorf and Cover (2005), Sliker (2007)), and 2) Caselli's (2005) estimate relative to a "benchmark economy." Following Section 2 and especially Eq. (7) we robustify the analysis in several ways. First, we add measurement error to investment data that the analyst uses in constructing capital stocks. Second, we allow for measurement error in estimating the growth rate which enters the BEA calculation as discussed in Section 2.2, by varying the sample over which this rate is estimated. Third, we allow for a trend in TFP growth which varies cyclically at low frequency.

It is important to state carefully the assumptions behind the construction of the alternative TFP growth measures. The analyst is assumed never to observe the true capital stock, but does observe gross investment, employment, GDP, and factor payments in each period. Under alternative scenarios, the analyst may or may not observe the current rate of capacity utilization or the depreciation rate. If unobservable, a constant quarterly depreciation rate is assumed (0.015). For the DS measure, we assume that the analyst cannot observe the user cost of capital (κ_t) in each period and employs a constant $\overline{\kappa}$ set equal to its average value over the entire sample realization.[21] For the GD measure, values of the constant ι is set to 0.0225. We employed the Malmquist index to estimate the initial condition of TFP growth using a procedure described in Appendix B.[22] As before, the basis of comparison is the root mean squared error (RMSE) for sample time series of 50 or 200 observations taken from 100 independent realizations of the stochastic growth model.

5.2. TFP measurement when investment and growth rates are measured with error

The quantitative significance of error in the measurement of investment expenditures used in the construction of capital stocks and the estimation of TFP is an important issue for all economies, as time-to-build and market valuation considerations drive a wedge between investment expenditures in the national income and product accounts and the expansion of the stock of effective capital.[23] In developing countries

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[20] To see that: $a_t \approx \ln\left(\frac{A_t}{A_t - 1}\right) = \ln\left(\frac{A_t/\overline{A}_t}{A_{t-1}/\overline{A}_t}\right) = \ln\left(\frac{(1+a)A_t/\overline{A}_t}{A_{t-1}/\overline{A}_{t-1}}\right) \approx \overline{a}_t + \ln\left(\hat{A}_t\right) - \ln\left(\hat{A}_{t-1}\right)$, where $\overline{a}_t \equiv \ln\left(\frac{\overline{A}_t}{\overline{A}_{t-1}}\right)$ is the underlying trend growth rate. If TFP grows at constant rate a, $a_t^{GD} \approx a + ln\left(\hat{A}_t\right) - ln\left(\hat{A}_{t-1}\right) = (1-\alpha)(g-n) + ln\left(\hat{A}_t\right) - ln\left(\hat{A}_{t-1}\right)$.

[21] The user cost of capital could be expressed as $p_{t-1}i_t + \delta p_{t-1} + [p_{t-1} - p_t]$ where i is the nominal interest rate and p is the price of investment goods. As Balk (2010) has noted, user cost is more difficult to compute when conventional neoclassical assumptions are relaxed. We investigated the relevance of measurement error by adding to $\overline{\kappa}$ a uniformly distributed random variable with a standard deviation equal to twice that of the US ex-post real interest rate (measured by either the prime lending rate or the 10-year US Treasury bond yield less the CPI inflation rate, quarterly data, 1960:1–2013:1). The results are not significantly different from the ones reported in Tables 3a and 3b.

[22] We also considered alternative initial conditions for a_0^{GD}. For example, we imposed zero or labor productivity growth as initial value. Because these assumptions are rather arbitrary and far away from the real initial productivity growth, the RMSE are larger than those implied by the Malmquist index.

[23] In the course of several quarters leading up to the completion of a large public infrastructure project, effective gross contributions to the productive capital stock will be zero although gross expenditure on investment are positive.

Table 3a
A horse race: RMSEs of stock-less versus traditional Solow–Törnqvist estimates of TFP growth (% per period) when investment is measured with error.

Mature economy (100 realizations, standard errors in parentheses)	A (Quarterly)		B (Quarterly)		C (Quarterly)		D (Annual)	
	T = 50	T = 200	T = 50	T = 200	T = 50	T = 200	T = 12	T = 50
a^{DS}	0.91	0.91	0.67	0.66	0.67	0.66	3.80	3.75
	(0.10)	(0.05)	(0.08)	(0.04)	(0.08)	(0.04)	(0.82)	(0.36)
a^{GD}	0.86	0.86	0.61	0.61	0.59	0.59	3.42	3.43
	(0.10)	(0.06)	(0.08)	(0.04)	(0.08)	(0.05)	(0.85)	(0.38)
a^{ST} with BEA estimate of K_0.								
$N = 8^*$	1.14	0.99	0.95	0.76	0.95	0.76	4.23	3.93
	(0.22)	(0.09)	(0.24)	(0.10)	(0.24)	(0.10)	(0.87)	(0.37)
$N = 20^*$	0.98	0.94	0.75	0.69	0.75	0.70	4.01	3.86
	(0.14)	(0.07)	(0.16)	(0.07)	(0.16)	(0.06)	(0.88)	(0.39)
$N = T^*$	0.94	0.92	0.70	0.67	0.70	0.67	3.85	3.80
	(0.12)	(0.06)	(0.12)	(0.05)	(0.12)	(0.05)	(0.87)	(0.38)
a^{ST} with Caselli's benchmark economy capital K_0 (BEA estimate)								
$N = 8^*$	1.14	0.99	0.94	0.76	0.94	0.76	4.21	3.93
	(0.23)	(0.09)	(0.25)	(0.10)	(0.25)	(0.10)	(0.88)	(0.37)
$N = 20^*$	0.98	0.94	0.75	0.69	0.75	0.69	4.00	3.86
	(0.14)	(0.07)	(0.16)	(0.07)	(0.16)	(0.06)	(0.89)	(3.93)
$N = T^*$	0.94	0.92	0.70	0.67	0.70	0.67	3.85	3.79
	(0.12)	(0.06)	(0.12)	(0.05)	(0.12)	(0.05)	(0.88)	(0.38)

* The value of g^I is based on the first N available quarterly observations (for annual data, $N = 2$ or $N = 5$).
When average $g^I < 0$, the average value over all the positive observations is used.
A: Analyst observes quarterly data $\{Y_t, N_t, I_t, \omega_t\}$.
B: Analyst observes quarterly data $\{Y_t, N_t, I_t, \omega_t, U_t\}$.
C: Analyst observes quarterly data $\{Y_t, N_t, I_t, \omega_t, U_t, \delta_t\}$.
D: Analyst observes annual time-aggregated data from Scenario A.

this problem is even more severe (Pritchett (2000)). Besides resource limitations for precise measurement of investment, investment goods prices are significantly distorted in developing and emerging economies (DeLong and Summers, 1992). Eq. (7) suggests that this could be a significant source of measurement error, entering in both the second and fourth terms of that expression. As Table 1 indicates, there are significant differences between the standard deviations of consumption and investment of the model economy and US data.

We implement this measurement error in a straightforward way. The analyst does not observe $\{I_t\}$, but rather a series $\{I_t^*\}$ given by

$$I_t^* = I_t(1 + \epsilon_t).$$

The measurement error process is i.i.d. with mean zero and constant variance σ_ϵ^2, which we calibrate to match the difference between Dejong and Dave's (2007) estimate of the standard deviation of measured US

Table 3b
A horse race: RMSEs of stock-less versus traditional Solow–Törnqvist estimates of TFP growth (% per period) when investment is measured with error.

Transition economy (100 realizations, standard errors in parentheses)	A (Quarterly)		B (Quarterly)		C (Quarterly)		D (Annual)	
	T = 50	T = 200	T = 50	T = 200	T = 50	T = 200	T = 12	T = 50
a^{DS}	3.19	1.80	2.30	1.30	2.30	1.31	4.18	4.00
	(0.22)	(0.10)	(0.17)	(0.08)	(0.17)	(0.08)	(0.66)	(0.35)
a^{GD}	4.95	3.12	4.50	2.25	4.01	2.59	6.02	5.48
	(0.39)	(0.10)	(0.17)	(0.13)	(0.33)	(0.15)	(1.04)	(0.54)
a^{ST} with BEA estimate of K_0.								
$N = 8^*$	3.62	2.01	2.70	1.50	2.51	1.41	4.58	4.17
	(0.37)	(0.17)	(0.34)	(0.16)	(0.25)	(0.12)	(0.72)	(0.38)
$N = 20^*$	3.40	1.90	2.47	1.29	2.42	1.37	4.39	4.11
	(0.28)	(0.13)	(0.23)	(0.11)	(0.23)	(0.11)	(0.74)	(0.38)
$N = T^*$	3.31	1.84	2.38	1.33	2.33	1.33	4.21	4.05
	(0.25)	(0.11)	(0.20)	(0.08)	(0.18)	(0.08)	(0.69)	(0.36)
a^{ST} with Caselli's benchmark economy capital K_0 (BEA estinate)								
$N = 8^*$	3.31	1.86	2.38	1.35	2.39	1.36	4.59	4.18
	(0.27)	(0.12)	(0.22)	(0.10)	(0.22)	(0.10)	(0.75)	(0.36)
$N = 20^*$	3.25	1.83	2.33	1.32	2.34	1.33	4.38	4.11
	(0.23)	(0.11)	(0.18)	(0.08)	(0.18)	(0.09)	(0.72)	(0.36)
$N = T^*$	3.23	1.82	2.32	1.31	2.32	1.31	4.25	4.05
	(0.23)	(0.11)	(0.18)	(0.08)	(0.18)	(0.08)	(0.70)	(0.36)

