FinMaP-Working Paper No.3

This project has received funding from the European Union's Seventh Framework Programme for research, technological development and demonstration under grant agreement no. 612955

FINMAP –

FINANCIAL DISTORTIONS AND MACROECONOMIC PERFORMANCE: EXPECTATIONS, CONSTRAINTS AND INTERACTION OF AGENTS

DATE: 04/29/2014

TITLE

Emergence of a Core-Periphery Structure in a Simple Dynamic Model of the Interbank Market

by: Thomas Lux

ABSTRACT

This paper studies a simple dynamic model of interbank credit relationships. Starting from a given balance sheet structure of a banking system with a realistic distribution of bank sizes, the necessity of establishing interbank credit connections 3merges from idiosyncratic liquidity shocks. Banks initially choose potential trading partners randomly, but form preferential relationships via an elementary reinforcement learning algorithm. As it turns out, the dynamic evolution of this system displays a formation of a core-periphery structure with mainly the largest banks assuming the roles of money center banks mediating between the liquidity needs of many smaller banks. Statistical analysis shows that this evolving interbank market shares virtually all of the salient characteristics of interbank credit relationship that have been put forth in recent literature. Preferential interest rates for borrowers with strong attachment to a lender may prevent the system from becoming extortionary and guarantee the survival of the small peripherical banks.

JEL-Classification: D85, G21, D 83
Keywords: liquidity, interbank market, network formation

AUTHORS

1. **Thomas Lux**
   Kiel University
   Department of Economics
   Chair of Monetary Economics and International Finance
   24098 Kiel, Germany

   Bank of Spain chair of Computational Finance
   Department of Economics,
   University Jaume I
   Castellón, Spain
   Email: lux@bwl.uni-kiel.de

Emergence of a Core-Periphery Structure in a Simple Dynamic Model of the Interbank Market*

Thomas Lux†

Abstract

This paper studies a simple dynamic model of interbank credit relationships. Starting from a given balance sheet structure of a banking system with a realistic distribution of bank sizes, the necessity of establishing interbank credit connections emerges from idiosyncratic liquidity shocks. Banks initially choose potential trading partners randomly, but form preferential relationships via an elementary reinforcement learning algorithm. As it turns out, the dynamic evolution of this system displays a formation of a core-periphery structure with mainly the largest banks assuming the roles of money center banks mediating between the liquidity needs of many smaller banks. Statistical analysis shows that this evolving interbank market shares virtually all of the salient characteristics of interbank credit relationship that have been put forth in recent literature. Preferential interest rates for borrowers with strong attachment to a lender may prevent the system from becoming extortionary and guarantee the survival of the small peripherical banks.

JEL Classification: **D85, G21, D83**
Keywords: **liquidity, interbank market, network formation**

*The research reported in this paper has been initiated during a visit at the Financial Stability Assessment Division of the European Central Bank during September 2013. The author is extremely grateful for the hospitality of the Division as well as very stimulating discussions with Christoffer Kok, Grzegorz Halaj, Mattia Montagna, Michael Wellman and Shu-Heng Chen. Helpful comments from the audience of various seminar and workshop presentations are also gratefully acknowledged. I am also very thankful for the research assistance of Ricardo Giglio. This report is part of a research initiative founded by the Leibniz Community. The research leading to these results has also received funding from the Spanish Ministry of Science and Innovation (ECO2011-23634), from Universitat Jaume I (P1.1B2012-27) and from the European Union Seventh Framework Programme (FP7/2007-2013) under grant agreement no. 612955.

†Corresponding author. Department of Economics, University of Kiel, Germany & Banco de España Chair in Computational Economics, Universitat Jaume I, Castellón, Spain. Email: lux@bwl.uni-kiel.de.

1 Introduction

While it had received only scarce attention for a long time, the interbank credit market has been in the focus of monetary policy authorities and financial economists since the outbreak of the still continuing financial crises in 2007/08. As it appeared from the contagion effects after the default of Lehman Brothers and the subsequent collapse of trading activity in unsecured money markets, interbank credit is a crucial component of the financial architecture of modern economies. Its disruption could lead to severe problems for the liquidity management of single institutions and the sudden rapture of established funding lines could trigger an avalanche of liquidity problems across the banking system. Macroprudential regulation and stress testing is paying more and more attention to the risks emerging from the various connections between financial institutions so as to make the system safer against interbank contagion effects. Understanding the structure of the existing network of credit relationships is, therefore, essential for an assessment of its potential risk. However, very little has been known until very recently about the topology of credit links between banks and its salient features.

In view of the events of the year 2008 and after, an increasing body of recent research has investigated the interbank market under the perspective of network theory. Indeed, interpreting the single banks as nodes and their credit relationships as links between nodes, one can represent such data in a straightforward way as a network, with credit volumes defining the so-called adjacency matrix of links between the underlying entities. With the surge of network research in the natural and social sciences, interbank data have occasionally been investigated from such a perspective even before the financial crisis, cf. Boss et al. (2004), Inaoka et al. (2007) or Soramäki et al. (2007). The predominant objective of this early literature had been to phenomenologically describe the data with popular network statistics such as the degree distribution, centrality of nodes etc. and to compare their structure with well-known canonical network models such as purely random, scale-free or small-world networks. Such a classification (if possible) could already provide important insights on the dangers of disruptions and inherent systemic risk as, for example, it is well-known that scale-free networks are in general robust to random disturbances but are highly vulnerable in the presence of targeted attacks on their most connected nodes.

Recently, attention has been shifting towards alternative models of the network structure that might be particular to socio-economic relationships and less so to phenomena in the natural world. A number of authors have argued that interbank relations might be akin to a core-periphery structure, a setting first proposed in sociology for networks of acquaintanceships (cf. Borgatti and Everett, 2000). The pioneering application of this model to interbank data is due to Craig and von Peter (2014). Using a data set on large loans and exposures between German banks they find a very stable set of banks forming the core of the system. They also found that alternative random and scale-free networks cannot explain the degree of stratification within the German banking system, and that total balance sheet size could predict how banks position themselves in the system.

Fricke and Lux (2014) have applied the core-periphery framework to data of the electronic platform e-MID that basically is used for short-term (overnight) liquidity provision. They also found that the structure of the networks derived from these data can be captured in a very robust way by a core-periphery model. Applying an asymmetric version of the CP framework they also find that banks' roles as borrowers and lenders in the money market can be very different. Distinguishing between their "in-coreness" and "out-coreness" they found both measures to be virtually uncorrelated. This asymmetry is inherited from a very asymmetric structure of the raw data: In particular, there is a high concentration of incoming links per bank with almost always only a small number of creditors, on which a single institute relies for liquidity provision within an extended time span. In contrast, the distribution of outgoing links is much more heterogeneous and broader ranging from many zero entries (no lending in the interbank market) to large numbers of borrowers.

A core-periphery (CP) analysis of the UK interbank market is provided by Langfield et al. (2013) who use a comprehensive data set on connections between UK banks with a detailed breakdown into a large number of financial instruments. Identifying banks' roles in different segments of this multi-layered network topology, they also find some heterogeneity of their 'coreness' in different markets. Van Lelyveld and in't Veld (2012) apply the core-periphery model to contractual obligations among Dutch banks at a quarterly frequency of reporting and also obtained a fit of the CP model in line with that reported in the papers discussed above. Summarizing this emergent literature, it appears that linkage structures between financial institutions can often be captured in a compact way by a core-periphery distinction or by assigning a 'coreness' statistic to the individual banks. Such an approach has been found to describe different data sets better than traditional network models inherited from the natural sciences and it has some economic appeal in that the network core mostly (though not exclusively) consists of large banks which assume the role of money center banks for the system.

