Animal Spirits, Financial Markets and Aggregate Instability*

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Abstract

People's animal spirits are a significant driver behind the fluctuations of the U.S. business cycle. This insight is demonstrated within an estimated artificial economy with financial market frictions. Animal spirits shocks account for around 40 percent of output fluctuations over the period from 1955 to 2014. Financial friction and technology shocks are considerably less important with best point estimates for both near 20 percent. We also find that the Great Recession, for the most part, was caused by adverse shocks to expectations.

* *Keywords:* Endogenous financial frictions, indeterminacy, animal spirits, business cycles, Bayesian estimation. *JEL classification:* **E32**, **E44**.

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1 Introduction

What are the shocks that cause macroeconomies to experience recurrent sequences of booms and slumps? The current paper attends to this question by presenting evidence on the sources of business cycles for the post-Korean War American economy. The results back the view that people's psychological motivations, a.k.a. animal spirits, provoke a significant portion of the fluctuations in aggregate real economic activity, causing around forty percent of U.S. output volatility. This insight is demonstrated within an artificial economy with financial market frictions. Likewise, our exercise suggests that it was chiefly adverse shocks to expectations that led to the Great Recession.

Models with credit market frictions have become popular since the Great Recession and this interest reflects the notion that disruptions to financial markets were the key factors behind this contraction. Building on earlier work, such as Kiyotaki and Moore (1997) as well as Bernanke et al. (1999), this research has shown how financial market frictions can amplify shocks to macroeconomic fundamentals by transforming small economic disturbances into large business cycles.[1] Del Negro et al. (2015) as well as Christiano et al. (2015), for example, extend New Keynesian models by financial market frictions to explain some key aspects of the recession.

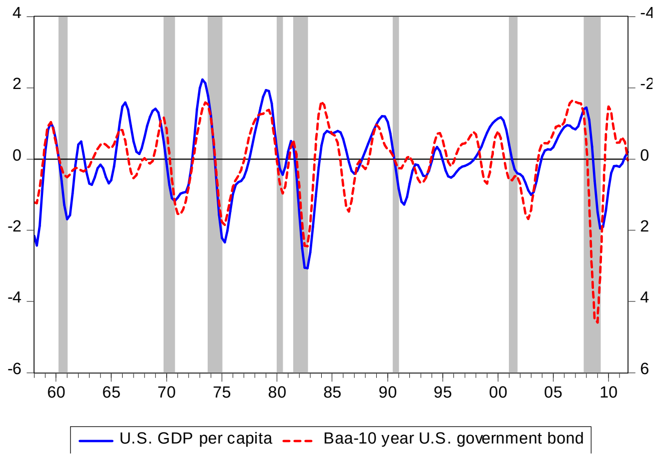


Figure 1: U.S. GDP and credit spread (on right-hand scale) at business cycle frequencies. Shaded areas indicated NBER recessions.

We depart from the aforementioned works twofold. First, the parametric space of the model includes multiple equilibria such that self-fulfilling nonfundamental stochastic shocks to beliefs can act as impulses of endogenous cycles. Second, unlike most existing work on such indeterminacy, the analysis concentrates on estimating the model and focuses on the empirical implications of the multiplicity by explicitly analyzing the business cycle variance contribution of animal spirits or belief shocks. The undertaking is implemented by building on a variant of Benhabib and Wang

[1]See also Nolan and Thoenissen (2009).

(2013).[2] Indeterminacy in this model is linked to the empirically observed countercyclical movement of financial market tightness. Figure 1 plots the pattern of financial market health and aggregate economic activity. Financial health is instrumented by the Baa Corporate Bond Spread which is displayed on an inverted scale and opposite the fluctuations of per capita GDP. The shaded areas in the figure correspond to NBER recessions. They highlight that financial conditions are not only countercyclical but also deteriorate markedly during most slumps.

In the artificial economy, the interaction of a time varying collateral constraint and a countercyclical markup spawns equilibrium indeterminacy, a condition that allows aggregate fluctuations to be caused by extrinsic changes to people's expectations. Moreover, in addition to such animal spirits shocks, the economy is buffeted by a parade of fundamental shocks. The model is estimated by full information Bayesian methods using quarterly U.S. data covering the period from 1955:I to 2014:IV. This approach follows various key contributions by Otrok (2001), Justiniano et al. (2011) as well as Schmitt-Grohé and Uribe (2012), who all, however, only explore the role of fundamental shocks as the engines of business cycles. The Bayesian estimation chooses disturbances so that the probability of empirical series (i.e. the observables) is maximized and the key result that ensues from this exercise is that animal spirits are important drivers of the repeated fluctuations of the U.S. macroeconomy. Specifically, by computing forecast error variance decompositions, we find that animal spirits account for about forty percent of U.S. output variations and for about two thirds of the fluctuations in investment. Disturbances that originate in the financial sector explain less than ten percent of output fluctuations. Moreover, historical decomposition of output growth shows that belief shocks have decidedly (although not exclusively) contributed to the sharp contraction in economic activity of the Great Recession that began at the end of 2007.

