Optimal Unemployment Insurance and International Risk Sharing[☆]

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(preliminary version, comments welcome)

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Abstract

In this paper, we discuss how unemployment insurance policy can be used to improve international risk sharing. We use a two-country real business cycle model augmented by a search labour market and incomplete financial markets and let the unemployment insurance scheme operate across both countries. We find that cross-country insurance through the unemployment insurance system can in principle be achieved without distorting national labour markets, and that international risk-sharing introduces a countercyclical element to the unemployment insurance tradeoff. When we calibrate our model to the Euro area, the desire to insure country-specific risk dominates and optimal benefits and replacement rates are countercyclical and lead to substantial welfare gains. By contrast, recent Eurozone policy proposals seem to have only limited effects.

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JEL: H2, J6, E32, E62)

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1. Introduction

European countries have recently experienced highly asymmetric business cycle movements. This development, together with the resulting asymmetric strains on public budgets, has renewed calls to introduce some form of public cross-country risk sharing, sometimes under the name of a "fiscal union". Indeed, a widely held view is that a common currency exacerbates the need for international risk sharing mechanisms, and that fiscal transfers become desirable when the private sector lacks effective risk sharing mechanisms (Mundell, 1961). Capital and labour markets are not nearly as integrated

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across member countries in the European Monetary Union as they are across states in the United States, so this seems to be a relevant concern for the Eurozone.

At the same time, high unemployment levels in many developed countries have renewed interest in the design of unemployment insurance. In the Eurozone in particular, policy makers have argued that the unemployment insurance system is a good and politically viable channel to share risk across countries.[1]

In this paper, we ask the question: If a group of countries were to introduce a common unemployment insurance system, what should this system look like? We answer this question using a two-country business cycle model with search frictions in labour markets. Financial markets are incomplete and labour is immobile across countries, so that there country-specific risk and idiosyncratic unemployment risk can only be partially privately insured. The government in each country maintains a mandatory unemployment insurance system providing benefits to unemployed workers and financed by a wage tax on the employed. On top of this, a supranational unemployment insurance agency is introduced which is able to administer an additional component of the unemployment insurance system. This component can vary across countries as a function of country-specific shocks.

We derive two theoretical insights from our analysis. Our first result is that a supranational unemployment insurance system can be used to insure against country-level risk without affecting unemployment levels. The intuition for this result is as follows. The effect of unemployment insurance on employment operates through the relative value of employment over unemployment, which determines search effort and wage bargaining. When a country is to receive a fiscal transfer, this relative value can be kept constant by simultaneously increasing the level of benefits and lowering the rate of contributions to the unemployment insurance system. The opposite can be done in the country which is to send the transfer. This way, the incentives for search and job creation are unchanged while resources have effectively been transferred across countries. In principle then, the classic objective of insuring workers and the newly introduced objective of international risk sharing can be separated in the design of unemployment insurance.

At the optimum however, these two objectives do interact. Our second result is that the presence of an international risk sharing motive introduces a countercyclical element to the optimal unemployment insurance policy. Here, the intuition is as follows. The classic unemployment insurance tradeoff for a social planner is between efficiency of employment and insurance. Too much insurance reduces search effort and/or job creation, while too little insurance harms risk-averse workers who cannot insure privately against unemployment risk. When international risk sharing is present, the planner is

[1] A harmonised unemployment insurance system within the Eurozone as a tool for international risk sharing has been suggested by the President of the European Council (van Rompuy, 2012), the International Monetary Fund (Blanchard *et al.*, 2014), the German Institute for Economic Research (Bernoth and Engler, 2013) and the French Advisory Council (Artus *et al.*, 2013). Brenke (2013) also discusses some of the drawbacks.

partially shielding local consumption from fluctuations in local output. When output falls, its share in local consumption falls as well. But this means that it becomes less costly to improve insurance, since local consumption is now less affected by it. The tradeoff between efficient employment and insurance shifts towards the latter. Therefore, insurance becomes more countercyclical than when the risk-sharing motive is absent.

We calibrate our model to the Euro area and compute the Ramsey-optimal policy. We then compare this with a policy of constant replacement rates and no international transfers (as they are currently in place in most countries). Our baseline simulation suggests that the optimal unemployment insurance policy is countercyclical, even when nationally optimal policies would prescribe procyclicality. The optimal policy is very far from the constant replacement rate-benchmark and involves sizeable changes in replacement rates and cross-country transfers over the business cycle. We also compare the optimal policy to a recent proposal by Artus *et al.* (2013) to implement a EU-wide unemployment insurance. We find that this proposal only has very small effects in our model. This is in contrast to the findings of Artus *et al.*, the main reason being that their calculations do not take into account that surpluses and deficits of their scheme will have to be financed by national governments. When we take these fiscal effects into account in our general equilibrium model, the effectiveness of the proposal is greatly reduced.

Our results have several limitations. First, the only relevant sources of employment fluctuations in our model are productivity shocks. The first-best outcome in this case, well-known from the international RBC literature, is to reduce employment in a country experiencing a negative shock and increase it elsewhere where it is more productive. In competitive equilibrium, this can be realised by making unemployment insurance more generous in the affected country and less generous elsewhere. In this sense, our analysis is a first pass at the policy design problem. Second, we abstract from sovereign debt and impose balanced budgets on national governments. When cyclical fluctuations are sufficiently short-lived, governments can effectively implement risk-sharing by financing countercyclical fiscal policy with debt, and it is not clear that the benefits of a supranational risk-sharing scheme are large. However, recent events in the Eurozone have made it clear that there are limits on the debt capacity of governments especially at times when risk sharing might be needed most. Our assumption of balanced budgets can be interpreted as an extreme form of a sovereign debt constraint. Third, we abstract from political moral hazard as in Persson and Tabellini (1996): It is plausible that a fiscal transfer mechanism reduces incentives for national governments to carry out structural reforms. This moral hazard problem is probably the main political reason for its opposition in the Eurozone. But ultimately, such concerns need to be weighed against the economic benefits of the mechanism. In this paper, we are concentrating on the latter.

The remainder of the paper is organised as follows. We briefly review the related literature in Section 2. In Section 1, we lay out a simplified version of our model with only two periods. This allows us to show our theoretical insights analytically and provide intuition. In Section 4, we lay out the full dynamic model that we use for quantitative analysis and calibrate it to the Euro area. Section 5 contains the numerical results from our calibrated model. Section 6 concludes.

2. Related literature

Our analysis relates to the literature on international risk sharing and fiscal union on one hand, and the literature on the design of optimal unemployment insurance on the other hand.

It is well known that the search externality in frictional labour markets can be corrected using unemployment insurance. Because of costly search, employment – and the corresponding fluctuations – may be too low or too high depending mainly on the relation of the workers' bargaining power to the matching elasticity. In the steady state, this can be resolved by correspondingly changing the outside option of workers through unemployment benefits (Hosios, 1990). When workers are risk-averse, the correction of the search externality needs to be weighed the provision of insurance. Fredriksson and Holmlund (2006) provide surveys of the literature on optimal unemployment insurance in static and steady-state situations. More recently, interest has emerged in unemployment insurance policies that depend on the state of the business cycle. Here, a central point of debate is whether benefits should become more generous in a recession in order to increase insurance (countercyclical policy), or less generous in order to mitigate the fall in employment (procyclical policy). Earlier contributions such as Kiley (2003) and Sanchez (2008) suggest that there is room for countercyclical unemployment benefit policy. A more recent contribution is Landais *et al.* (2015). They analyse a model with sticky wages and job rationing and find that a countercyclical policy is optimal as the effects of insurance on equilibrium unemployment is smaller in recessions. On the other side, Mitman and Rabinovich (2015) numerically compute optimal dynamic policies and show that the cyclical stance of the unemployment insurance is procyclical, in a setting when workers' outside option leads to inefficiently high wages. Moyen and Stähler (2014) analyse the optimal cyclicality of unemployment insurance holding its average level fixed. They show that there are situations in which unemployment insurance should be countercyclical even when wages are directly affected *and* the bargaining power of workers is too high relative to the Hosios condition. This is the case when steady-state unemployment benefits are too generous in light of the Hosios condition and marginal utility of consumption varies sufficiently much.[2]Jung and Kuester (2015) analyse first-best policy with sufficiently many fiscal instruments. They find that,

[2]Moyen and Stähler (2014) compare optimal benefit duration policy in Europe and the US. In their European calibration, the bargaining power of workers is larger than the matching efficiency, implying the optimal benefit to be negative in light of the Hosios condition. However, it is restricted to be positive. Additionally, rule-of-thumb households make average marginal utility of consumption fluctuate relatively much. It can be shown that steady-state benefits above optimum and relatively volatile marginal utility of consumption makes optimal benefit policy countercyclical even when bargaining power of workers is high already.

in recessions, benefits should rise – i.e be countercyclical – if, at the same time, hiring subsidies and layoff taxes rise. The latter two instruments increase the incentives to hire workers and decrease those to fire workers, which may compensate partly for increased wage costs when increasing benefits in recession. However, if the other two instruments are not available, they also find procyclical benefit policy to be optimal.

The literature exclusively analyses closed economies. Our paper analyses optimal policy in a context in which unemployment insurance operates across multiple countries and faces the additional objective of sharing cross-country risk.

Turning to the literature on fiscal unions, Leduc *et al.* (2009) have shown that, when asset markets are incomplete, country-specific productivity disturbances can have large uninsurable effects on wealth and, thus consumption paths. In a prominent recent paper, Farhi and Werning (2012) find that such uninsurable effects may be especially large in a currency union with nominal rigidities. They suggest forming a transfer union to insure against this risk. Many economists follow their view that, in federal unions, a (fiscal) transfer mechanism to at least compensate for the uninsurable effects due to nominal rigidities may be desirable. However, there is still some debate on how to ideally establish such a transfer mechanism or a fiscal union (see Bargain *et al.*, 2013 and Bordo *et al.*, 2011 for a discussion). In this paper, we show that a transfer mechanism is even desirable in a real model, and it can be implemented through a supranational unemployment insurance system without distorting national labour markets. Including nominal frictions to the model should only strengthen our results.

3. Simplified model

The intuition for our results can best be seen in a simple two-period model, where we can analytically prove our theoretical results and provide a graphical representation. The quantitative analysis is carried out in the next section.

3.1. Model setup

There are two countries, Home and Foreign. Home is inhabited by a mass $\omega \in (0, 1)$ of workers, while Foreign is inhabited by a mass $1 - \omega$ of workers. In each country, firms transform labour into consumption goods. Firms are owned by risk-neutral entrepreneurs. While consumption goods can be traded across countries in competitive markets, labour is immobile across countries and labour markets are subject to search frictions.

In the first period, all workers start out as unemployed and no production takes place. Agents can, however, trade assets with each other. Asset markets are incomplete, and we will spell out the precise market structure later on. In any event, the utility function of a worker in period 1 is as follows:

$$U = \mathbb{E}\left[nu\left(c_e\right) + \left(1 - n\right)u\left(c_u\right)\right] \tag{1}$$

where c_e is his consumption level if he turns out to be employed in period 2, and c_u his consumption level when he turns out to be unemployed. n is the employment level in period 2. We assume logarithmic utility $u(c) = \log(c)$.

