The Financial Transmission of Shocks in a Simple Hybrid Macroeconomic Agent Based Model*

Tiziana Assenza[a,b] Domenico Delli Gatti[a,c]

[a] *Complexity Lab in Economics (CLE), Department of Economics and Finance, Università Cattolica del Sacro Cuore, Milano,*

[b] *Amsterdam School of Economics, University of Amsterdam, CeNDEF,*

[c] *CESifo Group Munich,Germany*

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Abstract

Employing the methodology described in Assenza and Delli Gatti 2013 (AD2013 hereafter), in the present paper we build a macro multi-agent model described by a IS schedule, a Taylor Rule (TR) and a Phillips curve (AS curve). At the micro level we consider a corporate sector populated by heterogeneous firms in terms of financial conditions (net worth) that decide investment. Hence, aggregate investment is a function of the interest rate augmented by an aggregate or average *External Finance Premium (EFP)*, which in turn is a function of the moments of the distribution of firms' net worth. The moments of the distribution of firms' net worth, therefore, are (predetermined) state variables of the aggregate variables. Moreover, individual net worth is affected by the interest rate: the higher the interest rate, the lower realized profits and the lower net worth. A two-way feed back between the macroeconomic and the agent based model is at work. We want to study how do macroeconomic shocks propagates is such an economy. For each shock we will determine the change generated on output gap assuming an unchanged distribution of the net worth (*first round effect*) and the change in output gap due to a change in the distribution of net worth induced by the aggregate shock (*second round effect*). Therefore, the net effect on output gap will depend on the sign and the magnitude of these two effects.

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*Tiziana Assenza: e-mail: tiziana.assenza@unicatt.it; address: Università Cattolica del Sacro Cuore, Largo Gemelli, 1, 20123 Milano, Italy; phone number: +390272342483.
Domenico Delli Gatti: e-mail: domenico.delligatti@unicatt.it; address: Università Cattolica del Sacro Cuore, Largo Gemelli, 1, 20123 Milano, Italy; phone number: +390272342499.

1 Introduction

In a macroeconomic agent-based framework, an aggregate variable such as GDP is determined "from the bottom up" i.e., summing the output of a large number of heterogeneous firms. In other words GDP is a function of the entire *distribution* of agents' characteristics. The dynamic pattern of GDP, therefore, is an *emergent property* of the model which is determined by the complex interactions of myriads of heterogeneous actors. For example, in the presence of a financial friction, investment and output at the individual level may be constrained by net worth. Hence GDP will be a function, in the end, of the distribution of the firms' net worth. In this setting, thinking in macroeconomic terms – i.e. in terms of interrelated changes of aggregate variables – is *prima facie* impossible. When an aggregate shock occurs, it is extremely difficult to trace the transmission mechanism. In order to understand how a shock works its way through the web of micro interactions and affects macro variables, we have to rely on stories which may or may not be convincing.

In a previous paper (AD2013) we have proposed a methodology to deal with this issue which consists in building a *Hybrid Macroeconomic ABM*. By adopting an appropriate aggregation procedure – which we label the *Modified-Representative Agent* – we approximate the distribution of agents' characteristics by means of (at least) the (first and second) moments of the distribution. The moments of the distribution play the role of macroeconomic variables and therefore can be incorporated in a macroeconomic model.[1] In AD2013 we adopted this procedure in a IS-LM macroeconomic framework.

In this paper we build a multi-agent model of the corporate sector along the lines of AD2013 where firms characterized by heterogeneous financial conditions (net worth) decide investment. By applying the procedure mentioned above we determine an aggregate investment equation whose argument is the interest rate augmented by an aggregate or average *External Finance Premium (EFP)*, which in turn is affected by the moments of the distribution of firms' net worth. In the following we show, in fact, that the aggregate EFP is decreasing with the cross-sectional mean of the net worth distribution and increasing with the cross sectional variance. We use this investment equation in a macroeconomic framework which can be described by a IS schedule, a Taylor Rule (TR) and a Phillips curve. The latter provides the Aggregate Supply (AS) of the model. The moments of the distribution of firms' net worth, therefore, enter as (predetermined) state variables in the reduced form of the IS-AS-TR model. In each period, say t, given the moments of the distribution in period $t-1$, we can determine the equilibrium levels of the output gap, the inflation rate and the interest rate.

The distribution of net worth, however, is not constant. In order to determine how the distribution changes over time, we go back to the multi-agent model. We define the

[1] The procedure has already been used. See Agliari et al. (2000). It is thoroughly discussed and compared with other aggregation procedures in Gallegati et al. (2006) where it is labelled the Variant-Representative-Agent methodology.

law of motion of individual net worth, which turns out to be affected by the interest rate: the higher the interest rate, the lower realized profits and the lower net worth. The ABM boils down to a system of non-linear difference equations (one for each firm) which allow to trace the evolution over time of each and every element of the distribution of net worth. From the artificial data obtained through simulations we retrieve the evolution over time of the mean and the variance of the distribution of net worth which will impact on future endogenous variables. In a nutshell there is a two-way feed back between the macroeconomic and the agent based submodels: the equilibrium interest rate in t, which is affected by the moments of the distribution in t-1, will impact on the moments of the distribution in t, which will reverberate on the equilibrium interest rate in t+1 and so on...Changes over time of the moments drive the evolution of the equilibrium interest rate, the output gap and inflation.

We use the model to provide an answer to the following question: how do macroeconomic shocks reverberate in the macroeconomy? For each shock, we will provide a breakdown of the associated change of the output gap which can be represented as follows:

$$\begin{aligned}\Delta x &= \Delta x_{1st} + \Delta x_{2nd} \\ \Delta x_{2nd} &= \Delta x_{2nd,RA} + \Delta x_{2nd,HA}\end{aligned}$$

The expression Δx_{1st} is the direct or *first round* effect i.e., the change in x generated by the shock assuming that the distribution of net worth does not change.

In our setting, however, the distribution does change. There is also an indirect or *second round* effect represented by Δx_{2nd} that captures the change of the output gap due to the change in the distribution of net worth generated by the shock. The second round effect is determined by the effect of the shock on the average EFP. The indirect effect is due to a *financial transmission mechanism*, because it is entirely due to the change of net worth, a measure of financial robustness. It can be broken down, in turn, into two components: a Representative Agent (RA) component $\Delta x_{2nd,RA}$ and a Heterogeneous Agents (HA) component $\Delta x_{2nd,HA}$. The former is the indirect change in output which would occur if the individual EFP coincided with the average EFP while the latter incorporates also the effect of changes in the variance of the distribution

Given the chosen parameterization, we are able to quantify these effects. We consider three (permanent) shocks: (i) an expansionary fiscal shock (increase of Government expenditure); (ii) a monetary shock (increase of the exogenous component of the interest rate); (iii) a financial shock (increase of the exogenous component of the individual EFP).

In the following we anticipate the main results.

- In all the cases considered, the first round effect explains most of the actual change of the output gap.
- The second round effect is unambiguously negative both in the case of an expan-

sionary fiscal policy and in the case of a contractionary monetary policy. In both cases, in fact, the cross-sectional mean of the distribution of net worth goes down and the cross-sectional variance goes up: The first and second moments of the distribution are negatively correlated. Hence the EFP goes up. This is largely due to the consequences of the increase of the interest rate on the distribution.

- The second round effect is negative also in the case of a contractionary financial shock. In this case, however, both the cross-sectional mean and the variance go up: The moments are positively correlated. Hence the EFP goes up because the increase of the variance offsets the effects of the increase of the mean.
- The second round effect amplifies the effect of the monetary shock and the financial shock and mitigates the effect of the fiscal shock. In the latter case, in fact, the financial transmission mechanism contributes to crowding out.
- In the case of the fiscal and monetary shock, the HA component has the same sign of the RA component and explains 40% of the second round effect.
- In the case of the financial shock, the RA component is positive and the HA component is negative. The latter is much bigger in absolute value than the former. Therefore, it explains entirely the second round effect.

Of course the size of these effects is due to the particular configuration of parameters and to the modelling choices we adopted. Let's remind that there is only one source of heterogeneity in this model, i.e. the heterogeneity of firms' financial conditions.

The paper is organized as follows. Section 2 is devoted to the derivation of optimal investment on the part of financially heterogeneous firms. Section 3 presents a simpe IS-TR model which serves as an introduction to the complete model. Most of the mechanisms at work can be presented at this relatively simple level of abstraction. In section 4 we analyze the IS-AS-TR model. In section 5 we derive and discuss the dynamics generated by the model and the output of simulations. Sections 6, 7 and 8 are devoted to the analysis of the effects of the fiscal shock, the monetary shock and the financial shock respectively. Section 9 concludes.

2 Firms

Firms, indexed by $i = 1, 2, .., F$, produce a homogeneous final good by means of capital and labor in a competitive setting. They are heterogeneous with respect to their financial robustness captured by *equity* or *net worth* A_{it}. The distribution of the individual net worth changes over time. We will denote the mean and variance of the distribution of firms' equity in t with $A_t = E(A_{it})$ and $V_t = E(A_{it} - A_t)^2$ respectively. We will refer to this variables as the *cross-sectional* mean and variance.[2] Since the equity of each firm is endogenously determined by means of an individual law of motion[3], also the cross sectional mean and variance are time-varying. In the "long run" they settle at values A^* and V^*, the average over a certain time span of the cross sectional mean and variance, which we will interpret as steady states of the aggregate laws of motion of the cross sectional mean and variance. We will refer to A^* and V^* as the long run (cross-sectional) mean and variance.[4]

Firms rely on bank loans to finance investment. Therefore, they run the risk of bankruptcy. Banks extend credit to firms at an interest rate which takes this risk into account. They charge to each firm a specific interest rate which includes an *external finance premium* (Bernanke-Gertler, 1989, 1990) which is decreasing with individual net worth.

Each firm carries on production by means of a Leontief technology that uses labor and capital. The production function of the i-th firm is $Y_{it} = \min(\lambda N_{it}, \nu K_{it})$ where Y_{it}, N_{it} and K_{it} represent output, employment and capital, ν and λ are positive parameters which measure the productivity of capital and labour respectively.

The profit of the i-th firm in real terms in the current period ϕ_{it} is the difference between revenues $u_{it}Y_{it}$ and total costs, which consist of production costs $wN_{it} + r_{it}K_{it}$ and adjustment costs $\frac{1}{2}K_{it}^2$:

$$\phi_{it} = u_{i,t}Y_{it} - wN_{it} - r_{it}K_{it} - \frac{1}{2}K_{it}^2 \tag{2.1}$$

The firm faces an idiosyncratic shock u_{it} to revenue due, for instance, to a sudden change in preferences. u_{it} is a random variable distributed as a uniform over the interval $(0, 2)$ with expected value $E(u_{it}) = 1$. For simplicity we assume that the real wage w is given and constant. r_{it} is the real interest rate charged to firm i. In the following we will refer to this variable also as the individual cost of capital. Assuming 100% depreciation K_{it} is capital and investment. Adjustment costs are quadratic in investment (as usual in investment theory).