Previous work on (real) multiple equilibria economies has overwhelmingly remained in the theoretical realm and estimation exercises have been rare. Farmer and Guo (1995) is an early attempt to estimate a sunspot model using classical simultaneous equations methods. Pintus et al. (2016) and Pavlov and Weder (2017) perform full-information Bayesian estimations as in the present paper. While Pintus et al. build a model with financial market frictions, they do not establish a significant role for animal spirits. Financial markets are not featured in Pavlov and Weder and their study purposely excludes the Great Recession.

2 The Model

The artificial economy is a discrete-time adaptation of Benhabib and Wang (2013). The model features credit frictions in the form of endogenous borrowing constraints

[2] Azariadis et al. (2016), Liu and Wang (2014) and Harrison and Weder (2013) are other examples that combine multiple equilibria and financial frictions.

in a model of monopolistic competition in which, as usual, perfectly competitive firms produce final output by combining a continuum of differentiated intermediate inputs. Intermediate goods producing firms are collateral-constrained in how much they can borrow to finance their working capital needs. We modify the original model by incorporating a set of fundamental shocks which are frequently considered as key drivers of business cycles. The model's discussion will be relatively brief and it will concentrate on the alterations from the original setup.

2.1 Technology

A unit mass of monopolistic competitive firms has access to a constant returns technology that transforms capital services $\kappa_t(i)$ and labor hours $N_t(i)$ into intermediate, differentiated outputs $Y_t(i)$

$$Y_t(i) = \kappa_t(i)^{\alpha}(X_t N_t(i))^{1-\alpha} \qquad 0 < \alpha < 1.$$

Exogenous labor-augmenting technological progress X_t affects all firms equally. Its growth rate $\mu_t^x \equiv X_t/X_{t-1}$ evolves as a first-order autoregressive process

$$\ln \mu_t^x = (1-\rho_x)\ln \mu^x + \rho_x \ln \mu_{t-1}^x + \varepsilon_{x,t} \qquad 0 < \rho_x < 1$$

with $\varepsilon_{x,t} \backsim N(0, \sigma_x^2)$ and $\ln \mu^x$ is average growth rate. The firms rent factor services from the households at perfectly competitive prices W_t and r_t. Final output Y_t is a constant elasticity of substitution aggregator of a basket of intermediate inputs

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\lambda-1}{\lambda}} di\right)^{\frac{\lambda}{\lambda-1}} \qquad \lambda > 1.$$

Here λ denotes the elasticity of substitution between varieties. The monopolistic competitive firms generate profits by charging a mark-up over marginal costs. They must borrow for working capital needs. Imperfect enforcement requires a process to constrain borrowing by the value of the collateral. Specifically, firm i's total amount of debt is an intraperiod loan $B_t(i)$

$$B_t(i) = W_t N_t(i) + r_t \kappa_t(i)$$

and it is constrained by the value of the collateral, which is the output being produced, i.e.

$$W_t N_t(i) + r_t \kappa_t(i) \leq \theta_t \xi_t P_t(i) Y_t(i).$$

Under this credit constraint, if there is a default event, the lender has the right to recover a fraction of less than one of the firm's end-of-period value of output $P_t(i)Y_t(i)$.[3]

[3] Unlike the original model, our setup does not include fixed liquidation costs. Indeterminacy still holds. When we compare the two models using Bayesian estimation method, we find that the model without fixed costs is favored by the data.

The model features two financial frictions and their product $\theta_t \xi_t$ represents the artificial economy's financial tightness. Concretely, ξ_t refers to an endogenous credit constraint: the borrowing constrictions vary with the aggregate state of economic activity which reflects creditors' ability to pay back loans. In particular, ξ_t is an increasing function of the deviation of actual from balanced growth output

$$\xi_t = \tau \left(\frac{Y_t}{\overline{Y}_t} \right)^{\gamma}$$

with the parameter restrictions $0 < \tau < 1$ and $\gamma > 0$. The parsimonious formulation of ξ_t entails many micro-founded makeups without the need to confine itself to a particular one. For example, it can stand in for Benhabib and Wang's (2013) original setup with fixed liquidation costs. In addition to the endogenous component, exogenous disturbances θ_t affect financial health. These shocks originate in the financial sector as in Jermann and Quadrini (2012) and Liu et al. (2013). The collateral or financial shock θ_t evolves as

