Empirical Properties of Inflation Expectations and the Zero Lower Bound*

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Abstract

Recent papers studying survey data on inflation expectations find that households' average inflation expectation responds sluggishly to realized shocks to future inflation and that households have heterogeneous inflation expectations. In models with a zero lower bound on the nominal interest rate currently used for policy analysis, households' inflation expectations are not sluggish and not heterogeneous. Motivated by this tension, this paper solves a New Keynesian model with a zero lower bound and dispersed information on the household side. The model with sluggish and heterogeneous inflation expectations has the following properties: (1) the deflationary spiral in bad states of the world is less severe than under perfect information, (2) central bank communication about the current state of the economy affects consumption and the sign of the consumption effect depends on whether or not the zero lower bound binds, and (3) a commitment by the central bank to increase future inflation can reduce consumption. These effects are stronger in states of the world that have a smaller prior probability.

Keywords: zero bound, business cycles, monetary policy, information frictions. (*JEL*: D83, E31, E32, E52).

1   Introduction

Shortly after U.S. GDP contracted sharply in the fall of 2008 the zero lower bound on the federal funds rate became binding. In the following years, the U.S. Congress passed a major fiscal package and the Federal Reserve used announcements about the future path of the policy rate ("forward guidance") as one of its main policy tools. These policy actions are frequently justified by referring to three common theoretical results in the New Keynesian literature on the zero lower bound: (1) even a small negative shock can have large adverse effects when the zero lower bound binds, (2) a commitment by the central bank to future inflation stimulates current consumption, and (3) the government spending multiplier is larger when the zero lower bound binds.

It is important to recognize that in New Keynesian models with a binding zero lower bound, movements in inflation expectations by households play a crucial role for the amplification of shocks and the effectiveness of policies. Consumption has to satisfy the Euler equation for a nominal asset, and in most models, the economy eventually returns to the original deterministic steady state. Solving the log-linearized Euler equation forward then implies that the deviation of consumption from steady-state consumption is determined by the expected sum of current and future real interest rates and shocks to the Euler equation if the model contains such shocks. Furthermore, when the expected nominal interest rate in period t equals zero, the expected real interest rate in period t simply equals minus the expected inflation rate between periods t and t+1. Hence, when the zero lower bound constraint on monetary policy is expected to bind for an extended period, the amplification of shocks mainly comes from movements in inflation expectations and monetary and fiscal policy mainly have an effect on consumption through movements in inflation expectations.

In the context of New Keynesian models with a binding zero lower bound, it therefore seems particularly desirable to model inflation expectations in a way that is consistent with data. One pervasive feature of survey data on inflation expectations is that inflation expectations respond sluggishly to realized shocks to future inflation (see Coibion and Gorodnichenko (2012)). Another pervasive feature of survey data on inflation expectations is significant heterogeneity in inflation expectations across households (see, e.g., Armantier et al. (2011) for the inflation expectations survey of consumers conducted by the Federal Reserve Bank of New York). In any model with perfect information and rational expectations, all agents have the same expectation of aggregate inflation and this expectation responds instantly and one-for-one to realized shocks to future in-

flation. All agents have the same expectation of aggregate inflation, because all agents have the same information and the same perceived law of motion of the economy. The expectation of future inflation responds instantly and one-for-one to realized shocks to future inflation, because agents know the exact realization of the shock and have correct believes about how the shock affects the conditional mean of future inflation.

Motivated by the importance of inflation expectations in New Keynesian models with a binding zero lower bound and the tension between empirical and model properties of inflation expectations, this paper solves a New Keynesian model with imperfect information on the side of households. The assumption that households have imperfect information yields the sluggish response of households' inflation expectations. The additional assumption that households have different pieces of information yields the heterogeneity in households' inflation expectations.

The main properties of the model with sluggish and heterogeneous inflation expectations are as follows. First of all, the deflationary spiral in bad states of the world is less severe than under perfect information. Second, central bank communication about the current state of the economy (without any change in policy) affects consumption, and the sign of this consumption effect depends on whether or not the zero lower bound binds. Third, a commitment by the central bank to increase future inflation has less desirable effects than under perfect information and can even reduce consumption. These effects are stronger in states of the world that have a smaller prior probability.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 states the optimality conditions for households and firms. Section 4 solves the model under perfect information and under imperfect information. Section 5 presents results on monetary policy. Section 6 concludes.

2   Model

The economy consists of households, firms, and a government. The government in turn consists of a fiscal authority and a monetary authority. The monetary authority controls the nominal interest rate, but cannot lower the net nominal interest rate below zero. In contrast to the existing literature on the zero lower bound, households have imperfect information.

Households. The economy is populated by a continuum of households of mass one. Households are indexed by $i \in [0, 1]$. The preferences of household i are given by

$$E_0^i \left[\sum_{t=0}^{\infty} \beta^t e^{\xi_{i,t}} \left(\frac{C_{i,t}^{1-\gamma} - 1}{1 - \gamma} - L_{i,t} \right) \right],$$

where $C_{i,t}$ and $L_{i,t}$ are consumption and labor supply of household i in period t. The preference parameters satisfy $\beta \in (0, 1)$ and $\gamma > 0$. Here E_0^i is the expectation operator conditioned on information of household i in period zero and the variable $\xi_{i,t}$ is a preference shock.

To facilitate comparison to the pertinent literature (e.g., Eggertsson and Woodford (2003) and Christiano, Eichenbaum, and Rebelo (2011)), I study the response of the economy to a temporary change in households' desire to save. Formally, in period zero each household is hit by a preference shock $\xi_{i,0} \in \{\xi_L, \xi_H\}$ with $\xi_L < \xi_H$. In contrast to the existing literature, I assume that there is heterogeneity across households in the value of the preference shock. Let $1 - \lambda$ denote the mass of households with $\xi_{i,0} = \xi_L$ and let λ denote the mass of households with $\xi_{i,0} = \xi_H$. Moreover, in period zero, there are two possible aggregate exogenous states. The two aggregate states differ in terms of the mass of households who experience the high realization of the preference shock: $\lambda \in \{\lambda_{bad}, \lambda_{good}\}$ with $\lambda_{bad} < \lambda_{good}$. Let $\varphi \in (0, 1)$ denote the prior probability of the good state.

Since I am interested in the effects of imperfect information on the household side about the aggregate state of the economy, I need to introduce at least two possible aggregate states. To ensure that the own preference shock does not perfectly reveal the aggregate state, I assume that each realization of the preference shock is possible in both aggregate states, $0 < \lambda_{bad} < \lambda_{good} < 1$. Finally, note that heterogeneity in preference shocks across households also implies heterogeneity in beliefs across households so long as households use their own local conditions to form beliefs about the aggregate state. I specify the information structure below.

Following the pertinent literature, I consider two forms of persistence of the preference shock. To obtain a closed-form solution of the model, I first assume that the preference shock decays stochastically. Formally, in each period $t \geq 1$, the variable $\xi_{i,t}$ remains constant with probability μ and returns permanently to its normal value of zero with probability $1 - \mu$. The return to the normal value of zero occurs at the same time for all households. Later I assume that the preference shock decays deterministically. Formally, in each period $t \geq 1$, $\xi_{i,t} = \rho^t \xi_{i,0}$ with $\rho \in (0, 1)$.

