

Labor Market Participation, Unemployment and Monetary Policy*

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Abstract

In the present paper we study to what extent the introduction of endogenous participation in an otherwise standard DSGE model with matching frictions plays a role for business cycle dynamics and monetary policy. The contribution of the paper is threefold: first, we introduce the participation margin in a standard DSGE model with matching frictions and nominal rigidities and show that the model provides a good fit for employment and unemployment volatility, as well as participation volatility and correlation of participation with output with US data. Second, we show that in such a model, *and contrary to a model with exogenous participation*, a monetary authority becoming more aggressive in fighting inflation decreases the volatility of employment and unemployment. Finally, we show the role of search costs in shaping those results.

Keyword: matching frictions, endogenous participation, monetary policy.

JEL Codes: E24, E32, E52

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1 Introduction

The labor market participation margin over the business cycle has received surprisingly little attention by most of the literature. The assumption of inelastic labor force has become common practice since Andolfatto (1996) and Merz (1995), the first invoking matching frictions to explain the aggregate fluctuations of labor market variables. Also, the assumption has been imported by most of the recent vintage of business cycle models featuring both nominal rigidities and matching frictions¹.

Still, the participation rate to the labor market fluctuates at business cycle frequencies and it is pro-cyclical. For instance, over the period 1964-2007 in the United States, the standard deviation² of the participation rate relative to GDP is 0.20, while its correlation with GDP is 0.42. In addition, as pointed out by Barnichon and Figura (2010), participation accounts for one third of the variance of the unemployment rate over the business cycle.

In this paper we argue that models neglecting fluctuations of the labor force lead to incorrect conclusions about the effect of monetary policy on employment and unemployment. This finding has interesting implications. It is well known that switching from a flexible to a strict inflation targeting regime, the latter defined as complete stabilization of inflation around its target, magnifies the volatility of the unemployment rate, conditionally on technology shocks. See for instance Blanchard and Gali (2010). We show that a model with exogenous participation rates overstates the surge in volatility due to the policy change. The intuition is straightforward: missing the participation margin, involuntary unemployment can never be substituted with voluntary non-employment. To the extent that the labor force reacts to a change in monetary policy, exogenous participation models overlook the policy transmission channel acting through the participation decision. Here, we show that the incentives driving participation to the labor force respond to frictions, be they search frictions or nominal rigidities. In turn, monetary policy affects the relevance of frictions over the business cycle. As an implication, policy changes affect the participation rate, along with the job finding rate, by making the household more willing to move from unemployment to voluntary non-employment and the other way around. Conditionally on productivity shocks, the effect of monetary policy on the unemployment rate is considerably lower when participation is free to move. Under our calibration, which allows for other shocks in addition to productivity, such as demand shocks and labor supply shocks, unemployment volatility falls rather than rising when the central bank switches to a strict inflation targeting policy.

We address our questions in a standard model featuring matching frictions à la Mortensen and Pissarides (1999) and nominal price rigidity à la Calvo (1983), where we make costly the entry to the labor market by modelling home produc-

¹As emphasized by Gali (2010) this is in stark contrast with the earlier generation of New-Keynesian models, allowing for an elastic labor supply.

²We extract the business cycle component from the data in logs, by applying the Hodrick-Prescott filter with a conventional smoothing parameter of 1600. Further details are provided below.

tion activity and search activity as both requiring time. Non participant agents only are assumed to allocate all their time to home production, while participation to the labor market implies a loss in terms of time devoted to home production. Hence, entering the labor market entails a cost paid in exchange for a chance to be matched with a job. We define households' search cost as the loss due to a movement from non-participation to unemployment, relative to the movement from non-participation to employment. Following the tradition in this literature, we assume that agents belong to big families pooling members' home production activity, wage and unemployment benefits so as to achieve perfect consumption insurance against the idiosyncratic income risk brought about by unemployment fluctuations. This allows us to maintain a representative agent framework. We calibrate and solve the model and we compare it to a New-Keyensian model (NK henceforth) with Walrasian labor markets, a Mortensen and Pissarides (1999) model with aggregate fluctuations (DMP henceforth) and a model with sticky prices and matching frictions without participation decision (baseline NK-DMP henceforth), all models differing from their baseline version for the presence of home production only. In the calibration exercise, we make sure that our model is at least doing as well as an NK-DMP model augmented with home production technology without participation margin. In assessing both models we compare the simulated unconditional moments with their observed counterparts.

The main results of the paper can be summarized as follows. First, the household aims at replicating the level of home production that would be achieved in a model without labor market frictions. This is true under all parametrizations, be prices sticky or not. Then, the main incentive driving participation is to keep the marginal rate of substitution between market and non-market consumption at the ideal level that would prevail with Walrasian labor markets. Equivalently, matching frictions open a home production gap and participation is chosen so as to close the gap. Second, households' search costs determine the importance of finding rate fluctuations in the participation decision. When the search cost is high, changes in the finding rates are relatively less important. In fact, employment and unemployment are roughly equally expensive in terms of non-market activity. Finally, monetary policy affects the response of macro variables to shocks through the participation decision, in addition to the conventional demand side channel. For instance, conditionally on productivity shocks, strict inflation targeting increases the volatility of the participation rate, this fact dampening the surge in the standard deviation of the unemployment rate that would be observed under exogenous participation. Also, conditionally on preference shocks, strict inflation targeting generates larger flows to the labor force, dampening the volatility of the employment rate and its pro-cyclicality. Again, those effects of policy are not taken into account by a model with exogenous participation margin.

We conclude that a change in the monetary policy rule affects business cycle moments in a way that is overlooked by models abstracting from the participation margin. We also believe that we contribute to the literature by showing the importance of a proper calibration of households' search cost. In fact, the

search cost plays a role in shaping the relative importance of matching frictions in driving the participation decision and then the strength of the policy transmission channel acting through the labor force. In particular, the difference in the predicted policy outcome between our model and the baseline NK-DMP version falls in the search cost.

Our findings are relevant for and related to different strands of the literature on aggregate fluctuations. On the one hand, several papers focus on the behavior of labor market variables at business cycle frequencies. Typical references are Shimer (2005), Hall (2005), Chéron and Langot (2000) and Walsh (2005)³. On the other hand, few and recent contributions focussed on the participation margin. Garibaldi and Wasmer (2005) introduced participation in a static framework where agents differ in the value of their home productivity. Ebell (2008) modelled the participation margin in a real business cycle model. We follow her in adopting a modelling strategy that eliminates agents heterogeneity, but we do allow unemployed workers to take part in the home production activity. Given the importance of the households' search cost, both in terms of calibration and results, this is a crucial difference. Haefke and Reiter (2006) solve an heterogeneous agents model in a real business cycle framework, abstracting from monetary policy. Recently, Bruckner and Pappa (2010) introduced endogenous participation in a New Keynesian model to study the transmission of fiscal shocks on labor market variables. Like Ebell (2008), they also assume no home production from unemployed workers. Gali (2010) focuses on the interaction between matching frictions and optimal monetary policy in a New Keynesian model where the cost of search is high. The paper is silent about the channel we emphasize here. Christiano, Trabandt and Walentin (2010) introduce endogenous and unobservable search effort in a medium-scale New Keynesian model, where the household members are heterogenous in terms of disutility from working and consumption is not fully insured against unemployment fluctuations, so as to give members the incentive to exert effort. They show that in such an environment the Phillips curve can be rewritten as a function of the gap between actual and efficient unemployment. We view their contribution as complementary to ours, though some differences should be emphasized. First, we abstract from households members heterogeneity and the arising incentive problem. Second, we model job finding probability following the DMP tradition, hence as a general equilibrium outcome resulting from the interaction of participation and vacancy posting decisions. Also and more importantly, we differentiate from Christiano et al. (2010) in terms of research question. In fact, their contribution does not look at the implications of endogenous participation for the transmission channel of monetary policy. To conclude, it is worth pointing out that the participation decision in our framework is substantially different from the intensive margin of hours worked, such as the one studied in Sveen and Weinke (2008) or Trigari (2009). This is because the choice of hours is made *conditionally* on having en-

³See also Trigari (2009) on the persistence of monetary policy shocks, Christoffel and Linzert (2010) on inflation dynamics, Gertler and Trigari (2009) and Krause and Lubik (2007) on the role of wage rigidity and Thomas (2008) and Faia (2008) on optimal monetary policy.

tered the labor force and being in a match. It follows that the finding rate does not affect the hours decision, while it does affect the participation choice, since the job finding probability determines the expected marginal gain associated to entering the labor force. Different is the case of on the job search where search intensity is a function of the finding rate. This is the case made by Krause and Lubik (2010). However, Krause and Lubik (2010) focus on the intensive rather than on the extensive margin of participation, i.e. the labor force is endogenous only because of endogenous search effort on the job. In addition, they abstract from monetary policy.