$$\ln \theta_t = (1 - \rho_\theta) \ln \theta + \rho_\theta \ln \theta_{t-1} + \varepsilon_{\theta,t} \qquad 0 < \rho_\theta < 1$$

with $\varepsilon_{\theta,t} \backsim N(0, \sigma_\theta^2)$ and steady state value $\theta = 1$. The corresponding first-order conditions for the profit maximization problem involve

$$r_t \kappa_t(i) = \alpha \phi_t Y_t(i)$$

$$W_t N_t(i) = (1 - \alpha) \phi_t Y_t(i)$$

and

$$\frac{\lambda - 1}{\lambda} P_t(i) - \phi_t + \mu_t(i) \left[\theta_t \xi_t \frac{\lambda - 1}{\lambda} P_t(i) - \phi_t \right] = 0 \tag{1}$$

where ϕ_t stands for monopolistic firms' marginal costs and $\mu_t(i)$ denotes the multiplier associated with the borrowing constraint.

2.2 Preferences

Households are represented by an agent with the lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\ln(C_t - \Delta_t) - \varphi \frac{N_t^{1+\eta}}{1+\eta} \right) \qquad 0 < \beta < 1,\ \eta \geq 0 \text{ and } \varphi > 0$$

where β is the discount factor, C_t stands for consumption, and N_t for total hours worked. The functional form of the period utility ensures that the economy is consistent with balanced growth. The parameter φ denotes the disutility of working. The term Δ_t represents shocks to the agent's utility of consumption that generate urges to consume, as in Baxter and King (1991) and Weder (2006). The preference shock follows the autoregressive process

$$\ln \Delta_t = \rho_\Delta \ln \Delta_{t-1} + \varepsilon_{\Delta,t} \qquad 0 < \rho_\Delta < 1$$

with $\varepsilon_{\Delta,t} \backsim N(0, \sigma_\Delta^2)$. This shock is also a driver of the economy's labor wedge, i.e. the gap between the marginal rate of consumption-leisure substitution and the marginal product of labor. Hence, our estimation will allow a much wider interpretation than mere shocks to preferences – a more agnostic reading includes, for example, changes to monetary policy, taxes, or labor market frictions. Households own the physical capital stock K_t and decide on its utilization rate, u_t, thus $\kappa_t = u_t K_t$. The agent faces the period budget constraint

$$C_t + A_t I_t + T_t = W_t N_t + r_t u_t K_t + \Pi_t$$

and the law of motion for capital is

$$K_{t+1} = (1 - \delta_t) K_t + I_t.$$

The term I_t is investment spending and A_t represents a non-stationary investment-specific technology shock which affects the transformation of consumption goods into investment goods. In the model, the concept corresponds to the relative price of new investment goods in terms of consumption goods. The shock's growth rate μ_t^a evolves as

$$\ln \mu_t^a = (1 - \rho_a) \ln \mu^a + \rho_a \ln \mu_{t-1}^a + \varepsilon_{a,t} \qquad 0 < \rho_a < 1$$

with $\varepsilon_{a,t} \backsim N(0, \sigma_a^2)$, and $\ln \mu^a$ is the average growth rate. Lump-sum taxes are denoted by T_t. The rate of physical capital depreciation

$$\delta_t = \delta_0 \frac{u_t^{1+\nu}}{1+\nu} \qquad 0 < \delta_0 < 1 \text{ and } \nu > 0$$

is an increasing function in the utilization and $\nu > 0$ measures the elasticity of the depreciation rate with respect to capacity used. The first-order conditions are standard and delegated to the Appendix.

2.3 Government

The government purchases G_t units of the final output. G_t is neither productive nor does it provide any utility. The spending is financed by the lump-sum taxes. We model government's spending with a stochastic trend

$$X_t^G = (X_{t-1}^G)^{\psi_{yg}} (X_{t-1}^Y)^{1-\psi_{yg}} \qquad 0 < \psi_{yg} < 1$$

where ψ_{yg} governs the smoothness of the government spending trend relative to the trend in output. Then, detrended government spending is $g_t \equiv G_t / X_t^G$ and this follows the process

$$\ln g_t = (1 - \rho_g) \ln g + \rho_g \ln g_{t-1} + \varepsilon_{g,t} \qquad 0 \leq \rho_g < 1$$

with the shock's variance σ_g^2.