* The value of g^I is based on the first N available quarterly observations (for annual data, $N = 2$ or $N = 5$).
When average $g^I < 0$, the average value over all the positive observations is used.
A: Analyst observes quarterly data $\{Y_t, N_t, I_t, \omega_t\}$.
B: Analyst observes quarterly data $\{Y_t, N_t, I_t, \omega_t, U_t\}$.
C: Analyst observes quarterly data $\{Y_t, N_t, I_t, \omega_t, U_t, \delta_t\}$.
D: Analyst observes annual time-aggregated data from Scenario A.

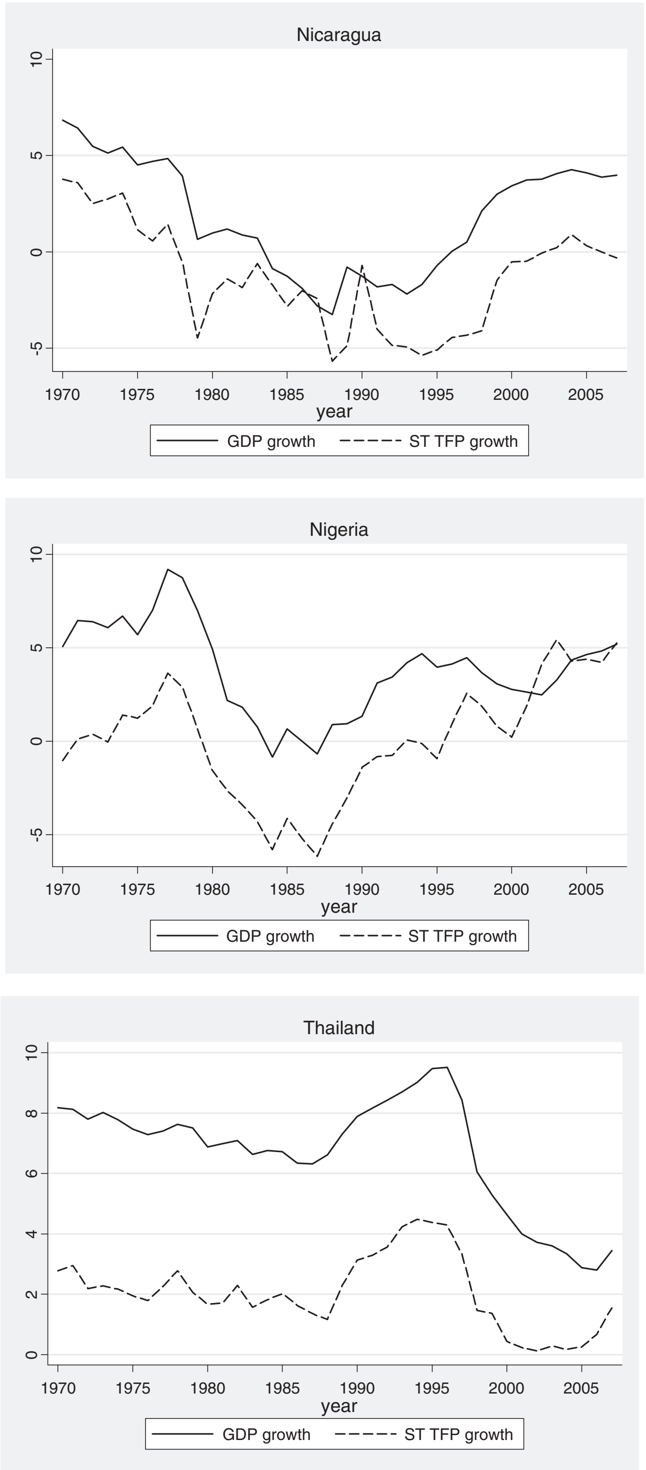

Fig. 2. 10-year moving averages of GDP growth and ST measure for three developing countries (% per annum).

investment around trend from Table 1 $\left(\hat{I}_t^*\right)$ and the variance of investment in the model $\left(\hat{I}_t\right)$ according to the formula

$$\sigma_\epsilon = \sqrt{var\left(\hat{I}_t^*\right) - var\left(\hat{I}_t\right)} = \sqrt{(4.23 * 0.0177)^2 - (2.08 * 0.0201)^2} = 0.062.$$

Because the original model is calibrated with data from the United States, where national income product accounts are of high quality, we also consider a measurement error of three times this magnitude, $\sigma_\epsilon = 0.2$. In order to enforce consistency with the rest of the model, this error occurs at the expense of investment's complement on the expenditure side of the GDP accounts, consumption, so that measurement precision of all other variables, including GDP, is assumed to be uncompromised.[24]

The first horse race presented in panels of Tables 3a and 3b demonstrates that in the presence of measurement error, measures which do not involve the capital stock can significantly outperform conventional Solow–Törnqvist residual. This improvement is significant in samples of 50 observations for both mature and transition economies, with both DS and GD measures outperforming the Solow residual in Scenario A by as much as one-third. For the GD approach, the estimate of initial TFP growth based on the Malmquist index makes a substantial contribution to RSME compared with simply assuming $ln\left(A_0/\bar{A}_0\right) = 0$.[25] In the B-scenario the RMSE is reduced by as much as 56% (BEA vs. GD). In the 200 quarter samples, however, all measures are similarly precise, especially when the full sample is used to estimate g^I. For transition economies (Table 3b), the improvement disappears for the GD measure and is attenuated, but still significant, ranging from 10 to 15% for DS. As would be expected, the RMSE improvement of the stock-less measures over the conventional Solow–Törnqvist residual estimates is inversely related to the relative importance of the initial condition and thus to the length of the sample time series. For mature economies, this improvement is independent of whether the BEA or Caselli estimate of initial capital stocks is employed.

5.3. Nonconstant steady state TFP growth

The synthetic data generated in Section 3 and used in the previous analysis assumed constant trend TFP growth. While this may be a good approximation for the very long run, in development applications, reality may be quite different. Fig. 2 plots 10-year moving averages of GDP growth (World Bank, in constant US dollars) from 1970 to 2007 along with the ST measure applied to the Heston–Summers Penn World Tables (version 7.0). Evidently TFP growth can oscillate at low frequencies, corresponding as much to political and regulatory developments as the implementation of technological innovations.

To model the impact such waves might have on TFP measurement, we replaced the deterministic TFP component ψ^t with $\psi^t[1 + \Phi\cos(\theta t + \theta_p)]$. We set the cycle amplitude $\Phi = 0.25$ (in logs) and the frequency $\theta = 0.0628$, which corresponds to a "long-wave" periodicity of 25 years for the deterministic part of the TFP process. For each of the hundred realizations, a phase shift θ_p was drawn from a uniform distribution on $[0, \frac{\pi}{25}]$. The results of this analysis for both the mature and transition economies are presented in panels of Tables 4a and 4b and show that a non-constant underlying trend growth rate increases the margin of error for all measures, but especially for the traditional Solow residual and when the number of observations used for computing g^I is small. This conclusion continues to hold when investment is measured with error at the same time (presented in Appendix C). Overall, we observe a systematic improvement in the RMSE, especially when using the DS measure. Even if our results frequently lie within the bounds of sampling error, their systematic tendency militates in favor of the capital-stock-less measures in very short samples and in economies far from the steady state.

6. Application: TFP growth in developing countries

As an empirical application, we use our proposed alternative measures to study the contribution of TFP to economic growth in developing

[24] If the income side is measured correctly, measurement errors with respect to investment will be perfectly negatively correlated with those of consumption.

[25] We also considered the Malmquist index (44) itself as an alternative measure of TFP growth in each period. We obtained similar, but inferior, results compared with the GD measure.

Table 4a
A horse race: RMSEs of stock-less versus traditional Solow–Törnqvist estimates of TFP growth (% per period) when trend TFP follows a low frequency wave pattern.