Assuming that labour is always abundant, we can write $Y_{it} = \nu K_{it}$ and $N_{it} = \dfrac{\nu}{\lambda}K_{it}$. Substituting these expressions into (2.1) and re-arranging we get:

$$\phi_{it} = \left(u_{it}\nu - w\frac{\nu}{\lambda} - r_{it}\right)K_{it} - \frac{1}{2}K_{it}^2 \tag{2.2}$$

[2]The net worth of the representative agent coincides with the cross sectional mean when the variance is zero. In other words the representative agent is the zero-variance average agent.

[3]See section 5.

[4]In symbols, the cross sectional mean in period t is $A_t = \frac{\sum_{i=1}^{F} A_{it}}{F}$ while the long run mean (over the time span which goes from t_0 to t_1) is $A^* = \frac{\sum_{t=t_0}^{t_1} A_t}{t_1 - t_0}$. Analogously, the cross sectional variance in t is $V_t = \frac{\sum_{i=1}^{F}(A_{it} - A_t)^2}{F}$ while the long run variance is $V^* = \frac{\sum_{t=t_0}^{t_1} V_t}{t_1 - t_0}$. The long run variance is not the variance of the time series of the cross sectional mean, i.e. it does not measure the amplitude of the fluctuation of the cross sectional mean.

We assume that the risk neutral firm chooses the stock of capital (and therefore of employment and output) to maximize expected profits $E(\phi_{it})$. Recalling that $E(u_{it}) = 1$, the problem of the firm can be written as

$$\max_{K_{it}} E(\phi_{it}) = \Big(\gamma - r_{it}\Big)K_{it} - \frac{1}{2}K_{it}^2$$

where $\gamma = \nu\Big(1 - \dfrac{w}{\lambda}\Big)$. We assume that $\lambda > w$ so that $\gamma > 0$. The solution to the problem above is

$$K_{it} = \gamma - r_{it} \tag{2.3}$$

We need to specify a functional form for the individual real cost of capital (r_{it}). We assume it is equal to the risk free interest rate augmented by an idiosyncratic *external finance premium (EFP)* f_{it}:

$$r_{it} = r_t + f_{it}, \tag{2.4}$$

Hence:

$$K_{it} = \gamma - (r_t + f_{it}) \tag{2.5}$$

Notice that, given the linear structure of the previous equation, the cross-sectional mean of capital $K_t :=< K_{it} >$ turns out to be:

$$K_t = \gamma - (r_t + f_t) \tag{2.6}$$

where $f_t =< f_{it} >$.

We assume that the EFP is decreasing with the firm's internal financial resources (or net worth) in the previous period:

$$f_{it} = \frac{\alpha}{A_{it-1}} \tag{2.7}$$

where $\alpha > 0$ is the exogenous component of the individual EFP, which is uniform across firms. In the Representative Agent (RA) case the external finance premium will be: $f_t^{RA} = \dfrac{\alpha}{A_{t-1}}$. Hence $K_t^{RA} = \gamma - \left(r_t + \dfrac{\alpha}{A_{t-1}}\right)$

Let's consider now the Heterogeneous Agents (HA) case. From a second order approximation of (2.7) around the cross-sectional mean, we get the following equation for the average EFP in the HA case:[5]

$$f_t^{HA} \approx \frac{\alpha}{A_{t-1}} + \frac{\alpha V_{t-1}}{A_{t-1}^3} \tag{2.8}$$

The average EFP in the HA case is the sum of two terms. The first one coincides with the EFP in the RA case and depends on the cross-sectional average (therefore we will refer to this component as the "RA term"), the second one captures the role of heterogeneity in the average EFP. The second term (which we will refer to as the"HA term") is affected both by the mean and the variance of the distribution of firm's equity.

Hence average investment will be:

$$K_t^{HA} = \gamma - \left(r_t + \tfrac{\alpha}{A_{t-1}} + \tfrac{\alpha V_{t-1}}{A_{t-1}^3} \right) \tag{2.9}$$

3 The fixprice IS-TR model

In this section we use a simple fixprice IS-TR model to understand the basics of the financial transmission in the presence of a shock. For simplicity, we will consider only a fiscal shock. The complete flexprice IS-AS-TR model, which will be presented and discussed in the following section, is a "natural" extension of the fixprice model. The financial transmission with flexible prices is a slightly more sophisticated version of the core mechanism which we will discuss in the following.

3.1 Equilibrium on the goods market: the IS equation

We consider a representative household who earns the wage (w) if employed and the unemployment subsidy (σ) otherwise, with $w > \sigma$. Denoting employment with N_t, the (constant) labor force with $\bar{N}$ and the (uniform) propensity to consume with c, aggregate consumption is $C_t = c[wN_t + \sigma(\bar{N} - N_t)]$, which can be rewritten as:

$$C_t = c[(w-\sigma)x_t + \sigma]\bar{N}, \tag{3.1}$$

where $x_t = N_t/\bar{N}$ represents the fraction of population which is employed. Therefore $1 - x_t$ is the unemployment rate. Since the technology is linear, x_t is also the ratio

[5] First, we approximate (2.7) around average net worth A_{t-1} by means of a second order Taylor expansion:

$$f_{it} \approx \frac{\alpha}{A_{t-1}} + \left.\frac{\partial f_{it}}{\partial A_{it-1}}\right|_{A_{t-1}} (A_{it-1} - A_{t-1}) + \frac{1}{2}\left.\frac{\partial^2 f_{it}}{\partial A_{it-1}^2}\right|_{A_{t-1}} (A_{it-1} - A_{t-1})^2$$

where $\left.\frac{\partial f_{it}}{\partial A_{it-1}}\right|_{A_{t-1}} = -\frac{\alpha}{A_{t-1}^2}$ and $\left.\frac{\partial^2 f_{it}}{\partial A_{it-1}^2}\right|_{A_{t-1}} = \frac{2\alpha}{A_{t-1}^3}$. Substituting these derivatives in the expression above, taking the expected value of the RHS and recalling that $E(A_{it-1} - A_{t-1}) = 0$ and $E(A_{it-1} - A_{t-1})^2 = V_{t-1}$ we obtain (2.8).

of current output to full-employment output. For simplicity, we will refer to x as the output ratio or output tout court in the following. Hence $x_t - 1$ is the output gap.

Aggregate production can be written as $Y_t = \lambda x_t \bar{N}$. Equilibrium on the market for final goods occurs when $Y_t = C_t + K_t + G$ where G is exogenous public expenditure. Plugging (3.1) and (2.6) in the equilibrium condition and rearranging we get:

$$x_t = m_0(x_0 - r_t - f_t) \tag{3.2}$$

where $m_0 := 1/\tau\bar{N}$; $\tau := \lambda - c(w - \sigma)$ and $x_0 := c\sigma\bar{N} + G + \gamma$. Since $\lambda > w$ then $\tau > 0$. (3.2) is the equation of the IS schedule on the (x_t, r_t) plane. Notice that the (time varying) average EFP f_t is a shift parameter of the curve. An exogenous change in Government expenditure yields an identical change in x_0 and a shift of the IS curve. Hence G, A_{t-1} and V_{t-1} are shifters of the IS curve. In figure 1 the downward sloping straight lines represent different IS schedules (depending on the level of Government expenditure, mean and variance of the distribution of firms' net worth).

Notice that, in a simple income-expenditure model (e.g. a model in which investment is not affected by changes of the interest rate) m_0 would be the multiplier of government expenditure (fiscal multiplier hereafter) i.e., the increase of the output ratio (reduction in the unemployment rate) associated to a one-unit increase of Government expenditure.

3.2 A simplified Taylor Rule (TR)

We need to define a monetary policy rule. In a fixprice economy, the real and nominal interest rates coincide $r_t = i_t$. In this section, we assume that the central bank adopts a simplified Taylor rule in which it responds only to the output gap $x_t - 1$:

$$r_t \quad = \quad r_n + \alpha_x(x_t - 1) \tag{3.3}$$

where r_n is the natural rate of interest.

(3.3) is the equation of the upward sloping TR schedule on the (x_t, r_t) plane (see figure 1.)

3.3 The macroeconomic equilibrium

The model consists of equations (3.2) and (3.3). This system can be solved for x_t and r_t given f_t. This is the short run or temporary macroeconomic equilibrium:

$$\begin{cases} x_t^* = m_1(x_1 - f_t) \\ \\ r_t^* = r_n + \alpha_x[m_1(x_1 - f_t) - 1] \end{cases} \tag{3.4}$$

where: $m_1 := \frac{1}{\bar{N}\tau + \alpha_x}$ and $x_1 := \gamma - r_n + \alpha_x + c\sigma\bar{N} + G$.

It is useful to conceive of x_1 as the sum of two terms: $a_0 = \gamma - r_n + \alpha_x$ and $a_1 = c\sigma\bar{N} +$

G. a_0 includes parameters concerning monetary policy while a_1 includes parameters concerning fiscal policy.

From (3.4) we infer that the interest rate goes down when the EFP goes up. Hence an increase of the EFP has opposite effects on the average cost of capital $r_t + f_t$. The direct effect is positive and the indirect effect (through the equilibrium interest rate) is negative. Which one prevails?

Using the definition of the equilibrium interest rate we get:

$$r_t + f_t = r_n - \alpha_x + \alpha_x m_1 x_1 + (1 - \alpha_x m_1) f_t \tag{3.5}$$

Notice that, from the definition of m_1 follows that $1 - \alpha_x m_1 > 0$. Hence we can conclude that the direct effect prevails: an increase of the EFP unambiguously increases the average cost of capital. Since investment in the aggregate is $K_t = \gamma - (r_t + f_t)$ (see equation (2.6)), investment unambiguously declines when the EFP goes up.

Equilibrium as defined in (3.4) is temporary. In fact x and r in t depend on the state of the EFP in t, which in turn depends on the cross sectional mean and variance in $t-1$. In a sense (3.4) can be thought as a "frame" which visualizes the macro-economy in t. As time goes by, a frame in t+1 follows the frame in t and so on. The entire "movie" is shown. The distribution of the firms' net worth changes and so does the EFP and x and r which depend on the EFP.

Let's assume the process is ergodic. Then at the end of the adjustment process (at the end of the movie) there will be a *long run distribution* of the firms' net worth. Let's denote with A^*, V^* the moments of the long run distribution. The long run macroeconomic equilibrium in the HA case (the final frame of the movie) therefore will be:[6]

$$\begin{cases} x^{HA} = m_1(x_1 - f^{HA}) \\ r^{HA} = r_n + \alpha_x[m_1(x_1 - f^{HA}) - 1] \end{cases} \tag{3.6}$$

where

$$f^{HA} = \frac{\alpha}{A^*} + \frac{\alpha V^*}{(A^*)^3} \tag{3.7}$$

In the absence of shocks the economy will show this same frame period after period (steady state). In figure 1 point A represents the equilibrium position when Government expenditure is G_0, and the distribution is characterized by long run cross sectional mean A_0 and variance V_0.If a shock – i.e. a sudden change of an exogenous variable – occurs, there will be a *direct effect* on the equilibrium values of the endogenous variables (x and r) and a process of adjustment of the distribution of the firms' net worth. A new long

[6]The RA case is the same as 3.6 without the HA term.

run equilibrium will be reached. The moments of the new long run distribution will be different from the original ones. Hence also the EFP will change. We can conclude that, on top of the direct effect, there will be an *indirect effect* of the shock which emerges in the long run and is due to the change of the EFP.