A few papers have started to try and provide theoretical explanations for such a structure: Hommes et al. (2013) look at the formation of CP networks under the perspective of game-theoretic concepts of network stability. They find that such a structure would not be stable in a system of homogenous banks, but could be stable under sufficient size heterogeneity. In a somewhat different vain, Castiglionesi and Navarro (2011) show the stability of a CP network as the equilibrium structure in a setting where banks have the choice between a safe investment strategy and a 'gambling' project. The CP structure then emerges as the optimal way for providing liquidity insurance with 'gamblers' being positioned in the periphery.

We approach the question of how a CP structure might emerge from a different theoretical perspective using an elementary dynamic model of the interbank market. The basic ingredients to our model are: (i) idiosyncratic liquidity shocks that hit all banks of a (closed) financial system in any period and that have to be evened out via the interbank market, (ii) a heterogeneous distribution of the balance sheet sizes of banks in accordance with empirical observations, (iii) a simple reinforcement-learning scheme that governs banks' decisions to contact other institutions as potential trading counterparts: If there has been a previous successful attempt at obtaining credit from a certain bank to overcome a liquidity shortage, the borrower will have a higher tendency of contacting this creditor again when another negative liquidity shock hits. If credit is denied, the 'trust' in this potential borrower will decline. Simulations show that this system quickly *self-organizes* into a core-periphery structure and also displays other realistic features found in interbank credit data. This finding suggests that the CP structure might be a natural consequence of a banking system with heterogeneous balance sheet size as we historically find it in industrialized economies. Not too much rationality and no knowledge of the complete structure of the system are, therefore, required on the part of liquidity managers, to

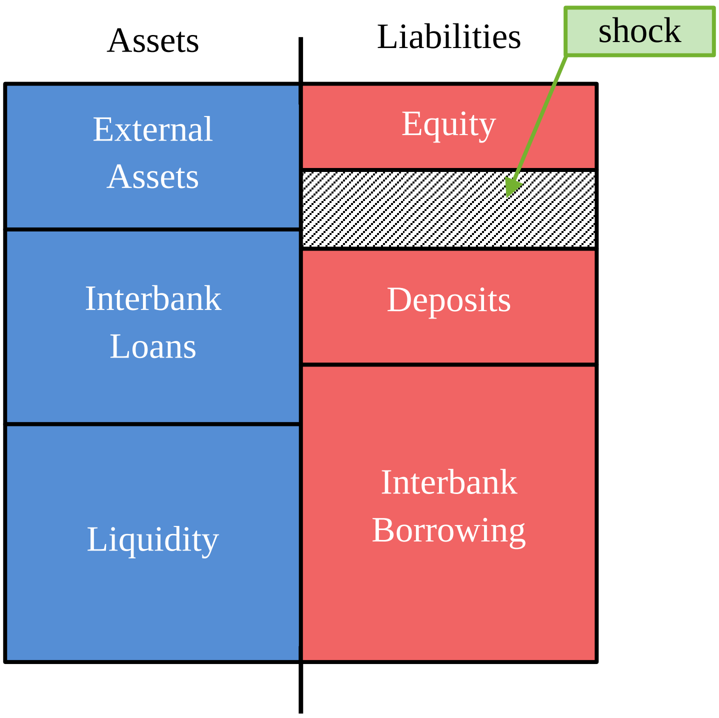


Figure 1: Balance Sheet Structure of Banks

deliberately create such a structure. On the contrary, single decentralized decisions based on past experiences would generically lead the overall system towards such a topology.

The present simple model is also, to the best of my knowledge, the first attempt to formulate a *dynamic* process of liquidity exchange within a heterogeneous banking system with fully specified balance sheet structure. It could, thus, be used as a starting point for various extensions studying other channels of connections between banks and could be used as a tool for dynamic stress tests in the presence of shocks to various types of assets and funding sources.

The paper proceeds as follows: sec. 2 provides more details on the structure of the model, while sec. 3 discusses the emergent CP structure and other properties of our simulations. Sec. 4 drops the previous restriction of zero interest rates and throws a glance on the role of interest rates heterogeneity in a core-periphery topology of the interbank market. Sec. 5 concludes.

2 Basic Building Blocks of the Model

We start with the stylized balance sheet structure as illustrated in Fig. 1. Bank i's total assets, A_i, consist of external assets e_i (outside the banking system), interbank loans l_i and liquidity m_i (which might sum up central bank deposits and cash reserves):

$$A_i = e_i + l_i + m_i. \tag{1}$$

Liabilities I_i are made up of deposits d_i, inter-bank borrowing b_i and bank's equity, η_i:

$$I_i = \eta_i + d_i + b_i. \tag{2}$$

Since we concentrate here on exchange of liquidity, not all of these positions will undergo changes. In particular, external assets are only included for completeness of the simplified balance sheet structure (but instead of liquidity shocks we could as well consider return shocks to e_i).

For simplicity, we will impose a certain structure as an initial condition on the balance sheets of all banks $i = 1, ..., N$ of our system. Namely, initially, the interbank market does not yet exist and so at time $t = 0 : l_i(0) = b_i(0)$ holds for all i.

We furthermore impose at time $t = 0$ the following structure on the entries of the balance sheet:

$$e_i(0) = \theta A_i,\ m_i(0) = (1-\theta)A_i,$$
$$\eta_i(0) = \gamma A_i,\ d_i(0) = (1-\gamma)A_i.$$

In the course of the evolution of our system, some of these quantities will not change in absolute value and others will. For instance, as long as no bankruptcies occur (and as long as we disregard interest for interbank loans), equity or bank capital η_i will remain constant in absolute value, but it might change, in fact, as a percentage of overall balance sheet size, A_i.

As concerns the distribution of the total balance sheet size A_i across banks, we mostly follow the empirical literature in using draws from a Pareto distribution to determine the size of the members of our banking system. That firm size distributions are highly skewed has been known for a long time and has recently been confirmed again, for example, by a comprehensive sample of U.S. companies (Axtell, 2001). The banking sector is no exception to this rule (cf. Ennis, 2001; Janicki and Prescott, 2006 for recent evidence for the U.S. and Bremus et al., 2012, for international evidence from a sample of banks in more than 70 countries). While there are different proposals in the literature for positive, skewed distributions to characterize firm sizes, we use here a Pareto distribution, i.e. we assume that the cumulative distribution function of the bank size distribution follows a law of the form

$$f(A_i) \sim A_i^{-\tau} \tag{3}$$

The reason to prefer the Pareto distribution is that it is a very parsimonious specification and that it is a generic way to capture the variability of large realizations of distributions with fat tails.[1]

Empirical evidence for firm size distributions speaks for values of the decay parameter of about 1 (Zipf's law). For banks Bremus et al. (2012) find a coefficient of ~ 3 for a sample of 11,476 banks from 73 countries, with country-specific estimates ranging down to about 1.4. In our simulations, we will consider bank size distributions within this range and for practical comparability of simulation runs, restrict the support to a finite interval of balance sheet sizes.

Once we have "created" our banking system at time $t = 0$, we expose banks to liquidity shocks. Basically, each bank is assumed to be hit by a liquidity shock $\sigma_i \epsilon_{i,t}$ with the size of the shock being proportional to its balance sheet size: $\sigma_i \sim A_i$, and $\epsilon_{i,t}$ standard Normally distributed innovations.[2] We introduce these shocks via deposits that will increase or decrease by the pertinent increment. Since a pure random shock would make deposits non-stationary,

[1] Statistical extreme value theory distinguishes between three kinds of generic behavior: exponential tails, hyperbolically decaying tails and distributions with finite endpoints. There is hardly any doubt that firm size distributions follow the second type of extremal behavior for which (3) captures the limiting behavior of *all* distributions that fall into this class, c.f. Reiss and Thomas (2007).

