

Linear Rational Expectations Models with Lagged Expectations: A Synthetic Method[☆]

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Abstract

This paper contains a solution and an estimation method for linear rational-expectations models with lagged expectations. The solution method is a synthetic approach, combining state-space and infinite-MA representations with a simple system of linear equations. The advantage lies in the particular combination of methods from the literature, providing faster execution, more general applicability, and more straightforward usage than existing algorithms. Bayesian estimation methods are employed without the Kalman filter using a recursive algorithm to evaluate the likelihood function and are used to compare small-scale sticky-information and sticky-price DSGE models. Standard truncation methods are shown to not generally be innocuous.

JEL classification: C32, C63, E37

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1. Introduction

This paper presents a method for solving and estimating linear rational-expectations models with lagged expectations. Though the method itself contains little novelty, it contributes to the literature by combining several different methods established in the literature into one coherent approach. The resulting algorithm performs at least as well as each individual method while maintaining generality. The freely-available software¹ strives to minimize computing and preprocessing time. I estimate simple sticky-information and sticky-price models using Bayesian methods, evaluating the likelihood function with an alternative to the Kalman filter. Two new contributions of this paper are the explicit consideration of models with infinite lagged expectations and the examination of truncation methods from the literature for such cases. The method and software should be of special interest to those interested in sticky information à la Mankiw and Reis (2002).

The solution method starts with the method of Taylor (1986), analyzing an infinite moving average solution. The undetermined coefficients approach yields a deterministic nonautonomous system of difference equations. After the largest expectational lag has been included, the system of difference equations becomes autonomous. Standard algorithms for solving potentially singular systems of difference equations are employed for the coefficients thereafter. Using the infinite moving average solution, the method will provide the unique, stable solution of the problem should it exist. The software provides the option of using the QZ method of Klein (2000) or the shuffle and eigenvalue method of Anderson and Moore (1985). The remaining coefficients are then determined by solving a block version of Mankiw and Reis's (2007) tridiagonal system. This particular synthesis eliminates the need for any manual reformulation and provides a computationally efficient algorithm that draws on preexisting algorithms with established properties.

For models with an infinite number of lagged expectations, e.g., models with the sticky-information Phillips Curve of Mankiw and Reis (2002), the method of this paper uses an explicit convergence criterion to determine when and how to truncate. This is advantageous as current analyses of models containing infinite lagged expectations generally truncate either arbitrarily or through a process of trial and error. The appropriate

¹MATLAB[®] software and examples are available on the author's website.

truncation point will depend not only on the specific model, but also on the specific choice of parameter values. Using an arbitrary truncation point can provide potentially misleading results and using the truncation point derived from one particular parameter combination is unlikely to be suitable when parameter values are changed.

I estimate a simple New Keynesian model with Bayesian likelihood methods, comparing a sticky-information Phillips curve with its sticky-price equivalent. I treat the entire sample as a single draw from a multivariate normal distribution and obtain the covariance matrix using spectral methods. Evaluating the likelihood function requires the determinant and inverse of this matrix, which are calculated recursively using Akaike's (1973) Levinson method for block Toeplitz matrices. I am able to avoid the use of the Kalman filter, which is desirable due to the potentially prohibitive size of the underlying state space when many lagged expectations are present. A similar Levinson algorithm, familiar in the time series literature for the solution of the Yule-Walker equations and other aspects of ARMA estimation, (see Moretting, 1984, for a review) was also used by Leeper and Sims (1994, p. 99) to evaluate the likelihood function. The resulting estimates show that the sticky-information model can fare favorably in comparison with the standard sticky-price model, especially in reproducing the empirical lead of the output gap over inflation, and that arbitrary truncation can reverse this assessment.

Many solution methods for solving linear rational expectation models can be found in the literature. For the analysis here, they can be split into two groups: those that explicitly allow for lagged expectations and those that do not. An incomplete list of the latter includes Blanchard and Kahn (1980), McCallum (1983), Anderson and Moore (1985), Binder and Pesaran (1995), King and Watson (1998), Uhlig (1999), Anderson (Forthcoming), Klein (2000), Sims (2001), and Christiano (2002). Although these methods can solve models with a finite number of lagged expectations, this requires the manual definition of dummy variables, see Binder and Pesaran (1995) or Sims (2001), to bring the system into canonical form.² The disadvantage is twofold. Firstly, defining dummy variables is tedious and prone to user error. Secondly, the computational burden from the increased number of variables can become prohibitive.

²Christiano (2002, p. 23) does allow for the information set to vary across equations, but to have varying information sets within one equation, dummy definitions would still be necessary.

There are several solution methods that operate directly with lagged expectations. Taylor (1986) analyzes solutions that take the form of an infinite moving average and Mankiw and Reis (2002) demonstrate how this solution form can be applied to models with lagged expectations in the absence of forward-looking variables. Zadrozny (1998) provides a general method for solving systems with a finite number of lagged expectations, but the absence of a software implementation, as noted by Anderson (2008, p. 96), would require substantial work on behalf of the modeler to use his method. Wang and Wen's (2006) method solves models with lagged expectations by combining standard state-space techniques with a fixed-point approach for expectational errors. Requiring the modeler to manually reformulate lagged expectations as expectational errors reintroduces the potential for user error. Their fixed-point approach is unnecessarily complicated and computationally burdensome. Finally, Mankiw and Reis (2007) provide a method that works entirely on the infinite moving average representation. By reducing the system of equations to a scalar second-order nonautonomous difference equation and imposing a boundary condition at a finite horizon, they reduce the problem to solving a tridiagonal system. While the method could be altered to avoid the reformulation into a scalar system, it is unclear how and when the boundary conditions for a vector of variables should be imposed in more general settings. None of these methods give an explicit criterion for how to proceed when lagged expectations reach back into the infinite past.

The paper is organized as follows. Section 2 presents the model to be analyzed. Section 3 derives the solution method. Section 4 examines the dangers associated with truncations. Section 5 compares the method and its performance with alternate methods. Section 6 presents the method used for estimation and Section 7 examines the importance of lagged expectations using estimated sticky-information and sticky-price New Keynesian models. Finally, Section 8 concludes.

2. Statement of the Problem

Log-linearized economic models can typically be represented by a system of linear expectational difference equations:

$$\begin{aligned}
0 = & \sum_{i=0}^I A_i E_{t-i} [Y_{t+1}] + \sum_{i=0}^I B_i E_{t-i} [Y_t] + \sum_{i=0}^I C_i E_{t-i} [Y_{t-1}] \\
& + \sum_{i=0}^I F_i E_{t-i} [W_{t+1}] + \sum_{i=0}^I G_i E_{t-i} [W_t]
\end{aligned} \tag{1}$$

$$W_t = \sum_{j=0}^{\infty} N_j \epsilon_{t-j}, \quad \epsilon_t \sim i.i.d. \mathcal{N}(0, \Omega) \tag{2}$$

$$\lim_{j \rightarrow \infty} \xi^{-j} E_t [Y_{t+j}] = 0, \quad \forall \xi \in \mathbb{R} \text{ s.t. } \xi > g^u, \text{ where } g^u \geq 1 \tag{3}$$

where Y_t is a $k \times 1$ vector of endogenous variables of interest, W_t an $n \times 1$ vector of exogenous processes stable with respect to ξ^3 and with given moving average coefficients $\{N_j\}_{j=0}^{\infty}$, and where $I \in \mathbb{N}_0$. It is assumed that there are as many equations, k , as endogenous variables in (1). Variables dated t are in the information set dated t .

Equation (1) represents the collection of log-linearized equilibrium equations. Equation (2) specifies the exogenous process W_t as a vector $MA(\infty)$ process. Equation (3) may be interpreted as a transversality condition derived as a condition from intertemporal maximization, where g^u is the maximal growth rate of endogenous variables (see Sims (2001) or Burmeister (1980) for discussions on the limitations of this interpretation).

3. Solution of the Problem

In the following, I shall differentiate between three cases: (i) $I = 0$, (ii) $0 < I < \infty$, and (iii) $I \rightarrow \infty$. The distinction between the first two serves to compare the solution here with methods in the literature for standard (i.e., without lagged expectations) formulations. The infinite case will need to be handled separately and provides a justification and criterion for appropriate truncations.

³E.g., for $g^u = 1$, unit roots in both the endogenous and exogenous variables are allowed.

Following Muth (1961) and Taylor (1986), the solution will take the form

$$Y_t = \sum_{j=0}^{\infty} \Theta_j \epsilon_{t-j}. \quad (4)$$

Inserting (4) and (2) into (1) yields

$$\begin{aligned} 0 = & \sum_{j=0}^{\infty} \left(\sum_{i=0}^{\min(I,j)} A_i \right) \Theta_{j+1} \epsilon_{t-j} + \sum_{j=0}^{\infty} \left(\sum_{i=0}^{\min(I,j)} B_i \right) \Theta_j \epsilon_{t-j} \\ & + \sum_{j=0}^{\infty} \left(\sum_{i=0}^{\min(I,j+1)} C_i \right) \Theta_j \epsilon_{t-j-1} + \sum_{j=0}^{\infty} \left(\sum_{i=0}^{\min(I,j)} F_i \right) N_{j+1} \epsilon_{t-j} \\ & + \sum_{j=0}^{\infty} \left(\sum_{i=0}^{\min(I,j)} G_i \right) N_j \epsilon_{t-j} \end{aligned} \quad (5)$$

Defining $\tilde{M}_j = \sum_{i=0}^{\min(I,j)} M_i$, for $M = A, B, C, F, G$, one can rewrite the foregoing as

$$\begin{aligned} 0 = & \sum_{j=0}^{\infty} \tilde{A}_j \Theta_{j+1} \epsilon_{t-j} + \sum_{j=0}^{\infty} \tilde{B}_j \Theta_j \epsilon_{t-j} + \sum_{j=0}^{\infty} \tilde{C}_{j+1} \Theta_j \epsilon_{t-j-1} \\ & + \sum_{j=0}^{\infty} \tilde{F}_j N_{j+1} \epsilon_{t-j} + \sum_{j=0}^{\infty} \tilde{G}_j N_j \epsilon_{t-j} \end{aligned} \quad (6)$$

Comparing coefficients, this yields the non-stochastic linear recursion

$$0 = \tilde{A}_j \Theta_{j+1} + \tilde{B}_j \Theta_j + \tilde{C}_j \Theta_{j-1} + \tilde{F}_j N_{j+1} + \tilde{G}_j N_j \quad (7)$$

with initial conditions, $\Theta_{-1} = 0$, and terminal conditions from (3), $\lim_{j \rightarrow \infty} \xi^{-j} \Theta_j = 0$. The initial conditions require Y_{t-1} to be independent of ϵ_t (i.e. variables from yesterday cannot respond to innovations today), leaving an additional k restrictions for the terminal conditions to determine.

3.1. Case 1: $I = 0$

This is the standard case without lagged expectations. Here $\tilde{M}_j = M_0$, for $M = A, B, C, F, G$ and thus (7) reduces to a recursion with constant coefficients

$$0 = A_0\Theta_{j+1} + B_0\Theta_j + C_0\Theta_{j-1} + F_0N_{j+1} + G_0N_j \quad (8)$$

This system of deterministic difference equations can be solved using standard methods. In Appendix A, I follow Klein (2000) and note how his approach can be adapted to the deterministic and potentially non-stationary system.