3.4 The fiscal multiplier in the short and in the long run

We will now illustrate how the financial transmission works in this simplified framework. In order to do so, suppose the economy is originally in point A in figure 1. In point A the long run distribution of the firms' equity is characterized by cross-sectional mean and variance A_0, V_0. Hence, the EFP can be written as $f_0 = \frac{\alpha}{A_0} + \frac{\alpha V_0}{(A_0)^3}$. The current state of Government expenditure, say G_0, is somehow associated to this long run shape of the distribution. We will elaborate on this later on. In symbols: $A_0 = A(G_0), V_0 = V(G_0)$.

Consider now the effects of an increase in Government spending $\Delta G = G_1 - G_0$.

An increase of Government expenditure translates into a change of the same magnitude of x_1, i.e. $\Delta G = \Delta x_1$. The first round effect of the fiscal shock on the output gap is $\Delta x_{1st} = m_1 \Delta G$ where m_1 is the short run fiscal multiplier. Notice that $m_1 < m_0$. To understand why, notice that a fiscal shock ($G_1 > G_0$) makes the IS schedule shift up along the upward sloping TR schedule so that a new equilibrium B will be reached characterized by a higher x and r. In fact the fiscal shock generates a *crowding out* effect due to the monetary tightening engineered by the central bank: when x increases, the central bank reacts by increasing r as shown by the Taylor rule. As a consequence investment shrinks and output goes down.

The increase of G changes also the average EFP through the effects of the interest rate on the long run mean and variance (see section 5 for a discussion). Therefore the absolute change of the output gap is:

$$\Delta x = m_1 \Delta G - m_1 \Delta f$$

where $\Delta f = f_1 - f_0 = \left(\frac{\alpha}{A_1} - \frac{\alpha}{A_0}\right) + \left(\frac{\alpha V_1}{(A_1)^3} - \frac{\alpha V_0}{(A_0)^3}\right)$, $A_1 = A(G_1), V_1 = V(G_1)$. The first term of the expression $\Delta x_{1st} = m_1 \Delta G$ represents the first round effect and is certainly positive. The second term $\Delta x_{2nd} = -m_1 \Delta f$ which represent the second round effect is negative if the EFP increases as a consequence of the fiscal shock (as we will assume in the following).

Notice that

$$\Delta x_{2nd,RA} = -m_1 \left(\frac{\alpha}{A_1} - \frac{\alpha}{A_0}\right)$$

$$\Delta x_{2nd,HA} = -m_1 \left(\frac{\alpha V_1}{(A_1)^3} - \frac{\alpha V_0}{(A_0)^3}\right)$$

In our setting, the fiscal shock affects the individual net worth through the increase in the interest rate, which makes each and every firm worse off. It is reasonable to assume that the cross sectional equity at the end of the adjustment process would be smaller. In other words we conjecture that $A_1 < A_0$. Hence $\Delta x_{2nd,RA}$ will be negative. In the figure, the economy moves from B to C because $A_1 < A_0$.

If the variance goes up, i.e. $V_1 > V_0$ also $\Delta x_{2nd,HA}$ will be negative. Indeed the EFP will unambiguously go up because the cross sectional mean decreases and the variance increases. In the figure, the economy moves from C to D which is the final position (steady state). The first round effect is positive and the second round effect, in this scenario, would be negative. The final effect would depend on the relative size of these effects. In the figure, the first round effect prevails.

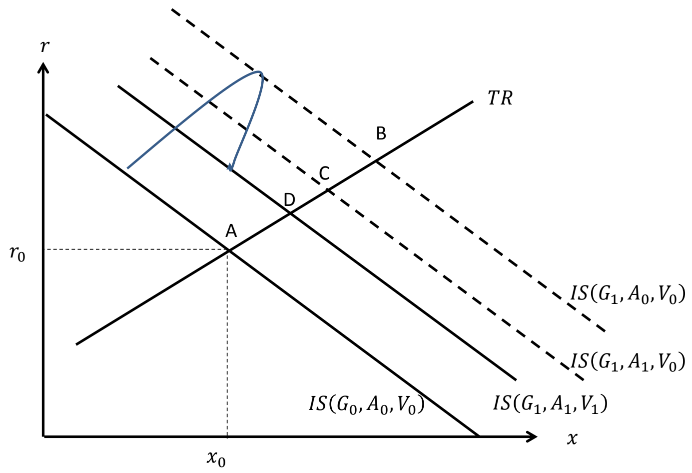

Figure 1

4 The flexprice IS-TR-PC model

Suppose now prices are flexible. The equilibrium on the goods market is described also in this context by the IS equation (3.2) but the real interest rate r does not coincide with the nominal interest rate. In fact $r_t = i_t - \pi_t$ where π is inflation. We should therefore rewrite the IS equation as follows:

$$x_t = m_0[x_0 - (i_t - \pi_t + f_t)] \tag{4.1}$$

The model will be closed by a Taylor Rule (TR) for the nominal interest rate and a Phillips curve or AS curve for inflation.

We assume the central bank adopts a standard Taylor rule to set the nominal interest rate:

$$i_t = r_n + (1+\alpha_\pi)\pi_t + \alpha_x(x_t - 1) \tag{4.2}$$

with $\alpha_\pi, \alpha_x > 0$. To close the model we assume inflation is proportional to the output gap according to a standard Phillips curve:

$$\pi_t = \theta x_t \tag{4.3}$$

with $\theta > 0$. Notice that expected inflation is left out of the picture, it does not show up either in the definition of the real interest rate or in the Phillips curve. The reason for this is simplicity. We want to explore the properties of this hybrid model before introducing the complications due to the formation of expectations in a setting characterized by heterogeneous agents. These issues will be dealt with in future developments of the model.

4.1 The macroeconomic equilibrium

We have now a system of three equations (4.1)(4.2)(4.3) in three unknowns, i.e. the nominal interest rate, inflation and the output gap.

To solve the model, we recall that, by definition, $r_t = i_t - \pi_t$. Plugging the Taylor rule in this equation we get the following equation for the real interest rate:

$$r_t = i_t - \pi_t = r_n + \alpha_\pi \pi_t + \alpha_x(x_t - 1). \tag{4.4}$$

Notice that (4.4) can be represented as an upward sloping line of the $(x_t; r_t)$ plane *parameterized to the level of inflation* (see, e.g, lower panel of figure 2).[7] Higher inflation will generate a higher real interest rate as the central bank reacts to inflation more than proportionately.[8]

Substituting (4.4) into (4.1) we get:

$$x_t = m_1(x_1 - f_t) - m_1 \alpha_\pi \pi_t \tag{4.5}$$

Equation (4.5) represents the *Aggregate Demand (AD)* function. It can be represented by a downward sloping line on the (x_t, π_t) plane.

The output gap, as determined from the demand side, is decreasing with inflation. Other things being equal, in fact, higher inflation pushes up the nominal and real interest rate and therefore depresses aggregate demand.

Notice that the mean and the variance of the distribution of firms' net worth are shift parameters of the AD curve: as the distribution changes over time the AD curve shifts.

[7]By construction, the simplified Taylor rule adopted in section 3.2 does not depend on inflation.

[8]In fact, the Taylor rule (4.2) incorporates the Taylor principle by construction.

The *Aggregate Supply (AS)* side of the model is represented by the Phillips curve, whereby inflation is increasing with the output gap. It can be represented by an upward sloping line on the $(x_t; \pi_t)$ plane.

Solving the AD-AS system consisting of equations (4.5) and (4.3) we get the temporary equilibrium of the flexprice model:[9]

$$\begin{cases} x_t^* = m_2\ m_1(x_1 - f_t) \\ \pi_t^* = \theta m_2\ m_1(x_1 - f_t) \\ r_t^* = r_n - \alpha_x + (\alpha_\pi\theta + \alpha_x)[m_2\ m_1(x_1 - f_t)] \end{cases} \tag{4.6}$$

where: $m_2 = \dfrac{1}{1 + m_1\alpha_\pi\theta}$.

In equilibrium, inflation and the interest rate are linear increasing functions of the output gap. All the variables are parameterized to the EFP.

Equilibrium is temporary. In fact x π and r in t will depend on the state of the EFP in t, which in turn depends on the moments of the distribution in $t-1$.

In the long run, i.e. when the distribution is characterized by the steady state levels of the mean and the variance of the distribution, A^* and V^*, the macroeconomic equilibrium is:

$$\begin{cases} x^{HA} = m_2\ m_1(x_1 - f^{HA}) \\ \pi^{HA} = \theta m_2\ m_1(x_1 - f^{HA}) \\ r^{HA} = r_n - \alpha_x + (\alpha_\pi\theta + \alpha_x)[m_2\ m_1(x_1 - f^{HA})] \end{cases} \tag{4.7}$$

where

$$f^{HA} = \frac{\alpha}{A^*} + \frac{\alpha V^*}{(A^*)^3} \tag{4.8}$$

In the upper panel of figure 2 point A represents the equilibrium inflation and output gap π_0 and x_0 when the moments of the long run distributions are A_0 and V_0. In the lower panel, we can read the equilibrium real interest rate, at the intersection of the IS curve (whose equation is (3.2)) and the Taylor rule, parameterized to the equilibrium level of inflation π_0.

[9] In the following we report the equilibrium real interest rate to make the graphical analysis below easy to follow. The equilibrium nominal interest rate can be easily retrieved by adding the equilibrium inflation rate to the equilibrium real interest rate.

4.2 The transmission of the fiscal shock in the flexprice model

Suppose the economy is initially in the steady state with Government expenditure G_0. This is represented by point A both in the upper panel (AD-AS framework) and the lower panel (IS-TR framework) of figure 2. The TR schedule in the lower panel is parameterized to the level of inflation π_0 determined in the upper panel. In A the long run distribution of the firms' equity is characterized by cross-sectional mean and variance A_0, V_0. Hence, the EFP can be written as $f_0 = \frac{\alpha}{A_0} + \frac{\alpha V_0}{(A_0)^3}$. The current state of Government expenditure, say G_0, is somehow associated to this long run shape of the distribution. We will elaborate on this later on. In symbols: $A_0 = A(G_0), V_0 = V(G_0)$.