[2] There is relatively little empirical evidence on the statistical characteristics of deposits and other balance sheet shocks of banks. While some dependency on size appears realistic, observations reported in Hester and Pierce (1975) suggest that the size of the shock should rather increase less than proportionally with banks' total balance sheet size. We also implemented our model with functional specifications (e.g., a square root dependency) in line with such behaviour, without observing qualitative changes of the outcome. The results reported in Hester and Pierce (1975) also speak in favour of mean-reversion of liquidity shocks.

we also add, realistically, a mean-reverting component so that deposits of each bank develop over time as:

$$d_{i,t+1} = \beta(\overline{d}_i - d_{i,t}) + \sigma_i \epsilon_{i,t} \quad with \quad \overline{d}_i = \theta A_i \tag{4}$$

To make the system as a whole completely conservative in terms of aggregate deposits, the mean across all banks is also subtracted from the ensemble of realizations of shocks at any time t so that the sum of deposits in the system remains constant. The mean-reversion and subtraction of the mean both help to avoid bankruptcies or illiquidity 'by chance' due to non-stationarity. In fact, in all specifications of the model presented in this section, no such case did occur over very long simulation rounds (see below). Further elaborations of this system could, nevertheless, be used to focus on bankruptcies, illiquidity and contagion effects after external disturbances.

The last ingredient of our system is an $N \times N$ matrix $\Omega = [\varphi_{ij}]$ of 'trust' coefficients that indicate the strength of the ties that have been established between banks via repeated contact in the interbank market. We start with a random assignment of 'trust' coefficients within the interval [0,1], and during the simulations increase the trust of bank i into j (φ_{ij}) if j agreed to extend credit to i when needed or decrease φ_{ij} if it declined to provide credit. Our assumption of reinforcement of existing business relationship is motivated by various findings of relationship lending practices between banks as documented, for example, in Cocco et al. (2009), or Finger and Lux (2014).

After setting up the balance sheet structure of the system, without any interbank connections at time $t = 0$, the subsequent simulations proceed in the following way:

1. In each period t, all banks are subject to liquidity shocks as formalized in eq. (4).

2. With these emerging imbalances, banks $i = 1, ..., N$ one after the other consider whether there is a need to balance their liquidity via the interbank market:

    2.1. If $d_{i,t} < 0$, bank i suffers from an outflow of liquidity. If it has sufficient own liquidity, it will simply absorb the liquidity shock through its own funds. This will be the case if the current liquidity position is above a lower threshold $\overline{m}_i$ that guarantees continuance of regular business operations or is simply imposed by authorities as a regulatory measure. If the deposit shock would push its liquidity below $\overline{m}_i$, it will try to make up for this loss by taking up interbank credit of volume $d_{i,t}$. To this end, it will contact other banks starting with the one with highest trust factor φ_{ij}. If this one agrees, i will have satisfied its needs, if not, it will turn to the one with second-highest φ_{ij} and so on. We will assume that the banks contacted will 'normally' agree to provide credit unless their own liquidity position would fall below their lower threshold $\overline{m}_j$. If threatened by a too high loss of its liquidity itself, j then would deny extension of credit to i and would suffer a loss in trust. In the presence of a negative liquidity shock, all previous borrowing in the interbank market will also be prolonged, and additional interbank credit will be sought. In case that no single other bank will be able to satisfy the liquidity needs of i, the amount of credit requested will be split up into different chunks. In particular, the bank with the highest trust factor will provide as much credit as affordable with its own liquidity provision and the remainder will be solicited from the next banks in i's ranking of trustworthiness. Our assumption of an aggregate balancing of all liquidity shocks makes sure that all liquidity needs of single banks can be satisfied in this way.

2.2. If $d_{i,t} > 0$, bank i will pay back existing interbank loans. If the sum of all its interbank liabilities is larger than $d_{i,t}$ the reduction will be proportional to the size of the loan. If it is able to pay back all loans in full, its balance sheet will actually increase by the remaining part of new deposits.

Note that in our interbank credit market, all contracts are borrower-initiated. This is not unrealistic: in the e-MID platform (the only data set that provides evidence on initiation of such trades) about 70 percent of all quotes come from potential borrowers.

Of course, many modifications of our baseline model could be thought of and have actually been investigated as sensitivity checks: one could change the sequence of events choosing, for example, banks with liquidity shocks randomly and one could vary all parameters of the model over relatively wide ranges. We have also assumed a zero interest rate for the time being so that we do not have to account for capital gains and losses through interbank credit. This restriction will be dropped in sec. 4.

3 The Simulated Banking System

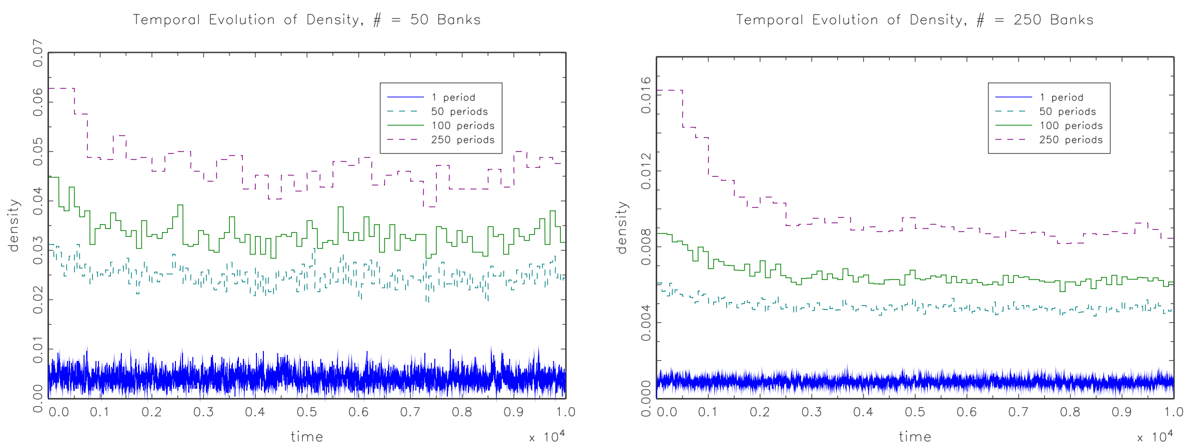
We now scrutinize the characteristics of an evolving banking system following the above rules of the game: initialization with a realistic size distribution and no interbank credit, recurring liquidity shocks over time, and rebalancing of liquidity via the interbank market. Given the conservative properties of the system (overall deposits and liquidity being constant) one would expect the system to display stationary behaviour along certain dimensions after an initial transitory phase. After this initial stage, the system should reach a kind of *statistical equilibrium* with remaining fluctuations of constant amplitude.

In the following we provide illustrations and statistics for Monte Carlo simulations of our artificial banking system with the following settings: The number of banks is chosen to be either $N = 50$ or $N = 250$. Initial balance sheet sizes are determined by draws from a truncated Pareto distribution over the range [5, 200] with power-law parameters from the set $\tau = \{1.2, 1.6, 2, 3\}$ to allow for different levels of heterogeneity (or granularity) of the banking system. Illustrations typically are provided for the case of $\tau = 1.2$ while in the tables we present key statistics for a number of parameter settings and additionally consider the case of a fully homogeneous banking system (initially identical sizes of banks). The fraction of external assets is initially set to $\theta = 0.9$ (hence $1 - \theta = 0.1$ is the ratio of liquidity reserves), and the ratio of equity to balance sheet size is fixed at $\gamma = 0.08$ (hence $1 - \gamma = 0.92$ is the initial ratio of deposits to total liabilities). The lower ratio of liquid reserves at which some action is required on the part of banks to remain solvent is $\bar{m}_i = 0.04A_i$. Finally, eq. (4) is implemented with parameters $\beta = 0.5$ and $\sigma_i = 0.025d_i(t)$ [3]. A large number of alternative scenarios have been tried as well, but without major qualitative changes of the statistical results.