2 The Model

The representative household consists of a continuum $[0, 1]$ of family members. Each of them can be employed, unemployed or non participant. Non participant family members allocate all their time to home production. Employed members spend all their time to work receiving a salary in exchange. Unemployed workers spend some of their time actively searching for a new job while the rest is used for home production. While unemployed, they are entitled to an unemployment benefit. Wages, unemployment benefits and home production are pulled together and redistributed equally within the family members so that they all enjoy the same level of consumption and home production⁴. Consumption and savings are decided at the household level, together with the choice of how many family members to let participate in the labor market.

The economy is characterized by two sectors⁵. In the final sector there is a continuum of retailers, each selling a differentiated good under monopolistic competition and using intermediated goods as the only input in production. Calvo price stickiness is assumed in this sector. In the intermediate sector infinitely many firms produce an homogeneous good under perfect competition and flexible prices. In order to produce each firm has to be matched with a worker. Firms are subject to a vacancy posting cost when searching for a worker. Existing matches can be exogenously discontinued at any time.

We choose to consider three shocks in the model. Disturbances to market goods production technology and to preferences are considered, as it is standard in the literature. In addition, we include shocks to home goods production technology that are correlated with market technology shocks. This is because we want to allow for the possibility that an improvement in market technology spreads to the home production sector. In the calibration exercise we then leave the data to choose the variances and the cross-correlation between home and market technology shocks so as to match unconditional simulated moments with their observed counterparts.

⁴See Andolfatto (1996) and Merz (1995).

⁵We use the two sectors set-up in order to keep the matching frictions separated from the price rigidity. See for example Sveen and Weinke (2008).

2.1 Households

A household is made up by a continuum $[0, 1]$ of family members. Let E_{t-1} be the employed members in period $t - 1$. When entering period t , a fraction ρ of those jobs will be exogenously discontinued. Among those, some may drop out of the labor force, if the household decides to reduce labor market participation, while the others will search for a new job. We assume instantaneous hiring⁶ i.e. searching workers matched with a firm will start working already in period t . Searching workers who will not be matched, will receive the unemployment status, be entitled to the unemployment benefit and take part to some home production⁷. Therefore, if N_t is the fraction of family members participating in the labor market, searching workers in period t are defined as⁸:

$$S_t = N_t - (1 - \rho)E_{t-1} \quad (2.1)$$

while non participant members are given by:

$$L_t = 1 - N_t \quad (2.2)$$

Let f_t be the job finding rate, that will be endogenously defined when solving the search and matching problem in the intermediate sector. Then, the evolution of employment reads as follows:

$$E_t = (1 - \rho)(1 - f_t)E_{t-1} + f_t N_t \quad (2.3)$$

Let $C_t \equiv \left[\int_0^1 C_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$ be a Dixit-Stiglitz aggregator of different varieties of goods. The optimal allocation of expenditure on each variety is given by $C_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} C_t$ where $P_t \equiv \left[\int_0^1 P_t(i)^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}}$. The representative household then maximizes the expected lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[Z_t \log(C_t) + \phi \frac{h_t^{1+\nu}}{1+\nu} \right] \quad (2.4)$$

subject to:

⁶Because of the assumption of sticky prices, production is demand driven in the short run. Therefore, firms need to have a margin of adjustment to supply as many goods as demanded at the prevailing price. In a model without capital, as standard in this literature, two are the possible options. Either introducing endogenous job destruction or allowing for instantaneous hiring. We decided for the second in order to keep the model as simple as possible. Since we calibrate the model at quarterly frequency, it also seems reasonable.

⁷Intuitively, we are assuming that the search process takes place at the beginning of the period so that, if matched with a vacancy, workers can produce immediately otherwise, they can use the rest of their time for home production.

⁸We are implicitly assuming that when reducing participation, there are always enough unemployed workers out of which to choose so that all workers who were employed in the previous period and whose job was not exogenously discontinued, will keep their job.

$$P_t C_t + R_t^{-1} D_t \leq D_{t-1} + W_t E_t + P_t b U_t + T_t \quad (2.5)$$

$$E_t = (1 - \rho)(1 - f_t)E_{t-1} + f_t N_t \quad (2.6)$$

$$N_t = E_t + U_t \quad (2.7)$$

$$h_t = [\xi_t(1 - E_t - \Gamma U_t)]^{1-\alpha_h} \quad (2.8)$$

taking as given the nominal interest rate R_t , the nominal wage W_t , the aggregate price of goods P_t and T_t , including lump-sum taxes and profits. b is the real unemployment benefit, D_t is a risk-free nominal bond paying one unit of currency in the following period, h_t represents the home production activity, $\nu < 0$ is the inverse of the home production elasticity, $0 < \Gamma < 1$ is the fraction of time that unemployed workers devote to the search activity and $\alpha_h \in [0, 1)$ allows for decreasing returns in the home production technology. Finally, Z_t is a shock to preferences and ξ_t is a shock to home production technology. Optimization implies a conventional Euler equation:

$$\beta R_t E_t \left\{ \frac{C_t}{C_{t+1}} \frac{Z_{t+1}}{Z_t} \frac{P_t}{P_{t+1}} \right\} = 1 \quad (2.9)$$

and the following equation:

$$\begin{aligned} & \left[\frac{1-f_t}{f_t} \right] \left(\frac{\phi \Gamma h_t^\nu C_t}{Z_t} \xi_t (1 - \alpha_h) h_t^{-\frac{\alpha_h}{1-\alpha_h}} - b \right) = \frac{W_t}{P_t} - \frac{\phi h_t^\nu C_t}{Z_t} \xi_t (1 - \alpha_h) h_t^{-\frac{\alpha_h}{1-\alpha_h}} \\ & + \beta E_t \left\{ \frac{C_t}{C_{t+1}} \frac{Z_{t+1}}{Z_t} \frac{(1-\rho)(1-f_{t+1})}{f_{t+1}} \left(\frac{\phi \Gamma h_{t+1}^\nu C_{t+1}}{Z_{t+1}} \xi_{t+1} (1 - \alpha_h) h_{t+1}^{-\frac{\alpha_h}{1-\alpha_h}} - b \right) \right\} \end{aligned} \quad (2.10)$$

2.1.1 Endogenous Participation

Rearranging the optimality condition (2.10) allows to gain some insight about the key determinants of the participation decision. After defining:

$$\Omega_t \equiv \frac{(1-f_t)}{f_t} \left[\frac{\phi \Gamma h_t^\nu C_t}{Z_t} \xi_t (1 - \alpha_h) h_t^{-\frac{\alpha_h}{1-\alpha_h}} - b \right] \quad (2.11)$$

(2.10) can be rewritten recursively as:

$$\Omega_t = \frac{W_t}{P_t} - \frac{\phi h_t^\nu C_t}{Z_t} \xi_t (1 - \alpha_h) h_t^{-\frac{\alpha_h}{1-\alpha_h}} + E_t \left\{ \frac{\beta C_t (1-\rho)}{C_{t+1}} \frac{Z_{t+1}}{Z_t} \Omega_{t+1} \right\} \quad (2.12)$$

Note that $\frac{\phi \Gamma h_t^\nu C_t}{Z_t} \xi_t (1 - \alpha_h) h_t^{-\frac{\alpha_h}{1-\alpha_h}} - b$ is the flow benefit of withdrawing one unemployed worker from the labor force and reallocating it to home production

in terms of consumption, net of the unemployment benefit. Also, the term $\left[\frac{1-f_t}{f_t}\right]$ is a wedge introduced by matching frictions capturing the extra change in home production, relative to a frictionless labor market, needed to increase employment by one unit. In fact, by manipulating the law of motion of employment, it is straightforward to get:

$$E_t = (1 - \rho)E_{t-1} + \frac{f_t}{1 - f_t}U_t \quad (2.13)$$

Not surprisingly, matching frictions introduce a wedge between employment and the participation decision. Such a wedge decreases in the job finding rate and it is strictly positive for a job finding rate lower than one.