The QZ method of Klein (2000) or Anderson and Moore's (1985) AIM method yields

$$\Theta_j = \alpha\Theta_{j-1} + \beta\Xi_j^u \quad (9)$$

a recursive form, along with the initial conditions, for the MA-coefficients of Y_t .

Following Blanchard and Kahn (1980), the solution will be unique if the system has exactly k eigenvalues greater than g^u in modulus, non-unique if there are more than k such eigenvalues, and non-existent if there are fewer than k such eigenvalues. With exactly k eigenvalues greater than g^u in modulus, the terminal conditions (2) should provide the missing k linear restrictions needed to complete the recursion.⁴

The algorithm uses standard recursive methods for potentially singular difference equation systems. Whereas current standard methods (e.g., Uhlig (1999) or Klein (2000)) solve for a recursive form for the endogenous variables themselves, the solution form here yields a recursive form for the infinite MA coefficients, following the representations proposed by Muth (1961) and Taylor (1986). This approach transforms the stochastic system of difference equations into a deterministic system in the impulse responses of endogenous variables to exogenous shocks ϵ_t . Both non-existence and non-uniqueness of the fundamental solution will be indicated by the non-existence or non-uniqueness of the stable manifold. The algorithm is mute on the form of the solution(s), should the MA representation be non-unique or non-existent. The infinite MA representation and

⁴A caveat is noted by Klein (2000): the explosive eigenvalues have to be "translatable" to the missing initial conditions. If the upper-left $k \times k$ block of Z from the QZ decomposition in Appendix A is invertible, however, this will not be a problem.

associated deterministic system of difference equations avoids any expansion of the state space (see Mankiw and Reis, 2002, p. 1325) and still admits the use of standard methods when lagged expectations are present. The following section will develop this case.

3.2. Case 2: $0 < I < \infty$

This is a deviation of the standard case examined in the literature. Here $\tilde{M}_j = \tilde{M}_I$, for $M = A, B, C, F, G$, and $\forall j \geq I$ and thus (7) reduces to a recursion with constant coefficients $\forall j \geq I$.

$$0 = \tilde{A}_I \Theta_{j+1} + \tilde{B}_I \Theta_j + \tilde{C}_I \Theta_{j-1} + \tilde{F}_I N_{j+1} + \tilde{G}_I N_j \quad (10)$$

Analogously to the foregoing section, if $s = k$ and if the k restrictions can be associated with the “missing” initial conditions,

$$\Theta_j = \alpha(I) \Theta_{j-1} + \beta(I) \Xi_j^u(I), \quad \forall j \geq I \quad (11)$$

a recursive solution for all MA-coefficients from I onward.⁵

The remaining coefficients can then be obtained as the solutions to

$$\begin{bmatrix} \tilde{B}_0 & \tilde{A}_0 & 0 & \dots & 0 \\ \tilde{C}_1 & \tilde{B}_1 & \tilde{A}_1 & 0 & \dots & 0 \\ 0 & \tilde{C}_2 & \tilde{B}_2 & \tilde{A}_2 & 0 & \dots & 0 \\ \vdots & & & & & & \vdots \\ 0 & \dots & 0 & \tilde{C}_{I-1} & \tilde{B}_{I-1} & \tilde{A}_{I-1} \\ 0 & \dots & 0 & -\alpha(I) & I \end{bmatrix} \begin{bmatrix} \Theta_0 \\ \Theta_1 \\ \Theta_2 \\ \vdots \\ \Theta_{I-1} \\ \Theta_I \end{bmatrix} = \begin{bmatrix} -\tilde{F}_0 N_1 - \tilde{G}_0 N_0 \\ -\tilde{F}_1 N_2 - \tilde{G}_1 N_1 \\ -\tilde{F}_2 N_3 - \tilde{G}_2 N_2 \\ \vdots \\ -\tilde{F}_{I-1} N_I - \tilde{G}_{I-1} N_{I-1} \\ \beta(I) \Xi_I^u(I) \end{bmatrix} \quad (12)$$

The left-hand side is a block extension of Mankiw and Reis’s (2007) tridiagonal structure, readily exploitable numerically. (See Golub and van Loan, 1989, p. 170)

As in the case when $I = 0$, the method here provides a linear recursion for the infinite MA coefficients for $j \geq I$. So long as I is finite, the inclusion of lagged expectations

⁵Where $\Xi_j^u(I)$ is given by (A.2), see Appendix A to compare with the solution from Section 3.1.

extends standard solution methods by a sparse system of equations for all coefficients up to I . Standard state-space methods, however, would extend the state space with dummy variables to capture the effects of lagged expectations.

3.3. Case 3: $I \rightarrow \infty$

Unlike the previous two cases, (7) cannot be reduced to a linear recursion with constant coefficients for $j \geq I$. Assuming that (where l and m denote row and column)

$$\lim_{j \rightarrow \infty} \left(\tilde{M}_j \right)_{l,m} = \left(\tilde{M}_\infty \right)_{l,m}, \text{ for } M = A, B, C, F, G; \text{ and } \forall l, m \quad (13)$$

exists and is finite, then there exists, by the definition of a limit in \mathbb{R}^1 , some $I(\delta)_{M,l,m}$ for each M, l , and m , such that

$$\forall \delta > 0, \exists I(\delta)_{M,l,m} \text{ s.t. } n > I(\delta)_{M,l,m} \Rightarrow \left| \left(\tilde{M}_n \right)_{l,m} - \left(\tilde{M}_\infty \right)_{l,m} \right| < \delta \quad (14)$$

and, thus, there exists some maximal $I(\delta)_{max} = \max\{I(\delta)_{M,l,m}\}$ such that

$$\forall \delta > 0, \exists I(\delta)_{max} \text{ s.t. } n > I(\delta)_{max} \Rightarrow \left| \left(\tilde{M}_n \right)_{l,m} - \left(\tilde{M}_\infty \right)_{l,m} \right| < \delta; \forall M, l, m \quad (15)$$

Therefore, (7) can be approximated as

$$0 = \tilde{A}_j \Theta_{j+1} + \tilde{B}_j \Theta_j + \tilde{C}_j \Theta_{j-1} + \tilde{F}_j N_{j+1} + \tilde{G}_j N_j, \quad 0 \leq j < I(\delta)_{max} \quad (16)$$

$$0 = \tilde{A}_\infty \Theta_{j+1} + \tilde{B}_\infty \Theta_j + \tilde{C}_\infty \Theta_{j-1} + \tilde{F}_\infty N_{j+1} + \tilde{G}_\infty N_j, \quad j \geq I(\delta)_{max} \quad (17)$$

This system is now analogous to the system in the foregoing section where I now equals $I(\delta)_{max}$ and can be solved using the methods presented there.

The main distinction is that the autonomous recursion is defined by the limiting coefficients ($I \rightarrow \infty$) and not by the finite $I = I(\delta)_{max}$ coefficients. As the behavior of the system in the limit is decisive for the application of (3) to ascertain whether additional restrictions to determine the system exist, the use of coefficients other than those of the limiting case can produce spurious results regarding asymptotic stability.

The existence and uniqueness of the stable manifold now depends on the eigenvalues of the system of these limiting coefficients. The assumption of element-wise limits in the

coefficient matrices rules out the possibility of asymptotically periodic coefficients and ensures that any desired degree of accuracy, through an appropriate choice of δ , can be achieved without endangering the asymptotic behavior of the recursion.

3.4. A Recursive Law of Motion

For a recursive law of motion, the infinite MA solution can be rewritten as

$$Y_t = \sum_{j=0}^{\infty} \Theta_j \epsilon_{t-j} = \sum_{j=0}^{I-1} \Theta_j \epsilon_{t-j} + \sum_{j=I}^{\infty} \Theta_j \epsilon_{t-j} \quad (18)$$

Assuming the MA coefficients of the exogenous process W_t follow the simple recursion $N_{j+1} = NN_j$ with all eigenvalues of N less than or equal to g^u ,⁶ a recursive law of motion can be derived as all MA coefficients after I follow an autonomous recursion.⁷ From equations (11) and (A.2), as well as Klein (2000, p. 1423),

$$\Theta_j = \alpha(I)\Theta_{j-1} + \beta(I)M(I)N^j, \quad j \geq I \quad (19)$$

where $vec(M(I)) = (N' \otimes S_{22}(I) - I \otimes T_{22}(I))^{-1} vec \left(Q_2(I) \begin{bmatrix} \tilde{F}_I N + \tilde{G}_I \\ 0 \end{bmatrix} \right)$.

Defining $U_t = \sum_{j=I}^{\infty} \Theta_j \epsilon_{t+I-j}$, the solution with a VAR(1) exogenous process is

$$\begin{aligned} Y_t &= \sum_{j=0}^{I-1} \Theta_j \epsilon_{t-j} + U_{t-I} \\ U_{t-I} &= \alpha(I)(U_{t-I-1} + \Theta_{I-1} \epsilon_{t-I}) + \beta(I)M(I)N^I W_{t-I} \\ W_t &= NW_{t-1} + \epsilon_t \end{aligned} \quad (20)$$

with U_{t-I} being the same as $E_{t-I}[Y_t]$. Note that if $I = 0$, the foregoing reduces to

$$\begin{aligned} Y_t &= \alpha(0)Y_{t-1} + \beta(0)M(0)W_t \\ W_t &= NW_{t-1} + \epsilon_t \end{aligned} \quad (21)$$

a standard form for the recursive law of motion.

⁶I.e., W_t is a stable VAR(1) process.

⁷Recalling from the foregoing section that $I = I(\delta)_{max}$ with infinitely lagged expectations.

4. The Perils of Premature Truncation

Solving linear rational expectations models when many lagged expectations appear in the structural equations generally entails a truncation of the number of lagged expectations included in the model. In this section, I shall explore the implications of two methods of truncation to show that truncation is not generally innocuous.

The sticky information model of Mankiw and Reis (2002), incorporating an infinite sum of lagged expectations, has presented the literature with an alternative to the sticky-price Phillips curve. Andrés et al. (2005), Keen (2007), and Wang and Wen (2006) are a few examples of models that combine forward-looking agents and an infinity of lagged expectations: all of them truncate this infinity with the truncation point ranging from 3, Andrés et al. (2005), to 50, Wang and Wen (2006). Kiley (2007, p. 112) compares sticky prices and sticky information empirically and notes, “[i]n practice, the longest information lag is truncated at four quarters.” Using a truncated version to draw inference on the infinite specification requires that the former yield results that do not differ substantially from the latter. The methods in the foregoing section allow for a clear picture (to machine precision) of the infinite specification, allowing for the assessment of the generally arbitrary truncations found in the literature. These arbitrary truncations can distort the dynamics of the model, even to an extent apparent by visual inspection.

The sticky-information Phillips curve of Mankiw and Reis (2002) is⁸

$$\pi_t = \frac{\lambda\alpha}{1-\lambda}y_t + \lambda \sum_{j=0}^{\infty} (1-\lambda)^j E_{t-j-1} [\pi_t + \alpha\Delta y_t] \quad (22)$$

with π_t , Δy_t , and y_t being the gross inflation rate, the growth of the output gap, and the output gap itself. Equation (22) is the sticky-information Phillips curve and, as it is the only equation here to contain lagged expectations, will be the focus of the examination of the consequences of truncation. Note that this model does not contain any forward-looking variables. As Mankiw and Reis (2002) show, this allows the MA coefficients to be solved for directly. This yields an analytical solution that facilitates comparison of truncation methods.

