Preliminary and Incomplete

Risk Sharing, Human Model, and Limited Contract Enforcement

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Abstract

This paper develops a tractable production model with limited enforceability of contracts, provides a characterization of recursive equilibria that facilitates the computation of equilibria substantially, and uses a calibrated version of the model to show that limited contract enforcement has large macroeconomic effects. In the model, there are a large number of long-lived, risk-averse households who can invest in risk-free physical capital and risky human capital. Households have access to a complete set of credit and insurance contracts, but their ability to use the available financial instruments is limited by the possibility of default (endogenous borrowing/short-sale constraints). A calibrated version of the model, which is consistent with a number of micro-level facts (individual labor income risk, distribution of wealth and earnings growth), generates an equilibrium in which a large group of households is borrowing constrained and bears a high level of consumption risk. Moreover, limited contract enforcement is an important macroeconomic issue in the sense that removing financial frictions generates large growth and welfare gains.

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I. Introduction

There is by now a large body of empirical work documenting that individual households face a substantial amount of idiosyncratic labor income risk, and that this risk has a strong impact on individual consumption.[1] In other words, the hypothesis of perfect risk sharing (complete and frictionless financial markets) is strongly rejected by the data. An influential literature has suggested that this lack of individual consumption insurance may be explained by one particular financial friction, namely limited contract enforcement.[2] However, recent work in the literature has shown that realistically calibrated macro models with physical capital and production yield almost perfect risk sharing.[3] In other words, the theory fails to explain the very fact is was meant to explain. Moreover, any policy conclusion drawn from this class of models will be quantitatively indistinguishable from the recommendation implied by the corresponding model with frictionless financial markets.

In this paper, we develop a tractable production model with limited enforceability of contracts, provide a characterization of recursive equilibria that facilitates the computation of equilibria substantially, and use a calibrated version of the model to show that limited contract enforcement has large macroeconomic effects.

Our model is a version of the type of human capital model that has been popular in

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[1]For the estimation of income risk, see, for example, MaCurdy (1982), Carroll and Samwick (1997), Meghir and Pistaferri (2004), and Storesletten, Telmer, and Yaron (2004). For the consumption response, see, for example, Cochrane (1990), Flavin (1981), Townsend (1994), and Blundell, Pistaferri, and Preston (2008).

[2]See, for example, Alvarez and Jermann (2000), Kehoe and Levine (1993,2001), Kocherlakota (1996), Ligon, Thomas, and Worrall (2002), and Thomas and Worrall (1988).

[3]See Krueger and Perri (2006) for a highly influential paper in this literature. Note that the equilibrium allocation in Krueger and Perri (2006) matches the observed cross-sectional distribution of consumption levels fairly well, but the model implies a negligible volatility of individual consumption growth.

the endogenous growth literature.[4] More specifically, we consider a production economy with an aggregate constant-returns-to-scale production function using physical and human capital as input factors. There are a large number (a continuum) of individual households with CRRA-preferences who can invest in risk-free physical capital and risky human capital. Human capital investment is risky due to shocks to the stock of human capital that follow a stationary Markov process with finite support. In the main part of the paper, we assume that all shocks are idiosyncratic, but we also discuss how our theoretical characterization result can be extended to the case in which idiosyncratic and aggregate shocks co-exists. Households have access to a complete set of credit and insurance contracts, but their ability to use the available financial instruments is limited by the possibility of default (endogenous borrowing/short-sale constraints).[5] In contrast to the previous literature, we allow for two dimensions of contract enforcement that are parameterized by two distinct continuous variables: enforcement through the threat of exclusion of defaulting households from future risk sharing until a stochastically determined future date (punishment I) and enforcement through the threat of seizing a fraction of the physical capital owned by defaulting households (punishment II).[6]

Our quantitative analysis proceeds in two steps. First, we show that punishment I is a very ineffective means of enforcing contracts for realistic values of the exclusion parameter (around 7-10 years of exclusion after default), but punishment II is highly effective for

[4]See Lucas (1988), Jones and Manuelli (1990), and Rebelo (1991) for early contributions to the endogenous growth literature emphasizing human capital.

[5]There is also a large literature on default with incomplete markets. See, for example, Livshits, MacGee, and Tertilt (2007) and Chatterjee, Corbae, Nakajima, and Rios-Rull (2007).

[6]Punishment I is the enforcement mechanism that has been at the center of attention in the literature quoted in footnote 2, though this literature has usually assumed that defaulting households are excluded from financial markets forever (but see also Krueger and Uhlig (2006)). In contrast, punishment II has been emphasized by the voluminous macroeconomic literature on collateral constraints (for example, Kiyotaki and Moore (1997)). In this sense, the current paper provides an integrated approach to these two strands of the literature.

households holding close to the average amount of wealth in the economy. Thus, limited contract enforcement only has significant macroeconomic effects if there is a non-negligible group of households with little financial wealth and a strong incentive to borrow/insure, a condition that is not met by previous quantitative work based on production models. In a second step, we consider a particular version of our model with three equally large groups of households (young, middle-aged, old) facing different average human capital returns (high, medium, low), and show that the calibrated model generates an equilibrium outcome in which young households would like to borrow and buy insurance, but cannot do so because of the possibility of default. Moreover, the model implies that limited contract enforcement is an important macroeconomic issue: removing the financial friction generates large growth and welfare gains.

The methodological contribution of this paper is to develop a tractable model and to provide a general characterization result for recursive equilibria. More specifically, previous quantitative work has encountered two problems that have made the computation of recursive equilibria a daunting task even for relatively simple model structures. First, the wealth distribution is in general a relevant state variable, which implies that for models with a continuum of agents recursive equilibria are the solution to a complicated infinite-dimensional fixed point problem.[7] Second, in models with production, the participation constraint may introduce non-convexities into the choice sets of individual households preventing a simple application of the first-order condition approach. For the model developed in this paper, we show that these two problems can be circumvented. More precisely, we show that the original equilibrium problem can be transformed into an equivalent equilibrium problem in which choice sets are convex and the wealth distribution has become irrelevant. We believe

[7] This first point equally applies to standard incomplete-market models. Of course, the tractability issue becomes most pressing in models with both aggregate and idiosyncratic shocks. Because of space limitations, we only briefly outline how to extend our characterization result to the this case.

that this characterization result will provide a powerful tool for the quantitative analysis of a wide range of interesting macroeconomic issues that have so far not been formally analyzed because of computational limitations.

II. Model

In this section, we develop the model and define the relevant equilibrium concept.[8]

a) Time and Uncertainty

Time is discrete and indexed by $t = 0, 1, \ldots$. There is a continuum of infinitely-lived households of unit mass. There is no aggregate risk and we will confine attention to stationary equilibria, but in section h) we discuss the extension to aggregate shocks. Idiosyncratic risk is represented by a Markov shock process, $\{s_t\}$, where each s_t takes on a finite number of possible values. We denote by $s^t = (s_1, \ldots, s_t)$ the history of idiosyncratic shocks up to period t (date-event, node) and let $\pi(s^t) = \pi(s_t|s_{t-1}) \ldots \pi(s_2|s_1)$ stand for the probability that s^t occurs. At time $t = 0$, the type of an individual household is characterized by his initial state, $x_0 = (k_0, h_0, s_0)$, where s_0 denotes the initial shock, k_0 the initial stock of physical capital, and h_0 the initial stock of human capital (note that s_0 is not included in s^t). We take as given an initial measure, π_0, over initial types.

b) Production

There is one all-purpose good that can be consumed, invested in physical capital, or invested in human capital. Production of this one good is undertaken by one firm (a large number of identical firms) that rents capital and labor in competitive markets and uses these

[8] With its emphasis on physical and human capital investment within a convex framework, the model is similar to Krebs (2003), but Krebs (2003) assumes exogenous market incompleteness and i.i.d. shocks. Wright (2003) considers an "AK" model with i.i.d. shocks and limited enforcement that, in a certain sense, is a simplified version of the current set-up.

input factors to produce output, Y_t, according to the aggregate production function $Y_t = F(K_t, H_t)$. Here K_t and H_t are the (aggregate) levels of physical and human capital employed by the firm. The production function, F, is a standard neoclassical function, that is, it has constant-returns-to-scale, satisfies a Inada condition, and is continuous, concave, and strictly increasing in each argument. Given these assumptions on F, the derived intensive-form production function, $f(\tilde{K}) = F(\tilde{K}, 1)$, is continuous, strictly increasing, strictly concave, and satisfies a corresponding Inada condition, where we introduced the "capital-to-labor ratio" $\tilde{K} = K/H$. Given the assumption of perfectly competitive labor and capital markets, profit maximization implies

$$\begin{aligned} r_k &= f'(\tilde{K}) \\ r_h &= f(\tilde{K}) + f'(\tilde{K})\tilde{K} \ , \end{aligned} \tag{1}$$

where r_k is the rental rate of physical capital and r_h is the rental rate of human capital. Note that r_h is simply the wage rate per unit of human capital and that we dropped the time index because of our stationarity assumption. Clearly, (1) defines rental rates as functions of the capital to labor ratio: $r_k = r_k(\tilde{K})$ and $r_h = r_h(\tilde{K})$.

c) Preferences

At time $t = 0$, the type of an individual household is defined by his initial state (k_0, h_0, s_0). Households have identical preferences over consumption plans, $\{c_t\}$, where $\{c_t\}$ denotes a sequence of functions (random variables), c_t, mapping shock histories, s^t, into consumption levels, $c_t(s^t)$. Similar notation will be used for investment plans (see below). Households are risk-averse and their preferences allow for a time-additive expected utility representation:[9]

$$U(\{c_t\}|s_0) \doteq E\left[\sum_{t=0}^{\infty} \beta^t u(c_t)|s_0\right] \ , \tag{2}$$

[9]The notation $E\left[\sum_{t=0}^{\infty} \beta^t u(c_t)|s_0\right]$ stands for $\sum_{s^t} \sum_{t=0}^{\infty} \beta^t u(c_t(s^t))\pi(s^t|s_0)$.

where the expectations in (2) is taken over all histories, s^t, keeping the initial type, (k_0, h_0, s_0), fixed. We assume that the one-period utility function exhibits constant relative risk aversion: $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$ for $\gamma \neq 1$ and $u(c) = lnc$ otherwise. In other words, we assume that preferences are homothetic.

d) Budget Constraint

Each household can invest in physical capital, k, or human capital, h. In addition, he can buy and sell a complete set of financial contracts (assets) with state-contingent payoffs. More specifically, there is one contract (Arrow security) for each state, and we denote by $a_{t+1}(s_{t+1})$ the quantity bought in period t (sold if negative) of the contract that pays off one unit of the good in period $t+1$ if s_{t+1} occurs. Given his initial type, (k_0, h_0, s_0), a household chooses a plan, $\{c_t, k_t, h_t, \vec{a}_t\}$, where the notation $\vec{a}$ indicates that in each period the household chooses a vector of contract holdings. A budget-feasible plan has to satisfy the sequential budget constraint

$$\begin{aligned} c_t + k_{t+1} + h_{t+1} + \sum_{s_{t+1}} a_{t+1}(s_{t+1}) q(s_{t+1}) &= (1 + r_k - \delta_k) k_t + (1 + r_h - \delta_h(s_t)) h_t + a_t(s_t) \\ c_t \geq 0 \;\; , \;\; k_{t+1} \geq 0 \;\; , \;\; h_{t+1} \geq 0 \qquad &, \end{aligned} \tag{3}$$

where $q(s_{t+1})$ is the price of a financial contract that pays off if s_{t+1} occurs and δ_k and $\delta_h(s_t)$ are the depreciation rates of physical and human capital, respectively. Note that (3) has to hold in realizations, that is, it has to hold for all histories, s^t.