We first depict the resulting *network density* which is defined by the number of existing links over all possible credit links in a fully connected network(which would amount to $N \cdot (N - 1)$). Fig. 2 shows this statistics for two specifications with $N = 50$ and $N = 250$ banks, respectively, and different aggregation periods for extraction of network statistics from

[3] This is in line with the orders of magnitude found in Hester and Pierce (1975).

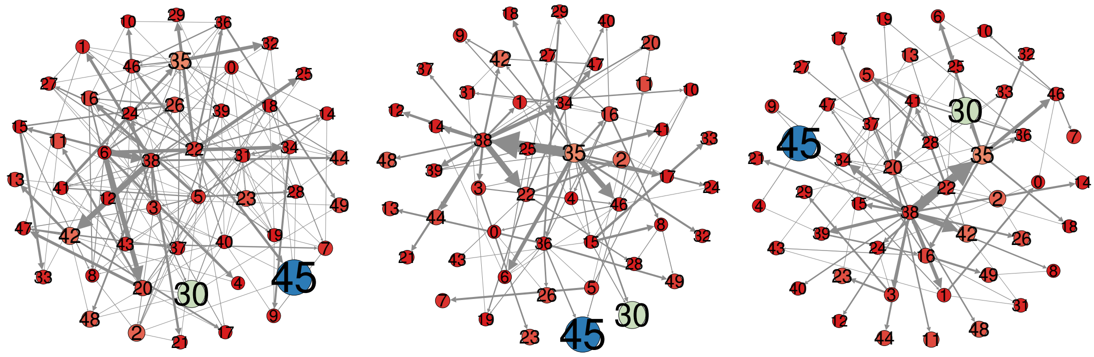
the unfolding development of the interbank market. Aggregating data is necessary, as each unit time period ("day") might only provide a snapshot of a random selection of links that get activated by the liquidity shocks occurring on that day. Similarly as with empirical data (Finger et al., 2013), longer aggregation periods might be preferable to have a better coverage of the network of preferential lending relationships. In line with this consideration, we see a higher density when moving from the "daily" level of activity to the longer aggregation horizons of 50, 100, and 250 periods. The plots speak of a development towards a statistical equilibrium in which either $\sim$ 5 percent ($N = 50$) or $\sim$ 1 percent (N=250) of all possible links will be active over the longest time horizons of 250 days, i.e. we observe a strong concentration of the interbank trading activity within a few of all possible lender-borrower combinations. This agrees with findings of actual interbank data that typically have very sparse adjacency matrices (i.e. with few non-zero entries). The higher variability in the case $N = 50$ is, of course, due to the smaller sample. Note also that the density shows an initial overshooting before converging to its long-run level. From the initially random assignment of trust coefficients, first many different links will be activated when the need arises while the lock-in to more stable relationships leads to a consolidation in later stages of the simulation.



(a) (b)

Figure 2: Temporal development of the density of the interbank network for the baseline scenario ($\tau = 1.2$ and all other parameters as given in the text).

Now, we move to an analysis of the emerging network structure. Fig. 3 shows network plots of the specification with $N = 50$ banks after $t = 100$, 5000, and 10000 rounds of interbank

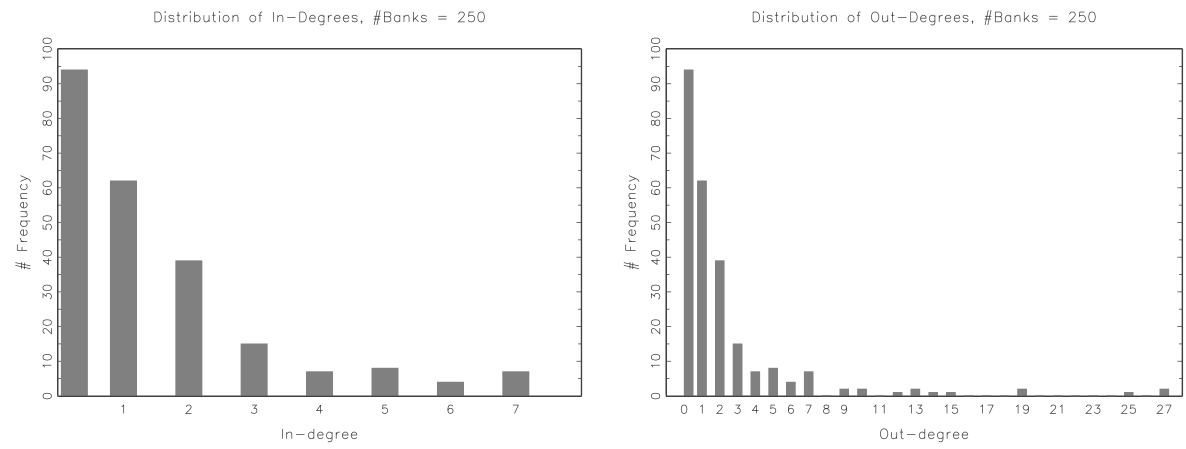


(a) (b) (c)

Figure 3: Network images constructed from interbank loans, snapshots at times $t = 100$(a), $5,000$ (b) and $10,000$ (c). The visualization have been prepared with the open source software Gephi, cf. Bastian et al. (2013).

activity. The size of the nodes reflects their balance sheet size. Similarly, the size of the links between them are proportional to their existing credit volumes in the particular period. As we observe, the network constructed from the interbank credit relations is relatively unstructured at the start of the simulation ($t = 100$), but evolves into a more hierarchical structure in which a few banks have many links while the remaining ones only have few connections. Some banks can easily be spotted as the presumptive ”’core” banks who serve a certain number of other, mostly smaller banks with interbank loans. The core banks will typically be the larger ones although there is no strict one-to-one relationship between size and core membership (statistics are provided below). When moving from $t = 5000$ to $t = 10,000$ we see that while many details might have changed, the general structure is still very much the same which speaks for convergence to a statistical equilibrium.

How can we intuitively explain this emergence of a hierarchical structure in the interbank market? Because of their sheer size, the larger banks have more buffer liquidity and so can serve the needs of a number of smaller banks when these are hit by negative liquidity shocks. Over time, the more frequent availability of large banks as lenders leads to the emergence of preferential lending relationships to a number of smaller client banks, their periphery. More

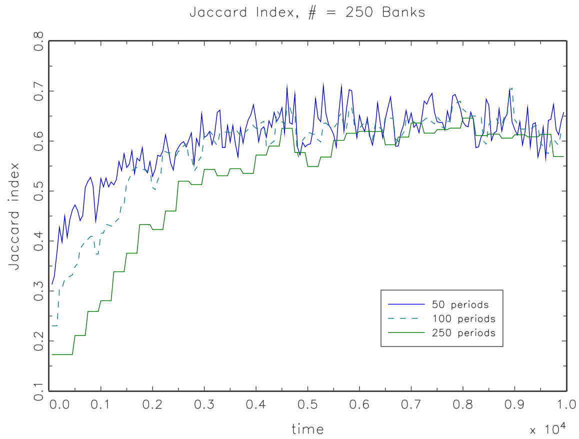


(a) (b)

Figure 4: Distribution of in-degrees and out-degrees across banks. This particular snapshot was taken from the last 250 periods of a simulation running over a total of 10,000 time steps.

and more, smaller banks restrain from trading with banks of the same size class when in need of liquidity since these more often have to turn them down if they suffer from a negative shock as well or, generally have insufficient liquidity due to their history of past shocks. Since also large banks do not have unlimited capacity for liquidity provision, and because of preferences of the smaller ones being driven by historical contingency, the large banks often acquire their own, partially overlapping and occasionally changing periphery.