We interpret (2.12) as the optimality condition for labor market participation: it states that the marginal benefit of increasing employment has to equalize its marginal cost, once the wedge due to frictions is taken into account. On the one hand, Ω_t is the utility loss implied by diverting from home production the extra fraction of population frictions require to marginally increase employment. On the other, the right hand side of (2.12) represents the household's marginal benefit, adding the wage premium over the marginal rate of substitution to the option value of getting an additional member into employment, Ω_{t+1} . A positive option value arises, as long as a match realized in the current period allows the household to save on the future search cost with a positive probability $1 - \rho$. Finally, note that if the wedge vanishes, the marginal rate of substitution between consumption and home production equals the real wage. We define such a situation as full participation, since non-employment is entirely voluntary.

Condition (2.12) links the participation decision to the job finding rate through home production. A raise in the finding rate shifts downwards the marginal cost of increasing employment, for any given level of the marginal rate of substitution. Therefore, everything else equal, home production has to fall as leisure would do in the baseline business cycle model with endogenous labor supply.

2.2 Firms

2.2.1 Intermediate Good Producers

There are infinitely many firms $j \in [0, 1]$ producing an homogeneous good under perfect competition and flexible prices and using labor as the only input in production. The labor market is characterised by matching frictions in the standard Mortensen and Pissarides (1999) framework. Firms have to search for a worker in the pool of searching workers. Posting a vacancy costs κ units of the final good C_t in each period. When the vacancy is filled, it produces:

$$X_t(j) = A_t \quad (2.14)$$

where the (log of) technology A_t is assumed to follow an AR(1) process: $\log(A_t) = \rho_a \log(A_{t-1}) + \xi_t^a$ with ξ_t^a being an i.i.d. shock with zero mean and variance σ_a .

We use a standard constant return to scale technology converting searching workers S_t and vacancies V_t into new matches M_t :

$$M_t = \omega V_t^{1-\gamma} S_t^\gamma \quad (2.15)$$

We define labor market tightness as $\theta_t \equiv \frac{V_t}{S_t}$, the job filling rate (i.e. the rate at which searching workers meet a vacancy) as $q_t \equiv \omega \theta_t^{-\gamma}$, and the job finding rate (i.e. the rate at which vacancies are filled) as $f_t \equiv \theta_t q_t$. Because of instantaneous hiring, once the vacancy is filled it is immediately productive. Let P_t^x be the price at which firms sell the homogenous good to the final goods producers. The value of a filled vacancy, V_t^J expressed in terms of the final consumption bundle P_t , is given by:

$$V_t^J = \frac{P_t^x}{P_t} A_t - \frac{W_t}{P_t} + (1 - \rho) E_t \{ Q_{t,t+1} V_{t+1}^J \} \quad (2.16)$$

where $Q_{t,t+1} \equiv \beta \frac{C_t}{C_{t+1}} \frac{Z_{t+1}}{Z_t}$. The free entry condition ensures that:

$$\frac{\kappa}{q_t} = V_t^J \quad (2.17)$$

Substituting (2.17) into (2.16) gives the job creation condition:

$$\frac{\kappa}{q_t} = \frac{P_t^x}{P_t} A_t - \frac{W_t}{P_t} + (1 - \rho) E_t \left\{ Q_{t,t+1} \frac{\kappa}{q_{t+1}} \right\} \quad (2.18)$$

Finally, the wage is determined solving a Nash bargaining problem between the firm and the worker. In order to do that we have to compute the surplus from employment keeping participation constant. This is given by⁹:

$$V_t^w = \frac{W_t}{P_t} - b - \frac{\phi h_t^\nu (1 - \Gamma) C_t}{Z_t} \xi_t (1 - \alpha_h) h_t^{-\frac{\alpha_h}{1 - \alpha_h}} + E_t \{ Q_{t,t+1} (1 - \rho) (1 - f_{t+1}) V_{t+1}^w \} \quad (2.19)$$

Let η be the firm's bargaining power. Then, the total surplus from the match is split according to the optimal sharing rule:

$$\eta V_t^w = (1 - \eta) V_t^J \quad (2.20)$$

Using the definitions of V_t^J and V_t^w in (2.20), together with the free entry (2.17) and the job creation condition (2.18), it is possible to derive the wage equation:

$$\begin{aligned} \frac{W_t}{P_t} = (1 - \eta) \frac{P_t^x}{P_t} A_t + \eta \left[b + \frac{\phi h_t^\nu (1 - \Gamma) C_t}{Z_t} \xi_t (1 - \alpha_h) h_t^{-\frac{\alpha_h}{1 - \alpha_h}} \right] \\ + (1 - \eta) (1 - \rho) E_t \{ Q_{t,t+1} \kappa \theta_{t+1} \} \end{aligned} \quad (2.21)$$

⁹See appendix for the derivation.

2.2.2 Final Goods Retailers

In the final good sector there are infinitely many producers of differentiated goods. Each is producing a variety $i \in [0, 1]$ using the following technology:

$$Y_t(i) = X_t(i)^{1-\alpha} \quad (2.22)$$

They face a downward sloping demand function¹⁰:

$$Y_t(i) = \left[\frac{P_t(i)}{P_t} \right]^{-\varepsilon} [C_t + \kappa V_t] \quad (2.23)$$

Under flexible prices the optimal pricing rule is given by:

$$\frac{P_t^*(i)}{P_t} = \frac{\varepsilon}{\varepsilon - 1} \frac{MC_t(i)}{P_t} \quad (2.24)$$

where $P_t^*(i)$ is the optimal price and $MC_t(i) = \frac{1}{1-\alpha} P_t^x X_t(i)^\alpha$ is the nominal marginal cost. Imposing symmetry equation (2.24) becomes:

$$1 = \frac{\varepsilon}{\varepsilon - 1} \frac{1}{1 - \alpha} \frac{P_t^x}{P_t} X_t^\alpha \quad (2.25)$$

When price rigidity à la Calvo (1983) is assumed, the pricing first order condition for a firm allowed to reoptimize in t is given by:

$$\sum_{T=0}^{\infty} \xi^T E_t \left\{ Q_{t,t+1} \frac{Y_{t+T}(i)}{P_{t+T}} \left[P_t^*(i) - \frac{\varepsilon}{\varepsilon - 1} MC_{t+T}(i) \right] \right\} = 0 \quad (2.26)$$

where ξ represents the probability of not changing the price in a given period. Log-linearizing (2.26) around the zero inflation symmetric steady state we obtain the New Keynesian Phillips Curve (NKPC):

$$\hat{\pi}_t = \beta \hat{\pi}_{t+1} + \lambda \hat{m}c_t \quad (2.27)$$

where $\lambda = \frac{(1-\xi)(1-\beta\xi)}{\xi}$ and lower case variables with a hat represent log-deviations from steady state.

2.3 Market Clearing Conditions

The aggregate production of the intermediate sector is given by:

$$X_t = \int_0^1 X_t(j) dj = A_t E_t \quad (2.28)$$

Integrating the demand of good i , (2.23) yields the conventional aggregate resource constraint:

¹⁰Remember that intermediate firms pay the vacancy posting cost in terms of final goods and therefore solve an expenditure minimization problem like the household.

$$Y_t = C_t + \kappa V_t \quad (2.29)$$

after defining aggregate output as:

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \quad (2.30)$$

Combining demand of final goods (2.23) with their production function and integrating delivers the aggregate production function:

$$Y_t = X_t^{1-\alpha} \Delta_t^{\alpha-1} \quad (2.31)$$

where the following definition applies:

$$\Delta_t = \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{\frac{-\varepsilon}{1-\alpha}} di \quad (2.32)$$

and Δ_t , bounded by 1 from below, is a measure of price dispersion.

2.4 Monetary Policy

We assume that the monetary policy follows a simple interest rate rule:

$$\log(R_t) = -\log(\beta) + \phi_\pi \hat{\pi}_t \quad (2.33)$$

3 Participation, Matching Frictions and Sticky Prices

3.1 Steady State and Calibration

We choose the parameters related to the NK part of the model as it is conventional in the literature. Then, the elasticity of substitution among varieties of the final good is set to 6 and the Calvo parameter to $\xi = 2/3$. We also maintain $\alpha = 1/3$ in the production function¹¹. Also, we restrict to the case of a deterministic steady state where inflation and productivity are constant and normalized to zero and one respectively. It follows that the relative price dispersion of the final goods is zero, while the relative price of the intermediate good is distorted only by monopolistic competition in the final good sector. Finally, the Taylor rule coefficient is set to 1.5.

¹¹Given the magnitude of the calibrated firms' search cost, 0.0045, this implies a steady state labor income share close to 2/3.