Mature economy (100 realizations, standard errors in parentheses)	A (Quarterly)		B (Quarterly)		C (Quarterly)		D (Annual)	
	T = 50	T = 200	T = 50	T = 200	T = 50	T = 200	T = 12	T = 50
a^{DS}	0.89	1.83	0.64	1.72	0.67	0.66	8.94	5.70
	(0.10)	(0.05)	(0.08)	(0.04)	(0.08)	(0.04)	(1.00)	(0.48)
a^{GD}	1.44	2.11	1.30	2.01	1.32	1.26	9.64	6.83
	(0.11)	(0.06)	(0.10)	(0.05)	(0.10)	(0.06)	(0.98)	(0.43)
a^{ST} with BEA estimate of K_0.								
$N = 8^*$	0.93	1.86	0.68	1.74	0.90	0.76	9.49	6.09
	(0.12)	(0.07)	(0.12)	(0.04)	(0.22)	(0.09)	(1.10)	(0.52)
$N = 20^*$	0.88	1.85	0.61	1.73	0.69	0.69	9.28	6.00
	(0.11)	(0.06)	(0.11)	(0.05)	(0.10)	(0.04)	(1.03)	(0.50)
$N = T^*$	0.85	1.85	0.58	1.73	0.61	0.67	9.20	5.93
	(0.11)	(0.06)	(0.10)	(0.04)	(0.10)	(0.05)	(1.07)	(0.49)
a^{ST} with Caselli's benchmark economy capital K_0 (BEA estinate)								
$N = 8^*$	0.95	1.86	0.70	1.75	0.95	0.78	9.61	6.14
	(0.13)	(0.06)	(0.13)	(0.04)	(0.25)	(0.10)	(1.13)	(0.54)
$N = 20^*$	0.89	1.85	0.71	0.70	0.62	1.73	9.37	6.04
	(0.11)	(0.06)	(0.13)	(0.06)	(0.10)	(0.04)	(1.05)	(0.50)
$N = T^*$	0.86	1.85	0.58	1.73	0.62	0.68	9.27	5.96
	(0.11)	(0.06)	(0.10)	(0.04)	(0.10)	(0.05)	(1.09)	(0.49)

* The value of g^I is based on the first N available quarterly observations (for annual data, $N = 2$ or $N = 5$).
When average $g^I < 0$, the average value over all the positive observations is used.
A: Analyst observes quarterly data $\{Y_t, N_t, I_t, \omega_t\}$.
B: Analyst observes quarterly data $\{Y_t, N_t, I_t, \omega_t, U_t\}$.
C: Analyst observes quarterly data $\{Y_t, N_t, I_t, \omega_t, U_t, \delta_t\}$.
D: Analyst observes annual time-aggregated data from Scenario A.

economies. We take inspiration from Klenow and Rodriguez-Clare (1997) and start with the standard growth accounting decomposition [26]:

$$\begin{aligned}\Delta \ln Y_t &= \Delta \ln A_t^j + \alpha \Delta \ln K_t^j + (1-\alpha)\Delta \ln N_t \\ &= \Delta \ln A_t^j + \Delta \ln X_t^j\end{aligned} \tag{14}$$

where $\Delta \ln A_t^j$ and $\Delta \ln X_t^j$ represent the approximate contributions of TFP and observable production factors to economic growth, respectively. Now let b_a^j and b_X^j stand for coefficients from univariate OLS regressions of each of the right-hand side components of Eq. (14) on observed GDP growth (with a constant term). It can be shown that

$$1 = b_a^j + b_X^j = \frac{Var\left(\Delta \ln A_t^j\right) + Var\left(\Delta \ln X_t^j\right) + 2Cov\left(\Delta \ln A_t^j, \Delta \ln X_t^j\right)}{Var(\Delta \ln Y_t)}. \tag{15}$$

The "KRC decomposition" answers the question: conditional on observing higher output growth, how much of that growth is associated with technology (b_a^j) and how much of it is associated with growth in factors of production (b_X^j)?[27] Jones (1997) has criticized the implicit (equal) allocation of covariance between $\Delta \ln X$ and $\Delta \ln A$ across the two components as arbitrary, and may lead to negative values. This limitation notwithstanding, for purposes of comparison we will apply this technique to the ST, DS and GD TFP growth measures, assigning the covariance term equally to both sources of growth:

$$b_a^j = \frac{Cov\left(\Delta \ln Y_t, \Delta \ln A_t^j\right) + Var\left(\Delta \ln A_t^j\right)}{Var(\Delta \ln Y_t)} \tag{16}$$

$$b_X^j = \frac{Cov\left(\Delta \ln Y_t, \Delta \ln X_t^j\right) + Var\left(\Delta \ln X_t^j\right)}{Var(\Delta \ln Y_t)}. \tag{17}$$

Using data from the Penn World Tables on output, employment, and investment, we construct the KRC decomposition for the following 26 high-growth developing countries: Angola, Chad, Ghana, Mozambique, Nigeria, South Africa, and Uganda; Cambodia, China, Hong Kong, India, Indonesia, Laos, Malaysia, Philippines, Singapore, South Korea, Taiwan, Thailand, Turkey, and Vietnam; Argentina, Brazil, Chile, and Peru. Capital is constructed using the PIM. The initial capital stock is estimated following the BEA procedure with the growth rate of investment g^I set equal to the annual average of country investment series for the entire sample. The values for capital share, depreciation, and the annual gross rental rate for capital are 0.33, 0.08 and 0.11, respectively for all countries.

Table 5 displays the KRC decomposition for three different periods (1975–1984, 1984–1995, and 1995–2007 as well as 1975–2007) and shows contribution of productivity b_a^j and observable factors (capital and labor) b_X^j according to the ST, DS and GD measures. Judging from the ST and DS measures, productivity can account for about 90% of the total output growth over the entire period, in line with the results reported by Klenow and Rodriguez-Clare (1997). In contrast, the GD measurement – which we have argued is less appropriate for developing countries presumably far from their steady state – suggests a much more modest contribution of total factor productivity. In Appendix D we report values for 88 countries of the PWT dataset, which includes a wider range of countries, including chronic poor growth performers such as Haiti and Zimbabwe. Our findings are even more pronounced in favor of the TFP-driven view of economic development.

In Fig. 3 we report average values of the three TFP measures computed over the period 1962–2007 for nine exemplary high growth countries from three continents: Africa (Ghana, Nigeria, and South Africa), Asia (Malaysia, Taiwan, and Thailand), and South America (Argentina,

[26] Klenow and Rodriguez-Clare (1997) apply their decomposition to levels of labor productivity. A lack of data (especially for wages and user costs) preclude consideration of the dual approach (Aiyar and Dalgaard (2005) and Hsieh (2002)). Moreover, the choice of user cost of capital can bias TFP estimates generated using for the dual approach (Young (1995a)).
[27] Klenow and Rodriguez-Clare (1997, p. 80).

Table 4b
A horse race: RMSEs of stock-less versus traditional Solow–Törnqvist estimates of TFP growth (% per period) when trend TFP follows a low frequency wave pattern.

Transition economy (100 realizations, standard errors in parentheses)								
	A (Quarterly)		B (Quarterly)		C (Quarterly)		D (Annual)	
	T = 50	T = 200	T = 50	T = 200	T = 50	T = 200	T = 12	T = 50
a^{DS}	3.19	2.40	2.30	2.05	2.30	1.31	7.96	6.70
	(0.22)	(0.08)	(0.17)	(0.05)	(0.17)	(0.05)	(0.75)	(0.38)
a^{GD}	5.18	3.70	2.99	2.95	4.28	2.86	10.37	6.76
	(0.37)	(0.18)	(0.22)	(0.13)	(0.32)	(0.14)	(0.98)	(0.44)
a^{ST} with BEA estimate of K_0.								
$N = 8^*$	3.46	2.52	2.52	2.14	2.37	1.36	7.84	6.57
	(0.27)	(0.10)	(0.21)	(0.07)	(0.22)	(0.10)	(0.75)	(0.37)
$N = 20^*$	3.28	2.45	2.38	2.11	2.42	1.37	7.88	6.62
	(0.24)	(0.09)	(0.22)	(0.13)	(0.23)	(0.11)	(0.77)	(0.44)
$N = T^*$	3.21	2.42	2.28	2.07	2.29	1.32	7.96	6.68
	(0.23)	(0.09)	(0.17)	(0.06)	(0.18)	(0.08)	(0.75)	(0.37)
a^{ST} with Caselli's benchmark economy capital K_0 (BEA estimate)								
$N = 8^*$	3.21	2.43	2.28	2.07	2.36	1.36	7.86	6.57
	(0.27)	(0.09)	(0.17)	(0.06)	(0.21)	(0.10)	(0.76)	(0.38)
$N = 20^*$	3.20	2.42	2.30	1.32	2.36	2.06	7.87	6.60
	(0.23)	(0.09)	(0.18)	(0.08)	(0.18)	(0.06)	(0.77)	(0.38)
$N = T^*$	3.19	2.42	2.28	2.06	2.29	1.32	7.90	6.65
	(0.23)	(0.09)	(0.17)	(0.06)	(0.18)	(0.08)	(0.76)	(0.37)

* The value of g^I is based on the first N available quarterly observations (for annual data, $N = 2$ or $N = 5$).
When average $g^I<0$, the average value over all the positive observations is used.
A: Analyst observes quarterly data $\{Y_t, N_t, I_t, \omega_t\}$.
B: Analyst observes quarterly data $\{Y_t, N_t, I_t, \omega_t, U_t\}$.
C: Analyst observes quarterly data $\{Y_t, N_t, I_t, \omega_t, U_t, \delta_t\}$.
D: Analyst observes annual time-aggregated data from Scenario A.