In the second period, firms post vacancies, workers are matched with firms and production takes place. In the Home country, the initial unemployment rate is $u = 1$ and the number of vacancies is v. The number of matches follows a Cobb-Douglas production function

$$m(u, v) = \kappa_m u^{\mu} v^{1-\mu} \tag{2}$$

and employment[3] at the end of the period is

$$n = m(1, \theta) \tag{3}$$

where $\theta = v/u$ is labour market tightness.

A firm that posts a vacancy incurs a cost κ_v. The probability that the vacancy is filled is $q(\theta) = \kappa_m \theta^{-\mu}$. In that case, the match produces output a and the worker gets paid a wage w. This wage is determined using Nash bargaining, where the bargaining power of workers is denoted ξ (the bargaining solution is described further below). A zero-profit condition for vacancy creation prescribes

$$\kappa_v = q(\theta)(a - w) \tag{4}$$

We denote by y aggregate output in the Home country net of vacancy costs:

$$y = an - \kappa_v v \tag{5}$$

The productivity a is a random variable which is only revealed in the second period.

Employed workers receive wages w which are taxed at the rate τ, while unemployed workers receive unemployment benefits b. Each worker might also receive income W_i from assets traded in the first period, where $i = e, u$ denotes his status as employed or unemployed. The individual and aggregate consumption levels are:

$$c_e = (1 - \tau) w + W_e \tag{6}$$
$$c_u = b + W_u \tag{7}$$
$$c = nc_e + (1 - n) c_u \tag{8}$$

The Foreign country has a similar structure to the Home country, but with possibly different parameters. We denote Foreign variables with an asterisk, e.g. b^* for foreign unemployment benefits. Home and foreign productivity (a, a^*) are the only sources of aggregate uncertainty.

Payroll taxes τ and benefits b are administered by an unemployment insurance

[3] Throughout the paper, quantity variables will be expressed in per capita terms unless otherwise indicated.

agency. We assume that the two countries are part of an insurance union, such that the agency operates across both countries. It has to run a balanced budget with the constraint:

$$\omega\left[(1-n)\,b-n\tau w\right]+(1-\omega)\left[(1-n^{*})\,b^{*}-n^{*}\tau^{*}w^{*}\right]=0 \tag{9}$$

In order to close the model, we have to specify the assets that agents can use in the first period to insure themselves against risk, and the unemployment insurance policies.

3.2. Social planner solution

Before looking at the competitive equilibrium, we first look at a benchmark social planner solution. A utilitarian social planner maximises the sum of worker utilities subject only to the resource constraint and the search friction by solving the following problem:

$$\max_{\begin{pmatrix} n,\theta,c^{e},c^{u} \\ n^{*},\theta^{*},c^{e*},c^{u*}\end{pmatrix}} \begin{array}{cc} \tilde{\omega} & \mathbb{E}\left[nu\left(c_{e}\right)+(1-n)\,u\left(c_{u}\right)\right] \\ +\,(1-\tilde{\omega}) & \mathbb{E}\left[n^{*}U\left(c_{u}^{*}\right)+(1-n^{*})\,U\left(c_{u}^{*}\right)\right] \end{array}$$

$$\begin{aligned} \text{s.t. } n &= \kappa_{v}\theta^{1-\mu} && (10)\\ n^{*} &= \kappa_{v}^{*}\left(\theta^{*}\right)^{1-\mu^{*}} && (11)\\ \omega\left(nc_{e}+(1-n)\,c_{u}\right) & \\ +\,(1-\omega)\left(n^{*}c_{e}^{*}+(1-n^{*})\,c_{u}^{*}\right) &= \omega\left(an-\kappa_{v}\theta\right) \\ &+\;(1-\omega)\left(a^{*}n^{*}-\kappa_{v}^{*}\theta^{*}\right) && (12) \end{aligned}$$

Here, $\tilde{\omega}$ is the relative weight the planner puts on workers in the Home country, which might be more or less than the size of its population ω. Within a country, all workers are ex-ante homogenous and so weighting of individual workers is inconsequential.[4] The first order conditions of the planner problem are standard:

$$\begin{aligned} \kappa_{v} &= \kappa_{m}\theta^{-\mu}\left(1-\mu\right)a && (13)\\ \kappa_{v}^{*} &= \kappa_{m}^{*}\left(\theta^{*}\right)^{-\mu^{*}}\left(1-\mu^{*}\right)a^{*} && (14)\\ c_{u} &= c_{e} && (15)\\ c_{u}^{*} &= c_{e}^{*} && (16)\\ \frac{\omega}{\tilde{\omega}}c_{e} &= \frac{1-\omega}{1-\tilde{\omega}}c_{e}^{*} && (17) \end{aligned}$$

The first two conditions are the Hosios conditions in each country, which determine the number of vacancies that maximise aggregate output net of vacancy costs. The remaining conditions prescribe full risk sharing within and across countries. The consumption

[4]The entrepreneurs owning firms can be ignored in the problem, since they are risk neutral and make zero profits in all states of the world.

levels of employed and unemployed workers within each country should be identical, and each country should consume a constant fraction of union output.

3.3. *Optimal policy with private insurance*

We now come back to the competitive equilibrium. Even when markets are complete, the competitive equilibrium generally doesn't implement the social planner solution because of search externalities. Throughout this paper, we assume some form of market incompleteness, since the focus of our paper is on how unemployment insurance can be used to overcome insufficient international risk sharing. In this section, we simply preclude any international risk sharing, but allow workers to insure privately against idiosyncratic unemployment risk at no cost. In Period 1, each worker $i \in [0, \omega]$ at Home can issue a claim on his future income. These claims can be traded in a competitive market within the country but not across countries. Since all workers are ex-ante identical, it is optimal for a worker to fully diversify his risk by selling his entire future income in exchange for a diversified portfolio of income of all other workers in his country. In this case, the consumption levels in Period 2 are

$$c_e = c_u = c \tag{18}$$

This allows us to solve for the Nash-bargained wage. The worker surplus from a match is

$$\mathcal{W} - \mathcal{U} = u'(c)\frac{\partial c}{\partial n} \tag{19}$$

and the firm surplus is

$$\mathcal{J} = a - w. \tag{20}$$

When workers have bargaining power ξ, the bargained wage is simply:

$$w(a, \rho) = \frac{\xi a}{\xi + (1-\xi)(1-\rho)} \tag{21}$$

where ρ is the net replacement rate, defined as

$$\rho = \frac{b}{(1-\tau)w}. \tag{22}$$

A higher replacement rate effectively means more generous unemployment insurance. This improves workers' outside option and drives up wages. It thereby lowers the incentives for job creation and reduces employment.

We want to know what the optimal unemployment insurance scheme looks like in this situation. With privately insured unemployment risk, the insurance agency has to mitigate three inefficiencies: search externalities in the Home and Foreign country and lack of international risk sharing. It also has three policy instruments: the Home and Foreign replacement rates and a cross-country transfer. This already suggests that there exists a policy that eliminates all inefficiencies. That is indeed the case.

We first note that the budget constraint of the unemployment insurance agency can be rewritten as

$$\begin{aligned} 0 = & \quad \omega\left[(1-n)\,b - n\tau w\right] + (1-\omega)\left[(1-n^*)\,b^* - n^*\tau^* w^*\right] \\ = & \quad \omega\,(c-y) + (1-\omega)\,(c^*-y^*) \end{aligned} \tag{23}$$

in which the replacement rates do not appear. That is, we can choose replacement rates ρ, ρ^* and a cross-country transfer $\omega\,(c-y)$ as a policy, and back out the necessary benefits b, b^*, τ, τ^* from the budget constraint and replacement rate definition.

Labour market outcomes in this model only depend on policy through the replacement rate and therefore, *a transfer of resources from one country to another can be implemented through the unemployment insurance system without affecting unemployment levels*. A positive transfer from Foreign to Home would be implemented by increasing benefits b to unemployed workers, and at the same time lowering payroll taxes τ on employed workers. This way, all workers get to consume more, but the net replacement rate ρ stays constant and the relative bargaining position of workers is unchanged.[5]

The replacement rates which satisfy the Hosios condition are

$$\rho = \frac{\mu - \xi}{\mu\,(1-\xi)},\, \rho^* = \frac{\mu^* - \xi^*}{\mu^*\,(1-\xi^*)} \tag{24}$$

and the optimal consumption with a social planner weight $\tilde{\omega}$ on the Home country is

$$c = \frac{\tilde{\omega}}{\omega}\left(\omega y + (1-\omega)\,y^*\right). \tag{25}$$

The planner weight $\tilde{\omega}$ is in principle indeterminate. Here, we determine it by the condition that transfers are zero in expectation, so that in Period 1, neither country expects to be subsidising the other country on average through the unemployment insurance system. Imposing $\mathbb{E}\left[c-y\right] = 0$ leads to a planner weight that is simply the expected share of Home output in union output

$$\tilde{\omega} = \frac{\mathbb{E}\left[\omega y\right]}{\mathbb{E}\left[\omega y + (1-\omega)\,y^*\right]} \tag{26}$$

and a transfer policy

$$c - y = \frac{\mathbb{E}\left[y\right]\mathbb{E}\left[y^*\right]}{\mathbb{E}\left[\frac{\omega}{1-\omega}y + y^*\right]}\left(\frac{y}{\mathbb{E}\left[y^*\right]} - \frac{y}{\mathbb{E}\left[y\right]}\right). \tag{27}$$

[5] This is a general result: An unemployment insurance scheme which can vary benefits and contributions in both countries has four policy instruments and one budget constraint, which means that it is possible to achieve three objectives, in particular leaving employment levels unchanged while implementing a cross-country fiscal transfer.

That is, the Home country receives a transfer when its output is below average, but has to pay a transfer when output in the Foreign country is below average. This policy perfectly replicates the social planner solution.

3.4. Optimal policy without private insurance

The previous case has illustrated how the unemployment insurance system can implement cross-country transfers orthogonally to unemployment levels. However, we have so far abstracted from the most important objective of unemployment insurance, namely to insure against unemployment. In the presence of uninsurable idiosyncratic unemployment risk, the optimal policy becomes genuinely second-best and tradeoffs emerge between all three policy objectives: maximising net output, providing insurance between employed and unemployed, and providing insurance across countries.

We now eliminate all asset trade in Period 1, which means that workers cannot insure any risk. In this case, the consumption levels in Period 2 are simply:

$$c_e = (1-\tau)\, w \tag{28}$$

$$c_u = b = \rho c_u \tag{29}$$

We solve again for the Nash-bargained wage, which now takes into account the curvature in the worker's utility function. The worker surplus from a match is

$$\mathcal{W} - \mathcal{U} = u\left(c_e\right) - u\left(c_u\right) \tag{30}$$

and the firm surplus is unchanged. When workers have bargaining power ξ, the bargained wage is now:

$$w\left(a,\rho\right) = \frac{\xi a}{\xi - (1-\xi)\log\rho}. \tag{31}$$

In this situation, the social planner allocation is no longer feasible. Providing full insurance against idiosyncratic unemployment risk clearly calls for $\rho = 1$, but in this case the worker gets to capture the whole surplus ($w = a$) and job creation completely collapses. Therefore, we have to solve for the Ramsey-optimal policy here.