Following the DMP literature, we calibrate as many parameters as possible so as to match the steady state of labor market variables with their observed unconditional mean.

The law of motion of employment (2.3) gives a relation between the steady state employment rate, the finding rate and the exogenous separation rate

$$\frac{E}{N} = \frac{f}{1 - (1 - \rho)(1 - f)} \quad (3.1)$$

We set the separation rate, ρ , to 0.1 following Shimer (2005) and by targeting an employment rate¹² of 0.9411 we recover the implied finding rate, 0.6572 per quarter, which is lower than in Shimer (2005). A lower finding rate is explained by the assumption of instantaneous hiring. In fact, workers can be matched in the same period they start to search, so that the model needs a lower f to replicate the same employment rate. The scaling parameter of the matching function, ω , is chosen in such a way that the job filling rate q is equal to $2/3$. This implies a steady state labor market tightness of about 1. These values are conventionally used in the literature, though it is worth noticing that all our results are robust to changes of the steady state of q and θ . In fact, as pointed out by Shimer (2005), the value of those variables is simply a matter of normalization.

Then, we are left with seven free parameters: the inverse of labor supply elasticity ν , the preference shifter ϕ , the cost of search Γ , workers' bargaining power $1 - \eta$, the unemployment benefit b , the elasticity of matches to vacancies $1 - \gamma$ and the cost of posting a vacancy κ . We set $\nu = -5$ implying a Frisch elasticity of labor supply equal to 0.2, keeping comparability with Gali (2010). Following Hagedorn and Manovskii (2008) and Gali (2010), we calibrate the cost of posting a vacancy κ by targeting vacancy costs per filled job as a fraction of the real wage. We choose 0.045 as a target, as in Gali (2010). To this end we use the job creation condition and the target to solve for the real wage at the steady state and parameter κ . Given the bargaining power of workers and the unemployment benefit, Γ is pinned down by the wage equation, after replacing the marginal rate of substitution with its steady state value:

$$\phi h^\nu c(1 - \alpha_h)h^{-\frac{\alpha_h}{1 - \alpha_h}} = \frac{fW/P + (1 - f)(1 - \beta(1 - \rho))b}{\Gamma(1 - f)(1 - \beta(1 - \rho)) + f} \quad (3.2)$$

We interpret Γ as the fraction of time devoted to home production, relative to the time endowment, that would be lost by moving a household member from non-participation to unemployment. By referring to time use survey evidence, Krueger and Mueller (2008) document that the amount of time per week spent looking for a job by the unemployed is on average about 2.5 hours. In addition, 220 minutes per week are devoted to home production. In the time use survey non-participants are not observed. However, if the sum of time devoted to home production and search by the unemployed is normalized to one and assumed

¹²The average employment and participation rate have been computed by using the same sample we consider to calibrate the variance and the cross-correlation of the shocks, as it is discussed below.

to be equal to the total amount of time devoted to home production by non-participants, moving from non-participation to unemployment would imply a loss of about 58 percent of home production time. Hence, we choose η and the unemployment benefit according to the following procedure. First, since Petrongolo and Pissarides (2001) and Mortensen and Nagypal (2007) we know that η and the replacement rate, though they are not easily identifiable from the data directly, have to lie respectively on the intervals $(0.3, 0.5)$ and $(0.2, 0.4)$. Then, we set η to the midpoint and we choose from the interval $(0.2, 0.4)$, the replacement rate $\frac{b}{W/P}$ implying a Γ that is as close as possible to 0.58. We obtain as a result $\frac{b}{W/P} = 0.4$ and $\Gamma = 0.45$. The value of ϕ is determined ex-post to implement the observed participation rate, $N_t = 0.6394$, while $\gamma = 0.6$ ensures that the Hosios condition holds.

3.2 The role of endogenous participation, sticky prices and the cost of search

The object of this section is to disentangle the incentives driving the participation decision. The intuition goes as follows. A shock can affect the marginal rate of substitution directly (e.g. preference shock) and/or indirectly through the presence of price rigidity and matching frictions, by changing the job finding rate and then the allocation of time between market and non-market activity. Households use the participation margin to keep the marginal rate of substitution between consumption and home production as close as possible to the one that would arise in a model with Walrasian labor markets.

We make the point by comparing the impulse responses of home production and participation to a positive one percent TFP shock for three versions of our model: a version with Walrasian labor markets and variable home production, to which we refer as the frictionless labor market case (or frictionless for short); a version with matching frictions but exogenous labor market participation; our full model with both matching frictions and endogenous labor market participation¹³. All versions feature the same steady state, apart from unemployment, which is constantly equal to zero, and vacancies and labor market tightness, which do not appear in the frictionless labor market model. We consider flexible as well as sticky prices. As it will be evident below, the interaction between marginal rate of substitution, matching frictions and nominal rigidities crucially depends on the cost of search, Γ and so does the participation decision. Hence, we consider, alongside the baseline calibration, also an alternative one with $1 - \eta = 0.95$.¹⁴ In fact, as shown in Table 1, the calibration strategy creates

¹³A detailed description of the model with exogenous participation is provided in the appendix. The only difference with our baseline version is that the labor force is assumed to be constant and equal to the observed participation rate.

¹⁴To meet the Hosios (1990) condition so that unemployment is efficient in the model, under this calibration we also change the matching function parameter to $\gamma = 0.95$. Keeping $\gamma = 0.6$, as in the baseline calibration, does not change the results.

a link between worker's bargaining power and the cost of search. In particular, under our baseline calibration, $\Gamma = 0.45$ while for the alternative calibration, $\Gamma = 1$ implying that unemployed workers spend all their time searching for a job and therefore cannot contribute to home production.¹⁵

Before inspecting impulse response functions, it is worth reminding the response of the economy to shocks in the frictionless model, which constitutes a useful benchmark. Conditionally on prices being flexible (Figure 1), with log utility, the income and substitution effects generated by a positive TFP shock exactly offset each other so that home production and participation do not move. Under sticky prices instead (Figure 2) the smaller reduction in prices induces the household to substitute less between market good and home production, relatively to the flexible price case and, as a result, participation declines in equilibrium while home production increases. In addition, it is also useful to recall that the search cost shapes the equilibrium relation between home production and participation when frictions are introduced. In the extreme case of $\Gamma = 1$, home production moves one to one with participation as

$$h_t = \xi_t(1 - N_t)^{1-\alpha_h}$$

Alternatively, if $\Gamma = 0$ home production moves one to one with employment

$$h_t = \xi_t(1 - E_t)^{1-\alpha_h}$$

It follows that if $\Gamma = 1$, given the participation decision, finding rates cannot affect the allocation of time. Participation to the market indeed is costly irrespectively of the employment status. In contrast, when $\Gamma = 0$, unemployment is costless in terms of home production, hence, given the participation decision, movements in the finding rate change the time allocated to the market.

Figure 1 displays the case of low and high bargaining power of workers, which in turn implies a low and high cost of search, under flexible prices. When the cost of search is low ($\eta = 0.4$) home production under exogenous participation is determined by frictions and it has to fall after a positive productivity shock, since the finding rate increases. This opens a gap with respect to the constant level that would be observed in the frictionless model. Then, active the participation margin, the household withdraws unemployed from the labor force, thus reducing participation. Matching frictions bind and the participation margin matters. When the cost of search is high ($\eta = 0.05$) instead, movements in the finding rate do not affect home production that is therefore constant even in the model with exogenous participation. Hence, a constant home production level, as in the case of the frictionless model, is achieved without the need to move participation. When prices are flexible and the cost of search is high the participation margin does not matter.

When prices are sticky (Figure 2) the TFP shock increases the price markup charged by firms. As a consequence, in the frictionless model the desired level of

¹⁵For example this calibration is close to the one considered by Gali (2010).

non-market activity also increases. However, if frictions are introduced and the search cost is low ($\eta = 0.4$), home production falls rather than increasing. Then the household reduces participation responding both to the undesirable reduction of home activity due to the increase of the finding rate and to the surge of the frictionless level of h_t . When the cost of search is high ($\eta = 0.05$) instead the finding rate does not matter for the allocation of time and participation responds to price rigidity only. This is the reason why the response of participation is muted, compared with the low search cost case.

To sum up, in response to a market technology shock, the finding rate always increases under our calibration, be prices sticky or not, so that home production falls if participation does not adjust. Given that the desired level of home production is constant under flexible prices and it increases under sticky prices, participation always declines to replicate the flexible labor market outcome and it does so by more when prices are sticky. However, when the search cost is high, participation always moves less, because finding rates play no role and the only driving force is price stickiness.