Households can save or borrow by holding (positive or negative amounts of) nominal government

bonds. Let $B_{i,t}$ denote the bond holdings of household i between periods t and $t+1$. The evolution of the bond holdings of household i between periods t and $t+1$ is given by the household's flow budget constraint

$$B_{i,t} = R_{t-1}B_{i,t-1} + W_{i,t}L_{i,t} + D_{i,t} - P_tC_{i,t} + Z_{i,t},$$

where R_{t-1} denotes the gross nominal interest rate on bond holdings between periods $t-1$ and t, $W_{i,t}$ is the nominal wage rate for labor supplied by household i in period t, and $D_{i,t}$ denotes the difference between dividends received by the household in period t and nominal lump-sum taxes paid by the household in period t. The term $P_tC_{i,t}$ is the household's consumption expenditure, where P_t denotes the price of the final good in period t, and the term $Z_{i,t}$ is a net transfer that is specified below. The household can save or borrow (i.e., bond holdings can be positive or negative). The household cannot run a Ponzi scheme. All households have the same initial bond holdings in period minus one.

For simplicity, I assume that households can trade state-contingent claims with one another in period minus one (i.e., when all households are still identical). Recall that each household is hit by a preference shock in period zero and the preference shocks of all households revert permanently back to zero in the stochastic period $T > 0$. The contingent claims are settled in that period T. A state-contingent claim specifies a payment to the household who purchased the claim that is contingent on the individual history of the household and the aggregate history of the economy (i.e., the claim is contingent on $\xi_{i,0}$, λ and T). The term $Z_{i,t}$ in the flow budget constraint is the net transfer associated with these state-contingent claims. This term equals zero in all periods apart from period T. The fact that agents can trade these state-contingent claims in period minus one implies that in equilibrium all households will have the same post-transfer wealth in period T. This simplifies the analysis because to solve for consumption of each household one does not have to keep track of the dynamics of the wealth distribution in periods $0 \leq t < T$. A similar assumption is made in Lucas (1990), Lorenzoni (2010), and Curdia and Woodford (2011).

Finally, let us turn to the information structure. Each household observes the realization of the own preference shock in period zero. In addition, there is sticky information, as in Mankiw and Reis (2002, 2006). In each period $0 \leq t < T$, a constant fraction $\omega \in [0, 1]$ of randomly selected households update their information sets. Households who have updated their information sets since period zero know the aggregate state perfectly. Households who have not updated their information

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sets since period zero form beliefs about the aggregate state based on $\xi_{i,0}$. The parameter ω controls the speed of information diffusion in society. When $\omega = 1$ all households have perfect information in every period. When $\omega = 0$ all households form beliefs about the aggregate state based on their own preference shock alone. When $\omega \in (0, 1)$, information about the exact size of the aggregate shock in period zero diffuses slowly in society. Finally, I assume that all households learn the aggregate history perfectly in period T. As usual, all households know the current date t and there is common knowledge about the structure of the economy.

Firms. There are final good firms and intermediate good firms. To isolate the effect of imperfect information on the household side, I assume that firms have perfect information. The final good is produced by competitive firms using the technology

$$Y_t = \left(\int_0^1 Y_{j,t}^{\frac{\psi-1}{\psi}} dj \right)^{\frac{\psi}{\psi-1}}.$$

Here Y_t denotes output of the final good and $Y_{j,t}$ denotes input of intermediate good j. The parameter $\psi > 1$ is the elasticity of substitution between intermediate goods. Final good firms have fully flexible prices. Profit maximization of firms producing the final good implies the following demand function for intermediate good j

$$Y_{j,t} = \left(\frac{P_{j,t}}{P_t} \right)^{-\psi} Y_t,$$

where $P_{j,t}$ is the price of intermediate good j and P_t is the price of the final good. The zero profit condition of final good firms implies

$$P_t = \left(\int_0^1 P_{j,t}^{1-\psi} dj \right)^{\frac{1}{1-\psi}}.$$

The intermediate good j is produced by a monopolist using the technology

$$Y_{j,t} = L_{j,t}^{\varrho},$$

with

$$L_{j,t} = \left(\int_0^1 L_{i,j,t}^{\frac{\eta-1}{\eta}} di \right)^{\frac{\eta}{\eta-1}}.$$

Here $Y_{j,t}$ is output, $L_{j,t}$ is composite labor input, and $L_{i,j,t}$ is type i labor input of this monopolist. The parameter $\varrho \in (0, 1]$ is the elasticity of output with respect to composite labor and the parameter $\eta > 1$ is the elasticity of substitution between different types of labor. Cost minimization of

the monopolist implies that the input of type i labor in period t equals

$$L_{i,j,t} = \left(\frac{W_{i,t}}{W_t}\right)^{-\eta} L_{j,t},$$

where

$$W_t = \left(\int_0^1 W_{i,t}^{1-\eta} di\right)^{\frac{1}{1-\eta}}.$$

Furthermore, the wage bill of firm j in period t equals $W_t L_{j,t}$ at the cost-minimizing labor mix. Monopolists producing intermediate goods are subject to a price-setting friction, as in Calvo (1983). Each monopolist can optimize its price with probability $1-\alpha$ in any given period. With probability α the monopolist producing good j sets the price

$$P_{j,t} = P_{j,t-1}.$$

How firms value profit in different states of the world is determined by the ownership structure. I assume that each monopolist is owned by a single household and takes the household's marginal utility of consumption as given, because the household also owns many other firms.[1]

Monetary policy. The monetary authority sets the gross nominal interest rate according to the rule

$$R_t = \max\left\{1, R\Pi_t^\phi\right\}.$$

Here $R = (1/\beta)$ is the nominal interest rate in the non-stochastic steady state with zero inflation, $\Pi_t = (P_t/P_{t-1})$ denotes the inflation rate, and $\phi > 1$ is a parameter. The monetary authority follows a Taylor rule as long as the Taylor rule implies a non-negative net nominal interest rate and the monetary authority sets the net nominal interest rate to zero otherwise.

Fiscal policy. The fiscal authority can purchase units of the final good and finance the purchases with current or future lump-sum taxes. The government flow budget constraint in period t reads

$$T_t + B_t = R_{t-1}B_{t-1} + P_t G_t.$$

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[1] To ensure that households are ex-ante identical in period minus one, ownership is assigned randomly in period zero. The individual history of a household in period T then consists of the realization of the preference shock and the realization of ownership in period zero. Since the state-contingent claims specify a payment that is contingent on the individual history, in equilibrium all households have the same post-transfer wealth in period T.

The government has to finance maturing nominal government bonds and any purchases of the final good, denoted G_t. The government can collect lump-sum taxes, denoted T_t, or issue new bonds. For most of the paper, I assume $G_t = 0$ in every period.

3 Household and firm optimality

This section states equations such as the household and firm optimality conditions and the New Keynesian Phillips curve, which are then used in the following section to derive the equilibrium of the model.

Households. The first-order conditions for consumption and the real wage rate read

$$C_{i,t}^{-\gamma} = E_t^i \left[\beta \frac{e^{\xi_{i,t+1}}}{e^{\xi_{i,t}}} \frac{R_t}{\Pi_{t+1}} C_{i,t+1}^{-\gamma} \right],$$

and

$$\tilde{W}_{i,t} = \frac{\eta}{\eta - 1} C_{i,t}^{\gamma},$$

where $\tilde{W}_{i,t} = (W_{i,t}/P_t)$ denotes the real wage rate for type i labor. For the moment, I assume that each household chooses consumption and sets a real wage rate to isolate the implications of imperfect information coming from the consumption Euler equation. When each household chooses consumption and sets a nominal wage rate, both the consumption Euler equation and the wage setting equation differ from their perfect-information versions, which amplifies the effects presented in the following section. I will discuss this point in more detail in the conclusion.