⁸The system is closed by an AR(1) process for the growth of money and the quantity equation in first-difference form.

Andrés et al. (2005, p. 1033) note that to make the model tractable, “[they] approximate it by truncating [lagged expectations in the Phillips curve] at three quarters.”⁹ Using this truncation¹⁰ alters equation (22) to

$$\pi_t = \frac{\lambda\alpha}{1-\lambda}y_t + \lambda \sum_{j=0}^3 (1-\lambda)^j E_{t-j-1} [\pi_t + \alpha\Delta y_t] \quad (23)$$

Kiley (2007, p. 112) follows a different truncation technique and states, “the probabilities of information arrival are constant in each period up to the truncation period, with the remaining mass of the probability distribution placed on the last period.” Following this truncation, equation (22) is rewritten as

$$\pi_t = \frac{\lambda\alpha}{1-\lambda}y_t + \lambda \left(\sum_{j=0}^2 (1-\lambda)^j E_{t-j-1} [\pi_t + \alpha\Delta y_t] + \frac{(1-\lambda)^3}{\lambda} E_{t-4} [\pi_t + \alpha\Delta y_t] \right) \quad (24)$$

[INSERT FIGURE 1 HERE]

Figure 1, clockwise from the upper-left panel, shows the impulse responses of inflation to a negative shock to the money growth rate, the impulse responses of the output gap to the same, the crosscorrelations of the output gap with contemporary inflation, and the autocorrelation of inflation for the two approximations and the original specification of Mankiw and Reis (2002).¹¹ As the model does not contain any forward-looking behavior, the initial responses of inflation and the output gap are the same in all three versions. The second truncation, equation (24), displays a sharp jump in the response of inflation four periods after the shock, due to the large weight attached to lagged expectations at this horizon. Neither of the truncations can reproduce the maximal response of inflation at seven quarters. The impulse response of the output gap shows the transition of the rate of convergence of the output gap from the first truncation, equation (23), to the

⁹Note that Andrés et al. (2005, p. 1038) interpret the parameter for the probability of the arrival of new information according to the non-truncated version and concludes that its estimated value “leads to an average duration slightly higher than six quarters” despite their truncation point.

¹⁰I extend the truncation point to four quarters as in Kiley (2007) for comparability.

¹¹The solutions in this section not labeled as a truncation are implemented using the method developed here with δ (see Section 3.3) set to floating point accuracy. This level of tolerance implies that the computer is no longer capable of distinguishing between the autonomous recursion from the limiting coefficients and the nonautonomous recursion continued past $I(\delta)_{max}$, see equation (16). As an anonymous referee pointed out, the results derived thusly for Mankiw and Reis’s (2002) model are indistinguishable from their original results.

second. The second truncation underestimates the crosscorrelation of the output gap with inflation and the autocorrelation of the latter. The first specification matches the autocorrelation of inflation within the displayed horizon remarkably well, though it misses the horizon of the lead of the output gap in the crosscorrelation.

When forward-looking behavior is added to the model, responses will depend on future trajectories. The first example of Wang and Wen (2006) will serve to illustrate the issue. They present a simple model with sticky information and monopolistic competition on the supply side (leading to a sticky-information Phillips curve) and capital accumulation, a cash-in-advance constraint, bond holdings, and labor and consumption decisions maximized intertemporally on the demand side.

[INSERT FIGURE 2 HERE]

Using the truncations presented above, the first row of Figure 2 shows the impulse responses of inflation and marginal costs, replacing the output gap in (22), to a positive unit innovation in the money growth rate. Both truncation methods¹² fail to match the peak response of inflation, with responses differing now both before and after truncation. The impulse responses of marginal costs demonstrate similar short-comings.

Wang and Wen (2006, p. 10) note, for their sticky-information model, that a truncation point of 20 provides “very good results.” Trabandt (2007) draws the same conclusion in his model - though with an explicit convergence criterion, see Section 5. There is, of course, nothing intrinsically special about a truncation point of 20. The second row of Figure 2 shows the consequences of the two truncation methods with truncation points 20 for the impulse response of inflation to a shock in money growth in the sticky-information model of Wang and Wen (2006) when θ ¹³ is increased from 0.8 to 0.9, Figure 2c, and to 0.95, Figure 2d. Both truncations miss the peak response of inflation and either over- or underestimate the autocorrelation of inflation.

Mankiw and Reis (2007) analyze a DSGE model with pervasive inattention. Their aggregate supply curve is given in terms of the price level instead of inflation:

$$p_t = \lambda \sum_{j=0}^{\infty} (1 - \lambda)^j E_{t-j} [p_t + x_t] \quad (25)$$

¹²Again, truncation is imposed at four quarters.

¹³Their equivalent to $1 - \lambda$ from (22) - the probability of not receiving an information update.

where p_t is the price level and x_t is a composite term comprising real marginal costs and desired markups. In their appendix, they show that the price level displays unit-root behavior in the limit. Figure 3 shows the impulse response of the price level to a shock in aggregate demand for the two truncation types with differing truncation points. Notice that first type of truncation fails to deliver the unit root, whereas the second type does. Examining (25), if the infinite sum is simply truncated (i.e., type one truncation) without correcting for the remaining probability mass (i.e., type two truncation), the supply curve would imply a long-run relation between the price level and the composite term. As both real marginal costs and desired markups are stationary, this forces the price level itself to become stationary. Using type two truncation delivers the unit root in prices regardless of the truncation point, as the limiting coefficients which contain the unit root are imposed after truncation. Premature truncation with the type two method, however, can still lead to an erroneous new steady state of the price level.

[INSERT FIGURE 3 HERE]

Different truncation methods can have different consequences, which themselves might differ when applied to different models or parameter settings. Both of the truncation methods presented above, as found in the literature, will converge to the true model as the truncation point is extended towards infinity. Knowing a priori when and how to truncate would seem difficult to ascertain. The method presented in this paper automatically calculates the truncation point given the tolerance parameter δ . As the setting changes, so too will the truncation point, eliminating the need for arbitrary truncations or processes of trial and error to deliver a clear picture of the model's dynamics.

5. Comparison of Solution Methods

In this section, I compare the solution method presented in Section 3 with three alternate methods in the literature for solving models with lagged expectations. The method dominates all three alternative solution methods in terms of computation time and/or implementation time on behalf of the user for given error tolerances. In the absence of lagged expectations, the method here collapses to the method of Klein (2000)¹⁴ and a comparison of its performance with other methods can be found in Anderson (2008).

¹⁴Or of Anderson and Moore (1985) if the option to solve using AIM is chosen.

For the comparison that follows, computation times and relative errors, following Golub and van Loan (1989, p. 54) and using the Euclidean norm, of impulse responses between the alternative solution methods and the method here are reported. Trabandt (2007) provides computation times but his software is not publicly available, so the comparison must restrict itself to comparing computation time across platforms. For both Wang and Wen (2006) and Mankiw and Reis (2007), software is publicly available from the authors' websites and is used to compare computation times and calculated impulse responses on the same platform.¹⁵

Trabandt (2007) uses the QZ implementation of Uhlig (1999) to solve a model with sticky information by expanding the state vector. That is, a variable $E_{t-1}[x_t]$ is modeled by defining $x_{t-1}^1 = E_{t-1}[x_t]$ and adding the additional equation $x_t^1 = E_t[x_{t+1}]$. While this method has the advantage of using standard methods, it requires the definition of an ever-increasing state vector by including additional variables and equations. This increases not only the dimension of the problem but also the programming time, as these lagged expectations must be manually redefined through additional variables. Notably, Trabandt's (2007) method is the only method other than the one presented here that works with an explicit convergence criterion for imposing truncation. The criterion adds additional lagged expectations one-by-one, re-solving the model until the solution does not change more than a specified tolerance. While preferable to an arbitrary truncation, this is computationally expensive compared with the criterion in Section 3.3, which determines the truncation point before solving the model, and it is not clear whether his method can be applied in other settings. Trabandt (2007, p. 18) requires three minutes to solve his model with twenty lagged expectations included; the method here requires about one-and-a-half hundredths of a second to do the same.

The computational disadvantage of methods based on state-vector expansion is due to the computation costs associated with a QZ decomposition, a function of the cube of the state vector. (Golub and van Loan, 1989, p. 404) Anderson and Moore's (1985) AIM method presents an alternative method and their method generally entails significant reductions in computation times. (Anderson, 2008, p. 102) Mathias Trabandt¹⁶ notes

¹⁵Platform used: Pentium® IV 3 GHz machine with 2 GB of RAM running MATLAB® version R2007a under Windows® XP 2002 SP 2

¹⁶Personal communication

that using the AIM method reduces his computation time to 1.75 seconds. Though an improvement, this is still more than two orders of magnitude slower than the method developed here. One advantage of the method presented in this paper, likewise of Wang and Wen (2006) and Mankiw and Reis (2007), comes from its division of the problem into an autonomous and a nonautonomous part. The dimensions of the autonomous recursion from Section 3.3 is invariant to the inclusion of lagged expectations.

Wang and Wen (2006) present a method for solving linear rational expectations models with lagged expectations that is very similar to the solution presented here in several ways. In contrast to the method here, the authors work directly with a recursion in state variables and solve for the forecast errors induced by lagged expectations. By approximating models with lagged expectations reaching back into the infinite past with a finite number of forecast errors, Wang and Wen (2006) impose the same condition that is imposed in the method presented here.¹⁷ The method in their paper, however, requires the modeler to reformulate lagged expectations into expectation errors, opening a window for user error. Furthermore, the combination of the recursion in state variables with forecast errors poses a more complicated fix-point problem than the tridiagonal problems posed by Mankiw and Reis (2007) working with the innovations representation. The fix-point problem seems to limit Wang and Wen’s (2006) method in terms of accuracy and is also a likely culprit for the rather excessive increase in computation time as $nlag$, the authors’ parameter for the number of lagged expectations included, is increased.

[INSERT FIGURE 4 HERE]

In Figure 4a, for varying values of δ (see Section 3.3) and $nlag$, the computation time and relative errors associated with the first example in Wang and Wen’s (2006) paper are compared. The relative errors contrast the impulse responses of the model’s variables to a shock to the rate of money growth with varying truncation points against the impulses obtained when using δ equal to floating-point accuracy and $nlag = 252$. This value of $nlag$ is used as Wang and Wen’s (2006) method required more than one hour with this value to calculate the solution. As can be seen in the figure, the method proposed here solves the model for a given relative error at least 100 times more quickly

¹⁷As pointed out by an anonymous referee, the software of Wang and Wen (2006) uses a different method than presented in their paper, truncating the lagged expectations themselves rather than the expectation errors.

than the method of Wang and Wen (2006).

The methods proposed by Trabandt (2007) and Wang and Wen (2006) differ subtly. Wang and Wen (2006) propose a solution for a finite number of lagged expectation errors, whereas Trabandt's (2007) algorithm applies to a finite number of lagged expectations. When a truncation is necessary, Trabandt's (2007) solution is equivalent to (23) and Wang and Wen's (2006) to (24).¹⁸ Truncating the lagged expectations themselves does not preserve the asymptotic qualities of the recursion. The methods proposed by Wang and Wen (2006) and implemented by this paper, by contrast, do preserve these asymptotic qualities. Although, in the limiting case when the truncation points go to infinity, all three approaches are theoretically equivalent, a method that preserves these asymptotic qualities would seem more appropriate for numerical application.