Brazil, and Peru). We display averages for the entire sample and two sub-samples (1962–1984 and 1985–2007). The growth measures confirm the non-constancy of TFP growth over the subperiods and a higher estimated TFP growth rate using the DS method compared with either the ST or the GD measure. These results further underscore the importance of TFP-driven growth in these high-growth countries. Higher values of the DS measure for the US as well as many other countries is consistent with a undermeasured (or overdepreciated) capital stock on the basis of the standard PIM method.[28]

In Fig. 4 we measure the contributions in growth relative to the US, following Jones (1997) to understand better the roles of TFP and factor-driven growth relative to the outer envelope of potential technological progress. Assuming that the world's technological frontier is growing at the US rate according to measure j, $a^{j,US}$, the "exceptional labor productivity growth" in country i, $g^{i}_{Y/N} - g^{US}_{Y/N}$ can be decomposed for each method into "exceptional TFP growth" with respect to the evolution of TFP, $a^{j,i} - a^{j,US}$, and "exceptional" growth in factor deepening ($g^{j,i}_{X} - g^{j,US}_{X}$) for the same countries considered in Fig. 3. While both observables and unobservables contribute to long run growth relative to the US, the ST measure appears to be associated with systematic overstatement of the contribution of observables (in our case, capital intensity only) to labor productivity growth in Asia in both periods as well as for Latin America and Africa. Our results thus supports the view that TFP movements are more important for explaining long-run growth – in both directions – than the standard Solow residual would lead us to believe. This hypothesis is especially plausible when one expands the interpretation of TFP to include the evolution of social capital, rule of law, human capital infrastructure and other determinants of growth (Hall and Jones (1999)).[29]

7. Conclusion

Over the past half-century, the Solow residual has attained widespread use in economics and management as a measurement of total factor productivity. Its popularity derives from its simplicity and independence of statistical methods. Despite universal acceptance of this measurement tool, its quantitative features have yet to be evaluated systematically, despite potentially severe measurement problems associated with capital stock, depreciation and utilization data. We have documented the quantitative significance of this error, as measured by the root mean squared error, in a synthetic data set. Our Solow residuals without capital stocks also confirm the assessment, now standard in the literature on growth and development, that growth in observable factor inputs contributes only modestly to explaining cross country variation in long-run economic growth.

We find that while measurement error of the Solow residual decreases with sample size, it remains a serious problem for short data

[28] Lower depreciation rates generally lead to lower mean TFP growth estimates with the DS method. In a previous version of this paper, we estimated and compared TFP growth for West German states using the value of δ backed out from published capital stocks estimates (roughly 5.6%) and the means of DS and ST methods differed by less than 0.5% per annum. For consistency, we employed the same depreciation rate used in the data generation process, which may be too high.

[29] In the appendix, we present detailed growth accounting relative to the US frontier for the following three periods: 1962–2007, 1962–1982, and 1983–2007. US per capita GDP growth in the three periods is as follows:

Period	a^{ST}	a^{DS}	a^{GD}
1962–2007	0.9	2.8	1.1
1962–1982	1.0	2.9	1.2
1983–2007	0.9	2.6	1.0

The outsized values of a^{DS} for the US may be surprising, but result from a common value of capital depreciation imposed on all countries in the sample (8% per annum). Using a value of δ estimated for Germany in a previous version of this paper ($\delta = 5.6\%$, see footnote 28), US estimates for a^{DS} are considerably lower (1962–2007: 2.0%; 1962–1982: 2.2%; 1983–2007: 1.8%).

Table 5
Decomposing growth in developing countries: The role of productivity and observable factors.

Period	ST		DS		GD	
	b_a^{ST}	b_X^{ST}	b_a^{DS}	b_X^{DS}	b_a^{GD}	b_X^{GD}
1975–1984	86.8	3.2	87.8	12.2	62.4	37.6
1984–1995	95.6	4.4	95.0	5.0	67.2	32.8
1995–2007	92.1	7.9	97.9	2.1	65.7	34.3
1975–2007	91.8	8.2	94.4	5.6	65.0	35.0

sets or economies in a developmental take-off phase. Thus, the Solow residual is least accurate in applications for which TFP measurements are most valuable: studying the medium term effects of sweeping institutional reforms, the transition to a market economy, the introduction of ICT capital in the production process, or the role of weightless assets such as advertising goodwill and knowledge acquired through R&D expenditures (Corrado et al. (2009)).

Both proposed alternatives to the Solow–Törnqvist measures can be thought of as a "marginalization" of the error carried forward in the capital stock across time. Most recent investment expenditures are most likely to be properly valued at acquisition cost and to be fully utilized. Our methods could be applied to a number of investment context and types, thus broadening the scope and appeal of applied TFP measurement.

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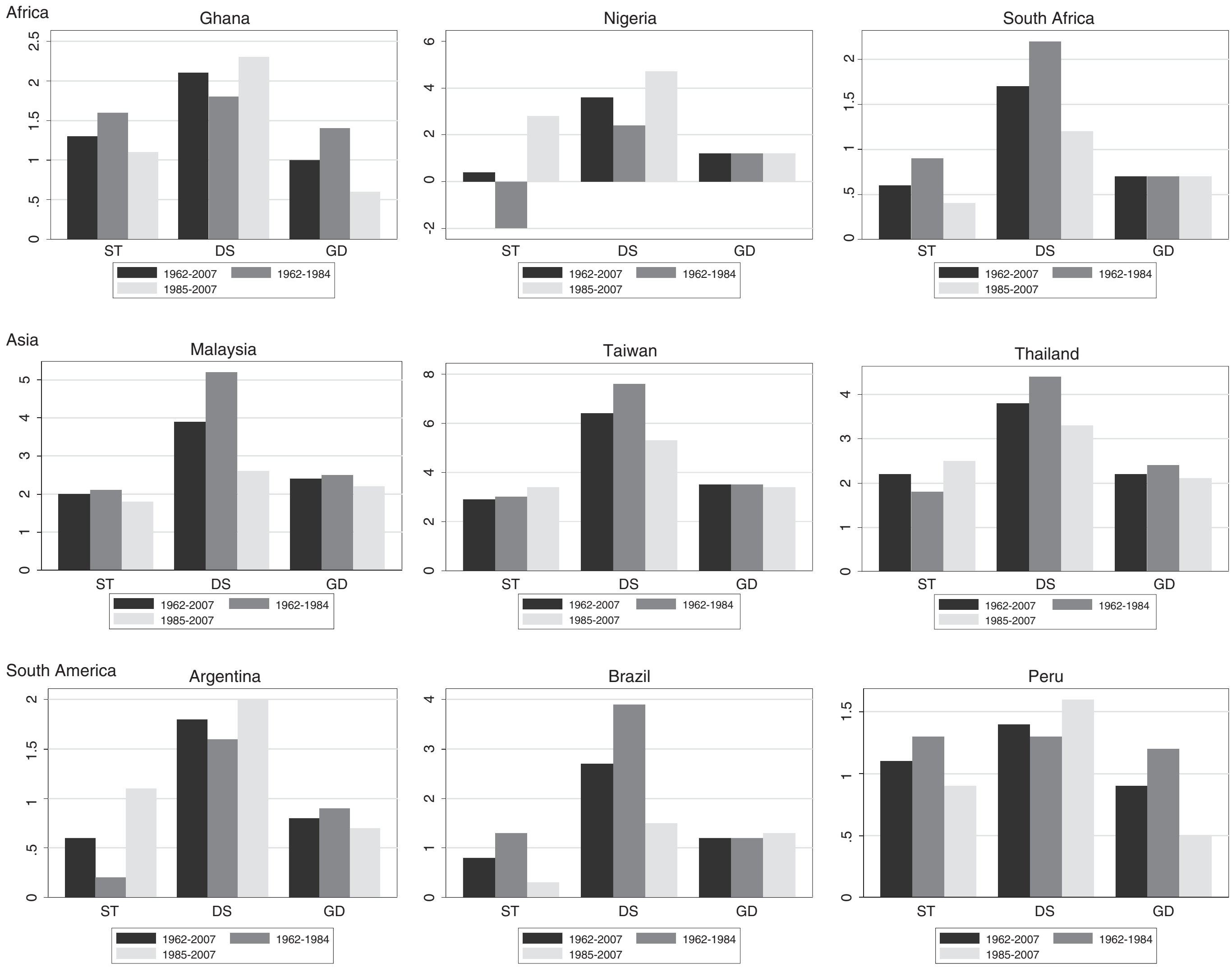

Fig. 3. TFP growth measurement in a sample of developing countries 1962–2007: A comparison.