Let small letters denote log-deviations from the non-stochastic steady state with zero inflation. Log-linearizing the last two equations around the non-stochastic steady state yields

$$c_{i,t} = E_t^i \left[-\frac{1}{\gamma} \left(\xi_{i,t+1} - \xi_{i,t} + r_t - \pi_{t+1} \right) + c_{i,t+1} \right], \tag{1}$$

and

$$\tilde{w}_{i,t} = \gamma c_{i,t}. \tag{2}$$

Furthermore, when households set nominal wage rates, the wage setting equation reads

$$w_{i,t} = \gamma c_{i,t} + E_t^i \left[p_t \right]. \tag{3}$$

Firms. An intermediate good firm j that can adjust its price in period t and is owned by household i sets the price

$$X_{j,t}^i = \underset{P_{j,t} \in \mathbb{R}_{++}}{\arg\max} E_t \left[\sum_{s=t}^{\infty} (\alpha\beta)^{s-t} \left(\frac{e^{\xi_{i,s}} C_{i,s}^{-\gamma} P_t}{e^{\xi_{i,t}} C_{i,t}^{-\gamma} P_s} \right) \left(P_{j,t} \left(\frac{P_{j,t}}{P_s} \right)^{-\psi} Y_s - W_s \left(\left(\frac{P_{j,t}}{P_s} \right)^{-\psi} Y_s \right)^{\frac{1}{\varrho}} \right) \right].$$

Log-linearizing the first-order condition for the adjustment price around the non-stochastic steady state with zero inflation yields

$$x_{j,t}^i = (1 - \alpha\beta) E_t \left[\sum_{s=t}^{\infty} (\alpha\beta)^{s-t} \left(p_s + \frac{1}{1 + \frac{1-\varrho}{\varrho}\psi} (w_s - p_s) + \frac{\frac{1-\varrho}{\varrho}}{1 + \frac{1-\varrho}{\varrho}\psi} y_s \right) \right].$$

Note that the log-linearized adjustment price is independent of who owns the firm and is the same for all adjusting firms. Therefore, one can drop the superscript i and the subscript j. Furthermore, the last equation can be stated in recursive form as

$$x_t = (1 - \alpha\beta) \left(p_t + \frac{1}{1 + \frac{1-\varrho}{\varrho}\psi} (w_t - p_t) + \frac{\frac{1-\varrho}{\varrho}}{1 + \frac{1-\varrho}{\varrho}\psi} y_t \right) + \alpha\beta E_t [x_{t+1}].$$

Log-linearizing the equation for the price of the final good given in Section 2 and using the fact that adjusting firms are selected randomly and the log-linearized adjustment price is the same for all firms yields

$$p_t = \int_0^1 p_{j,t} dj = \alpha p_{t-1} + (1 - \alpha) x_t.$$

Using the last equation to substitute for the adjustment prices x_t and x_{t+1} in the previous equation and rearranging yields

$$\pi_t = \frac{(1 - \alpha)(1 - \alpha\beta)}{\alpha} \left(\frac{1}{1 + \frac{1-\varrho}{\varrho}\psi} (w_t - p_t) + \frac{\frac{1-\varrho}{\varrho}}{1 + \frac{1-\varrho}{\varrho}\psi} y_t \right) + \beta E_t [\pi_{t+1}]. \tag{4}$$

Finally, log-linearizing the equation for the wage index presented in Section 2 yields

$$w_t = \int_0^1 w_{i,t} di.$$

Substituting the log-linearized wage index and the wage setting equation (2) into equation (4) and using $y_t = c_t$ yields the standard New Keynesian Phillips curve

$$\pi_t = \frac{(1 - \alpha)(1 - \alpha\beta)}{\alpha} \frac{\gamma + \frac{1-\varrho}{\varrho}}{1 + \frac{1-\varrho}{\varrho}\psi} c_t + \beta E_t [\pi_{t+1}], \tag{5}$$

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where c_t denotes aggregate consumption of the final good. Using instead the wage setting equation (3) yields a modified version of the New Keynesian Phillips curve

$$\pi_t = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\left(\frac{\gamma + \frac{1-\varrho}{\varrho}}{1 + \frac{1-\varrho}{\varrho}\psi}c_t + \frac{1}{1 + \frac{1-\varrho}{\varrho}\psi}\left(\bar{E}_t\left[p_t\right] - p_t\right)\right) + \beta E_t\left[\pi_{t+1}\right], \tag{6}$$

where $\bar{E}_t\left[p_t\right] = \int_0^1 E_t^i\left[p_t\right]di$ denotes the households' average expectation of the price level. In the following, let κ denote the coefficient on consumption in the New Keynesian Phillips curve (5).

4 Equilibrium

4.1 Perfect information

This subsection solves the model under perfect information to obtain a benchmark for comparison and to illustrate that movements in inflation expectations by households play a crucial role for the propagation of shocks.

Perfect information is a special case of the model, because when $\omega = 1$ all households learn the exact size of the aggregate shock already in period zero. Following the pertinent literature (e.g., Eggertsson and Woodford (2003), Christiano, Eichenbaum, and Rebelo (2011)), I consider equilibria with the following two properties: (i) consumption, inflation, and the nominal interest rate are constant from period zero until the preference shocks revert permanently back to zero, and (ii) the economy is in the non-stochastic steady state with zero inflation thereafter (i.e., $c_{i,t} = c_t = \pi_t = r_t = 0$ for all $t \geq T$).

It is an equilibrium that all households have the same consumption level in period T because all households have the same post-transfer wealth in period T. The reason is the trade in state-contingent claims in period minus one. Furthermore, it is straightforward to verify that $c_{i,t} = c_t = \pi_t = r_t = 0$ for all $t \geq T$ satisfies the consumption Euler equation (1) with $\xi_{i,t} = \xi_{i,t+1} = 0$, the New Keynesian Phillips curve (5), and the monetary policy rule presented in Section 2.

Before solving for consumption, inflation, and the nominal interest rate in periods $0 \leq t \leq T-1$, let us simplify notation. Since these variables are constant over time but depend on the aggregate state, I replace the subscript t by the subscript $s \in \{bad, good\}$. Furthermore, since all variables in the consumption Euler equation remain constant with probability μ and revert to the steady-state

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value with probability $1 - \mu$ and since all households have perfect information, the consumption Euler equation (1) reduces to

$$c_{i,s} = -\frac{1}{\gamma} \left[(\mu - 1) \xi_{i,0} + r_s - \mu \pi_s \right] + \mu c_{i,s}.$$

Integrating across households yields aggregate consumption of the final good in state s

$$c_s = -\frac{1}{\gamma} \left[(\mu - 1) \bar{\xi}_s + r_s - \mu \pi_s \right] + \mu c_s, \qquad (7)$$

where $\bar{\xi}_s = (1 - \lambda_s) \xi_L + \lambda_s \xi_H$ denotes the cross-sectional mean of the preference shock in state s. Moreover, the New Keynesian Phillips curve (5) reduces to

$$\pi_s = \kappa c_s + \beta \mu \pi_s. \qquad (8)$$

The monetary policy rule reads

$$r_s = \max \left\{ -\ln (R), \phi \pi_s \right\}. \qquad (9)$$

If monetary policy is not constrained by the zero lower bound (i.e., $\max \left\{ -\ln (R), \phi \pi_s \right\} = \phi \pi_s$), substituting equations (8)-(9) into equation (7) and rearranging yields the following expression for aggregate consumption in state s

$$c_s = \frac{\frac{1}{\gamma} \bar{\xi}_s}{1 + \frac{\frac{1}{\gamma} (\phi - \mu)}{1 - \mu} \frac{\kappa}{1 - \beta \mu}}. \qquad (10)$$