Mankiw and Reis (2007) develop a solution method from the MA representation that differs in two major respects from the method presented here. Firstly, they reduce the problem to a univariate second-order nonautonomous difference equation. Secondly, Mankiw and Reis (2007) solve the model by imposing a boundary condition prematurely (i.e. in finite time). Reducing the system to a scalar system requires considerable work on behalf of the modeler and, as such, is liable to user error. Furthermore, it is not clear that every model can be reduced to a scalar second-order difference equation and absent such a reduction it is not obvious that all the boundary conditions ought to be imposed at the same point. The method developed here neither requires manual reduction nor does it impose a univariate structure. Both the method of Mankiw and Reis (2007) and the method here exploit readily available and fast implementations of Gaussian elimination to solve a (block) tridiagonal system. That the autonomous recursion consistent with the limiting coefficients is imposed instead of the boundary conditions themselves allows fewer nonautonomous coefficients to be added to achieve a given relative error.

In Figure 4b, for varying values of δ (see Section 3.3) and N , the authors' parameter for the number of MA coefficients included before the boundary conditions are imposed, the computation time and relative errors associated with solving the model in Mankiw and Reis (2007) are compared. The method of this paper entirely avoids the several

¹⁸The method actually implemented by Wang and Wen (2006) in their software, however, is equivalent to (23), see footnote 17.

pages of “tedious algebra” from the technical appendix of Mankiw and Reis (2007) to arrive at their solution. That Mankiw and Reis’s (2007) method solves the model more quickly than the method presented here for large relative errors is likely due to the initial fixed costs of the higher level of generality of the method here. The method presented in this paper, however, requires a smaller increase in computational time for a given increase in the level of accuracy; at some level of accuracy, the method here surpasses that of Mankiw and Reis (2007) in terms of computation time. Numerical limitations on the QZ decomposition, as discussed by Anderson (2008, p. 103), can be reduced by solving for the limiting recursion using Anderson and Moore’s (1985) AIM method, as can be seen in Figure 4b. Less than two seconds are needed to solve the model using the convergence criterion $\delta = \text{eps}(0)$, including thrice as many lagged expectations in half as much time. As the method derived here is not model specific and does not require any manual reformulation on behalf of the user, it appears to be at an advantage.

When the model to be analyzed possesses no lagged expectations, the method here fits within the class of solution methods used throughout the literature. For models with lagged expectations, the method derived in this paper is superior to current models with respect to computation and/or implementation times. The method here is non-model-specific and can be readily applied to existing and new DSGE models both with and without lagged expectations efficiently.

6. Likelihood Estimation

The use of Bayesian likelihood methods to explore the unknown posterior distribution of DSGE models’ parameters given the data and the researcher’s prior beliefs has been gaining popularity. (See An and Schorfheide, 2007, for an overview.) The iterative nature of the methods highlights the advantages of having a fast solution method: the solution will need to be recalculated thousands if not millions of times.

One difficulty in implementing likelihood methods, aside from the calculation of a recursive solution as presented in Section 3, lies in the evaluation of the likelihood function. The Kalman filter, frequently used to obtain a prediction error decomposition of the likelihood function, is undesirable here. Using the Kalman filter would require the state space of the recursive laws of motion in Section 3.4 to be expanded to accommodate

a first-order form. The resulting recursion would calculate products and inverses on a dimension equal to the expanded state space. An alternative approach is developed here based on the Toeplitz structure of the covariance of stationary time series to yield an iterative method for evaluating the likelihood function by treating the sample as a single draw from a multivariate normal distribution.¹⁹ It should be noted that as the standard model (i.e., without lagged expectations) is included as a special case of the class of models considered here, the method can be used as an alternative to the Kalman filter for the likelihood estimation of standard DSGE models.²⁰

The recursive laws of motion in Section 3.4 will provide the basis for estimation. The block-Levinson-type algorithm in Appendix B relies on there being a stationary innovations representation of the observables as discussed in Anderson and Moore (1979, Ch. 9), so I restrict the eigenvalues of $\alpha(I)$ and N in the recursive law of motion (20) to lie inside the unit circle. Given this assumption of stationarity and that of $n \geq p$ mean-zero, serially independent, normal innovations ϵ_t , T observations on X_t , a $p \times 1$ linear function of Y_t given by $X_t = HY_t$, are normally distributed with mean zero and non-singular block Toeplitz covariance matrix

$$\Psi = \begin{bmatrix} \Gamma_0 & \Gamma'_1 & \dots & \Gamma'_{T-2} & \Gamma'_{T-1} \\ \Gamma_1 & \Gamma_0 & \dots & \Gamma'_{T-3} & \Gamma'_{T-2} \\ \vdots & & \ddots & & \vdots \\ \Gamma_{T-2} & \Gamma_{T-3} & \dots & \Gamma_0 & \Gamma'_1 \\ \Gamma_{T-1} & \Gamma_{T-2} & \dots & \Gamma_1 & \Gamma_0 \end{bmatrix} \quad (26)$$

where Γ_k is the k th autocovariance matrix given by $E[X_t X'_{t-k}] = E[X_{t+k} X'_t]$. (See Hamilton, 1994, equations 10.2.1 and 10.2.2 on pp. 261-262)

The log-likelihood of a vector of underlying parameters ϑ given the data is thus

$$\mathcal{L}(\vartheta|X) = -0.5pT \ln(2\pi) - 0.5 \ln(\det(\Psi(\vartheta))) - 0.5 X' \Psi(\vartheta)^{-1} X \quad (27)$$

¹⁹Similarly to Leeper and Sims (1994), though the authors truncate their calculations. Additionally, details on the algorithm are provided here to make it more accessible to the literature.

²⁰Whether or under what conditions this would actually be desirable remains a subject of future research. In a preliminary work, Schmitt-Grohé and Uribe (2007) express the likelihood of a standard DSGE model in a format compatible with this section - but without guidance as to how it might be evaluated numerically.

where $X = [X_1' X_2' \dots X_T']'$.

If a prior density $\mathcal{P}(\vartheta)$ for the underlying parameters is given, the log of the posterior density of the underlying parameters given the data is

$$\ln(\mathcal{P}(\vartheta|X)) \propto \mathcal{L}(\vartheta|X) + \ln(\mathcal{P}(\vartheta)) \quad (28)$$

Given (26) and $\mathcal{P}(\vartheta)$, only two potentially challenging quantities need to be calculated: $\ln(\det(\Psi(\vartheta)))$ and $X'\Psi(\vartheta)^{-1}X$. Appendix B provides details of the recursive algorithm used to calculate these two quantities. The algorithm incorporates the calculations into Akaike's (1973) iterative method for the inversion of block Toeplitz matrices. Neither Ψ nor its inverse is either stored or calculated in full and the dimensions of the calculations are invariant to the size of the underlying state space.

All that remains, then, is a method of deriving the sequence of autocovariance matrices needed for the likelihood calculations. Under the stationarity assumptions, (20) can be rewritten using the lag operator L as

$$\begin{aligned} Y_t &= \left[\Theta(L) + (I_k - \alpha L)^{-1} \left(\alpha \Theta_{I-1} + \beta M N^I (I_n - N L)^{-1} \right) L^I \right] \epsilon_t \\ W_t &= (I_n - N L)^{-1} \epsilon_t \end{aligned} \quad (29)$$

where $\Theta(L) = \sum_{j=0}^{I-1} \Theta_j L^j$, I_k and I_n are $k \times k$ and $n \times n$ identity matrices, and α , β , and M refer to $\alpha(I)$, $\beta(I)$, and $M(I)$.²¹ Thus, the autocovariance generating function of $X_t = H Y_t$ is given by

$$\begin{aligned} G_X(z) &= H \left[\Theta(z) + (I_k - \alpha z)^{-1} \left(\alpha \Theta_{I-1} + \beta M N^I (I_n - N z)^{-1} \right) z^I \right] \Omega \\ &\quad \times \left[\Theta(z^{-1}) + (I_k - \alpha z^{-1})^{-1} \left(\alpha \Theta_{I-1} + \beta M N^I (I_n - N z^{-1})^{-1} \right) z^{-I} \right]' H' \end{aligned} \quad (30)$$

whence the spectrum (Hamilton, 1994, pp. 268-269) and, through an inverse Fourier transform, the autocovariance matrices can be calculated. (Uhlig, 1999, p. 49)

²¹It is to be understood here and in the following that all matrices are potentially functions of the underlying parameters ϑ .

7. Estimating Sticky Information and Sticky Prices

A simple example will illustrate the estimation method.²² As a baseline, consider the following basic New Keynesian model²³

$$\pi_t = \frac{1-\lambda}{\lambda} (\xi y_t + w_t^{pc}) + (1-\lambda) \sum_{j=0}^{\infty} \lambda^j E_{t-j-1} [\pi_t + \xi \Delta y_t + \Delta w_t^{pc}] \quad (31)$$

$$y_t = E_t [y_{t+1}] + a_1 (R_t - E_t [\pi_{t+1}]) + w_t^{is} \quad (32)$$

$$R_t = \phi_R R_{t-1} + (1 - \phi_R) (\phi_\pi \pi_t + \phi_y y_t) + w_t^{mp} \quad (33)$$

where π_t denotes inflation, y_t the output gap and R_t the nominal interest rate. The first equation is Mankiw and Reis's (2002) sticky-information Phillips curve, the second a standard dynamic IS-curve derived from the Euler equation associated with household intertemporal optimization, and the third a Taylor rule with interest rate smoothing. w_t^{pc} and w_t^{is} are AR(1) processes with persistence parameters ρ_{pc} and ρ_{is} and innovation variances σ_{pc}^2 and σ_{is}^2 . w_t^{mp} is serially uncorrelated with variance σ_{mp}^2 . All three of the exogenous processes are assumed to be mutually independent and normally distributed.

The parameters outside the exogenous processes are λ the probability for price setters of not receiving an information update, ξ the degree of strategic complementarities in price setting, a_1 whose inverse for the purposes here can be interpreted as the coefficient of relative risk aversion, ϕ_R the degree of interest rate smoothing, ϕ_π the degree of inflation targeting, and ϕ_y the degree of output-gap targeting.

Two additional variants of the model will also be estimated. First, a version of (31) truncated according to the first type of truncation in Section 4 after three lagged expectations, following Andrés et al. (2005, p. 1033). Second, a sticky-price model will

²²Comparing estimation methods and results for sticky information from the literature is beyond the scope of this paper- The interested reader is directed to the overview provided by Reis (2009, pp. 18-19).

²³See Trabandt (2007) for a first principles' derivation; here, monetary policy is defined here through control of the nominal interest rate and a markup shock has been added to the Phillips curve.

also be estimated, replacing (31) with the sticky-price Phillips curve

$$\pi_t = \frac{(1-\lambda)(1-\beta\lambda)}{\lambda} (\xi y_t + w_t^{pc}) + \beta E_t [\pi_{t+1}] \quad (34)$$

where λ now refers to the probability for price setters of not being able to update their prices and β the discount factor. (E.g., Woodford, 2003) As the latter is present only in the sticky-price model, it is fixed at 0.99 and not included in the estimation procedure. The baseline version uses the solution method from Section 3.3 with the tolerance parameter δ set to 1e-5 and the truncated variant and sticky-price model will serve the investigation of the importance of sticky information and lagged expectations

The priors used are identical in all specifications, coincide primarily with those found in Smets and Wouters (2007), and can be found in Table 1. λ is centered at 0.5, implying an average information or price update every two quarters; the mean of ξ is set at 0.25, conservative with respect to the value set by Mankiw and Reis (2002) and those discussed by Woodford (2003, Ch. 3), and is restricted to imply strategic complementarities;²⁴ and α_1 is set with mean one (log utility) and a wide variance.