An increase of Government expenditure yields the first round effect on the output gap is $\Delta x_{1st} = m_2 m_1 \Delta G$ where $m_2 m_1$ is the short run fiscal multiplier. Since $m_2 < 1$, the short run fiscal multiplier is smaller in a flexible price setting. To understand why, notice that a fiscal shock ($G_1 > G_0$) makes the AD schedule shift up along the upward sloping AS schedule so that a new equilibrium B will be reached characterized by a higher x and π. Therefore in the lower panel both the IS and TR curves shift up. The shift of the IS curve is determined by the increase of G, the shift of the TR schedule is determined by the increase of inflation. The latter effect is of course absent in the fixprice model. Hence the increase of the interest rate (and the crowding out effect) is magnified in the flexprice case. The central bank, in fact, reacts not only to the increase of output but also to the increase of inflation due to the demand shock.

The increase of G changes also the average EFP through the effects of the interest rate on the long run mean and variance (see section 5 for a discussion). Therefore the absolute change of the output gap is:

$$\Delta x = m_2 m_1 \Delta G - m_1 \Delta f$$

where $\Delta f = f_1 - f_0$. The second term $\Delta x_{2nd} = -m_2 m_1 \Delta f$ which represent the second round effect is negative if the EFP increases as a consequence of the fiscal shock (as we will assume in the following).

Notice that

$$\Delta x_{2nd,RA} = -m_2 m_1 \left(\frac{\alpha}{A_1} - \frac{\alpha}{A_0} \right)$$

$$\Delta x_{2nd,HA} = -m_2 m_1 \left(\frac{\alpha V_1}{(A_1)^3} - \frac{\alpha V_0}{(A_0)^3} \right)$$

In the presence of flexible prices, we can replicate the same line of reasoning of the fixprice case. The main difference with respect to the fixprice model is that in the flexprice case, in principle all the effects are smaller in size.

It is reasonable to assume that the cross sectional equity at the end of the adjustment process will be smaller: $A_1 < A_0$. Hence $\Delta x_{2nd,RA}$ will be negative. In the figure, the economy moves from B to C.

If the variance goes up, i.e. $V_1 > V_0$ also $\Delta x_{2nd,HA}$ will be negative. Indeed the EFP will unambiguously go up because the cross sectional mean decreases and the variance increases. In the figure, the economy moves from C to D which is the final position (steady state). The first round effect is positive and the second round effect, in this scenario, would be negative. The final effect would depend on the relative size of these effects.

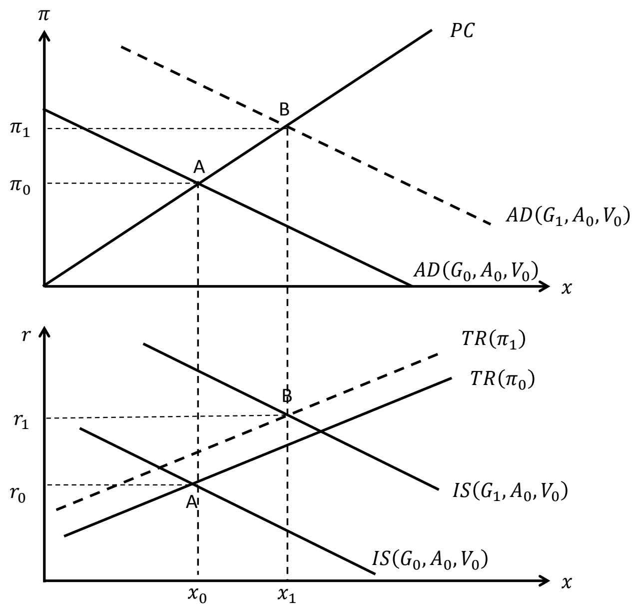

Figure 2

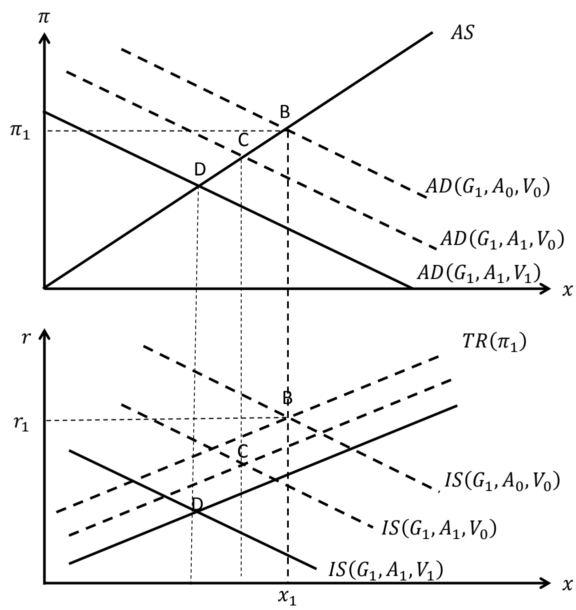

Figure 3

5 The agent based model

5.1 The law of motion of individual net worth

At the end of period t, once goods have been sold and profits realized, the firm decides on dividends and net worth accumulation. For simplicity we assume that a fraction δ of the net worth of the previous period A_{it-1} will be paid out as dividends to shareholders, while the firm devotes realized profits to the accumulation of net worth. By definition, therefore, net worth in period t is $A_{it} = (1-\delta)A_{it-1}+\phi_{it}$. Plugging (2.1) in the expression above we get:

$$A_{it} = \left(u_{it}\nu - w\frac{\nu}{\lambda} - r_{it}\right)K_{it} - \frac{1}{2}K_{it}^2 \tag{5.1}$$

Substituting (2.5) for the optimal level of investment, rearranging and simplifying we get:

$$A_{it} = (1-\delta)A_{it-1} + \nu\left(u_{it} - \frac{w}{\lambda}\right)(\gamma - r_{it}) + \frac{1}{2}(r_{it}^2 - \gamma^2) \tag{5.2}$$

There is only one endogenous variable in (5.2), namely the individual cost of capital r_{it}. The impact of a change in the cost of capital on net worth is in principle uncertain. It is easy to see, however, that it is negative (as our intuition would suggest) if $(\nu w/\lambda)+r_{it} < \nu u_{it}$. In words: an increase of the cost of capital depresses net worth if the interest rate is relatively low and/or the idiosyncratic shock is relatively high.

The individual cost of capital is the sum of the interest rate (controlled by the central bank) and the individual EFP. Taking into account the solution for the equilibrium real interest rate (4.6), the individual cost of capital turns out to be

$$r_{it} = r^* + f_{it} = r_n - \alpha_x + (\alpha_\pi\theta + \alpha_x)[m_2\ m_1(x_1 - f_t)] + f_{it} \tag{5.3}$$

where

$$\begin{aligned} f_t &= \frac{\alpha}{A_{t-1}} + \frac{\alpha V_{t-1}}{A_{t-1}^3} \\ f_{it} &= \frac{\alpha}{A_{it-1}} \end{aligned}$$

The individual cost of capital, therefore, depends both on the individual and on the average EFP. While an increase of the individual EFP pushes obviously up the individual cost of capital, an increase of the average EFP affects negatively the individual cost of capital (through the equilibrium interest rate). This sounds strange but is perfectly understandable, given the context. OTBE, in fact, an increase of the average EFP pushes down both the output gap and inflation. Hence the equilibrium interest rate (governed by the Taylor rule) goes down and brings down the individual cost of capital.

Plugging (5.3) in (5.2) and taking into account the definition of the average and the individual EFP we conclude that the law of motion of individual net worth is a non-linear function of the mean and the variance of the distribution of net worth. A *macroeconomic externality* is at work here: changes in the shape of the distribution of net worth, captured by the moments, impact upon the individual net worth. We can succinctly represent this law of motion as follows:

$$A_{it} = f(A_{it-1}, A_{t-1}, V_{t-1}; u_{it-1}, Z) \qquad i = 1, 2, ...F \tag{5.4}$$

where Z represents exogenous variables and parameters. Since there is one law for each firm, in the end we have a system of F non-linear difference equations subject to idiosyncratic shocks.

Given the complexity of the system it is not possible to compute a closed form solution, hence, we need to build a simple Agent Based Model (ABM) and make use of computer simulations to assess the dynamic properties of the economy we are investigating. Notice moreover that the dynamics of net worth is constrained by a lower bound: When A_{it} reaches zero, in fact the firm goes bankrupt. The ABM will incorporate this condition. The bankrupt firm will be replaced by a new (entrant) firm with a pre-specified endowment (initial net worth). In the following subsection we will describe the baseline scenario of the ABM.

5.2 The baseline scenario

We consider an economy populated by $F = 1000$ firms over a time span of $T = 1000$ periods (here the time scale can be thought of as a quarter).

In table 1 we report the configuration of the 13 parameters in the baseline scenario.

Table 1: Parameters' value in numerical simulations

Variable	Symbol	Numerical Value
Finance premium	α	0.2
Average productivity of capital	ν	0.5
Average productivity of labor	λ	4
Wage rate	w	0.5
Unemployment subsidy	σ	0.3
Natural real interest rate	r_n	0.03
Total labor force	$\bar{N}$	3
Propensity to consume	c	0.8
Central bank sensibility to output gap	α_x	0.5
Central bank sensibility to inflation rate	α_π	0.5
AS curve slope	θ	0.04
Dividend yeld	δ	0.1
Public expenditure	G	10

In each period, say t, through simulations we generate a distribution of firms' net worth and compute the mean A_t and variance V_t. This determines also the average EFP in t+1: f_{t+1}. Plugging these numbers into the reduced form of the model (4.6) we get the the output gap, inflation and the interest rate in t+1. Using r_{t+1}, given A_{it}, from (5.2) we get A_{it+1}, and the cycle starts again. We therefore can retrieve the time series of the cross-sectional mean and variance of the net worth over 1000 periods, and the associated values for the output gap, inflation and the interest rate.

Notice that a certain fraction of the population of firms will go out of business every period because of bankruptcy and will be replaced. Bankruptcies and replacement, of course, will impact on the shape of the distribution, hence on the moments and the average EFP.

In figure 4. we show the HP filtered time series of (a): the output gap, (b):mean and variance of the distribution of net worth, (c):interest rate, (d):EFP, (e):inflation rate, (f): bankruptcies.

Each time series fluctuates irregularly around a long run mean (not shown). Notice that the interest rate and inflation are highly correlated with the output gap (which is a straightforward conclusion on the basis of the reduced form of the model). The mean and variance are negatively correlated. The average EFP is highly positively (negatively) correlated with the variance (mean) of the distribution of net worth. The fraction of the population of firms which go bankrupt is highly correlated with the average EFP.

Notice that there is a long run mean also of the cross-sectional mean and variance. Therefore there is a long run distribution.

We conjecture that changes in the parameters will affect the dynamics both at high and low frequency. To capture the effects of changes in policy parameters in the long run we propose a method to compute the long run mean and variance (and therefore the long run mean of the output gap, inflation and the interest rate). Our strategy is as follows. We assume that the dynamics of mean and the variance of the net worth can be approximately described by the following linear system:

$$A_t = \alpha_1 + \alpha_2 A_{t-1} + \alpha_3 V_{t-1} \tag{5.5}$$

$$V_t = \beta_1 + \beta_2 A_{t-1} + \beta_3 V_{t-1} \tag{5.6}$$

Given the simulated time series, we run an OLS regression on 900 simulated data (we discard the transient of the first 100 periods) to numerically estimate the coefficients of the linear system (5.5). In table 2 we report the estimated coefficients (significant at 5% level).