By contrast, if monetary policy is constrained by the zero lower bound (i.e., $\max \left\{ -\ln (R), \phi \pi_s \right\} = -\ln (R)$), substituting equations (8)-(9) into equation (7) and rearranging yields[2]

$$c_s = \frac{\frac{1}{\gamma} \bar{\xi}_s + \frac{\frac{1}{\gamma}}{1 - \mu} \ln (R)}{1 - \frac{\frac{1}{\gamma} \mu}{1 - \mu} \frac{\kappa}{1 - \beta \mu}}. \qquad (11)$$

Finally, the zero lower bound is not binding in state s if $\bar{\xi}_s \geq \bar{\xi}_{crit}$, whereas the zero lower bound is binding in state s if $\bar{\xi}_s < \bar{\xi}_{crit}$. Here[3]

$$\bar{\xi}_{crit} = -\frac{1 + \frac{\frac{1}{\gamma} (\phi - \mu)}{1 - \mu} \frac{\kappa}{1 - \beta \mu}}{\frac{\frac{1}{\gamma} \phi}{1 - \mu} \frac{\kappa}{1 - \beta \mu}} \frac{1}{1 - \mu} \ln (R). \qquad (12)$$

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[2] Following common practice in the literature on the zero lower bound, I assume that parameters are such that the denominator in equation (11) is positive. See, e.g., Woodford (2011), Section IV.A.

[3] Substituting equations (8) and (10) into $\phi \pi_s = -\ln (R)$ and solving for $\bar{\xi}_s$ yields equation (12). In addition, substituting equations (8) and (11) into $\phi \pi_s = -\ln (R)$ and solving for $\bar{\xi}_s$ also yields equation (12).

An important insight in the existing literature on the zero lower bound is that if the zero lower bound is binding, the drop in consumption can be arbitrarily large. Formally, a positive denominator on the right-hand side of (11) in combination with the condition $\bar{\xi}_s < \bar{\xi}_{crit}$ implies a negative numerator on the right-hand side of (11). Furthermore, the denominator on the right-hand side of (11) is a difference between two positive numbers that can be arbitrarily small in absolute value.

To understand why the fall in consumption can be so large, I propose the following decomposition. The aggregated consumption Euler equation (7) can be written as

$$c_s = \frac{1}{\gamma}\bar{\xi}_s - \frac{\frac{1}{\gamma}}{1-\mu}r_s + \frac{\frac{1}{\gamma}}{1-\mu}\mu\pi_s.$$

Aggregate consumption in state s equals the sum of three terms: The first term is the direct effect of the preference shock on consumption, the second term is the effect of the nominal interest rate on consumption, and the third term is the effect of expected inflation on consumption. Substituting in the equilibrium nominal interest rate when the zero lower bound is binding (i.e., $r_s = -\ln(R)$) and equilibrium inflation when the zero lower bound is binding yields

$$c_s = \frac{1}{\gamma}\bar{\xi}_s + \frac{\frac{1}{\gamma}}{1-\mu}\ln(R) + \underbrace{\frac{\frac{1}{\gamma}}{1-\mu}\mu\frac{\kappa}{1-\beta\mu}\frac{\frac{1}{\gamma}\bar{\xi}_s + \frac{\frac{1}{\gamma}}{1-\mu}\ln(R)}{1 - \frac{\frac{1}{\gamma}\mu}{1-\mu}\frac{\kappa}{1-\beta\mu}}}_{\text{expected inflation}}.$$

The first term is negative and simply reflects the direct effect of the average preference shock on aggregate consumption. The second term is positive because the monetary authority can lower the nominal interest rate to some extent relative to its steady-state value. The third term is negative and reflects the indirect effect of the average preference shock on aggregate consumption coming from movements in inflation expectations.

The reason why the fall in consumption can be arbitrarily large for a given size of the shock is the third term. The model of this subsection predicts that the fall in consumption can be arbitrarily large for a given size of the shock, because the drop in inflation expectations can be arbitrarily large for a given size of the shock. All the amplification of the shock comes from movements in inflation expectations. How we model inflation expectations therefore seems crucial for results concerning dynamics at the zero lower bound.

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4.2 Imperfect information

To match the empirical findings that households' average inflation expectation responds sluggishly to realized shocks to future inflation and that households' inflation expectations are heterogeneous, let us introduce imperfect information on the household side (i.e., $\omega \in [0, 1)$). To understand the implications of sluggish and heterogeneous household inflation expectations as clearly as possible, consider first the special case of the model where households learn about the aggregate state only from their own preference shock (i.e., $\omega = 0$). This version of the model can be solved analytically and is useful for developing intuition. The main results carry over to the case of slow information diffusion about the aggregate state (i.e., $\omega \in (0, 1)$), which is treated in the next subsection.

I again consider equilibria of the following form: consumption, inflation, and the nominal interest rate are constant over time in periods $0 \leq t \leq T-1$ and the economy is in the non-stochastic steady state with zero inflation in periods $t \geq T$.

It is straightforward to solve for aggregate consumption of the final good in periods $0 \leq t \leq T-1$ in state $s \in \{bad, good\}$. The consumption Euler equation (1) can be written as

$$c_i = -\frac{1}{\gamma} \left[(\mu - 1) \xi_{i,0} + E^i \left[r_S - \mu \pi_S \right] \right] + \mu c_i,$$

where $E^i \left[r_S - \mu \pi_S \right]$ is household i's expectation of the real interest rate. Integrating across households yields aggregate consumption in state s

$$c_s = -\frac{1}{\gamma} \left[(\mu - 1) \bar{\xi}_s + \bar{E}_s \left[r_S - \mu \pi_S \right] \right] + \mu c_s, \tag{13}$$

where $\bar{\xi}_s = (1 - \lambda_s) \xi_L + \lambda_s \xi_H$ is the cross-sectional mean of the preference shock in state s and $\bar{E}_s \left[r_S - \mu \pi_S \right]$ denotes the average expectation of the real interest rate in state s. The latter equals

$$\bar{E}_s \left[r_S - \mu \pi_S \right] = \bar{p}_s^{good} \left(r_{good} - \mu \pi_{good} \right) + \bar{p}_s^{bad} \left(r_{bad} - \mu \pi_{bad} \right), \tag{14}$$

where $\bar{p}_s^{good}$ denotes the average probability that households assign to being in the good state when the economy is actually in state s, and $\bar{p}_s^{bad}$ denotes the average probability that households assign to being in the bad state when the economy is actually in state s. Formally,

$$\bar{p}_s^{good} = (1 - \lambda_s) p_L^{good} + \lambda_s p_H^{good}, \quad \bar{p}_s^{bad} = 1 - \bar{p}_s^{good}.$$

Here p_H^{good} denotes the probability that a high type assigns to the good state and p_L^{good} denotes the probability that a low type assigns to the good state. Of course, under perfect information,

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$\bar{p}_{bad}^{good} = \bar{p}_{good}^{bad} = 0$, i.e., households assign zero probability to the wrong state. However, under imperfect information, households assign some probability to the wrong state.