2.4 Equilibrium

In symmetric equilibrium, $\kappa_t(i) = u_t K_t$, $N_t(i) = N_t$, $P_t(i) = P_t = 1$, $Y_t(i) = Y_t$ and $\Pi_t(i) = \Pi_t = Y_t - W_t N_t - r_t u_t K_t$, hold and (1) becomes

$$\frac{\lambda - 1}{\lambda} - \phi_t + \mu_t \left[\theta_t \xi_t \frac{\lambda - 1}{\lambda} - \phi_t \right] = 0. \tag{2}$$

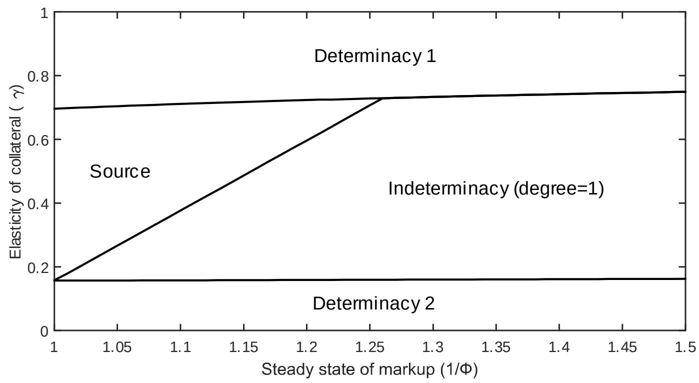
From (2), and if $\theta_t \xi_t \frac{\lambda-1}{\lambda} < \phi_t < \frac{\lambda-1}{\lambda}$, the financial constraint binds, thus,

$$\phi_t = \theta_t \xi_t = \tau \theta_t \left(\frac{Y_t}{\bar{Y}_t} \right)^{\gamma}.$$

In the steady state, τ equals marginal costs thus it is not a free parameter.

2.5 Self-fulfilling dynamics

The detrended and linearized economy is solved numerically (using standard parameters as listed in Table 1). We assume that the credit constraint is always binding. Figure 2 maps the local dynamics' zones in the $\gamma - \phi^{-1}-$space. If market power is small, i.e. the inverse of the marginal cost ϕ^{-1} is close to one and the credit limit is constant, i.e. the curvature parameter γ is small, the economy's dynamics are unique. However, combinations of market power and a procyclical credit limit delivers indeterminacy. The indeterminacy mechanism operates via an upwardly sloping wage-hours locus similar to many animal spirits models.[4] Then, how can pessimistic expectations about the future create problems? If people believe that the future is worse, they will attempt to work more hours. In terms of the labor market equilibrium, this change in expectations will shift the labor supply curve out. But the pessimistic expectations will lead households to decrease the lending to firms. This contraction of credit will tighten the firms' borrowing constraints; costs and markups will rise and the individual labor demand schedules move leftwards. As a consequence, the economy's wage-hours-locus is upwardly sloping. In equilibrium, the outward shift of labor supply will result in lower employment and in a drop in aggregate production, in sum, the low animal spirits will be self-fulfilling.



[4]See for example, Farmer and Guo (1994) or Wen (1998).

Figure 2: Parameter space for dynamics.

3 Estimation

The artificial economy's local dynamics can become indeterminate. Our next step is to discuss how animal spirits are introduced into the model, to present the data that is employed in the analysis, as well as to outline the full information Bayesian estimation of the artificial economy. Finally, we compare the estimated shocks to corresponding empirical measures.

If there are many rational expectations equilibria in the model economy, this continuum is a device to introduce animal spirits. To do this, we break down the forecast error of output

$$\eta_t^y \equiv \hat{y}_t - E_{t-1}\hat{y}_t$$

into fundamental and non-fundamental components, as suggested by Lubik and Schorfheide (2003):

$$\eta_t^y = \Omega_x \varepsilon_t^x + \Omega_a \varepsilon_t^a + \Omega_\Delta \varepsilon_t^\Delta + \Omega_g \varepsilon_t^g + \Omega_\theta \varepsilon_t^\theta + \varepsilon_t^b.$$

The parameters Ω_x, Ω_a, Ω_Δ, Ω_g and Ω_θ determine the effect of technological progress, investment-specific technology, preferences, government spending and collateral shocks on the expectations error. This break-down leaves the belief shock ε_t^b as a residual. The last equation promulgates a strict definition of animal spirits: they are orthogonal to the other disturbances, thus independent of economic fundamentals.