The following figures and tables show that the simulations also agree with empirical data in more details, and essentially are able to reproduce all those statistical features of the interbank market that have been highlighted in recent literture (cf. Craig and von Peter, 2014, Finger et al., 2013, Anand et al., 2013). Fig. 4 illustrates that there is a pronounced difference in the distribution of incoming links (number of creditors per bank) and out-going links (number of borrowers). While the in-degree distribution has very limited support (in our example from a setting with $N = 250$ banks, the maximum is 7), the out-degree distribution is much broader and also much more skewed: about 50 percent of all banks have zero out-degree, i.e. they do not lend at all in the interbank market while some have up to a maximum of 27 borrowers. While these and the following figures are computed from one realization of the interbank network (typically the network reconstructed from the trading activity over the last 250 time steps of simulation lasting for 10,000 periods), the statistics obtained from large number of Monte Carlo simulations



(a)

Figure 5: Temporal development of the Jaccard index for different aggregation levels over a typical simulation run.

confirm their representativeness. Tables 1 and 2 display means and standard deviations of the previous and some other statistics over 100 simulation runs for different scenarios, respectively. The tables can also be compared to those of Table 1 in Anand et al. (2013), where the authors compare statistics of a static network formation algorithm with those of the empirical data studied by Craig and von Peter (2014). In terms of their qualitative features, the later are also very close to those extracted from the e-MID overnight market (Finger et al., 2013; Fricke et al., 2013). Our Table 1 shows results for a banking system with $N = 50$ banks whereas Table 2 exhibits the same set of results for a larger system with $N = 250$. Statistics are computed for different parameters of the size distribution of banks' balance sheets, eq. (3), namely $\tau = 1.2, 1.6, 2$ and 3 as well as for a banking system with (initially) uniform balance sheet sizes. $\tau = 1.2$ is closest to traditional empirical findings of a Zipf's law of the firm size distribution while for the size distribution of banks available results hover in the range 1 to 3, cf. Bremus et al. (2012). Simulations with $\tau = 1.2$ lead to a very heterogeneous banking system composed of many small and a few large banks whose total assets may be up to 50 times larger than those of the smallest market participants. With larger τ, this heterogeneity gets less and less pronounced, and it is completely absent in the last case of uniform initial balance sheet sizes.[4]

[4]Due to deposit shocks, some heterogeneity also emerges over time in this case, but it remains of very limited magnitude.

Both Tables 1 and 2 show the mean and standard deviation of the average in-degree (which by definition is equal to the average out-degree) as well as the maximum in-degree and out- degree across 100 simulations of each scenario, for aggregation horizons of $50, 100$ and 250 periods. As might be expected, the numbers typically increase slightly with the time horizon used to compute network statistics, but their relatively moderate increase also shows that most links are preserved over time so that higher horizons have a much smaller number of links than what one would obtain from adding up randomly created networks.

Interestingly, the increase of the number of banks from 50 to 250 is not reflected in a similar increase of the average and maximum in-degrees and out-degrees. Rather, these quantities appear to be relatively little affected by the size of the system. Hence, the intrinsic logic of the link formation based on the need for liquidity exchange seems to lead to a certain concentration of links as well as a pronounced asymmetry between incoming and outcoming links that is relatively insensitive to the overall size of the system. A glance at the pertinent statistics for the case of uniform balance sheet size shows, that in this case, the pronounced asymmetry between in-degree distribution and out-degree distribution does not emerge. While one does not find complete symmetry, both the mean and dispersion of outgoing links are much lower than in any of the settings with heterogeneous bank size distribution.

Also shown in the tables is the *Jaccard* index which is a measure of persistence of links over time: It is defined as the number of links that survive from one period to the next divided by all links that are observed in at least one of both periods. The numbers reported in Tables 1 and 2 for this statistics show again little sensitivity with respect to the overall size of the system ($N = 50$ vs. $N = 250$) and the length of the period used to compute network statistics. There is a bit of sensitivity with respect to the power-law index τ of the bank size distribution with variation of the Jaccard index between ~ 0.65 for $\tau = 1.2$ and ~ 0.5 at $\tau = 3.0$. These numbers are in perfect agreement with what Fricke et al. (2013) report for monthly to yearly aggregation levels of the e-MiD overnight banking data. In both cases, at least 50 percent or more of the existing links are maintained on average from one quarter to the next. Note that numbers are smaller for the case of a uniform initial distribution of balance sheet sizes, but still speak of some preferential banking relationship formation even without size-heterogeneity. Fig. 5 provides an illustration of the development of the Jaccard index over the course of one simulation run for the different sampling periods (again for the case $\tau = 1.2$). It clearly indicates that persistent links emerge endogenously from the network formation between banks. In the unstructured setting upon initialization of the system, the Jaccard index is close to zero, but over time bi-lateral ties become stronger and stronger and lead to a sharp increase of the index. When it has reached its "equilibrium" level, only fluctuations around some constant value are observed due to the dynamics of liquidity shocks.

The next set of statistic we display only for the aggregation level of 250 periods (since there is relatively little difference to the case of 50 and 100 periods), but record their realisation at specific points in time of the simulation: at $t = 250$, $t = 5,000$ and $t = 10,000$. The first of these satistics is what is called *assortativity* in network research. Shown in the tables is a simple measure of assortativity: the correlation between the degrees (sum of in-degrees and out-degrees) of the two sides connected by a link. Typical findings for interbank data point to negative assortativity (disassortativity, cf. Finger et al., 2013) indicating that typically at one end we find a well-connected entity while at the other end of the link its partner is an entity with relatively few connections. We again confirm that this feature prevails in our model in the later stages ($t = 5,000$ and $t = 10,000$) when it has settled down at its statistical equilibrium. Initially ($t = 250$), in contrast, this feature is absent so that disassortativity can again be classified as an emerging property of our system after the onset of the dispersed activity of its actors. The numbers obtained at the later periods are again in good agreement with those reported for empirical data.

Since the finding of disassortative link formation motivates application of core-periphery models to interbank data, we next try to identify in our simulations the number of "core" banks on the base of a dichotomic core-periphery distinction. The typical idealized pattern of a discrete core-periphery model assumes that the core should be fully connected while there should be no links between peripherical banks. In the simplest version, an objective function is minimized to identify the core members that penalizes both missing links between core members and existing links between peripherical agents while being indifferent to links between core and periphery (of which there should be at least one per periphery bank to be connected to the network at all). We use the recently proposed fast algorithm by Lip (2011) to identify the number of core members at different times during each simulation. Numbers are again relatively robust for variations of τ and converge in the long run to a core of about 10 percent of all banks for a system size $N = 50$, and about 4 percent for $N = 250$. While there is no clear benchmark for this number (Anand et al., 2013, report a core fraction of 2.5 percent for their sample of 1800 German banks, while Fricke and Lux, 2014, find a stable core of about 25 to 30 percent of the banks trading in e-MID), the constancy over time is in line with these previous findings. Core size is, however, not a distinguishing feature between banking systems with heterogeneous balance sheet structure against those with uniform size distribution as shown by the last block of results in Tables 1 and 2.

Finally, we consider the average degree of dependency of single banks on their most prominent lender and borrower, as well as the relationships between net lending positions and the size of a bank or its centrality in the system. In all heterogeneous banking systems we find a large difference in the dependency on the most important borrower and lender. For example, for 50 banks and $\tau = 1.2$, a bank typically receives 85 percent of its interbank credit from the same counterparty, but its most important borrower accounts for only 20 percent of the credit extended. These numbers change only slightly for less heterogeneous systems (higher τ) and larger system size ($N = 250$), and they are again in very good agreement with empirical numbers (cf. Anand et al., 2013). However, the asymmetry vanishes for the case of uniform balance sheet sizes.