The budget constraint (3) assumes that physical capital can be accumulated by investing $k_{t+1} - (1 - \delta_k) k_t$. Similarly, human capital can be accumulated by investing $h_{t+1} - (1 - \delta_h(s_t)) h_t$. The budget constraint (3) makes four implicit assumptions about the accumulation of human capital. First, it lumps together general human capital (education, health) and specific human capital (on-the-job training). Second, it follows Jones and Manuelli (1990) and Rebelo (1991) by focusing on the direct monetary costs of human capital investment. In contrast, Ben-Porath (1967) and Lucas (1988) consider the indirect costs that arise when households have to allocate a fixed amount of time between work and human capital investment.[10] However, it can be shown that the current formulation is equivalent to a set-up where households have access to a linear human capital production technology and allocate a fixed amount of time between work and human capital investment. Third, our current formulation of the household problem neglects labor-leisure decision, but extending the model to allow for a labor-leisure choice is straightforward. Fourth, (3) does not impose a non-negativity constraint on human capital investment ($x_{hit} \geq 0$). In our numerical applications this inequality is always satisfied so that adding this constraint would not change the equilibrium (see section III for details), but this is not a general property of the model.

The random variable δ_{ht} represents uninsurable idiosyncratic labor income risk. A negative human capital shock could be due to the loss of firm- or sector-specific human capital subsequent to job termination (worker displacement). The budget constraint (3) assumes that the wage payment is received in each period, but it is straightforward extension of the current model to allow the wage rate, r_h, to depend on the idiosyncratic shock (unemployment). A decline in health (disability) provides a second example for a negative human capital shock. In this case, both general and specific human capital might be lost. Internal promotions and upward movement in the labor market provide two examples of positive human capital shock.

It is convenient to introduce new variables that emphasize that individual households solve a standard inter-temporal portfolio choice problem (with additional participation con-

[10] Trostel (1993) provides a careful analysis of the different types of human capital investment and estimates that for the US about one half of the cost of human capital investment is direct expenditures on human capital investment goods.

straints). To this end, introduce the following variables:

$$\begin{aligned}
w_t &= k_t + h_t + \sum_{s_t} q_{t-1}(s_t)a_t(s_t) \\
\theta_{kt} &= \frac{k_t}{w_t} \ , \ \ \theta_{ht} = \frac{h_t}{w_t} \ , \ \ \theta_{at}(s_t) = \frac{a_t(s_t)}{w_t} \\
1 + r_t &= (1 + r_k - \delta_k)\theta_{kt} + (1 + r_h - \delta_h(s_t))\theta_{ht} + \theta_{at}(s_t)
\end{aligned} \tag{4}$$

In (4) the variable w_t stands for beginning-of-period wealth consisting of real wealth, $k_t + h_t$, and financial wealth, $\sum_{s_t, S_t} q_{t-1}(s_t)a_t(s_t)$. The variable $\theta_t = (\theta_{kt}, \theta_{ht}, \vec{\theta}_{at})$ denotes the vector of portfolio shares and $(1 + r)$ is the gross return to investment. Using the new notation, the budget constraint (4) reads

$$\begin{aligned}
w_{t+1} &= (1 + r_t)\, w_t - c_t \\
1 &= \theta_{kt} + \theta_{ht} + \sum_{s_t} q_{t-1}(s_t)\theta_{at}(s_t) \\
c_t &\geq 0 \ , \ \ w_t \geq 0 \ , \ \ \theta_{kt} \geq 0 \ , \ \ \theta_{ht} \geq 0 \ .
\end{aligned} \tag{5}$$

Clearly, (5) is the budget constraint corresponding to an intertemporal portfolio choice problem with linear investment opportunities and no exogenous source of income.

So far, we have not imposed any restrictions on trading of financial assets. In this paper, we augment the sequential budget constraint by the following short-sale constraints:

$$\theta_{at}(s_t) \geq -\bar{\theta}_a(s_t) \ , \tag{6}$$

where $\bar{\theta}(s_t)$ is a number that will be chosen large enough so that it will not bind in equilibrium. In this case, (6) is equivalent to a no-Ponzi-scheme condition if the interest rate is positive. However, in contrast to the no-Ponzi-scheme condition, the short-sale constraint (6) has three advantages. First, it allows us to consider equilibria with non-positive interest rates. Second, it nicely fits into a recursive formulation of the problem. Finally, it will be useful for the proof of proposition 2.

In this paper, we confine attention to equilibria in which financial contracts are priced in a risk-neutral manner,

$$q_t(s_{t+1}) \quad = \quad \frac{\pi(s_{t+1}|s_t)}{1+r_f} \ , \tag{7}$$

where r_f is the interest rate on financial transactions. Note that this interest rate is in general different from the rate of return on physical capital investment, $r_k - \delta_k$. The pricing equation (7) can be interpreted as a zero-profit condition. More precisely, consider financial intermediaries that sell insurance contracts to individual households and invest the proceeds in the risk-free asset that can be created from the complete set of financial contracts yielding a certain return r_f. Given that financial intermediaries face linear investment opportunities and assuming no quantity restrictions on the trading of financial contracts for financial intermediaries, equilibrium requires that financial intermediaries make zero profit, namely condition (7).

e) Participation/Enforcement Constraint

In addition to the standard budget constraint, the household has to satisfy a sequential enforcement (participation) constraint, which ensures that at no point in time individual households have an incentive to default on their financial obligations. More precisely, individual consumption plans have to satisfy

$$E\left[\sum_{n=0}^{\infty}\beta^n u(c_{t+n})|w_0,\theta_0,s_0,s^t\right] \geq V_d(w_t,\theta_{kt},\theta_{ht},s_t) \ , \tag{8}$$

where V_d is the value function of a defaulting household (autarky value) defined as follows.

A household who defaults in period t, all the short and long positions in financial assets are canceled, $\theta_{at}(s_t) = 0$, which is the reason why the function V_d only depends on $(\theta_{kt}, \theta_{ht})$. Further, there are two types of punishment for default. First, upon default households lose a fraction ϕ of their physical capital (but keep all their human capital). Second, defaulting households will be excluded from participation in financial markets, $\theta_{a,t+n}(s_{t+n}, S_{t+n}) = 0$, until a stochastically determined future date that occurs with probability $(1-p)$ in each period, that is, the probability of remaining in (financial) autarky is p.[11] After regaining access to financial markets, the household's expected continuation value is $V^e(w, \theta, s)$, where (w, θ, s) is the individual state at the time of regaining access. For the individual household the function V^e is taken as given, but we will close the model and determine this function endogenously by requiring that $V^e = V$, where V is the equilibrium value function associated with the maximization problem of a household who participates in financial markets. In other words, we assume rational expectations.[12] Note that the two parameters ϕ and p correspond to two different dimensions of contract enforcement (punishment I and punishment II), and that an increase in either parameter amounts to an improvement in contract enforcement.

Finally, with respect to the use of physical and human capital after default, we consider two different scenarios. In the first case, defaulting households still participate in the market for physical and human capital, that is, they rent out their physical and human capital at the going market rate. In the second case, households become self-employed and use their physical and human capital to produce according to the neoclassical production function (1). In either case, a household in autarky faces an investment return function $r_{dt} = r_d(\theta_{kt}, \theta_{ht})$, where $r_d = \theta_k(r_k - \delta_k) + \theta_h(r_h - \delta_h(s))$ in the first case and $r_d = F(\theta_k, \theta_h) - \delta_k\theta_k - \delta_h(s)\theta_h$ in the second case.

In the appendix, we show that if condition (A4) is satisfied and the expected value

[11] The previous literature has usually assumed $p = 1$ (permanent autarky). There is also a literature that only rules out short positions after default (see Hellwig and Lorenzoni (2009) for a recent example). *Mutatis mutandis*, our theoretical results will also go through for this case.

[12] See also Krueger and Uhlig (2006) for a similar approach. Note that the credit (default) history of an individual household is not a state variable affecting the expected value function, V^e. Thus, we assume that credit (default) history of households is information that cannot be used for contracting purposes.

function has the functional form

$$V^e(w,\theta_k,\theta_h,s) \;=\; \begin{cases} \tilde{V}^e(s)\,(1+r_d(\theta_k,\theta_h,s))^{1-\gamma}\,w^{1-\gamma} & if\;\gamma\;\neq\;1 \\ \tilde{V}^e(s)\;+\;\frac{1}{1-\beta}log(1+r_d(\theta_k,\theta_h,s))\;+\;\frac{1}{1-\beta}logw & otherwise \end{cases}\;, \tag{9}$$

then the autarky value function is given by

$$V_d(w,\theta_k,\theta_h,s) \;=\; \begin{cases} \tilde{V}_d(s)\,(1+r_d(\theta_k,\theta_h,s)-\phi\theta_k)^{1-\gamma}\,w^{1-\gamma} & if\;\gamma\;\neq\;1 \\ \tilde{V}_d(s)\;+\;\frac{1}{1-\beta}log(1+r_d(\theta_k,\theta_h,s)-\phi\theta_k)\;+\;\frac{1}{1-\beta}logw & otherwise \end{cases}\;, \tag{10}$$

where the intensive-form value function, $\tilde{V}_d$, is the unique solution to the intensive-form Bellman equation (A3). The proof is a standard application of dynamic programming with unbounded payoff (utility) function. Note that r_d enters into our specification of V^e, which amounts to the assumption that in the period of re-entering financial markets households use their physical and human capital according to the autarky specification. Note also that the equilibrium condition $V^e = V$ boils down to the condition $\tilde{V}^e = \tilde{V}$, where $\tilde{V}$ is the intensive-form value function of a household with access to financial markets.

f) Aggregate State

In this paper, we focus on recursive equilibria. In the main part of the paper, we consider the case of no aggregate shocks and confine attention to stationary recursive equilibria, but in section h) we outline how to extend the analysis to the general case with both idiosyncratic and aggregate shocks.

For the concept of a recursive equilibrium, the definition of the relevant state space is essential. The state of an individual household is (w,θ,s), where (w,θ) is the endogenous part of the individual state. For the aggregate state, it turns out that Ω is a sufficient statistic, where $\Omega \in \mathbb{R}^n$ is the finite-dimensional vector (distribution) of relative wealth shares of a household with current shock s_t:

$$\Omega_t(s_t) \;\doteq\; \frac{E\left[(1+r_t)w_t|s_t\right]}{E\left[(1+r_t)w_t\right]}\;.$$

Note that $\sum_{s_t} \Omega_t(s_t)\pi(s_t) = 1$ and that $(1+r_t)w_t$ is individual wealth including current asset payoffs.

