# The Role of Labour Markets for Fiscal Policy Transmission*

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**Abstract**

This paper identifies how frictions in the labour market shape the responses of private consumption, employment and the real wage to government spending shocks. The open economy New Keynesian DSGE model is extended by labour market frictions of the Mortensen-Pissarides type and a detailed description of fiscal policy. The nature of offsetting fiscal measures is found to be critical for the effects of fiscal stimulus, due to the different effects of different tax instruments on the labour market. Specifically, shifting the debt-stabilizing burden towards the distortionary labour tax has detrimental effects on the labour market outcome and on general economic performance. The results indicate that wage rigidity increases the effectiveness of fiscal policy in the short term but leads to a worse longer term development including unemployment above steady state levels. The analysis suggests that a closer look at the functioning of labour markets may help to identify fiscal policy transmission channels not captured by the standard New Keynesian model.

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# 1 Introduction

This paper studies the transmission of fiscal policy in the presence of labour market frictions. In order to address the question we extend the standard open-economy New Keynesian (NK) business cycle model in two dimensions: a detailed formulation of fiscal policy, and labour market matching frictions along the lines of Mortensen and Pissarides (MP). We consider a small monetary union member state following Galí and Monacelli (2005).[1]

Fiscal policy is back at the centre of the policy debate. After the implementation of huge fiscal stimulus packages to counter the effects of the financial crisis, the focus has shifted on the alternative ways to pay back the resulting large increases in government debt. At the same time, there is continuing uncertainty, both in the empirical and theoretical literature, on what the effects of fiscal policy really are. The positive effect of increased government spending on output is widely acknowledged. But the magnitude of the output multiplier as well as effects on especially private consumption and the real wage are still debated. Private consumption, as the largest component of aggregate demand, is a key determinant of the size of the government spending multiplier.

The New Keynesian model in its standard form predicts a negative response of private consumption to government spending shocks. The basic mechanism of adjustment is the wealth effect which reduces private lifetime resources. As a consequence, households reduce their demand for consumption and leisure, if both are normal goods. The negative effect on private consumption is, however, typically smaller than in RBC models, because, when prices are rigid, firms increase labour demand as they respond to increased aggregate demand (see e.g. Linnemann and Schabert (2003)). The responses of employment and the real wage to fiscal shocks have received much less attention than effects on output and private consumption. In the New Keynesian framework, the increase in labour demand together with the increase in labour supply can drive up the real wage or at least make it fall by a smaller amount, and employment may increase.

We focus on the effects of government spending shocks on private consumption, employment and the real wage, and identify how frictions in the labour market shape these responses in the New Keynesian framework. The approach is closest to Monacelli, Perotti and Trigari (2010) who investigate output and unemployment fiscal multipliers in an RBC model with labour market matching. They consider New Keynesian features as one extension to their baseline model. Recent research on monetary policy in the presence of labour market frictions (see e.g. Christoffel, Kuester, Linzert (2009)) is also a close reference, and indicates that these frictions may have an important role in shaping the economy's response to shocks.

We consider specifically different fiscal policy instruments that can be used to finance the public debt that results from increased government spending. This approach is motivated by the early finding by Baxter and King (1993) that the chosen financing scheme is a crucial assumption for the effects of fiscal policy. Since that finding, this question has received surprisingly little attention in the otherwise abundant literature on the effects of government

[1]This paper is related to a larger modelling project where the objective is to build a framework for the macroeconomic analysis of the Finnish economy. The choice of the theoretical framework is, therefore, guided by specific country characteristics such as the requirements of Euro area membership, the wage negotiation tradition and rigid nominal wages, as well as a fairly high labour taxation.

spending. More recently, however, e.g. Bilbiie and Straub (2004) have recognised that the way fiscal shocks are financed, shapes the response to a government spending shock in a New Keynesian model as well. Corsetti, Meier and Müller (2009), in turn, analyze a policy where not all stimulus is financed by tax increases but also by reductions in spending over time, in a small open-economy NK model, and find that this spending reversal significantly alters the impact of increased public spending.

The role of labour market rigities is inspected especially in the case of wage rigidity, introduced with the help of a staggered bargaining framework that follows Gertler and Trigari (2009). This is because, in addition to being an intuitively important element in the modelling of a small euro area member country, rigid wages have been found to be a central explanation for the volatile behavior of unemployment in business cycles (see Shimer (2010)).

Our main findings can be summarized as follows. First, the effects of fiscal shocks in our baseline model are in line with the standard New Keynesian model. Output increases, and the response of private consumption is negative and small. Labour market frictions contribute to the intuitive explanation of the rise in employment and the real wage. Firms see their future profit opportunities rise and open new vacancies increasing labour demand. At the same time, the negative wealth effect both increases the supply of hours by each employed worker and increases the relative value from employment for all workers. Labour supply increases along both the intensive and extensive margin. Real wages rise.

Second, the assumption of the chosen offsetting fiscal measure is found to be critical for the effects of fiscal stimulus, due to the different effects of different tax instruments on the labour market. Most importantly, shifting the debt-stabilizing burden towards the distortionary labour tax has detrimental effects on the labour market outcome and on general economic performance. The wage bargaining model implies that, as the debt-stabilizing tax rule becomes operative, the higher proportional tax rate feeds through to a higher negotiated wage. Specifically, the bargained wage rises to compensate workers for the otherwise falling net income. The higher wage directly implies higher labour costs to firms which reduce the number of open vacancies and unemployment starts rising. Due to this subsequent fall in employment the contraction in private consumption is larger than when public debt is adjusted through lump-sum taxes. Interestingly, financing debt by raising the consumption tax has less negative consequences on the labour market than the labour tax because consumption taxes have a smaller negative effect on the total surplus from employment.

Third, wage rigidity increases the magnitude of the responses of labour market variables. Vacancies react more strongly to the initial stimulus, since firms' expected profits are larger when their labour costs do not rise. This is in line with the intuition backed by the literature on labour markets and business cycles, but in contrast to Monacelli, Perotti and Trigari (2010). The main differences is the assumption on price rigidity and the behavior of the real interest rate. In the New Keynesian framework, as opposed to an RBC model, rigid prices give rise to a labour demand effect as witnessed by increased vacancy creation. Combined with rigid prices rigid wages amplify the labour demand effect of fiscal stimulus on employment since firms' expected profits rise more than with flexible wages. In addition, in this model, the real interest rate always falls in response to a government spending shock because the rise in prices in the small currency union member state is not counteracted by tightening monetary policy by the currency union's central bank. This effect is large enough to overturn the upward pressure on the real interest rate caused by the rise in the shadow value of wealth.

Furthermore, our results indicate that while wage rigidity would seem to make fiscal policy more effective in the short term, in the longer term, the gradual increase in the wage causes a prolonged increase in unemployment to above the steady state level. Public debt stays higher and the negative effect of private consumption larger than when wages are flexible.

The remainder of the paper is organised as follows: Section 2 describes the model, Section 3 describes the parametrization and steady state of the model, Section 4 presents the simulation results and elaborates on the transmission mechanisms of fiscal shocks, and Section 5 concludes.

# 2 The model

## 2.1 General features

The model considers a small monetary union member state and builds in this respect on Galí and Monacelli (2005). As in Corsetti, Meier and Müller (2009), however, we close the model by assuming a debt-elastic interest rate instead of complete asset markets. The home country is modelled along standard New-Keynesian practise comprising households, firms and a public sector. For simplicity, capital is not included as a factor of production.

The framework is augmented by a Mortensen and Pissarides (MP) search and matching labour market model (Mortensen and Pissarides, 1994; Pissarides 2000). The structure of the standard labour market matching model has been amended with some key features that have, in more recent literature, been found useful in capturing the data and explaining the so-called unemployment volatility puzzle.[2] There is an emerging consensus that labour market frictions, wage rigidities and staggered price setting together are needed to explain movements in unemployment, and the effects of monetary policy shocks (see e.g. Blanchard and Galí (2008), Christoffel et al. (2008)). These features are taken to be important also for analyzing fiscal policy.

The present model adds rigidity in the adjustment of wages in the form of staggered bargaining initially developed by Gertler and Trigari (2006,2009), and applied in Gertler, Sala and Trigari (2008) and Christoffel, Kuester and Linzert (2008). One advantage of this approach is that wage rigidity gets the explicit interpretation of longer wage contracts. Lengthening the duration of wage contracts makes wages in each period less responsive to economic conditions, and shifts adjustment to the labour quantity side.

[2]Shimer (2005) argues that the MP model in its standard form does not sufficiently reproduce the relatively smooth behavior of wages and relatively volatile behavior of labor market variables observed in the data. Shimer argued that the problem arises because, in the standard model, the wage is renegotiated in every period by Nash bargaining and is thereby let to adjust very easily to changes in the economic environment. The volatility of wages absorbs a large part of the fluctuation that is actually observed in employment variables. In the growing body of literature that has attempted to explain the problem, also known as the unemployment volatility puzzle, the focus has accordingly been on ways to amplify the response of vacancies and unemployment to shocks. The range of alternative models proposed to solve the unemployment volatility puzzle include both flexible and rigid wage variants and have been summarized in e.g. Hall (2005).

In our framework, there is only one worker per firm, and the wage and price setting decisions are separated from each other. Labour market frictions arise in the intermediate good sector. The wholesale firms buy intermediate goods and re-sell them to the final goods sector. Wholesale firms operate under monopolistic competition and set prices subject to Calvo rigidities. Final goods are produced from domestic and imported intermediate inputs under perfect competition.

The other extension of the model concerns the public sector. The government policy instruments include a lump-sum tax, a proportional wage tax paid by the employees, wage taxes paid by the employers in the form of social security contributions, unemployment benefits and other government transfers as well as a consumption tax. The tax instruments react to changes in the debt-to-output ratio according to simple fiscal feedback rules. Government spending is subject to shocks.

## 2.2 Preferences

As in similar kinds of models, we adopt the representative or large household interpretation. This implies perfect consumption insurance, a key assumption needed to embed the MP model in a GE framework. Household members perfectly insure each other against variations in labour income due to their labour market status. This tackles the problem whereby households are identical but not all of their members are employed. As a result, the employment and unemployment rates are identical at the household level and across the population at large (see e.g. Merz (1995)).

The representative household maximizes its expected lifetime utility

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \frac{(C_t - \varkappa C_{t-1})^{1-\varrho}}{1-\varrho} - \delta n_t \frac{(h_t)^{1+\phi}}{1+\phi} \right] \right\} \tag{1}$$

where $C_t$ is final good consumption in period $t$, $\varkappa \in (0,1)$ indicates an external habit motive, $C_{t-1}$ stands for aggregate consumption in the previous period, $h_t$ are hours worked, and $\delta$ is a scaling parameter for the disutility of work. The inverses of $\varrho$ and $\phi$ are the elasticities of intertemporal substitution and of labour supply respectively. The household's (real) budget constraint is

$$\begin{aligned} & (1+\tau_t^c) C_t + \frac{B_t}{P_t} + \frac{B_t^*}{P_t} \\ = \; & n_t \frac{w_t}{P_t} h_t (1-\tau_t) + (1-n_t) b \\ & + \frac{TR_t}{P_t} + R_{t-1} \frac{B_{t-1}}{P_t} + R_{t-1}^* p\left(b_{t-1}^*\right) \frac{B_{t-1}^*}{P_t} + \frac{P_{H,t}}{P_t} D_t + \frac{P_{H,t}}{P_t} n_t \Psi \end{aligned} \tag{2}$$

The left-hand side of the equation describes the expenditures of the household. Consumption $C_t$ is subject to a proportional tax $\tau_t^c$. The household can buy two kinds of nominal one-period bonds, domestic $B_t$ and foreign $B_t^*$ which form the portfolio of its financial assets and are both denominated in the common monetary union currency. Domestic bonds are issued by the domestic government for which they represent debt. The right hand side describes the household's income sources which consist of after-tax real wage $n_t \frac{w_t}{P_t} h_t (1 - \tau_t)$, unemployment benefits $(1 - n_t) b$, lump-sum transfers $\frac{TR_t}{P_t}$, profit from firm ownership $D_t$, and of fixed costs of production $n_t \Psi$ which accrue to consumers who own the firms. Income is also received in the form of repayment of last period's domestic or foreign bond purchases. $R_t = (1 + r_t^n)$ stands for the gross nominal return on domestic bonds. The interest rate paid or earned on foreign bonds by domestic households $R_{t-1}^* p\left(b_{t-1}^*\right)$ consists, in turn, of the common currency union gross interest rate $R_{t-1}^*$ which, for the small member state is taken to be exogenous, and a country-specific risk premium $p\left(b_{t-1}^*\right)$. The risk premium is assumed to be increasing in the aggregate level of foreign real debt as a share of domestic output ($b_t^* = \frac{B_t^*}{P_t Y_t}$).[3]

We leave aside for a moment the labour supply decision, which will be dealt with in the section describing the labour market, below. Optimal allocations are characterized by the following conditions

$$\Lambda_t = \frac{\lambda_t}{(1 + \tau_t^c)} \tag{3}$$

$$\Lambda_t = \beta E_t \left[ \Lambda_{t+1} \frac{R_t}{\pi_{t+1}} \right] \tag{4}$$

$$\Lambda_t = \beta E_t \left[ \Lambda_{t+1} \frac{R_{t-1}^* p\left(b_{t-1}^*\right)}{\pi_{t+1}} \right] \tag{5}$$

where $\lambda_t = (C_t - \varkappa C_{t-1})^{-\varrho}$ is the marginal utility of consumption and $\pi_{t+1} = \frac{P_{t+1}}{P_t}$ is CPI inflation. The discount factor is the same for all optimizing agents in the economy and is hereafter defined throughout the paper as $\beta_{t,t+s} = \beta^s \frac{\Lambda_{t+s}}{\Lambda_t}$.

Combining the Euler conditions for domestic and foreign assets yields a modified uncovered interest rate parity relation where no risk is associated with exchange rate movements, as both domestic and foreign bonds are denominated in the same currency.

$$R_t = R_t^* p\left(b_t^*\right) \tag{6}$$

This arbitrage relation says that, as domestic and foreign bonds perfectly substitute each other, their nominal returns to the consumers have to be equal in equilibrium.

The risk premium on foreign bond holdings $p\left(b_t^*\right)$ follows the function

$$p\left(b_t^*\right) = \exp\left[-\gamma_{b^*}\left(b_t^* - \bar{b}\right)\right], \text{ with } \gamma_{b^*} > 0 \tag{7}$$

This should ensure the stability and determinacy of equilibrium in a small member state of the monetary union model [4]. In the steady state, the risk premium is assumed to be equal

[3] This is the debt-elastic interest rate assumption which is one of the mechanisms suggested by Schmitt-Grohé and Uribe (2003) to close a small open economy model. Note that with the current notation a negative (positive) deviation of the stock of foreign bonds from the steady state zero level implies that the home country as a whole becomes a net borrower (lender), and faces a positive (negative) risk premium.

[4] As Galí and Monacelli (2005) point out, along with accession to the monetary union the small member state no longer meets the Taylor principle since variations in its inflation that result from idiosyncratic

to one, and the domestic and foreign interest rates are the same. After loglinearization the arbitrage relation gets the form

$$\widehat{R}_t = \widehat{R}^*_t - \gamma_{b^*}\widehat{b}^*_t$$

## 2.3 The labour market

The labour market brings together workers and intermediate good firms.

### 2.3.1 Unemployment, vacancies and matching

The measure of successful matches $m_t$ is given by the matching function

$$m_t(u_t, v_t) = \sigma_m u_t^{\sigma} v_t^{1-\sigma} \tag{8}$$

where $u_t$ and $v_t$ are the aggregate measures of unemployed workers and vacancies. $m_t$ is the flow of matches during a period $t$, and $u_t$ and $v_t$ are the stocks at the beginning of the period. The matching function is, as usual, increasing in both vacancies and unemployment, concave, and homogeneous of degree one (see e.g. Petrongolo and Pissarides (2001)). The Cobb-Douglas form implies that $\sigma$ is the elasticity of matching with respect to the stock of unemployed people, and $\sigma_m$ represents the efficiency of the matching process. The probabilities that a vacancy will be filled and that the unemployed person finds a job are respectively

$$q_t^F = q_t^F(\theta_t) = \frac{m_t}{v_t} = \sigma_m \theta_t^{-\sigma} \tag{9}$$

$$q_t^W = \theta_t q_t^F(\theta_t) = \frac{m_t}{u_t} = \sigma_m \theta_t^{1-\sigma} \tag{10}$$

and the inverse of these probabilities is the mean duration of vacancies and unemployment.

$\theta_t = \frac{v_t}{u_t}$ is labour market tightness. The tighter the labour market is, or the less there are unemployed people relative to the number of open vacancies (i.e. larger $\theta_t$), the smaller the probability that the firm succeeds in filling the vacancy and the larger the probability that the unemployed person finds a job. Similarly, a decrease in the number of vacancies relative to unemployment (smaller $\theta_t$) implies that the unemployed person has a smaller probability to find a job.