We also find that the degree of centrality of a bank (simply measured by the sum of its in- and out-connections relative to the number of all links of a fully connected system) is highly correlated with size, and also that its net lending position is similarly highly correlated with its size, again in close conformity with empirical data. These correlations are always in the range of 0.8 to 0.9 in heterogeneous banking systems, but vanish completely for the homogeneous banking system. Figs. 6 and 7 provide examples from single simulation runs for the association between size and centrality/net banking position. The later relationship is particularly interesting as it amounts to a pronounced asymmetry of the interbank market emerging out of a completely symmetric setting: While all banks are subject to mean-reverting shocks with mean zero, the smaller banks over time mostly assume the role of net borrowers, while the larger banks become net lenders. The reason is that small banks due to their limited capacity often have no borrowers or only few themselves, while larger ones become core banks and collect around them a periphery of smaller ones whom they provide with credit when needed.

4 The Role of Interest Rates

So far we have abstracted from interest payments on interbank liabilities. While a full-fletched analysis of interest rate formation is outside the scope of the present paper, there is one aspect of the core-periphery model that we would like to have a closer look at. Namely, the formation of a hierarchical structure of the interbank market with some emerging money center banks and peripheral ones that are predominantly borrowers leads to a pronounced asymmetry in net lending positions in the interbank market as illustrated in Fig. 7. It can easily be imagined, that this leads to similarly asymmetric interest rate flows and, in the absence of countervailing effects in other areas of activity, would drain off funds from the small borrower banks to the larger core banks or money market centers. Introducing a constant interest rate, we indeed immediately verify that this particular structure of the interbank market has a strong tendency towards redistribution of capital between banks despite the perfect symmetry of their liquidity shocks. Fig. 8a illustrates this effect by the development of the equity ratios of the 5 largest and the 5 smallest banks of a system with a total of 250 banks. In order not to condemn those banks who are seldomly lenders themselves to rapid extinction, we also introduce interest revenues on liquid assets (which would, for example, be interest on overnight deposits at the central bank). At the same time, we impose (somewhat unrealistically) outflow of funds via dividends with the same rate of return to keep the system conservative in its overall size. If we impose a homogeneous interest rate for interbank loans above the rate obtained for central bank deposits, we nevertheless find a monotonic redistribution of capital from the smaller to the large banks, cf. Fig. 8a, that will eventually end up with the bankruptcy of the smaller ones.

However, there might be a countervailing force. Interest rates agreed on in the interbank market are contract-specific and, among other factors, depend on the strength of the existing

Statistics of the Interbank Market, N=50

alpha	1.2			1.6			2.0			3.0			uniform		
Aggregation level	50	100	250	50	100	250	50	100	250	50	100	250	50	100	250
Density	0.023	0.028	0.035	0.023	0.029	0.037	0.023	0.030	0.040	0.025	0.033	0.045	0.029	0.041	0.059
	(0.002)	(0.003)	(0.004)	(0.002)	(0.003)	(0.004)	(0.002)	(0.003)	(0.005)	(0.002)	(0.003)	(0.005)	(0.002)	(0.003)	(0.003)
Avg. In-degree	1.127	1.396	1.770	1.141	1.427	1.852	1.172	1.477	1.977	1.239	1.631	2.273	1.457	2.036	2.959
	(0.116)	(0.135)	(0.192)	(0.125)	(0.172)	(0.225)	(0.115)	(0.161)	(0.244)	(0.112)	(0.152)	(0.229)	(0.114)	(0.125)	(0.141)
Max. In-degree	2.570	3.120	4.060	2.670	3.230	4.280	2.860	3.290	4.420	3.060	3.580	4.770	3.360	4.010	5.250
	(0.714)	(0.756)	(1.033)	(0.697)	(0.737)	(1.064)	(0.779)	(0.844)	(0.966)	(0.547)	(0.741)	(0.930)	(0.523)	(0.541)	(0.687)
Max. Out-degree	15.740	17.420	19.060	15.980	17.820	19.560	14.970	16.910	18.660	12.410	14.280	16.140	5.340	6.200	7.620
	(4.775)	(4.637)	(4.259)	(6.307)	(5.931)	(5.972)	(6.114)	(5.864)	(6.101)	(3.352)	(3.429)	(3.294)	(0.879)	(1.155)	(1.071)
Jaccard index	0.640	0.665	0.655	0.607	0.643	0.639	0.576	0.616	0.619	0.505	0.564	0.593	0.416	0.497	0.566
	(0.090)	(0.059)	(0.043)	(0.088)	(0.072)	(0.050)	(0.089)	(0.069)	(0.044)	(0.071)	(0.047)	(0.041)	(0.046)	(0.043)	(0.030)
Measured at time	250	5,000	10,000	250	5,000	10,000	250	5,000	10,000	250	5,000	10,000	250	5,000	10,000
Assortativity	-0.096	-0.312	-0.347	-0.079	-0.267	-0.311	-0.073	-0.224	-0.262	-0.063	-0.174	-0.202	-0.040	-0.088	-0.066
	(0.044)	(0.090)	(0.097)	(0.043)	(0.098)	(0.105)	(0.046)	(0.089)	(0.089)	(0.042)	(0.067)	(0.075)	(0.048)	(0.070)	(0.081)
Core size	0.178	0.120	0.117	0.179	0.118	0.119	0.180	0.121	0.119	0.180	0.123	0.124	0.185	0.121	0.120
	(0.011)	(0.013)	(0.014)	(0.012)	(0.014)	(0.015)	(0.011)	(0.012)	(0.013)	(0.008)	(0.009)	(0.010)	(0.010)	(0.004)	(0.003)
Dep. on lender		0.851			0.840			0.819			0.765			0.650	
		(0.042)			(0.048)			(0.050)			(0.047)			(0.024)	
Dep. on borrower		0.196			0.241			0.310			0.431			0.603	
		(0.063)			(0.078)			(0.080)			(0.077)			(0.033)	
Correlation size-centrality		0.916			0.909			0.896			0.860			0.036	
		(0.032)			(0.038)			(0.042)			(0.048)			(0.143)	
Correlation size-net lending		0.869			0.866			0.858			0.816			0.030	
		(0.083)			(0.082)			(0.083)			(0.088)			(0.150)	

Table 1: Note: All statistics are mean values (with standard deviations in parentheses) of 100 Monte Carlo simulations each running over 10,000 time steps of the interbank market dynamics with the pertinent parameter setting. The statistics for the density, in-degree, out-degree and the Jaccard index are extracted at the end of each simulation run for aggregation levels of 50, 100 and 250 periods, respectively, and are averaged over each set of 100 Monte Carlo replications. Statistics for assortativity and core sizes are extracted only for the aggregation level of 250 periods, but are extracted at times 250, 5,000 and 10,000 from each simulation to show the difference between the initial transient dynamics and the fluctuations in statistical equilibrium. The table shows mean and standard deviation of these statistics over the 100 Monte Carlo replications, with the same set of parameters. The last set of statistics (dependence on largest lender and borrower as well as correlations between size and centrality/net lending) are extracted only for 250-period aggregates at the end of each simulation run, and averaged again over simulations.