In a recursive equilibrium, the evolution of the endogenous aggregate state variable is given by an endogenous law of motion $\Omega_{t+1} = \Phi(\Omega_t)$. Moreover, rentals rates and the interest rate on financial transactions become functions of the aggregate state:

$$\begin{aligned} r_{k,t+1} &= r_k(\Omega_t) \\ r_{h,t+1} &= r_h(\Omega_t) \\ r_{ft} &= r_f(\Omega_t) \end{aligned} \tag{11}$$

In a stationary recursive equilibrium, the stationarity condition

$$\Omega = \Phi(\Omega) \tag{12}$$

is satisfied. This stationarity condition in conjunction with (8) implies that rental rates and the interest rate are time-independent. Thus, individual households face constant aggregate investment returns. Given these aggregate returns and the idiosyncratic shock process, each household chooses a plan, $\{w_t, \theta_t, c_t\}$, that maximizes expected lifetime utility (2) subject to the budget constraint (5,6) and the participation constraint (8).

g) Equilibrium

In equilibrium, a household of type (w_0, θ_0, s_0) chooses a plan, $\{c_t, w_t, \theta_t\}$, where $\theta_t = (\theta_{kt}, \theta_{ht}, \vec{\theta}_{at})$. The collection of plans, one for each initial type, defines a global plan or allocation. Given a global plan, we find the aggregate capital stock held by all households as follows:[13]

$$\begin{aligned} K_{t+1} &= E\left[k_{t+1}\right] \\ &= E\left[\theta_{k,t+1} w_{t+1}\right] \end{aligned} \tag{13}$$

[13]The notation $E\left[x_{t+1}\right]$ is short-hand for $\int_{w_0,\theta_0,s_0} \sum_{s^t} x_{t+1}(w_0, \theta_0, s_0, s^t)\pi(s^t|s_0)\pi(w_0, \theta_0, s_0)$ for any household-specific variable x. Similar notation will be used throughout the paper.

Note that $\theta_k w$ is simply the physical capital stock of an individual household and $\theta_h w$ the corresponding human capital stock. In equilibrium, the level of physical and human capital demanded by the firm must be equal to the corresponding aggregate levels supplied by households. Because of the constant-returns-to-scale assumption, only the ratio of physical to human capital is pinned down by this market clearing condition. That is, in equilibrium we must have

$$\tilde{K}_{t+1} = \frac{E\left[\theta_{k,t+1} w_{t+1}\right]}{E\left[\theta_{h,t+1} w_{t+1}\right]} \,, \tag{14}$$

where $\tilde{K}$ is the capital-to-labor ratio chosen by the firm.

The second market clearing condition requires that no resources are created or destroyed by trading in financial contracts. In other words, the total value of all financial asset holdings has to sum to zero:

$$\sum_{s_{t+1}} E\left[q_t(s_{t+1})\theta_{a,t+1}(s_{t+1}) w_{t+1}\right] \; = \; 0 \,. \tag{15}$$

Straightforward calculation shows that the two market clearing conditions in conjunction with the budget constraint (5) imply the standard aggregate resource constraint (goods market clearing). Moreover, using the pricing condition (7), the asset market clearing condition (15) reads:

$$E\left[\theta_{a,t+1} w_{t+1}\right] \; = \; 0 \,. \tag{16}$$

To sum up, we have the following definition of equilibrium:

Definition A stationary recursive equilibrium is a law of motion, Φ, for the aggregate state variable, Ω, a pricing function (11) mapping the state of the economy, Ω, onto (r_k, r_h, r_f), an expected value function, V^e, and a household policy function, g,[14] so that
i) Utility maximization of households: for each household type, $(w_0, \theta_0, s_0) = (w_0, \theta_0, s_0)$, the

[14]The function g defines next period's endogenous state as a function of this period's endogenous state and this period's exogenous shock:$(w_{t+1}, \theta_{t+1}) = g(w_t, \theta_t, s_t)$.

household policy function, g, generates a plan, $\{c_t, w_t, \theta_t\}$, that maximizes expected lifetime utility (2) subject to the sequential budget constraint (5), the short-sale constraint (6), and the sequential participation constraint (8).

ii) Profit maximization of firms: aggregate capital-to-labor ratio and rental rates satisfy the first-order conditions (1).

iii) Financial intermediation: financial contracts are priced according to (7)

iv) Market clearing: equations (14) and (16) hold.

v) Rational expectations: $V^e = V$ and Φ is the law of motion induced by g.

vi) Stationarity: condition (12) is satisfied.

h) Extension: Aggregate Shocks

Suppose now that there are idiosyncratic shocks, s, and aggregate shocks, S, and that uncertainty is described by a stationary joint Markov process $\{s_t, S_t\}$ with transition probabilities denoted by $\pi(s_{t+1}, S_{t+1}|s_t, S_t)$. The relevant aggregate state then becomes (Ω_t, S_t), where Ω_t is defined as before. In a recursive equilibrium, the evolution of the endogenous aggregate state variable is given by an endogenous law of motion $\Omega_{t+1} = \Phi(\Omega_t, S_t, S_{t+1})$. Further, rentals rates and the interest rate on financial transactions become functions of the aggregate state: $r_{k,t+1} = r_k(\Omega_t, S_t, S_{t+1}) r_{h,t+1} = r_h(\Omega_t, S_t, S_{t+1}) r_{ft} = r_f(\Omega_t, S_t)$. The definition of a recursive equilibrium is, *mutatis mutandis*, as before.

A straightforward, but rather tedious extension of the subsequent theoretical analysis shows that a modified version of our general characterization result (proposition 4) still holds. In particular, recursive equilibria can be computed by solving a convex problem that is independent of the wealth distribution, though clearly the finite-dimensional distribution of relative wealth, Ω, still enters into the equilibrium conditions. Notice, however, that in many application the relevant part of Ω is low-dimensional. More precisely, we will show below that for i.i.d. shocks the portfolio choice of households becomes shock-independent, which implies in conjunction with the intensive-form market clearing conditions (22,23) that the corresponding marginal distribution does not enter into the equilibrium determination (see section d) of the theoretical section for details). Thus, if the idiosyncratic shock has several components, $s = (s_1, ..., s_n)$, then we have to keep track only of the marginal distribution over those components that are serially correlated. For instance, in our example 1 of the quantitative section (one-type case) the set of all relevant marginal distributions is null-dimensional and in example 2 (three-type case) the set corresponding set is two-dimensional.

III. Theoretical Results

In this section, we present the main theoretical results. We begin with the proof of the functional-form result (10) for the value function, V_d, of a household in financial autarky (proposition 1). We then prove the principle of optimality and the equivalence between intensive-form Bellman equation and extensive-form Bellman equation for the maximization problem of an individual household with access to financial markets, and therefore with participation constraint (proposition 2 and 3). Based on this result and the nature of market clearing, we then show the equivalence between stationary recursive equilibria and intensive-form equilibria (proposition 4). Finally, we analyze the empirically important case of i.i.d. human capital shocks and discuss the computation of equilibria using our characterization result.

a) Household problem in financial autarky

In our previous discussion, we have already stated that the solution to the household maximization problem in financial autarky is particularly simple and given by (12). We now formally state and proof this result:

Proposition 1. Suppose condition (A4) is satisfied and the expected value function, V^e, is continuous. Then the value function, V_d, associated with the maximization problem of an individual household in financial autarky is the unique continuous solution to the Bellman equation (A3). If the expected value function, V^e, has the functional form (9), then the value function, V_d, has the functional form (10), where the intensive-form value function, $\tilde{V}_d$, is the unique solution to the intensive-form Bellman equation (A6).

Proof. See appendix.

b) Household problem with financial market access

For the maximization problem of a household with access to financial markets facing the sequential participation constraint (8), we conjecture that the solution is recursive in the state variable (w, θ, s). To be more precise, consider the Bellman equation

$$\begin{aligned} V(w,\theta,s) &= \max_{w',\theta'} \left\{ u\left((1+r(\theta,s))w - w'\right) + \beta \sum_{s''} V\left(w',\theta',s'\right)\pi(s'|s) \right\} \\ s.t. \quad 1 &= \theta'_k + \theta'_h + \sum_{s'} \frac{\pi(s'|s)\theta'_a(s')}{1+r_f} \\ 0 \leq w' &\leq (1+r(\theta,s))w \quad , \quad \theta'_k \geq 0 \quad , \quad \theta'_h \geq 0 \quad , \quad \theta'_a(s') \geq -\bar{\theta}_a(s') \\ V(w',\theta',s') &\geq V_d(w',\theta',s') \end{aligned} \tag{17}$$

and the corresponding Bellman operator, T, defined in the canonical way. Proposition 2 below shows that the principle of optimality applies if the condition that for all s

$$\begin{aligned} \forall\theta' &: \beta \sum_{s'} \left(1+r(\theta',s')\right)^{1-\gamma} \pi(s'|s) < 1 \quad if \;\; 0 < \gamma < 1 \\ \exists\theta' &: \beta \sum_{s'} \left(1+r(\theta',s')\right)^{1-\gamma} \pi(s'|s) < 1 \quad if \;\; \gamma > 1 \end{aligned} \tag{18}$$

holds.[15] The proposition shows further how the value function, which is identical with the maximal solution of the Bellman equation (17), can be found by iterating on the solution to the Bellman equation without participation constraint:

[15]Note that for the log-utility case, no condition of the type (18) is required.

Proposition 2. Suppose that condition (18) is satisfied and that the law of motion, Φ, and the value function of a household in financial autarky, V_d, are continuous. Let T stand for the operator associated with the Bellman equation (17). Then

i) There is a unique continuous solution, V_0, to the Bellman equation (18) without participation constraint.

ii) $\lim_{n\to\infty} T^n V_0 = V_\infty$ exists and is the maximal solution to the Bellman equation (17)

iii) V_∞ is the value function, V, of the sequential household maximization problem.

Proof. See appendix.

Comment on the proposition; method of proof (monotone operator theorem).