Table 2: Estimated coefficients

Coefficient	α_1	α_2	α_3	β_1	β_2	β_3
Value	0.4291	0.5942	0.5086	0.2185	-0.1007	0.2688

From the estimated linear system we can compute the steady state for the mean and the variance of the net worth $A^* = 1.2212$ and $V^* = 0.1306$. Notice that the computed

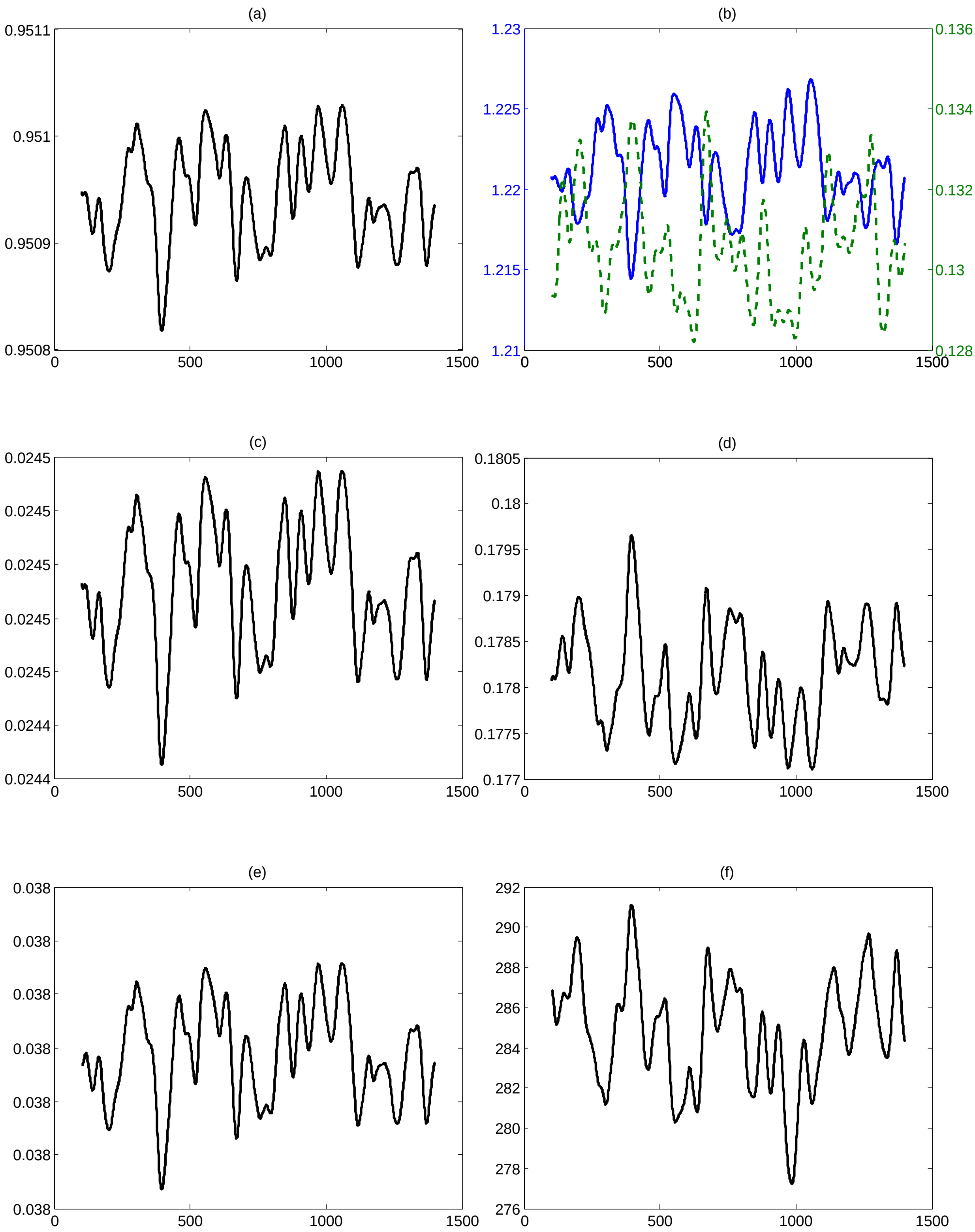

Figure 4: The benchmark scenario. Filtered time series (Hodrick-Prescott filter). Panel (a): output gap; panel (b): y-left axes mean net worth (solid line), y-right axes variance of the net worth (dashed line); panel (c): real interest rate; panel (d): risk premium; panel (e): inflation rate; panel (f): average bankruptcies (in levels).

steady state is indeed very close to the long run average of the cross-sectional mean and variance. The extraction of the dynamics from the artificial time series therefore allows to provide an accurate description of the actual long run position. In the long run the actual distribution of the firms' net worth converges to a stable distribution characterized by a mean and a variance equal to A^* and V^* . Given the steady state cross sectional mean and variance we can compute the steady state for the risk premium $f^* = 0.1781$. This is approximately the long run mean of the average EFP. Therefore we determine through the ABM the crucial figure to compute the long run position of the macroeconomic variables, namely the output gap, inflation and the interest rate. Steady states are reported in tab.3, in the long run the unemployment rate is about 5%, inflation slightly less than 4%and the real interest rate is around 2.5%.

Table 3: Steady state values

A^*	V^*	f^*	x^*	r^*	π^*	cv^*	Av. bank.
1.2212	0.1306	0.1781	0.9509	0.0245	0.0380	0.2959	28.4%

In order to assess the properties of the steady state, it is useful to rewrite the system (5.5) as follows:

$$\Delta A = \alpha_1 - (1 - \alpha_2)A_t + \alpha_3 V_t \tag{5.7}$$

$$\Delta V = \beta_1 + \beta_2 A_t - (1 - \beta_3)V_t \tag{5.8}$$

where $\Delta A := A_{t+1} - A_t$ and $\Delta V := V_{t+1} - V_t$ In the limiting case of continuous time this may be conceived as a linear system of differential equations in the state variables A and V. As shown in table 2 the autoregressive coefficients are smaller than one, therefore the trace of the Jacobian of the system above is always negative. Since $\beta_2 < 0$, moreover, the determinant is positive, hence the resulting steady state is stable.

In order to represent the system graphically, we impose $\Delta A = 0$ in the first equation and get the AA demarcation line, i.e. the locus of (V, A) pairs such that A is constant. Moreover, we impose $\Delta V = 0$ in the second equation and get the VV demarcation line, i.e. the locus of (V, A) pairs such that V is constant. Substituting the estimated coefficients in 5.7, we get the following equations for the AA and VV curves:

$$A = 1.0574 + 1.2533V$$

$$A = 2.1698 - 7.2612V$$

The AA curve is upward sloping while the VV curve is downward sloping. At the intersection point between the two curves (E) we determine the steady state A^* and V^* (see figure 5).

In the next three sections we will show the effects of a fiscal shock (namely a permanent increase of Government expenditure) a monetary shock (a permanent increase of

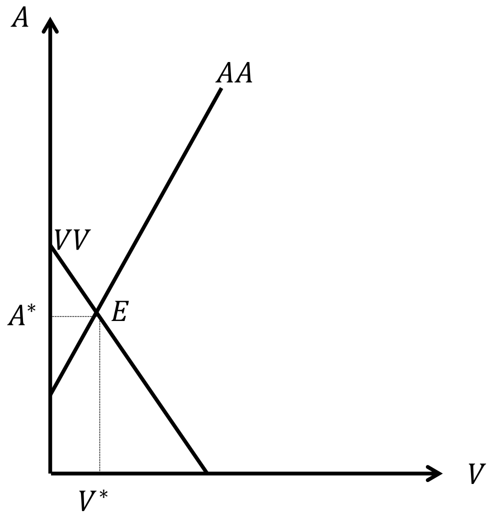

Figure 5: Phase diagram of the linear system (5.5)

the natural interest rate) and a financial shock (a permanent increase of the α parameter in the definition of the EFP). We will focus on the effects on the output gap. From the reduced form (4.6) in fact, it is easy to see that inflation and the interest rate are increasing linear transformations of the output gap.

6 A fiscal shock

In this section we explore the effects of an expansionary permanent fiscal policy shock captured by an increase of Government expenditure from $G = 10$ (as in the baseline scenario) before $T = 600$ to $G' = 12$ from $T = 600$ on. The other parameters remain unchanged.

The results are shown in figure 6. A clear change in regime occurs in T=600. The output gap, which fluctuated around 95% before the shock, jumps up and starts fluctuating around a much higher long run average characterized by over-employment. The inflation rate and the interest rate go in the same direction. The increase of the interest rate affects the accumulation of individual net worth. Therefore also the distribution changes. The long run cross sectional mean (variance) of net worth goes down (up). Hence the average EFP unambiguously goes up and so do bankruptcies.

The transmission mechanism of a fiscal shock can be characterized as follows. As Government expenditure increases, the output gap and the inflation rate go up on impact. Hence the central bank raises the interest rate. As a consequence the cost of capital for each and every firm increases and investment goes down (this is the standard crowding out effect). However, as in all Keynesian models, the expansionary effect of an increase of Government expenditure is bigger in size than the standard crowding out effect. In the graphical representation of the model, the macroeconomy moves from A to B (see