3.3 Impulse responses and second moments

Now that the incentives behind the participation decision have been clarified, we focus on the endogenous participation model and explore more in detail the transmission of the different shocks under the baseline calibration. To this purpose we first look at impulse response functions. Then, we compare the simulated unconditional moments with the ones observed in the data. Given that our final goal is to evaluate the policy predictions, relative to a matching frictions model with exogenous participation, we make sure that two versions of the model, one with and one without participation margin, are able to replicate business cycle evidence unconditionally. To assess the empirical fit of the model, we also need to choose the parameters governing the distribution of the shocks. Hence, we conclude the section with a calibration exercise.

Figure 3 shows the responses of several variables to a positive market productivity shock. Consistently with the discussion outlined above, participation falls and the household substitutes unemployment with voluntary non-employment to increase the level of home production. Both the increase of the finding rate and the outflow from the labor force drive down the unemployment rate, making it more responsive than it would be under exogenous participation.

Figure 4 considers the case of a positive preference shock to the market good. This shock influences directly the marginal rate of substitution inducing the household to demand less of home production and more of the market good. As a consequence, participation increases and the unemployment rate falls, though less than it would with exogenous participation. In fact, the surge in participation counterbalances the rise in the finding rate. Now movements in the participation margin dampen the reaction of labor market variables to a preference shock, the opposite to the case of a market productivity shock.

Figure 5 displays the case of higher productivity in the home production

technology. In terms of marginal rate of substitution this shock is very similar to the positive preference shock as it induces households to demand more of the market good, thus pushing output up. However, the improved technology makes feasible to produce more of the home goods with the same number of non-participants/unemployed. Then, now participation increases by more than with the previous shock, since it entails a lower loss in terms of home production. The increased labor supply pushes wages and prices down. Employment increases but not enough to compensate the surge in participation and the unemployment rate increases as well, even though unemployment falls.

Finally, Table 2 presents the results of our calibration exercise by showing the empirical moments, the simulated moments for the model with endogenous participation and the simulated moments for the model with exogenous participation. We use US data over the period 1964:1-2006:3. The sample start coincides with Gertler and Trigari (2009) and Krause and Lubik (2007) and the whole sample is the same as Christoffel and Kuester (2008). All data are from the Federal Reserve Bank of St. Louis' database FRED II and we apply a Hodrick-Prescott filter with a conventional smoothing parameter of 1600 to extract the business cycle component from the data in logs. Seasonality has been removed before filtering. We set the serial correlation of all shocks to 0.9 and then we calibrate the standard deviation of market technology, home production technology and preference shocks, and the cross-correlation of market and home technology so as to minimize the average distance of the simulated unconditional moments from their empirical counterparts. In particular, we consider the following targets: the standard deviation of output, the standard deviation of employment and of the unemployment rate relative to output and the correlation of the unemployment rate with output. We make a grid search to simultaneously determine the value of parameters minimizing the distance of the model from the data. It is evident that both models account well for business cycle fluctuations, though the version with exogenous participation performs slightly worse in terms of employment volatility relative to output. In order to evaluate the baseline version with endogenous participation, we look at the standard deviation of the participation rate relative to output and the correlation of the participation rate with output, two moments that we have *not* targeted. It is evident that the predictions of the model in terms of participation are well in line with the evidence.

4 Participation and Monetary Policy

The purpose of this section is twofold. On the one hand, we aim at assessing the relevance of the participation margin for predictions about the effects of monetary policy on volatilities and co-movements of macro variables. This is an interesting exercise, since the incentives driving participation interact with frictions. In turn, monetary policy affects the role of frictions in shaping the response of macro variables to shocks. As a consequence, it is natural to expect that the presence of the participation margin creates an additional transmission

channel of monetary policy, overlooked by the current literature. On the other hand, we investigate the robustness of the results to different values of households' search cost. We believe that the latter experiment is also interesting. In fact, we learnt from the previous analysis that the search cost determines the importance of labor market frictions relatively to nominal rigidities in driving the participation choice. This fact implies that the effects of monetary policy, partly acting through participation, may not be independent of the search cost.

To this end we compare the predictions of the endogenous and exogenous participation models for two values of ϕ_π , 1.5 and 100. Hence, we focus on the effect on macro moments of a policy switch from a flexible to a strict inflation targeting regime, where $\phi_\pi = 100$ implements the flexible price allocation. For each of the two models, we keep structural parameters at the value minimizing the distance from the data of the model predicted moments. This is because we want to give both models the same chance to fit the data unconditionally. However, note that parameters across the two models only differ in terms of standard deviations of the shocks and of the cross correlation between home and market technology. As an implication, *conditionally* on each of the shocks, differences in the predicted moments across models do not depend on the calibration. Therefore, when looking at the conditional moments, differences in the impact of the policy switch are *entirely* due to the presence or the lack of the participation margin.

The experiment shows that a change in the monetary policy rule affects business cycle moments in a way that is overlooked by models abstracting from the participation margin. In fact, when people can optimally reallocate time between market and home production activity, they choose to do so in such a way that the effect of a monetary policy regime change may be dampened, as in the case of market technology shocks, or magnified, as in the case of preferences or home technology shocks.

Tables 3-5 report the moments of macro variables for the case of $\phi_\pi = 1.5$ and strict inflation targeting, conditioning on one shock at a time, market technology, home technology and preference shocks respectively. Table 6 reports the same moments unconditionally, when all shocks hit.

As it is evident from Table 3, conditionally on market productivity shocks, strict inflation targeting magnifies the volatility of employment and unemployment rates in both models. This is because replicating the flexible price equilibrium eliminates inefficient fluctuations in price mark-ups and boosts the sensitivity of aggregate demand to market productivity. It follows that the positive response of vacancy posting and of the finding rates are higher. However, the exogenous participation model over-predicts the surge in the volatility of labor market variables. When the household indeed chooses the participation rate, the volatility of the labor force also increases. Under our calibration of the search cost the main incentive driving the participation decision is the matching friction rather than price mark-ups. Constant the participation rate, the stronger response of the finding rate induced by the regime switch would lead to a reallocation of time from home to market that the household dislikes. Hence, for the

household it is optimal to substitute unemployed with non-participant members and she does so, to a greater extent when monetary policy is more aggressive. The fall in the number of searching workers counterbalances the rise in vacancy posting. As a result, employment, the employment rate and the unemployment rate are less volatile, relative to a world where the household cannot adjust the participation margin.

Table 4 displays the case of home productivity shocks. In both models the volatility of output is higher under strict inflation targeting. As with market technology shocks, when price rigidity vanishes, demand and thus output are more responsive to productivity, due to the elimination of mark-ups time variation. However, under exogenous participation, the volatility of all macro variables varies proportionally with output volatility, so that the standard deviation of labor market variables relative to output does not change. When the participation margin is active, the picture is different. Table 4 makes straightforward to see that the different reaction of employment and unemployment rates to the policy change across models is entirely due to the participation margin. In fact, as well as in the case of exogenous participation, when the size of the labor force can be adjusted, the volatility of employment relative to output does not change. Still, employment and unemployment rates behave differently. A positive home production shock increases participation and the unemployment rate more than proportionally relative to output, so that the unemployment rate is always pro-cyclical irrespectively of the monetary policy regime. Since more aggressive monetary policy reduces the response of participation to the shock, then employment and unemployment rate fluctuations are also dampened.

As reported in Table 5, under preference shocks and absent the participation margin, again the policy rule does not affect macro moments relative to output. In contrast, when the household can choose the participation rate, the shock triggers a flow into the labor force and, under the baseline policy, an increase of output and vacancies, both driving down the unemployment rate. Strict inflation targeting magnifies the volatility of participation. As an implication, the larger flows to the labor force make the volatility of the employment rate and its correlation with output smaller. Under our calibration, the effect is so large that switching from flexible to strict inflation targeting changes the sign of the correlation between employment rate and output from positive to negative.

Finally, Table 6 replicates the experiment when all shocks hit. In this case the endogenous participation model predicts a fall in unemployment rate volatility when switching from flexible to strict inflation targeting. The exogenous participation model predicts the opposite. The result follows directly from the conditional analysis performed above. Under market technology shocks the exogenous participation model over-predicts the surge in unemployment volatility. In addition, it overlooks its fall, conditionally on home technology shocks. This facts explain the mistake in the policy evaluation of unconditional moments.