Using proposition 2 and an induction argument, we next show (proposition 3 below) that the value function, V, has the functional form

$$V(w,\theta,s) = \begin{cases} \tilde{V}(s)(1+r(\theta,s))^{1-\gamma}w^{1-\gamma} & if\ \gamma \neq 1 \\ \tilde{V}(s) + \frac{1}{1-\beta}log\,(1+r(\theta,s)) + \frac{1}{1-\beta}w & otherwise \end{cases} \tag{19}$$

and that the corresponding optimal policy function, g, is

$$c(w,\theta,s) = \begin{cases} \tilde{c}(s)(1+r(\theta,s))w & if\ \gamma \neq 1 \\ (1-\beta)(1+r(\theta,s))w & otherwise \end{cases} \tag{20}$$

$$w'(w,\theta,s) = \begin{cases} (1-\tilde{c}(s)(1+r(\theta,s))w & if\ \gamma \neq 1 \\ \beta(1+r(\theta,s))w & otherwise \end{cases}$$

$$\theta'(w,\theta,s) = \theta'(s)\ .$$

In other words, the value function has the functional form of the underlying utility function, consumption and next-period wealth are linear functions of this-period wealth, and next-period portfolio choices only depend on the current shock. Moreover, proposition 3 also shows that the intensive-form value function, $\tilde{V}$, together with the optimal consumption and portfolio choices, $\tilde{c}$ and θ, can be found by solving an intensive-form Bellman equation that reads

$$\begin{aligned}
\tilde{V}(s) \quad &= \quad \max_{\tilde{c},\theta'} \left\{ \frac{\tilde{c}^{1-\gamma}}{1-\gamma} + \beta(1-\tilde{c})^{1-\gamma} \sum_{s'} \left(1 + r(\theta', s')\right)^{1-\gamma} \tilde{V}(s')\pi(s'|s) \right\} \\
& s.t. \quad 1 \ = \ \theta'_k + \theta'_h + \sum_{s'} \frac{\theta'_a(s')\pi(s'|s)}{1 + r_f} \\
& \qquad 0 \ \leq \ \tilde{c} \ \leq \ 1 \ , \ \ \theta'_k \geq 0 \ , \ \ \theta'_h \geq 0 \ , \ \ \theta'_a(s') \geq \bar{\theta}_a(s') \\
& \qquad \left(\frac{\tilde{V}(s')}{\tilde{V}_d(s')} \right)^{\frac{1}{1-\gamma}} (1 + r(\theta', s')) \ \geq \ 1 + r_d(\theta'_k, \theta_h, s') - \phi\theta'_k
\end{aligned} \tag{21}$$

for $\gamma \neq 1$ and

$$\begin{aligned}
\tilde{V}(s) \quad &= \quad \max_{\theta'} \left\{ log(1-\beta) \ + \ \frac{\beta}{1-\beta} log\beta \ + \ \frac{\beta}{1-\beta} \sum_{s'} log(1 + r(\theta', s')\pi(s'|s) \right. \\
& \qquad \left. + \beta \sum_{s'} \tilde{V}(s')\pi(s'|s) \right\} \\
& s.t. \quad 1 \ = \ \theta'_k + \theta'_h + \sum_{s'} \frac{\theta'_a(s')\pi(s'|s)}{1 + r_f} \\
& \qquad \theta'_k \geq 0 \ , \ \ \theta'_h \geq 0 \ , \ \ \theta'_a(s') \geq \bar{\theta}_a(s') \\
& \qquad e^{(1-\beta)\left(\tilde{V}(s') - \tilde{V}_d(s')\right)} (1 + r(\theta', s')) \ \geq \ 1 + r_d(\theta'_k, \theta'_h, s') - \phi\theta'_k
\end{aligned}$$

for the log-utility case.

Proposition 3. Suppose that condition (18) is satisfied, the law of motion, Φ, is continuous, and the autarky value function has the functional form (10) (the expected value function, V^e, has the functional form (9)). Then the value function, V, has the functional form (19) and the optimal policy function is given by (20). Moreover, the intensive-form value function, $\tilde{V}$, and the corresponding optimal consumption and portfolio choices, $\tilde{c}$ and θ', are the maximal solution to the intensive-form Bellman equation (21). This maximal solution is obtained by iterating on the solution, $\tilde{V}_0$, of the intensive-form Bellman equation (21) without participation constraint:

$$\tilde{V} = \lim_{n \to \infty} \tilde{T}^n \tilde{V}_0 \ ,$$

where $\tilde{T}$ is the operator associated with the intensive-form Bellman equation (21)

Proof Appendix.

Note that proposition 3 cannot simply be proved by the guess-and-verify method we have used to prove proposition 1 for the case of financial autarky. The reason is that there may be multiple solutions to the Bellman equation (17). In other words, the operator associated with the Bellman equation is monotone, but not a contraction. However, proposition 3 ensures that we have indeed found the value function associated with the original utility maximization problem, and also provides us with a iterative method to compute this solution. Note further that the constraint set in (21) is linear since the return functions are linear in θ. Thus, the constraint set is convex and we have transformed the original utility maximization problem into a convex problem. In other words, the non-convexity problem alluded to in the introduction has been resolved.

c) Intensive-form equilibrium

Proposition 3 shows how to rewrite the maximization problem of individual households into a recursive problem that is wealth-independent. One implication of the intensive-form representation of the individual maximization problem is that optimal portfolio choices are wealth independent. This result, in turn, implies that the market clearing conditions (14) and (16) can be re-written as intensive-form market clearing conditions that are independent of the wealth distribution. More precisely, (14) is equivalent to the intensive-form market clearing condition

$$\tilde{K}' = \frac{\sum_s \theta'_k(s)(1 - \tilde{c}(s))\Omega(s)\pi(s)}{\sum_s \theta'_h(s)(1 - \tilde{c}(s))\Omega(s)\pi(s)} \tag{22}$$

and (16) is equivalent to

$$0 \;=\; \sum_{s,s'} \theta_a'(s';s)(1-\tilde{c}(s))\pi(s'|s)\pi(s)\Omega(s)\;. \tag{23}$$

Moreover, using the definition of Ω and the optimal policy function, g, defined by (20), we also show that the law of motion for the Ω-distribution induced by g together with the stationarity condition read:

$$\Omega(s') \;=\; \frac{\sum_s (1-\tilde{c}(s))(1+r(\theta'(s),s'))\Omega(s)\pi(s)}{\sum_s\sum_{s'}(1-\tilde{c}(s))(1+r(\theta'(s),s'))\Omega(s)\pi(s,s')}\;. \tag{24}$$

In sum, a recursive equilibrium can be found by solving (21)-(24) if the solution satisfies for all s the condition

$$\beta\sum_{s'}\left(1+r(\theta',s')\right)^{1-\gamma}\pi(s'|s) \;<\; 1\;. \tag{25}$$

Note that inequality (25) simply ensures that condition (18) is satisfied so that proposition 2 is applicable.

Proposition 4. Suppose that $(\theta,\tilde{c},\tilde{V},r_f)$ is an intensive-form equilibrium, that is, the consumption-portfolio choice $(\tilde{c},\theta)$ together with the intensive-form value function $\tilde{V}$ are the maximal solution to the intensive-form Bellman equation (21) satisfying condition (25), the market clearing conditions (22) and (23) are satisfied, and the stationary version of the law of motion (24) holds. Then $(g,\tilde{V},r_f,\Phi)$ is a stationary recursive equilibrium, where g is the individual policy function associated with $(\theta,\tilde{c})$ and Φ the aggregate law of motion induced by $(\theta,\tilde{c})$.

Proof. See appendix.

d) The i.i.d. case

For an i.i.d. shock process, $\{s_t\}$, the Bellman equation determining the intensive-form value function becomes:

$$
\begin{aligned}
\tilde{V} \;&=\; \max_{\tilde{c},\theta'} \left\{ \frac{\tilde{c}^{1-\gamma}}{1-\gamma} + \beta(1-\tilde{c})^{1-\gamma}\tilde{V} \sum_{s'} \left(1 + r(\theta', s')\right)^{1-\gamma} \pi(s') \right\} \\
s.t. \;\; 1 \;&=\; \theta_k' + \theta_h' + \sum_{s'} \frac{\theta_a'(s')\pi(s')}{1+r_f} \\
& 0 \;\le\; \tilde{c} \;\le\; 1 \;\;,\;\; \theta_k' \ge 0 \;\;,\;\; \theta_h' \ge 0 \;\;,\;\; \theta_a'(s') \ge \bar{\theta}_a(s', S') \\
& \left(\frac{\tilde{V}}{\tilde{V}_d}\right)^{\frac{1}{1-\gamma}} \left(1 + r(\theta', s')\right) \;\ge\; 1 + r(\theta_k', \theta_h, 0, s') - \phi\theta_k'
\end{aligned}
\tag{26}
$$

for $\gamma \neq 1$ and

$$
\begin{aligned}
\tilde{V} \;&=\; \max_{\theta'} \left\{ log(1-\beta) \,+\, \frac{\beta}{1-\beta} log\beta \,+\, \frac{\beta}{1-\beta} \sum_{s'} log(1 + r(\theta', s'))\pi(s') + \beta\tilde{V} \right\} \\
s.t. \;\; 1 \;&=\; \theta_k' + \theta_h' + \sum_{s',S'} \frac{\theta_a'(s')\pi(s')}{1+r_f} \\
& \theta_k' \ge 0 \;\;,\;\; \theta_h' \ge 0 \;\;,\;\; \theta_a'(s') \ge \bar{\theta}_a(s') \\
& e^{(1-\beta)\left(\tilde{V}-\tilde{V}_d\right)} \left(1 + r(\theta', s')\right) \;\ge\; 1 + r(\theta_k', \theta_h', 0, s') - \phi\theta_k'
\end{aligned}
$$

Note that the intensive-form value function, $\tilde{V}$, is now a real number. The optimal policy function associated with the solution to this Bellman equation is a state-independent portfolio and consumption choice θ and $\tilde{c}$. These choices will depend on the aggregate capital-to-labor ratio and the interest rate: $\theta' = \theta'(\tilde{K}', r_f)$ and $\tilde{c} = \tilde{c}(\tilde{K}', r_f)$. The market clearing condition

$$
\begin{aligned}
\tilde{K}' \;&=\; \frac{\theta_k'}{\theta_h'} \\
1 \;&=\; \theta_k' + \theta_h'
\end{aligned}
\tag{27}
$$

pins down the equilibrium value of $\tilde{K}'$ and r_f. Note that we also have $\sum_{s'} \theta_a(s')\pi(s') = 0$.

In terms of individual consumption, i.i.d. shocks to human capital imply that consumption growth is i.i.d. and given by

$$\frac{c_{t+1}}{c_t} = (1-\tilde{c})\,(1+\theta_k(r_k-\delta_k)+\theta_h(r_h-\delta_h(s_{t+1})))\ . \tag{28}$$

Thus, the effect of human capital shocks, s_t, on consumption is weakened for two reasons. First, households can self-insure through their own savings, and in equation (30) the shock is therefore multiplied by labor's share in income, θ_h. Second, households buy insurance contracts, and this is represented by the term $\theta_a(s)$.

e) Computation of equilibrium

To be written

IV. Quantitative Results

We now discuss the quantitative implications of the model. We consider two versions of the general model: one version with one type of households facing i.i.d. human capital shocks (one-type economy), and one with three types of households (young, middle-aged, and old) facing i.i.d. human capital shocks. For each version, we first specify functional forms and assigns values to all relevant parameters of the model (calibration), and then report the equilibrium amount of consumption insurance for different values of the contract enforcement parameters. We further report the growth and welfare effects of improving contract enforcement.

a) One-type economy: Calibration

We assume that workers have logarithmic utility functions, $u(c) = logc$, and that the production function is Cobb-Douglas: $f(\tilde{k}) = A\tilde{k}^{\alpha}$. We use $\alpha = .36$ to match capital's share in income, annual (average) depreciation rates of physical and human capital of $\delta_k = .06$ and $\delta_h = .04$, respectively. We assume that human capital shocks are i.i.d. so that all aggregate variables grow at the common rate $\beta(1+\bar{r})-1$, where $\bar{r}=\theta_k(r_k-\delta_k)+\theta_h(r_h-\delta_h)$. We further assume that human capital shocks are log-normally distributed: $\delta_h(s_t) = \delta_h + s_t$ with $E[s_t]=0$ and $log(1+\bar{r}+s_t) \sim N(log(1+\bar{r})-\sigma^2/2, \sigma^2)$. We choose $\sigma = .15$ to match observed labor income risk (see below). The values of the remaining fundamental parameters A and β are chosen so that the model is roughly consistent with the US evidence along the two dimensions saving and growth. More specifically, we require that per capita growth is $g = .02$ and that the implied saving rate is $s_k = x_{kt}/y_t = .20$.