Statistics of the Interbank Market, N=250

alpha	1.2			1.6			2.0			3.0			uniform		
Aggregation level	50	100	250	50	100	250	50	100	250	50	100	250	50	100	250
Density	0.004	0.005	0.007	0.004	0.006	0.007	0.005	0.006	0.008	0.005	0.007	0.009	0.006	0.008	0.013
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.000)	(0.000)	(0.001)	(0.000)	(0.000)	(0.000)
Avg. In-degree	1.106	1.323	1.717	1.121	1.380	1.819	1.145	1.451	1.980	1.233	1.647	2.372	1.445	2.055	3.162
	(0.047)	(0.058)	(0.093)	(0.052)	(0.074)	(0.116)	(0.058)	(0.089)	(0.139)	(0.059)	(0.097)	(0.139)	(0.052)	(0.057)	(0.092)
Max. In-degree	3.300	4.010	5.870	3.520	4.320	6.430	3.540	4.710	6.980	3.970	4.780	7.180	4.270	5.480	7.950
	(0.704)	(0.969)	(1.390)	(0.674)	(1.043)	(1.328)	(0.744)	(0.967)	(1.279)	(0.745)	(0.883)	(1.184)	(0.617)	(0.785)	(0.869)
Max. Out-degree	27.790	30.800	33.450	30.270	33.980	36.390	31.280	35.310	38.160	26.820	31.460	34.200	6.660	7.950	9.830
	(4.195)	(4.219)	(4.578)	(4.843)	(4.763)	(4.985)	(6.050)	(6.791)	(6.416)	(7.866)	(8.128)	(7.743)	(1.121)	(1.298)	(1.248)
Jaccard index	0.671	0.664	0.620	0.629	0.635	0.584	0.579	0.598	0.551	0.498	0.530	0.489	0.405	0.445	0.401
	(0.041)	(0.032)	(0.029)	(0.048)	(0.035)	(0.030)	(0.041)	(0.037)	(0.028)	(0.031)	(0.030)	(0.024)	(0.020)	(0.019)	(0.013)
Measured at time	250	5,000	10,000	250	5,000	10,000	250	5,000	10,000	250	5,000	10,000	250	5,000	10,000
Assortativity	-0.046	-0.242	-0.254	-0.037	-0.194	-0.207	-0.029	-0.153	-0.166	-0.023	-0.103	-0.111	-0.009	-0.039	-0.040
	(0.022)	(0.047)	(0.051)	(0.022)	(0.048)	(0.050)	(0.019)	(0.038)	(0.040)	(0.019)	(0.031)	(0.032)	(0.022)	(0.036)	(0.038)
Core size	0.052	0.038	0.039	0.052	0.038	0.038	0.052	0.036	0.037	0.052	0.035	0.036	0.051	0.031	0.029
	(0.003)	(0.002)	(0.003)	(0.002)	(0.003)	(0.003)	(0.003)	(0.003)	(0.002)	(0.002)	(0.002)	(0.003)	(0.002)	(0.002)	(0.002)
Dep. on lender		0.852			0.838			0.811			0.745			0.615	
		(0.019)			(0.023)			(0.028)			(0.026)			(0.011)	
Dep. on borrower		0.244			0.316			0.388			0.502			0.580	
		(0.041)			(0.046)			(0.045)			(0.034)			(0.017)	
Correlation size-centrality		0.929			0.919			0.905			0.868			0.012	
		(0.015)			(0.019)			(0.024)			(0.030)			(0.072)	
Correlation size-net lending		0.866			0.880			0.877			0.845			0.020	
		(0.035)			(0.040)			(0.041)			(0.048)			(0.069)	

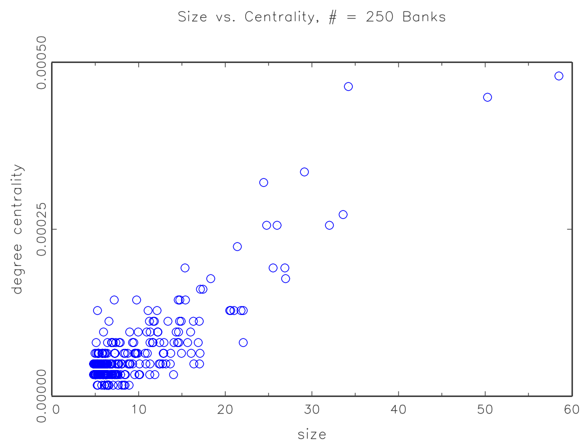
Table 2: Note: All statistics are mean values (with standard deviations in parentheses) of 100 Monte Carlo simulations each running over 10,000 time steps of the interbank market dynamics with the pertinent parameter setting. The statistics for the density, in-degree, out-degree and the Jaccard index are extracted at the end of each simulation run for aggregation levels of 50, 100 and 250 periods, respectively, and are averaged over each set of 100 Monte Carlo replications. Statistics for assortativity and core sizes are extracted only for the aggregation level of 250 periods, but are extracted at times 250, 5,000 and 10,000 from each simulation to show the difference between the initial transient dynamics and the fluctuations in statistical equilibrium. The table shows mean and standard deviation of these statistics over the 100 Monte Carlo replications, with the same set of parameters. The last set of statistics (dependence on largest lender and borrower as well as correlations between size and centrality/net lending) are extracted only for 250-period aggregates at the end of each simulation run, and averaged again over simulations.

lending relationship between both parties, cf. Cocco et al. (2009). Given the empirical evidence on relationship lending it appears reasonable to assume that a strong mutual relationship is rewarded by below-average lending rates. In order to capture this effect, we simply let interest rates be a negative function of the trust coefficients φ_{ij}. Fig. 8b shows the outcome of a simulation with this modification and otherwise identical parameters and initial conditions as with the setting of Fig. 8a. Note also that we designed the negative dependency on φ_{ij} in a way to preserve the same mean value of the interbank lending rates. As it turns out, in this case the initial divergence of equity ratios of large and small banks levels out at some point and the overall system seems to become stationary again. This indicates that the development of preferential lending relationships with below market rates helps to compensate to some extend for the exploitative tendency of the emerging money center topology. While in the initial stage of the simulation of Fig 8b no strong lending relationships have been formed and the dynamics is close to that of Fig. 8a, after the 'warming up phase' of the banking system, the heterogeneity of interbank interest rates allows equity to be preserved even by the peripherical banks.

Hence, besides the role of relationship lending to overcome monitoring and default risk problems at the micro level, it might also play an important role as an automatic stabilizing mechanism at the system level. The present section only points out this tendency. Since we have simply imposed an ad-hoc relationship-dependent rule for the lending rate, no further mechanism exists that would guarantee that the resulting degree of heterogeneity of lending rates is sufficient in the long run to neutralise completely the asymmetric flows of interest payments within the banking system. It would be interesting to replace the present ad hoc rule for relationship-based interest rates by a behavioural analysis along the lines of the bargaining approach put forward by Halaj and Kok (2013) to see whether systematic tendencies towards tolerable levels of interest rates emerge that guarantee survival of the peripherical borrowers.