In the beginning of each period, a fraction of matches will be terminated with an exogenous probability $\rho_t \in (0, 1)$. The separation rate evolves according to the autoregressive process

shocks will have an infinitesimal effect on union-wide inflation, and will thus induce little or no response from the union's central bank. According to the Taylor principle, in order to guarantee the uniqueness of the equilibrium, the central bank would have to adjust the nominal interest rates more than one-for-one with changes in inflation (see e.g. Woodford (2003))

$$\log(\rho_t) = (1-\nu_\rho)\log(\rho) + \nu_\rho \log\left(\rho_{t-1}\right) + \epsilon_t^\rho, \text{ where } \nu_\rho \in (0,1),\ \epsilon_t^\rho \overset{iid}{\sim} N\left(0, \sigma_\rho^2\right)$$

Labour market participation is characterised as follows. The size of the labour force is normalised to one. The number of employed workers at the beginning of each period is

$$n_t = (1-\rho_t)\, n_{t-1} + m_{t-1} \tag{11}$$

where the first term on the right hand side represents those workers who were employed already in the previous period and whose jobs have survived beginning-of-period job destruction, and the second term covers those workers who got matched in the previous period and become productive in the current period. After the exogenous separation shock, the separated workers return to the pool of unemployed workers and start immediately searching for a job. The number of unemployed is $u_t = 1 - n_t$.

In the steady state an equal amount of jobs are created and destructed:

$$JC = JD \Longleftrightarrow m = \rho n \tag{12}$$

### 2.3.2 Wage bargaining

Job creation takes place when a worker and a firm meet and agree to form a match at a negotiated wage. The wage that the firm and the worker choose must be high enough that the worker wants to work in the job, and low enough that the employer wants to hire the worker. These requirements define a range of wages that are acceptable to both the firm and the worker. The unique equilibrium wage.is, however, the outcome of a bargain between the worker and the firm.

The structure of the staggered multiperiod contracting model follows Gertler, Sala and Trigari (2008) but includes also the intensive margin of adjustment of the labour input (hours worked per worker) as well as distortionary taxes. For comparison, the period-by-period bargaining outcome is presented in the appendix. The idea of staggered wage bargaining is analogous to Calvo price setting. Rigidity is created by assuming that a fraction $\gamma$ of firms are not allowed to renegotiate their wage in a given period. As a result, all workers in those firms receive the wage paid the previous period $w_{t-1}$ partially indexed to inflation. The constant probability that firms are allowed to renegotiate the wage is labeled $1-\gamma$. Accordingly, $\frac{1}{1-\gamma}$ is the average duration of a wage contract. Thus, the combination of wage bargaining and Calvo price setting allows to give an intuitive interpretation to the source of wage rigidity instead of more or less ad hoc formulations. Period-by-period bargaining corresponds to the special case of $\gamma = 0$.

As in the standard Mortensen-Pissarides model, it is assumed that match surplus, the sum of the worker and firm surpluses, is shared according to efficient Nash bargaining. In the baseline model, wages and hours are negotiated simultaneously. The firm and the worker choose the nominal wage and the hours of work to maximize the weighted product of their net return from the match. When wages are rigid, it is assumed that as they become productive, new matches enter the same Calvo scheme for wage-setting than existing matches. This is an important assumption for wage rigidity to have an effect on job creation. Gertler and Trigari (2009) argue that after controlling for compositional effects there are no differences in the flexibility of new and existing worker's wages.[5]

The contract wage $w_t^*$ is chosen to solve

$$\max\left[H_t(r)\right]^{\eta}\left[J_t(r)\right]^{1-\eta} \tag{13}$$

subject to the random renegotiation probability. $H_t(r)$ and $J_t(r)$ are the matching surpluses of renegotiating workers and firms respectively, and $0 \leq \eta \leq 1$ is the relative measure of workers' bargaining strength. The value equations describing the worker's and the firm's surplus from employment are the key determinants of the outcome of the wage bargain.

**Workers** The value to the worker of being employed consists of after-tax labour income, the disutility from working, expressed in marginal utility terms, and the expected present value of his situation in the next period. In the case of non-renegotiation, the past nominal wage is partially indexed to CPI inflation $[\pi_t^{\varepsilon_w}(\pi^{1-\varepsilon_w})]$ as in Smets and Wouters (2003) or Christoffel, Kuester and Linzert (2009).

$$\begin{aligned}W_t(r) \;=\;& \frac{w_t^*}{P_t}h_t(1-\tau_t)-\frac{g(h_t)}{\Lambda_t}\\&+E_t\beta_{t,t+1}\left(1-\rho_{t+1}\right)\left[\gamma W_{t+1}\left(w_t^*\left[\pi_t^{\varepsilon_w}\left(\pi^{1-\varepsilon_w}\right)\right]\right)+(1-\gamma)W_{t+1}(w_{t+1}^*)\right]\\&+E_t\beta_{t,t+1}\rho_{t+1}U_{t+1}\end{aligned} \tag{14}$$

The value to the worker of being unemployed is

$$U_t(r)=b+E_t\beta_{t,t+1}\left[q_t^W W_{x,t+1}+\left(1-q_t^W\right)U_{t+1}\right] \tag{15}$$

where the first term on the RHS is the value of the outside option to the worker, i.e. the unemployment benefit $b$, and the second term gives the expected present value of either working or being unemployed in the following period. Unemployed workers do not need to take into account the probability of job destruction even if they get matched because of the timing assumption. A match that has not yet become productive cannot be destroyed. Note that the value for the worker who is currently unemployed to move from unemployment to employment next period is $W_{x,t+1}$, the expected *average* value of being employed, because matching is a random process. New matches are subject to the same bargaining scheme as existing matches, and therefore the new worker does not have a priori knowledge of whether the firm he will start working for will be allowed to renegotiate its wage. Combining these value equations gives the expression for worker surplus

[5] E.g. Pissarides (2009) and Haefke et al (2008) argue the opposite: that wages of newly hired workers are volatile unlike wages for ongoing job relationships. This would mean that there is wage rigidity, but not of the kind that leads to more volatility in unemployment fluctuations.

$$
\begin{aligned}
H_t(r) &= W_t(r) - U_t(r) \\
&= \frac{w_t^*}{P_t} h_t \left(1-\tau_t\right) - \frac{g\left(h_t\right)}{\Lambda_t} - b \\
&\quad + E_t \beta_{t,t+1} \left(1-\rho_{t+1}\right) \left[\gamma H_{t+1}(w_t^* \left[\pi_t^{\varepsilon_w} \left(\pi^{1-\varepsilon_w}\right)\right]) + \left(1-\gamma\right) H_{t+1}(w_{t+1}^*)\right] \\
&\quad - q_t^W E_t \beta_{t,t+1} H_{x,t+1}
\end{aligned} \tag{16}
$$

**Intermediate firms** For the firm, the value of an occupied job is equal to the profit of the firm in the current period net of payroll taxes $s_t$, and the expected future value of the job.is

$$
\begin{aligned}
J_t(r) &= x_t f\left(h_t\right) - \frac{w_t^*}{P_t} h_t \left(1+s_t\right) - \Psi \\
&\quad + E_t \beta_{t,t+1} \left(1-\rho_{t+1}\right) \left[\gamma J_{t+1}(w_t^* \left[\pi_t^{\varepsilon_w} \left(\pi^{1-\varepsilon_w}\right)\right]) + \left(1-\gamma\right) J_{t+1}(w_{t+1}^*)\right]
\end{aligned} \tag{17}
$$

where $x_t$ is the relative price of the intermediate sector's good, and $f\left(h_t\right) = z_t {h_t}^{\alpha}$ is match output. The marginal product of labour is accordingly $mpl_t = \alpha z_t {h_t}^{\alpha-1} = \alpha \frac{f(h_t)}{h_t}$. In addition to labour costs, the firm faces a per-period fixed cost of production $\Psi$ which is independent of hours worked and defined in real terms. At the economy's level, fixed costs are proportional to the number of employed workers. Labour-augmenting productivity $z_t$ is identical for all matches and follows

$$
\log(z_t) = \left(1-\nu_z\right) \log\left(z\right) + \nu_z \log\left(z_{t-1}\right) + \epsilon_t^z, \text{ where } \nu_z \in (0,1),\ \epsilon_t^z \overset{iid}{\sim} N\left(0, \sigma_z^2\right)
$$

The value to the firm of an open vacancy is

$$
V_t = -\kappa_t + E_t \beta_{t,t+1} q_t^F \left[\gamma J_{t+1}(w_t \left[\pi_t^{\varepsilon_w} \left(\pi^{1-\varepsilon_w}\right)\right]) + \left(1-\gamma\right) J_{t+1}(w_{t+1}^*)\right] + E_t \beta_{t,t+1} \left(1 - q_t^F\right) V_{t+1} \tag{18}
$$

The value of a vacancy consists of an exogenous hiring cost $\kappa_t$, and of the expected value from future matches. In equilibrium, all profit opportunities from new jobs are exploited so that the equilibrium condition for the supply of vacant jobs is $V_t = 0$. With each firm having only one job, profit maximization is equivalent to this zero-profit condition for firm entry. Setting the equation for $V_t$ as zero in every period gives:

$$
\frac{\kappa_t}{q_t^F} = E_t \beta_{t,t+1} \left[\gamma J_{t+1}(w_t \left[\pi_t^{\varepsilon_w} \left(\pi^{1-\varepsilon_w}\right)\right]) + \left(1-\gamma\right) J_{t+1}(w_{t+1}^*)\right] \tag{19}
$$

This vacancy posting condition equates the marginal cost of adding a worker (real cost times mean duration of vacancy) to the discounted marginal benefit from a new worker. After taking into account the free entry condition, the firm surplus reduces to $J_t$.

For later use, it is useful to note that the total real profits of the intermediate sector firms, which are paid to the families that own them, is

$$D_t^I = \int_0^{n_t} \left[ x_t z_t {h_{it}}^{\alpha} - \frac{w_{it}^*}{P_t} h_{it} \left(1 + s_t\right) - \Psi \right] di - \kappa_t v_t \tag{20}$$

**Multiperiod bargaining set up** Unlike with period-to-period bargaining, in the presence of staggered contracting, firms and workers have to take into account the impact of the contract wage on the expected future path of firm and worker surplus. Accordingly, the first order condition for wage-setting is given by:

$$\eta \Delta_t J_t \left(r\right) = \left(1 - \eta\right) \Sigma_t H_t \left(r\right) \tag{21}$$

where the partial derivatives of the surplus equations w.r.t. the wage $\Delta_t = P_t \frac{\partial H_t(r)}{\partial w_t}$ and $\Sigma_t = -P_t \frac{\partial J_t(r)}{\partial w_t}$ denote the effect of a rise in the *real* wage on the worker surplus and (minus) the effect of a rise in the real wage on the firm's surplus respectively (see Appendix for details).

$$\Delta_t = h_t \left(1 - \tau_t\right) + E_t \beta_{t,t+1} \left(1 - \rho_{t+1}\right) \gamma \left[\pi_t^{\varepsilon_w} \left(\pi^{1-\varepsilon_w}\right)\right] \pi_{t+1}^{-1} \Delta_{t+1} \tag{22}$$

$$\Sigma_t = h_t \left(1 + s_t\right) + E_t \beta_{t,t+1} \left(1 - \rho_{t+1}\right) \gamma \left[\pi_t^{\varepsilon_w} \left(\pi^{1-\varepsilon_w}\right)\right] \pi_{t+1}^{-1} \Sigma_{t+1} \tag{23}$$

These expressions can be interpreted as the discounting factors for the worker and the firm (respectively) for evaluating the value of the future stream of wage payments. As wage contracts extend over multiple periods, agents have to take into account also the *future* probabilities of not being allowed to renegotiate the wage, or of not surviving exogenous destruction. In the one firm - one worker setup, used in this paper, the discounting factors would be equal across agents unless distortionary taxes were breaking this symmetry[6]. With staggered bargaining, labour taxes enter the discounting factor equations of the agents implying that workers and firms also take into account the future path of taxation in their negotiating behaviour. As is apparent from the loglinearized forms of the discounting factors, presented in the Appendix, both the worker's and the firm's marginal tax rate effectively reduce the worker's relative bargaining power, and consequently his share of the surplus. This effect on the division of match surplus is amplified by staggered bargaining. In the limiting case of efficient bargaining, $\gamma = 0$, the partial derivatives of the surpluses w.r.t. the wage

[6] In Gertler and Trigari (2009), this is not the case. Differences in the worker's and the firm's optimization perspectives, a "horizon effect", arises because large firms take into account possible changes in future hiring rates. The effect of distortionary taxes is different. Proportional tax rates influence the *division* of the total surplus from a job in equilibrium, irrespective of the bargaining horizon (see Pissarides (2000), chapter 9).

reduce to $\Delta_t = h_t(1-\tau_t)$, and $\Sigma_t = h_t(1+s_t)$, and the first order condition accordingly reduces to its period-by-period counterpart $\eta(1-\tau_t)J_t = (1-\eta)(1+s_t)H_t$.

Given that the probability of wage adjustment is i.i.d., and all matches at renegotiating firms end up with the same wage $w_t^*$, the evolution of the nominal *average hourly* wage in the economy can be expressed as a convex combination of the contract wage and the average wage across the matches that do not renegotiate, after taking into account the indexation scheme.

$$w_{t+1} = (1-\gamma)w_{t+1}^* + \gamma\int_0^{n_t}\frac{w_{it}}{n_t}\left[\pi_t^{\varepsilon_w}\left(\pi^{1-\varepsilon_w}\right)\right]di \tag{24}$$

**Wage dynamics** The staggered bargaining framework has implications on the behavior of workers and firms. To describe wage dynamics in the presence of staggered contracting, we will develop loglinear expressions for the relevant wage equations in the same way as in Gertler, Sala and Trigari (2008). The contract wage is solved by first linearizing the first order condition

$$\widehat{J}_t(r) + \widehat{\Delta}_t = \widehat{H}_t(r) + \widehat{\Sigma}_t \tag{25}$$

and then plugging into the FOC the value equations and discounting factors for the worker and the firm respectively in their loglinearized form. The latter, as well as the derivation of the contract wage, are presented in detail in the Appendix. The resulting contract wage is

$$\widehat{w}_t^* = [1-\iota]\,\widehat{w}_t^0(r) + \iota E_t\left(\widehat{\pi}_{t+1} - \varepsilon_w\widehat{\pi}_t\right) + \iota E_t\widehat{w}_{t+1}^* \tag{26}$$

where $\iota = \overline{\beta}(1-\overline{\rho})\gamma$. This is the optimal wage set at time $t$ *by all matches that are allowed to renegotiate their wage.* As is usual with Calvo contracting, it depends on a wage target $w_t^0(r)$ and next period's optimal wage. As the probability of not being able to renegotiate the wage approaches zero $\gamma \longrightarrow 0$, $\iota \longrightarrow 0$, and the contract wage, $w_t^*$, approaches the period-by-period Nash wage.

Unlike in the more conventional set up of New Keynesian models, where Calvo wage contracting is combined with a monopolistic supplier of labour, the target wage here also includes a spillover effect that brings about additional rigidity on top of that implied by the Calvo scheme alone. Gertler and Trigari (2006) show how these spillover effects result from wage bargaining. The target wage can be decomposed into two parts

$$\widehat{w}_t^0(r) = \widehat{w}_t^0 + \varphi_H\Gamma E_t\left[\widehat{w}_{t+1} - \widehat{w}_{t+1}^*\right] \tag{27}$$

where $\varphi_H\Gamma = \frac{(1-\eta)\beta q^w}{(1-\iota)}$ is the spillover effect[7]. The spillover coefficient is positive, indicating that when the expected average market wage $E_t\widehat{w}_{t+1}$ is higher than the expected contract wage $E_t\widehat{w}^*_{t+1}$, (indicating unusually good labour market conditions) this raises the target wage in the negotiations. Thus, wage rigidity and the resulting employment dynamics are not only a product of staggered wage setting, but also of the spillover effects from the Nash bargaining process.

The spillover-free component of the target wage is of exactly the same form than the period-by-period negotiated wage, only adjusted for the multiperiod discounting factors.

$$\begin{aligned}\widehat{w}^0_t &= \varphi_x\left(\widehat{x}_t + \widehat{mpl}_t\right) + \varphi_m\widehat{mrs}_t + \varphi_H E_t\left(\widehat{q}^W_{t+1} + \widehat{H}_{t+1}\left(w^*_{t+1}\right) + \widehat{\beta}_{t,t+1}\right) \\ &\quad -\varphi_h\widehat{h}_t - \varphi_s\widehat{s}_t + \varphi_\tau\widehat{\tau}_t + \varphi_D E_t\left[\widehat{\Sigma}_{t+1} - \widehat{\Delta}_{t+1}\right] + \widehat{P}_t \end{aligned} \tag{28}$$

Finally, combining all the relevant elements of the wage bargaining outcome, yields a second-order difference equation for the evolution of the average wage (see Appendix)

$$\widehat{w}_t = \lambda_b\left(\widehat{w}_{t-1} + \varepsilon_w\widehat{\pi}_{t-1} - \widehat{\pi}_t\right) + \lambda_0\widehat{w}^0_t + \lambda_f E_t\left(\widehat{w}_{t+1} + \widehat{\pi}_{t+1} - \varepsilon_w\widehat{\pi}_t\right) \tag{29}$$

Due to staggered contracting, $\widehat{w}_t$ depends on the lagged wage $\widehat{w}_{t-1}$, the spillover-free target wage $\widehat{w}^0_t$, and the expected future wage $E_t\widehat{w}_{t+1}$.