The choice of $\sigma = .15$ is made to ensure consistency with the empirical results of a number of micro studies on labor income risk. More specifically, in the model economy labor income of an individual household in period t is given by $y_{ht} = r_h h_t$, so that the growth rate of labor income is equal to the growth rate of human capital: $y_{h,t+1}/y_{ht} = h_{h,t+1}/h_t$. Changes in human capital, in turn, are driven by two forces: the shock s_t and changes in human capital investment. In the model, human capital is immediately adjustable at no cost, so that the model tends to understate the volatility of labor income. To address this issue, we neglect this margin of adjustment when calibrating the model, which can be done by replacing actual human capital investment by its mean value. This yields the following expression for equilibrium labor income growth,

$$\frac{y_{h,t+1}}{y_{ht}} \approx \beta(1+\bar{r}-s_t) \ ,$$

where the approximation sign indicates that we have replaced x_{ht} by $E[x_{ht}]$.

Taking logs in (31) yields:

$$log y_{h,t+1} = log y_{th} + d + \epsilon_t, \tag{29}$$

where $d = log\beta + log(1+\bar{r}) - \sigma^2/2$ and $\{\epsilon_t\}$ is a sequence of i.i.d. random variables with $\epsilon_t \sim N(0,\sigma^2)$. Hence, the logarithm of labor income follows (approximately) a random walk with drift d and innovation term ϵ_t.[16] The random walk specification is often used by the empirical literature to model the permanent component of labor income risk (Carroll and Samwick (1997), Meghir and Pistaferri (2004), and Storesletten et al. (2004)). Thus, their estimate of the standard deviation of the error term for the random walk component of annual labor income corresponds to the value of σ. In our baseline model we use $\sigma = .15$, which lies on the lower end of the spectrum of estimates found by the empirical literature. For example, Carroll and Samwick (1997) find .15, Meghir and Pistaferri (2004) estimate .19, and Storesletten et al. (2004) have .25 (averaged over age-groups and, if applicable, over business cycle conditions). All these studies use labor income before transfer payments, which is the relevant variable from our point of view.

There are at least two reasons why the above approach might underestimate human capital risk. First, a constant $\sigma = .15$ represents less uncertainty than a σ that fluctuates with business cycle conditions and has a mean of .15. Second, the assumption of normally distributed innovations understates the amount of idiosyncratic risk households face if the actual distribution has a fat lower tail. For strong evidence for such a deviation from the normal-distribution framework, see Geweke and Keane (2000). Further, the literature on the long-term consequences of job displacement (Jacobson, LaLonde, and D. Sullivan (1993)) has found wage losses of displaced workers that are somewhat larger than suggested by our mean-variance framework.

There are, however, also arguments that the current approach might overestimate human capital risk. First, we assumes that all of labor income is return to human capital investment. However, if some component of labor income is independent of human capital investment

[16] We have ϵ_t instead of ϵ_{t+1} in equation (29), and the latter is the common specification for a random walk. However, this is not a problem if the econometrician observes the idiosyncratic depreciation shocks with a one-period lag. In this case, (29) is the correct equation from the household's point of view, but a modified version of (29) with ϵ_{t+1} replacing ϵ_t is the specification estimated by the econometrician.

and if this component is random (random endowment of genetic skills), then some part of the variance of labor income is not human capital risk. Second, ignoring job mobility the empirical literature cited above attributes wage hikes due to improved firm-worker matches to income risk, a point that has been emphasized by Low, Meghir, and Pistaferri (2008).

Finally, the two parameters measuring the two dimensions of contract enforcement, ϕ and p, will be allowed to vary between zero and one.

b) Quantitative Results: One-type economy

We first consider the amount of risk sharing as a function of the two enforcement parameters p and ϕ. We measure the degree of risk sharing as the ratio of the standard deviation of consumption growth in two different economies: one without financial markets (no risk sharing) and one with financial markets and enforcement constraints. This measure varies between 0 and 1 and an increase means that more risk sharing is possible in equilibrium. We report all results for two different scenarios with respect to the consequences of default. In the first scenario, we assume that defaulting households lose access to financial markets, but continues to supply physical and human capital in the corresponding factor markets (financial autarky). In the second scenario, defaulting households lose access to financial markets and use their physical and human capital for home production (total autarky).

Figure 1 shows the amount of risk sharing for different values of p holding the other enforcement parameter constant at $\phi = 0$. From this figure we infer that contract enforcement through the exclusion from financial market participation is not very effective. More specifically, if a defaulting household spends on average 7 years in (financial) autarky, $1 - p = 1/7$, a choice that is motivated by the features of the US bankruptcy code (see Livshits et al. (2007)), then there is almost no risk sharing in equilibrium, that is, consumption volatility in equilibrium is high. Further, figure 2 shows that the welfare cost associated with this consumption volatility is substantial. Finally, even if defaulting households are excluded from financial markets forever, $p = 1$, individual households buy only a limited amount of insurance. Thus, consumption volatility and the corresponding welfare losses are still substantial.

Figure shows our results for different values of ϕ keeping the other enforcement parameter constant at $1 - p = 1/7$. From figure 3 we can see that enforcement through the seizure of capital is highly effective.[17] More specifically, for $\phi = 0$ there is very little risk sharing in equilibrium, but if defaulting households lose only fifty percent of their capital, $\phi = .5$, risk sharing is very close to complete and the corresponding volume of insurance is large. In other words, the equilibrium allocation is almost indistinguishable from the equilibrium allocation without enforcement frictions. Finally, if all the capital is seized, $\phi = 1$, then we obtain full risk sharing in equilibrium. Note that $\phi = 1$ is the parameter value chosen in Krueger and Perri (2006).

c) Three-type economy: Calibration

To be written

[17] This result is true for any value of the enforcement parameter p. In particular, it is also true if results barely change if we set $p = 1$ (no return to financial markets) or $p = 1$ (immediate return to financial markets).

Appendix

Proof of proposition 1

In the case of default, a household faces the budget constraint

$$\begin{aligned} w_{t+n+1} &= [1 + r_{d,t+n}]\, w_{t+n} - c_{t+n} \\ 1 &= \theta_{k,t+n} + \theta_{h,t+n} \qquad\qquad (1) \\ c_{t+n} &\geq 0 \;\; , \;\; w_{t+n} \geq 0 \;\; , \;\; \theta_{k,t+n} \geq 0 \;\; , \;\; \theta_{h,t+n} \geq 0 \; , \end{aligned}$$

where we allow for two specifications for the investment return after default, r_d. In the first specification, we assume that after default the household continues to supply physical and human capital in competitive factor markets so that $r_d = \theta_k(r_k - \delta_k) + \theta_h(r_h - \delta_h(s))$. In the second case, we assume that a household in financial autarky has access to a backyard technology (self-employment) so that $r_d = F(\theta_k, \theta_h) - \delta_k\theta_k - \delta_h(s)\theta_h$.

For given expected value function V^e, a household who defaults in period t chooses a continuation plan, $\{c_{t+n}, w_{t+n}, \theta_{k,t+n}, \theta_{h,t+n}\}$, that maximizes

$$u(c_t) + \sum_{n=1}^{\infty} \beta^n p^{n-1} \left(pE[u(c_{t+n})] + (1-p)E[V^e(w_{t+n}, \theta_{k,t+n}, \theta_{h,t+n}, s_{t+n})] \right) \qquad (2)$$

subject to the sequential budget constraint (A1), where p stand for the probability that the household remains in autarky. The corresponding Bellman equation reads:

$$\begin{aligned} V_d(w, \theta_k, \theta_h, s) \quad &= \quad \max_{c, w', \theta'_k, \theta'_h} \left\{ u(c) + \beta \sum_{s', S'} \left(pV_d(w', \theta'_k, \theta'_h, s') \right. \right. \\ &\qquad \left. + (1-p)V^e\,(w', \theta'_k, \theta'_h, s') \right) \pi(s'|s) \} \qquad (3) \\ &s.t. \;\; 1 \; = \; \theta'_k + \theta'_h \;\; , \;\; c = (1 + r_d(\theta_k, \theta_h, s))w - w' \geq 0 \;\; , \;\; \theta'_k \geq 0 \;\; , \;\; \theta'_h \geq 0 \; . \end{aligned}$$

Using a contraction mapping argument, in the technical appendix we show that (A3) has a unique continuous solution if V^e is continuous and if the condition

$$\forall \theta' \; : \; \beta \sum_{s''} \left(1 + r_d(\theta'_k, \theta'_h, s')\right)^{1-\gamma} \pi(s'|s) \;\; < \;\; 1 \quad if \;\; 0 \; < \; \gamma \; < \; 1 \qquad (4)$$

$$\exists \theta' \; : \; \beta \sum_{s'} \left(1 + r_d(\theta_k', \theta_h', s')\right)^{1-\gamma} \pi(s'|s) \;\; < \;\; 1 \quad if \;\; \gamma \; > \; 1$$

is satisfied for all s. Furthermore, the technical appendix also shows that this solution is also the value function associated with the sequential maximization problem of a defaulting household (principle of optimality). Hence, we can confine attention to the Bellman equation (A3) when dealing with the individual household problem after default.