The Ramsey planner solves the following problem:

$$\max_{\substack{n,\theta,c^e,\rho,\\ n^*,\theta^*,c^{e*},\rho^*}} \quad \begin{array}{ll} \tilde{\omega} & \mathbb{E}\left[nu\left(c_e\right) + (1-n)\, u\left(\rho c_e\right)\right] \\ +\,(1-\tilde{\omega}) & \mathbb{E}\left[n^* u\left(c_e^*\right) + (1-n^*)\, u\left(\rho^* c_e^*\right)\right] \end{array}$$

$$\text{s.t. } n = \kappa_m \theta^{1-\mu} \tag{32}$$

$$n^* = \kappa_m^* \left(\theta^*\right)^{1-\mu^*} \tag{33}$$

$$\kappa_v = \kappa_m \theta^{-\mu}\left(a - w\left(a,\rho\right)\right) \tag{34}$$

$$\kappa_v^* = \kappa_m^* \left(\theta^*\right)^{-\mu^*}\left(a^* - w^*\left(a^*,\rho^*\right)\right) \tag{35}$$

$$\begin{aligned} \omega\left(n+(1-n)\rho\right)c_e & \\ +\left(1-\omega\right)\left(n^*+(1-n^*)\rho^*\right)c_e^* &= \omega w\left(a,\rho\right)n \\ &+ \left(1-\omega\right)w^*\left(a^*,\rho^*\right)n^* \end{aligned} \tag{36}$$

Here, we have substituted out many of the equilibrium conditions of the competitive equilibrium. In particular, we have made use of the fact stated before that choosing an unemployment insurance policy (b, b^*, τ, τ^*) subject to the insurance agency's budget constraint is equivalent to choosing replacement rates and consumption levels $(\rho, \rho^*, c_e, c_e^*)$subject to the aggregate resource constraint. As before, we choose the social planner $\tilde{\omega}$ such that any transfers made across countries net out in expectation: $\mathbb{E}\left[c-y\right]=0$.

As we have written it, the problem has eight choice variables and five constraints, leaving three degrees of freedom. These correspond of course to the three policy instruments ρ, ρ^* and the cross-country transfer $c-y$. The first order condition determining the optimal transfer is as follows:

$$c-y=\frac{\mathbb{E}\left[y\right]\mathbb{E}\left[y^*\right]}{\mathbb{E}\left[\frac{\omega}{1-\omega}y+y^*\right]}\left(\frac{y^*}{\mathbb{E}\left[y^*\right]}-\frac{y}{\mathbb{E}\left[y\right]}\right). \tag{37}$$

This is the exact same condition as in the previous case of private insurance against unemployment risk: Each country at optimum consumes a constant share of union output. The Home country receives a transfer when its output is below average, but has to pay a transfer when output in the Foreign country is below average.[6] However, this now only holds for *average* consumption in a country. The consumption of each worker need not be proportional to union output.

The central equation in this section is the first order condition with respect to the replacement rate. For the Home country, it reads as follows (the expression is symmetric for the Foreign country):

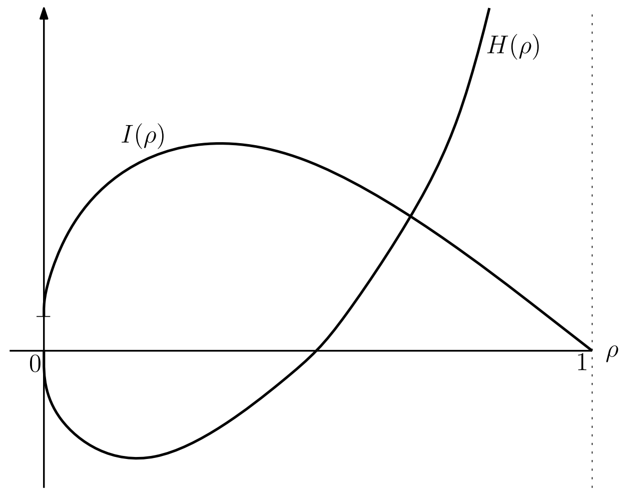
$$\underbrace{\frac{(1-n)(1-\rho)}{n+(1-n)\rho}-\epsilon_\rho^n\left(\log\rho+\frac{1-\rho}{n+(1-n)\rho}\right)}_{=:I(\rho)}=\underbrace{-\epsilon_\rho^y\frac{1}{n}\frac{y}{c}}_{=:H(\rho)} \tag{38}$$

where $\epsilon_\rho^n=\frac{dn}{d\rho}\frac{\rho}{n}$ is the elasticity of Home employment with respect to the Home replacement rate, and $\epsilon_\rho^y=\frac{dy}{d\rho}\frac{\rho}{y}$ is the elasticity of net Home output with respect to the Home replacement rate.

This condition has an intuitive interpretation. The left-hand side, which we call $I(\rho)$, is the marginal benefit of insurance when raising the replacement rate, at a fixed quantity of output available to the country. By raising ρ, the unemployed's marginal utility increases relative to average marginal utility. This is the first term on the left-hand

[6]This result is due to our assumption of logarithmic utility.

Figure 1: Optimal replacement rate for constant a and y/c.



side of Equation (38). At the same time, a higher ρ reduces employment (through higher wages and lower job creation) which shifts the composition of the workforce towards the unemployed. This means that one marginal worker suffers a utility loss, which is the "$\log \rho$" term in the left-hand side of Equation (38). It also implies a composition effect on the insurance budget, captured by the remaining term on the left-hand side. The right-hand side, which we call $H(\rho)$, is the marginal cost of raising the replacement rate in terms of net output lost (output minus vacancy costs).

The determination of the optimal replacement rate is graphically depicted in Figure 1, which plots the functions $H(\rho)$ and $I(\rho)$.[7] We can see that the insurance term $I(\rho)$ is positive and only equals zero at $\rho = 1$. Intuitively, holding output constant it is always desirable to increase the replacement rate until full insurance is achieved. The efficiency term $H(\rho)$ is first negative and then turns positive, approaching plus infinity at $\rho \rightarrow 1$. Intuitively, when ρ is too high, there is too little job creation and the amount of resources available for consumption can be increased by lowering replacement rates, thereby lowering bargained wages and increasing job creation. In this case, $\epsilon_{\rho}^{y} < 0$ and therefore $H(\rho)$ is also positive. As $\rho \rightarrow 1$, output collapses to zero and the marginal utility from lowering the replacement rate becomes infinite. Conversely, when ρ is too low, there is too much vacancy posting and the amount of resources available for consumption can be increased by raising replacement rates. In this case, $H(\rho)$ is negative.

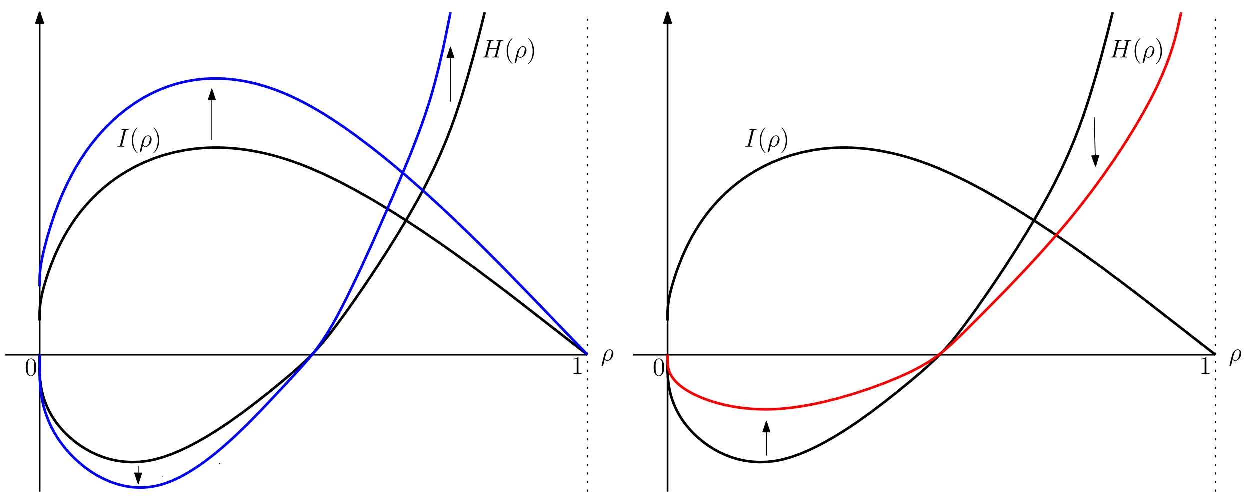
The optimal replacement rate lies at the intersection between the two curves. We can already see that under the optimal policy, employment is always inefficiently low. Since the benefit of insurance is always positive, the optimal ρ is higher than what the Hosios condition $H(\rho) = 0$ would call for.

What happens to the optimal replacement rate when shocks to a or a^* hit the econ-

[7]Proposition 2 in the appendix proves that the shape of the I and H curves are indeed as depicted.

Figure 2: Change in the optimal replacement rate in a recession.

(a) Reduction in a. (b) Reduction in y/c.



omy? We first keep the ratio y/c constant (one can imagine a closed-economy situation in which $y/c = 1$) and look at the effect of a reduction in productivity a. The effects are depicted in in Panel (a) of Figure 2.

A reduction in a increases the insurance term $I(\rho)$ and also the absolute value of the efficiency term $H(\rho)$. Intuitively, holding total resources constant, a decrease in productivity increases individual risk and therefore the social benefit to insure: $I(\rho)$ shifts up for any value of ρ. At the same time, total resources available also decrease, and so the marginal utility of increasing output towards its efficient level also increases: $H(\rho)$ increases in magnitude for any value of ρ. These two forces work in opposite directions on the replacement rate, so that the overall effect is ambiguous.

So far, we have kept the ratio y/c constant, but at optimum it is jointly determined with the replacement rate. The risk-sharing condition (37) prescribes that average consumption c of the Home country be proportional to union output, not just its own output. If Home produces more, then consumption rises less than one-for-one: y/c is increasing in y and decreasing in y^*. It is this risk-sharing aspect which is novel to the literature on optimal unemployment insurance.

The presence of international risk-sharing makes the replacement rate more countercyclical. This can be seen easily from the optimality condition (38). When a falls, Home's output will relatively low compared to union output, and y/c will drop. Panel (b) of Figure 2 shows that the optimal replacement rate rises as a response, introducing a countercyclical element to the optimal policy. When y/c falls, the efficiency term $H(\rho)$ gets compressed towards zero, which can be seen directly from Equation (38). Intuitively, Home's output is now relatively less important compared to union output. Therefore, it becomes less important to ensure that this output is at its efficient level. The trade-off between efficiency and insurance shifts towards the latter and the replacement rate becomes more generous.[8]

We can show (see Proposition 5 in the appendix) that *in the limit as $\omega \to 0$, the Home replacement rate when y/c is varied optimally is decreasing in Home productivity a*. The smaller ω, the better Home country risk can be hedged and as the country's size approaches zero relative to the entire risk-sharing union, average consumption can be completely shielded from output fluctuations. In this case, a fall in a unambigously raises the replacement rate. Nevertheless, the Ramsey planner's tradeoff between efficiency and insurance does not disappear – the optimal replacement rate always remains below one.