#### 2.3.3 Determining hours of work

While matches are restrained to renegotiate the wage only with a given exogenous probability, hours per worker can be *renegotiated at each point in time.* With efficient Nash bargaining, optimal hours of work can be found from the following first order condition obtained by differentiating the Nash maximand w.r.t hours

$$(1-\tau_t)\,x_t f_{h,t} = (1+s_t)\,\frac{g'(h_t)}{\Lambda_t}$$

where $f_{h,t}$ is, as before, the marginal product of the labour input i.e. hours, and which, using the expressions for the production and utility functions, can be written as

$$(1-\tau_t)\,x_t mpl_t = (1+s_t)\,mrs_t\,(1+\tau^c_t) \tag{30}$$

This optimality condition equates the value of marginal product to the marginal rate of substitution between work and leisure, and resembles, thus, to the corresponding condition in a competitive labour market. However, with labour market frictions, while the hourly wage is such that the marginal cost to the worker from working is equal to the marginal gain to the firm, neither of these measures needs to be equal to the wage. It is important to observe that the optimality condition for hours determines the optimal hours per worker, i.e. the intensive margin of labour adjustment. This individual labour input of a worker is

[7] In Gertler and Trigari's (2006) original framework, there is also an indirect spillover effect because the expected hiring rate of the large renegotiating firm affects the bargaining outcome. In the present one worker per firm setup that effect disappears.

determined *irrespective of the wage.* But the model also allows for labour adjustment in the number of workers, as defined by the vacancy posting condition and the matching function.

## 2.4 Final good firms

There are two types of final goods firms. One produces private consumption goods and the other type of final goods firm produces public consumption goods[8].

### 2.4.1 Private consumption good

The private consumption good is a composite of intermediate goods distributed by a continuum of monopolistically competitive wholesale firms at home and abroad. Wholesale firms, their products and prices are indexed by $i \in [0,1]$. Final good firms operate under perfect competition and purchase both domestically produced intermediate goods $y_{H,t}(i)$ and imported intermediate goods $y_{F,t}(i)$. They minimize expenditure subject to the following aggregation technology

$$C_t = \left[ (1-W)^{\frac{1}{\varpi}} \left( \left[ \int_0^1 y_{H,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \right)^{\frac{\varpi-1}{\varpi}} + W^{\frac{1}{\varpi}} \left( \left[ \int_0^1 y_{F,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \right)^{\frac{\varpi-1}{\varpi}} \right]^{\frac{\varpi}{\varpi-1}} \tag{31}$$

where $\varpi$ measures the trade price elasticity, or elasticity of substitution between domestically produced intermediate goods and imported intermediate goods in the production of final goods for given relative prices, and $W$ is the weight of imports in the production of final consumption goods.The parameter $\varepsilon > 1$ is the elasticity of substitution across the differentiated intermediate goods produced and distributed within a country.

The optimization problem determining the allocation of expenditure between the individual varieties of domestic and foreign intermediate goods yields the following demand curves facing each wholesale firm

$$y_{H,t}(i) = \left( \frac{p_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} Y_{H,t} \tag{32}$$

$$y_{F,t}(i) = \left( \frac{p_{F,t}(i)}{P_{F,t}} \right)^{-\varepsilon} Y_{F,t} \tag{33}$$

where $P_{H,t}$ and $P_{F,t}$ are the aggregate price indexes for the domestic and foreign intermediate goods respectively

$$P_{H,t} = \left[ \int_0^1 p_{H,t}(i)^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}} \tag{34}$$

[8]This is a standard assumption in New Open Economy Macro Models that assess fiscal policy. E.g. in Obstfeld and Rogoff's (1996) extension of the Redux model, government spending is introduced as a basket of public consumption goods aggregated in the same way as for private consumption.

$$P_{F,t} = \left[ \int_0^1 p_{F,t}(i)^{1-\varepsilon} \, di \right]^{\frac{1}{1-\varepsilon}} \tag{35}$$

To determine the optimal allocation between the domestic and imported intermediate goods, the final good firm minimizes costs $P_{H,t}Y_{H,t}+P_{F,t}Y_{F,t}$ subject to its production function or aggregation constraint. This yields the demands for the domestic and foreign intermediate good *bundles* by domestic final good producers

$$Y_{H,t} = (1-W)\left(\frac{P_{H,t}}{P_t}\right)^{-\varpi} C_t \tag{36}$$

$$Y_{F,t} = W\left(\frac{P_{F,t}}{P_t}\right)^{-\varpi} C_t \tag{37}$$

where $P_t$ is the home country's aggregate price index, or consumption price index

$$P_t = \left((1-W)\, P_{H,t}^{1-\varpi} + W P_{F,t}^{1-\varpi}\right)^{\frac{1}{1-\varpi}} \tag{38}$$

At the level of individual intermediate goods the law of one price holds[9]. That, together with the assumption that the weight of the home country good in the foreign consumer price index is infinitesimally small, implies that $P_{F,t}$ is equal to the foreign CPI $P_t^*$ (see Galí-Monacelli (2005)).

### 2.4.2 Public consumption good

The public consumption good is composed of only domestic intermediate goods $g_t(i)$. This assumption implies, contrary to e.g. the Redux model, full home bias in government spending. This simplifying assumption can be supported by the observation from input-output tables that the use of foreign intermediate goods in government spending is significantly lower than in private consumption.

$$G_t = \left[ \int_0^1 g_t(i)^{\frac{\varepsilon-1}{\varepsilon}} \, di \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{39}$$

Each wholesale firm $i$ selling intermediate goods to the public consumption good producer faces the following demand schedule

$$g_t(i) = \left(\frac{p_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} G_t \tag{40}$$

[9] Note, however, that due to home bias in consumption the basket of consumed goods may differ in the two areas, and therefore purchasing power parity does not hold.

## 2.5 Wholesale firms and price setting

The wholesale firms buy the homogeneous intermediate goods at nominal price $p_{H,t}x_t$ per unit and transform them one-to-one into the differentiated product. As in most models that incorporate labour market matching into the NK framework, the price setting decision is separated from the wage setting decision to maintain the tractability of the model[10]. Price rigidities arise at the wholesale level while search frictions and wage rigidity only affect directly the intermediate goods sector.

There is Calvo-type stickiness in price-setting and the relative price of intermediate goods $x_t$ coincides with the real marginal cost faced by wholesale firms. In each period, the wholesale firm can adjust its price with a constant probability $1-\xi$ which implies that prices are fixed on average for $\frac{1}{1-\xi}$ periods. The wholesale firm's optimization problem is to maximize expected future discounted profits by choosing the sales price $p_{H,t}(i)$, taking into account the pricing frictions and the demand curve they face. It is assumed that the wholesale firm sells the home-country intermediate goods for the same price for domestic and foreign final goods producers, and for the domestic government.

The first order condition for the pricing decision of a wholesale firm that reoptimizes at $t$ is

$$E_t \sum_{s=0}^{\infty} \xi^s \beta_{t,t+s} \left[ \left( \frac{p_{H,t}(i)}{P_{H,t+s}} \right) y_{t+s}(i) - x_{t+s} y_{t+s}(i) \right] = 0 \tag{41}$$

where $y_t(i)$ is the demand of firm $i$'s product by domestic private consumption good firms, foreign private consumption good firms and the domestic government as outlined in the previous section

$$y_t(i) = y_{H,t}(i) + y^*_{H,t}(i) + g_t(i) = \left( \frac{p_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} Y_t^D$$

where $Y_t^D$ stands for total demand for domestic intermediate goods. All wholesale firms are identical except that they may have set their current price at different dates in the past. However, in period $t$, if they are allowed to reoptimize their price, they all face the same decision problem and choose the same optimal price $p^*_{H,t}$. Using the definition of the discount factor and rearranging, the FOC can be rewritten as

$$E_t \sum_{s=0}^{\infty} \xi^s \beta^s \frac{\lambda_{t+s}}{\lambda_t} \left[ (1-\varepsilon) \left( \frac{p^*_{H,t}}{P_{H,t+s}} \right) + \varepsilon x_{t+s} \right] \left( \frac{1}{p^*_{H,t}} \right) \left( \frac{p^*_{H,t}}{P_{H,t+s}} \right)^{-\varepsilon} Y^D_{t+s} = 0 \tag{42}$$

which can be solved for $\frac{p^*_{H,t}}{P_{H,t}}$ to yield the following pricing equation

[10] A number of extensions merge the intermediate and retail sectors so that there are interactions between wage and price setting at the level of the individual firm. E.g. Christoffel et al. (2009) assess the implications of that specification for inflation dynamics.

$$\frac{p^*_{H,t}}{P_{H,t}} = \left(\frac{\varepsilon}{\varepsilon-1}\right)\frac{E_t\sum_{s=0}^{\infty}\xi^s\beta^s\frac{\lambda_{t+s}}{\lambda_t}x_{t+s}\left(\frac{P_{H,t+s}}{P_{H,t}}\right)^{\varepsilon}Y^D_{t+s}}{E_t\sum_{s=0}^{\infty}\xi^s\beta^s\frac{\lambda_{t+s}}{\lambda_t}\left(\frac{P_{H,t+s}}{P_{H,t}}\right)^{\varepsilon-1}Y^D_{t+s}} \tag{43}$$

where $\frac{\varepsilon}{\varepsilon-1} = \mu$ is the flexible-price markup. This is the standard Calvo result. In the absence of price rigidity, the optimal price would reduce to a constant markup over marginal costs. Log-linearizing the FOC around the steady state yields the New Keynesian Phillips Curve where domestic inflation depends on marginal costs and expected future inflation

$$\widehat{\pi}_{H,t} = \nu\widehat{x}_t + \beta E_t\widehat{\pi}_{H,t+1} \tag{44}$$

where $\nu = \frac{(1-\xi)(1-\xi\beta)}{\xi}$.

Total real profits of the wholesale sector firms are

$$D^R_t = \int_0^{n_t}\left[\left(\frac{p_{H,t}(i)}{P_{H,t}} - x_t\right)y_t\left(i\right)\right]di \tag{45}$$

## 2.6 Fiscal policies

The public sector's role in this economy is to collect taxes and use them to finance unemployment benefits and lump-sum transfers as well as government spending $G_t$. If expenditures in any period are larger than income it can finance the deficit by issuing bonds which are repaid in the next period. The various tax instruments in use are the labour tax on workers $\tau_t$, payroll taxes on firms $s_t$, and a consumption tax $\tau^c_t$. Lump-sum transfers $TR_t$ may also be altered in response to changes in spending. The government budget constraint is

$$n_tw_th_t(\tau_t + s_t) + \tau^c_tP_tC_t + B_t = P_{H,t}G_t + P_tbu_t + TR_t + R_{t-1}B_{t-1} \tag{46}$$

Accordingly, the government real debt $b_t = \frac{B_t}{P_t}$, evolves as

$$b_t = R_{t-1}\frac{b_{t-1}}{\pi_t} + \frac{P_{H,t}}{P_t}G_t + bu_t + \frac{TR_t}{P_t} - n_t\frac{w_t}{P_t}h_t(\tau_t + s_t) - \tau^c_tC_t \tag{47}$$

Fiscal policy is assumed to obey a rule whereby the chosen fiscal variable is adjusted to changes in debt as a fraction of steady state output. On the revenue side, we consider four alternative tax instruments: the lump-sum tax, consumption tax and the labour taxes on the employer and the employee. The rules relate the change in the policy instrument from its steady state level to the deviation of real debt from its target level

$$TAX_t = \overline{TAX} + \Omega_d \left( \frac{b_{t-1}}{Y_{t-1}} - \frac{\overline{b}}{\overline{Y}} \right) \tag{48}$$

where $TAX_t = \tau_t^{LS}, \tau_t^c, \tau_t, s_t$ and $\Omega_d$ is the sensitivity of the tax instrument with respect to the government debt-to-output ratio. Government spending is characterised by the following autoregressive process

$$\log(G_t) = (1 - \rho_G) \log(\overline{G}) + \rho_G \log(G_{t-1}) + \epsilon_t^G, \text{ where } \rho_G \in (0, 1), \ \epsilon_t^G \overset{iid}{\sim} N\left(0, \sigma_G^2\right)$$

where $\epsilon_t^G$ is the government spending shock.

## 2.7 Equilibrium

For each intermediate good, supply must equal total demand. The demand for good $i$ is, as shown previously, $y_t\left(i\right) = \left(\frac{p_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} Y_t^D$, where $Y_t^D$ is total demand for domestic intermediate goods by domestic and foreign final goods firms and the domestic government. Using the expressions for the demands for domestic intermediate good *bundles* derived previously, this can be written as

$$y_t\left(i\right) = \left(\frac{p_{H,t}\left(i\right)}{P_{H,t}}\right)^{-\varepsilon} \left\{ \left(1 - W\right) \left(\frac{P_{H,t}}{P_t}\right)^{-\varpi} C_t + W \left(\frac{P_{H,t}}{P_t^*}\right)^{-\varpi} C_t^* + G_t \right\} \tag{49}$$

Following Galí and Monacelli (2005) defining an index for aggregate domestic demand $Y_t^D = \left[\int_0^1 y_t\left(i\right)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}$ allows us to rewrite this as

$$Y_t^D = \left(1 - W\right) \left(\frac{P_{H,t}}{P_t}\right)^{-\varpi} C_t + W \left(\frac{P_{H,t}}{P_t^*}\right)^{-\varpi} C_t^* + G_t$$

Aggregate demand for domestic intermediate goods has to equal their aggregate supply minus the resources lost to vacancy posting, leading to the home economy's aggregate resource constraint

$$Y_t = \left(1 - W\right) \left(\frac{P_{H,t}}{P_t}\right)^{-\varpi} C_t + W \left(\frac{P_{H,t}}{P_t^*}\right)^{-\varpi} C_t^* + G_t + \kappa_t v_t \tag{50}$$

While the above equation states that in equilibrium domestic output has to equal its usage as consumption, exports and government spending, market-clearing in the intermediate good sector also requires

$$Y_t = n_t z_t {h_t}^{\alpha} \tag{51}$$

The net foreign asset position is determined by the trade balance - the difference between domestic output and domestic consumption.

$$B_t^* - R_{t-1}^* p\left(b_{t-1}^*\right) B_{t-1}^* = P_{H,t} Y_t - P_t C_t - P_{H,t} G_t - P_{H,t} \kappa_t v_t \tag{52}$$

This relation is obtained by combining the consumers' budget constraint, the government's budget constraint and the economy's aggregate resource constraint as well as the equation for total dividends accrued to households, i.e. the sum of the profits in the intermediate and wholesale sectors

$$D_t = Y_t - n_t \frac{w_t^*}{P_t} h_t \left(1 + s_t\right) - n_t \Psi - \kappa_t v_t \tag{53}$$

# 3 Parameterization and steady state of the model

The parameter values are chosen mostly on the basis of existing literature, and are summarized in table 1. For preferences and the labour market part, they follow mainly Christoffel-Kuester-Linzert (2008) who use quarterly data from 1984Q1 to 2006Q4 for the euro area and for the open economy Corsetti, Meier and Müller (2009).

The quarterly discount factor is $\beta = 0.992$ which corresponds to an annual interest rate of $3,3\%$. The labour supply, or Frish elasticity $(\frac{1}{\phi})$, is set to 0.2. This is in the middle range of values implied by most microeconomic studies which estimate this elasticity to be between 0 and 0.5 (see Card (1994)) for a survey). Much higher elasticities have been generally used in the business cycle literature because macro elasticities account also for the variation in the employment rate[11]. The quarterly separation rate is calibrated at $\rho = 0.04$. The labour elasticity of production parameter is set to $\alpha = 0.66$ which implies decreasing returns to scale in the intermediate goods production sector, and a labour share of 60 percent. The unemployment benefit parameter is calibrated at $b = 0.4$, and generates a net replacement rate of 75 percent, defined as the ratio of net unemployment benefits to average net (after-tax) income from work $\frac{b}{w_t h_t (1-\tau)}$. This is sligthly higher than e.g. the OECD's "Benefits and Wages" publication suggests for Finland. There, the average net replacement rate over 60 months of unemployment for Finland is 70 percent, averaging over four different family types. The unemployment benefit is not assumed to be proportional to the wage nor to be indexed to inflation. As Christoffel et al. (2008) note, in labour market matching models, there is a trade-off between obtaining a reasonable labour share and a plausible replacement rate. Further, Costain and Reiter (2008) show that a real business cycle model augmented with labour market matching can be made consistent with either business cycle facts or the effects of labour market policies but not both. The assessment of the chosen parameters in the light of these considerations is important especially in empirical work but that is not the focus of the current paper.

[11] See Fiorito, R. - Zanella, G. (2008) for a recent comparison of micro and macro elasticities of labor supply. They estimate an individual elasticity of about 0.1 and an aggregate elasticity of about 1.