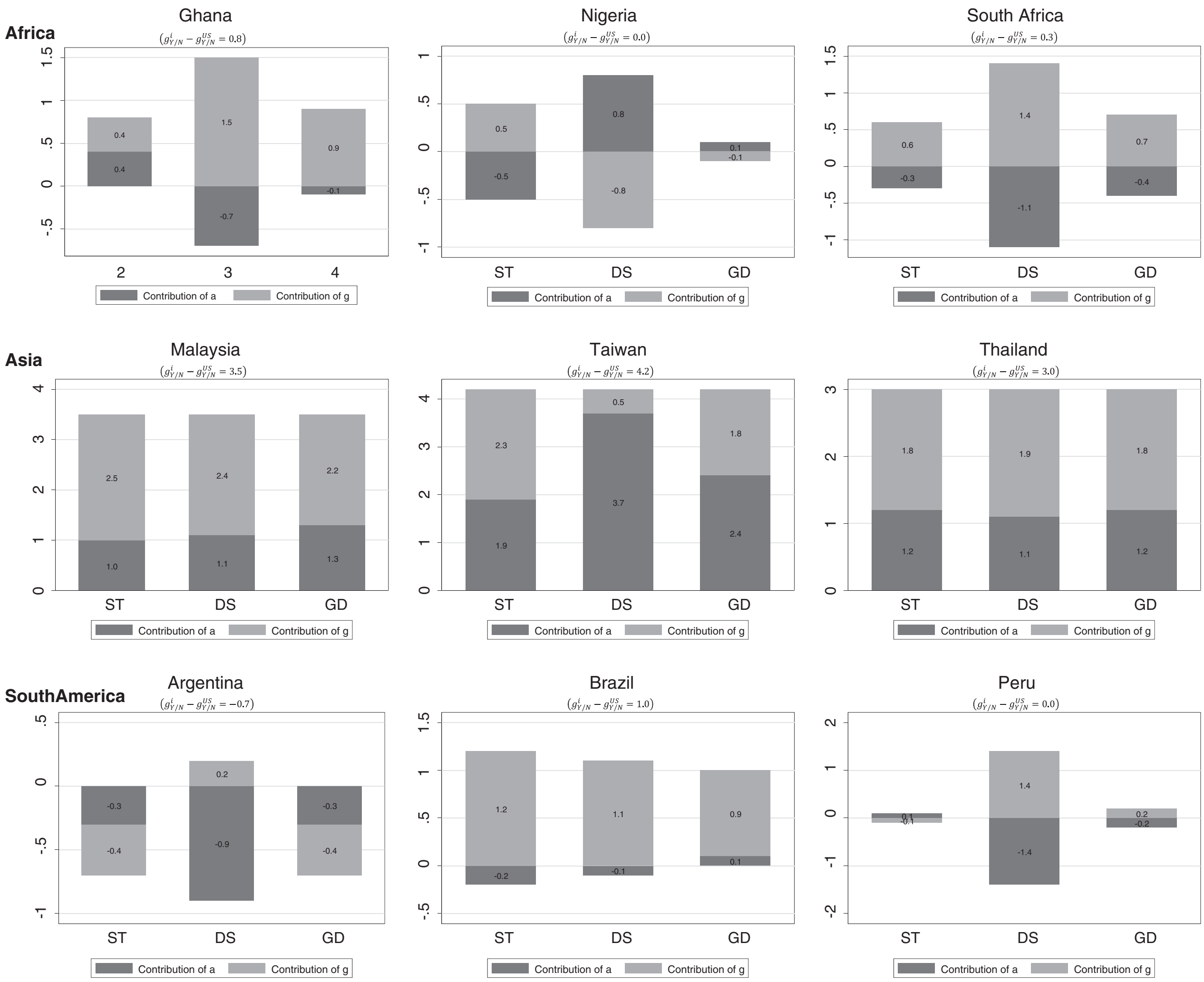

Fig. 4. Growth accounting relative to the frontier (US) in high-growth developing countries 1962–2007.

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Appendix A. The stochastic growth model

A1. Technology

Productive opportunities in this one-good economy evolve as a trend-stationary stochastic process. Total factor productivity $\{A_t\}$ is embedded in a standard constant returns production function of capital services and labor inputs, and evolves for $t = 1,2,...$ according to

$$A_t = \psi^{t(1-\rho)} A^{\rho}_{t-1} e^{\epsilon_t}, \tag{A.1}$$

where $\psi > 1$, $|\rho| < 1$, A_0 is given and ϵ_t is white noise. Output is given by the Cobb–Douglas specification

$$Y_t = A_t (U_t K_t)^{\alpha} N_t^{1-\alpha}, \tag{A.2}$$

where $U_t \in (0,1)$ denotes the utilization rate of capital ("capacity utilization").

In this version of the model, output can either be consumed or invested in productive capacity ("capital"). Starting from a given initial K_0, capital evolves according to Eq. (3), where the rate of depreciation is an increasing, convex function of capacity utilization

$$\delta_t = \frac{B}{\chi} U_t^{\chi} \tag{A.3}$$

with $B > 0$ and $\chi > 1$. We deviate from Wen (1998) and Harrison and Weder (2006) by adding a scale parameter B, which allows us to match both the mean and variance of the model's simulated capacity utilization with data from actual economies.

A2. Households

Households own capital and labor and sell factor services to firms in competitive factor markets. Facing sequences of wages $\{\omega_t\}^{\infty}_{t=0}$ and user cost of capital $\{\kappa_t\}^{\infty}_{t=0}$, the representative household chooses paths of consumption $\{C_t\}^{\infty}_{t=0}$, labor supply $\{N_t\}^{\infty}_{t=0}$, capital utilization $\{U_t\}^{\infty}_{t=0}$, and capital in the next period $\{K_{t+1}\}^{\infty}_{t=0}$ to maximize the expected present value of lifetime utility:

$$\max_{\{C_t\},\{N_t\},\{K_{t+1}\},\{U_t\}} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln C_t + \frac{\theta}{1-\eta}\left[(1-N_t)^{1-\eta} - 1\right]\right\} \tag{A.4}$$

subject to an initial condition for the capital stock K_0, the periodic budget restriction for $t = 0,1...$

$$C_t + K_{t+1} - (1-\delta_t)K_t = \omega_t N_t + \kappa_t U_t K_t, \tag{A.5}$$

and the dependence of capital depreciation on utilization given by Eq. (A.3). The period-by-period budget constraint restricts consumption and investment to be no greater than gross household income from labor ($\omega_t N_t$) and capital ($\kappa_t U_t K_t$).

A3. Firms

Firms in this perfectly competitive economy are owned by the representative household. The representative firm employs labor N_t and hires capital services $U_t K_t$ to maximize profits subject to the constant returns production function given by Eq. (A.2). Note that for the firm, capital service input is the product of the capital stock and its utilization rate; the firm is indifferent to whether these services stem from the extensive or intensive margin.

A4. First order conditions, decentralized equilibrium and steady state

We now summarize the first order conditions for optimal behavior of households and firms and characterize the decentralized market equilibrium, which in this regular economy is unique. Dynamic behavior can be approximated by log-linearized versions of these equilibrium conditions around the model's unique steady state growth path. Along that path, output, consumption, investment and capital stock all grow at a constant rate $g = \psi^{\frac{1}{1-\alpha}} - 1$, while total factor productivity grows at rate $\psi - 1$. Population growth is set to zero; employment, capital utilization and interest rates are trendless.