The data used is taken from Smets and Wouters (2005) for the United States for 1970:1-2002:2. The observables are: the effective federal funds rate expressed on a quarterly basis (R_t), 100 times the log difference of the implied deflator of GDP (π_t) and the HP-filtered series of 100 times the log of real GDP divided by the civilian population over 16 (y_t). All series are demeaned.

The estimation procedure follows the Random-Walk Metropolis Algorithm from An and Schorfheide (2007, p. 131) with the likelihood evaluated with the algorithm in Section 6 instead of the Kalman filter. For each model following Mankiw and Reis (2007), I generate 5 chains of 50,000 draws discarding the first 30,000. This gives, for each model, a total of 250,000 draws calculated and 100,000 draws kept and mixed after checking the chains for convergence.²⁵ The truncation point varies in the baseline model with the minimum and maximum over all the draws being 17 and 184 respectively.

²⁴As noted by Keen (2007), this is not a wholly innocuous assumption, as the hump-shaped behavior of inflation in Mankiw and Reis (2002) disappears if price-setting decisions are strategic substitutes. See Woodford (2003, Ch. 3) for a defense of the assumption of strategic complementarities in price setting.

²⁵The chains for the truncation did not appear to have converged. The covariance matrix was recalculated using the 100,000 draws and 5 new chains that indicated convergence were generated.

The algorithms presented here are able to obtain these 250,000 draws in just under 7 hours for the baseline sticky-information model, requiring less than one-tenth of a second to solve the model, calculate the sequence of autocovariance matrices, and calculate the log-likelihood for each draw. These seven hours were split roughly equally between these three operations. Obtaining the draws for the sticky-price model required 5 hours. More computational time is required with lagged expectations, but not prohibitively so.

The differences in terms of computation times presented in Section 5 are starkly highlighted by the estimation example here. From Section 5, the method of, e.g., Wang and Wen (2006) is two orders of magnitude slower than the method of this paper. This would translate to well over a week to replicate the estimation and, as the computation time required by their method increases quickly in the truncation point, likely much longer. Trabandt (2007, p. 22), in a robustness exercise, recalculates his solution for some 5,000 draws from uniform parameter spaces and notes that “somewhat more than two weeks” are required. When facing such computational burdens, the option of truncating after just a few lagged expectations would seem appealing. However, as the comparison of the baseline results with those from the truncated model will show, this can bias the estimates and is, with the methods here, unnecessary.

As was shown in Section 4, the accuracy of a truncation can deteriorate as λ is increased. The methods provided in this paper automatically adjust the truncation point as needed based on the tolerance parameter. Estimating the truncated model with a higher but still fixed truncation point might still be inaccurate for some parameter combination, producing an erroneous likelihood calculation; and might use too high a truncation point for other combinations, unnecessarily burdening the computations.

[INSERT TABLE 1 HERE]

The estimates can be found in Table 1. The data is generally informative about the parameters, the main exception being ξ . The primary differences between the two sticky-information models occur in the estimates in the Phillips curve: λ , ρ_{pc} , and σ_{pc} . This is not at all surprising given the potentially altered persistence of variables due to truncation, as shown in Section 4, and that it is the Phillips curve that is being truncated here. The priors and posteriors under the three specifications can be found in Figure 5 for these parameters. The posterior mean of the truncated model implies that firms,

on average, receive an information update about once every 4 quarters, whereas the estimate in the baseline model is about once every 3 quarters. The posterior distribution of truncated model is skewed to the right with many accepted draws with a high degree of information rigidity; exactly the region for this parameter where the truncation is most likely to give inaccurate results. Truncation induces varying degrees of inaccuracy as different regions are explored by the estimation procedure.

[INSERT FIGURE 5 HERE]

The notable differences in the estimates for the sticky-price model are ρ_{is} and ρ_{pc} , with the sticky-price model placing more persistence into the PC shocks and less into the IS shocks. The priors and posteriors for these parameters are shown in Figure 5. The estimate of λ implies that firms update their prices once every 4 quarters on average.

Overall, the posterior estimates for all three specifications are similar. Relative risk aversion, the inverse of a_1 , is consistently estimated to be about 3.4, between the estimates of, say, Rabanal and Rubio-Ramirez (2005) and Smets and Wouters (2005). The degree of interest-rate smoothing is consistent with the estimates of Smets and Wouters (2005) and Clarida et al. (2000), with the elasticities with respect to inflation and the output gap within standard estimates. The models predict a degree of persistence in the exogenous processes similar to Rabanal and Rubio-Ramirez (2005), but without any of near unit-roots in Smets and Wouters (2005) or Smets and Wouters (2007).

[INSERT FIGURE 6 HERE]

Turning to the second moments, the autocorrelations of inflation, the output gap, and the nominal interest rate can be found in Figure 6. The baseline sticky-information model fairs best, but not appreciably so, in replicating the autocorrelations of inflation and the output gap, but substantially overestimates the autocorrelation of the nominal interest rate. The truncated sticky-information and sticky-price models deliver very similar results in this dimension. In terms of the crosscorrelations, only the baseline sticky-information model replicates the observed lead of the output gap over inflation, with the truncated and sticky-price models failing to generate the hump shape in the crosscorrelelogram. The sticky-price model is the only model that generates the negative correlation between current inflation and future output gaps, but it maintains this prediction at all horizons. All three models replicate the shape of the the crosscorrelelogram of

inflation and the nominal interest rate, but the baseline model overestimates the degree of crosscorrelation throughout. Both the baseline and the truncated sticky-information models do a reasonable job replicating the crosscorrelation of the output gap with the nominal interest rate, but they miss the negative lead of the interest rate over the output gap. The sticky price model predicts much less correlation than is found in the data.

[INSERT FIGURE 7 HERE]

The source of the lead of the output gap over inflation can be seen in Figure 7, the impulse response of inflation to a unit IS shock. In the baseline model, the maximal impact of a demand shock on inflation occurs about 8 quarters after impact.²⁶ With truncation after three lagged expectations, the truncated model is not even capable of creating such a delay despite the higher degree of information persistence implied by its estimates. The posterior estimates put the maximal response on impact of the shock and the upper bound of the credible set displays a sharp peak at the truncation point, a visual cue to a distortive truncation. The internal propagation method of the sticky-information model relies on the moving average terms generated by the lagged expectations. On impact of the shock, those firms receiving an information update set their pricing plans realizing that as time progresses more firms will become aware of the persistent shock - it is the interplay between the shock's persistence and the degree of information rigidity that produces the hump shape. Instead of front-loading future desired price increases, as in the sticky-price model, firms fix a plan for future prices given current information. In the truncated model, current information includes the fact that inflation will begin returning to its zero steady-state at an exponential rate after the truncation point, severely limiting the potential inertia in inflation.

[INSERT FIGURE 8 HERE]

Apart from the IS shock, all three models display qualitatively identical impulse responses; the impulse responses for the baseline model can be found in Figure 8. The maximal response of inflation to a monetary shock occurs on impact. All three models are unable to reproduce the hump-shaped responses to a monetary shock with output leading inflation as found by Christiano et al. (2005). As Coibion (2006) show, the hump-shaped response of inflation to a monetary shock in the sticky-information model found

²⁶The maximal response of the output gap for all three specifications occurs on impact.

by Mankiw and Reis (2002) is sensitive to whether monetary policy is defined over the money supply or the nominal interest rate. In their model with pervasive information stickiness, Mankiw and Reis (2007) find that a monetary shock can produce a hump-shaped response of inflation even under an interest-rate rule. As Reis (2009) illustrates, however, this requires substantial persistence in the monetary shock itself, and a much greater degree of interest-rate smoothing than was estimated would be needed here.²⁷ The failure to reproduce the hump shapes should not be taken as too serious a failure, as Christiano et al. (2005, pp. 2-3) note that wage rigidity is the critical nominal friction and the inclusion of variable capital utilization is crucial for this result - neither of these rigidities are present in the estimated models. Trabandt (2007) notes that Mankiw and Reis's (2002) monetary shock can be interpreted as a nominal income shock and, in that sense, the estimates here confirm the results of Mankiw and Reis (2002), albeit not directly through the monetary shock.

[INSERT TABLE 2 HERE]

By and large, the baseline sticky-information and sticky-price models agree as to the relative contribution of shocks in the forecast variances. The variance decompositions can be found in Table 2 and the PC shock is the primary driver of inflation, the IS shock of the output gap, and the MP shock of the interest rate at lower forecast horizons. Through the high persistence of the PC and IS shocks, the MP shock loses importance as the horizon is increased. In the sticky-price model, the PC shock is more persistent than the IS shock and gains importance relative to the latter at higher horizons for the output gap and the nominal interest rate. In the sticky-information model, the internal propagation of IS shocks in inflation, and thereby in the nominal interest rate, drives the difference to the sticky-price model. At the posterior mean, the IS shock is more important relative to the sticky-price model and its relative importance is increasing as the the horizon is increased. The width of the credible sets indicate, however, that there is a high degree of variability in this internal propagation method with results much closer to those of the sticky-price model contained within the 90% credible set.

Log marginal data densities can be found in the last line of Table 1. Though not

²⁷Though a higher degree of interest-rate smoothing would counteract the momentum in the response to the other shocks. Likewise for the PC shock, a higher degree of persistence in the exogenous process than was estimated would produce a humped-shaped response in inflation.

conclusive, the results are tantalizing, with the ranking of sticky information and sticky prices reversed by truncation. Although the baseline model reproduced the lead of the output gap over inflation, it was less successful than the sticky-price model in other dimensions. It is not surprising that the baseline model does not fare overwhelmingly better than the sticky-price model in terms of posterior odds. Additionally, the sticky-information model reveals a greater degree uncertainty in the dynamics, reflected in the wider credible sets for the responses to an IS shock and in the variance decompositions.

Truncation can alter the dynamics predicted by a model with lagged expectations and estimates that try to match these dynamics are vulnerable to making biased inferences. The estimates here have shown that taking the sticky-information model seriously can lead to a different assessment of the model's ability to match the data and that, using the methods developed here, sticky-information models can be readily analyzed.

8. Conclusion

I have derived a method for solving linear rational expectations models with lagged expectations using standard methods for the autonomous recursion of the MA coefficients and a sparse block tridiagonal system of equations for the nonautonomous coefficients, thus combining several known methods into one coherent approach. The method explicitly allows for models with lagged expectations reaching back into the infinite past, providing a formal justification for when and how to truncate based on a convergence criterion. The method performs favorably in comparison with existing methods, minimizing computing and preprocessing time, while avoiding the problems associated with an arbitrary truncation.

I have also presented likelihood methods for estimation that avoid the Kalman filter. This is desirable as lagged expectations makes the state-space necessary for the use of the Kalman filter prohibitively large, negating the gains made with the solution method. The comparison of simple New Keynesian models with sticky information and prices favored sticky information marginally with truncation reversing the ordering.

The solution and estimation methods derived here allow researchers to analyze models with lagged expectations quickly and easily. As such, they should facilitate the further investigation of models such as the sticky-information model.