Also, *the Home replacement rate is increasing in Foreign productivity a^**, as this variable affects Equation (38) only through a lower ratio y/c. When the Foreign country experiences a drop in productivity, maximising Home output now matters relatively more for the Ramsey planner, and the tradeoff between efficiency and insurance shifts towards the former. The Home replacement rate therefore becomes less generous.

This concludes the presentation of our two main theoretical results: Unemployment can be used to share risk across countries through transfers without affecting national unemployment levels; and at the optimum, international risk sharing introduces a countercyclical component to the optimal insurance policy.

4. Model for quantitative analysis

While the stylised model of the previous section illustrated the relevant tradeoffs of supranational unemployment insurance, we would like to know whether our results survive in a more general setting. We therefore set up a dynamic model in the spirit of the simple model above, and include several additional features such as search effort on behalf of workers and imperfect substitutability between Home and Foreign goods. We calibrate the model to Eurozone data and numerically solve for the Ramsey-optimal policies.

4.1. Model setup

Time is discrete at $t = 0, 1, 2, ...$ As before, a unit mass of workers and firms populates the economy, where $\omega \in (0, 1)$ workers live in the Home country and $(1 - \omega)$ workers live in the Foreign country. We describe the model setup in the Home country. The structure of the Foreign economy is identical to the Home country (up to potentially different parameter values).

[8] Proposition 4 in the appendix provides a formal proof.

4.1.1. Workers

A worker in the Home country can be employed or unemployed (indexed by $j = e, u$). It maximises expected lifetime utility

$$U_j = \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t \left(u\left(c_{jt}\right) - k\left(e_{jt}\right)\right)\right]$$
$$c_{jt}, e_{jt} \geq 0$$
$$u\left(c_{jt}\right) = \frac{\left(c_{jt}\right)^{1-\gamma}}{1-\gamma}, \gamma \geq 0$$
$$k\left(e_{jt}\right) = \frac{\left(e_{jt}\right)^{1+\phi}}{1+\phi}, \phi > 0$$

$\mathbb{E}[\cdot]$ is the expectations operator, $\beta \in (0,1)$ is the discount factor. e_{jt} is effort spent on job search (which is zero when $j = u$) and $k\left(e_t^j\right)$ is the convex cost of job search. γ is the coefficient of relative risk aversion, and C_{jt} denotes expenditure on a consumption basket. This basket consists of goods produced in the Home and Foreign country and is given by

$$c_{jt} = \left(\psi\left(c_{jt,H}\right)^{\sigma} + (1-\psi)\left(c_{jt,F}\right)^{\sigma}\right)^{1/\sigma} \tag{39}$$

where $c_{jt,H}$ is the amount of goods consumed and produced at Home, while $c_{jt,F}$ is the amount of goods consumed at Home but produced in Foreign. The parameter $\sigma \in (-\infty, 1)$ governs the elasticity of substitution between foreign and domestic goods, which is constant at $1/(1-\sigma)$, and the parameter ψ represents the relative valuation of Home goods.[9]

We assume that there are no international trade costs, so the law of one price holds for both goods. We normalise the price of the Home good to unity and let p_t be the price of Foreign goods. Thus, p_t equals the terms of trade of the Home country. Next, we define the *consumer price index* (CPI) at Home by $P_t = \left(c_{jt,H} + p_t c_{jt,F}\right)/c_{jt}$. Utility maximisation implies that

$$\frac{c_{jt,H}}{c_{jt,F}} = \left(p_t \frac{\psi}{1-\psi}\right)^{\frac{1}{1-\sigma}} \tag{40}$$

$$P_t = \left(\psi^{\frac{1}{1-\sigma}} + (1-\psi)^{\frac{1}{1-\sigma}} p_t^{-\frac{\sigma}{1-\sigma}}\right)^{-\frac{1-\sigma}{\sigma}} \tag{41}$$

[9]In the case of unitary elasticity of substitution ($\sigma = 0$), the consumption basket is of the Cobb-Douglas form $c_{jt} = \left(c_{jt,H}\right)^{\psi}\left(c_{jt,F}\right)^{1-\psi}$, so that the expenditure share on Home goods is exactly ψ. A situation where $\phi > \omega$ then corresponds to home bias in consumption.

We still need to specify workers' budget constraints and the financial assets they have access to. We want to capture an incomplete market setting in which workers can neither obtain perfect insurance of their idiosyncratic unemployment risk, nor perfect insurance of country-specific risk. Here, we will consider an extreme case in which workers do not have access to savings at all and simply consume their income each period. Intermediate forms of market incompleteness would certainly add realism, but at the cost of tractability of the model in the presence of heterogenous agents.

Employed workers receive the real wage w_t of which an amount τ_t of payroll taxes is deducted. The unemployed receive unemployment insurance benefits b_t. Also, workers receive the profits from firms (described further below). We have to take a stance on how firm profits are distributed in the presence of heterogeneity, and assume that all agents own equal shares of the firms in their country which pay an aggregate profit π_t.[10] Thus, in each period the real value of an employed worker's consumption basket is simply his after-tax real wage plus profits, while the unemployed consume their unemployment benefit plus profits:

$$c_{et} = (w_t - \tau_t + \pi_t) / P_t \tag{42}$$

$$c_{ut} = (b_t + \pi_t) / P_t. \tag{43}$$

We are now ready to solve the agents' optimisation problem. Define the worker value functions as follows:

$$\mathcal{W}_t = u(c_{et}) + \beta \mathbb{E}_t \{(1-s)\mathcal{W}_{t+1} + s\mathcal{U}_{t+1}\} \tag{44}$$

$$\mathcal{U}_t = u(c_{ut}) + \beta \mathbb{E}_t \{(1 - f_{t+1}e_{t+1})\mathcal{U}_{t+1} + f_{t+1}e_{t+1}\mathcal{W}_{t+1} - k(e_{t+1})\}. \tag{45}$$

Then maximising the utility of the unemployed with respect to effort leads to the following optimality condition:

$$k'(e_t) = f_t(\mathcal{W}_t - \mathcal{U}_t). \tag{46}$$

4.1.2. Firms

Each country produces a distinct good. In the Home country, a representative firm produces the Home good using a production technology which is linear in labour:

$$y_t = a_t n_t. \tag{47}$$

Employment is subject to search frictions. The firm needs to post a number of vacancies v_t, each of which leads to successful matching with a worker with probability q_t. The vacancy filling rate is taken as given by the firm. Successful matches start production in the next period. At the same time, existing matches are destroyed at the exogenous rate

[10] Since firms discount profits at the household rate, holding the firm portfolio effectively does give agents a form of savings through firms' intertemporal decisions.

s.

The firm needs to pay its workers a wage w_t (expressed in units of domestic goods), and it needs to pay a cost for each vacancy, which takes the form of a constant quantity of domestically produced goods κ_v. Its profits are given by

$$\pi_t = (a_t - w_t)\, n_t - \kappa_v v_t. \tag{48}$$

The firm maximises the discounted sum of profits

$$\mathbb{E} \sum_{t=0}^{\infty} Q_{0,t} \pi_t$$

where $Q_{s,t}$ is the discount factor between times s and t. Since the firm is owned in parts by employed and unemployed workers, it is not obvious what discount factor the firm should use. As in Jung and Kuester (2015), we set the firm discount factor to a weighted average of the worker discount factors:

$$Q_{s,t} = \beta^{t-s} \frac{n_t u'(c_{et}) + (1 - n_t)\, u'(c_{ut})}{n_s u'(c_{es}) + (1 - n_s)\, u'(c_{us})} \frac{P_s}{P_t}. \tag{49}$$

We denote by $\mathcal{J}_t$ the value of a filled job:

$$\mathcal{J}_t = a_t - w_t + (1 - s)\, \mathbb{E}_t Q_{t,t+1} \left[\mathcal{J}_{t+1} \right]. \tag{50}$$

The optimality condition of the firm with respect to vacancy creation then takes the familiar form:

$$\kappa_v = q_t \mathcal{J}_t. \tag{51}$$

4.1.3. Matching and wage determination

At the beginning of period t, a fraction u_t of workers at Home are unemployed. We assume that labour is immobile across countries, so that workers can only search for jobs domestically. The number of total new hires is determined by the number of searching workers u_t, the search effort e_t of these workers, and the number of vacancies v_t. Workers and vacancies are then randomly matched according to a standard Cobb-Douglas matching function

$$m_t = \kappa_e \left(e_t u_t \right)^{\mu} v_t^{1-\mu} \tag{52}$$

where κ_e is a matching efficiency parameter and μ is the elasticity of matches with respect to unemployment. Defining labour market tightness as $\theta_t = v_t / e_t u_t$, the probability that a vacancy gets filled, and the probability that a worker putting in one unit of search effort finds a job, are given by:

$$q_t = m_t / v_t = \kappa^e \theta_t^{-\mu} \tag{53}$$

$$f_t = m_t / u_t = \kappa^e \theta_t^{1-\mu}. \tag{54}$$

Separation occurs at the exogenous rate s. Unemployed workers who separate have to wait one period before they can start searching again. In this period, they are not participating. Accordingly,the law of motion for employment and unemployment are given as follows:

$$n_t = (1-s)n_{t-1} + q_t v_t \tag{55}$$

$$u_t = 1 - n_{t-1}. \tag{56}$$

The wage paid to workers is determined by Nash bargaining in which workers and firms share the surplus from matching according to

$$\max_{w_t} \left(\mathcal{W}_t - \mathcal{U}_t\right)^{\xi} \mathcal{J}_t^{1-\xi}$$

where ξ is the bargaining power of workers. Due to the curvature of the utility function, a closed-form solution for the wage does not exist, but is implicitly given by the first-order condition:

$$\mathcal{W}_t - \mathcal{U}_t = \frac{\xi}{1-\xi}\frac{u'\left(c_{et}\right)}{P_t}\mathcal{J}_t. \tag{57}$$

4.1.4. Government

Unlike in the simple model of the previous section, we explicitly spell out national governments as well as a supranational unemployment insurance agency, each of which independently manage their finances.

The government in the Home country gains revenue exclusively from payroll taxes τ_{gt}. These taxes are used to fund benefits for unemployed workers b_{gt} as well as government expenditure g_t. Government expenditure is spent entirely on domestically produced goods.[11] The government has to balance its budget every period. Its budget constraint writes

$$g_t + u_t b_{gt} = \tau_{gt} n_t. \tag{58}$$

The supranational unemployment insurance agency can likewise administer a component of unemployment insurance. This agency also has to balance its budget every period. It collects payroll taxes τ_{xt} and disburses unemployment benefits b_{xt} in the Home country, payroll taxes τ^*_{xt} and disburses unemployment benefits b^*_{xt} in the Foreign country. The agency's budget constraint writes

$$\omega\left(1-n_t\right) b_{xt} + (1-\omega)\left(1-n_t^*\right) p_t b_t^* = \omega n_t \tau_{xt} + (1-\omega)\, n_t^* p_t \tau_{xt}^*. \tag{59}$$

Total taxes on employed workers and total benefits received by unemployed workers,

[11]Our setup implicitly assumes that any utility workers receive from government expenditure is separable from market consumption, so that we can ignore it in the utility function.

and the net replacement rate are then given by:

$$\tau_t = \tau_{gt} + \tau_{xt} \tag{60}$$

$$b_t = b_{gt} + b_{xt} \tag{61}$$

In our benchmark calibration, the supranational agency is inactive ($b_{xt} = b^*_{xt} = \tau_{xt} = \tau^*_{xt} = 0$) and national governments target a constant replacement rate $\rho_t = \bar{\rho}$ and $\rho^*_t = \bar{\rho}^*$. Since this situation is close to the current system in place in the Eurozone, we refer to this situation as the "status quo".