The wholesale sector is calibrated in line with the literature so that the markup is at a conventional value of $\mu = \frac{\varepsilon}{\varepsilon - 1} = 1.1$. The Calvo parameter is $\xi = 0.75$ on the basis of CKL calibration from the Eurosystem Inflation Persistence Network. The average duration of prices is accordingly 4 quarters. As to wages, they are assumed to be renegotiated every one and a half years, implying $\gamma = 0.83$.

**Table 1: Parameter values**

| Parameter | *Value* | Explanation |
|---|---|---|
| Preferences | | |
| $\beta$ | .992 | Time-discount factor |
| $\phi$ | 5 | Labour supply (Frish) elasticity $\frac{1}{\phi}$ of 0.2 |
| $\varrho$ | 1.5 | Risk aversion |
| $\varkappa$ | 0.6 | External habit persistence |
| Labour market | | |
| $\alpha$ | 0.66 | Labour elasticity of production |
| $\sigma$ | 0.6 | Elasticity of matches w.r.t. unemployment |
| $\sigma_m$ | 0.5 | Efficiency of matching |
| $\rho$ | 0.04 | Exogenous quarterly job destruction rate |
| $\eta$ | 0.6 | Bargaining power of workers |
| $b$ | 0.4 | Unemployment benefits |
| $\kappa$ | 0.068 | Vacancy posting costs |
| $z$ | 2.27 | Technology, targets output $Y = 1$ |
| $\gamma$ | 0.83 | Pr(no renegotiation), avg duration of wage contracts of 6 qrts |
| $\Psi$ | 0.24 | Fixed cost of production |
| $\varepsilon_w$ | 0 | Wage indexation; no indexation in baseline model |
| Wholesale sector | | |
| $\varepsilon$ | 11 | Elasticity of substitution, implies a markup of 10 percent |
| $\xi$ | 0.75 | Calvo stickiness of prices, average duration of 4 qrts |
| $\nu \left( = \frac{(1-\xi)(1-\beta\xi)}{\xi} \right)$ | 0.085 | Coefficient of marginal costs in NK Phillips curve |
| Final goods sector | | |
| $(1 - W)$ | 0.75 | Home bias in final goods production |
| $\varpi$ | 0.66 | Trade price elasticity |
| $\gamma_{b^*}$ | 0.005 | Debt-elasticity of interest rates |

The steady state values of the model variables implied by the current parameterization can be found in table 2. The steady state equations of the model are in turn provided in appendix A. In the steady state, output is normalized to one, so that GDP components can be interpreted as shares of GDP. The working force is also normalised to one so that the steady state unemployment level is 9 percent. A symmetric open economy steady state is assumed where consumption levels are initially the same at home and abroad and both the trade balance and net foreign asset holdings are zero. As no capital is included in the model, output components private consumption and government consumption (and the tiny amount of resources lost to vacancy posting) are scaled so that private consumption accounts for 71 percent of steady state output and government consumption is 28 percent.

The steady state tax rates for labour and consumption are computed as ten year historical averages of corresponding tax rates in Finland times the model-implied tax base for that tax category. Accordingly, labour taxes for the employee and the employer respectively amount to 30 percent and 25 percent times the wage bill and the consumption tax rate corresponds to an average of 19 percent times the size of private consumption. The government's steady state debt to GDP ratio is set at 45 percent, close to the current value for the so-called EMU debt for Finland.

**Table 2**

| Variable | Value | Description |
|---|---|---|
| $Y$ | 1 | Output |
| $C$ | 0.71 | Consumption |
| $u$ | 0.09 | Unemployment rate |
| $\kappa v$ | 0.003 | Total vacancy costs |
| $n$ | 0.91 | Employment |
| $qw$ | 0.4 | Probability of finding a job |
| $qf$ | 0.7 | Probability of finding a worker |
| $b/(wh(1-\tau))$ | 0.78 | Net replacement rate |
| $nwh$ | 0.60 | Wage bill |
| Fiscal policy | | |
| $\tau^C$ | 0.13 | Consumption tax |
| $\tau$ | 0.18 | Labour tax rate on employee |
| $s$ | 0.15 | Employers' social security contribution |
| $TR$ / $\tau^{LS}$ | 0.03 | Lump-sum tax |
| $d/Y$ | 0.45 | Government debt to GDP ratio |
| $G$ | 0.29 | Government spending |
| $\rho_G$ | 0.8 | Autocorrelation of government spending |
| $\epsilon_t^G$ | 0.05 | Government spending shock |

# 4 Model evaluation

## 4.1 Steady state properties

The majority of papers which have augmented the New Keynesian business cycle model with search and matching frictions in the labour market do not incorporate distortionary taxation in their framework. Monacelli, Perotti and Trigari (2010), however, look at this feature as one extension to their RBC model. To understand the working of the model and as a background for the dynamic simulations, it is useful to look at how distortionary taxes and unemployment benefits affect the steady state of the model.

Comparative statics of the tax and benefit parameters, for given values of vacancy posting costs and fixed costs of maintaining a filled vacancy, reveal[12] that cutting wage taxes,

[12] Calculations available from the author upon request.

employers' social security benefits or the unemployment benefit level all decrease significantly equilibrium unemployment and the average duration of unemployment spells ($\frac{1}{q^w}$), and increase the aggregate output of the economy, as expected in a standard MP model (see Pissarides (2000)). The mechanism for all these policy instruments is the same: they decrease the net replacement rate (in the case of payroll taxation indirectly through an increase in the wage rate), making work relatively more attractive. The working of this channel depends, of course, on the assumption that unemployment benefit is not taxed in the same proportion as the wage or otherwise directly indexed to the wage rate. As the workers' threat point in the wage bargain decreases, they agree to negotiate a lower wage. Lower labor costs encourage firms to post more vacancies resulting in higher employment rates. At the same time, tightness in the labor market increases, and contributes, through a higher wage, to restoring the equilibrium. These results, thus, support the strand of literature which emphasizes the role of employment-friendly institutions for the determination of equilibrium labor market outcomes.

Labour taxation also decreases the total surplus from employment in equilibrium as can be seen from the following steady state equation

$$S = xf(h) - \Psi - (\tau + s)\, wh - \frac{g(h)}{\Lambda} - b + \left[1 + \frac{(1-\tau)(1-qw)\,\eta}{(1+s)(1-\eta)}\right] \frac{\kappa}{q}$$

In addition, the present model also has the equilibrium property of the standard MP model, that proportional taxes affect the *division* of match surplus[13]. Both the wage tax on the worker and the employer's contribution to social security reduce the worker's relative share of total match surplus, which would be just equal to his bargaining power $\eta$ if these taxes were set to zero.

In the recent literature on labour markets and business cycles, summarized by Shimer (2010), the magnitude of the match surplus has been identified as one factor contributing to explaining the unemployment volatility puzzle. The intuition is that a smaller surplus reacts more to technology shocks of equal size, and this translates into increased volatility of labour market variables. This would suggest that higher tax rates or unemployment benefits, by contributing to a smaller match surplus, would help to improve the cyclical properties of the model, at least in the presence of technology shocks.

As Monacelli, Perotti and Trigari explain, government spending shocks increase the surplus from employment. The temporary increase in government spending is interpreted, by intertemporally optimizing consumers, as a future rise in taxes, and consequently as a fall in their lifetime resources. This effect is captured by a rise in $\lambda$ which, by the surplus equation, all else equal, decreases the relative disutility from working and increases the total surplus from employment. The government spending shock will affect economic outcomes through a real interest rate channel which will be explained in the next section.

[13] This can be seen by inspecting the steady state equations for the worker's and the firm's *share of total surplus* that are obtained by rewriting the first order condition for wage setting $H = \frac{\eta(1-\tau)}{(1-\eta)(1+s)+\eta(1-\tau)} S$ and $J = \frac{(1-\eta)(1+s)}{(1-\eta)(1+s)+\eta(1-\tau)}$

## 4.2 Dynamic simulations

In the following, we analyze the transmission mechanism of fiscal policy in the presence of frictional labour markets with either flexible or rigid wages. Specifically, we assess the effects of government spending shocks because these are at the centre of the debate on the effects of fiscal policy. Special emphasis is put on how the public debt resulting from a spending increase is paid back. Different debt-stabilizing fiscal policy scenarios are therefore assessed, to see whether labour market frictions have different implications for different fiscal policy instruments. Tax instruments are assessed in isolation in order to identify the mechanisms at work with each instrument - instead of a more realistic scenario where fiscal policy consists of a combination of instruments.

After having been identified by Baxter and King (1993) as a crucial assumption for the effects of fiscal policy in an RBC model, the chosen financing scheme has received surprisingly little attention in the otherwise abundant literature on the effects of government spending. However, recently Bilbiie and Straub (2004) recognised that the way fiscal shocks are financed shapes the response to a government spending shock in a New Keynesian model. Corsetti, Meier and Müller (2009) argue that not all public spending is financed with increases in tax rates, and analyze a policy that reduces spending over time in response to an initial rise in public debt in a small open-economy NK model. They find that this spending reversal enhances the expansionary impact of increased public spending. Galí et al. (2007) identify as crucial factors for the effects of government spending shock in a NK model: the share of liquidity constrained consumers, the extent of price rigidity, the persistence of the government spending shock and the intertemporal path of taxation (i.e. how strongly and quickly taxes react to debt and deficit).

In the following simulations, the positive government spending shock generates public debt which is gradually paid back following alternative fiscal feedback rules written on lump-sum taxes, labour taxes or consumption taxes. As a baseline, we analyze an increase in government spending corresponding to an approximately 1 percent increase of steady state output, with distortionary labour taxes and the consumption tax kept constant, and with flexible wages. The resulting public debt is brought back to its steady state level by allowing lump-sum taxes to increase, as commonly assumed in most other papers. Then, to reveal the specific properties of the present model, two other tax instruments are considered: the labour tax on employees and the consumption tax. The effects of wage rigidity and the relative importance of some other parameters are assessed separately.

### 4.2.1 The baseline response with a lump-sum tax rule

The baseline response to a positive government spending (solid line in Figure 1) is in line with results obtained from standard New Keynesian models (see e.g. Linnemann and Schabert (2003)). The rise in government demand has a positive effect on output. Because of full home bias in government consumption, the multiplier is directly proportional to the share of government spending in GDP and the size of the shock. The effect on private consumption is negative but small. The negative wealth effect, caused by the perceived fall in lifetime income, produces an initial drop in private consumption and an increase of hours worked per person.

Because prices are not fully flexible, the increase in government demand is larger than the decrease in private consumption, and aggregate demand rises. The increase in aggregate demand raises the expected returns of firms from a filled vacancy. Due to the timing assumption of the matching process, vacancies increase on impact but employment only starts to rise (unemployment starts to fall) from the next period on, as new matches become productive[14]. The combined increase in both labour demand and supply drives up the negotiated wage. Also the real wage rises contemporaneously, in line with recent findings by e.g. Pappa (2009).

An important feature differentiating the responses to a government spending shock in this model from the conventional closed-economy NK models, is that there is no endogenous monetary policy response that would counteract the effect of fiscal policy. The rise in the prices of the home country.would, in the presence of a central bank following the Taylor rule, be compensated more than one-for-one by an increase in the nominal interest rate, implying an increase in the real interest rate. Here the rise in government spending leads unambiguously to a terms of trade appreciation (rise in the price level) and to a fall in the real interest rate, attenuating the negative response of consumption.

Importantly, matching frictions add a new transmission channel of fiscal policy to the labour market. While the wealth effect raises the supply of individual hours worked (intensive margin) in the same way as in standard NK models, in this framework the tightening of the consumer budget constraint also affects the *total surplus from employment*, and therefore vacancy creation[15]. In particular, the increase in total surplus due to the increase in the marginal value of wealth encourages firms to open more vacancies. As both wages and the labour input (along both the intensive and the extensive margin) increase, the initial negative response of consumption is reversed.

In addition, as in Monacelli, Perotti and Trigari (2010), the rise in the shadow value of wealth drives up the real interest rate, producing a fall in the discounted marginal benefit from new vacancies. This channel works to decrease vacancy posting but is not significant enough to overturn the positive response of vacancies to the government spending shock.

The effect of increased government spending on the trade balance and on the terms-of-trade appear similar to what e.g. Kim and Roubini (2003) or Müller (2006) find. An increase in government spending appreciates the terms of trade and increases net exports. The terms of trade appreciation is natural in the presence of full home bias in government consumption: the export price index - which in this framework is just the domestic price index (because of producer pricing) - rises relative to the foreign price index which is not affected by fiscal stimulus in the small member state.

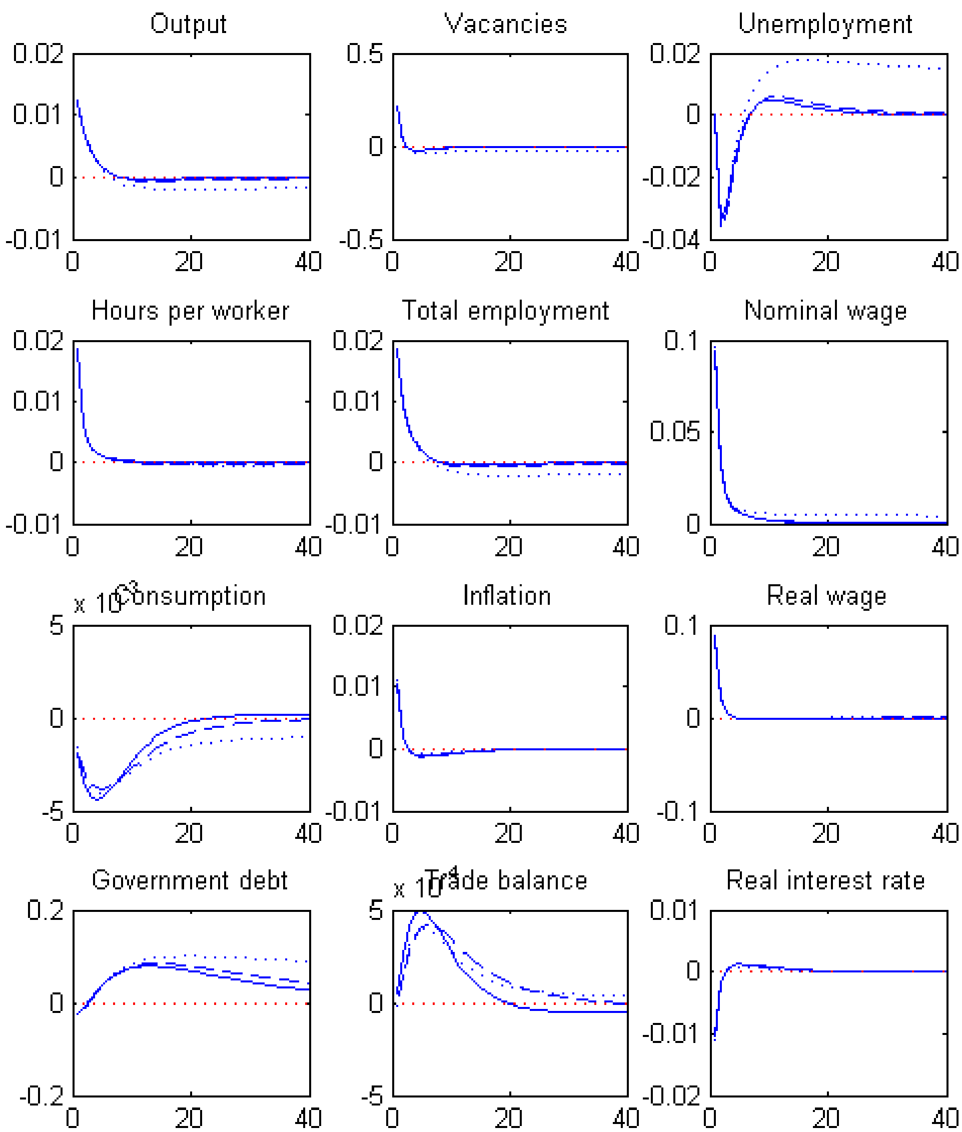
As to the trade balance, there are two counteracting forces. On the one hand, the trade balance improves because the value of trade increases, but on the other hand higher prices of

[14] The timing assumption is the same as in the standard Mortensen-Pissarides model. All labour adjustment in the first period after the shock is through the intensive margin, hours worked per person, which may cause them to overreact compared to what is observed in business cycle data. Blanchard and Galí (2006) introduced contemporaneous hiring into a business cycle matching model, whereby new matches become immediately productive, shifting labour adjustment to the extensive margin, the number of workers.

[15] This effect is similar to the marginal value of time channel in Monacelli, Perotti and Trigari (2010).

home-produced goods have a negative effect on the trade balance through the substitution channel. Here the former effect dominates. The latter effect tends to be larger the higher the home bias in private consumption and the higher the intratemporal elasticity of substitution between the home and foreign good.