Appendix A. Application of Klein's (2002) QZ Method

Use the QZ decomposition to find unitary matrices Q and Z and upper-triangular matrices S and T such that $\begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} \begin{bmatrix} 0 & -A_0 \\ I & 0 \end{bmatrix} \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = S$ and $\begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} \begin{bmatrix} C_0 & B_0 \\ 0 & I \end{bmatrix} \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = T$. The decomposition will be arranged such that the first s eigenvalues are those less than or equal to g^u , satisfying (3), and the remaining ones, given by the generalized eigenvalues of $S_{22}z - T_{22}$ following Klein (2000, p. 1415), are greater than g^u . Assuming $s = k$ and Z_{11} is of full rank,

$$\Xi_j^u = - \sum_{k=0}^{\infty} [T_{22}^{-1} S_{22}]^k T_{22}^{-1} Q_2 \begin{bmatrix} F_0 N_{j+1+k} + G_0 N_{j+k} \\ 0 \end{bmatrix} \quad (\text{A.1})$$

and, following Theorem 5.1 of Klein (2000, p. 1417) where $\alpha = Z_{21}Z_{11}^{-1}$ and $\beta = Z_{22} - Z_{21}Z_{11}^{-1}Z_{12}$, this delivers (9).

For Section 3.2, the system (8) is replaced by the system (10). This yields, analogous to the foregoing, (11), where

$$\Xi_j^u(I) = - \sum_{k=0}^{\infty} [T_{22}(I)^{-1} S_{22}(I)]^k T_{22}(I)^{-1} Q_2(I) \begin{bmatrix} \tilde{F}_I N_{j+1+k} + \tilde{G}_I N_{j+k} \\ 0 \end{bmatrix} \quad (\text{A.2})$$

Klein (2000) demonstrates, with a stationary VAR(1) exogenous process, that (A.1) can be reduced to a geometric sum. To meet the assumption in his appendix, the exogenous process need not be stationary. To see this, assume that $N_{j+1} = N N_j$ and define

$$H_k = \Phi^k \Delta N^k, \quad \Phi = T_{22}^{-1} S_{22}, \quad \Delta = T_{22}^{-1} Q_2 \begin{bmatrix} \tilde{F}_I N^{j+1} + \tilde{G}_I N^j \\ 0 \end{bmatrix} \quad (\text{A.3})$$

then $H_{k+1} = \Phi H_k N \Rightarrow \text{vec}(H_{k+1}) = (N' \otimes \Phi) \text{vec}(H_k)$. The stability of this recursion is determined by $\text{eig}(N' \otimes \Phi) = \text{vec}(\text{eig}(N')' \text{eig}(\Phi))$. As, by definition, $|\text{eig}(\Phi)| < \frac{1}{g^u}$ then $|\text{eig}(N' \otimes \Phi)| < 1$ so long as $|\text{eig}(N')| \leq g^u$. Thus, if the moving-average coefficients of the exogenous process follow a recursion that itself satisfies the uniform growth restriction, (A.1) and (A.2) meet Klein's (2000) assumption and are well defined.

Appendix B. Recursive Algorithm for Computing the Log-Likelihood

Akaike (1973) provides a recursive algorithm for inverting block Toeplitz matrices. As Ψ , defined in (26), is additionally symmetric, the first block column contains all the information necessary for the calculations. Following Akaike (1973, p. 237), define

$$\Psi_T = \begin{bmatrix} \Psi_{T-1} & \hat{a}_{T-1} \\ \hat{r}_{T-1} & \Gamma_0 \end{bmatrix} = \begin{bmatrix} \Gamma_0 & \tilde{a}_{T-1} \\ r_{T-1} & \Psi_{T-1} \end{bmatrix} \quad (\text{B.1})$$

where $\hat{r}_{T-1} = [\Gamma_{T-1} \ \Gamma_{T-2} \ \dots \ \Gamma_1]$, $\hat{a}_{T-1} = \tilde{r}'_{T-1}$, $\tilde{a}_{T-1} = [\Gamma'_1 \ \dots \ \Gamma'_{T-2} \ \Gamma'_{T-1}]$, $r_{T-1} = \tilde{a}'_{T-1}$. The inverse of Ψ_T can be likewise defined recursively, noting symmetry, as

$$\Psi_T^{-1} = \begin{bmatrix} \Psi_{T-1}^{-1} + \hat{j}_{T-1} s_{T-1}^{-1} \tilde{i}_{T-1} & \hat{j}_{T-1} \\ \tilde{i}_{T-1} & s_{T-1} \end{bmatrix} = \begin{bmatrix} \Psi_{T-1}^{-1} + \tilde{i}'_{T-1} s_{T-1}^{-1} \tilde{i}_{T-1} & \tilde{i}'_{T-1} \\ \tilde{i}_{T-1} & s_{T-1} \end{bmatrix} \quad (\text{B.2})$$

From Akaike (1973, p. 239), the determinant of $\Psi \equiv \Psi_T$ is calculated recursively with

$$\ln(\det(\Psi_T)) = \ln(\det(\Psi_{T-1})) + \ln(\det(s_{T-1}^{-1})) \quad (\text{B.3})$$

and the quadratic form $X' \Psi^{-1} X \equiv \tilde{X}'_T \Psi_T^{-1} \tilde{X}_T$, using (B.2), with

$$\tau_T \equiv \tilde{X}'_T \Psi_T^{-1} \tilde{X}_T = [\tilde{X}'_{T-1} \ X'_T] \begin{bmatrix} \Psi_{T-1}^{-1} + \tilde{i}'_{T-1} s_{T-1}^{-1} \tilde{i}_{T-1} & \tilde{i}'_{T-1} \\ \tilde{i}_{T-1} & s_{T-1} \end{bmatrix} \begin{bmatrix} \tilde{X}_{T-1} \\ X_T \end{bmatrix}$$

where $\tilde{X}_T = [X'_1 X'_2 \dots X'_T]'$. Multiplying out yields

$$\tau_T = \tau_{T-1} + \tilde{X}'_{T-1} \tilde{i}'_{T-1} s_{T-1}^{-1} \tilde{i}_{T-1} \tilde{X}_{T-1} + X'_T \tilde{i}_{T-1} \tilde{X}_{T-1} + \tilde{X}'_{T-1} \tilde{i}'_{T-1} X_T + X'_T s_{T-1} X_T \quad (\text{B.4})$$

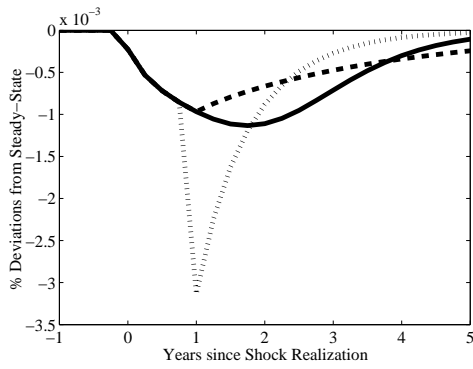
Equations (B.3) and (B.4) along with the recursions from Akaike (1973, p. 238)²⁸ provide a recursive algorithm for calculating the log-likelihood, requiring as input only the data and the sequence of autocovariances.

²⁸Note that Akaike (1973, p. 238) provides recursions for and using $s_T^{-1} \tilde{i}_T$ and $q_T^{-1} \tilde{f}_T$. Postmultiplying his equations 3.18 and 3.20 with E_T , defined by his equation 2.2, and redefining his equations 3.23 and 3.24 in terms of $s_T^{-1} \tilde{i}_T$ and $q_T^{-1} \tilde{f}_T$ provide the necessary recursions.

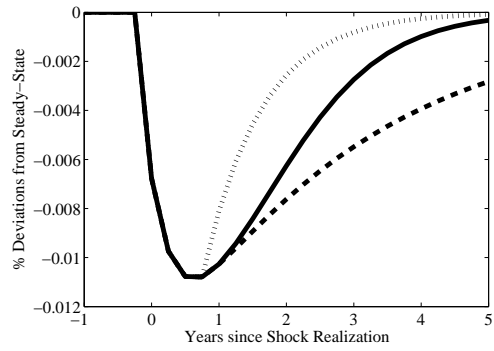
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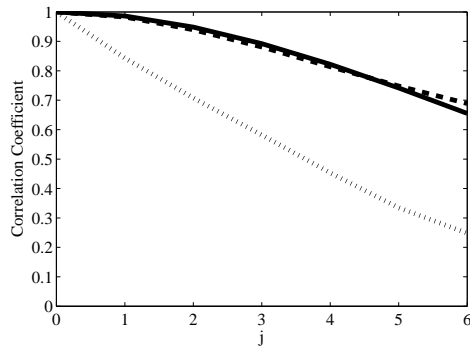
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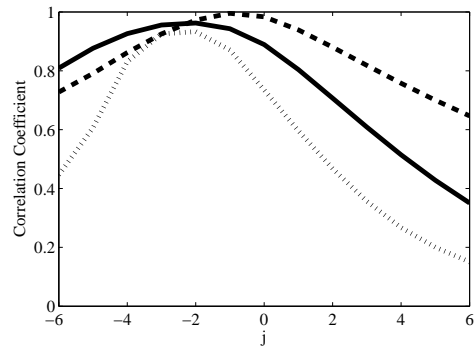
(a) Impulse Response of Inflation to a Shock in Money Growth



(b) Impulse Response of the Output Gap to a Shock in Money Growth

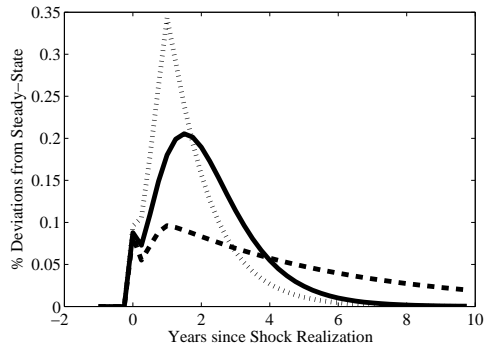


(c) Autocorrelation of Inflation

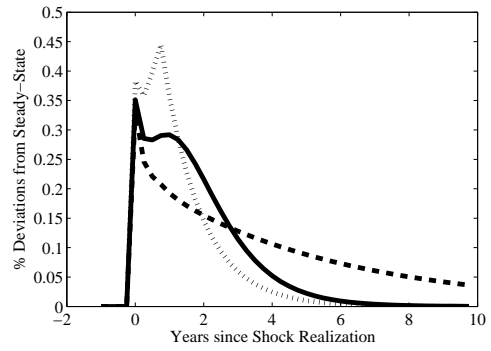


(d) Crosscorrelation of the Output Gap at $t+j$ with Inflation at t

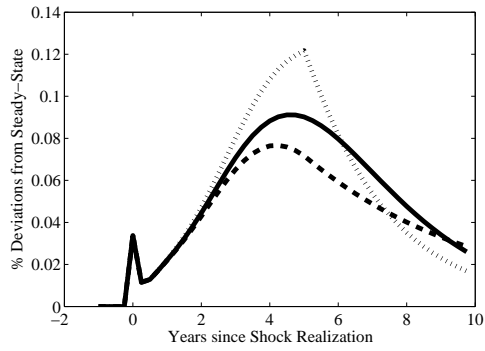
Figure 1: Consequences of Truncation in the Model of Mankiw and Reis (2002); Solid Line - Method Here, Dashed - Truncation Type 1, Dotted - Truncation Type 2