Let λ_t denote the Lagrange multiplier corresponding to the periodic resource constraint (A.5). The first order conditions for the household are, for $t \geq 0$:

$$C_t : \lambda_t = \frac{1}{C_t} \tag{A.6}$$

$$K_{t+1} : \lambda_t = \beta E_t\left[\lambda_{t+1}\left((1-\delta_{t+1}) + \kappa_{t+1} U_{t+1}\right)\right] \tag{A.7}$$

$$N_t : \theta(1-N_t)^{-\eta} = \lambda_t \omega_t \tag{A.8}$$

$$U_t : BU_t^{\chi-1} = \kappa_t. \tag{A.9}$$

First-order conditions for the firms

$$N_t : (1-\alpha)A_t(U_t K_t)^{\alpha} N_t^{-\alpha} = \omega_t \tag{A.10}$$

$$U_t K_t : \alpha A_t U_t^{\alpha-1} K_t^{\alpha-1} N_t^{1-\alpha} = \kappa_t \tag{A.11}$$

the production function

$$Y_t = A_t U_t^{\alpha} K_t^{\alpha} N_t^{1-\alpha} \tag{A.12}$$

and the aggregate resource constraint (since $\omega_t N_t + \kappa_t U_t K_t = Y_t$).

$$K_{t+1} = (1-\delta_t)K_t + Y_t - C_t \tag{A.13}$$

The equilibrium of this decentralized economy is defined as the sequences of wages $\{\omega_t\}$, rental prices for capital $\{\kappa_t\}$, output $\{Y_t\}$, consumption $\{C_t\}$, employment $\{N_t\}$, capital stocks $\{K_{t+1}\}$, and the capacity utilization rate $\{U_t\}$ such that Eqs. (A.10), (A.11), (A.12) and (A.13) hold for $t \geq 0$ plus a suitable transversality condition to guarantee that the capital stock path is consistent with utility maximization. The equilibrium of the problem will be, by the first and second welfare theorems, unique and equivalent to the one chosen by a social planner with the objective of solving the utility of the representative household.

A5. Detrended version of equilibrium

Steady state values of the model's variables are denoted by an upper bar. In the steady state $\overline{X}_{t+1} = (1+g)\overline{X}_t$ for $X \in \{C, I, Y, K\}$ and $\overline{A}_{t+1} = \psi \overline{A}_t$. We define detrended values of the variables of interest such that $\widetilde{X}_t \equiv X_t / \overline{X}_t$. The following equations characterize the equilibrium of this transformed economy:

$$\theta \frac{\widetilde{C}_t}{(1-N_t)^{\eta}} = (1-\alpha)A_t U_t^{\alpha} \widetilde{K}_t^{\alpha} N_t^{-\alpha} \tag{A.14}$$

$$1 = E_t\left[\beta \frac{\widetilde{C}_t}{\psi \widetilde{C}_{t+1}} R_{t+1}\right] \tag{A.15}$$

$$\begin{aligned} &\alpha \widetilde{A}_t \left(\frac{\widetilde{K}_t}{N_t}\right)^{\alpha-1} = BU_t^{\chi-\alpha} \\ &\psi \widetilde{K}_{t+1} = (1-\delta_t)\widetilde{K}_t + \widetilde{Y}_t - \hat{C}_t. \end{aligned} \tag{A.16}$$

The first equation characterizes intratemporal optimality of time across alternative uses in production and leisure; the second is the familiar Euler equation which arbitrages expected intertemporal rates of substitution and transformation in expectation, where the latter is defined by $R_{t+1} = \alpha A_t \left(U_t \widetilde{K}_t\right)^{\alpha-1} N_t^{1-\alpha}$ and represents the gross rate of return on holding a unit of capital from period t to period $t+1$. The last equation is the periodic resource constraint of the economy, given the production function and competitive factor remuneration. Given that this economy fulfills the conditions of the first welfare theorem, it would also characterize the optimal choice of a central planner solving Eq. (A.4) subject to the resource constraint (A.5) and the initial condition K_0.

A6. The steady state

To solve for the non-stochastic steady state, let $A_t = 1$ and $\widetilde{X}_{t+1} = \widetilde{X}_t = \overline{X}$. We obtain the following equations:

$$\frac{\theta \overline{C}}{(1-\overline{N})^{\eta}} = (1-\alpha)\overline{U}^{\alpha} \overline{K}_t^{\alpha} \overline{N}^{-\alpha} \tag{A.17}$$

$$1 = \frac{\beta}{\psi} \overline{R} \tag{A.18}$$

$$\alpha \left(\frac{\overline{K}}{\overline{N}}\right)^{\alpha-1} = B\overline{U}^{\chi-\alpha} \tag{A.19}$$

A7. Log linearization

Using the convention that $\hat{X} = (X-\overline{X})/\overline{X}$ denotes deviations from steady state values, the log-linearized first order condition for labor supply can be written as

$$\hat{C}_t - \left(\alpha + \frac{N}{1-N}\eta\right)\hat{N}_t = \hat{A}_t + \alpha\left(\hat{U}_t + \hat{K}_t\right). \tag{A.20}$$

The resource constraint is:

$$\frac{\overline{C}}{\overline{K}}\hat{C}_t + \psi \hat{K}_{t+1} = (1-\delta)\hat{K}_t - \chi \hat{U}_t + \alpha \frac{\overline{Y}}{\overline{K}}\hat{K}_t + (1-\alpha)\frac{\overline{Y}}{\overline{K}}\overline{N}\hat{N}_t + \frac{\overline{Y}}{\overline{K}}\hat{A}_t \tag{A.21}$$

and the Euler equation becomes

$$0 = E_t\left[\hat{C}_t - \hat{C}_{t+1} + \beta \overline{r}\left[\hat{A}_{t+1} - (1-\alpha)\left(\hat{K}_{t+1} - \hat{N}_{t+1}\right) - \chi \hat{U}_t\right]\right]. \tag{A.22}$$

A8. Model calibration and generation of the synthetic dataset

We calibrate the model to a quarterly setting using values typically used for simulating the US time series in the literature and discussed in Prescott (1986) or King and Rebelo (1999). The values chosen for the parameters are presented in Table A1.

Table A1
Stochastic growth model: parameters and calibration values.

Parameter	Definition	Value
β	Utility discount factor (quarterly)	0.985
$\overline{R}$	Average real interest factor (quarterly)	1.015
$\overline{A}$	Technology	1
$\overline{\delta}$	Depreciation rate of physical capital	0.015
α	Capital elasticity in production	0.36
η	Elasticity of periodic utility to leisure	0.85
θ	Utility weight for leisure/consumption	2.1
$\psi = (1+g)^{1-\alpha}$	Constant growth factor of technology	1.0075
B	Level parameter for capital depreciation rate	0.0425
χ	Elasticity of depreciation to capacity utilization	1.9
ρ	Autocorrelation of the log of TFP term A_t	0.95

Appendix B. The Malmquist Index

B1. The basics

The Malmquist index is one of the most commonly used indices in data envelopment analysis and an alternative way for computing productivity and efficiency changes in production functions.[30] Proposed by Caves et al. (1982) as a reintreptation of an index introduced by Malmquist (1953), it is the ratio of two distance output functions $D_O^t(x, y)$ (Shepard (1970)) at time t and $t+1$:

$$M_{CCD}^t = \frac{D_O^t\left(x^{t+1}, y^{t+1}\right)}{D_O^t\left(x^t, y^t\right)} \tag{B.1}$$

where the numerator is represented by the maximal proportional change in outputs required to obtain the combination (x^{t+1}, y^{t+1}) feasible in relation to the technology at time t. Färe et al. (1989) consider an alternative measure of Eq. (B.1):

$$M_{FGLR}^{t+1} = \frac{D_O^{t+1}\left(x^{t+1}, y^{t+1}\right)}{D_O^{t+1}\left(x^t, y^t\right)} \tag{B.2}$$

and propose a new version of the Malmquist index, defined as the geometric mean of Eqs. (B.1) and (B.2):

$$M_0\left(x^{t+1}, y^{t+1}, x^t, y^t\right) = \left[\left(\frac{D_O^t\left(x^{t+1}, y^{t+1}\right)}{D_O^t\left(x^t, y^t\right)}\right)\left(\frac{D_O^{t+1}\left(x^{t+1}, y^{t+1}\right)}{D_O^{t+1}\left(x^t, y^t\right)}\right)\right]^{\frac{1}{2}}. \tag{B.3}$$

In addition, Färe et al. (1992) rewrite Eq. (B.3) yielding an efficiency and a technological term:

$$M_0\left(x^{t+1}, y^{t+1}, x^t, y^t\right) = \left(\frac{D_O^{t+1}\left(x^{t+1}, y^{t+1}\right)}{D_O^t\left(x^t, y^t\right)}\right)\left[\left(\frac{D_O^t\left(x^t, y^t\right)}{D_O^{t+1}\left(x^t, y^t\right)}\right)\left(\frac{D_O^t\left(x^{t+1}, y^{t+1}\right)}{D_O^{t+1}\left(x^{t+1}, y^{t+1}\right)}\right)\right]^{\frac{1}{2}} \tag{B.4}$$

where the term $\left[\left(\frac{D_O^t\left(x^t, y^t\right)}{D_O^{t+1}\left(x^t, y^t\right)}\right)\left(\frac{D_O^t\left(x^{t+1}, y^{t+1}\right)}{D_O^{t+1}\left(x^{t+1}, y^{t+1}\right)}\right)\right]^{\frac{1}{2}}$ measures the contribution of technological change and is equivalent to the Törnqvist index.[31]

[30] For a review of the index numbers used in productivity analysis, see Thanassoulis et al. (2008).