The New Keynesian Phillips curve and the monetary policy rule are again given by equations (8) and (9). An important difference to the case of perfect information is that the outcome in the bad state depends on the outcome in the good state, because the average expectation of the real interest rate in the bad state depends on the real interest rate in the good state. See equation (14). For this reason, one cannot solve the model state by state and one has to distinguish three cases: (i) the zero lower bound binds in both states, (ii) the zero lower bound binds in no state, and (iii) the zero lower bound binds only in the bad state.

First, consider the case where the zero lower bound binds in both states. Stating equation (13) for the good state and the bad state and substituting in equation (14), equation (8), and $r_{good} = r_{bad} = -\ln(R)$ yields a system of two equations in the two unknowns c_{good} and c_{bad}. The solution is

$$c_{good} = \frac{\frac{1}{\gamma}\bar{\xi}_{good} + \frac{\frac{1}{\gamma}}{1-\mu}\ln(R)}{1 - \frac{\frac{1}{\gamma}}{1-\mu}\frac{\mu\kappa}{1-\beta\mu}} - \bar{p}_{good}^{bad}\frac{\frac{\frac{1}{\gamma}}{1-\mu}\frac{\mu\kappa}{1-\beta\mu}}{1 - \frac{\frac{1}{\gamma}}{1-\mu}\frac{\mu\kappa}{1-\beta\mu}}\left(c_{good} - c_{bad}\right),$$
(15)

and

$$c_{bad} = \frac{\frac{1}{\gamma}\bar{\xi}_{bad} + \frac{\frac{1}{\gamma}}{1-\mu}\ln(R)}{1 - \frac{\frac{1}{\gamma}}{1-\mu}\frac{\mu\kappa}{1-\beta\mu}} + \bar{p}_{bad}^{good}\frac{\frac{\frac{1}{\gamma}}{1-\mu}\frac{\mu\kappa}{1-\beta\mu}}{1 - \frac{\frac{1}{\gamma}}{1-\mu}\frac{\mu\kappa}{1-\beta\mu}}\left(c_{good} - c_{bad}\right),$$
(16)

where

$$c_{good} - c_{bad} = \frac{\frac{\frac{1}{\gamma}\left(\bar{\xi}_{good}-\bar{\xi}_{bad}\right)}{1-\frac{\frac{1}{\gamma}}{1-\mu}\frac{\mu\kappa}{1-\beta\mu}}}{1 + \left(\bar{p}_{good}^{bad} + \bar{p}_{bad}^{good}\right)\frac{\frac{\frac{1}{\gamma}}{1-\mu}\frac{\mu\kappa}{1-\beta\mu}}{1-\frac{\frac{1}{\gamma}}{1-\mu}\frac{\mu\kappa}{1-\beta\mu}}} > 0.$$
(17)

When the zero lower bound binds in both states, imperfect information on the household side increases consumption in the bad state. To see this, note that the first term on the right-hand side of equation (16) equals consumption in the bad state under perfect information and the average probability that households assign to the good state when they are in the bad state is positive. In the bad state, households now assign positive probability to a state with a higher inflation rate and thus a lower real interest rate. This reduces the fall in consumption in the bad state. The sluggish response of household inflation expectations keeps consumption high in the bad state.

Second, let us turn to the case where the zero lower bound binds in no state. Stating equation (13) for the good state and the bad state and substituting in equation (14), equation (8), and

$r_s = \phi\pi_s$ in both states yields a system of two equations in the two unknowns c_{good} and c_{bad}. The solution is

$$c_{good} = \frac{\frac{1}{\gamma}\bar{\xi}_{good}}{1 + \frac{1}{\gamma}\frac{1}{1-\mu}\frac{(\phi-\mu)\kappa}{1-\beta\mu}} + \bar{p}_{good}^{bad}\frac{\frac{1}{\gamma}\frac{1}{1-\mu}\frac{(\phi-\mu)\kappa}{1-\beta\mu}}{1 + \frac{1}{\gamma}\frac{1}{1-\mu}\frac{(\phi-\mu)\kappa}{1-\beta\mu}}\left(c_{good} - c_{bad}\right), \tag{18}$$

and

$$c_{bad} = \frac{\frac{1}{\gamma}\bar{\xi}_{bad}}{1 + \frac{1}{\gamma}\frac{1}{1-\mu}\frac{(\phi-\mu)\kappa}{1-\beta\mu}} - \bar{p}_{bad}^{good}\frac{\frac{1}{\gamma}\frac{1}{1-\mu}\frac{(\phi-\mu)\kappa}{1-\beta\mu}}{1 + \frac{1}{\gamma}\frac{1}{1-\mu}\frac{(\phi-\mu)\kappa}{1-\beta\mu}}\left(c_{good} - c_{bad}\right), \tag{19}$$

where

$$c_{good} - c_{bad} = \frac{\frac{\frac{1}{\gamma}\left(\bar{\xi}_{good} - \bar{\xi}_{bad}\right)}{1 + \frac{1}{\gamma}\frac{1}{1-\mu}\frac{(\phi-\mu)\kappa}{1-\beta\mu}}}{1 - \left(\bar{p}_{good}^{bad} + \bar{p}_{bad}^{good}\right)\frac{\frac{1}{\gamma}\frac{1}{1-\mu}\frac{(\phi-\mu)\kappa}{1-\beta\mu}}{1 + \frac{1}{\gamma}\frac{1}{1-\mu}\frac{(\phi-\mu)\kappa}{1-\beta\mu}}} > 0. \tag{20}$$

When the zero lower bound binds in no state, imperfect information on the household side reduces consumption in the bad state. The first term on the right-hand side of equation (19) equals consumption in the bad state under perfect information and $\bar{p}_{bad}^{good} > 0$ due to imperfect information. In the bad state, households again assign positive probability to a state with a higher inflation rate, and the Taylor principle implies that a higher inflation rate is associated with a higher real interest rate. Households believe that the central bank might still be fighting high inflation. As a result, the sluggish response of inflation expectations amplifies the fall in consumption in the bad state.

The sign and the magnitude of the effect of household dispersed information on consumption depend on whether or not the zero lower bound is binding. The sign depends on whether or not the zero lower bound is binding, because movements in household inflation expectations are *destabilizing* when the zero lower bound is binding, whereas movements in household inflation expectations are *stabilizing* when the Taylor principle is satisfied. The magnitude depends on whether or not the zero lower bound is binding, because consumption choices of different households are *strategic complements* when the zero lower bound is binding, whereas consumption choices of different households are *strategic substitutes* when the Taylor principle is satisfied. It is well known from the literature on dispersed information that dispersed information has larger effects in games of strategic complementarity than in games of strategic substitutability.