We conclude this section by repeating part of our previous analysis for a different value of the search cost. For the sake of concision, we restrict our attention to few moments, the volatility of employment and participation rates.

Also, we only focus on market technology shocks. However, our conclusions are general and carry over to other moments and shocks.

Table 7 shows the moments for the case of a high households' search cost, 0.99. Hence, price mark-ups are the only force driving the participation choice, since movements in the finding rate per se do not open any home production gap. We know from the impulse response analysis that in this case the participation margin is less relevant, compared to our benchmark. It follows that, the exogenous participation model over-predicts the impact of the policy change on the employment rate by less than under the baseline calibration.

Overall, our experiments point to two important conclusions: neglecting participation leads to mistakes in assessing the impact of policy on macro variables; the size of those mistakes is decreasing in households' search cost. Nevertheless, the latter conclusion does not weaken the former one. In fact, small mistakes are obtained for an implausibly high value of households' search costs, i.e. when it is assumed that moving from non-participation to unemployment implies a loss in home production, which is as large as the one suffered by members moving from non-participation to employment.

5 Conclusions

We introduced endogenous participation in an otherwise standard New Keynesian model with matching frictions. We used this laboratory economy to study how the introduction of the participation margin changes the way shocks are transmitted to the economy compared to other two cases: frictionless labor market with endogenous participation; matching frictions with exogenous participation. In particular, we showed that switching from a flexible to a strict inflation targeting regime has remarkably different implications on second moments once the participation margin is introduced. It increases employment and unemployment rate volatilities, conditionally on a TFP shock. However, the introduction of endogenous participation dampens such a surge in volatility compared to a model with (constant) exogenous participation. The same switch in monetary policy decreases the volatility of employment and unemployment rate conditionally on a home productivity or a preference shock, while it does not when participation is exogenous. Finally, once all shocks are considered, a policy of strict inflation targeting decreases the volatility of employment, unemployment rate and employment rate in our model while it increases them when participation is exogenous and constant.

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A Value of Employment

Let us rewrite utility recursively:

$$U_t = Z_t \log(C_t) + \phi \frac{[\xi_t(1 - E_t - \Gamma(N_t - E_t))]^{(1-\alpha_h)(1+\nu)}}{1 + \nu} + \beta E_t \{U_{t+1}\} \quad (\text{A.1})$$

We want to compute $\frac{\partial U_t}{\partial E_t}$ taking into account (2.3) and (2.5):

$$\frac{\partial U_t}{\partial E_t} = \frac{Z_t}{C_t} \left[\frac{W_t}{P_t} - b \right] - \phi h_t^\nu (1 - \Gamma) \xi_t (1 - \alpha_h) h_t^{-\frac{\alpha_h}{1-\alpha_h}} + \beta E_t \left\{ \frac{\partial U_{t+1}}{\partial E_t} \right\} \quad (\text{A.2})$$

Note that:

$$\begin{aligned} \frac{\partial U_{t+1}}{\partial E_t} &= (1 - \rho)(1 - f_{t+1}) \left[\frac{Z_{t+1}}{C_{t+1}} \left(\frac{W_{t+1}}{P_{t+1}} - b \right) - \phi h_{t+1}^\nu (1 - \Gamma) \xi_{t+1} (1 - \alpha_h) h_{t+1}^{-\frac{\alpha_h}{1-\alpha_h}} + \beta E_{t+1} \left\{ \frac{\partial U_{t+2}}{\partial E_{t+1}} \right\} \right] \\ &= (1 - \rho)(1 - f_{t+1}) \frac{\partial U_{t+1}}{\partial E_{t+1}} \end{aligned} \quad (\text{A.3})$$

therefore we can rewrite (A.2) as:

$$\frac{\partial U_t}{\partial E_t} = \frac{Z_t}{C_t} \left[\frac{W_t}{P_t} - b \right] - \phi h_t^\nu (1 - \Gamma) \xi_t (1 - \alpha_h) h_t^{-\frac{\alpha_h}{1-\alpha_h}} + E_t \left\{ (1 - \rho)(1 - f_{t+1}) \beta \frac{\partial U_{t+1}}{\partial E_{t+1}} \right\} \quad (\text{A.4})$$

Let $V_t^w \equiv \frac{\partial U_t}{\partial E_t} / U_{c,t} = \frac{\partial U_t}{\partial E_t} \frac{C_t}{Z_t}$ be the surplus from employment in terms of current consumption of the final good. Then,

$$V_t^w = \frac{W_t}{P_t} - b - \phi h_t^\nu (1 - \Gamma) \frac{C_t}{Z_t} \xi_t (1 - \alpha_h) h_t^{-\frac{\alpha_h}{1-\alpha_h}} + E_t \left\{ (1 - \rho)(1 - f_{t+1}) \beta \frac{C_t}{C_{t+1}} \frac{Z_{t+1}}{Z_t} V_{t+1}^w \right\} \quad (\text{A.5})$$

that coincides with equation (2.19) in the text.

Figure 1: Comparison between endogenous participation with matching frictions (CG), endogenous participation without matching frictions (No Frictions) and exogenous participation with matching frictions (Exogenous) under two calibrations of the search cost: high ($\eta = 0.05$) and low ($\eta = 0.4$) - Flexible Prices

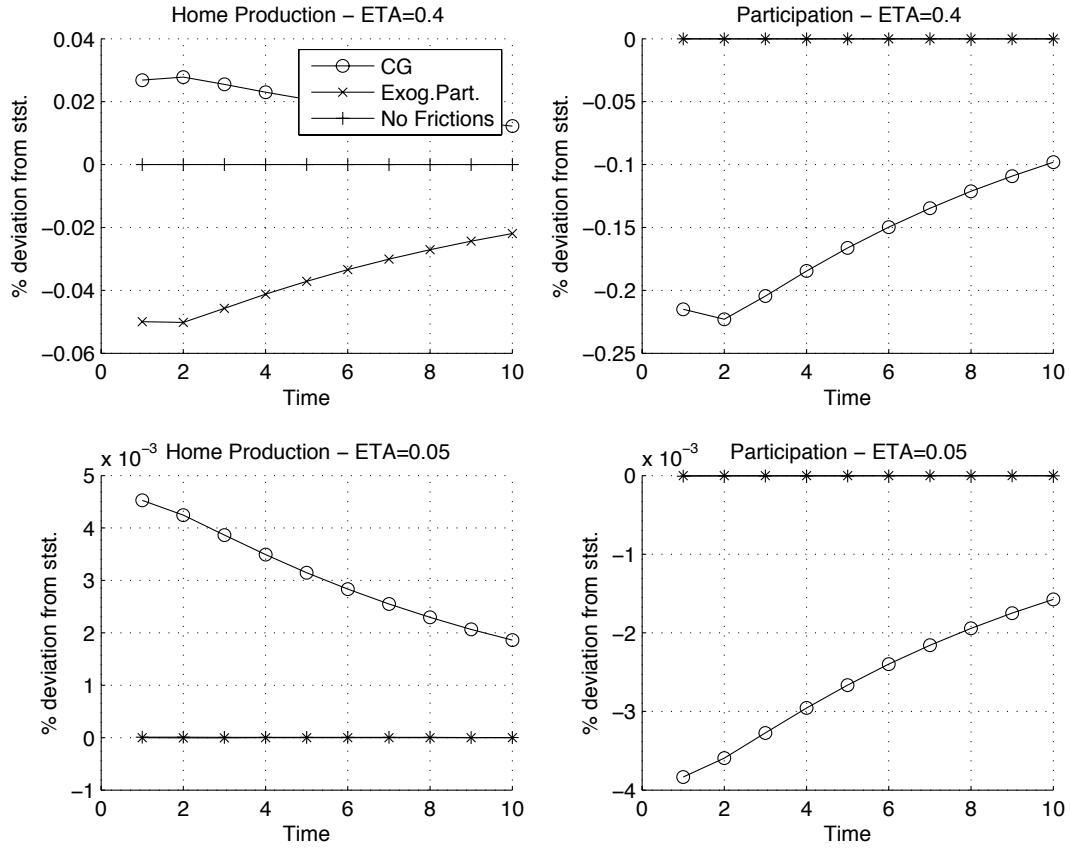


Figure 2: Comparison between endogenous participation with matching frictions (CG), endogenous participation without matching frictions (No Frictions) and exogenous participation with matching frictions (Exogenous) under two calibrations of the search cost: high ($\eta = 0.05$) and low ($\eta = 0.4$) - Sticky Prices