4.1.5. Market clearing and shocks

The market clearing conditions for consumption goods produced in each country take the form:

$$\omega\left(y_t - \kappa^v v_t - g_t\right) = \omega\left(n_t c_{et,H} + (1-n_t)\, c_{ut,H}\right) + (1-\omega)\left(n^*_t c^*_{et,H} + (1-n^*_t)\, c^*_{ut,H}\right) \tag{62}$$

$$(1-\omega)\left(y^*_t - \kappa^{*v} v^*_t - g^*_t\right) = \omega\left(n_t c_{et,F} + (1-n_t)\, c_{ut,F}\right) + (1-\omega)\left(n^*_t c^*_{et,F} + (1-n^*_t)\, c^*_{ut,F}\right) \tag{63}$$

Finally, the exogenous shocks in our model are persistent shocks to productivity and government spending. We rule out permanent shocks for reasons discussed in the next subsection. The processes for government spending and productivity in the Home country are as follows:

$$\log g_t = \rho_g \log g_{t-1} + \left(1-\rho_g\right) \log \bar{g} + \varepsilon_{gt} \tag{64}$$

$$\log a_t = \rho_a \log a_{t-1} + (1-\rho_a) \log \bar{a} + \varepsilon_{at} \tag{65}$$

In particular, we rule out any permanent shocks. This choice is not innocuous in our model, because it has implications for optimal risk sharing. The first best allocation in our model would completely shield domestic consumption from domestic employment and instead tie it to union output. In the presence of permanent shocks that differentially affect the long-run level of GDP in each country, this would effectively prescribe permanent transfers from the country with higher per capita income to the one with lower per capita income, and the Ramsey-optimal policy would then implement this prescription by permanent fiscal transfers. We do not see much practical relevance in such an extreme form of risk sharing, not even to speak of the political viability. We therefore focus exclusively on cyclical shocks. In this case, any cross-country transfers under the Ramsey planner will always fall back to zero in expectation.

4.2. Optimal policy

Our goal is to characterise the optimal unemployment insurance policy of the government sector. We can consolidate the two national governments and the supranational agency by aggregating the budget constraints as in the simplified model:

$$\omega\left(\left(1-n\right)b-n\tau\right)+\left(1-\omega\right)\left(\left(1-n^{*}\right)pb^{*}-n^{*}p\tau^{*}-\right)=0 \tag{66}$$

Effectively, the government sector has control over Home and Foreign benefits and taxes and faces one budget constraint, which implies three degrees of freedom. We express these degrees of freedom in terms of the Home and Foreign replacement rates $(\rho, \rho^{*})$and the transfer from Foreign to Home as a fraction of Home GDP:

$$T_{t}=\frac{\left(\left(1-n\right)b-n\tau\right)}{y_{t}} \tag{67}$$

A social planner who only faces the economy's resource constraints and search frictions in labour markets would simply equate marginal utilities of all workers in both countries (up to constant multiplying factors) and implement the efficient level of job creation as prescribed by the Hosios condition. If agents were able to privately insure their idiosyncratic unemployment risk within each country, this allocation could be implemented by setting replacements rates to satisfy the Hosios condition and equalise marginal utilities across countries by appropriate transfers. But in our setup, there are not enough policy instruments to neutralise all three sources of inefficiency (undiversified idiosyncratic unemployment risk, search externalities and undiversified cross-country risk).

We therefore solve for the Ramsey-optimal policy where the planner maximises the following objective function:

$$\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^{t}\left(\begin{array}{c}\tilde{\omega}\left(n_{t}u\left(c_{et}\right)+\left(1-n_{t}\right)u\left(c_{ut}\right)\right)\\+\left(1-\tilde{\omega}\right)\left(n_{t}^{*}u\left(c_{et}^{*}\right)+\left(1-n_{t}^{*}\right)u\left(c_{ut}^{*}\right)\right)\end{array}\right)\right] \tag{68}$$

subject to all the equilibrium conditions of the economy. As in the simple model, we choose the planner weight on the Home country $\tilde{\omega}$ to rule out permanent transfers from one country to another:

$$\mathbb{E}\left[T_{t}\right]=0. \tag{69}$$

4.3. Calibration and model-data comparison

We set the discount factor β to the standard value of 0.99 which yields an annual interest rate of 4 percent. The parameter σ is set to 0.736, corresponding to an elasticity of substitution between Home and Foreign goods of 3.9, which matches the European average of estimates reported in Corbo and Osbat (2013). Given that value, we calculate a value for the home good preference ψ of 0.56 to meet the corresponding average estimates of trade openness from Balta and Delgado (2009). The curvature of consumption γ is set to 1.5 as reported in Smets and Wouters (2003) and the search effort parameter is set to $\phi = 1$, corresponding to a unitary search effort elasticity. The effort scaling parameter κ_{e} is set to 0.692 to normalise steady state effort to unity.

Table 1: Baseline calibration.

Parameter	Symbol	Value
Country size	ω	0.5
Preferences		
Discount factor	β	0.99
Risk aversion	γ	1.5
Preference home/foreign goods	σ	0.736
Relative valuation of home goods	ψ	0.56
Inverse elasticity of search effort	ϕ	1
Effort cost scaling	κ_e	0.692
Labour market		
Matching elasticity	η	0.5
Matching efficiency	κ_e	0.692
Bargaining power	ξ	0.3
Separation rate	s	0.03
Vacancy costs	κ_v	0.711
Technology		
Steady-state level	$\bar{a}$	1
Autocorrelation of productivity	ρ_A	0.61
Std. dev. of productivity	σ_A	0.0069
Government spending		
Steady-state level	$\bar{g}$	0.182
Autocorrelation of government spending	ρ_G	0.79
Std. dev. of government spending	σ_A	0.0047
Policy		
Net replacement rate	ρ	0.65

We set the matching elasticity μ to the conventional value 0.5 according to estimates by Burda and Wyplosz (1994). The bargaining power of workers ξ is set lower, to 0.3. We target a steady-state unemployment rate of 9% and a quarterly vacancy-filling probability of 70% following Christoffel *et al.* (2009). A quarterly job finding rate of 30% is targeted in line with evidence provided by Elsby *et al.* (2013) for a number of European countries. The quarterly separation rate is then deduced from the implied steady state restrictions as $s = 0.030$. We also know that, in the steady state, the number of matches must be equal to the number of separations. This allows us to calculate the matching efficiency $\kappa_m = 0.458$ and vacancy posting costs $\kappa_v = 0.711$.

We set the technology shock persistence to $\rho_a = 0.95$ and its standard deviation such that the output's standard deviations obtained from our model matches the standard deviation of output in the data ($\sigma_a = 0.0069$). The government spending AR(1) process is parameterised to match detrended government expenditure data as in Christoffel *et al.* ($\rho_g = 0.79$, $\sigma_g = 0.0047$).

Finally, the net replacement rate is set at $\rho = 65\%$ that is the average across EMU countries taking into account short and long run benefits, again following Christoffel *et al.*. The calibration of the model is summarised in Table 1.

5. Results

5.1. Moments

Table 2 reports several second moments of the calibrated model at the status quo policy and compares them to the data. Only the standard deviation of output is calibrated to match the data. While the persistence of fluctuations in the model matches the data relatively well, the model suffersfrom a counterfactually low unemployment volatility. This is of course a well-known problem (Shimer, 2005): With Nash bargaining, wages track movements in productivity too closely and the job creation rate is almost acyclical. Indeed, the real wage in the model is more volatile than in the data and almost perfectly correlated with output. We present an alternative specification with rigid wages further below.

Table 2: Second order moments in benchmark calibration.

Variable	Std. dev.	rel. to real GDP	Corr. with real GDP	1st autocorr.
Real GDP y_t/P_t	0.87 [0.87]	1.00 [1.00]	1.00 [1.00]	0.72 [0.88]
Real wage w_t/P_t	0.86 [0.57]	1.01 [0.66]	0.99 [0.36]	0.72 [0.80]
Unemployment u_t	0.04 [0.40]	0.05 [0.47]	-0.49 [-0.86]	0.92 [0.95]

Second moments as obtained from simulating a linear approximation of the model at benchmark calibration. Corresponding moments in the data in parenthesis (from the ECB AWM database, 1984Q1-2008Q1). The second column reports the standard deviation, the third column reports the standard deviation relative to real GDP, the fourth column reports the cross-correlation with real GDP, and the last column reports the first order autocorrelation. Real GDP and real wage are in logarithms. All series are HP-filtered with smoothing parameter 1600.

Table 3: Cyclicality of the optimal unemployment insurance policy.

	Corr. with y_t/P_t	Corr. with y_t^*/P_t^*
Optimal transfer/GDP T_t	-0.61	0.61
Optimal replacement rate ρ_t	-0.13	0.88
Optimal replacement rate ρ_t (no transfers)	0.91	0.15

Correlation coefficients of simulated model data, unfiltered.

We numerically calculate the Ramsey-optimal policy and report its cyclical stance in Table 3.

The first row of the table shows the correlation of transfers T_t to the Home country (as a percentage of Home GDP) with Home and Foreign GDP, respectively. As expected, transfers correlate negatively with Home and positively with Foreign GDP, in order to insure local consumption from changes in local production. The second row reports the cyclicality of the replacement rate, which goes in the same direction: The replacement rate becomes more generous in a recession in which Home output falls relative to Foreign output. When Home productivity is relatively low, the local benefit from efficient production is lower compared to that of increased insurance. However, when Foreign GDP falls, the replacement rate at Home drops. This increases foreign output which is then partly transferred to finance foreign consumption.

In the third row, we constrain the Ramsey planner to not transfer any resources between countries, i.e. we impose $T_t = 0$. This corresponds to the optimal policy carried out by national unemployment insurance policies only, with the supranational scheme absent, and effectively shuts down the international risk sharing dimension of policy design. Consistent with the intuition of the simple model, the replacement rate loses its countercyclicality in the absence of cross-country transfers and the correlation with Home GDP becomes positive.