Third, consider the case where the zero lower bound binds only in the bad state. Stating equation (13) for the two states and substituting in equations (14), (8), $r_{good} = \phi\pi_{good}$, and $r_{bad} =$

$-\ln(R)$ yields a system of two equations in the two unknowns c_{good} and c_{bad}. The solution can be stated as

$$c_{good} = \frac{\frac{1}{\gamma}\bar{\xi}_{good}}{1 + \frac{1}{\gamma}\frac{(\phi-\mu)\kappa}{1-\mu}} + \bar{p}_{good}^{bad}\frac{\frac{1}{\gamma}}{1 + \frac{1}{\gamma}\frac{(\phi-\mu)\kappa}{1-\mu}} \left[(r_{good} - \mu\pi_{good}) - (r_{bad} - \mu\pi_{bad})\right], \qquad (21)$$

and

$$c_{bad} = \frac{\frac{1}{\gamma}\bar{\xi}_{bad} + \frac{\frac{1}{\gamma}}{1-\mu}\ln(R)}{1 - \frac{1}{\gamma}\frac{\mu\kappa}{1-\mu}\frac{1}{1-\beta\mu}} - \bar{p}_{bad}^{good}\frac{\frac{1}{\gamma}}{1 - \frac{1}{\gamma}\frac{\mu\kappa}{1-\mu}\frac{1}{1-\beta\mu}} \left[(r_{good} - \mu\pi_{good}) - (r_{bad} - \mu\pi_{bad})\right], \qquad (22)$$

with

$$(r_{good} - \mu\pi_{good}) - (r_{bad} - \mu\pi_{bad}) = \frac{\frac{(\phi-\mu)\kappa}{1-\beta\mu}\frac{\frac{1}{\gamma}\bar{\xi}_{good}}{1+\frac{1}{\gamma}\frac{(\phi-\mu)\kappa}{1-\mu}\frac{1}{1-\beta\mu}} - \left[-\ln(R) - \frac{\mu\kappa}{1-\beta\mu}\frac{\frac{1}{\gamma}\bar{\xi}_{bad}+\frac{\frac{1}{\gamma}}{1-\mu}\ln(R)}{1-\frac{1}{\gamma}\frac{\mu\kappa}{1-\mu}\frac{1}{1-\beta\mu}}\right]}{1 - \bar{p}_{good}^{bad}\frac{\frac{1}{\gamma}\frac{(\phi-\mu)\kappa}{1-\mu}\frac{1}{1-\beta\mu}}{1+\frac{1}{\gamma}\frac{(\phi-\mu)\kappa}{1-\mu}\frac{1}{1-\beta\mu}} - \bar{p}_{bad}^{good}\frac{-\frac{1}{\gamma}\frac{\mu\kappa}{1-\mu}\frac{1}{1-\beta\mu}}{1-\frac{1}{\gamma}\frac{\mu\kappa}{1-\mu}\frac{1}{1-\beta\mu}}}. \qquad (23)$$

The numerator on the right-hand side of equation (23) equals the real interest rate in the good state under perfect information minus the real interest rate in the bad state under perfect information (the term in square brackets). The denominator on the right-hand side of (23) is positive. Hence, when the numerator is positive, the real rate is higher in the good state than in the bad state, and imperfect information on the household side reduces consumption in the bad state. By contrast, when the numerator is negative, the real rate is lower in the good state than in the bad state, and imperfect information on the household side increases consumption in the bad state. Finally, when the perfect-information real rate is the same in the two states, imperfect information on the household side has no effect on consumption.[4]

When the zero lower bound binds only in the bad state under perfect information, the difference between the perfect-information real interest rate in the good state and the perfect-information real interest rate in the bad state can be positive, zero, or negative. The reason is the following. The kink in the monetary policy rule (9) implies that the real interest rate in state s is a non-monotonic function of inflation in state s. Furthermore, inflation in state s is a monotonic function of consumption in state s and consumption in state s is larger in the good state than in the bad state.

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[4]In this special case, the real interest rate is the same in the two states, but consumption differs across states and inflation differs across states. Hence, households' average inflation expectation is too high in the bad state and too low in the good state, and households have heterogeneous inflation expectations.

One interesting aspect of the case where the zero lower bound binds only in the bad state is that now individual consumption of the final good depends on several aspects of the conditional CDF of inflation. Individual consumption of the final good depends on the conditional mean of inflation as well as the conditional probability assigned to the bad state times a linear function of inflation in the bad state. To see this, substitute the monetary policy rule (9) into the individual consumption Euler equation above equation (13). As a result, aggregate consumption of the final good depends on the average expectation of inflation and the average probability assigned to the bad state times a linear function of inflation in the bad state. Nevertheless, one can solve the model analytically. See equations (21)-(23).

The analytical solution in this subsection invites comparative static exercises. I would like to emphasize one exercise that I find useful to think about recent events. Suppose the good state is a shock that would cause a severe recession under perfect information, while the bad state is a shock that would cause the worst recession in a hundred years under perfect information. Since the bad state is so unusual, agents assign a small prior probability to it (i.e., $1 - \theta$ is very small). Furthermore, suppose the zero lower bound binds in both states. Since agents know that they are either in the good state or in the bad state, agents know they are in a severe recession and the zero lower bound binds, but agents do not know the exact size of the shock. I believe this simple example captures some features of the U.S. economy in December 2008.

The probability that a high type assigns to the good state by Bayes' law equals

$$p_H^{good} = \frac{\lambda_{good}\theta}{\lambda_{good}\theta + \lambda_{bad}(1 - \theta)},$$

while the probability that a low type assigns to the good state by Bayes' law equals

$$p_L^{good} = \frac{(1 - \lambda_{good})\theta}{(1 - \lambda_{good})\theta + (1 - \lambda_{bad})(1 - \theta)}.$$

The average probability assigned to the good state when the economy is in the bad state equals

$$\bar{p}_{bad}^{good} = (1 - \lambda_{bad})p_L^{good} + \lambda_{bad}p_H^{good}.$$

The average probability assigned to the bad state when the economy is in the good state equals

$$\bar{p}_{good}^{bad} = (1 - \lambda_{good})p_L^{bad} + \lambda_{good}p_H^{bad}.$$

Of course, $\bar{p}_{bad}^{good}$ is an increasing function of the prior probability of the good state, θ, while $\bar{p}_{good}^{bad}$ is a decreasing function of the prior probability of the good state, θ. Hence, when the bad state has a small prior probability, household imperfect information has a large positive effect on consumption in the bad state (equation (16)), but only a small negative effect on consumption in the good state (equation (15)).

4.3 Imperfect information with information diffusion

The model can also be solved when there is slow information diffusion about the aggregate state (i.e., $\omega \in (0,1)$). As before, all households observe their own preference shocks in period zero and learn what the aggregate state has been in period T. In addition, in every period a constant fraction $\omega \in (0,1)$ of randomly selected households updates their information to perfect information.

The equations characterizing equilibrium can be written as a linear difference equation

$$A_t x_t = b + B x_{t+1},\tag{24}$$

where

$$x_t = \begin{pmatrix} c_{good,t}^{hi} \\ c_{bad,t}^{hi} \\ c_{good,t}^{li} \\ c_{bad,t}^{li} \\ c_t^{hu} \\ c_t^{lu} \\ c_{good,t} \\ c_{bad,t} \\ \pi_{good,t} \\ \pi_{bad,t} \end{pmatrix}.\tag{25}$$

There are four types of households in any given period $0 \leq t < T$, because a household has had a high or a low preference shock in period zero and has perfect or imperfect information in period t. In the following, a household with a high preference shock in period zero and perfect information in period t is called a "high, informed" household in period t, a household with a high preference shock in period zero and imperfect information in period t is called a "high, uninformed" household in period t, and so on. The first and second equation in (24) are the consumption Euler equation

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of a "high, informed" household in the good state and the bad state, respectively. The third and fourth equation in (24) are the consumption Euler equation of a "low, informed" household in the good state and the bad state, respectively. The fifth and six equation in (24) are the consumption Euler equation of a "high, uninformed" household and a "low, uninformed" household, respectively. The next two equations characterize aggregate consumption in the good state and the bad state, respectively, by aggregating across households. Since the fraction of households with perfect information changes over time, the coefficients in these two equations are a function of t. For this reason, the matrix A_t is a function of t. Finally, the last two equations in (24) are the New Keynesian Phillips curve in the good state and the bad state, respectively.