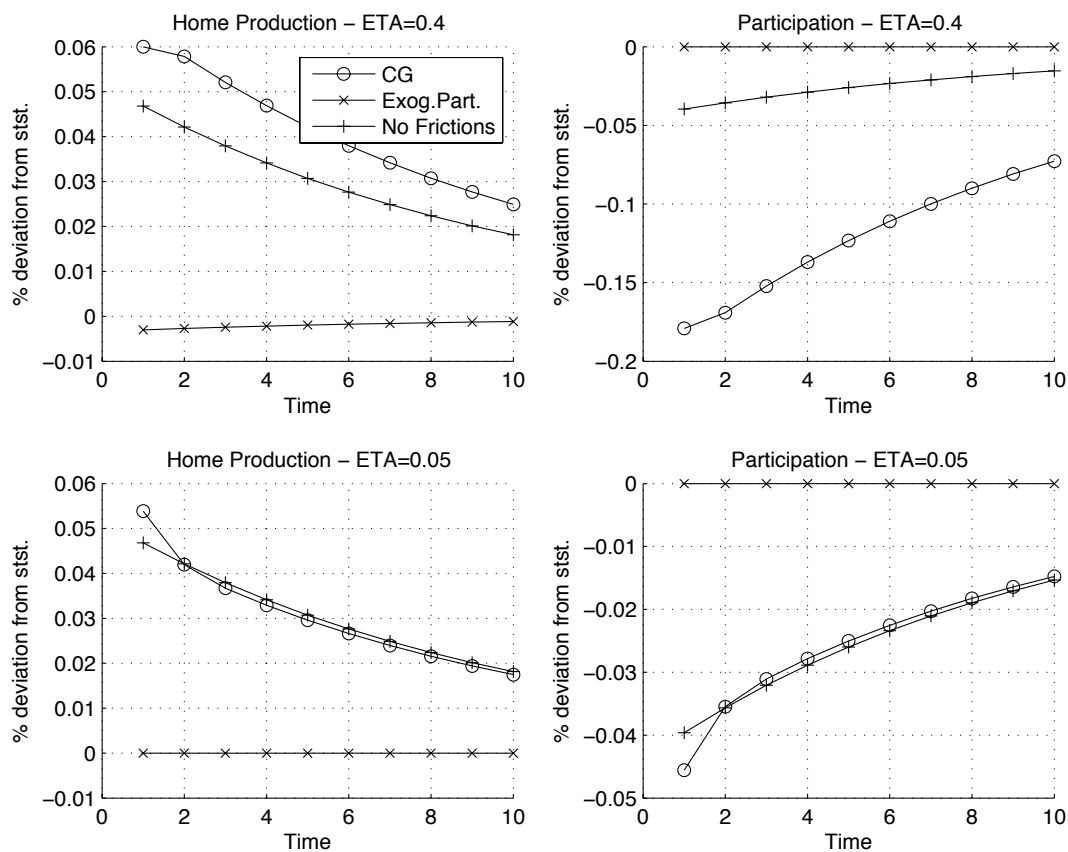


Figure 3: Impulse Responses to a market production TFP shock

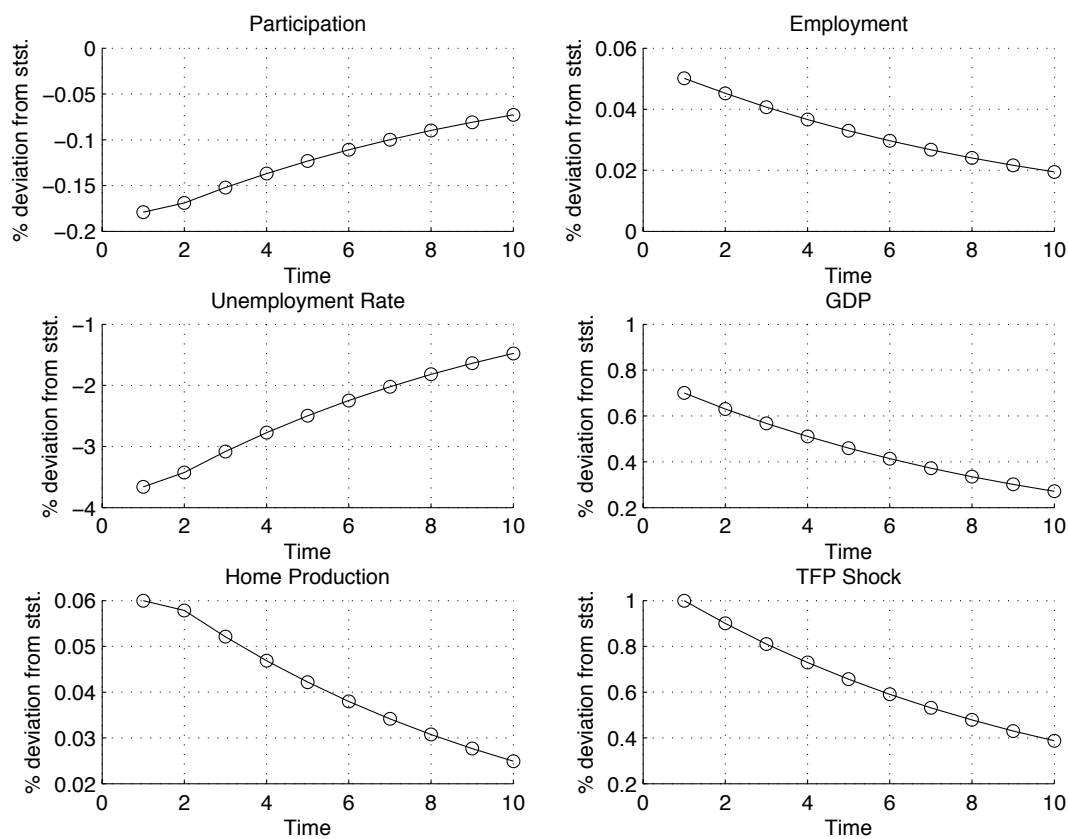


Figure 4: Impulse Responses to a preference shock

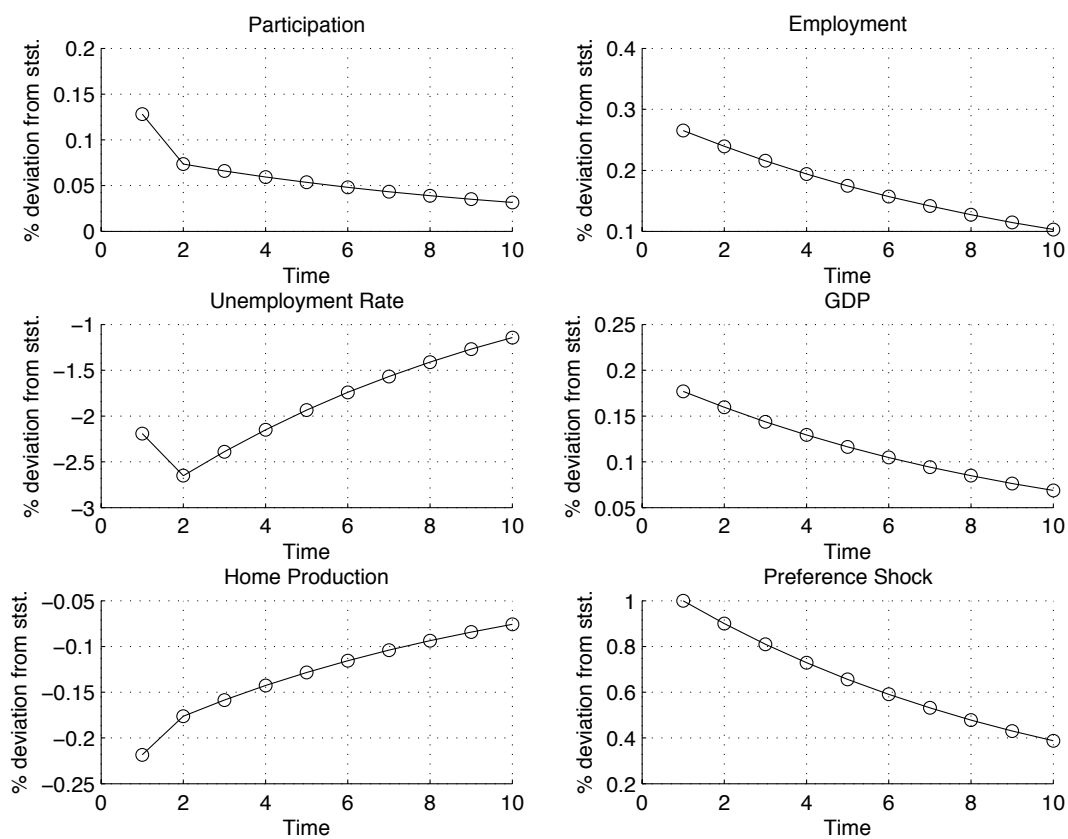


Figure 5: Impulse Responses to a home productivity shock

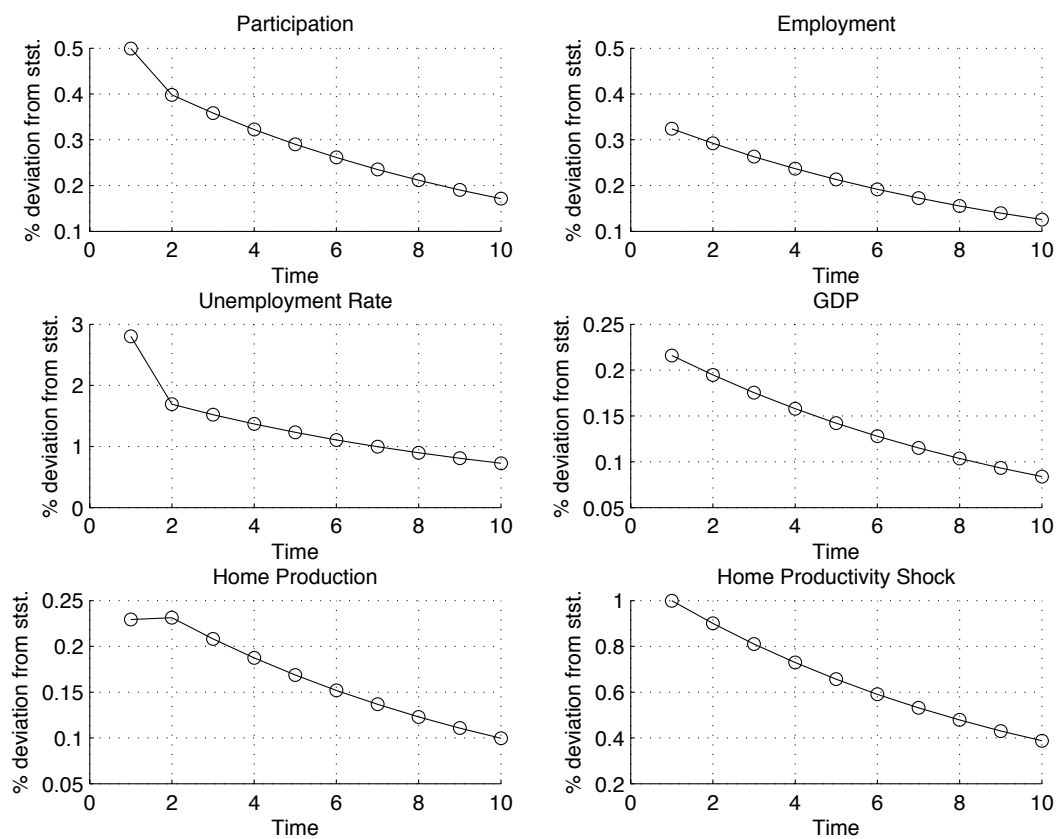


Table 1: Relation between worker's bargaining power ($1 - \eta$) and the cost of search Γ implied by the calibration strategy.

Workers' bargaining power	0.95	0.6
Γ	0.9997	0.4457

Table 2: Percentage standard deviations of output and of selected unconditional moments (relative to output) in the data, the endogenous and the exogenous participation models. Both models have been calibrated so as to give the best possible fit for the first 4 moments.

Unconditional Moments	Data	Endogenous	Exogenous
Output volatility	1.53	1.43	1.56
Unemployment rate volatility	7.40	7.36	7.55
Employment volatility	0.63	0.67	0.47
Correlation of Unemployment rate with Output	-0.85	-0.75	-1
Participation rate volatility	0.20	0.24	-
Correlation of Participation with Output	0.42	0.56	-
Calibrated Parameters			
st.dev. market TFP		0.0070	0.0074
st.dev. home TFP		0.0037	0.0070
st.dev. preference shock		0.0147	0
$corr_{A,AH}$		0.9474	1

Table 3: Percentage standard deviations of output and of selected moments (relative to output) in the endogenous and exogenous participation models, conditionally on market technology shocks. The table reports the value of moments under strict inflation targeting. In parenthesis it is reported the value for an inflation coefficient equal to 1.5 in the Taylor rule.

Moments Conditional on MTFP Shocks	Endogenous	Exogenous
Output volatility	1.24 (1.12)	1.17 (1.08)
Unemployment rate volatility	8.33 (5.40)	2.07 (0.12)
Employment volatility	0.20 (0.07)	0.13 (0.008)
Employment rate volatility	0.52 (0.34)	0.13 (0.008)
Participation rate volatility	0.32 (0.27)	0 (0)
Correlation of Participation with Output	-0.99 (-0.99)	0 (0)
Correlation of Unemployment rate with Output	-1 (-1)	-1 (-1)

Table 4: Percentage standard deviations of output and of selected moments (relative to output) in the endogenous and exogenous participation models, conditionally on home technology shocks. The table reports the value of moments under strict inflation targeting. In parenthesis it is reported the value for an inflation coefficient equal to 1.5 in the Taylor rule.

Moments Conditional on HTFP Shocks	Endogenous	Exogenous
Output volatility	0.20 (0.18)	0.52 (0.49)
Unemployment rate volatility	7.21 (9.63)	23.97 (23.97)