5.2. Impulse responses

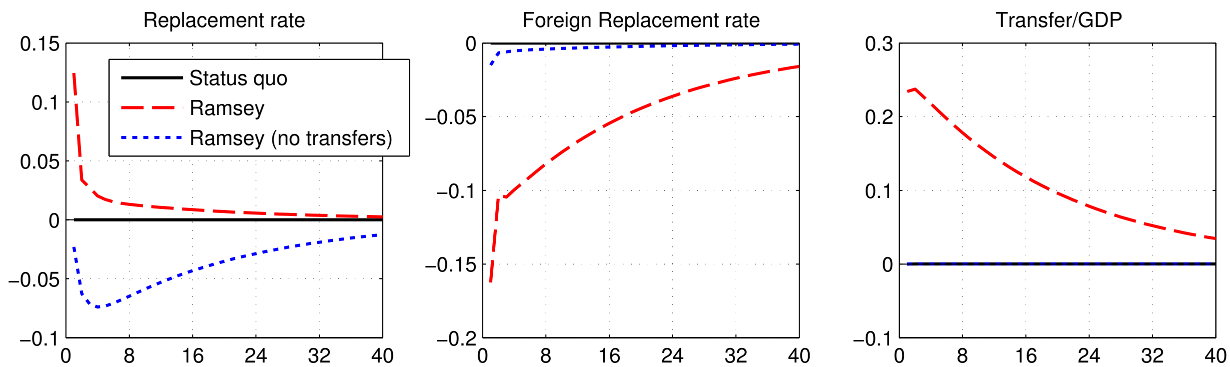
The effects of the Ramsey policy can be illustrated further by looking at impulse response functions. Figure 3 shows impulse responses to a negative productivity shock.

Panel (a) shows the response of the policy instruments which reflects the correlations presented above. Under the status quo (solid black line), replacement rates are constant and transfers between countries are zero. Under the Ramsey-optimal policy (red dashed line), the Home country receives a transfer from Foreign which amounts to more than 0.2% of Home GDP on impact. At the same time, the Home replacement rate becomes more generous while the Foreign rate becomes less generous as Home productivity falls. By contrast, when we shut down international risk sharing by imposing zero transfers (blue dotted line), the Home replacement rate falls instead (thereby becoming procyclical) while the Foreign rate barely reacts.

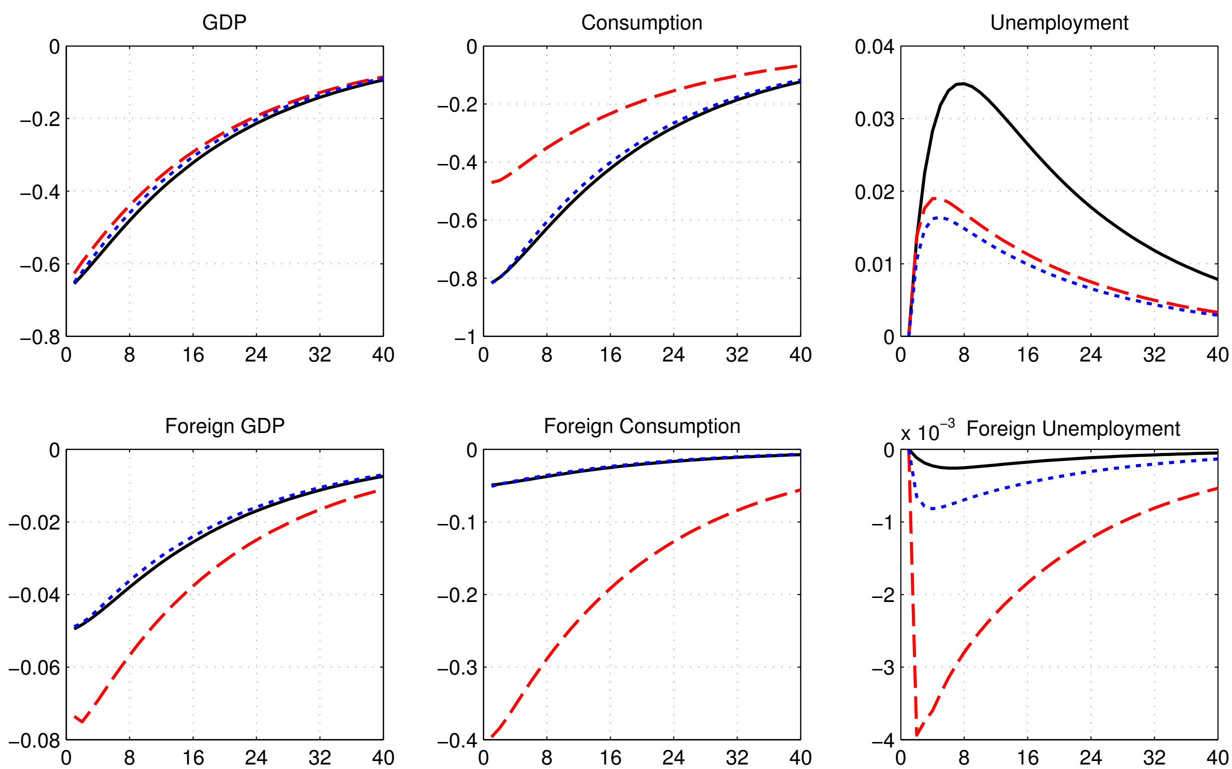
Panel (b) shows the effects of the shock on Home and Foreign output, consumption and the unemployment rate. Looking at the unemployment rate first, the planner achieves a smaller rise in unemployment. This is not surprising since low bargaining

Figure 3: Impulse responses, negative Home productivity shock.

(a) Policy instruments.



(b) Aggregate outcomes.



Impulse responses to a one-standard deviation negative Home productivity shock $\varepsilon_{At} = -\sigma_A = -0.69\%$. Replacement rates ρ_t, Foreign replacement rate ρ_t^*, Home transfer T_t , unemployment u_t and Foreign unemployment u_t^* are in percentage point deviation from steady-state. GDP y_t/P_t, Foreign GDP y_t^*/P_t^*, consumption c_t and foreign consumption c_t^* are in 100*log deviation from steady state.

power $\xi < \mu$ with Nash bargaining is known to lead to inefficiently volatile unemployment relative to the first-best. But the comparison between the Ramsey policy with and without transfers is revealing: without transfers, the rise in unemployment is even lower. This is a reflection of the much lower replacement rate which increases search effort and reduces bargained wages. Foreign unemployment falls in all cases as the Foreign country experiences a positive terms of trade shock. Under the optimal policy, this fall is amplified by a lower replacement rate. In all cases however, the Foreign unemployment rate reacts very little.

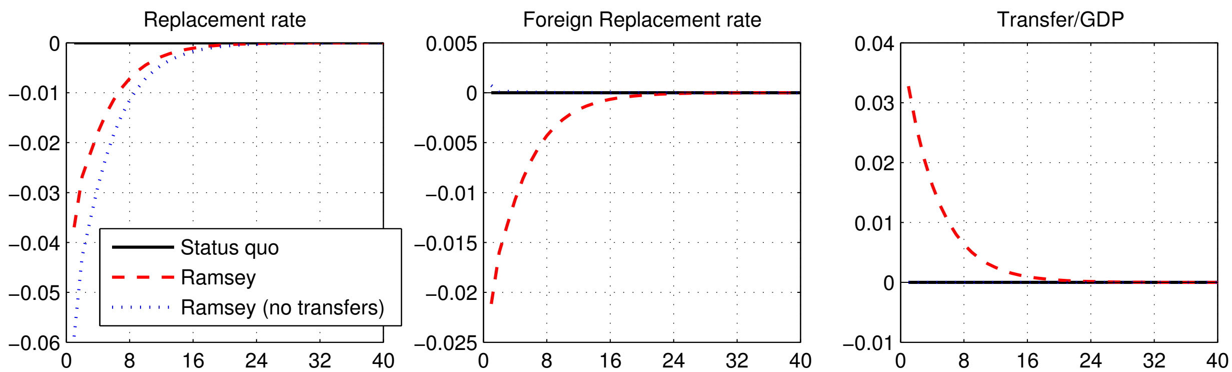
The response of output is dominated by the fall in productivity itself and does not change much across policies. Unless transfers are made between countries, this also holds for consumption. But the optimal policy with transfers does transfer significant resources from Foreign to Home, leading to a smaller reduction in consumption at Home but a sharper reduction in Foreign.

Figure 4 shows impulse responses to a positive shock to Home government spending. Under the status quo, the increase in government spending crowds out consumption and investment in vacancies, leading to a rise in unemployment and a fall in output under the status quo policy, as can be seen from Panel (b) of the figure. Consumption falls at Home and to a lesser extent in Foreign (since Foreign consumers also demand Home goods). The Ramsey-optimal policy reduces the replacement rate to mitigate the rise in unemployment, and effecs a transfer from Foreign to Home to shield consumption from the crowding-out effect. For this shock, transfers are actually procyclical since real GDP rises at Home and falls in Foreign. This goes against our general finding of countercyclical transfers, but is a natural consequence of our assumption that any utility from government expenditure is separable from utility of market consumption, so that the social planner essentially treats an increase in Home government spending as a loss of resources.

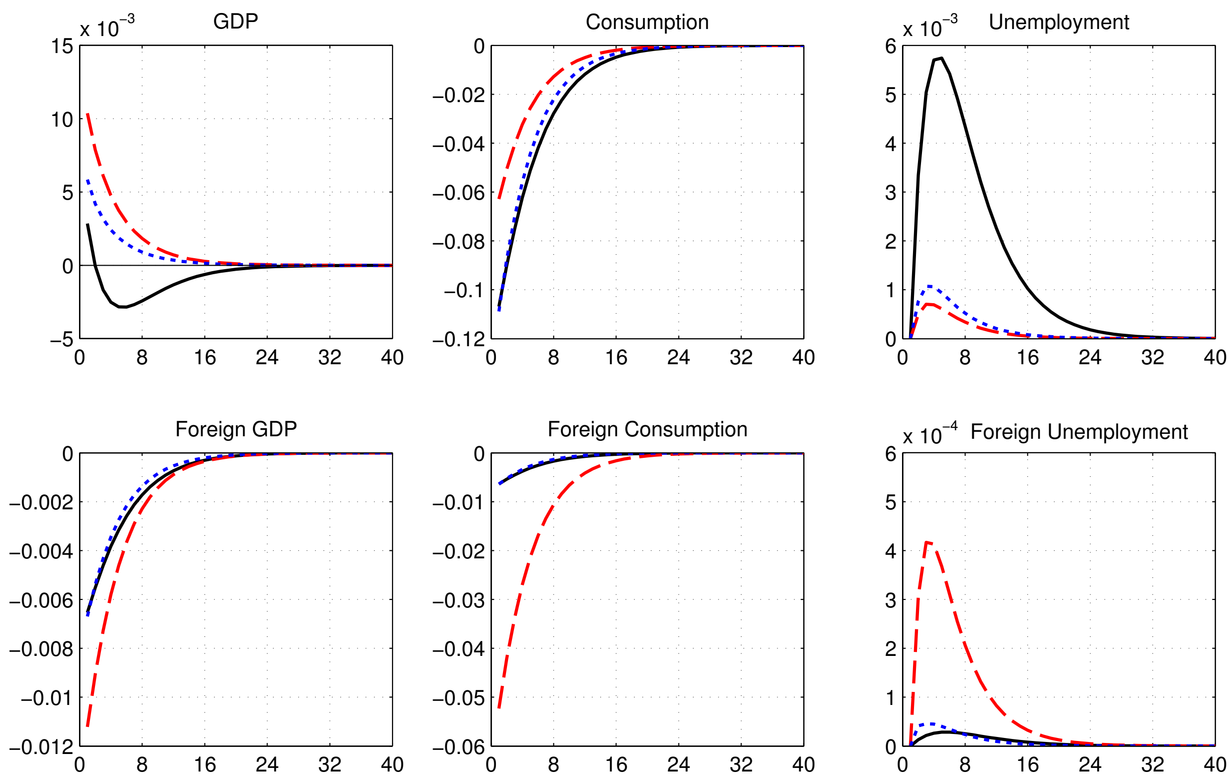
We finish this section with a discussion of a policy proposal by Artus *et al.* (2013). They advocate a European unemployment insurance that pays unemployed workers a 20% net replacement rate, with national benefits reduced by the same amount so that the overall benefits are unaffected. This is financed by a contribution by the employed that is "set to 20% of the aggregate payroll multiplied by the structural unemployment rate in the country". The first thing to note about this proposal is that it does not run a balanced budget, and does not even take into account an intertemporal budget constraint. If we were to reproduce this policy in our model, even with perfectly known structural (steady-state) unemployment rates the financial position of the European unemployment scheme would have a unit root. Therefore, it is not clear at all how Artus *et al.* conclude that the scheme would "avoid any permanent transfer" unless additional rules are put in place that determine the sharing of deficits and surpluses among member countries of their scheme. In our model, we computed the effects of their proposal under the assumption that a constant fraction of the surplus/deficit of the scheme is born by each country, with a country's weight proportional to its steady-state share in union-wide GDP. We found that the effects were extremely small. However, this result is certainly due to the fact that the fluctuations in unemployment rates are counter-

Figure 4: Impulse responses, positive Home government spending shock.