Suppose the expected value function has the functional form (11). Given this functional form for V^e, a simple guess-and-verify approach shows that the optimal policy function for a household in financial autarky is

$$c_d(w, \theta_k, \theta_s, s) \;\; = \;\; \begin{cases} \tilde{c}_d(s)(1 + r_d(\theta_k, \theta_h, s)w & if \; \gamma \; \neq \; 1 \\ (1-\beta)(1 + r_d(\theta_k, \theta_h, s))w & otherwise \end{cases} \tag{5}$$

$$w_d'(w, \theta_k, \theta_h, s) \;\; = \;\; \begin{cases} (1 - \tilde{c}_d(s))(1 + r_d(\theta_k, \theta_h, s))w & if \; \gamma \; \neq \; 1 \\ \beta(1 + r_d(\theta_k, \theta_h, s))w & otherwise \end{cases}$$

$$\theta_k'(w, \theta_k, \theta_h, s) \;\; = \;\; \theta_k'(s)$$

$$\theta_h'(w, \theta_k, \theta_h, s) \;\; = \;\; \theta_h'(s)$$

and that the corresponding value function has the functional form (12), where the intensive-form value function, $\tilde{V}_d$, is the solution to the following intensive-form Bellman equation:

$$\begin{aligned} \tilde{V}_d(s) \quad = \quad & \max_{\tilde{c}, \theta_k', \theta_h'} \left\{ \frac{\tilde{c}^{1-\gamma}}{1-\gamma} + \beta p (1-\tilde{c})^{1-\gamma} \sum_{s'} \left(1 + r_d(\theta_k', \theta_h', s')\right)^{1-\gamma} \tilde{V}_d(s') \pi(s'|s) \right. \\ & \left. + \beta(1-p)(1-\tilde{c})^{1-\gamma} \sum_{s'} \left(1 + r^e(\theta_k', \theta_h', s')\right)^{1-\gamma} \tilde{V}^e(s') \pi(s'|s) \right\} \end{aligned} \tag{6}$$

$$s.t. \quad 1 \; = \; \theta_k' + \theta_h' \;\; , \;\; 0 \leq \; \tilde{c} \; \leq \; 1 \;\; , \;\; \theta_k' \geq 0 \;\; , \;\; \theta_h' \geq 0$$

for $\gamma \neq 1$ and

$$\tilde{V}_d(s) \quad = \quad \max_{\theta_k', \theta_h'} \left\{ log(1-\beta) \; + \; \frac{\beta}{1-\beta} log\beta \; + \; \frac{\beta}{1-\beta} \sum_{s'} log(1 + r_d(\theta_k', \theta_h', s')) \pi(s'|s) + \right.$$

$$\beta \sum_{s'} \left(p\tilde{V}_d(s') + (1-p)\tilde{V}^e(s') \right) \pi(s'|s) \Big\}$$

$$s.t. \quad 1 \; = \; \theta'_k + \theta'_h \;\; , \;\; \theta'_k \geq 0 \;\; , \;\; \theta'_h \geq 0$$

for the log-utility case. This proves proposition 1.

Proof of Proposition 2

To simplify the notation, denote the endogenous individual state vector as $x_t = (w_t, \theta_t)$. Further, define a payoff function as $F(x_t, s_t, x_{t+1}) = u((1 + r(\theta_t, s_t))w_t - w_{t+1})$, and a feasibility correspondence as

$$\begin{aligned} \Gamma(x_t, s_t) \;\; = \;\; & \Big\{ x_{t+1} \in \mathbf{X} \mid \theta_{k,t+1}(s_t) + \theta_{h,t+1}(s_t) + \sum_{s_{t+1}} \frac{\theta_{a,t+1}(s_{t+1}; s_t)\pi(s_{t+1}|s_t)}{1 + r_f} = 1 \; , \\ & 0 \; \leq \; w_{t+1} \; \leq \; (1 + r(\theta_{t+1}, s_t))w_t \; , \\ & \theta_{k,t+1} \geq 0 \;\; , \;\; \theta_{h,t+1} \geq 0 \;\; , \;\; \theta_{a,t+1}(s_{t+1}; s_t) \geq \bar{\theta}_a(s_{t+1}; s_t) \; . \Big\} \end{aligned} \tag{7}$$

Using this notation, the household maximization problem reads

$$\begin{aligned} max \quad & E\left[\sum_{t=0}^{\infty} \beta^t F(x_t, s_t, x_{t+1}) | x_0, s_0 \right] \\ s.t. \quad & x_{t+1} \; \in \; \Gamma(x_t, s_t) \\ & E\left[\sum_{n=0}^{\infty} \beta^n F(x_{t+n}, s_{t+n}, x_{t+n+1}) | x_0, s^t \right] \; \geq \; V_d(x_t, s_t) \end{aligned} \tag{8}$$

The corresponding Bellman equation reads:

$$\begin{aligned} V(x, s) \quad = \quad & \max_{x'} \left\{ F(x, s, x') + \beta \sum s' V(x', s') \pi(s'|s) \right\} \\ & s.t. \quad x' \; \in \; \Gamma(x, s) \\ & \qquad V(x', s') \; \geq \; V_d(x', s') \end{aligned} \tag{9}$$

Define an operator, T, that maps semi-continuous functions into semi-continuous functions as

$$\begin{aligned} TV(x,s) &= \max_{x'} \{F(x,s,x') + \beta E[V(x',s')|s]\} \qquad (10) \\ s.t. &\quad x' \in \Gamma(x,s) \\ &\quad V(x's') \; \geq \; V_d(x',s') \; . \end{aligned}$$

As in the proof of proposition 1, a contraction mapping argument shows that there is a unique continuous solution to the Bellman equation (A9) without participation constraint, which we denote by V_0, if F is continuous, ii) Γ is compact-valued and continuous, and (18) holds. In the technical appendix to this paper, we also show that $V_\infty = \lim_{n\to\infty} T^n V_0$ exists, is equal to the maximal solution of the Bellman equation (A9), and is the value function of the sequential maximization problem (A8) if the following four conditions hold: i) F is continuous, ii) Γ is compact-valued and continuous, iii) for all states, (x,s), there exists a feasible plan, π, for the sequential problem (A2) so that the corresponding expected lifetime utility (payoff) is greater than $-\infty$, and iv) for any given state, (x,s), the value function of the sup-problem without participation constraints satisfies $V_0^*(x,s) < +\infty$. Thus, to prove proposition 2 it suffices to show that conditions i)-iv) hold.[18]

The continuity of the payoff function, F, is obvious. The correspondence, Γ, is compact-valued since portfolio-choices, θ', are elements of a closed and bounded subset of $\mathbb{R}^m$. Closedness follows from the fact that the set is defined by equalities and weak inequalities. The set is bounded from below because of the short-sale constraints (6) and it is then bounded from above because of the budget constraint. Continuity of the correspondence Γ is also straightforward to show. A standard argument shows that conditions iii) and iv) hold if condition (18) is satisfied. This proves proposition 2.

[18]Rustichini (1998) consider a class of dynamic programming problems with participation constraint (incentive compatability constraint) and possibly unbounded utility. However, he requires bi-convergence, which is always satisfied if lifetime-utility is bounded for all feasible paths (Streufert, 1990). Unfortunately, in our problem with $\gamma \geq 1$ the requirement of lower convergence is not satisfied, so that Rustichini (1998) is not directly applicable.

Proof of Proposition 3

As before, let V_0 be the solution of the Bellman equation (A9) without the participation constraint. Simple guess-and-verify shows that V_0 has the following functional form:

$$V_0(w, \theta, s) \;=\; \begin{cases} \tilde{V}_0(s)\,(1 + r(\theta, s))^{1-\gamma}\, w^{1-\gamma} & if\ \gamma \neq 1 \\ \tilde{V}_0(s) + \frac{1}{1-\beta} log(1 + r(\theta, s)) + \frac{1}{1-\beta} w & otherwise \end{cases} \tag{11}$$

where $\tilde{V}_0$ is the solution to the intensive-form Bellman equation (21) without participation constraint. Let the operator T be defined as in (A10). We show by induction that if $V_n = T^n V_0$ has the functional form, then $V_{n+1} = T^{n+1} V_)$ has the functional form. For $n = 0$ the claim is true because V_0 has the functional form. Suppose now V_n has the functional form. We then have

$$\begin{aligned} V_{n+1}(w, \theta, s) \;&=\; TV_n(w, \theta, s) \\ &=\; \max_{w', \theta'} \left\{ \frac{((1 + r(\theta, s))w - w')^{1-\gamma}}{1 - \gamma} + \sum_{s'} \tilde{V}_n(s')(1 + r(\theta', s'))^{1-\gamma}(w')^{1-\gamma}\pi(s'|s) \right\} \\ s.t.\quad 1 \;&=\; \theta'_k + \theta'_h + \sum_{s'} \frac{\theta'_a(s')\pi(s'|s)}{1 + r_f} \\ &\quad 0 \leq w' \leq (1 + r(\theta, s))w \ , \ \theta'_k \geq 0 \ , \ \theta'_h \geq 0 \ , \ \theta'_a(s') \geq \bar{\theta}_a(s') \\ &\quad \tilde{V}_n(s')\,(1 + r(\theta'_k, \theta'_h, \theta_a(s')))^{1-\gamma}\,(w')^{1-\gamma} \\ &\quad \geq \tilde{V}_d(s')(1 + r(\theta'_k, \theta'_h, 0, s') - \phi\theta'_k(s))^{1-\gamma}(w')^{1-\gamma} \end{aligned} \tag{12}$$

for $\gamma \neq 1$ and

$$\begin{aligned} V_{n+1}(w, \theta, s) \;&=\; TV_n(w, \theta, s) \\ &=\; \max_{w', \theta'} \Bigg\{ log\,(1 + r(\theta, s))w - w') + \beta \sum_{s'} \tilde{V}_n(s')\pi(s'|s) \\ &\qquad + \frac{\beta}{1 - \beta} \sum_{s'} log(1 + r(\theta', s'))\pi(s'|s) + \frac{\beta}{1 - \beta} log\, w' \Bigg\} \\ s.t.\quad 1 \;&=\; \theta'_k + \theta'_h + \sum_{s'} \frac{\theta'_a(s')\pi(s'|s)}{1 + r_f} \end{aligned}$$

$$
\begin{aligned}
& 0 \;\leq\; w' \;\leq\; (1+r(\theta,s))w \;\;,\;\; \theta'_k \geq 0 \;\;,\;\; \theta'_h \geq 0 \;\;,\;\; \theta'_a(s') \geq \bar{\theta}_a(s') \\
& \tilde{V}_n(s') \;+\; \frac{1}{1-\beta} log(1+r(\theta'_k,\theta'_h,\theta'_a(s'),s')) \;+\; \frac{1}{1-\beta} log\, w' \\
& \geq\; \tilde{V}_d(s') \;+\; \frac{1}{1-\beta} log(1+r(\theta'_k,\theta'_h,0,s')) \;+\; \frac{1}{1-\beta} log\, w'
\end{aligned}
$$

for the log-utility case. Clearly, the solution to the maximization problem defined by the right-hand-side of (A9) has the form

$$
\begin{aligned}
w'_{n+1} &= (1-\tilde{c}_{n+1}(s))(1+r(\theta_{n+1},s))w \\
\theta'_{n+1} &= \theta'_{n+1}(s) \;,
\end{aligned}
\tag{13}
$$

where the subscript $n+1$ indicates step $n+1$ in the iteration. Thus, we have

$$
V_{n+1}(w,\theta,s) \;=\; \begin{cases} \tilde{V}_{n+1}(s)\,(1+r(\theta,s))^{1-\gamma}\, w^{1-\gamma} & if\ \gamma\ \neq\ 1 \\ \tilde{V}_{n+1}(s) \;+\; \frac{1}{1-\beta} log(1+r(\theta,s)) \;+\; \frac{1}{1-\beta} w & otherwise \end{cases} \;,
$$

where $\tilde{V}_{n+1}$ is defined accordingly.