[31] Diewert and Fox (2010) derive a relationship between the Malmquist and the Törnqvist indexes under increasing returns to scale.

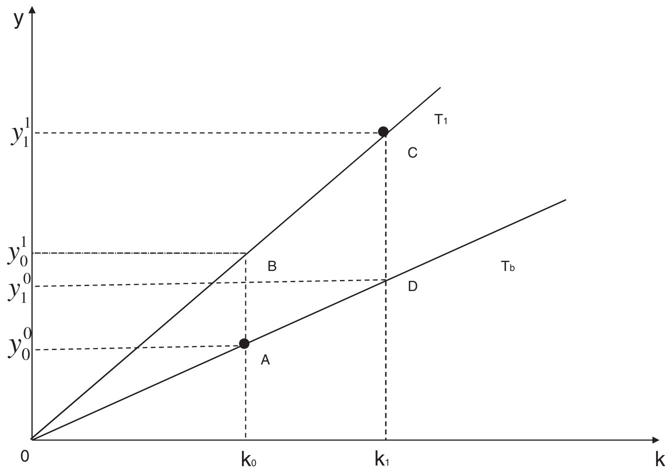

Fig. B1. Construction of the Malmquist index in the full efficiency case.

Assuming a case with one output and two inputs, it is possible to normalize by labor so as only one input in the production function, so that $y_t = \frac{Y_t}{N_t}$ and $k_t = \frac{K_t}{N_t}$.

Fig. B1 is a graphical representation of the Malmquist index for an economy in the presence of constant return to scale and full efficiency: four data points provide a measure of technology change (from T_0 to T_1), which contributes to move from point A, i.e., the amount of output produced at time 0 $y_0^0 \equiv f_0(k_0)$, to point C, i.e., the production in the second period $y_1^1 \equiv f_1(k_1)$. To do so, TFP growth is decomposed into the input accumulation and the information on the counterfactuals, point D, which depicts production using the technology at time 0 with the amount of input used at time 1 ($y_1^0 \equiv f_0(k_1)$), and point B, i.e., the amount produced with input at time 0 and technology used at time 1 ($y_0^1 \equiv f_1(k_0)$), where, for each y_i^j is the amount produced with input at time j and technology at time k. Assuming constant returns and full efficiency, the log of the Malmquist index equals the log of the geometric mean of the average products in the first two periods, or

$$\ln M_0^1 = \frac{1}{2}\ln\left(\frac{y_1^1}{y_0^0}\frac{y_1^0}{y_0^1}\right) = \underbrace{\frac{1}{2}\ln\left(\frac{y_1^1}{y_0^0}\right)}_{KNOWN} + \underbrace{\frac{1}{2}\ln\left(\frac{y_1^0}{y_0^1}\right)}_{UNKNOWN}. \tag{B.5}$$

The Malmquist index puts a bound on possible evolution of TFP from period 0 to period 1, even when the capital stock is poorly measured or unobservable. In our case we estimate the initial condition $\hat{a}_0^{GD}$ considering the production and the steady state at time 0:

$$\hat{a}_0^{GD} = \underbrace{\frac{1}{2}\ln\left(\frac{y_0^0}{\bar{y}_0^0}\right)}_{KNOWN} + UNKNOWN. \tag{B.6}$$

Consider first the extreme case in which there is no deviation from the steady state for capital accumulation in period 0, i.e. $k_0 = \bar{k}_1$ and $\hat{a}_0^{GD} = \frac{1}{2}\ln\left(\frac{y_0^0}{\bar{y}_0^0}\right)$; in the other extreme, capital accumulation is identical to the growth of labor productivity, i.e. $\hat{a}_0^{GD} = \ln\left(\frac{y_0^0}{\bar{y}_0^0}\right)$. We will employ the midpoint of these two values. We also consider the construction of the index when of negative deviation from the steady state: in this case, the lower bound is represented by the extreme case when capital accumulation is equal to the growth of labor productivity, i.e. $\hat{a}_0^{GD} = \ln\left(\frac{y_0^0}{\bar{y}_0^0}\right)$, while the upper bound is given by $\hat{a}_0^{GD} = \frac{1}{2}\ln\left(\frac{y_0^0}{\bar{y}_0^0}\right)$.

B3. The Malmquist index when capacity utilization is observed

If data on capacity utilization are available, we can rewrite Eq. (B.5) for the first two periods in the case of full efficiency extending De Borger and Kerstens (2000):

$$M_0^1 = \frac{CU_0(k_0, U_0k_0, y_0)}{CU_1(k_1, U_1k_1, y_1)}\sqrt{\frac{D_0(k_1, y_1)}{D_0(k_0, y_0)}} \tag{B.7}$$

where $CU_t(k_t, U_tk_t, y_t) = \frac{D_t(U_tk_t, y_t)}{D_t(k_t, y_t)} \le 1$ is the output efficiency measure removing any existing technical inefficiency attributable to idle capacity. If production function is given by Eq. (A.2), we can rewrite $CU_t(k_t, U_tk_t, y_t) = \frac{A_t\left[(U_tK_t)^{\alpha}N_t^{1-\alpha}\right]/N_t}{A_t\left[K_t^{\alpha}N_t^{1-\alpha}\right]/N_t} = U_t^{\alpha}$ and recompute Eq. (B.5) as

$$\ln M_0^1 = \frac{1}{2}\ln\left(\frac{y_1^1 y_1^0}{y_0^0 y_0^1}\right) + \alpha \ln\frac{U_0}{U_1} = \underbrace{\frac{1}{2}\ln\left(\frac{y_1^1}{y_0^0}\right) + \alpha \ln\frac{U_0}{U_1}}_{KNOWN} + \underbrace{\frac{1}{2}\ln\left(\frac{y_1^0}{y_0^1}\right)}_{UNKNOWN} \tag{B.8}$$

In our case, we consider

$$\hat{a}_0^{GD} = \underbrace{\frac{1}{2}\ln\left(\frac{y_0^0}{y_0^0}\right) + \alpha \ln\frac{U_0}{U_0}}_{KNOWN} + UNKNOWN \tag{B.9}$$

Appendix C

Table C1
A horse race: RMSEs of stock-less versus traditional Solow–Törnqvist estimates of TFP growth (% per period) when investment is measured with error and trend TFP follows a low frequency wave pattern.

Mature economy (100 realizations, standard errors in parentheses)								
	A (Quarterly)		B (Quarterly)		C (Quarterly)		D (Annual)	
	T = 50	T = 200	T = 50	T = 200	T = 50	T = 200	T = 12	T = 25
a^{DS}	0.95	1.86	0.67	1.73	0.67	0.66	8.96	5.71
	(0.10)	(0.12)	(0.08)	(0.04)	(0.08)	(0.04)	(1.01)	(0.49)
a^{GD}	1.50	2.09	1.32	2.04	1.32	1.26	9.66	6.84
	(0.08)	(0.08)	(0.10)	(0.05)	(0.10)	(0.06)	(0.99)	(0.43)
a^{ST} with BEA estimate of K_0.								
$N = 8^*$	1.06	1.89	0.85	1.77	0.91	0.77	9.56	6.13
	(0.20)	(0.08)	(0.22)	(0.06)	(0.22)	(0.09)	(1.18)	(0.56)
$N = 20^*$	0.91	1.86	0.66	1.74	0.68	0.69	9.32	6.01
	(0.12)	(0.07)	(0.12)	(0.05)	(0.11)	(0.05)	(1.07)	(0.51)
$N = T^*$	0.87	1.85	0.61	1.74	0.61	0.67	9.24	5.94
	(0.11)	(0.07)	(0.09)	(0.05)	(0.10)	(0.05)	(0.41)	(0.50)
a^{ST} with Caselli's benchmark economy capital K_0 (BEA estinate)								
$N = 8^*$	1.09	1.89	0.89	1.78	0.95	0.78	9.69	6.18
	(0.22)	(0.08)	(0.24)	(0.06)	(0.25)	(0.10)	(1.22)	(0.58)
$N = 20^*$	0.91	1.86	0.66	1.74	0.68	0.69	9.41	6.06
	(0.12)	(0.07)	(0.12)	(0.05)	(0.11)	(0.05)	(1.09)	(0.52)
$N = T^*$	0.87	1.85	0.61	1.74	0.61	0.67	9.31	5.97
	(0.11)	(0.07)	(0.09)	(0.05)	(0.10)	(0.05)	(1.11)	(0.50)

* The value of g^I is based on the first N available quarterly observations (for annual data, $N = 2$ or $N = 5$).
When average $g^I < 0$, the average value over all the positive observations is used.
A: Analyst observes quarterly data $\{Y_t, N_t, I_t, \omega_t\}$.
B: Analyst observes quarterly data $\{Y_t, N_t, I_t, \omega_t, U_t\}$.
C: Analyst observes quarterly data $\{Y_t, N_t, I_t, \omega_t, U_t, \delta_t\}$.
D: Analyst observes annual time-aggregated data from Scenario A.