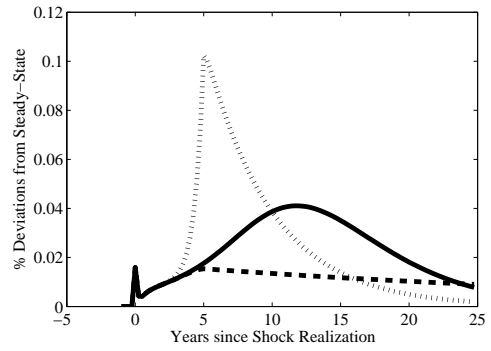
(a) Impulse Response of Inflation, $\theta = 0.8$



(b) Impulse Response of Marginal Costs, $\theta = 0.8$



(c) Impulse Response of Inflation, $\theta = 0.9$



(d) Impulse Response of Inflation, $\theta = 0.95$

Figure 2: Consequences of Truncation ($I=20$) in the First Model of Wang and Wen (2006); Impulse Responses to a Shock in Money Growth; Solid Line - Method Here, Dashed - Truncation Type 1, Dotted - Truncation Type 2

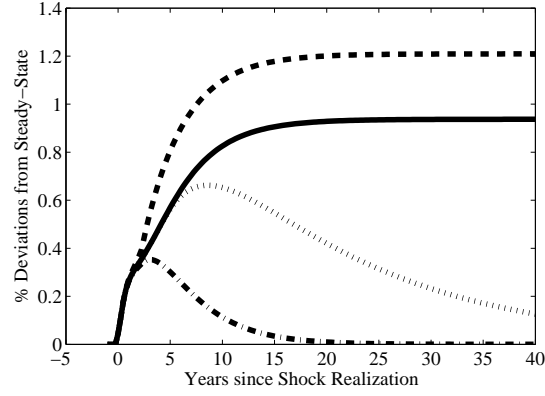
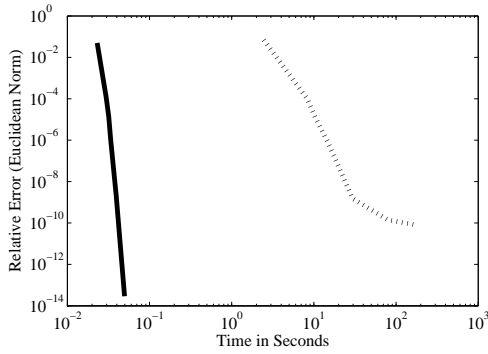
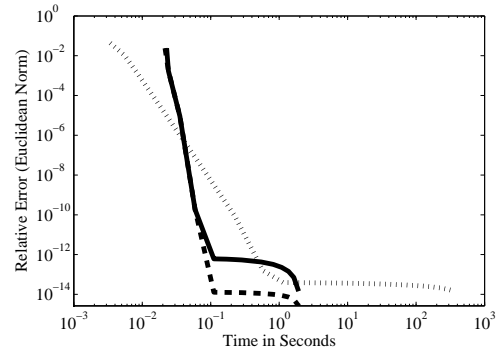


Figure 3: Impulse Response of the Price Level in the Model of Mankiw and Reis (2007); Truncation Type 2 (Solid Line - $I=100$, Dashed - $I=10$), Truncation Type 1 (Dotted - $I=20$, Dash-Dotted - $I=10$)

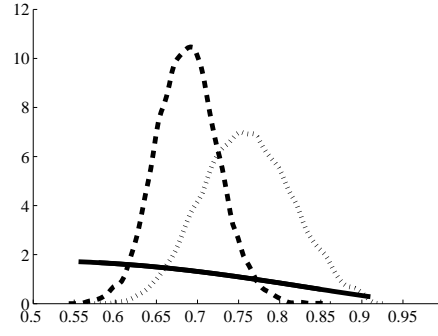


(a) Solid Line - Method Here (QZ), Dotted - Wang and Wen (2006)

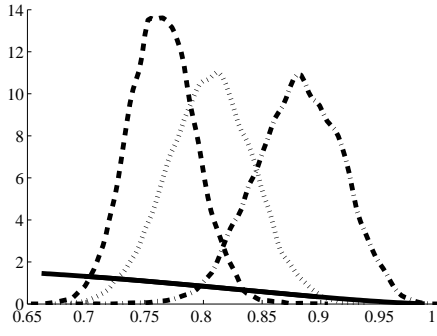


(b) Solid Line - Method Here (QZ), Dashed - Method Here (AIM), Dotted Mankiw and Reis (2007)

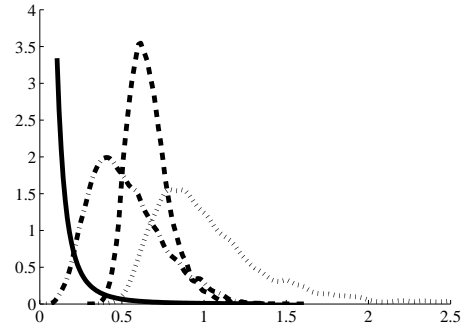
Figure 4: Computation Time versus Accuracy, Log Scale



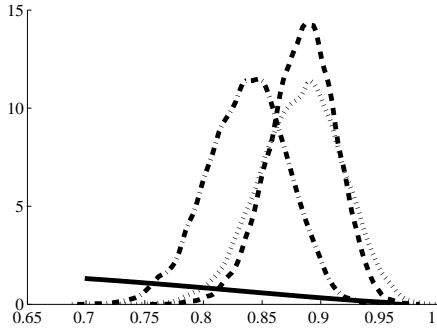
(a) Prior and Posterior Densities of λ



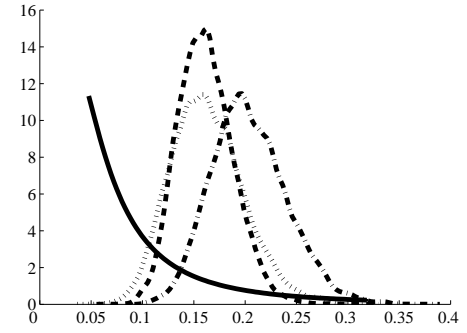
(b) Prior and Posterior Densities of ρ_{pc}



(c) Prior and Posterior Densities of σ_{pc}

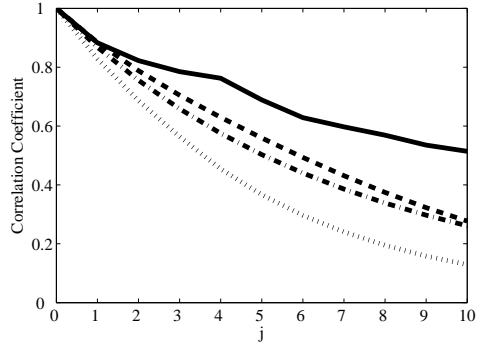


(d) Prior and Posterior Densities of ρ_{is}

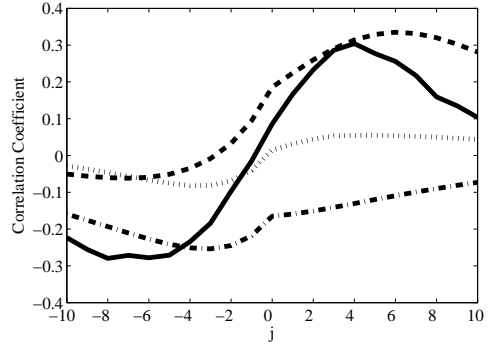


(e) Prior and Posterior Densities of σ_{is}

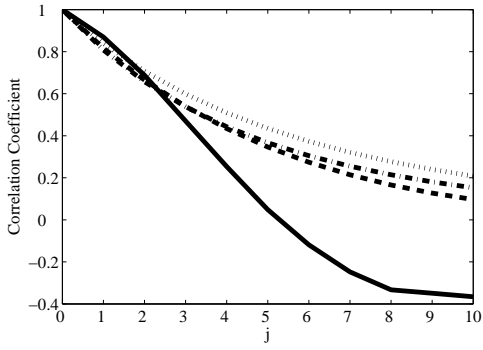
Figure 5: Selected Priors and Posterior Densities; Solid - Prior, Dashed - Baseline Model, Dotted - Truncation, Dash-Dotted - Sticky Prices



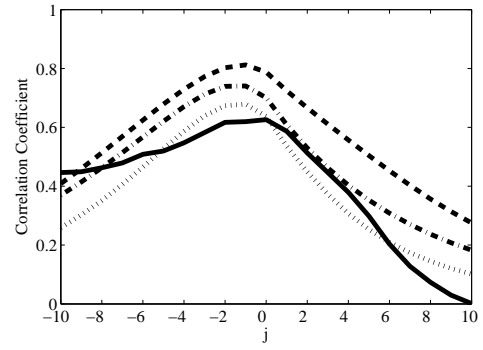
(a) Autocorrelation of Inflation



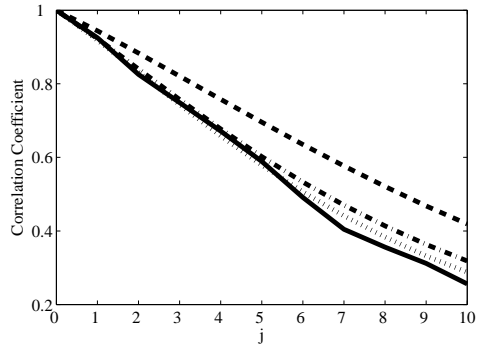
(b) Crosscorrelation of Inflation at $t+j$ with the Output Gap at t



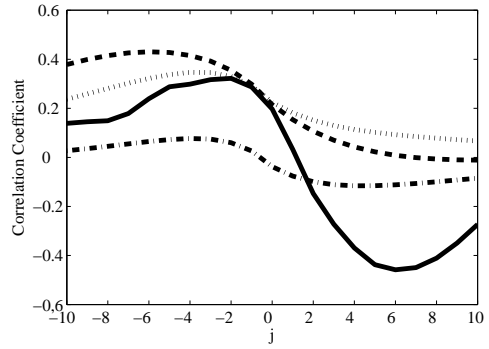
(c) Autocorrelation of the Output Gap



(d) Crosscorrelation of Inflation at $t+j$ with the Nominal Interest Rate at t

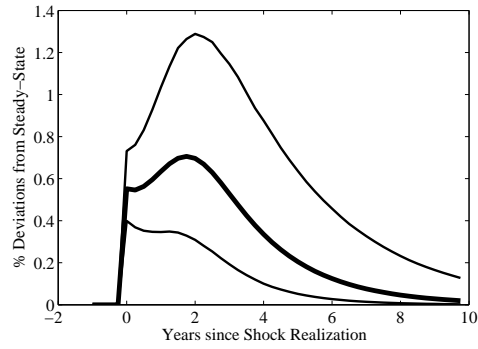


(e) Autocorrelation of the Nominal Interest Rate

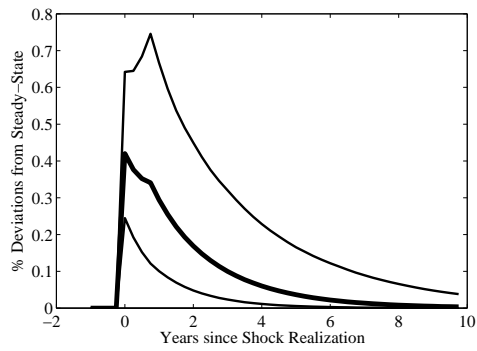


(f) Crosscorrelation of the Output Gap at $t+j$ with the Nominal Interest Rate at t