One period corresponds to one quarter. I assume a long-run annual real interest rate of 4% and set $\beta = 0.99$. The intertemporal elasticity of substitution is $(1/\gamma) = 1$ and the elasticity of output with respect to labor is $\varrho = (2/3)$. These are the most common values in the business cycle literature. The elasticity of substitution between intermediate goods is $\psi = 10$, which implies a long-run markup of 11%, a common target in the New Keynesian literature. The probability that a firm cannot adjust its price in a given quarter is $\alpha = 0.66$, implying that one third of prices change per quarter, a value consistent with micro evidence on prices once sales prices have been removed. See Nakamura and Steinsson (2008). For these parameters, the slope of the New Keynesian Phillips curve is $\kappa = 0.045$. I set $\phi = 1.5$, which is the most standard value for the coefficient on inflation in a Taylor rule.

Coibion and Gorodnichenko (2012) estimate impulse responses of inflation and inflation expectations to shocks. Under the null of perfect-information rational expectations, inflation expectations should adjust to a realized shock by the same amount as the conditional mean of future inflation. The assumption of perfect information implies that the shock is in the information set of the agents. The assumption of rational expectations implies that agents understand how the shock affects future inflation. By contrast, Coibion and Gorodnichenko (2012) find that the responses of inflation expectations to shocks are dampened and delayed relative to the responses of inflation to shocks. That is, after an inflationary shock, inflation expectations rise by less than future inflation and this difference becomes smaller over time and eventually converges to zero. This result is obtained for all four types of shocks they consider (technology shocks, news shocks, oil shocks, unidentified shocks),

inflationary and disinflationary shocks, and different types of agents.[5] As a next step, Coibion and Gorodnichenko (2012) estimate the degree of information rigidity that matches the empirical speed of response of inflation expectations to shocks. In the context of a sticky information model, the estimated degree of information rigidity corresponds to the fraction of agents that do not update their information sets in a given period.[6] On page 143 they write: "This procedure yields estimates between 0.86 and 0.89 for technology, news, and oil price shocks as well as for unidentified shocks." Based on these estimates, I set $\omega = 1 - 0.875$.[7]

Let us turn to the preference shock parameters. I set the persistence of the preference shocks to $\mu = 0.8$, which is a common value in the New Keynesian literature on the zero lower bound. I set $\xi_H = -0.05$ and $\xi_L = -0.075$, which implies that the shock term $(1 - \mu)\xi_{i,0}$ in the consumption Euler equation equals -1% for a high type and -1.5% for a low type. I set the fraction of high types in the good state to $\lambda_{good} = (3/4)$ and the fraction of high types in the bad state to $\lambda_{bad} = (1/4)$. The shock term $(1 - \mu)\bar{\xi}_s$ in the aggregated Euler equation thus equals -1.125% in the good state and -1.375% in the bad state. Under perfect information, the zero lower bound is marginally binding in the good state and clearly binding in the bad state (for comparison, $(1 - \mu)\bar{\xi}_{crit} = -1.09\%$). Furthermore, under perfect information, consumption drops by 4% in the good state and by 13% in the bad state. Hence, I think of the good state as a shock that would create a serious recession under perfect information, while I think of the bad state as a shock that would create the worst recession since World War II under perfect information. Finally, I set $\theta = 0.9$. That is, the prior probability of the bad state equals 10%. This seems a reasonable value given that recessions with

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[5] For professional forecasters, the inflation forecasts are from the Survey of Professional Forecasters and the time sample is 1976-2007. For households, the inflation forecasts are from the Michigan Survey of Consumers and the time sample is 1976-2007. For each group, Coibion and Gorodnichenko (2012) study the response of the average forecast.

[6] Coibion and Gorodnichenko (2012) also consider noisy signal models. In the baseline noisy signal model, in every period each agent observes a noisy signal about the current inflation rate with i.i.d. noise. In the context of the baseline noisy signal model, the estimated degree of information rigidity corresponds to the weight on the prior that agents carry over from the previous period. A particular variance of noise yields this value of the weight.

[7] These estimates are for professional forecasters. Coibion and Gorodnichenko (2012) obtain slightly lower estimates of the degree of information rigidity for households (see their Table 4). [On the other hand, ... Explain the effect of agents learning from their own discount factor shock.]

a 13 percent fall in consumption are rare.
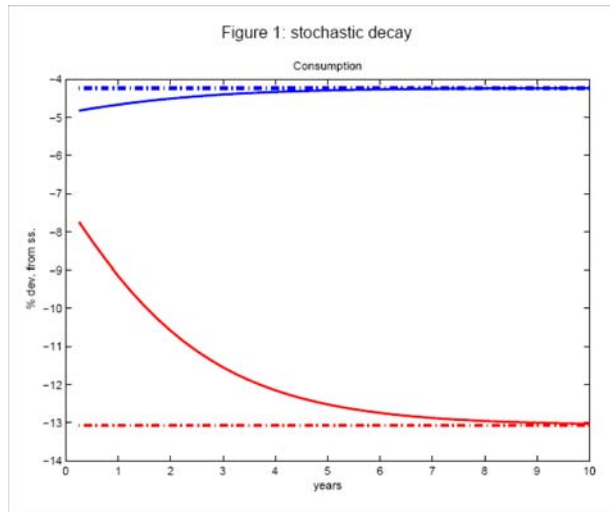

Figure 1: stochastic decay

Figure 1 shows the solution of the model for these parameter values. For these parameter values, the zero lower bound binds in both states under perfect information and also under slow information diffusion. The dashed lines in Figure 1 show equilibrium aggregate consumption under perfect information in the good state and in the bad state. The solid lines show equilibrium aggregate consumption in the case of slow information diffusion. The sluggish response of households' inflation expectations keeps consumption high in the bad state. Communicating the aggregate state perfectly to households (without any change in current or future policy) would reduce aggregate consumption in the bad state because it would lower the average inflation expectation of households in the bad state.

4.4 Deterministic decay

Finally, the model can also be solved with deterministic decay instead of stochastic decay. In the system of equations (24) one simply has to replace $\xi_{i,0}$ by $\rho^t \xi_{i,0}$ in the six period t consumption Euler equations, set μ close to one, and make a guess concerning the number of periods for which the zero lower bound binds in the good state and in the bad state. If the guess turns out to be incorrect, the guess is updated until a fixed point is reached.

Figure 2 presents a numerical example. [Add parameter values.] The red lines are aggregate consumption and the nominal interest rate in the case of slow information diffusion and deterministic decay. Initially consumption in the bad state is almost flat over time, because learning by households

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about the bad aggregate state and deterministic decay of the bad aggregate state are two forces working in opposite direction. The sluggish response of inflation expectations keeps consumption high in the bad state.
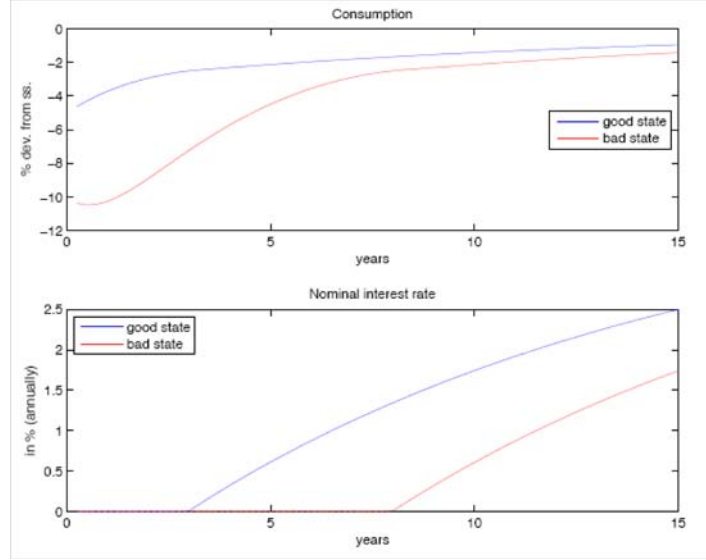
5 Monetary policy

5.1 Central bank communication about the current state

In this model, central bank communication about the current state of the economy can affect consumption, because the current state of the economy is not common knowledge. It turns out that the sign of the effect of this communication on consumption depends on whether or not the zero lower bound binds.