Figure 1. The dynamic effects of a government spending shock: baseline vs. alternative debt-stabilizing fiscal rules. Note: baseline (rigid line), labour tax rule (dotted line), and consumption tax rule (dashed line)



### 4.2.2 Alternative fiscal policy scenarios

In figure 1, two other debt-stabilizing scenarios are presented: one where the labour tax on the employee is increased in order to finance the initial increase in government spending, and another where repayment is done through increases in the consumption tax. The coefficient for the sensitivity of the tax instrument with respect to debt $\Omega_d$ is set so that the initial tightening effect of the fiscal rule is equal across different tax instruments.

The results show that shifting the debt-stabilizing burden towards the distortionary labour tax (dotted line) significantly changes the responses of the economy to the positive fiscal policy shock. Most importantly, after the initial identical shock, as soon as the labour tax rule becomes operative, the higher proportional tax rate feeds through to higher wages. In the wage bargaining framework included in the labour market matching model, the gradual increases in the labour tax rate imply an increase in the target wage of negotiators (as can be verified from the dynamic target wage equation)[16]. As a result, the bargained nominal wage stays above its steady state level for a prolonged period of time to compensate workers for the otherwise falling net income. The higher wage directly implies higher labour costs to firms which decrease open vacancies and unemployment starts rising. The corresponding fall in employment makes the private consumption response more negative than when public debt is adjusted through lump-sum taxes, despite the.higher wage.

When consumption taxes, in turn, are used to stabilize debt after an increase in government spending, the negative labour market reactions are smaller. Output and vacancies rise as much as with the lump-sum tax rule, but government debt returns more quickly to its steady state level due to the larger tax base. Even the response of private consumption is less negative than in the case of the labour tax rule although consumption is directly taxed.

The results show the importance of how the increased public spending is financed. Due to the detailed description of the labour market we are able to identify important transmission channels of fiscal policy, depending on the chosen financing scheme, that have not yet received much attention in the existing literature.

We also investigated a similar government spending shock using the labour tax on the employer as the stabilizing instrument. The results are very similar than when using the fiscal rule on the employee's labour tax. The only significant difference is that the negotiated wage does not rise in the same way as when the incidence of increased labour taxation is on the worker (indeed, the dynamic equation for the target wage shows that an increase in the employer's social security contribution has a negative direct effect on the target wage), leaving the worker's net income and the firm's labour cost approximately the same across these two scenarios. As labour costs are, however, raised directly by the tax on employers, the labour market outcome with employer contributions as the debt-stabilizing tool is similar with falling employment and rising unemployment. The simulations are available on request.

Automatic stabilizers are at work in the present setup. The initial expansion of output and the accompanying improvement in employment after a government spending shock increase the government's labour tax revenues and decreases expenditure on unemployment

[16] In the presence of wage rigidity, an increase in the labour tax rate raises the target wage in the negotiation directly, as with period-by-period bargaining, but also through the negative effect on the worker discount factor.

benefits. However, consumption tax revenue falls as private consumption decreases and government debt increases significantly and persistently. Indeed, debt-stabilizing fiscal rules are needed to help bringing debt back to its steady state level in a reasonable time frame. Because of the small initial size of lump-sum taxes in the government budget compared to government spending, the increase in taxes dictated by that rule is relatively ineffective in restraining indebtedness unless a higher debt-sensitivity coefficient is assumed. The present rules are calibrated so that irrespective of the rule in force the initial fiscal policy tightening implied by the rule is approximately equal in all cases.

### 4.2.3 Wage rigidity

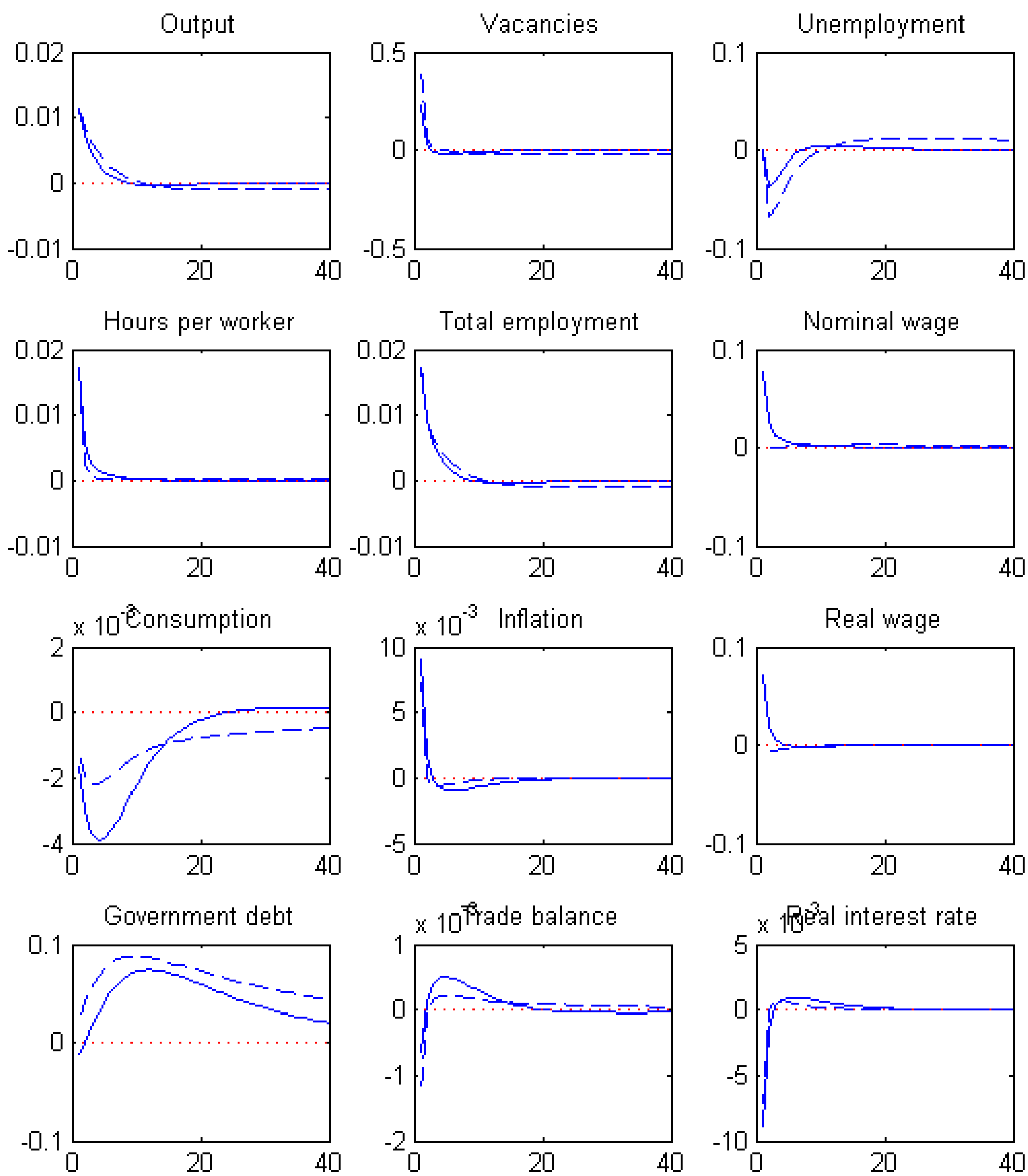
Figure 2 shows the results for the baseline model where lump-sum taxes react to public debt with wages being negotiated, instead of period-by-period, on average once every sixth quarter.

Making the wage more rigid increases the magnitude of the responses of labour market variables. Vacancies now react more strongly to the initial stimulus, since firms' expected profits are larger when their labour costs do not rise. Also the unemployment response to the shock is somewhat larger on impact. This is in line with the literature on labour markets and business cycles, which has stressed that, wage rigidity affects the cyclicality of labour market variables because it influences the firms' expected gains from the match. Compared to flexible wages, when wages are rigid, firms' expected profits rise more in upturns and fall more in downturns. The more favourable labour market reaction in the short-term to fiscal stimulus contributes to consumption falling less than in the baseline. However, in the longer term, as the wage adjusts upward, vacancies and employment start to fall and unemployment rises as shown by the right tails of the corresponding impulse response functions. Output and privat consumption remain lower than their steady state levels for a prolonged period of time.

The above result is in contrast to Monacelli, Perotti and Trigari (2010) who find that (real) wage rigidity[17] *dampens* the effect of government spending shocks on hiring, because wage rigidity increases the total surplus from the match by raising the firm's reservation wage, but also by lowering the worker's reservation wage. Of these two counteracting effects, the latter dominates in their simulations, lowering the Nash bargained wage. Since the shock also decreases the firm's share of the surplus, it discourages hiring. One important difference in their framework, compared to this model, is that they do not combine wage rigidity with price rigidity. The latter is needed to generate a rise in the real wage (the combined effect of increased labour demand and labour supply). When prices are rigid, the profit opportunities of firms are larger, and the net effect on vacancy posting is positive.

[17] Introduced as a simple wage adjustment rule, instead of as the result of staggered bargaining.

Figure 2. The dynamic effects of a government spending shock: flexible wages (rigid line) vs rigid wages $\gamma = 0.83$ (dotted line).



### 4.2.4 Relative importance of other parameters

The above simulations and comparisons support the finding from earlier literature that the degree of price rigidity is a crucial parameter in shaping the economy's response to fiscal policy shocks. In figure 3, the baseline model is simulate for two different degrees of price rigidity. With more flexible prices, the effects of fiscal stimulus are significantly dampened. The output multiplier shrinks and unemployment is nearly unaffected. The response of vacancies is more or less flat and privat consumtion reacts more negatively.

It is known from earlier contributions (see e.g. Linnemann and Schabert (2003)) that the real interest rate is the crucial variable for the adjustment to fiscal shocks because it determines the consumption path and, consequently, the magnitude of the aggregate demand effect. As shown in the dynamic simulations of the model, the small monetary union member state framework ensures that domestic prices rise but the nominal interest rate does not react to the speeding up of inflation. As a result, the real interest rate falls and attenuates the fall in private consumption. The model is closed by the debt-elastic interest rate assumption but the calibration of sensitivity parameter of the interest rate to the increase in foreign indebtedness implies that this is a purely technical assumption. Assuming a sufficiently more aggressive elasticity parameter would eventually reverse the response of the real interest rate to the spending shock.

While labour taxes were found to have significant equilibrium effects, lowering them by 3 percentage points hardly affects the dynamics of the model after a government spending shock.

# 5 Concluding remarks

This paper contributes to the ongoing debate about the effects of fiscal policy by analyzing government spending shocks under alternative fiscal rules and rigid labour markets. For this purpose, we have introduced fiscal policy and labour market matching frictions into an open-economy New Keynesian DSGE. The link between fiscal policy and the labour market was introduced with the help of distortionary labour taxes which directly influence the behavior of firms and workers on the matching market. The framework was adapted to the small currency union member country case, and additional rigidity in wage determination was introduced with the help of Gertler and Trigari's (2009) staggered bargaining framework.

We find that the effects of fiscal shocks, in the model with labour market frictions, are similar to those obtained from standard New Keynesian models. Fiscal stimulus has an expansionary effect on output, and a small but negative effect on private consumption. The detailed description of the labour market, however, helps to better understand the transmission mechanisms of fiscal policy to private consumption, employment and the real wage. The negative response of private consumption is driven by the negative wealth effect but counteracted by a positive employment response, brought about by increasing real wages and increasing labour supply along both the intensive and extensive margin.

The results show that the assumption of the offsetting fiscal measure is critical for the effects of fiscal stimulus. Specifically, shifting the debt-stabilizing burden towards the distortionary labour tax makes the effects significantly more negative. Most importantly, the wage bargaining model, included in the labour market matching framework, implies that, as the tax rule becomes operative, the higher proportional tax rate is internalized in the wage negotiation process. The bargained nominal wage rises to compensate workers for the otherwise falling net income. The higher wage directly implies higher labour costs to firms which decrease open vacancies and unemployment starts rising. The fall in employment implies a stronger contraction in private consumption compared with the more standard case of lump-sum tax adjustment. This would lead to the main, rather general, conclusion that, in an economy with labour market rigidities, withdrawing fiscal stimulus by means of increased labour taxes has detrimental effects on growth and employment dynamics. Interestingly, the response of private consumption to fiscal stimulus is not as negative if, instead of labour taxes, consumption taxes are used to consolidate the debt. This is because they have a smaller adverse effect on the labour market. In particular, they lower less the total surplus from employment than labour taxes.

Wage rigidity was found to increase the magnitude of the responses of labour market variables. Vacancies react more strongly to the initial stimulus, since firms' expected profits are larger when their labour costs do not rise. This is in line with the intuition backed by the literature on labour markets and business cycles, but in contrast to Monacelli, Perotti and Trigari (2010). The differences can, however, be explained in the light of different assumptions on price rigidity and the behavior of the real interest rate. Furthermore, our results indicate that while wage rigidity would seem to make fiscal policy more effective in the short term, in the longer term, the gradual increse in the wage causes a prolonged increase of unemployment to above the steady state level. Public debt stays higher and the negative effect of private consumption larger than when wages are flexible.

While the analysis conducted highlighted important transmission channels of fiscal policy not captured by standard New Keynesian models, the more precise quantitative effects of our fiscal policy simulations are still work in progress. Some sensitivity analysis was made with respect to the degree of price stickiness and to the behavior of the real interest rate, but the implications of different features of the labour market, other than wage rigidity, for the economy's response to government spending shocks is also work in progress. Recent literature suggests that, in addition to price rigidities, the economy should be modelled as "non-Ricardian" to account for important transmission channels of fiscal policy. A move in that direction could be, for example, the inclusion of rule-of-thumb consumers that has found to be important for the effects of fiscal policy (see e.g. Galí, Lopez-Salido and Valles (2007)). This is left for future work.

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# A Appendix

## A.1 Steady state of the model economy

Euler equation

$$\beta = \frac{1}{R}$$

Marginal utility of consumption

$$\lambda = (C - \varkappa C)^{-\varrho}$$

Marginal utilility of wealth

$$\Lambda = \frac{\lambda}{(1 + \tau^c)}$$

Interest rate on foreign bonds

$$R^* = R$$

FOC of retail firm

$$x = \frac{1}{\mu} = \frac{\varepsilon - 1}{\varepsilon}$$

Matches

$$m = \sigma_m u^\sigma v^{1-\sigma}$$

Employment

$$\rho n = m$$

Unemployment

$$u = 1 - n$$

Probability of finding a worker

$$q^F = \frac{m}{v}$$

Probability of finding a job

$$q^W = \frac{m}{u}$$

Labour market tightness

$$\theta = \frac{v}{u}$$

FOC for hours

$$(1-\tau)\, xmpl = (1+s)\, mrs$$

where

$$mpl = \alpha z h^{\alpha-1} \text{ and } mrs = \frac{\delta h^{\phi}}{\lambda}$$

Economy-wide resource constraint

$$\begin{aligned} Y &= (1-W)\, C + WC^{*} + G + \kappa v \\ &= C + G + \kappa v, \text{ in the symmetric steady state} \end{aligned}$$

Government budget constraint

$$(1-R)\, B = G + bu + TR - nwh(\tau+s) - \tau^{c} C$$

Market clearing / aggregate output

$$Y = nzh^{\alpha}$$

Wage

$$w = \frac{\eta}{(1+s)} \left[\frac{xmpl}{\alpha} + \frac{\kappa\theta}{h} - \frac{\Psi}{h}\right] + \frac{(1-\eta)}{(1-\tau)} \left[\frac{mrs}{(1+\phi)} + \frac{b}{h}\right]$$

Job creation condition

$$\kappa = q^{F} \beta J$$

where the firm surplus

$$J = \frac{1}{1-\beta\,(1-\rho)} \left[xzh^{\alpha} - wh\,(1+s) - \Psi\right]$$

Worker surplus

$$H = \frac{1}{1-\beta\,(1-\rho-q^{W})} \left[wh\,(1-\tau) - \frac{mrsh}{(1+\phi)} - b\right]$$

Worker discount factor

$$\overline{\Delta} = \frac{\overline{h}\,(1-\overline{\tau})}{1-\overline{\beta}\,(1-\overline{\rho})\,\gamma}$$

Firm discount factor

$$\overline{\Sigma} = \frac{\overline{h}\,(1+s)}{1-\overline{\beta}\,(1-\overline{\rho})\,\gamma}$$

## A.2 Model dynamics

The dynamics of the model are obtained by taking a log-linear approximation around a deterministic steady state.