From proposition 2 we know that $V_\infty = \lim_{n\to\infty} T^n V_0$ exists and that it is the maximal solution to the Bellman equation (A9) as well as the value function of the corresponding sequential maximization problem (A8). Since the set of functions with this functional form is a closed subset of the set of semi-continuous functions, we know that V_∞ has the functional form. This prove proposition 3.

Proof of Proposition 4

From proposition 3 we know that individual households maximize utility subject to the budget constraint and participation constraint if condition (18) is satisfied. It is easy to see that condition (18) is satisfied if the proposed portfolio choice, θ', satisfies condition (25). Thus, it remains to show that the intensive-from market clearing conditions (22) and (23) are equivalent to the market clearing conditions (14) and (16) and that the law of motion (24) describes the equilibrium evolution of the relative wealth distribution.

The aggregate stock of physical capital is

$$\begin{aligned}
K_{t+1} &= E\left[\theta_{k,t+1} w_{t+1}\right] \\
&= E\left[E\left[\theta_{k,t+1}(1-\tilde{c}_t)(1+r_t) w_t | s_t\right]\right] \\
&= E\left[\theta_{k,t+1}(1-\tilde{c}_t) E\left[(1+r_t) w_t | s_t\right]\right] \\
&= E\left[(1+r_t) w_t\right] \ \frac{E\left[\theta_{k,t+1}(1-\tilde{c}_t) E\left[(1+r_t) w_t | s_t\right]\right]}{E\left[(1+r_t) w_t\right]} \\
&= E\left[(1+r_t) w_t\right] \ E\left[\theta_{k,t+1}(1-\tilde{c}_t) \Omega(s_t)\right] \ ,
\end{aligned} \tag{14}$$

where the second line follows from the budget constraint, the third line from the law of iterated expectations, the fourth line from the fact that $\theta_{k,t+1}$ and $\tilde{c}_t$ are independent of wealth and s^{t-1}, and the last line from the definition of Ω. A similar expression holds for the aggregate stock of human capital, H_{t+1}, held by households. This proves the equivalence of the intensive-form market clearing condition (22) with the market clearing condition (14).

For the aggregate value of all financial asset holdings we find:

$$\begin{aligned}
E[\theta_{a,t+1} w_{t+1}] &= E[\theta_{a,t+1}(1-\tilde{c}_t)(1+r_t) w_t] \\
&= E[E[\theta_{a,t+1}(1-\tilde{c}_t)(1+r_t) w_t | s_t]] \\
&= E[\theta_{a,t+1}(1-\tilde{c}_t) E[(1+r_t) w_t | s_t]] \\
&= E\left[(1+r_t) w_t\right] \ \frac{E[\theta_{a,t+1}(1-\tilde{c}_t) E[(1+r_t) w_t | s_t]]}{E\left[(1+r_t) w_t\right]} \\
&= E\left[(1+r_t) w_t\right] \ E\left[\theta_{a,t+1}(1-\tilde{c}_t) \Omega(s_t)\right] \ .
\end{aligned} \tag{15}$$

where the first line follows from the budget constraint, the second line from the law of iterated expectations, the third line from the fact that $\theta_{a,t+1}$ and $\tilde{c}_t$ are independent of wealth and s^{t-1}, and the last line from the definition of Ω. This proves the equivalence of the intensive-form market clearing condition (23) with the market clearing condition (16).

Finally, the law of motion for Ω can be found as:

$$\Omega_{t+1}(s_{t+1}) = \frac{E\left[(1+r_{t+1}) w_{t+1} | s_{t+1}\right]}{E\left[(1+r_{t+1}) w_{t+1}\right]} \tag{16}$$

$$
\begin{aligned}
&= \frac{E\left[(1+r_{t+1})(1-\tilde{c}_t)(1+r_t)w_t|s_{t+1}\right]}{E\left[(1+r_{t+1})(1-\tilde{c}_t)(1+r_t)w_t\right]} \\
&= \frac{E\left[E\left[(1+r_{t+1})(1-\tilde{c}_t)(1+r_t)w_t|s_t, s_{t+1}\right]|s_{t+1}\right]}{E\left[E\left[(1+r_{t+1})(1-\tilde{c}_t)(1+r_t)w_t|s_t\right]\right]} \\
&= \frac{E\left[(1+r_{t+1})(1-\tilde{c}_t)E\left[(1+r_t)w_t|s_t, s_{t+1}\right]|s_{t+1}\right]}{E\left[(1+r_{t+1})(1-\tilde{c}_t)E\left[(1+r_t)w_t|s_t\right]\right]} \\
&= \frac{E\left[(1+r_{t+1})(1-\tilde{c}_t)E\left[(1+r_t)w_t|s_t, s_{t+1}\right]|s_{t+1}\right]}{E\left[(1+r_{t+1})(1-\tilde{c}_t)E\left[(1+r_t)w_t|s_t\right]\right]} \times \\
&\quad \frac{E\left[(1+r_t)w_t\right]}{E\left[(1+r_t)w_t\right]} \\
&= \frac{E\left[(1+r_{t+1})(1-\tilde{c}_t)\Omega_t(s_t)|s_{t+1}\right]}{E\left[(1+r_{t+1})(1-\tilde{c}_t)\Omega_t(s_t)\right]},
\end{aligned}
$$

where the second line follows from the budget constraint, the third line from the law of iterated expectations, the fourth line from the fact that θ_{t+1} and $\tilde{c}_t$ are independent of wealth and s^{t-1}, and the last line from the definition of Ω. This completes the proof of proposition 4.