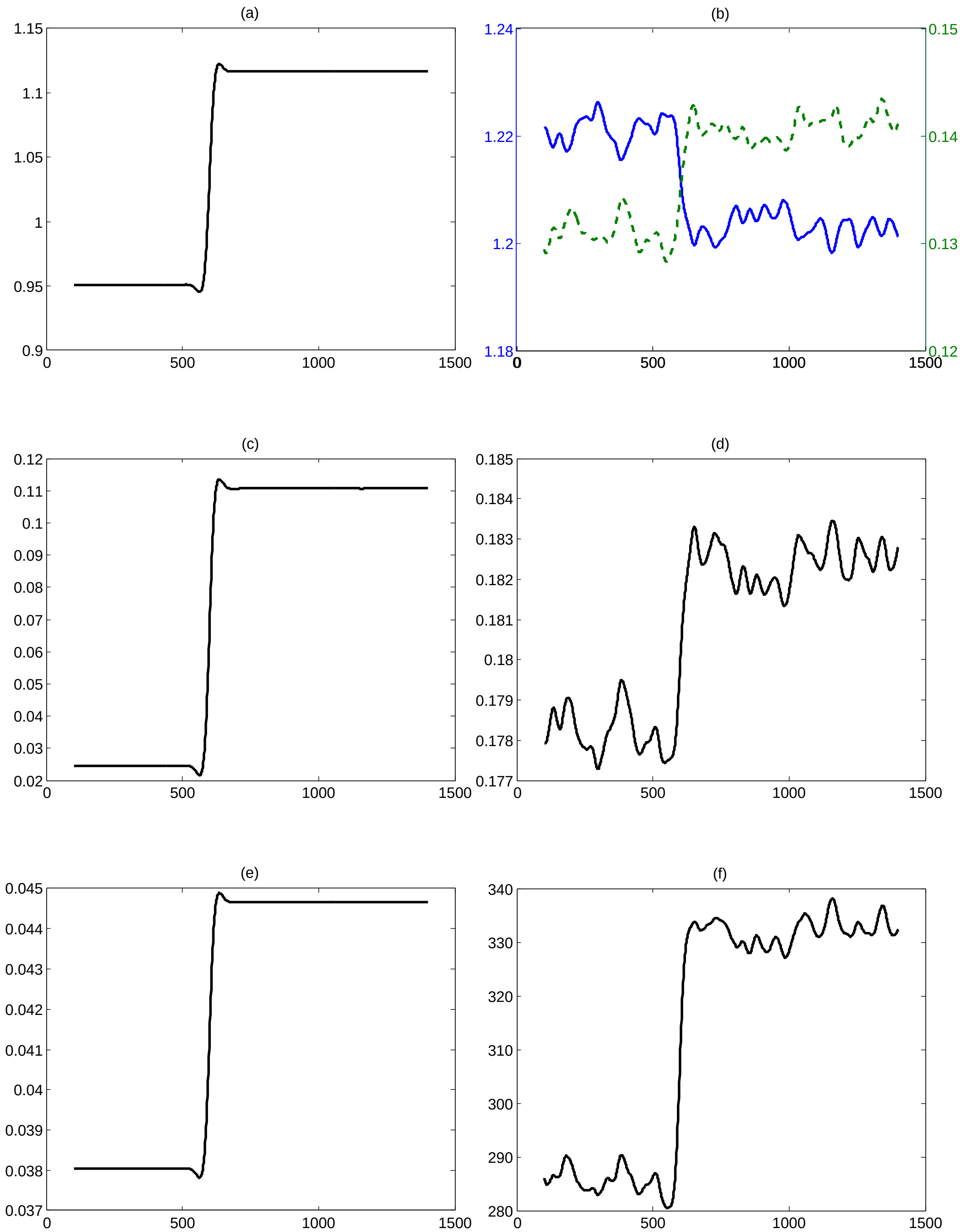

Figure 6: The effects of a fiscal shock. Hodrick-Prescott (HP) filtered time series. Panels show the following time series. (a): output gap; (b): cross sectional mean of net worth (solid line, y-left axis), cross sectional variance of net worth (dashed line, y-right axis); (c): interest rate; (d): average EFP; (e): inflation rate; (f): number of bankruptcies.

figure 2).

The novelty of the present model is the *indirect effect* of the fiscal shock which is due to the impact of the increase of the interest rate on the distribution of net worth. The increase of the interest rate, in fact, hits the accumulation of net worth, for each of the surviving firms.[10] Since net worth goes down, the individual EFP goes up. Moreover the number of bankruptcies increases (as shown in panel (f) of the figure) because some firms which were already on the verge of bankruptcy end up with negative net worth. Therefore A decreases and V increases, inducing an increase of the average EFP f. The increase of average EFP has an additional adverse effect on investment and the output gap. All in all, we can conclude that the positive impact on the output gap of an increase in Government expenditure is mitigated by more stringent credit terms due to the increase of the interest rate decided by the central bank and by the additional crowding out effect following the increase of the EFP f.

In order to quantify these effects we proceed as follows. First we estimate the αs and βs of the system (5.5) using simulated data from period 101 until period $t = 599$ when the simulated model corresponds to the benchmark one. Second we run the OLS regression of our artificial dataset from period $T = 600$, when the fiscal shock occurs, until the end of the time horizon. In tab.4 we report coefficients (significant at 5% level) before and after the shock.

Table 4: Estimated coefficients

$G = 10$	α_1	α_2	α_3	β_1	β_2	β_3
	0.3563	0.6465	0.5763	0.3039	-0.1614	0.1841
$G = 12$	α_1	α_2	α_3	β_1	β_2	β_3
	0.5139	0.5182	0.4662	0.2724	-0.1359	0.2254

Using the estimated coefficients we can determine the AA and VV curves before and after period T=600 when the fiscal shock occurs. They are:[11]

$$\begin{aligned}
AA(G) : A &= 1.0079 + 1.6303V \\
VV(G) : A &= 1.8829 - 5.0551V \\
AA(G') : A &= 1.0667 + 0.9676V \\
VV(G') : A &= 2.0044 - 5.6998V
\end{aligned}$$

Due to the expansionary fiscal shock the AA curve becomes flatter (the slope decreases of almost 50%) while the VV curve becomes slightly steeper, triggering a trajec-

[10] As shown above, this is true only if the individual interest rate is relatively low and/or the idiosyncratic shock is relatively high.

[11] As shown in table 4 the autoregressive coefficients, both before and after period T=600 are smaller than 1, therefore the trace of the Jacobian is always negative. Moreover, since $\beta_2 < 0$ in both regimes, the determinant is positive, hence the resulting steady states are stable.

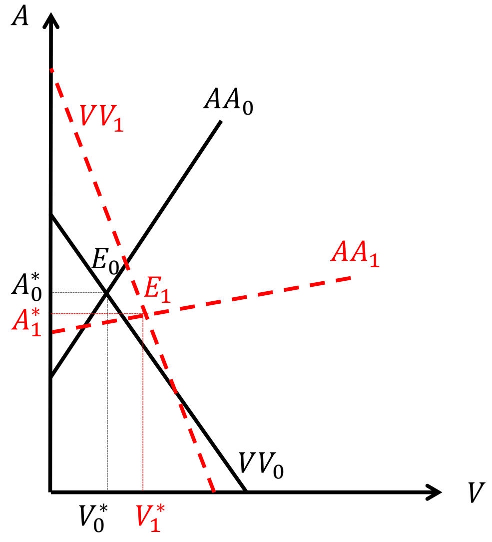

Figure 7: Demarcation lines AA and VV: Effects of a fiscal shock

tory of decreasing A and increasing V which starts at the original steady state E_0. As shown in figure 7, in the new steady state (E_1) the average net worth is smaller while the variance of the net worth is greater than before the shock: $A_1 < A_0$, $V_1 > V_0$.

Given the steady state cross sectional mean and variance, we can compute the steady state EFP f_1. Finally, substituting f_1 in the reduced form of the macroeconomic model we determine the steady state for the macroeconomic variables namely, the output gap, the inflation rate and the interest rate. Results are shown in tab.5

Table 5: Steady state values

$G = 10$	A^*	V^*	f^*	x^*	r^*	π^*	cv^*	Av. bank.
	1.2213	0.1309	0.1781	0.9509	0.0245	0.0380	0.2962	28.5%
$G' = 12$	A^*	V^*	f^*	x^*	r^*	π^*	cv^*	Av. bank.
	1.2027 $\downarrow$	0.1407 $\uparrow$	0.1825 $\uparrow$	1.1167 $\uparrow$	0.1107 $\uparrow$	0.0447 $\uparrow$	0.3118 $\uparrow$	33.2% $\uparrow$

As a consequence of the fiscal policy shock ($\Delta G = 2$), there is a sizable increase of the long run output gap $\Delta x \approx 0.1660$.

We can decompose the effect of an increase of Government expenditure on the output gap as shown in section 4.2.[12] The absolute change of the output gap is:

$$\Delta x = \underbrace{m_2 m_1 \Delta G}_{\Delta x_{1st}} \underbrace{- m_2 m_1 \left(\frac{\alpha}{A_1} - \frac{\alpha}{A_0} \right)}_{\Delta x_{2nd,RA}} \underbrace{- m_2 m_1 \left[\frac{\alpha V_1}{(A_1)^3} - \frac{\alpha V_0}{(A_0)^3} \right]}_{\Delta x_{2nd,HA}} \tag{6.1}$$

Given the numerical values of the parameters and of the steady state moments of

[12] Similar exercises can be carried out for the other endogenous variables.

the distribution before and after the shock we can assess quantitatively the size of each effect.

- The first round effect is positive and sizable: $\Delta x_{1st} = 0.1661$;
- The RA component of the second round effect is negative since the steady state cross sectional mean decreases: $\Delta x_{2nd,RA} = -2.1035 \times 10^{-4}$
- The HA component of the second round effect is also negative because the steady state cross sectional variance increases: $\Delta x_{2nd,HA} = -1.4981 \times 10^{-4}$

The first round effect explains most of the change in the output gap, being three orders of magnitude bigger than the indirect effect. The direct effect is driven by the shere size of the shock: a 20% increase of government expenditure. The HA component of the second round effect is of the same order of magnitude of the RA effect and represents more than 40% of the entire second round effect.

7 A monetary shock

In this section we explore the effects of a contractionary permanent monetary policy shock captured by an increase of the exogenous component of the Taylor rule, i.e. the natural rate, from $r_n = 0.03$ (as in the baseline scenario) before $T = 600$ to $r'_n = 0.06$ from $T = 600$ on.

The results are shown in figure 8. As a consequence of the shock, in T=600 the output gap goes down and starts fluctuating around a slightly lower long run mean. The inflation rate goes in the same direction. The interest rate, on the contrary goes up. Also the distribution changes. The long run cross sectional mean (variance) of net worth goes down (up). Hence the EFP unambiguously goes up and so do bankruptcies.

The transmission mechanism of a monetary shock can be described as follows. The exogenous increase of the policy rate has the direct effect of pushing down the output gap and inflation. As to the interest rate, there are two effects of the shock: i) a positive effect which coincides with the shock, i.e. the increase of the natural interest rate; ii) a negative effect due to the downward adjustment of the interest rate engineered by the central bank and dictated by the Taylor rule. The net effect is positive i.e., the interest rate unambiguously increases. This is the 1st round effect of the shock on the interest rate.