(a) Policy instruments.



(b) Aggregate outcomes.



Impulse responses to a one-standard deviation positive Home government spending shock $\varepsilon_{Gt} = \sigma_G = 0.47\%$. Replacement rates ρ_t, Foreign replacement rate ρ_t^*, Home transfer T_t , unemployment u_t and Foreign unemployment u_t^* are in percentage point deviation from steady-state. GDP y_t/P_t, Foreign GDP y_t^*/P_t^*, consumption c_t and foreign consumption c_t^* are in 100*log deviation from steady state.

factually small. The Artus *et al.* proposal effectively bases the size of transfers on the difference between actual and structural unemployment rate, which is too small in our calibrated model to produce any appreciable effects.

5.3. Alternative specification with rigid wages

One weakness of our benchmark calibration is the very low volatility of unemployment. In this section, we report results from an alternative specification of the model in which we make wages (measured in units of domestically produced goods) completely rigid. That is, we assume

$$w_t = \bar{w}. \tag{70}$$

We keep the parameter values of our benchmark calibration as reported in Table 1, and choose the value of $\bar{w}$ such that the standard deviation of the unemployment rate matches exactly that in the data. We also adjust the standard deviation of the productivity shock σ_A to match the volatility of output in the data as before. This leads to a value of $\sigma_A = 0.0041$.

Table 4 compares the second moments to the data for our alternative specification. Unsurprisingly, the improvement in the behaviour of the unemployment rate now

Table 4: Second order moments, rigid wage specification.

Variable	Std. dev.	rel. to real GDP	Corr. with real GDP	1st autocorr.
Real GDP y_t/P_t	0.87 [0.87]	1.00 [1.00]	1.00 [1.00]	0.82 [0.88]
Real wage w_t/P_t	0.07 [0.57]	0.09 [0.66]	-0.63 [0.36]	0.90 [0.80]
Unemployment u_t	0.40 [0.40]	0.47 [0.47]	-0.71 [-0.86]	0.93 [0.95]

Second moments as obtained from simulating a linear approximation of the model at benchmark calibration. Corresponding moments in the data in parenthesis (from the ECB AWM database, 1984Q1-2008Q1). The second column reports the standard deviation, the third column reports the standard deviation relative to real GDP, the fourth column reports the cross-correlation with real GDP, and the last column reports the first order autocorrelation. Real GDP and real wage are in logarithms. All series are HP-filtered with smoothing parameter 1600.

comes at the expense of a counterfactually smooth wage rate.

Table 5 reports the cyclicalities of the optimal policy. One can see from the first row of the table that our main result from the previous specification is unchanged even with completely rigid wages: The optimal Home replacement rate is countercyclical with respect to Home GDP. However, here it is also slightly negatively correlated with Foreign GDP. When we recalculate the optimal policy zero cross-country transfers imposed (second row), we find again that the Home replacement rate is less strongly correlated with output. The optimal transfer (third row) also remains countercyclical.

Finally, we now report the properties of the Artus *et al.* (2013) proposal as described above. With realistic fluctuations in the unemployment rate, this policy proposal can in principle have sizeable effects. We report the cyclicality of this policy in the last row of Table 5. As expected, the transfer is indeed countercyclical.

Table 5: Cyclicality of the optimal unemployment insurance policy, rigid wage specification.

	Corr. with y_t/P_t	Corr. with y_t^*/P_t^*
Optimal replacement rate ρ_t	-0.45	-0.13
Optimal replacement rate ρ_t (no transfers)	-0.38	-0.01
Optimal transfer/GDP T_t	-0.46	0.46
Transfer/GDP T_t (Artus et al. proposal)	-0.66	0.64

Correlation coefficients of simulated model data, unfiltered.

Here, we only report impulse response functions for a productivity shock, as this shock is the main driver of output fluctuations in the model. Figure reports the impulse responses to a negative Home productivity shock. As before, the solid black line is the response under the status quo policy (constant replacement rate, no transfers) and the dashed red line is the response under the Ramsey-optimal policy. Here, we compare this policy to the Artus *et al.* proposal (green dotted line).

Panel (a) shows the optimal response of the policy instruments. As with flexible wages, the optimal Home replacement rate rises when Home productivity falls, and the Home country receives a transfer from Foreign. The magnitudes of the responses are much larger than with flexible wages though: The Home replacement rate rises by 5 percentage points on impact, and remains 0.5 percentage point above its steady state for more than two years. The optimal transfer is also very large and amounts to more than 0.6% of Home GDP on impact with a slow decay.

Panel (b) reveals that the higher replacement rate only initially leads to a higher rise in unemployment, with the unemployment rate being lower than under the status quo after one year. The fall in real GDP is also mitigated. Foreign unemployment also rises, albeit by a much smaller amount, reflected in a small drop in Foreign GDP. The transfer of resources from Home to Foreign makes that Home consumption does not drop as much and even rises on impact, while Foreign consumption falls significantly.

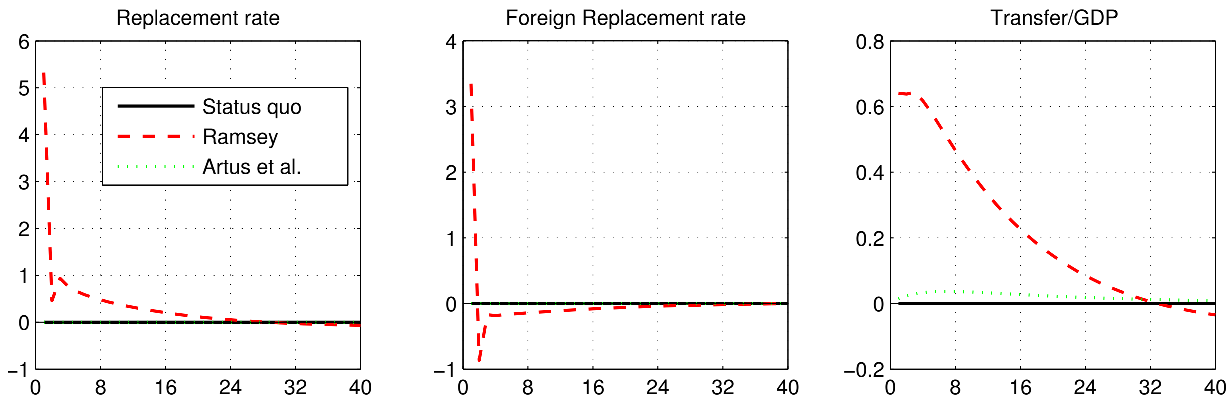
The impact of the Artus *et al.* (2013) proposal (green dashed line in the figure) remains very limited even with unemployment fluctuations as large as in the data. The reduction in Home consumption, for example, is somewhat smaller than under the status quo policy, but the difference is very small. In the light of this result, the benefit of this proposal seems therefore questionable.

6. Conclusions

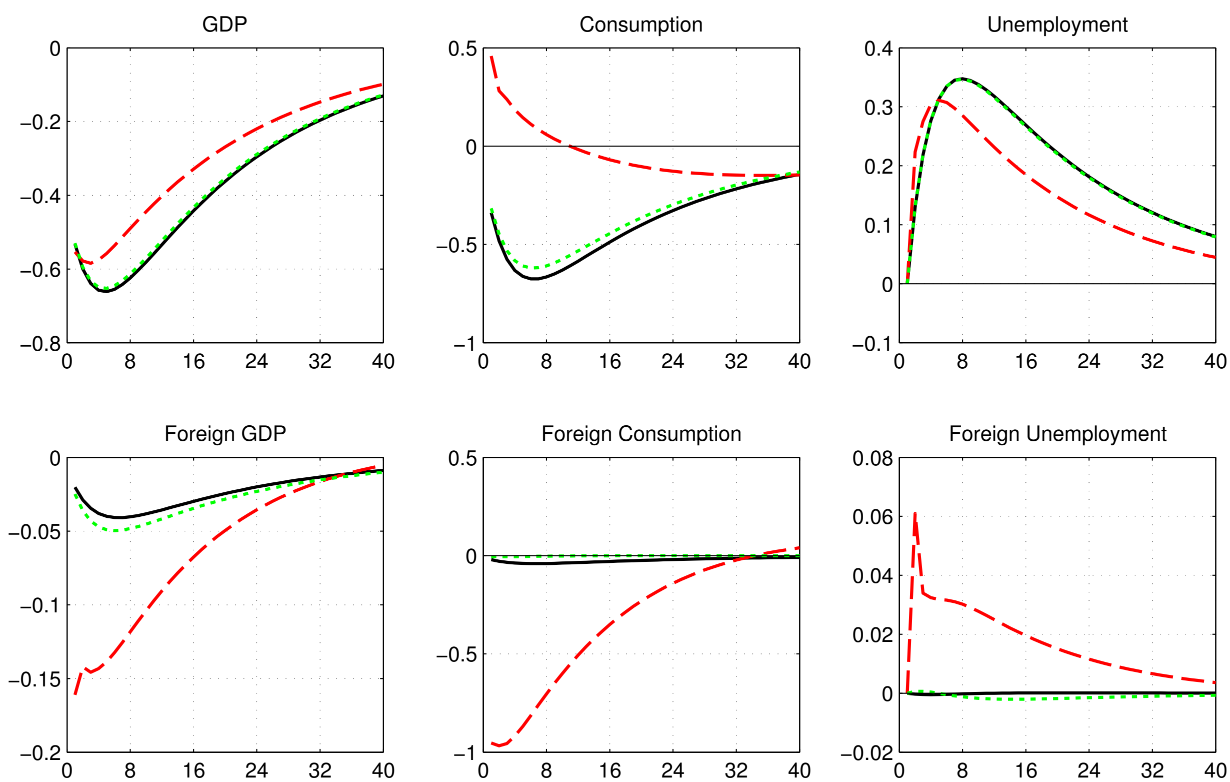
In this paper, we used an international business cycle model augmented by frictional labour markets and incomplete financial markets to discuss optimal unemployment insurance policy operating across multiple countries. This adds an additional objective of international risk sharing to the optimal policy problem, on top of the classic tradeoff between efficient employment and insurance of unemployment risk. We have shown that cross-country insurance through the unemployment insurance system can in principle be achieved without affecting unemployment levels; and that the desirability of interna-

Figure 5: Impulse responses, negative Home productivity shock, rigid wage specification.