I model central bank communication as an increase in $\bar{p}_{bad}^{bad}$ and $\bar{p}_{good}^{good}$ (i.e., in both states, households assign a higher probability to the correct state because of central bank communication.) In particular, I assume that a constant fraction $\zeta \in (0,1)$ of randomly selected households receive perfect information in period zero in both states. The corresponding reductions in $\bar{p}_{bad}^{good} = 1 - \bar{p}_{bad}^{bad}$ and $\bar{p}_{good}^{bad} = 1 - \bar{p}_{good}^{good}$ have the following effect on aggregate consumption.[8] When the zero lower bound binds in both states, consumption in the good state increases and consumption in the bad state falls (equations (15)-(17)). When the zero lower bound binds in no state, these effects are

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[8]The original probabilities $\bar{p}_{bad}^{good}$ and $\bar{p}_{good}^{bad}$ are multiplied by a factor of $(1 - \zeta)$.

reversed (equations (18)-(20)). Finally, when the zero lower bound binds only in the bad state, the sign of the effect of central bank communication about the current state of the economy on consumption depends on the sign of the numerator in equation (23). As explained in the previous section, the sign of the effect of household imperfect information on consumption depends on whether or not the zero lower bound binds. As a result, the sign of the effect of reducing this information friction depends on whether or not the zero lower bound binds.

5.2 Central bank communication about future policy

Next consider forward guidance. Suppose that the zero lower bound binds in both states and the economy is in the bad state (i.e., households know that the zero lower bound binds, but households do not know the exact size of the aggregate shock.) The central bank decides to commit to a higher inflation target $\bar{\pi} > 0$ for periods $t \geq T$. The central bank communicates this policy to households and explains why it has chosen this policy.

If the communication does not reach households, consumption in the bad state still equals

$$
c_{bad} = \frac{\frac{1}{\gamma}\bar{\xi}_{bad} + \frac{\frac{1}{\gamma}}{1-\mu}\ln\left(R\right)}{1 - \frac{\frac{1}{\gamma}}{1-\mu}\frac{\mu\kappa}{1-\beta\mu}} + \bar{p}_{bad}^{good}\frac{\frac{\frac{1}{\gamma}}{1-\mu}\frac{\mu\kappa}{1-\beta\mu}}{1 - \frac{\frac{1}{\gamma}}{1-\mu}\frac{\mu\kappa}{1-\beta\mu}}\left(c_{good} - c_{bad}\right),
$$

where $c_{good} - c_{bad}$ is given by equation (17). Since households' inflation expectations do not change, households' consumption choices do not change. If the communication does reach households and households understand that the economy is in the bad state (i.e., $\bar{p}_{bad}^{good} = 0$), consumption in the bad state equals

$$
c_{bad} = \frac{\frac{1}{\gamma}\bar{\xi}_{bad} + \frac{\frac{1}{\gamma}}{1-\mu}\ln\left(R\right) + \frac{\frac{1}{\gamma}}{1-\beta\mu}\bar{\pi} + \bar{c}}{1 - \frac{\frac{1}{\gamma}}{1-\mu}\frac{\mu\kappa}{1-\beta\mu}},
$$

where $\bar{c} \geq 0$ denotes consumption in the non-stochastic steady state with inflation rate $\bar{\pi} > 0$. Consumption in the first case is larger than consumption in the second case if and only if

$$
\mu\bar{p}_{bad}^{good}\frac{\frac{1}{\gamma}\kappa}{1-\beta\mu}\frac{\frac{1}{\gamma}\left(\bar{\xi}_{good} - \bar{\xi}_{bad}\right)}{1 - \left(1 - \bar{p}_{good}^{bad} - \bar{p}_{bad}^{good}\right)\frac{\frac{1}{\gamma}}{1-\mu}\frac{\mu\kappa}{1-\beta\mu}} > (1-\mu)\left(\frac{\frac{1}{\gamma}}{1-\beta\mu}\bar{\pi} + \bar{c}\right).
$$

Hence, a commitment by the central bank to increase future inflation can reduce or increase current consumption in the bad state. The reason is that household inflation expectations depend on both beliefs about the current state of the economy and beliefs about inflation after the recession ends.

Finally, if the commitment to a higher inflation target does not reveal any information about the current state of the economy (e.g., because the commitment has always been in place or is considered a good idea in general), then consumption in the bad state equals

$$c_{bad} = \frac{\frac{1}{\gamma}\bar{\xi}_{bad} + \frac{\frac{1}{\gamma}}{1-\mu}\ln(R) + \frac{\frac{1}{\gamma}}{1-\beta\mu}\bar{\pi} + \bar{c}}{1 - \frac{\frac{1}{\gamma}}{1-\mu}\frac{\mu\kappa}{1-\beta\mu}} + \bar{p}_{bad}^{good}\frac{\frac{\frac{1}{\gamma}}{1-\mu}\frac{\mu\kappa}{1-\beta\mu}}{1 - \frac{\frac{1}{\gamma}}{1-\mu}\frac{\mu\kappa}{1-\beta\mu}}\left(c_{good} - c_{bad}\right),$$

where $c_{good} - c_{bad}$ is given by equation (17). This consumption level in the bad state is the highest consumption level among the three cases. Maybe this outcome can be achieved through careful communication by the central bank.

6   Conclusion

Motivated by the importance of inflation expectations in New Keynesian models with a binding zero lower bound and the tension between empirical and model properties of inflation expectations, this paper solves a New Keynesian model with imperfect information on the side of households. The assumption that households have imperfect information yields the sluggish response of inflation expectations to realized shocks to future inflation (Coibion and Gorodnichenko (2012)). The additional assumption that households have different pieces of information yields the heterogeneity in inflation expectations across households (Armantier et al. (2011)).

The main properties of the model with sticky and heterogeneous inflation expectations are as follows. First, the deflationary spiral in bad states of the world is less severe than under perfect information. Second, communication by the central bank about the current state of the economy (without any change in current or future policy) affects consumption, and the sign of this effect depends on whether or not the zero lower bound binds. Third, a commitment by the central bank to increase future inflation has a smaller effect than under perfect information and can even reduce current consumption. These effects are stronger in states of the world that have a smaller prior probability. Finally, the results suggest that careful communication by the central bank could potentially minimize the negative effects of policy announcements on current consumption.

Throughout the paper, I have assumed that households set real wage rates. When households set nominal wage rates instead, nominal marginal costs in the bad state of the world are higher because households' expectations of the price level are too high. I conjecture that this further

reduces the fall in inflation and the fall in consumption in the bad state of the world.

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