The increase of the interest rate has a negative impact on net worth, for each of the surviving firms. The individual EFP goes up. Moreover the number of bankruptcies increases. Therefore A decreases and V increases, inducing an increase of the average EFP f. The increase of average EFP has an additional adverse effect on investment and the output gap. We can conclude that the negative impact on the output gap of

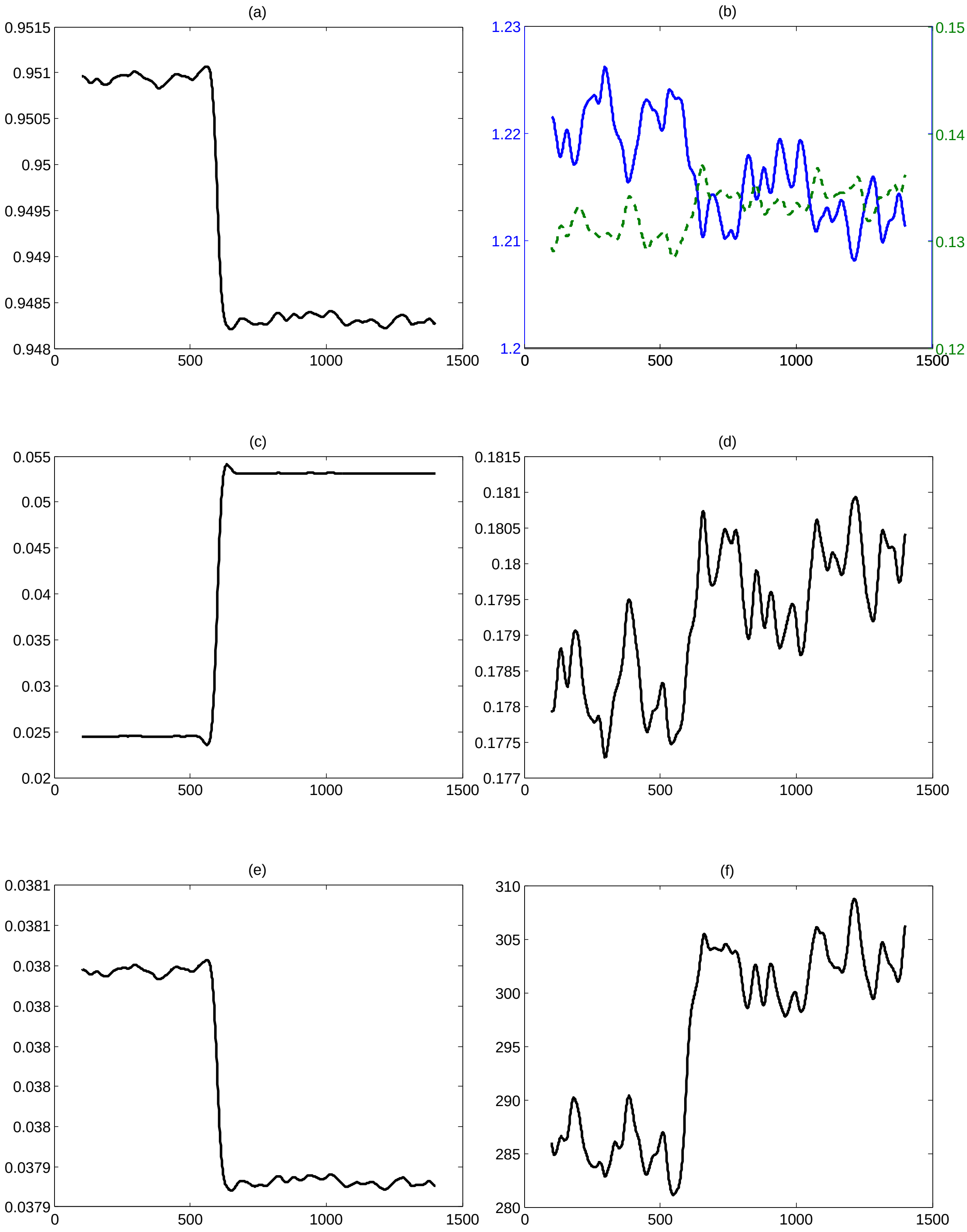

Figure 8: The effects of a monetary shock. Hodrick-Prescott (HP) filtered time series. Panels show the following time series. (a): output gap; (b): cross sectional mean of net worth (solid line, y-left axis), cross sectional variance of net worth (dashed line, y-right axis); (c): interest rate; (d): average EFP; (e): inflation rate; (f): number of bankruptcies.

an increase of the policy interest rate is amplified by more stringent conditions to access the credit market (via the increase in the average EFP).

In order to quantify these effects we proceed as in the previous section. First we estimate the αs and βs of the system ... using simulated data from period 101 until period $t = 599$ when the simulated model corresponds to the benchmark one. Second we run the OLS regression of our artificial dataset from period $T = 600$, when the monetary shock occurs, until the end of the time horizon. In tab.6 we report the coefficients (significant at 5% level) before and after the shock.

Table 6: Estimated coefficients

$r_n = 0.03$	α_1 0.3563	α_2 0.6465	α_3 0.5763	β_1 0.3039	β_2 -0.1614	β_3 0.1841
$r_n = 0.06$	α_1 0.4208	α_2 0.5972	α_3 0.5071	β_1 0.2622	β_2 -0.1321	β_3 0.2410

Using the estimated coefficients we can determine the AA and VV curves before and after the monetary shock. They are:

$$\begin{aligned} AA(r_n) : A &= 1.0079 + 1.6303V \\ VV(r_n) : A &= 1.8829 - 5.0551V \\ AA(r'_n) : A &= 1.0447 + 1.2589V \\ VV(r'_n) : A &= 1.9849 - 5.7456V \end{aligned}$$

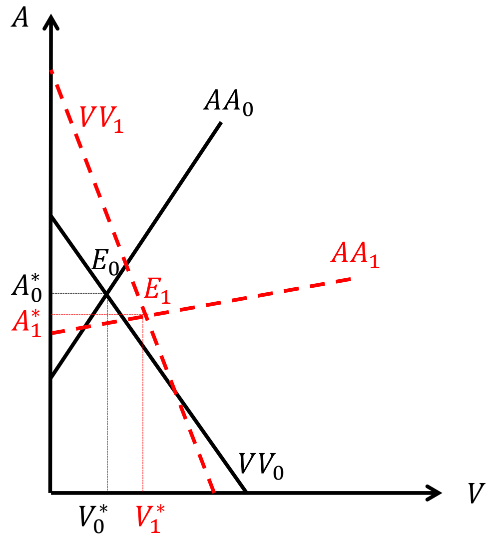

Figure 9: Demarcation lines AA and VV: the effects of a monetary policy shock

E_0 in figure 9 represents the steady state before the monetary policy shock. After the increase of the natural interest rate the AA curve becomes flatter and the VV curve steeper, triggering a pattern of decreasing A and increasing V. In the new steady state (E_1) the average net worth is smaller while the variance of the net worth is greater than in the original steady state. The contractionary monetary shock induces shifts of the demarcation curves that are qualitatively similar to the shifts generated by an expansionary fiscal policy.

Having computed the steady state of the mean and the variance of the distribution, we compute the steady state of the average EFP. Finally, substituting the steady state EFP in the macro-dynamical system we compute the equilibrium long run values of the endogenous variables: output gap, inflation rate and interest rate. The results are shown in tab.7

Table 7: Steady state values

$r_n = 0.03$	A^*	V^*	f^*	x^*	r^*	π^*	cv^*	Av. ban.
	1.2213	0.1309	0.1781	0.9509	0.0245	0.0380	0.2962	28.5%
$r_n = 0.06$	A^*	V^*	f^*	x^*	r^*	π^*	cv^*	Av. bank.
	1.2137 ↓	0.1342 ↑	0.1798 ↑	0.9483 ↓	0.0531 ↑	0.0379 ↓	0.3019 ↑	30.2% ↑

From the steady state values we notice that in the long run the output gap decreases slightly ($\Delta x = 0.0026$) as a consequence of the shock ($\Delta r_n = r_n' - r_n = 0.03$). We can break down this change as follows:

$$\Delta x = \underbrace{-m_2 m_1 \Delta r_n}_{\Delta x_{1st}} \underbrace{-m_2 m_1 \left(\frac{\alpha}{A_1} - \frac{\alpha}{A_0} \right)}_{\Delta x_{2nd,RA}} \underbrace{-m_2 m_1 \left[\frac{\alpha V_1}{(A_1)^3} - \frac{\alpha V_0}{(A_0)^3} \right]}_{\Delta x_{2nd,HA}} \tag{7.1}$$

Given the numerical values of the parameters, we can conclude that an increase of the natural interest rate $\Delta r_n = 0.03$ leads to the following:

- A negative mild direct effect ($\Delta x_{1st} = -2.4917 \times 10^{-3}$).
- A negative indirect RA effect ($\Delta x_{2nd,RA} = -8.5169 \times 10^{-5}$) due to the decrease of the cross sectional mean.
- A negative indirect HA effect ($\Delta x_{2nd,HA} = -5.3234 \times 10^{-5}$) due to the increase of the cross-sectional variance.

The first round effect explains most of the change in the output gap, being two orders of magnitude bigger than the indirect effect. The HA component of the second round effect is of the same order of magnitude of the RA effect and represents almost 40% of the entire second round effect.

8 A financial shock

In this section we explore the effects of a contractionary permanent financial shock captured by an increase of the exogenous component of the EFP from $\alpha = 0.2$ to $\alpha' = 0.4$.

The results are shown in figure 10. As a consequence of the shock, in T=600 the output gap goes down and starts fluctuating around a lower long run mean. The inflation rate and the interest rate go in the same direction. Also the distribution changes. The long run level of both the cross sectional mean and the variance of net worth go up. In principle the effect on the EFP is uncertain. However, simulations show that the EFP unambiguously goes up and so do bankruptcies. This indirect effect exacerbates the negative repercussions of the financial shock on the output gap and inflation.

The transmission mechanism of a financial shock can be characterized as follows. As α increases all the firms experience an increase of the cost of capital due to the increase of f_i, hence investment goes down and so does aggregate demand, the output gap and inflation. The interest rate is therefore steered by the central bank down. There are two opposite effects on the individual cost of capital: f_i goes up on impact and r goes down as a consequence of the reaction of the central bank (notice that we are keeping the distribution unchanged). However, the former effect offsets the latter. In fact we observe that in the period in which α rises the number of bankruptcies increases (as shown in panel (f) of fig. 10). As the number of bankruptcies increases both A and V increase, inducing an increase of the average EFP f: The increase of the variance offsets the effect of the increase of the cross-sectional mean on the average EFP. The increase in f leads to a further decrease in the output gap (as shown in in panel (a) of fig. 10). We can conclude that the negative impact on the output gap of an increase in the *individual* EFP is amplified by the further increase in the *average* EFP.

As in the previous sections, we estimate the αs and βs of the dynamical system (5.5) given the new artificial dataset. First we estimate the coefficients from period 101 until period $t = 599$ with $\alpha = 0.2$. Then we run the OLS regression from period $T = 600$ on, with $\alpha' = 0.4$. In tab.8 we report the estimated coefficients (significant at 5% level).