We now estimate the model, allowing all six shocks to matter. The approach attributes the contribution of each shock to aggregate fluctuations. The estimation uses quarterly U.S. data running from 1955:I to 2014:IV and includes seven observable time series: (i) the log difference of real per capita GDP, (ii) real per capita consumption, (iii) real per capita investment, (iv) real per capita government spending, (v) the relative price of investment, (vi) the log difference of per capita hours worked from its sample mean, as well as (vii) the credit spread from its sample mean. We instrument financial market conditions by a credit spread similar to Christiano et al. (2014). In particular, Christiano et al. make use of the difference between the interest rate on Baa corporate bonds and the ten-year US government bond rate. The Appendix provides a full description of the data and its construction. The corresponding measurement equation is

$$\begin{bmatrix} \ln Y_t - \ln Y_{t-1} \\ \ln C_t - \ln C_{t-1} \\ \ln A_t I_t - \ln A_{t-1} I_{t-1} \\ \ln G_t - \ln G_{t-1} \\ \ln A_t - \ln A_{t-1} \\ \ln N_t - \ln \bar{N} \\ \text{credit spread} \end{bmatrix} = \begin{bmatrix} \hat{y}_t - \hat{y}_{t-1} + \hat{\mu}_t^y \\ \hat{c}_t - \hat{c}_{t-1} + \hat{\mu}_t^y \\ \hat{\imath}_t - \hat{\imath}_{t-1} + \hat{\mu}_t^y \\ \hat{g}_t - \hat{g}_{t-1} + \hat{a}_t^g - \hat{a}_{t-1}^g + \hat{\mu}_t^y \\ \hat{\mu}_t^a \\ \hat{N}_t \\ -x * \phi * \hat{\phi}_t \end{bmatrix} + \begin{bmatrix} \ln \mu^y \\ \ln \mu^y \\ \ln \mu^y \\ \ln \mu^y \\ \ln \mu^a \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \varepsilon_{y,t}^{me} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \varepsilon_{s,t}^{me} \end{bmatrix}$$

where $a_t^g \equiv X_t^G/X_t^Y = (a_{t-1}^g)^{\psi_{yg}}(\mu_t^y)^{-1}$. In the last measurement equation, x is the scale parameter only appearing in the measurement equation to adjust the difference of the volatilities (that is, units) between the model frictions and the observable variable. Both output growth and credit spread are measured with errors $\varepsilon_{y,t}^{me}$ and $\varepsilon_{s,t}^{me}$ which are i.i.d. innovations with mean zero and standard deviation σ_y^{me} and σ_s^{me}, respectively. Allowing for a measurement error to output is a way to circumvent stochastic singularity (e.g. Schmitt-Grohé and Uribe, 2012). A measurement error to the spread can account for any mis-measurement in the data, especially when only a proxy is observed (e.g. Justiniano et al., 2011). Both measurement errors are restricted to absorb not more than ten percent of the variance of the corresponding observables.

Prior to the estimation, we fix a number of parameters. This set of parameters is calibrated following the literature and is based on national accounts data averages. We only address some of these calibrations (all are listed in completion in Table 1). The elasticity of substitution parameter λ is set at ten, as in Dotsey and King (2005) and Cogley and Sbordone (2008). The average government spending share in GDP, G/Y, is calibrated at 21 percent, a number which we take from national accounts. The quarterly growth rates of per capita output μ^y and the relative price of investment μ^a are set equal to their sample averages of 1.0041 and 0.9949. Finally, the household's first-order conditions determine the elasticity of the depreciation rate from $\nu = (\mu^k/\beta - 1)/\delta$.

The other model parameters are estimated. Our prior assumptions are summarized in Table 2. The parameters estimated here include the steady state marginal cost ϕ (or equivalently the inverse of the mark-up), the elasticity of collateral γ, the scale parameter x, the parameters that describe the stochastic processes and the standard deviation of the measurement error. A beta distribution is adopted for the steady-state marginal cost ϕ and its value falls between 0.83 and 0.9, so that the steady-state markup varies from around 11 to 20 percent. The range of marginal costs is chosen for two reasons. First, the empirically estimated markup falls in this range (see for example Cogley and Sbordone, 2008). Second, the upper value of ϕ is further restricted by the inequality constraints $\xi\frac{\lambda-1}{\lambda} < \phi < \frac{\lambda-1}{\lambda}$ for the financial constraint to bind.[5] We set the prior mean for x to match the standard deviation of the smoothed endogenous financial frictions in the model without any financial information (data and shock) and the standard deviation of the demeaned spread data. We adopt an inverse gamma distribution for the prior. For the persistence parameters we use a beta distribution and the standard deviations of the shocks follow an inverse gamma distribution. The prior distributions for the expectational parameters Ω_x, Ω_a,

[5] To land in the prior region of γ, we calculate the required region of γ to generate indeterminacy given each value of ϕ, as shown in Figure 1. We choose the minimum value 0.16 as the lower bound of the prior region, while 0.607 is the upper limit. This range will guarantee that we can cover the complete indeterminacy region. Since our model is indeterminate, during the MCMC, all proposed draws from the determinacy and source regions were discarded.