(a) Policy instruments.



(b) Aggregate outcomes.



Impulse responses to a one-standard deviation negative Home productivity shock $\varepsilon_{At} = -\sigma_A = -0.69\%$. Replacement rates ρ_t, Foreign replacement rate ρ_t^*, Home transfer T_t , unemployment u_t and Foreign unemployment u_t^* are in percentage point deviation from steady-state. GDP y_t/P_t, Foreign GDP y_t^*/P_t^*, consumption c_t and foreign consumption c_t^* are in 100*log deviation from steady state.

tional risk-sharing introduces a countercyclical element to the optimal unemployment insurance policy. Calibrated to Eurozone data, our model implied that the international risk-sharing component dominates in the design of optimal policy, making it countercyclical overall. The optimal policy prescribes siginificant transfers between countries as well as countercyclical replacement rates. By contrast, recent policy proposals seem to have only a limited impact on business cycle dynamics and international risk sharing.

There are several directions in which our findings could be extended. First, we currently employ a very stylised model of the Eurozone economy, with symmetrical countries, no private or public savings possibilities and no nominal rigidities, thereby abstracting from many potentially relevant factors for the optimal policy design. In these dimensions, our analysis can certainly be refined further. Second, the optimal policy we compute here is one in which the planner has perfect knowledge of the structure of the economy. One of the most difficult issues in implementing a policy such as the one we compute here is that the structurual rate of unemployment and many other factors can only be reliably estimated in hindsight if at all. It would therefore be useful to see whether simple policy rules that are more easily implementable under imperfect information can approximate our optimal policy well.

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Appendix A. Propositions for Section 1

The optimal replacement rate in the absence of private risk sharing satisfies Equation (37)in the main text:

$$\underbrace{\frac{(1-n)(1-\rho)}{n+(1-n)\rho} - \epsilon_\rho^n \left(\log\rho + \frac{1-\rho}{n+(1-n)\rho}\right)}_{=:I(\rho)} = \underbrace{-\epsilon_\rho^y \frac{1}{n} x}_{=:H(\rho)}$$

where $x = y/c$. Here, we prove the properties of the optimal policy as discussed in the text. Throughout, we make the following assumption:

Assumption 1. *I is strictly concave, H and Hy are strictly convex in ρ on $[0,1]$.*

We numerically verified Assumption 3 for a wide range of parameters, and conjecture that Assumption (1) always holds true. The limit behaviour of the functions at the edges is easy to prove and together with our assumption determines the shape of the curves in the main text.

Proposition 2. *$I(0) = \frac{1-\bar{n}}{\bar{n}}$ where $\bar{n} = \kappa_m \left(\frac{\kappa_m}{\kappa_v} a\right)^{(1-\mu)/\mu}$ and $I(1) = 0$. Also, holding y/c constant, $H(0) = H\left(exp\left(-\frac{1-\mu}{\mu}\frac{\xi}{1-\xi}\right)\right) = 0$, $\lim_{\rho\to 1} = +\infty$, and $H'(\rho)$ is strongly convex in $[0,1]$.*

Proof. We start with the insurance term $I(\rho)$. At the limit when $\rho \to 0$, we have $w \to 0$ and $n = \kappa_m \left(\frac{\kappa_m}{\kappa_v}(a-w)\right)^{(1-\mu)/\mu} \to \bar{n}$. Therefore:

$$\frac{(1-n)(1-\rho)}{n+(1-n)\rho} \overset{\rho\to 0}{\longrightarrow} \frac{1-\bar{n}}{\bar{n}}.$$

The remaining term of $I(\rho)$ must therefore go to zero. Indeed,

$$\begin{aligned}\epsilon_\rho^n = \frac{dn}{d\rho}\frac{\rho}{n} &= -\frac{a}{a-w}\frac{1-\mu}{\mu}\frac{w^2}{a^2}\frac{1-\xi}{\xi} \\ &= \frac{1}{\log\rho}\frac{1-\mu}{\mu}\frac{\xi}{\xi-(1-\xi)\log\rho}\end{aligned}$$

and therefore

$$\begin{aligned}&\epsilon_\rho^n \left(\log\rho + \frac{1-\rho}{n+(1-n)\rho}\right) \\ =& \frac{1-\mu}{\mu}\frac{\xi}{\xi-(1-\xi)\log\rho}\left(1+\frac{1-\rho}{n+(1-n)\rho}\frac{1}{\log\rho}\right) \overset{\rho\to 0}{\longrightarrow} 0.\end{aligned}$$

For the case $\rho \to 1$, the first term clearly disappears:

$$\frac{(1-n)(1-\rho)}{n+(1-n)\rho} \stackrel{\rho\to 1}{\longrightarrow} 0$$

and for the second term, we have:

$$\frac{1-\mu}{\mu}\frac{\xi}{\xi-(1-\xi)\log\rho}\left(1+\frac{1-\rho}{n+(1-n)\rho}\frac{1}{\log\rho}\right) \stackrel{\rho\to 0}{\longrightarrow} \frac{1-\mu}{\mu}\left(1+\lim_{\rho\to 1}\frac{1-\rho}{\log\rho}\right) = 0.$$

Next, we turn to the $H(\rho)$ function. As $\rho \to 0$, $n \to \bar{n} > 0$ and and $w \to 0$. Therefore

$$-\epsilon_\rho^y \frac{1}{n} = -\frac{1}{n}\frac{w}{a}\left(\frac{1-\xi}{\xi}+\frac{1-\mu}{\mu}\frac{1}{\log\rho}\right) \stackrel{\rho\to 0}{\longrightarrow} 0.$$

And as $\rho \to 1$, $w \to 1$ and $n \to 0+$, so that

$$-\frac{1}{n}\frac{w}{a}\left(\frac{1-\xi}{\xi}+\frac{1-\mu}{\mu}\frac{1}{\log\rho}\right) \stackrel{\rho\to 1}{\longrightarrow} +\infty.$$

□

Proposition 3. *The optimal replacement rate is unique and strictly between* $\exp\left(\frac{\mu}{1-\mu}\frac{1-\xi}{\xi}\right)$ *and one.*

Proof. Since $f(\rho) = H(\rho) - I(\rho)$ is continuous on $[0,1]$ and a strictly concave by Assumption (1), it crosses zero at most twice. But $f(0) > 0$ and $\lim_{\rho\to} -\infty$, so there is a unique interior solution ρ^* to $f(\rho) = 0$. Since $I(0) > I(1) = 0$ and I is strictly concave, $I(\rho) > 0 \forall \rho \in (0,1)$ and the optimum has $H(\rho^*) > 0$. Since H is a strictly convex function, $H(0) = 0$ and $\lim_{\rho\to 1} H(\rho) = +\infty$ and , $H(\rho_0) = 0$ for exactly one $\rho_0 \in (0,1)$ and $\rho^* > \rho_0$. Finally, $H\left(\exp\left(\frac{\mu}{1-\mu}\frac{1-\xi}{\xi}\right)\right) = 0$. □

Proposition 4. *The optimal replacement rate is strictly decreasing in y/c.*

Proof. Define $x = y/c$. Taking the total derivative of the optimality condition with respect to x, we have

$$0 = \frac{\partial I}{\partial x} - \frac{\partial H}{\partial x} + \frac{\partial I}{\partial \rho}\frac{d\rho}{dx} - \frac{\partial H}{\partial \rho}\frac{d\rho}{dx}$$
$$\Leftrightarrow \frac{d\rho}{dx} = -\frac{\frac{\partial I}{\partial x} - \frac{\partial H}{\partial x}}{\frac{\partial I}{\partial \rho} - \frac{\partial H}{\partial \rho}}.$$

Clearly, $dI/dx = 0$ and at the optimal ρ, we have $dH/dx = H(\rho)/x > 0$. Furthermore, we know that $I(0) > H(0)$ and $I(\rho) = H(\rho)$ only once, so it must be the case that $dH/d\rho > dI/d\rho$ at the optimal ρ. Therefore $d\rho/dx < 0$. □

Proposition 5. *In the limit as $\omega \to 0$, the optimal replacement rate is unique, strictly below one and strictly decreasing in a as y/c is chosen optimally.*

Proof. As $\rho \to 0$, the risk-sharing condition (37) becomes

$$c = \frac{\mathbb{E}[y]}{\mathbb{E}[y^*]} y^*.$$

The optimal choice of ρ when y/c is chosen optimally can now be described as

$$I(\rho) = \tilde{H}(\rho)$$

$$\text{where } \tilde{H}(\rho) = H(\rho) \frac{\mathbb{E}[y^*]}{\mathbb{E}[y]} \frac{y}{y^*}.$$

By Assumption (1), $\tilde{H}$ is a strictly convex function. The behaviour of $\tilde{H}$ at zero is

$$\tilde{H}(0) = H(0) \frac{\mathbb{E}[y^*]}{\mathbb{E}[y]} \frac{\lim_{\rho \to 0} wn}{y^*} = H(0) \cdot 0 = 0.$$

For the limit at one, we note

$$\tilde{H}(\rho) \frac{\mathbb{E}[y]}{\mathbb{E}[y^*]} y^* = -\frac{w^2}{a} \left(\frac{1-\xi}{\xi} + \frac{1-\mu}{\mu} \frac{1}{\log \rho} \right) \overset{\rho \to 1}{\longrightarrow} +\infty$$

since $w \to a$ as $\rho \to 1$. Therefore, the optimal ρ when y/c is chosen optimally has the same properties that we used before holding y/c constant. In particular, the optimal replacement rate is unique and strictly below one. Also, we have $d\tilde{H}/d\rho > dI/d\rho$ at the optimal ρ as in Proposition (4). Taking the total derivative again, we have

$$\frac{d\rho}{da} = -\frac{\frac{\partial I}{\partial a} - \frac{\partial \tilde{H}}{\partial a}}{\frac{\partial I}{\partial \rho} - \frac{\partial \tilde{H}}{\partial \rho}}$$

where the denominator of the fraction is negative, so $d\rho/da$ has the same sign as its enumerator. The derivatives of I and $\tilde{H}$ with respect to productivity a are:

$$\begin{aligned} \frac{\partial I}{\partial a} &= \frac{\partial I}{\partial n} \frac{\partial n}{\partial a} \\ &= \frac{\partial n}{\partial a} \left(\frac{1-\rho}{n + (1-n)\rho} \right)^2 \left(\frac{w}{a} \frac{1-\mu}{\mu} \frac{1}{\log \rho} - \frac{1}{1-\rho} \right) < 0 \end{aligned}$$

and

$$\frac{\partial \tilde{H}}{\partial a} = \frac{\tilde{H}}{a} > 0.$$

Therefore $d\rho/da < 0$. □