Table C2
A horse race: RMSEs of stock-less versus traditional Solow–Törnqvist estimates of TFP growth (% per period) when investment is measured with error and trend TFP follows a low frequency wave pattern.

Transition economy (100 realizations, standard errors in parentheses)								
	A (Quarterly)		B (Quarterly)		C (Quarterly)		D (Annual)	
	T = 50	T = 200	T = 50	T = 200	T = 50	T = 200	T = 12	T = 25
a^{DS}	3.43	2.43	2.30	2.06	2.30	1.31	9.94	6.19
	(0.17)	(0.08)	(0.17)	(0.06)	(0.17)	(0.06)	(0.86)	(0.40)
a^{GD}	5.61	3.79	3.00	3.00	4.28	2.86	15.69	10.04
	(0.33)	(0.28)	(0.23)	(0.13)	(0.33)	(0.14)	(1.29)	(0.57)

Table C2 (*continued*)

Transition economy (100 realizations, standard errors in parentheses)

	A (Quarterly)		B (Quarterly)		C (Quarterly)		D (Annual)	
	T = 50	T = 200	T = 50	T = 200	T = 50	T = 200	T = 12	T = 25
a^{ST} with BEA estimate of K_0.								
$N = 8^*$	3.55	2.56	2.62	2.18	2.41	1.38	10.66	6.65
	(0.37)	(0.14)	(0.34)	(0.11)	(0.22)	(0.10)	(1.04)	(0.46)
$N = 20^*$	3.33	2.47	2.38	2.11	2.37	1.35	10.33	6.51
	(0.27)	(0.11)	(0.22)	(0.07)	(0.23)	(0.11)	(0.90)	(0.40)
$N = T^*$	3.24	2.44	2.31	2.08	2.30	1.32	10.27	6.43
	(0.24)	(0.09)	(0.19)	(0.06)	(0.18)	(0.08)	(0.94)	(0.40)
a^{ST} with Caselli's benchmark economy capital K_0 (BEA estinate)								
$N = 8^*$	3.28	2.46	2.34	2.09	2.36	1.35	10.76	6.70
	(0.27)	(0.10)	(0.22)	(0.07)	(0.20)	(0.10)	(1.03)	(0.46)
$N = 20^*$	3.24	2.44	2.31	2.08	2.30	1.32	10.45	6.56
	(0.24)	(0.09)	(0.19)	(0.06)	(0.18)	(0.08)	(0.93)	(0.41)
$N = T^*$	3.20	2.44	2.31	2.08	2.30	1.32	10.34	6.46
	(3.20)	(2.43)	(2.29)	(2.07)	(2.29)	(1.32)	(0.95)	(0.40)

* The value of g^I is based on the first N available quarterly observations (for annual data, $N = 2$ or $N = 5$).
When average $g^I < 0$, the average value over all the positive observations is used.
A: Analyst observes quarterly data $\{Y_t, N_t, I_t, \omega_t\}$.
B: Analyst observes quarterly data $\{Y_t, N_t, I_t, \omega_t, U_t\}$.
C: Analyst observes quarterly data $\{Y_t, N_t, I_t, \omega_t, U_t, \delta_t\}$.
D: Analyst observes annual time-aggregated data from Scenario A.

Appendix D. Empirical application

Table D1
Decomposing growth in developing countries: The role of productivity and observable factors: Larger sample (all 88 countries listed in Table D2).

Period	ST		DS		GD	
	b_a^{ST}	b_X^{ST}	b_a^{DS}	b_X^{DS}	b_a^{GD}	b_X^{GD}
1975–1984	95.3	4.7	96.8	3.2	76.8	23.2
1984–1995	96.2	3.8	96.8	3.2	92.2	7.8
1995–2007	95.4	4.6	97.0	3.0	93.0	7.0
1975–2007	95.4	4.6	96.8	3.2	86.1	13.9

Table D2
List of all countries used in the PWT dataset.

Central America	South America	Africa	Asia
Bahamas	Argentina	Angola	Bahrain
Barbados	Bolivia	Benin	Bangladesh
Belize	Brazil	Botswana	Bhutan
Costa Rica	Chile	Burkina Faso	China Version 1
Dominican Republic	Colombia	Burundi	Hong Kong
El Salvador	Ecuador	Cameroon	India
Guatemala	Guyana	Chad	Indonesia
Haiti	Paraguay	Comoros	Iran
Honduras	Peru	Congo, Dem. Rep.	Iraq
Jamaica	Suriname	Congo, Republic of	Israel
Mexico	Uruguay	Cote d'Ivoire	Jordan
Nicaragua	Venezuela	Egypt	Korea, Republic of
Panama		Ethiopia	Laos
Puerto Rico		Gabon	Malaysia
Trinidad &Tobago		Gambia, The	Mongolia
		Ghana	Nepal
		Guinea	Oman
		Kenya	Pakistan
		Lesotho	Philippines
		Liberia	Singapore
		Madagascar	Sri Lanka
		Malawi	Syria
		Mali	Thailand
		Mauritania	
		Mauritius	
		Morocco	
		Mozambique	
		Namibia	

(*continued on next page*)

Table D2 (*continued*)

Central America	South America	Africa	Asia
		Niger	
		Nigeria	
		Rwanda	
		Senegal	
		Sierra Leone	
		Somalia	
		South Africa	
		Sudan	
		Swaziland	
		Tanzania	
		Togo	
		Tunisia	
		Uganda	
		Zambia	
		Zimbabwe	

Table D3
Growth accounting relative to the frontier (US) in high-growth developing countries 1962–2007.

	$g^{i}_{Y/N} - g^{US}_{Y/N}$	$a^{ST,i} - a^{ST,US}$	$g^{ST,i}_{X} - g^{ST,US}_{X}$	$a^{DS,i} - a^{DS,US}$	$g^{DS,i}_{X} - g^{DS,US}_{X}$	$a^{GD} - a^{GD,US}$	$g^{GD,i}_{X} - g^{GD,US}_{X}$
1962–2007							
Africa							
Ghana	0.8	0.4	0.4	−0.7	1.5	−0.1	0.9
Nigeria	0.0	−0.5	0.5	0.8	−0.8	0.1	−0.1
South Africa	0.3	−0.3	0.6	−1.1	1.4	−0.4	0.7
Asia							
Malaysia	3.5	1.0	2.5	1.1	2.4	1.3	2.2
Taiwan	4.2	1.9	2.3	3.7	0.5	2.4	1.8
Thailand	3.0	1.2	1.8	1.1	1.9	1.2	1.8
South America							
Argentina	−0.7	−0.3	−0.4	−0.9	0.2	−0.3	−0.4
Brazil	1.0	−0.2	1.2	−0.1	1.1	0.1	0.9
Peru	0.0	0.1	−0.1	−1.4	1.4	−0.2	0.2
1962–1982							
Africa							
Ghana	0.4	0.6	−0.2	−1.1	1.5	0.2	0.2
Nigeria	−1.3	−3.0	1.7	−0.6	−0.7	0.0	−1.3
South Africa	0.8	−0.1	0.9	−0.8	1.6	−0.5	1.3
Asia							
Malaysia	4.0	1.1	2.9	2.2	1.8	1.3	2.7
Taiwan	5.4	2.1	3.3	4.7	0.7	2.4	3.0
Thailand	3.6	0.8	2.8	1.5	2.1	1.2	2.4
South America							
Argentina	−1.0	−0.8	−0.2	−1.3	0.3	−0.3	−0.7
Brazil	2.4	0.4	2.0	0.9	1.5	0.0	2.4
Peru	0.0	0.3	−0.3	−1.7	1.7	0.0	0.0
1983–2007							
Africa							
Ghana	1.2	0.2	1.0	−0.3	1.5	−0.4	1.6
Nigeria	1.2	1.8	−0.6	2.0	−0.9	0.2	1.0
South Africa	−0.3	−0.6	0.3	−1.4	1.1	−0.3	0.0
Asia							
Malaysia	3.0	0.9	2.1	0.0	3.0	1.2	1.8
Taiwan	3.1	1.8	1.3	2.6	0.5	2.4	0.7
Thailand	2.4	1.6	0.8	0.7	1.7	1.1	1.3
South America							
Argentina	−0.3	0.1	−0.4	−0.6	0.3	−0.3	0.0
Brazil	−0.3	−0.6	0.3	−1.1	0.8	0.3	−0.6
Peru	0.1	0.0	0.2	−1.0	1.1	−0.5	0.6

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