Euler equation

$$\widehat{\Lambda}_t = E_t\left(\widehat{\Lambda}_{t+1} + \widehat{R}_t - \widehat{\pi}_{t+1}\right)$$

Shadow value of wealth

$$\widehat{\Lambda}_t = \widehat{\lambda}_t - \frac{\overline{\tau}^c}{(1+\overline{\tau}^c)}\left(\widehat{\tau}^c_t - \widehat{\tau}^c_{t+1}\right)$$

Marginal utility of consumption

$$\widehat{\lambda}_t = -\frac{\varrho}{(1-\varkappa)}\left(\widehat{C}_t - \varkappa\widehat{C}_{t-1}\right)$$

Interest rates

$$\widehat{R}_t = \widehat{R}^*_t - \gamma_{b^*}\widehat{b}^*_t$$

Matching function

$$\hat{m}_t = \sigma\hat{u}_t + (1-\sigma)\,\hat{v}_t$$

Employment dynamics

$$\hat{n}_t = (1-\bar{\rho})\,\hat{n}_{t-1} + \frac{\bar{m}}{\bar{n}}\hat{m}_{t-1} - \bar{\rho}\hat{\rho}_t$$

Unemployment

$$\widehat{u}_t = -\frac{1-\overline{u}}{\overline{u}}\hat{n}_t$$

Transition probabilities

$$\hat{q}^F_t = \hat{m}_t - \hat{v}_t$$

$$\hat{q}^W_t = \hat{m}_t - \hat{u}_t$$

labour market tightness

$$\widehat{\theta}_t = \hat{v}_t - \hat{u}_t$$

FOC for hours worked

$$(1-\overline{\tau})\,\overline{xmpl}\,(\hat{x}_t + m\hat{p}l_t) - \overline{\tau x mpl}\widehat{\tau}_t = (1+\overline{s})\,\overline{mrs}m\hat{r}s_t + \overline{smrs}\widehat{s}_t + \overline{mrs\tau^c}\,(1+\overline{s})\,\widehat{\tau}_t^c$$

$$\iff \hat{x}_t = m\hat{r}s_t - m\hat{p}l_t + \frac{\overline{\tau}}{(1-\overline{\tau})}\widehat{\tau}_t + \frac{\overline{s}}{(1+\overline{s})}\widehat{s}_t + \frac{\overline{\tau}^c}{(1+\overline{\tau}^c)}\widehat{\tau}_t^c$$

where

$$m\hat{p}l_t = \hat{z}_t - (1-\alpha)\,\hat{h}_t$$

and

$$m\hat{r}s_t = \phi\hat{h}_t - \hat{\lambda}_t$$

New Keynesian Phillips Curve

$$\widehat{\pi}_{H,t} = \nu\widehat{x}_t + \beta E_t\widehat{\pi}_{H,t+1}$$

where $\widehat{\pi}_{H,t} = \widehat{P}_{H,t} - \widehat{P}_{H,t-1}$ is domestic inflation

First order condition for wage setting

$$\widehat{J}_t(w_t^*) + \widehat{\Delta}_t = \widehat{H}_t(w_t^*) + \widehat{\Sigma}_t$$

Firm surplus

$$\begin{aligned}
\widehat{J}_t(w_t^*) \quad = \quad & \frac{\overline{x}\overline{mplh}}{\alpha\overline{J}}\left(\widehat{x}_t + \widehat{mpl}_t + \widehat{h}_t\right) - \frac{\overline{wh}\,(1+\overline{s})}{\overline{J}}\left(\widehat{w}_t^* - \widehat{P}_t + \widehat{h}_t\right) - \frac{\overline{w}\overline{h}\overline{s}}{\overline{J}}\widehat{s}_t \\
& -\overline{\beta}\overline{\rho}E_t\widehat{\rho}_{t+1} + \overline{\beta}\,(1-\overline{\rho})\,E_t\left(\widehat{J}_{t+1}\left(w_{t+1}^*\right) + \widehat{\beta}_{t,t+1}\right) \\
& -\frac{\overline{\beta}\,(1-\overline{\rho})\,\gamma}{1-\overline{\beta}\,(1-\overline{\rho})\,\gamma}\frac{\overline{wh}\,(1+\overline{s})}{\overline{J}}E_t\left(\widehat{w}_t^* + \varepsilon_w\widehat{\pi}_t - \widehat{w}_{t+1}^* - \widehat{\pi}_{t+1}\right)
\end{aligned}$$

Worker discount factor

$$\widehat{\Delta}_t = (1-\iota)\,\widehat{h}_t - \frac{(1-\iota)\,\overline{\tau}}{(1-\overline{\tau})}\widehat{\tau}_t + \iota E_t\left(\widehat{\beta}_{t,t+1} + \varepsilon_w\widehat{\pi}_t - \widehat{\pi}_{t+1} + \widehat{\Delta}_{t+1}\right) - \overline{\beta}\overline{\rho}\gamma E_t\widehat{\rho}_{t+1}$$

Worker surplus

$$\begin{aligned}\widehat{H}_t(w_t^*) &= \frac{\overline{wh}\,(1-\overline{\tau})}{\overline{H}}\left(\widehat{w}_t^* - \widehat{P}_t + \widehat{h}_t\right) - \frac{\overline{wh}\overline{\tau}}{\overline{H}}\widehat{\tau}_t - \frac{1}{1+\phi}\frac{\overline{mrs}\overline{h}\,(1+\overline{\tau}^c)}{\overline{H}}\left[\widehat{mrs}_t + \widehat{h}_t\right] \\ &\quad -\frac{\overline{mrs}\overline{h}\overline{\tau}^c}{(1+\phi)\,\overline{H}}\widehat{\tau}_t^c - \overline{\beta}\overline{q}^W E_t\left(\hat{q}_t^W + \widehat{H}_{x,t+1} + \widehat{\beta}_{t,t+1}\right) \\ &\quad +\overline{\beta}\,(1-\overline{\rho})\,E_t\left(\widehat{H}_{t+1}\left(w_{t+1}^*\right) + \widehat{\beta}_{t,t+1}\right) - \overline{\beta}\overline{\rho}E_t\widehat{\rho}_{t+1} \\ &\quad +\frac{\overline{\beta}\,(1-\overline{\rho})\,\gamma}{1-\overline{\beta}\,(1-\overline{\rho})\,\gamma}\frac{\overline{wh}\,(1-\overline{\tau})}{\overline{H}}E_t\left(\widehat{w}_t^* + \varepsilon_w\widehat{\pi}_t - \widehat{w}_{t+1}^* - \widehat{\pi}_{t+1}\right)\end{aligned}$$

Firm discount factor

$$\widehat{\Sigma}_t = (1-\iota)\,\widehat{h}_t + \frac{(1-\iota)\,\overline{s}}{(1+\overline{s})}\widehat{s}_t + \iota E_t\left(\widehat{\beta}_{t,t+1} + \varepsilon_w\widehat{\pi}_t - \widehat{\pi}_{t+1} + \widehat{\Sigma}_{t+1}\right) - \overline{\beta}\overline{\rho}\gamma E_t\widehat{\rho}_{t+1}$$

Optimal contract wage

$$\widehat{w}_t^* = [1-\iota]\,\widehat{w}_t^0\,(r) + \iota E_t\left(\widehat{\pi}_{t+1} - \varepsilon_w\widehat{\pi}_t\right) + \iota E_t\widehat{w}_{t+1}^*$$

Target wage

$$\widehat{w}_t^0(r) = \widehat{w}_t^0 + \varphi_H\Gamma E_t\left[\widehat{w}_{t+1} - \widehat{w}_{t+1}^*\right]$$

Spillover-free target wage

$$\begin{aligned}\widehat{w}_t^0 &= \varphi_x\left(\widehat{x}_t + \widehat{mpl}_t\right) + \varphi_m\widehat{mrs}_t + \varphi_H E_t\left(\widehat{q}_t^W + \widehat{H}_{t+1}\left(w_{t+1}^*\right) + \widehat{\beta}_{t,t+1}\right) \\ &\quad +\varphi_h\widehat{h}_t - \varphi_s\widehat{s}_t + \varphi_\tau\widehat{\tau}_t + \varphi_D E_t\left[\widehat{\Sigma}_{t+1} - \widehat{\Delta}_{t+1}\right] + \widehat{P}_t\end{aligned}$$

Average wage

$$\widehat{w}_t = (1-\gamma)\,\widehat{w}_t^* + \gamma\left(\widehat{w}_{t-1} - \widehat{\pi}_t + \varepsilon_w\widehat{\pi}_{t-1}\right)$$

or

$$\widehat{w}_t = \lambda_b\widehat{w}_{t-1} + \lambda_0\widehat{w}_t^0 + \lambda_f E_t\widehat{w}_{t+1}$$

Vacancy posting condition

$$\begin{aligned}\widehat{\kappa}_t - \hat{q}_t^F &= E_t\left(\widehat{J}_{t+1}(r) + \widehat{\beta}_{t,t+1}\right) \\ &+ \frac{\gamma}{1-\iota}\frac{\overline{wh}(1+\overline{s})}{\overline{J}}E_t\left(\widehat{w}_{t+1}^* + \widehat{\pi}_{t+1} - \widehat{w}_t - \varepsilon_w\widehat{\pi}_t\right)\end{aligned}$$

Trade balance

$$\widehat{TB}_t = \widehat{Y}_t - \overline{C}\widehat{C}_t + W\overline{C}\left(\widehat{P}_{H,t} - \widehat{P}_t^*\right) - \overline{G}\widehat{G}_t - \overline{\kappa v}\left(\widehat{\kappa}_t + \widehat{v}_t\right)$$

Economy-wide resource constraint (not updated)

$$\begin{aligned}\widehat{Y}_t &= (1-W)\overline{C}\widehat{C}_t + W\overline{C}^*\left(\widehat{C}_t^* + \varpi\widehat{P}_t^*\right) - \left[(1-W)\overline{C}\varpi W\right]\left(\widehat{P}_{H,t} - \widehat{P}_t^*\right) \\ &-\overline{C}\varpi W\widehat{P}_{H,t} + \overline{G}\widehat{G}_t + \overline{\kappa v}\left(\widehat{\kappa}_t + \widehat{v}_t\right)\end{aligned}$$

Consumer price index

$$\widehat{P}_t = (1-W)\widehat{P}_{H,t} + W\widehat{P}_t^*$$

Evolution of debt / Government budget constraint

$$\begin{aligned}\overline{b}\widehat{b}_t &= \overline{Rb}(\widehat{R}_{t-1} + \widehat{b}_{t-1} - \widehat{\pi}_t) + \overline{G}\left(\widehat{P}_{H,t} - \widehat{P}_t + \widehat{G}_t\right) + b\overline{u}\widehat{u}_t + \overline{TR}(\widehat{TR}_t - \widehat{P}_t) \\ &-\overline{nw}\overline{h}\left(\overline{\tau} + \overline{s}\right)\left(\widehat{n}_t + \widehat{w}_t - \widehat{P}_t + \widehat{h}_t\right) - \overline{nw}\overline{h}\overline{\tau}\widehat{\tau}_t - \overline{nw}\overline{h}\overline{s}\widehat{s}_t - \overline{\tau}^c\overline{C}\left(\widehat{\tau}_t^c + \widehat{C}_t\right)\end{aligned}$$

Market clearing / aggregate output (not updated)

$$\widehat{Y}_t + \widehat{P}_t - \widehat{P}_{H,t} = \widehat{n}_t + \hat{z}_t + \alpha\hat{h}_t$$

## A.3 Period-by-period Nash bargaining

In the standard MP model, it is assumed that total match surplus, $S_t = (W_t - U_t) + (J_t - V_t)$, the sum of the worker and firm surpluses is shared according to efficient Nash bargaining where wages and hours are negotiated simultaneously. The firm and the worker choose the wage and the hours of work to maximize the weighted product of the worker's and the firm's net return from the match.

$$\max_{w,h} (H_t)^{\eta} (J_t)^{1-\eta}$$

where $0 \leq \eta \leq 1$ is the relative measure of workers' bargaining strength.

The worker surplus gets the following form.

$$H_t = W_t - U_t = \frac{w_t^*}{P_t} h_t (1 - \tau_t) - \frac{g(h_t)}{\Lambda_t} - b + E_t \beta_{t,t+1} \left(1 - \rho_{t+1} - q_t^W\right) H_{t+1}$$

and the firm surplus is (after taking into account the free entry condition $V_t = 0$)

$$J_t = x_t f(h_t) - \frac{w_t^*}{P_t} h_t (1 + s_t) - \Psi + E_t \beta_{t,t+1} \left(1 - \rho_{t+1}\right) J_{t+1}$$

The first-order condition for wage-setting is

$$\eta \frac{\partial H_t}{\partial w_t} J_t = (1 - \eta) \frac{\partial J_t}{\partial w_t} H_t$$

$$\Longleftrightarrow \eta (1 - \tau_t) J_t = (1 - \eta)(1 + s_t) H_t$$

which would, without taxes, correspond to the simple surplus splitting result where the total surplus from the match is shared according to the bargaining power parameter $\eta$.

The optimality condition for wage-setting can be rewritten as a wage equation that includes only contemporaneous variables by substituting the value equations into the Nash FOC, and making use of the expressions for the production and utility functions.

$$\frac{w_t}{P_t} = \frac{\eta}{(1 + s_t)} \left[ \frac{x_t mpl_t}{\alpha} - \frac{\Psi}{h_t} \right] + \frac{(1 - \eta)}{(1 - \tau_t)} \left[ \frac{mrs_t (1 + \tau_t^c)}{(1 + \phi)} + \frac{b}{h_t} + \frac{q_t^W}{h_t} E_t \beta_{t,t+1} H_{t+1} \right] \quad (54)$$

where $w_t$ is the nominal hourly wage in a match. Further using the Nash first order condition for next period and the job creation condition, allows to write it as

$$\frac{w_t}{P_t} = \frac{\eta}{(1 + s_t)} \left[ \frac{x_t mpl_t}{\alpha} + \frac{\kappa_t \theta_t}{h_t} - \frac{\Psi}{h_t} \right] + \frac{(1 - \eta)}{(1 - \tau_t)} \left[ \frac{mrs_t (1 + \tau_t^c)}{(1 + \phi)} + \frac{b}{h_t} \right]$$

The wage equation is a convex combination of what the worker contributes to the match (the first square brackets) and what he has to give up in terms of disutility from supplying hours of work. Since workers and firms are homogeneous and all matches adjust their wages every period, they will all choose the same wage. The economy's wage bill is this wage rate times the total number of hours worked in the economy. It is clear from the wage equation that the introduction of taxes works to decrease the worker's relative effective bargaining power from $\eta$ to $\frac{\eta}{(1+s_t)}$. Consequently, economic conditions get a smaller weight in wage determination.

## A.4 Dynamics with wage rigidity

The derivation of the wage under staggered contracting follows Gertler, Sala and Trigari (GST) (2008). The Nash first order condition is in this case

$$\eta\Delta_t J_t\left(w_t^*\right) = (1-\eta)\,\Sigma_t H_t\left(w_t^*\right)$$

where the effect of a rise in the *real* wage on the worker's surplus is

$$\begin{aligned}
\Delta_t &= P_t \frac{\partial H_t\left(w_t\right)}{\partial w_t} \\
&= h_t\left(1-\tau_t\right) + E_t\beta_{t,t+1}\left(1-\rho_{t+1}\right)\gamma P_{t+1}\frac{\partial H_{t+1}\left(w_t\left[\pi_t^{\varepsilon_w}\left(\pi^{1-\varepsilon_w}\right)\right]\right)}{\partial w_t} \\
&= h_t\left(1-\tau_t\right) + E_t\beta_{t,t+1}\left(1-\rho_{t+1}\right)\gamma(\left[\pi_t^{\varepsilon_w}\left(\pi^{1-\varepsilon_w}\right)\right]h_{t+1}\left(1-\tau_{t+1}\right) \\
&\quad + E_t\beta_{t+1,t+2}\varsigma_{t+1,t+2}\gamma P_{t+2}\frac{\partial H_{t+2}\left(w_t\left[\pi_{t+1}^{\varepsilon_w}\left(\pi^{1-\varepsilon_w}\right)\pi_t^{\varepsilon_w}\left(\pi^{1-\varepsilon_w}\right)\right]\right)}{\partial w_t})... \\
&= E_t\sum_{s=0}^{\infty}\beta_{t,t+s}\varsigma_{t,t+s}\gamma^s\left[\left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{\varepsilon_w}\left(\pi^{1-\varepsilon_w}\right)^s\right]h_{t+s}\left(1-\tau_{t+s}\right)
\end{aligned}$$

$$\Longleftrightarrow \Delta_t = h_t\left(1-\tau_t\right) + E_t\beta_{t,t+1}\left(1-\rho_{t+1}\right)\gamma\left[\pi_t^{\varepsilon_w}\left(\pi^{1-\varepsilon_w}\right)\right]\pi_{t+1}^{-1}\Delta_{t+1}$$

And similarly for the firm

$$\Sigma_t = -P_t\frac{\partial J_t\left(w_t\right)}{\partial w_t} = h_t\left(1+s_t\right) + E_t\beta_{t,t+1}\left(1-\rho_{t+1}\right)\gamma\left[\pi_t^{\varepsilon_w}\left(\pi^{1-\varepsilon_w}\right)\right]\pi_{t+1}^{-1}\Sigma_{t+1}$$

The dynamic contract wage equation is solved by first linearizing the FOC for wage setting, and then substituting the linearized worker and firm surplus equations as well as the above discount factors in their loglinearized form (see GST (2008) for more details).