Ω_Δ, Ω_g and Ω_θ are uniform, thus agnostic about their values. Endogenous priors prevent overpredicting the model variances (as in Christiano et al., 2011). We use the Metropolis-Hastings algorithm to obtain one million draws from the posterior for each of the two chains, discard half of the draws, and adjust the scale in the jumping distribution to achieve a 25-30 percent acceptance rate for each chain.

The last two columns of Table 2 present the posterior means of the estimated parameters, along with their 90 percent posterior probability intervals. The parameters are precisely estimated as is evidenced by the percentiles. The estimated steady state of marginal cost implies a steady state markup of twenty percent. Preference, government spending and collateral shocks exhibit a high degree of persistence. The autocorrelation of the non-stationary technology shock is low, but it is not inconsistent with the moderate values commonly found in the literature.

Table 3 reports second moments of the main macroeconomic variables calculated using U.S. data and compares these moments to those obtained from model simulations at the posterior mean. The model underpredicts the volatility, but it matches fairly well the relative standard deviations, autocorrelations and the variables' cross-correlations with output. Table 4 displays the contribution of each structural shock, which we list in the first row, to the variances of key macroeconomic variables. The decomposition suggests that animal spirits shocks ε_t^b are the most important source of U.S. aggregate fluctuations. These shocks account for over forty percent of output growth. The Appendix presents a more detailed analysis of the beliefs driven cycle in the spirit of Burns and Mitchell (1946). The other aggregate demand shocks play a lesser role and the contribution of the two technology shocks is small at no more than twenty percent. For investment, the vast majority of its variations comes from animal spirits suggesting that much of the spending is driven by entrepreneurial sentiments. The credit spread (i.e. the financial frictions) is mainly driven by stochastic financial factors – by about forty percent.[6]

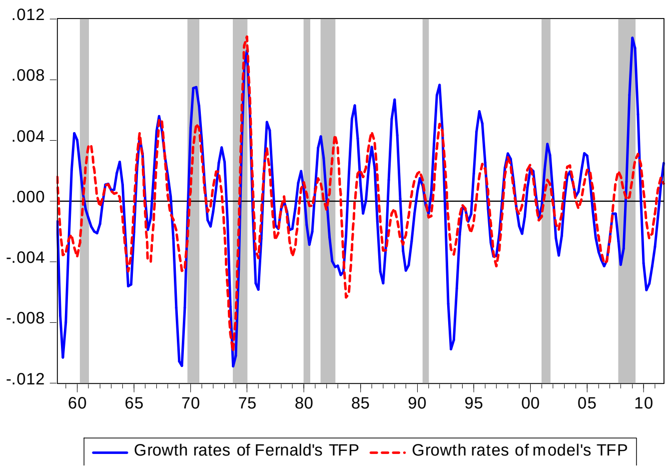


Figure 3: Fernald's vs Model's total factor productivity

[6] We also estimate the model using loan data and animal spirits remain important.

We identify the shocks by estimating in a system and it is thus fair to ask if the shocks are meaningfully labelled. More concretely, do the shocks share resemblance with empirical series that are computed with orthogonal information sets? To begin with, the estimated model's total factor productivity series is compared with Fernald's (2014) total factor productivity series for the United States.[7] Fernald's series are widely considered as the gold standard for this variable for which he adjusts for variations in factor utilization (labor effort and the workweek of capital) as well as labor skills. The results are reassuring as shown in Figure 3. Both productivity series not only have similar amplitudes, but their contemporaneous correlation comes in at 0.65. Next, Figure 4 compares the index of estimated confidence and the U.S. Business Confidence index (band-pass filtered to concentrate on the relevant frequencies). Clearly, the empirical confidence index is influenced by a raft of fundamentals and non-fundamentals, thus, it is not exactly clear how the empirical data would map our theoretical notion of animal spirits. Yet, the two confidence series are strongly correlated with coefficient 0.64 and we interpret the relationship in Figure 4 as endorsing our estimation and as supporting the case that estimated shocks reflect variations in people's expectations about the future path of the economy.[8] Furthermore, our estimated disturbances share similarity to Angeletos et al.'s (2016, Figure 8) confidence shocks. While Angeletos et al. argue in a unique-equilibrium model, we interpret the resemblance as complementary stories of the business cycle.

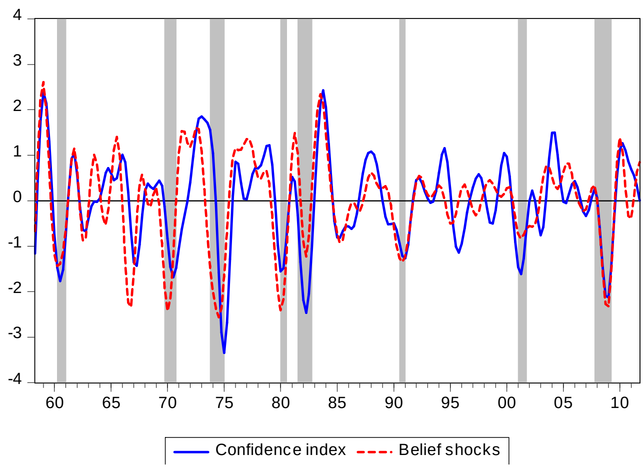


Figure 4: Business confidence index vs animal sprits shocks (normalized data).