5 Conclusions

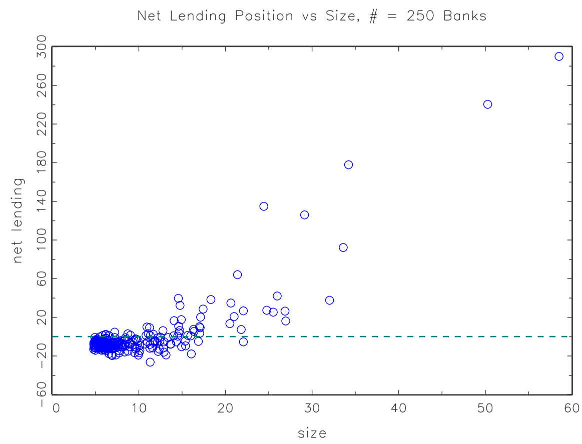
We have formalized an elementary model of interbank loans in a dynamic model where banks have to continuously rebalance their assets and liabilities in the presence of shocks to deposits. Keeping external assets and liabilities constant in the aggregate, we could study the evolution



(a)

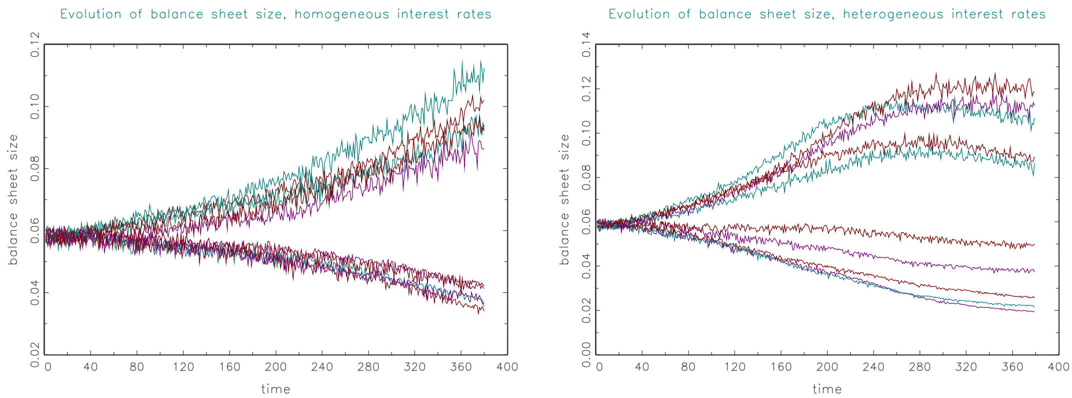
Figure 6: Size vs. centrality, last window of 250 observations of a simulation with a total of 10,000 time steps.

of the interbank model in a stochastically stable environment. We found that for heterogeneous balance sheet sizes, a self -organization of the network of interbank loans towards an asymmetric core-periphery structure unfolds that bears close similarity to established structural features of interbank credit relationships as they transpire from recent literature. Not too much rationality and information is, therefore, needed on the side of liquidity managers to enable the formation of such a hierarchical interbank system. Rather than being formed purposefully, in our model it emerges as a macroscopic profile that is created unintentionally by the dispersed activity under bounded rationality at he micro level. We also find that with a dependency of lending rates on the strength of established relationship between two banks, the asymmetry of interest flows from the peripherical to money center banks can at least to some extend be neutralised via heterogeneous interest rates. While extremely stylized, the model combines a dynamic stochastic environment with permanent shocks to the balance sheet of individual banks with a fully articulated accounting structure for an ensemble of banks with full consistency of all stocks and flows, and could, therefore, serve as a starting point to study a variety of questions via endogeneization of additional balance sheet items and additional links between banks.



(a)

Figure 7: Net lending positions vs size, last window of 250 observations of a simulation with a total of 10,000 time steps.



(a) (b)

Figure 8: Development of equity ratios of the five largest and five smallest banks in a setting with homogeneous (a) and heterogeneous (b) interest rates. The parameters have been choosen so to have sufficient trading activity on the interbank market to observe some discernible effect on equity over computationally affordable simulation runs. In particular, both simulations use the following parameter: $N = 250, \tau = 3, a = 50, b = 1000, \theta = 0.94, \gamma = 0.06, \beta = 0.75$. Interest rates were set at 0.01/100 for liquid assets and 0.015/100 for interbank loans in the homogeneous case, whereas in the heterogeneous case the lending rate for a credit from j to i was assumed to be $(2 - \varphi_{ij})/1000$ leading to interbank interest rates between 0.01/100 and 0.02/100.

References

Anand, K., Craig, B. and von Peter, G. (2013) *Fill in the Blanks: Network Structure and Interbank Contagion*. Working Paper, Bank of Canada.

Axtell, R. L. (2001) Zipf distribution of US firm sizes, *Science*, 293(5536), 1818-1820.

Bastian, M., Heymann, S. and M. Jacomy (2009) *Gephi: An Open Source Software for Exploring and Manipulation Networks*. AAAI Publications. Third international AAAI Conference on Weblogs and Social Media.

Borgatti, S. P. and M. G. Everett (2000) Models of core/periphery structures, *Social Networks*, 21(4), 375–395.

Boss, M., Elsinger, H., Summer, M. and S. Thurner (2004) Network topology of the interbank market, *Quantitative Finance*, 4(6), 677–684.

Bremus, F., C. M. Buch, N. Russ and M. Schnitzer (2012) *Big Banks and Macroeconomic Outcomes: Theory and Cross-Country Evidence of Granularity*. Manuscript, DIW, Berlin.

Castiglionesi, F. and N. Navarro (2011) *Fragile Financial Networks*. Manuscript, Tilburg University.

Cocco, J. F., Gomes, F. J. and N. C. Martins (2009) Lending relationships in the interbank market, *Journal of Financial Intermediation*, 18(1), 24–48.

Craig, B. R. and G. von Peter (2014) Interbank tiering and money center banks. *Journal of Financial Intermediation*, in press.

Ennis, H. (2001) On the size distribution of banks, *Federal Reserve Bank of Richmond Economic Quarterly*, 87(4), 1–25.

Finger, K. and T. Lux (2013) *Friendship between Banks: An Application of the Actor-Oriented Model of Network Formation to Interbank Credit Relations*. Working paper, University of Kiel.

Finger, K., Fricke, D., and Lux, T. (2013) On assortative and disassortative mixing in scale-free networks: The case of interbank credit networks. *Computational Management Science*, 10, 187–211

Finger, K., Fricke, D., and Lux, T. (2013) *On Assortative and Disassortative Mixing in Scale-Free Networks: The Case of Interbank Credit Networks*. Kiel Institute for the World Economy.

Fricke, D. and T. Lux (2014) Core-periphery structure in the overnight money market: Evidence from the e-mid trading platform. *Computational Economics* (in press).

Halaj, G. and C. Kok (2013) *Modelling Emergence of the Interbank Networks.* Working paper, European Central Bank.

Hester, D. D. and Pierce, J. L. (1975) *Bank Management and Portfolio Behavior.* Yale University Press.

Hommes, C., M. van der Leij and D. in 't Veld (2013) *The Formation of a Core-Periphery Network in Interbank Markets.* Manuscript, University of Amsterdam.

Inaoka, H., Ninomiya, T., Taniguchi, K., Shimizu, T. and H. Takayasu (2004) *Fractal Network Derived from Banking Transaction? An Analysis of Network Structures Formed by Financial Institutions.* Bank of Japan Working Papers, 4.

Janicki, H. and E. N. Prescott (2006) Changes in the size distribution of US banks: 1960-2005, *Federal Reserve Bank of Richmond Economic Quarterly*, 92(4), 291–316.

Langfield, S., Z. Liu and T. Ota (2012) *Mapping the UK Interbank System.* Manuscript. UK Financial Sevices Authority.

Lip, S. (2011) *A Fast Algorithm for the Discrete Core-Periphery Bipartitioning Problem.* Working Paper, University of Cambridge.

Reiss, R. D. and M. Thomas (2007) *Statistical Analysis of Extreme Values: With Applications to Insurance, Finance, Hydrology and Other Fields.* Springer.

Shin, H. S. (2009) Securitisation and Financial Stability, *The Economic Journal*, 119(536), 309–332.

Soramäki, K., Bech, M. L., Arnold, J., Glass, R. J. and W. E. Beyeler (2007) The topology of interbank payment flows, *Physica A: Statistical Mechanics and its Applications*, 379(1), 317–333.

van Lelyveld, I. and D. in't Veld (2012) *Finding the Core: Network Structure in Interbank Markets.* Working Paper no. 348, De Nederlandsche Bank, Amsterdam.