First order condition

$$\widehat{J}_t(w_t^*) + \widehat{\Delta}_t = \widehat{H}_t(w_t^*) + \widehat{\Sigma}_t$$

where the loglinear forms of the discount factors are

$$\widehat{\Delta}_t = (1-\iota)\,\widehat{h}_t - \frac{(1-\iota)\,\overline{\tau}}{(1-\overline{\tau})}\widehat{\tau}_t + \iota E_t\left(\widehat{\beta}_{t,t+1} + \varepsilon_w\widehat{\pi}_t - \widehat{\pi}_{t+1} + \widehat{\Delta}_{t+1}\right) - \overline{\beta}\overline{\rho}\gamma E_t\widehat{\rho}_{t+1}$$

$$\widehat{\Sigma}_t = (1-\iota)\,\widehat{h}_t + \frac{(1-\iota)\,\overline{s}}{(1+\overline{s})}\widehat{s}_t + \iota E_t\left(\widehat{\beta}_{t,t+1} + \varepsilon_w\widehat{\pi}_t - \widehat{\pi}_{t+1} + \widehat{\Sigma}_{t+1}\right) - \overline{\beta}\overline{\rho}\gamma E_t\widehat{\rho}_{t+1}$$

and the expressions for $\widehat{J}_t(w_t^*)$ and $\widehat{H}_t(w_t^*)$ can be found as follows

**Worker surplus** The worker surplus can be written as

$$\begin{aligned} H_t(w_t^*) \;=\; & \frac{w_t^*}{P_t}h_t\,(1-\tau_t) - \left[\frac{g\,(h_t)}{\Lambda_t} + b + E_t\beta_{t,t+1}q_t^W H_{x,t+1}\right] \\ & + E_t\beta_{t,t+1}\left(1-\rho_{t+1}\right)H_{t+1}(w_{t+1}^*) \\ & + \gamma E_t\beta_{t,t+1}\left(1-\rho_{t+1}\right)\left[H_{t+1}(w_t^*\left[\pi_t^{\varepsilon_w}\left(\pi^{1-\varepsilon_w}\right)\right]) - H_{t+1}(w_{t+1}^*)\right] \end{aligned}$$

In the last term, evaluate the expression $E_t\left[H_{t+1}(w_t^*\left[\pi_t^{\varepsilon_w}\left(\pi^{1-\varepsilon_w}\right)\right]) - H_{t+1}(w_{t+1}^*)\right]$

$$\begin{aligned} & E_t\left[H_{t+1}(w_t^*\left[\pi_t^{\varepsilon_w}\left(\pi^{1-\varepsilon_w}\right)\right]) - H_{t+1}(w_{t+1}^*)\right] \\ = \; & E_t\left[\frac{w_t^*\left[\pi_t^{\varepsilon_w}\left(\pi^{1-\varepsilon_w}\right)\right]}{P_{t+1}} - \frac{w_{t+1}^*}{P_{t+1}}\right]h_{t+1}\,(1-\tau_{t+1}) \\ & + \gamma E_t\beta_{t+1,t+2}\varsigma_{t+1,t+2}\left[H_{t+2}(w_t^*\left[\pi_{t+1}^{\varepsilon_w}\left(\pi^{1-\varepsilon_w}\right)\left[\pi_t^{\varepsilon_w}\left(\pi^{1-\varepsilon_w}\right)\right]\right]) - H_{t+2}(w_{t+1}^*\left[\pi_{t+1}^{\varepsilon_w}\left(\pi^{1-\varepsilon_w}\right)\right])\right] \end{aligned}$$

When linearized, this expression gets the following form

$$\begin{aligned} & E_t\left[\widehat{H}_{t+1}(w_t^*\left[\pi_t^{\varepsilon_w}\left(\pi^{1-\varepsilon_w}\right)\right]) - \widehat{H}_{t+1}(w_{t+1}^*)\right] \\ = \; & \frac{\overline{wh}\,(1-\overline{\tau})}{\overline{H}}E_t\left[\widehat{w}_t^* + \varepsilon_w\widehat{\pi}_t - \widehat{w}_{t+1}^* - \widehat{\pi}_{t+1}\right] \\ & + \overline{\beta}\,(1-\overline{\rho})\,\gamma E_t\left[\widehat{H}_{t+2}(w_t^*\left[\pi_{t+1}^{\varepsilon_w}\left(\pi^{1-\varepsilon_w}\right)\left[\pi_t^{\varepsilon_w}\left(\pi^{1-\varepsilon_w}\right)\right]\right]) - \widehat{H}_{t+2}(w_{t+1}^*\left[\pi_{t+1}^{\varepsilon_w}\left(\pi^{1-\varepsilon_w}\right)\right])\right] \end{aligned}$$

Iterating forward this can be further simplified to yield

$$
\begin{aligned}
& E_t \left[ \widehat{H}_{t+1}(w_t^* \left[ \pi_t^{\varepsilon_w} \left( \pi^{1-\varepsilon_w} \right) \right]) - \widehat{H}_{t+1}(w_{t+1}^*) \right] \\
= \; & \frac{1}{1-\overline{\beta}\left(1-\overline{\rho}\right)\gamma} \frac{\overline{wh}\left(1-\overline{\tau}\right)}{\overline{H}} E_t \left[ \widehat{w}_t^* + \varepsilon_w \widehat{\pi}_t - \widehat{w}_{t+1}^* - \widehat{\pi}_{t+1} \right]
\end{aligned}
$$

With the help of the above expression, the loglinear formulation of the worker surplus is found to be

$$
\begin{aligned}
\widehat{H}_t \;=\; & \frac{\overline{wh}\left(1-\overline{\tau}\right)}{\overline{H}} \left( \widehat{w}_t^* - \widehat{P}_t + \widehat{h}_t \right) - \frac{\overline{wh}\overline{\tau}}{\overline{H}} \widehat{\tau}_t - \frac{1}{1+\phi} \frac{\overline{mrs}\overline{h}\left(1+\overline{\tau}^c\right)}{\overline{H}} \left[ \widehat{mrs}_t + \widehat{h}_t \right] \\
& - \frac{\overline{mrs}\overline{h}\overline{\tau}^c}{\left(1+\phi\right)\overline{H}} \widehat{\tau}_t^c - \overline{\beta}\overline{q}^W E_t \left( \hat{q}_t^W + \widehat{H}_{x,t+1} + \widehat{\beta}_{t,t+1} \right) \\
& + \overline{\beta}\left(1-\overline{\rho}\right) E_t \left( \widehat{H}_{t+1}\left(w_{t+1}^*\right) + \widehat{\beta}_{t,t+1} \right) - \overline{\beta}\overline{\rho} E_t \widehat{\rho}_{t+1} \\
& + \frac{\overline{\beta}\left(1-\overline{\rho}\right)\gamma}{1-\overline{\beta}\left(1-\overline{\rho}\right)\gamma} \frac{\overline{wh}\left(1-\overline{\tau}\right)}{\overline{H}} E_t \left( \widehat{w}_t^* + \varepsilon_w \widehat{\pi}_t - \widehat{w}_{t+1}^* - \widehat{\pi}_{t+1} \right)
\end{aligned}
$$

where as shown in Gertler and Trigari (2006) up to a first order approximation $\widehat{H}_{x,t+1} = \widehat{H}_{t+1}\left(w_{t+1}\right)$.

**Firm surplus** The firm surplus can be written as

$$
\begin{aligned}
J_t(w_t^*) \;=\; & x_t f\left(h_t\right) - \frac{w_t^*}{P_t} h_t \left(1+s_t\right) + E_t \beta_{t,t+1} \left(1-\rho_{t+1}\right) J_{t+1}(w_{t+1}^*) \\
& + \gamma E_t \beta_{t,t+1} \left(1-\rho_{t+1}\right) \left[ J_{t+1}(w_t^* \left[ \pi_t^{\varepsilon_w} \left( \pi^{1-\varepsilon_w} \right) \right]) - J_{t+1}(w_{t+1}^*) \right]
\end{aligned}
$$

In the last term, evaluate the expression $E_t \left[ J_{t+1}(w_t^* \left[ \pi_t^{\varepsilon_w} \left( \pi^{1-\varepsilon_w} \right) \right]) - J_{t+1}(w_{t+1}^*) \right]$

$$
\begin{aligned}
& E_t \left[ J_{t+1}(w_t^* \left[ \pi_t^{\varepsilon_w} \left( \pi^{1-\varepsilon_w} \right) \right]) - J_{t+1}(w_{t+1}^*) \right] \\
= \; & -E_t \left[ \frac{w_t^* \left[ \pi_t^{\varepsilon_w} \left( \pi^{1-\varepsilon_w} \right) \right]}{P_{t+1}} - \frac{w_{t+1}^*}{P_{t+1}} \right] h_{t+1} \left(1+s_{t+1}\right) \\
& + \gamma E_t \beta_{t+1,t+2} \varsigma_{t+1,t+2} \left[ J_{t+2}(w_t^* \left[ \pi_{t+1}^{\varepsilon_w} \left( \pi^{1-\varepsilon_w} \right) \pi_t^{\varepsilon_w} \left( \pi^{1-\varepsilon_w} \right) \right]) - J_{t+2}(w_{t+1}^* \left[ \pi_{t+1}^{\varepsilon_w} \left( \pi^{1-\varepsilon_w} \right) \right]) \right]
\end{aligned}
$$

When linearized this expression gets the following form

$$
\begin{aligned}
& E_t\left[\widehat{J}_{t+1}(w_t^*\left[\pi_t^{\varepsilon_w}\left(\pi^{1-\varepsilon_w}\right)\right]) - \widehat{J}_{t+1}(w_{t+1}^*)\right] \\
= \; & -\frac{\overline{wh}\,(1+\overline{s})}{\overline{J}} E_t\left[\widehat{w}_t^* + \varepsilon_w \widehat{\pi}_t - \widehat{w}_{t+1}^* - \widehat{\pi}_{t+1}\right] \\
& + \overline{\beta}\,(1-\overline{\rho})\,\gamma E_t\left[\widehat{J}_{t+2}(w_t^*\left[\pi_{t+1}^{\varepsilon_w}\left(\pi^{1-\varepsilon_w}\right)\pi_t^{\varepsilon_w}\left(\pi^{1-\varepsilon_w}\right)\right]) - \widehat{J}_{t+2}(w_{t+1}^*\left[\pi_{t+1}^{\varepsilon_w}\left(\pi^{1-\varepsilon_w}\right)\right])\right]
\end{aligned}
$$

Iterating forward this can be further simplified to yield

$$
\begin{aligned}
& E_t\left[\widehat{J}_{t+1}(w_t^*\left[\pi_t^{\varepsilon_w}\left(\pi^{1-\varepsilon_w}\right)\right]) - \widehat{J}_{t+1}(w_{t+1}^*)\right] \\
= \; & -\frac{1}{1-\overline{\beta}\,(1-\overline{\rho})\,\gamma}\frac{\overline{wh}\,(1+\overline{s})}{\overline{J}} E_t\left[\widehat{w}_t^* + \varepsilon_w \widehat{\pi}_t - \widehat{w}_{t+1}^* - \widehat{\pi}_{t+1}\right]
\end{aligned}
$$

Finally, as with worker surplus, the loglinear formulation of the firm surplus can be found with the help of the above expression

$$
\begin{aligned}
\widehat{J}_t \; = \; & \frac{\overline{x}\overline{mplh}}{\alpha\overline{J}}\left(\widehat{x}_t + \widehat{mpl}_t + \widehat{h}_t\right) - \frac{\overline{wh}\,(1+\overline{s})}{\overline{J}}\left(\widehat{w}_t^* - \widehat{P}_t + \widehat{h}_t\right) - \frac{\overline{wh}\overline{s}}{\overline{J}}\widehat{s}_t \\
& - \overline{\beta}\overline{\rho} E_t \widehat{\rho}_{t+1} + \overline{\beta}\,(1-\overline{\rho})\,E_t\left(\widehat{J}_{t+1}\left(w_{t+1}^*\right) + \widehat{\beta}_{t,t+1}\right) \\
& + \frac{\overline{\beta}\,(1-\overline{\rho})\,\gamma}{1-\overline{\beta}\,(1-\overline{\rho})\,\gamma}\frac{\overline{wh}\,(1+\overline{s})}{\overline{J}} E_t\left(\widehat{w}_{t+1}^* + \widehat{\pi}_{t+1} - \widehat{w}_t^* - \varepsilon_w \widehat{\pi}_t\right)
\end{aligned}
$$

**The Contract wage** Inserting the expressions for the worker and firm surpluses, as well as those for the discount factors, into the FOC yields (after collecting the wage terms to the left-hand side and using the Nash FOC for next period)

$$
\begin{aligned}
\Longrightarrow\quad & \left[\frac{\overline{wh}\left(1-\overline{\tau}\right)}{\overline{H}}+\frac{\overline{wh}\left(1+\overline{s}\right)}{\overline{J}}\right]\widehat{w}_t^{*} \\
& +\frac{\iota}{\left(1-\iota\right)}\left[\frac{\overline{wh}\left(1-\overline{\tau}\right)}{\overline{H}}+\frac{\overline{wh}\left(1+\overline{s}\right)}{\overline{J}}\right]E_t\left(\widehat{w}_t^{*}+\varepsilon_w\widehat{\pi}_t-\widehat{w}_{t+1}^{*}-\widehat{\pi}_{t+1}\right) \\
= \quad & \frac{\overline{xmplh}}{\alpha\overline{J}}\left(\widehat{x}_t+\widehat{mpl}_t\right)+\frac{1}{1+\phi}\frac{\overline{mrs}\overline{h}\left(1+\overline{\tau}^c\right)}{\overline{H}}\left(\widehat{mrs}_t\right)+\frac{\overline{mrs}\overline{h}\overline{\tau}^c}{\left(1+\phi\right)\overline{H}}\widehat{\tau}_t^c \\
& +\overline{\beta}\left(1-\overline{\rho}\right)E_t\left[\widehat{J}_{t+1}\left(w_{t+1}^{*}\right)+\widehat{\beta}_{t,t+1}\right]-\overline{\beta}\left(1-\overline{\rho}\right)E_t\left[\widehat{J}_{t+1}\left(w_{t+1}^{*}\right)+\widehat{\beta}_{t,t+1}+\widehat{\Delta}_{t+1}-\widehat{\Sigma}_{t+1}\right] \\
& +\overline{\beta}\overline{q}^{W}E_t\left(\widehat{q}_t^{W}+\widehat{H}_{x,t+1}+\widehat{\beta}_{t,t+1}\right) \\
& -\left\{\left[\frac{\overline{wh}\left(1-\overline{\tau}\right)}{\overline{H}}+\frac{\overline{wh}\left(1+\overline{s}\right)}{\overline{J}}\right]-\frac{\overline{xmplh}}{\alpha\overline{J}}-\frac{\overline{mrs}}{1+\phi}\frac{\overline{h}\left(1+\overline{\tau}^c\right)}{\overline{H}}\right\}\widehat{h}_t \\
& -\left[\frac{\overline{whs}}{\overline{J}}+\frac{\left(1-\iota\right)\overline{s}}{\left(1+\overline{s}\right)}\right]\widehat{s}_t+\left[\frac{\overline{wh\tau}}{\overline{H}}-\frac{\left(1-\iota\right)\overline{\tau}}{\left(1-\overline{\tau}\right)}\right]\widehat{\tau}_t+\left[\frac{\overline{wh}\left(1-\overline{\tau}\right)}{\overline{H}}+\frac{\overline{wh}\left(1+\overline{s}\right)}{\overline{J}}\right]\widehat{P}_t \\
& +\left[\overline{\beta}\left(1-\overline{\rho}\right)\gamma-\overline{\beta}\left(1-\overline{\rho}\right)\right]E_t\widehat{\Delta}_{t+1}-\left[\overline{\beta}\left(1-\overline{\rho}\right)\gamma-\overline{\beta}\left(1-\overline{\rho}\right)\right]E_t\widehat{\Sigma}_{t+1}
\end{aligned}
$$

where $\iota=\overline{\beta}\left(1-\overline{\rho}\right)\gamma$. Dividing by the term $\left[\frac{\overline{wh}(1-\overline{\tau})}{\overline{H}}+\frac{\overline{wh}(1+\overline{s})}{\overline{J}}\right]=\frac{\overline{wh}(1+\overline{s})}{\eta\overline{J}}=\frac{\overline{wh}(1-\overline{\tau})}{(1-\eta)\overline{H}}$, and using the steady state equations for $\overline{\Delta}$ and $\overline{\Sigma}$, and for the Nash FOC allows us to rewrite the contract wage equation in the following simpler form

$$
\begin{aligned}
\Longrightarrow\quad & \widehat{w}_t^{*}+\frac{\iota}{1-\iota}E_t\left(\widehat{w}_t^{*}+\varepsilon_w\widehat{\pi}_t-\widehat{w}_{t+1}^{*}-\widehat{\pi}_{t+1}\right) \\
= \quad & \varphi_x\left(\widehat{x}_t+\widehat{mpl}_t\right)+\varphi_m\widehat{mrs}_t+\varphi_H E_t\left(\widehat{q}_t^{W}+\widehat{H}_{x,t+1}+\widehat{\beta}_{t,t+1}\right) \\
& -\varphi_h\widehat{h}_t-\varphi_s\widehat{s}_t+\varphi_\tau\widehat{\tau}_t+\varphi_{\tau^c}\widehat{\tau}_t^c+\varphi_D E_t\left[\widehat{\Sigma}_{t+1}-\widehat{\Delta}_{t+1}\right]+\widehat{P}_t \\
= \quad & \widehat{w}_t^{0}(r)
\end{aligned}
$$

where $\widehat{w}_t^{0}(r)$ is the target wage in the bargain, and its coefficients are

$$
\begin{aligned}
\varphi_x &= \frac{\overline{xmpl}\eta}{\alpha\overline{w}\left(1+\overline{s}\right)},\quad \varphi_m=\frac{\overline{mrs}\left(1-\eta\right)\left(1+\overline{\tau}^c\right)}{\left(1+\phi\right)\overline{w}\left(1-\overline{\tau}\right)},\quad \varphi_H=\frac{\left(1-\eta\right)\overline{H}}{\overline{wh}\left(1-\overline{\tau}\right)}\overline{\beta}\overline{q}^{W},\quad \varphi_{\tau^c}=\frac{\overline{mrs}\left(1-\eta\right)\overline{\tau}^c}{\left(1+\phi\right)\overline{w}\left(1-\overline{\tau}\right)} \\
\varphi_h &= \left\{1-\frac{\overline{xmpl}\eta}{\alpha\overline{w}\left(1+\overline{s}\right)}-\frac{\overline{mrs}\left(1-\eta\right)\left(1+\overline{\tau}^c\right)}{\left(1+\phi\right)\overline{w}\left(1-\overline{\tau}\right)}\right\},\quad \varphi_s=\frac{\eta\overline{s}}{\left(1+\overline{s}\right)}\left[1+\frac{\left(1-\iota\right)\overline{J}}{\overline{wh}\left(1+\overline{s}\right)}\right] \\
,\quad \varphi_\tau &= \frac{\left(1-\eta\right)\overline{\tau}}{\left(1-\overline{\tau}\right)}\left[1-\frac{\left(1-\iota\right)\overline{H}}{\overline{wh}\left(1-\overline{\tau}\right)}\right],\text{ and }\varphi_D=\left[\overline{\beta}\left(1-\overline{\rho}\right)\left(1-\gamma\right)\frac{\eta\overline{J}}{\overline{wh}\left(1+\overline{s}\right)}\right]
\end{aligned}
$$