Employment volatility	1.5 (1.5)	1.5 (1.5)
Employment rate volatility	0.45 (0.60)	1.5 (1.5)
Participation rate volatility	1.89 (2.10)	0 (0)
Correlation of Participation with Output	0.99 (1)	0 (0)
Correlation of Unemployment rate with Output	0.84 (0.98)	-1 (-1)

Table 5: Percentage standard deviations of output and of selected moments (relative to output) in the endogenous and exogenous participation models, conditionally on preference shocks. The table reports the value of moments under strict inflation targeting. In parenthesis it is reported the value for an inflation coefficient equal to 1.5 in the Taylor rule.

Moments Conditional on Preference Shocks	Endogenous	Exogenous
Output volatility	0.29 (0.60)	0 (0)
Unemployment rate volatility	7.56 (15.91)	23.97 (23.97)
Employment volatility	1.5 (1.5)	1.5 (1.5)
Employment rate volatility	0.47 (1.00)	1.5 (1.5)
Participation rate volatility	1.92 (0.52)	0 (0)
Correlation of Participation with Output	0.99 (0.98)	0 (0)
Correlation of Unemployment rate with Output	0.85 (-0.99)	-1 (-1)

Table 6: Percentage standard deviations of output and of selected unconditional moments (relative to output) in the endogenous and exogenous participation models. The table reports the value of moments under strict inflation targeting. In parenthesis it is reported the value for an inflation coefficient equal to 1.5 in the Taylor rule.

Unconditional Moments	Endogenous	Exogenous
Output volatility	1.46 (1.43)	1.69 (1.56)
Unemployment rate volatility	6.52 (7.35)	8.79 (7.57)
Employment volatility	0.48 (0.67)	0.55 (0.47)
Employment rate volatility	0.41 (0.46)	0.55 (0.47)
Participation rate volatility	0.40 (0.24)	0 (0)
Correlation of Participation with Output	0.13 (0.56)	0 (0)
Correlation of Unemployment rate with Output	-0.90 (-0.76)	-1 (-1)

Table 7: Percentage standard deviations of output and of selected moments (relative to output) in the endogenous and exogenous participation models, conditionally on market technology shocks. Here we depart from the baseline calibration and we assume a high households' search cost, equal to 0.99. The table reports the value of moments under strict inflation targeting. In parenthesis it is reported the value for an inflation coefficient equal to 1.5 in the Taylor rule.

Moments	Endogenous		Exogenous	
	Values	Δ %	Values	Δ %
Employment rate volatility	0.1100 (0.0699)	52.47%	0.1107 (0.0726)	64.42%
Unemployment rate volatility	1.7682 (1.1606)	52.35%	1.7580 (1.0689)	64.46%
Participation rate volatility	0.0048 (0.0583)		0 (0)	