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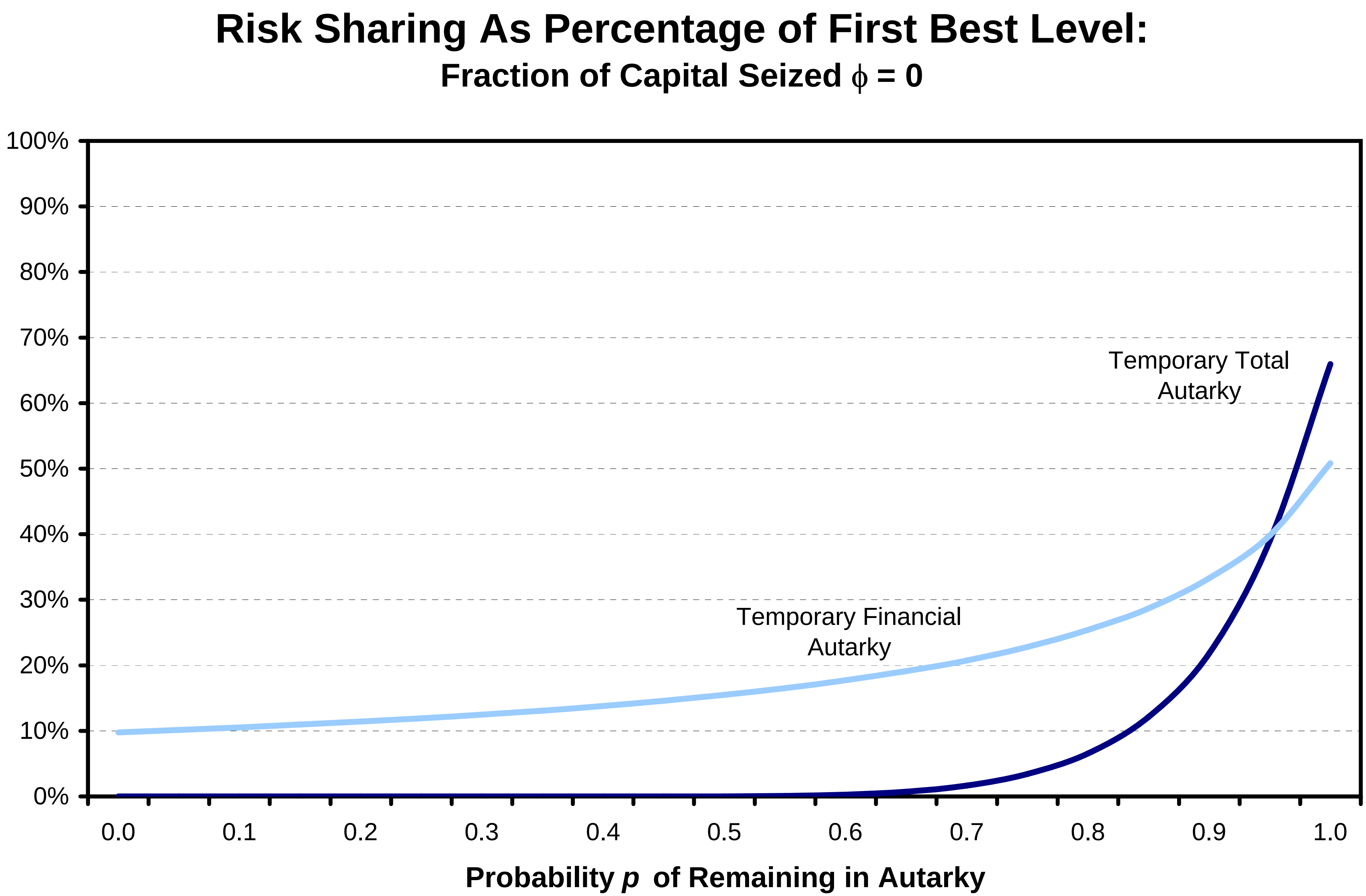
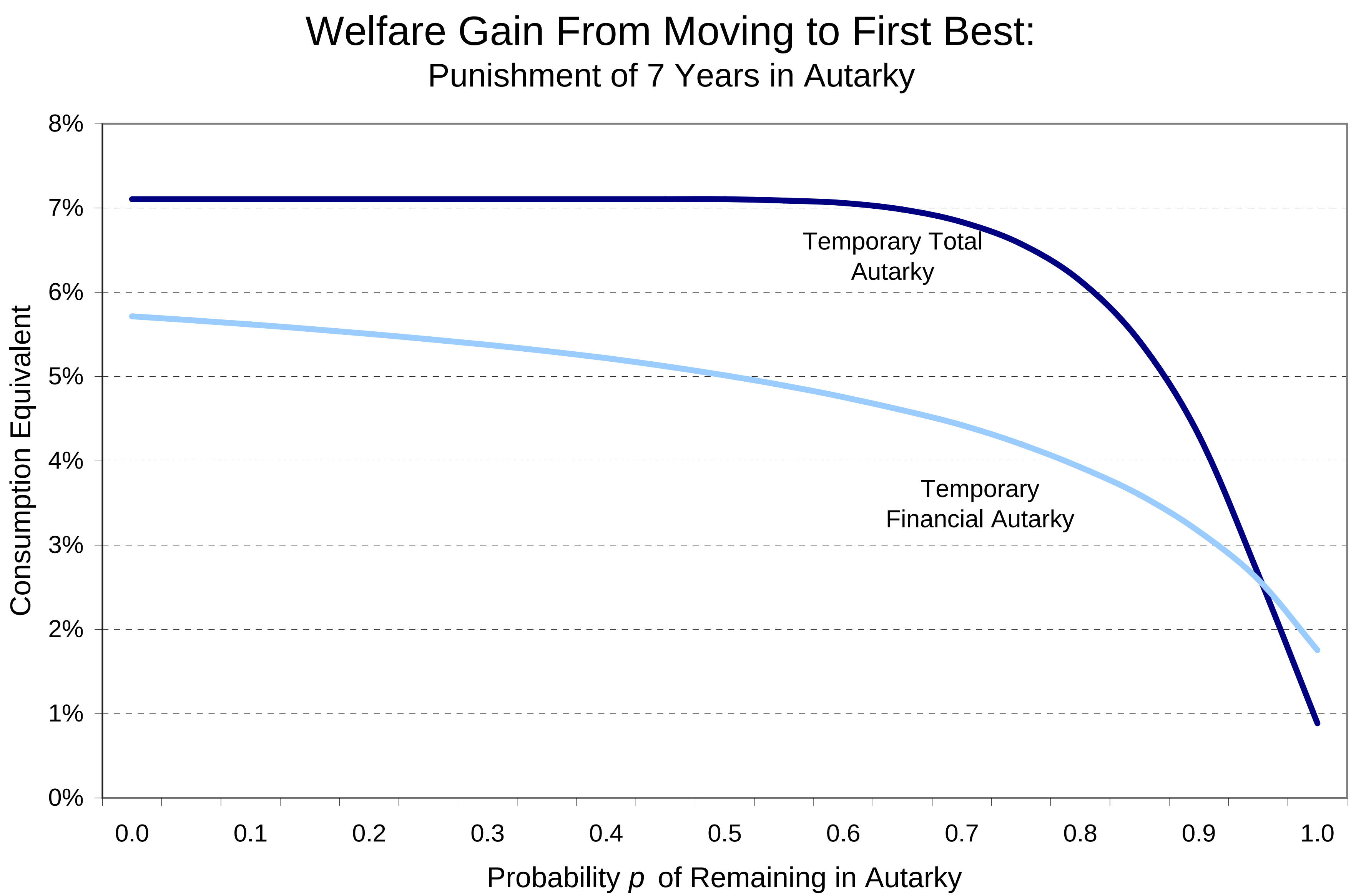
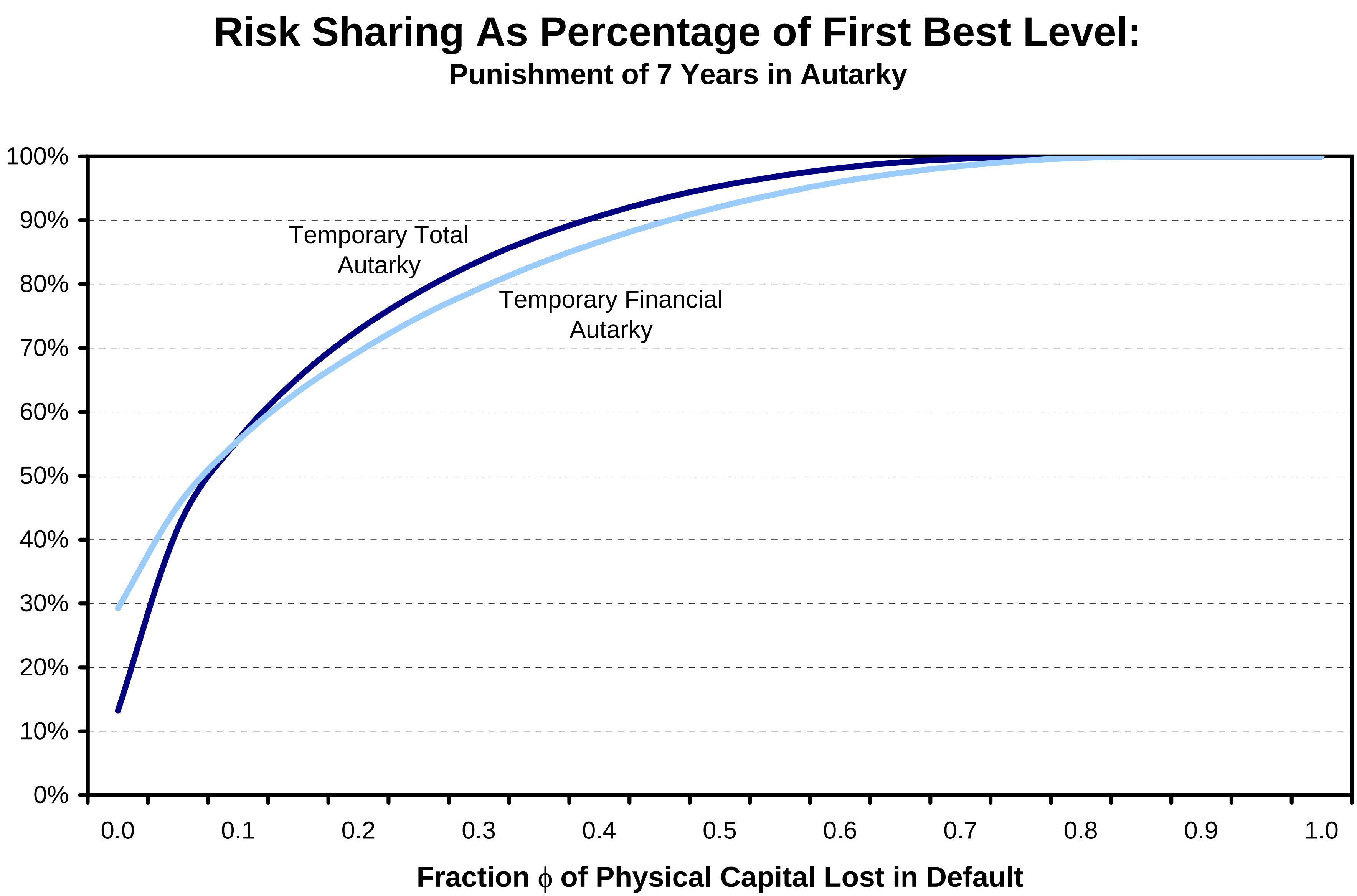
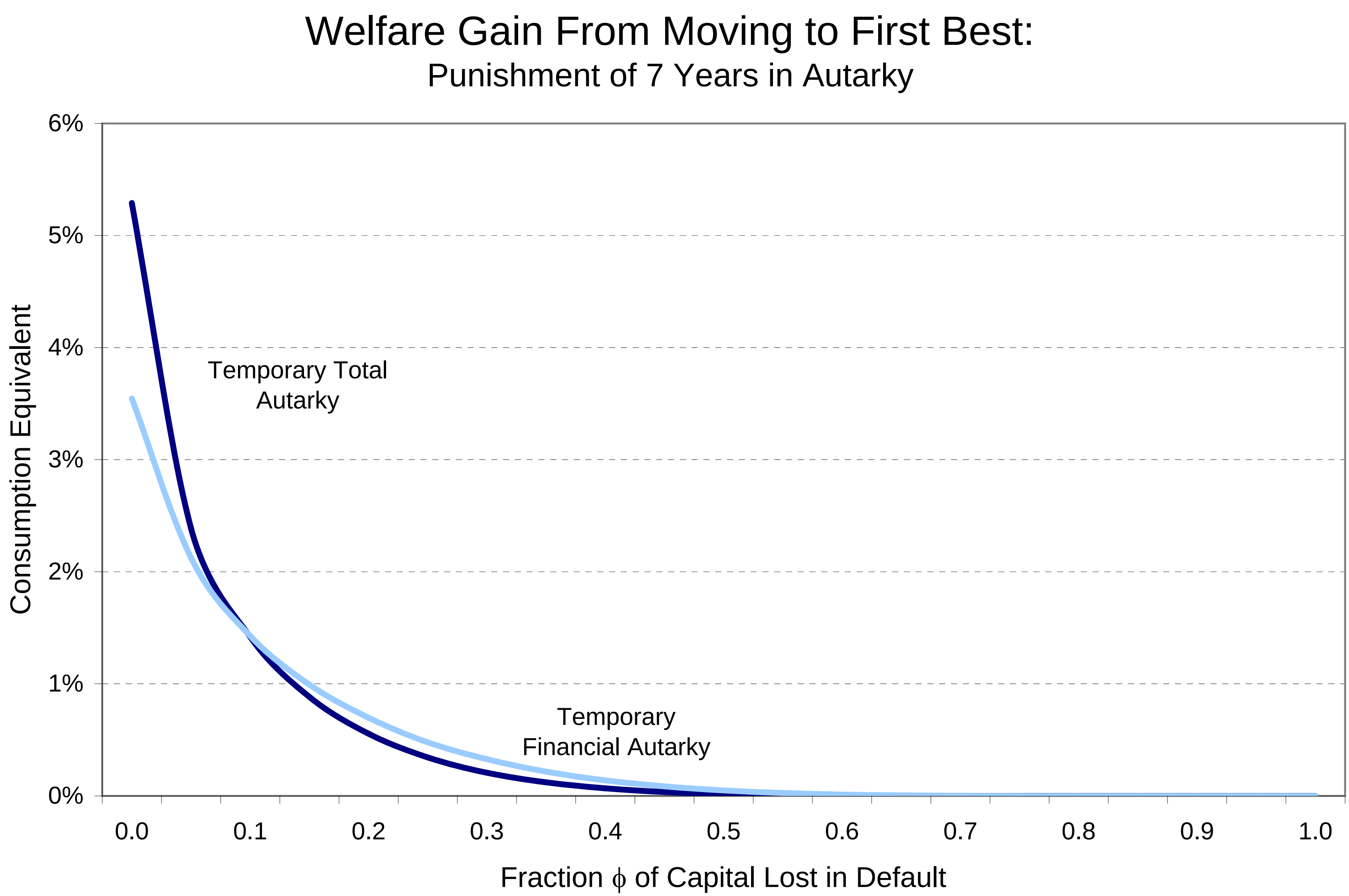

Risk Sharing As Percentage of First Best Level:
Fraction of Capital Seized ϕ = 0
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Probability *p* of Remaining in Autarky
Temporary Total Autarky
Temporary Financial Autarky



Welfare Gain From Moving to First Best:
Punishment of 7 Years in Autarky
Consumption Equivalent
8%
7%
6%
5%
4%
3%
2%
1%
0%
Temporary Total Autarky
Temporary Financial Autarky
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Probability p of Remaining in Autarky



Risk Sharing As Percentage of First Best Level:
Punishment of 7 Years in Autarky
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Fraction ϕ of Physical Capital Lost in Default
Temporary Total Autarky
Temporary Financial Autarky


Welfare Gain From Moving to First Best:
Punishment of 7 Years in Autarky
Consumption Equivalent
6%
5%
4%
3%
2%
1%
0%
Temporary Total Autarky
Temporary Financial Autarky
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Fraction ϕ of Capital Lost in Default