Table 8: Estimated coefficients

	α_1	α_2	α_3	β_1	β_2	β_3
$\alpha = 0.2$	0.3563	0.6465	0.5763	0.3039	-0.1614	0.1841
	α_1	α_2	α_3	β_1	β_2	β_3
$\alpha = 0.4$	0.7933	0.3605	0.1704	0.6643	-0.4485	0.8774

Using the estimated coefficients, we can determine the equations of the AA and VV

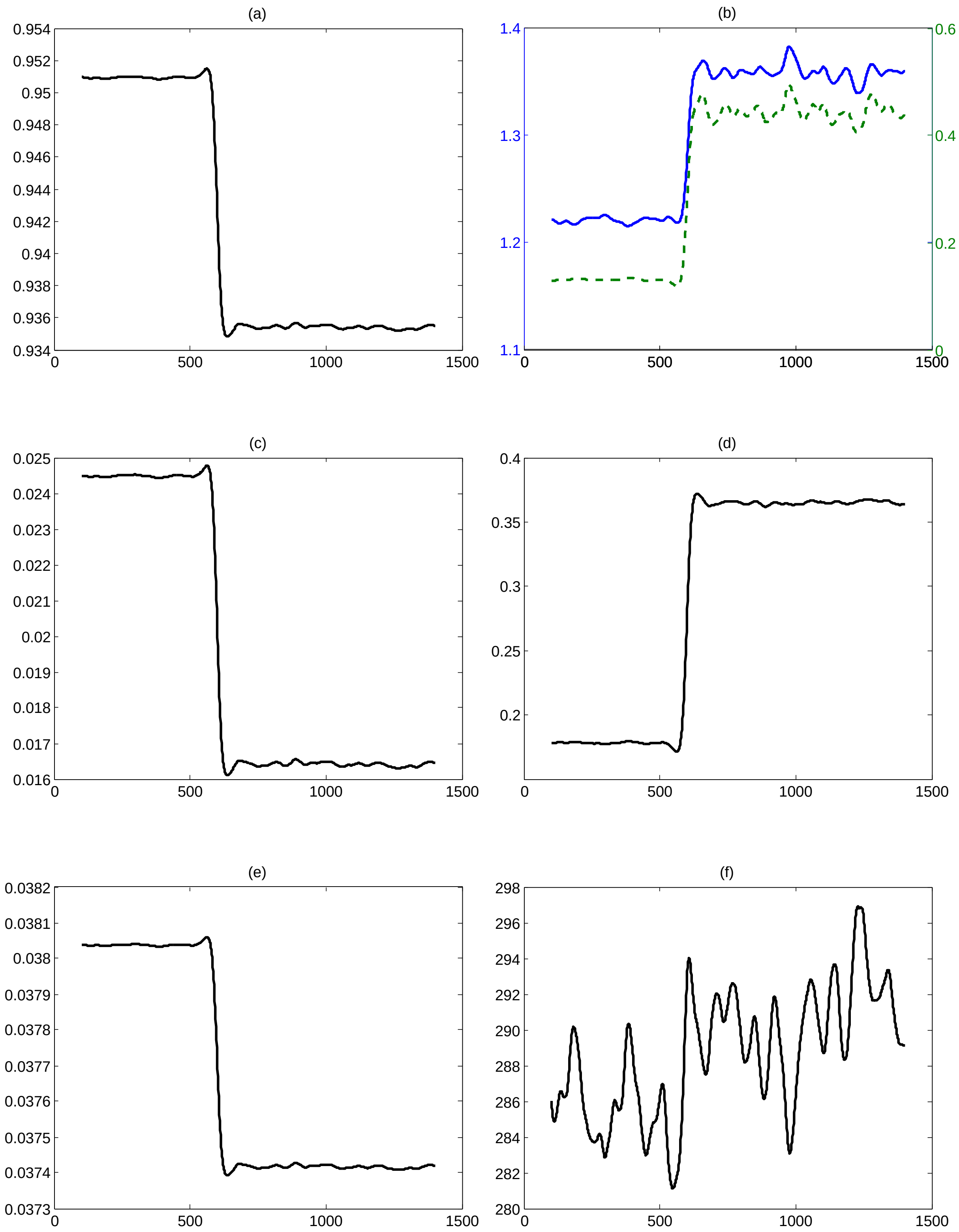

Figure 10: The effects of a financial shock. Hodrick-Prescott (HP) filtered time series. Panels show the following time series. (a): output gap; (b): cross sectional mean of net worth (solid line, y-left axis), cross sectional variance of net worth (dashed line, y-right axis); (c): interest rate; (d): average EFP; (e): inflation rate; (f): number of bankruptcies.

curves before and after the shock. They are:

$$\begin{aligned} AA(\alpha): A &= 1.0079 + 1.6303V \\ VV(\alpha): A &= 1.8829 - 5.0551V \\ AA(\alpha'): A &= 1.2405 + 0.2665V \\ VV(\alpha'): A &= 1.4812 - 0.2734V \end{aligned}$$

As shown in figure 11, the AA curve shifts up while the VV curve shifts down. Moreover both the AA and VV curves become flatter. The shock triggers a trajectory of increasing A and V. In the new steady state (E_1) both the cross sectional mean and variance of the net worth are higher than in the old one (as already shown in table 9 from the simulation results). It is worth noting, however, that while average net worth goes up moderately, the variance of net worth triples. The overall effect on the EFP is positive and sizable: the average EFP doubles.

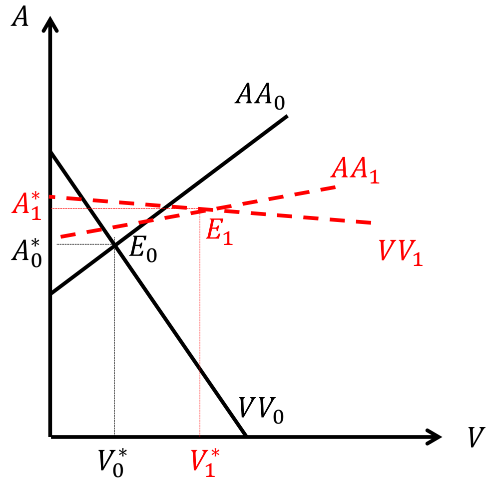

Figure 11: Demarcation lines AA and VV: the effects of a financial shock

Once the new steady state for the EFP is computed, we can retrieve the steady state for the endogenous variables from (4.6). Results are shown in tab.9.

Table 9: Steady state values

$\alpha = 0.2$	A^* 1.2213	V^* 0.1309	f^* 0.1781	x^* 0.9509	r^* 0.0245	π^* 0.0380	cv^* 0.2962	Av. bank. 28.5%
$\alpha = 0.4$	A^* 1.3593 ↑	V^* 0.4458 ↑	f^* 0.3653 ↑	x^* 0.9354 ↓	r^* 0.0164 ↓	π^* 0.0374 ↓	cv^* 0.4912 ↑	Av bank. 29% ↑

From the steady state values we notice that in the long run the output gap goes down from approximately 95% (with $\alpha = 0.2$) to 93.5% (with $\alpha' = 0.4$): $\Delta x = -0.0155$

From the reduced form (4.6), noticing that the parameter alpha shows up only in the EFP, we obtain:

$$\Delta x = -m_2 m_1 \Delta f$$

where $\Delta f = f_1 - f_0 = \frac{\alpha'}{A_1} + \frac{\alpha' V_1}{(A_1)^3} - \left(\frac{\alpha}{A_0} + \frac{\alpha V_0}{(A_0)^3}\right)$. It is worth noting that in the case of a financial shock, differently from the case of changes in fiscal and monetary policy,[13] the shock propagates through changes in the EFP only. The EFP, in turn, changes not only because of the change in α, but also because the distribution of firms' net worth changes. Therefore, we can break down the change in f into two components:

$$\Delta f = \underbrace{(\alpha' - \alpha)\frac{f_0}{\alpha}}_{\Delta f_{1st}} + \underbrace{\alpha'\left(\frac{f_1}{\alpha'} - \frac{f_0}{\alpha}\right)}_{\Delta f_{2nd}}$$

The *first round effect* of the shock on the average EFP Δf_{1st} is computed keeping the mean and the variance unchanged (f_0 is a function of A_0 and V_0 only). Recalling the definition of EFP we obtain:

$$\Delta f_{1st} = (\alpha' - \alpha)\frac{1}{A_0}\left(1 + \frac{V_0}{(A_0)^2}\right)$$

The *second round effect* Δf_{2nd} takes changes in the distribution into account. It can be decomposed as follows:

$$\Delta f_{2nd} = \underbrace{\alpha'\left(\frac{1}{A_1} - \frac{1}{A_0}\right)}_{\Delta f_{2nd,RA}} - \underbrace{\alpha'\left(\frac{V_1}{(A_1)^3} - \frac{V_0}{(A_0)^3}\right)}_{\Delta f_{2nd,HA}}$$

The effects of the shock on the output gap are proportional to the effects on the EFP. It is easy to see that:

$$\Delta x = \underbrace{-m_2 m_1 \Delta f_{1st}}_{\Delta x_{1st}} \underbrace{-m_2 m_1 \Delta f_{2nd,RA}}_{\Delta x_{2nd,RA}} \underbrace{-m_2 m_1 \Delta f_{2nd,HA}}_{\Delta x_{2nd,HA}} \tag{8.1}$$

Given the numerical values of the parameters, we can conclude that an increase of the parameter α ($\Delta\alpha = 0.2$) leads to the following change of the output gap:

- a negative direct effect ($\Delta x_{1st} = -1.4795 \times 10^{-2}$);
- a positive indirect RA effect ($\Delta x_{2nd,RA} = 2.761689 \times 10^{-3}$)
- a negative indirect HA effect ($\Delta x_{2nd,HA} = -3.4667 \times 10^{-3}$)

[13] Fiscal and monetary parameters are incorporated in the convolution of parameters denoted with x_1.

The second round effect is of one order of magnitude smaller than the first round effect. The RA effect is positive while the HA effect is negative but the former is much smaller than the absolute value of latter. The HA effect therefore accounts for the negative sign of the second order effect.

9 Conclusion

In this paper we have pursued further a line of research on Hybrid Macroeconomic ABMs which allows to resume macroeconomic thinking in a multiagent context. We consider a population of firms characterized by heterogeneous financial conditions. Each firms chooses the optimal level of investment in the presence of a financial friction. Hence individual investment depends on individual financial robustness captured by net worth. We aggregate individual investment by means of a stochastic procedure which resorts to the first and second moments of the distribution of net worth. Aggregate investment therefore will be affected by the interest rate and by the first and second moments of the distribution. We use this behavioral aggregate equation in the context of an IS-AS-TR framework where the IS curve is augmented by the moments of the distribution. Therefore, in equilibrium, the interest rate, inflation and the output gap will be functions of the moments mentioned above. The evolution over time of individual net worth turns out to be a function of the cross-sectional mean and variance (through the equilibrium interest rate). We simulate the model to understand the statistical properties of the results. We explore the consequences of three types of shocks. Thanks to our modelling strategy we are able to disentangle the first round effect of a shock (keeping the distribution unchanged) and the second round effect and to distinguish the specific role played by heterogeneity in the latter.

- In all the scenarios considered (fiscal shock, monetary shock, financial shock), the first round effect explains most of the actual change of the output gap.
- The second round effect amplifies the effect of a contractionary monetary shock and of the financial shock and mitigates the effect of the expansionary fiscal shock. In the latter case, in fact, the financial transmission mechanism contributes to crowding out.
- In the case of the fiscal and monetary shock, heterogeneity explains 40% of the second round effect.
- In the case of the financial shock, the entire second round effect is due to heterogeneity.

The benchmark model lends itself to a wide range of possible extensions, such as the explicit consideration of income and wealth inequality among households.

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