4 Checks of robustness

In this section, we consider several robustness checks: (i) Lubik and Schorfheide's (2003) representation of a belief shock is compared to Farmer et al.'s (2015) formulation, (ii) we go through alternative observables to measure financial markets' health,

[7]Growth of total factor productivity in our model is given by $(1-\alpha)(\hat{\mu}_t^x + \ln \mu^x)$.

[8]The correlation of the estimated sunspot shocks and Fernald's TFP series is -0.17.

(iii) Fernald's (2014) TFP data is added to the observables, and (iv) permanent technology shocks are replaced by transitory shocks.

To demonstrate the robustness of the above insights, we follow the approach of Farmer et al. (2015) in which the animals spirits shock is simply the forecast error, i.e. $\eta_t^Y = \varepsilon_t^s$, with variance σ_η^2. Intuitively, since output is forward looking, this expectation error should be correlated with fundamental shocks. Yet, it is also a sunspot shock, as it can cause movements in economic activity without any shifts to fundamentals. Assuming a uniform distribution, we thus estimate the correlations between η_t^Y and the fundamental shocks. The priors for the other parameters are kept the same as in the baseline model. As can be seen from Tables 2 and 5, our estimation results are robust to the formation of the expectation error. The posterior distributions are almost identical and the closeness of the log-data densities confirms that the goodness of fit between the models is equivalent.[9]

The next robustness check concerns the choice of the observed spread when instrumenting financial markets' conditions. We thus consider the sensitivity to using various alternative spreads. In particular, we sequentially explore if (i) the Baa-Aaa spread, (ii) the Baa-Federal funds rate spread or (iii) the Gilchrist and Zakrajšek's (2012) spread yield significantly different results in the estimation. We report the variance decompositions only. The results for the Baa-Aaa and Baa-Federal funds rate spreads are reported in Tables 6 and 7. Animal spirits continue to stand out as the main driver of the business cycle. The tables suggest that they account for about 40 percent of the U.S. output fluctuations. Only when using Gilchrist and Zakrajšek's (2012) spread do financial shocks' contributions climb to slightly over ten percent.[10]

Next, we add total factor productivity to the catalog of observables. Fernald's (2014) continuously updated data is the natural series to choose from. Fernald adjusts for variations in factor utilization (labor and capital) and includes adjustment for quality or composition. Most of these influences are not part of the artificial economy and we thus add one more measurement error on total factor productivity (at not more than ten percent). Table 9 shows that the previous results remain robust. Animal spirits continue to cause the bulk of U.S. output fluctuations. The technology shocks' contributions are lower, with a best point estimate near 10 percent.

Finally, we replace permanent technology shocks by transitory shocks. Then, the production technology is given by

$$Y_t = Z_t K_t^\alpha (\mu^t N_t)^{1-\alpha}$$

and the growth rate of labor augmenting technological progress is deterministic at the constant rate μ, as in King et al. (1988). We permit temporary changes in total

[9] Second moments and variance decompositions are virtually identical and are not presented to conserve space.

[10] However, this change appears to be mainly the outcome of a shorter sample given the spread's availability. To confirm this, we re-estimated the model using the other spreads, but only covering the 1973 to 2014 period. As expected, the results then came out very similar to Table 8's

factor productivity through Z_t, which follows a first-order autoregressive process

$$\ln Z_t = (1-\rho_z)\ln Z + \rho_z \ln Z_{t-1} + \varepsilon_{z,t} \qquad 0 < \rho_z < 1.$$

The model estimation delivers similar posterior means of the parameters as the baseline estimation and they are thus not reported here (see Appendix). Noteworthy is, however, the estimate for ρ_z at 0.997. While high, this number is consistent with Ireland (2001), for example. The variance decompositions of the stationary technology shocks model are reported in Table 10. Technology shocks account for about 17 percent of GDP volatility. However, animal spirits remain the most critical driver of aggregate fluctuations and they continue to explain roughly forty percent of output growth variations. Now, which specification of technology is favored by data? This question is answered in Table 11 which compares the model fits of the two alternatively specified models. Data strongly prefers a version of the model in which total factor productivity has a stochastic trend. We have conducted further robustness checks that are, however, delegated to the Appendix to conserve space.