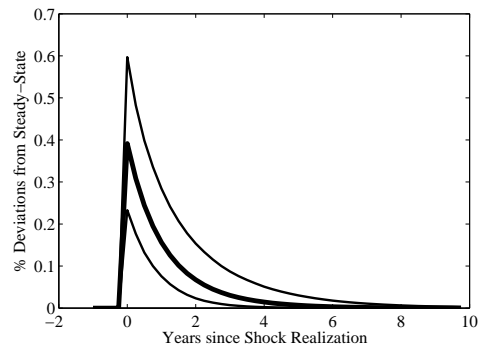
Figure 6: Selected Empirical and Posterior Statistics; Solid - Data, Dashed - Baseline Model, Dotted - Truncation, Dash-Dotted - Sticky Prices



(a) Impulse Response - Baseline Model

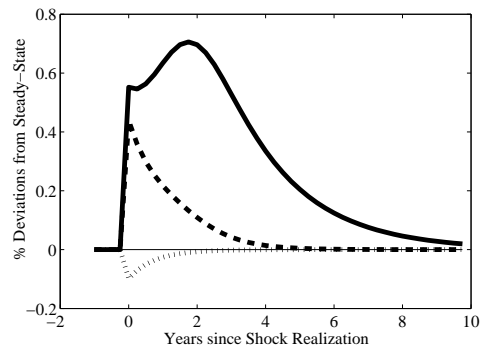


(b) Impulse Response - Truncation

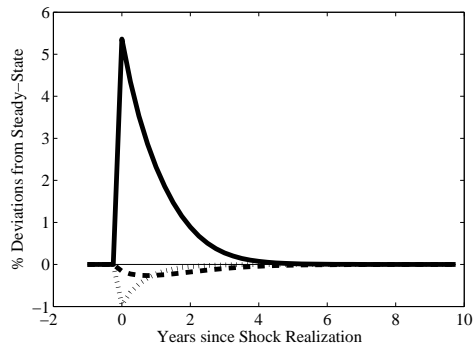


(c) Impulse Response - Sticky Prices

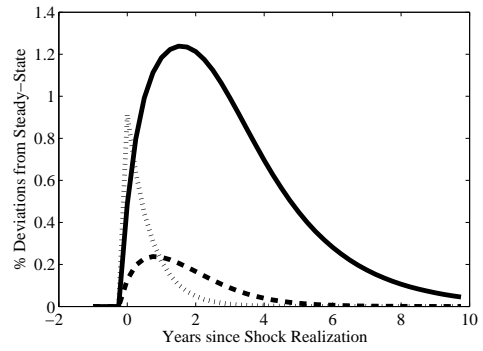
Figure 7: Impulse Responses of Inflation to a Unit IS Shock; Heavier Weighted Lines - Evaluated at the Posterior Mean, Lighter Weighted Lines - 10% and 90% Bounds on the Credible Set of Impulse Responses



(a) Inflation



(b) The Output Gap



(c) The Nominal Interest Rate

Figure 8: Impulse Responses to Unit Shocks in the Baseline Model; Solid - IS, Dashed - PC, Dotted - MP; Evaluated at the Posterior Mean

Table 1: Priors and Posteriors of Parameters

| | | Prior | | Posterior (Baseline) | | | | Posterior (Truncation) | | | | Posterior (Sticky Prices) | | | | |
|---------------------------|------------------------|--------------------|-------|----------------------|----------|------|--------|------------------------|-----------|------|--------|---------------------------|-----------|------|--------|------|
| | | Type | Mean | Std. | Mean | 5% | Median | 95% | Mean | 5% | Median | 95% | Mean | 5% | Median | 95% |
| 39 | λ | \mathcal{B} | 0.5 | 0.2 | 0.69 | 0.63 | 0.69 | 0.75 | 0.76 | 0.67 | 0.76 | 0.85 | 0.74 | 0.66 | 0.75 | 0.81 |
| | ξ | \mathcal{B} | 0.25 | 0.05 | 0.23 | 0.16 | 0.22 | 0.31 | 0.25 | 0.17 | 0.24 | 0.33 | 0.22 | 0.14 | 0.22 | 0.30 |
| | $\frac{1}{a_1}$ | \mathcal{N} | 1 | 0.75 | 3.40 | 2.60 | 3.38 | 4.27 | 3.39 | 2.57 | 3.37 | 4.26 | 3.49 | 2.70 | 3.48 | 4.33 |
| | ϕ_π | \mathcal{N} | 1.5 | 0.125 | 1.32 | 1.16 | 1.32 | 1.49 | 1.31 | 1.14 | 1.31 | 1.48 | 1.33 | 1.16 | 1.33 | 1.50 |
| | ϕ_y | \mathcal{N} | 0.125 | 0.05 | 0.25 | 0.18 | 0.25 | 0.31 | 0.25 | 0.19 | 0.25 | 0.32 | 0.26 | 0.19 | 0.26 | 0.32 |
| | ρ_{is} | \mathcal{B} | 0.5 | 0.2 | 0.88 | 0.84 | 0.89 | 0.93 | 0.88 | 0.82 | 0.88 | 0.94 | 0.84 | 0.78 | 0.84 | 0.89 |
| | ρ_{pc} | \mathcal{B} | 0.5 | 0.2 | 0.76 | 0.72 | 0.76 | 0.81 | 0.81 | 0.75 | 0.81 | 0.87 | 0.88 | 0.82 | 0.88 | 0.94 |
| | ρ_R | \mathcal{B} | 0.5 | 0.2 | 0.76 | 0.72 | 0.76 | 0.80 | 0.77 | 0.72 | 0.77 | 0.81 | 0.78 | 0.73 | 0.78 | 0.81 |
| | σ_{is}^ϵ | \mathcal{G}^{-1} | 0.1 | 2 | 0.16 | 0.12 | 0.16 | 0.21 | 0.16 | 0.11 | 0.16 | 0.22 | 0.20 | 0.15 | 0.20 | 0.27 |
| | σ_{pc}^ϵ | \mathcal{G}^{-1} | 0.1 | 2 | 0.66 | 0.49 | 0.64 | 0.89 | 1.03 | 0.62 | 0.95 | 1.74 | 0.52 | 0.23 | 0.49 | 0.94 |
| σ_{mp}^ϵ | \mathcal{G}^{-1} | 0.1 | 2 | 0.27 | 0.24 | 0.26 | 0.30 | 0.26 | 0.24 | 0.26 | 0.30 | 0.26 | 0.24 | 0.26 | 0.29 | |
| Log Marginal Data Density | | | | | -230.398 | | | | -234.0005 | | | | -233.1548 | | | |

Note: Data comprises 1970:1-2002:2; \mathcal{N} - Normal, \mathcal{B} - Beta, \mathcal{G}^{-1} - Inverse Gamma; Std. - Standard Deviation; The effective prior is truncated at the determinacy boundary and appropriately normalized. The log marginal densities are calculated using Geweke's (1999) modified harmonic mean with the 100,000 posterior draws.

Table 2: Variance Decompositions

| Sticky Information (Baseline Model) | | | | | | | | | |
|-------------------------------------|-----------|---------|-----------|------------|-----------|-----------|-----------------------|----------|----------|
| Horizon | Inflation | | | Output Gap | | | Nominal Interest Rate | | |
| | IS | PC | MP | IS | PC | MP | IS | PC | MP |
| 1 | 8.59 | 90.68 | 0.74 | 90.87 | 1.33 | 7.80 | 8.57 | 9.86 | 81.57 |
| | (5.1,14) | (85,94) | (0.4,1.3) | (88,93) | (0.5,2.5) | (5.9,11) | (5.8,12) | (7.2,12) | (77,86) |
| 2 | 10.03 | 89.31 | 0.66 | 90.59 | 2.43 | 6.98 | 16.98 | 17.82 | 65.21 |
| | (5.9,16) | (83,94) | (0.4,1.2) | (87,93) | (1.1,4.1) | (5.2,9.6) | (12,23) | (13,21) | (59,73) |
| 5 | 16.05 | 83.46 | 0.49 | 88.76 | 5.61 | 5.63 | 36.21 | 29.41 | 34.38 |
| | (8.6,27) | (72,91) | (0.3,0.9) | (84,92) | (3,8.7) | (4.1,7.9) | (27,46) | (22,35) | (28,43) |
| 10 | 28.00 | 71.63 | 0.37 | 86.36 | 8.58 | 5.06 | 51.99 | 28.86 | 19.15 |
| | (13,47) | (53,87) | (0.2,0.8) | (80,91) | (4.6,13) | (3.6,7.1) | (38,66) | (19,40) | (14,27) |
| 15 | 34.03 | 65.64 | 0.33 | 85.68 | 9.33 | 4.99 | 58.94 | 25.71 | 15.35 |
| | (16,57) | (43,84) | (0.2,0.7) | (79,91) | (5,15) | (3.6,7) | (42,74) | (14,39) | (10,23) |
| ∞ | 36.63 | 63.06 | 0.32 | 85.57 | 9.45 | 4.98 | 62.66 | 23.51 | 13.83 |
| | (16,64) | (36,83) | (0.1,0.7) | (79,91) | (5,15) | (3.5,7) | (43,80) | (11,37) | (7.7,21) |
| Sticky Prices | | | | | | | | | |
| Horizon | Inflation | | | Output Gap | | | Nominal Interest Rate | | |
| | IS | PC | MP | IS | PC | MP | IS | PC | MP |
| 1 | 5.92 | 93.79 | 0.29 | 90.77 | 1.50 | 7.73 | 7.72 | 10.77 | 81.52 |
| | (2.9,13) | (86,97) | (0.1,0.8) | (87,93) | (0.3,3.0) | (5.8,11) | (5.2,11) | (7.1,12) | (78,87) |
| 2 | 5.52 | 94.24 | 0.25 | 89.82 | 2.94 | 7.24 | 14.01 | 20.34 | 65.64 |
| | (2.6,12) | (87,97) | (0.1,0.7) | (86,93) | (1,5.0) | (5.5,10) | (9.9,20) | (14,22) | (61,74) |
| 5 | 4.73 | 95.08 | 0.19 | 85.94 | 7.80 | 6.26 | 24.12 | 38.90 | 36.99 |
| | (2.1,11) | (89,98) | (0.1,0.6) | (81,91) | (3.5,12) | (4.7,9.0) | (18,33) | (28,42) | (32,47) |
| 10 | 4.21 | 95.64 | 0.16 | 81.32 | 13.06 | 5.62 | 26.74 | 49.08 | 24.18 |
| | (1.8,10) | (89,98) | (0.0,0.5) | (74,89) | (6.2,19) | (4.1,8.2) | (20,39) | (35,55) | (20,33) |
| 15 | 4.05 | 95.81 | 0.15 | 79.56 | 14.99 | 5.45 | 26.46 | 52.05 | 21.49 |
| | (1.7,10) | (89,98) | (0.0,0.5) | (71,88) | (7.0,23) | (3.9,8) | (19,40) | (36,60) | (17,30) |
| ∞ | 3.98 | 95.88 | 0.15 | 78.77 | 15.84 | 5.39 | 25.98 | 53.44 | 20.58 |
| | (1.5,10) | (89,98) | (0.0,0.5) | (68,88) | (7.3,26) | (3.8,7.8) | (17,41) | (36,65) | (15,29) |

Note: Entries are given in percent. The main entries were evaluated at the posterior mean and may not add up to 100 due to rounding. The entries in parentheses give the 5% and 95% bounds for the posterior credible set and were calculated cell-by-cell.