The target wage $\widehat{w}_t^{0}(r)$ is of the same form than the period-by-period negotiated wage, adjusted for the new bargaining weights. The equation for the contract wage can be further rewritten as

$$\frac{1}{(1-\iota)}\widehat{w}_t^* = \widehat{w}_t^0(r) + \frac{\iota}{(1-\iota)}E_t(\widehat{\pi}_{t+1} - \varepsilon_w\widehat{\pi}_t) + \frac{\iota}{(1-\iota)}E_t\widehat{w}_{t+1}^*$$

$$\Longleftrightarrow \widehat{w}_t^* = [1-\iota]\,\widehat{w}_t^0(r) + \iota E_t(\widehat{\pi}_{t+1} - \varepsilon_w\widehat{\pi}_t) + \iota E_t\widehat{w}_{t+1}^*$$

This is the optimal contract wage set at time $t$ *by all matches that are allowed to renegotiate their wage.* As is usual with Calvo-type contracting, it depends on a wage target $w_t^0(r)$ and next period's optimal wage.

**The spillover effect** To derive the spillover effect, consider the worker surplus with *optimal (contract) wage* versus the expected *average market wage* in the same way as above

$$E_t\widehat{H}_{t+1}(w_{t+1}) = E_t\widehat{H}_{t+1}(w_{t+1}^*) + \frac{\overline{wh}(1-\overline{\tau})}{(1-\iota)\overline{H}}E_t\left(\widehat{w}_{t+1} - \widehat{w}_{t+1}^*\right)$$

Denoting $\frac{\overline{wh}(1-\overline{\tau})}{(1-\iota)\overline{H}} = \Gamma$ and substituting the above expression in the target wage equation gives

$$\begin{aligned}\widehat{w}_t^0(r) &= \varphi_x\left(\widehat{x}_t + \widehat{mpl}_t\right) + \varphi_m\widehat{mrs}_t + \varphi_H E_t\left(\widehat{q}_t^W + \widehat{H}_{t+1}(w_{t+1}^*) + \widehat{\beta}_{t,t+1} + \Gamma E_t\left[\widehat{w}_{t+1} - \widehat{w}_{t+1}^*\right]\right)\\ &\quad+\varphi_h\widehat{h}_t - \varphi_s\widehat{s}_t + \varphi_\tau\widehat{\tau}_t + \varphi_{\tau^c}\widehat{\tau}_t^c + \varphi_D E_t\left[\widehat{\Sigma}_{t+1} - \widehat{\Delta}_{t+1}\right] + \widehat{P}_t\end{aligned}$$

$$\Longleftrightarrow \widehat{w}_t^0(r) = \widehat{w}_t^0 + \varphi_H\Gamma E_t\left[\widehat{w}_{t+1} - \widehat{w}_{t+1}^*\right]$$

where the target wage $\widehat{w}_t^0(r)$ - the wage the firm and its worker would agree to if they are allowed to renegotiate, *and if firms and workers elsewhere remain on staggered multiperiod wage contracts* - is a sum of the wage that would arise *if all matches were negotiating wages period-by-period* $\widehat{w}_t^0$ and the spillover effect $\varphi_H\Gamma E_t\left[\widehat{w}_{t+1} - \widehat{w}_{t+1}^*\right]$.

**Evolution of the average wage** To derive the appropriate loglinear expression for the evolution of the average wage, first collect the necessary elements from previous calculations

1) The contract wage

$$\widehat{w}_t^* = [1-\iota]\,\widehat{w}_t^0(r) + \iota E_t(\widehat{\pi}_{t+1} - \varepsilon_w\widehat{\pi}_t) + \iota E_t\widehat{w}_{t+1}^*$$

2) The average wage

$$\widehat{w}_t = (1-\gamma)\,\widehat{w}_t^* + \gamma\,(\widehat{w}_{t-1} - \widehat{\pi}_t + \varepsilon_w \widehat{\pi}_{t-1})$$

3) The target wage

$$\widehat{w}_t^0(r) = \widehat{w}_t^0 + \varphi_H \Gamma E_t \left[\widehat{w}_{t+1} - \widehat{w}_{t+1}^*\right]$$

First, insert the target wage in the contract wage equation

$$\widehat{w}_t^* = [1-\iota]\left(\widehat{w}_t^0 + \varphi_H \Gamma E_t \left[\widehat{w}_{t+1} - \widehat{w}_{t+1}^*\right]\right) + \iota E_t\,(\widehat{\pi}_{t+1} - \varepsilon_w \widehat{\pi}_t) + \iota E_t \widehat{w}_{t+1}^*$$

Then update the average wage equation by one period and take expectations

$$E_t \widehat{w}_{t+1} = (1-\gamma)\,E_t \widehat{w}_{t+1}^* + \gamma\,(\widehat{w}_t - E_t \widehat{\pi}_{t+1} + \varepsilon_w \widehat{\pi}_t)$$

$$\Longleftrightarrow E_t \widehat{w}_{t+1}^* = \frac{1}{(1-\gamma)}\left(E_t \widehat{w}_{t+1} - \gamma\,(\widehat{w}_t - E_t \widehat{\pi}_{t+1} + \varepsilon_w \widehat{\pi}_t)\right)$$

Use this expression to eliminate $E_t \widehat{w}_{t+1}^*$ from the contract wage equation

$$\begin{aligned} \widehat{w}_t^* \;=\; & [1-\iota]\left(\widehat{w}_t^0 + \varphi_H \Gamma E_t \widehat{w}_{t+1} - \varphi_H \Gamma \left[\frac{1}{(1-\gamma)}\left(E_t \widehat{w}_{t+1} - \gamma \widehat{w}_t + \gamma E_t\,(\widehat{\pi}_{t+1} - \varepsilon_w \widehat{\pi}_t)\right)\right]\right) \\ & + \iota E_t\,(\widehat{\pi}_{t+1} - \varepsilon_w \widehat{\pi}_t) + \iota \left[\frac{1}{(1-\gamma)}\left(E_t \widehat{w}_{t+1} - \gamma \widehat{w}_t + \gamma E_t\,(\widehat{\pi}_{t+1} - \varepsilon_w \widehat{\pi}_t)\right)\right] \end{aligned}$$

$$\begin{aligned} \widehat{w}_t^* \;=\; & (1-\iota)\,\widehat{w}_t^0 + (1-\iota)\,\varphi_H \Gamma E_t \widehat{w}_{t+1} - (1-\iota)\,\varphi_H \Gamma \frac{1}{(1-\gamma)} E_t \widehat{w}_{t+1} \\ & + [1-\iota]\,\varphi_H \Gamma \frac{\gamma}{(1-\gamma)}\,(\widehat{w}_t - E_t \widehat{\pi}_{t+1} + \varepsilon_w \widehat{\pi}_t) + \iota\,(E_t \widehat{\pi}_{t+1} - \varepsilon_w \widehat{\pi}_t) \\ & + \frac{\iota}{(1-\gamma)} E_t \widehat{w}_{t+1} - \frac{\iota\gamma}{(1-\gamma)}\,(\widehat{w}_t - E_t \widehat{\pi}_{t+1} + \varepsilon_w \widehat{\pi}_t) \end{aligned}$$

$$\begin{aligned} \Longleftrightarrow \quad & \widehat{w}_t^* = (1-\iota)\,\widehat{w}_t^0 + \left[(1-\iota)\,\varphi_H \Gamma \frac{\gamma}{(1-\gamma)} - \iota \frac{\gamma}{(1-\gamma)}\right](\widehat{w}_t - E_t \widehat{\pi}_{t+1} + \varepsilon_w \widehat{\pi}_t) \\ & + \iota\,(E_t \widehat{\pi}_{t+1} - \varepsilon_w \widehat{\pi}_t) \\ & + \left[(1-\iota)\,\varphi_H \Gamma - (1-\iota)\,\varphi_H \Gamma \frac{1}{(1-\gamma)} + \iota \frac{1}{(1-\gamma)}\right] E_t \widehat{w}_{t+1} \end{aligned}$$

Denote $\zeta = (1-\iota)\varphi_H\Gamma$, and use the above equation to eliminate $\widehat{w}_t^*$ from the average wage equation (equation 2)

$$\begin{aligned}\widehat{w}_t &= (1-\gamma)(1-\iota)\widehat{w}_t^0 + (\zeta\gamma - \iota\gamma)(\widehat{w}_t - E_t\widehat{\pi}_{t+1} + \varepsilon_w\widehat{\pi}_t) \\ &\quad + (1-\gamma)\iota(E_t\widehat{\pi}_{t+1} - \varepsilon_w\widehat{\pi}_t) \\ &\quad + [(1-\gamma)\zeta - \zeta + \iota]E_t\widehat{w}_{t+1} + \gamma(\widehat{w}_{t-1} - \widehat{\pi}_t + \varepsilon_w\widehat{\pi}_{t-1})\end{aligned}$$

$$\begin{aligned}[1-\gamma(\zeta-\iota)]\widehat{w}_t &= (1-\gamma)(1-\iota)\widehat{w}_t^0 - \gamma(\zeta-\iota)(E_t\widehat{\pi}_{t+1} - \varepsilon_w\widehat{\pi}_t) + (1-\gamma)\iota(E_t\widehat{\pi}_{t+1} - \varepsilon_w\widehat{\pi}_t) \\ &\quad + [(1-\gamma)\zeta - \zeta + \iota]E_t\widehat{w}_{t+1} + \gamma(\widehat{w}_{t-1} - \widehat{\pi}_t + \varepsilon_w\widehat{\pi}_{t-1})\end{aligned}$$

Finally, after dividing by $[1-\gamma(\zeta-\iota)]$, the dynamic average wage equation can be expressed as

$$\Longleftrightarrow \widehat{w}_t = \lambda_b(\widehat{w}_{t-1} - \widehat{\pi}_t + \varepsilon_w\widehat{\pi}_{t-1}) + \lambda_0\widehat{w}_t^0 + \lambda_f E_t(\widehat{w}_{t+1} + \widehat{\pi}_{t+1} - \varepsilon_w\widehat{\pi}_t)$$

$$\begin{aligned}\text{where } \lambda_b &= \frac{\gamma}{[1-\gamma(\zeta-\iota)]},\ \lambda_0 = \frac{(1-\gamma)(1-\iota)}{[1-\gamma(\zeta-\iota)]},\ \text{and } \lambda_f = \frac{\iota-\gamma\zeta}{[1-\gamma(\zeta-\iota)]}, \\ \text{with } \zeta &= (1-\iota)\varphi_H\Gamma,\ \iota = \overline{\beta}(1-\overline{\rho})\gamma,\ \Gamma = \frac{\overline{wh}(1-\overline{\tau})}{(1-\iota)\overline{H}},\ \varphi_H = \frac{(1-\eta)\overline{H}\overline{\beta}\overline{q}^W}{\overline{wh}(1-\overline{\tau})} \text{as previously denoted.}\end{aligned}$$

Figure 3. Impulse responses to a government spending shock with different degrees of price rigidity. Solid line: baseline, $\xi = 0.75$, dotted line $\xi = 0.25$

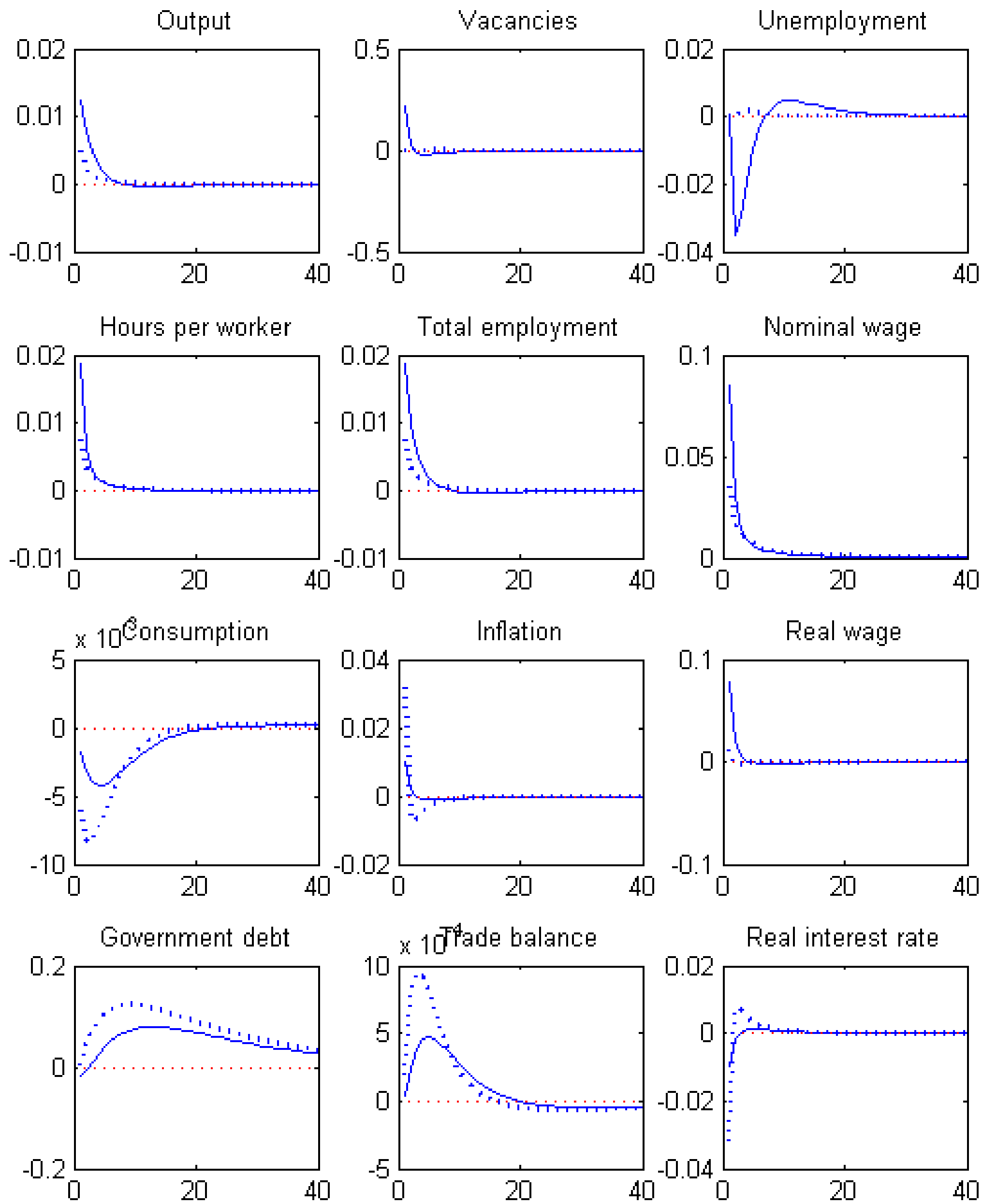

Figure 4. Impulse responses to a government spending shock financed by raising labour taxes with flexible wages (solid line) and rigid wages (dotted line)