5 A closer look at the Great Recession

From 2007 to 2009, the U.S. economy was in the turmoil of a severe recession. The Great Recession was the single-worst economic contraction since the 1930s, with economic activity diving after various financial institutions collapsed. One of the aims of the recent financial friction models is to identify the sources of the crisis. We follow this line and look more closely into the years 2007 to 2009.

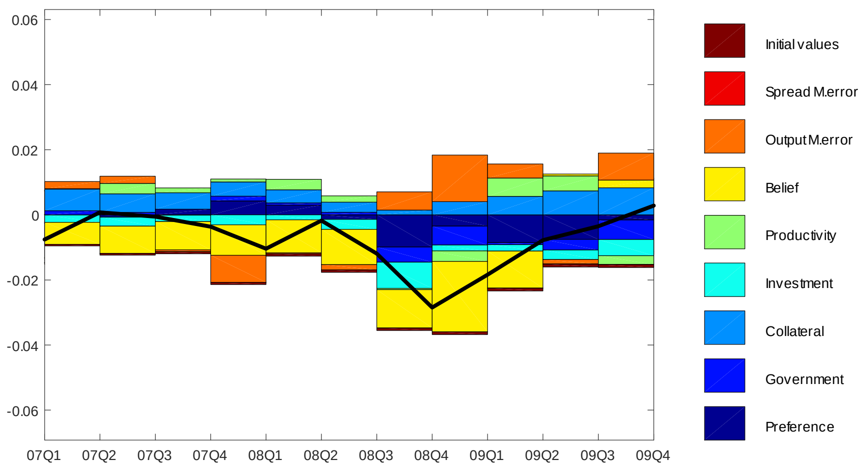


Figure 5: Historical decomposition of output growth

To begin with, we plot in Figure 5 the historical decomposition of the structural shocks to output growth over the 2007:I to 2009:IV period. The figure suggests that it was foremost pessimistic expectations that contracted the economy from the end of 2007 onwards. Thus, the data favors the interpretation that the Great Recession was closely associated with self-fulfilling beliefs. Our interpretation of events goes like this: when the run-up of real estate prices came to an end, when banks curbed lending and tightened credit, when investors stopped borrowing, then this occurred because people were expecting worsening business conditions and higher defaults. In other words, people became pessimistic and, as a consequence of the effect on financial markets, this pessimism became self-fulfilling. Thus, our results do not necessarily contradict Christiano et al.'s (2015) account of the Great Recession. Their study finds that the steep decline of aggregate economic activity was overwhelmingly caused by (exogenous) financial frictions. What our analysis suggests is, however, that a sudden drop in people's animal spirits found its catalyst in financial markets. In this reading of events, the financial sector propagated gloomy animal spirits into a full-blown financial crisis and disastrous macroeconomic collapse ensued.

Moreover, Brinca et al. assert that

> "[...] considering the period from 2008 until the end of 2011, [our] results imply that the Great Recession in the United States should be thought of as primarily a labor wedge recession." [Brinca et al., 2017, 1042]

What does the labor wedge look like in the artificial economy's benchmark version? The model's labor wedge is driven by fluctuations of both the markup as well as stochastic preferences. It is plotted along with the data equivalent in Figure 6. Clearly, the two series show high conformity. The artificial wedge explains 75 percent of the data wedge's plunge during 2008 and 2009 and it charts a tepid recovery over the 2010 to 2014 period. Thus, to an extent, our model is not inconsistent with Brinca et al.'s (2017) interpretation of the Great Recession.

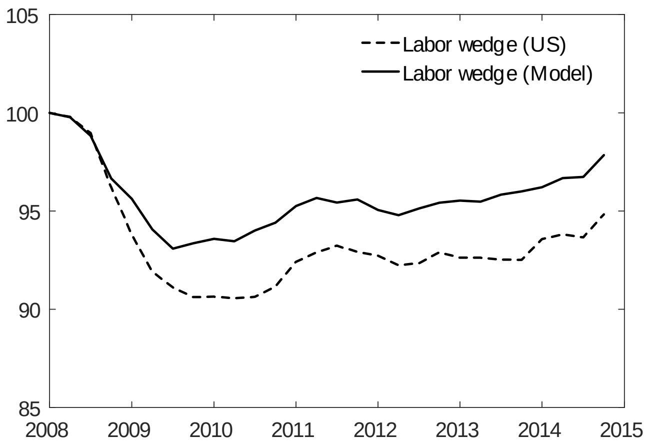


Figure 6: The artificial labor wedge during the Great Recession.

6 Does data prefer indeterminacy?

So far we have restricted the analysis to the parameter space with multiple equilibria, yet a natural question arises: does data in fact favor a model with indeterminacy? To answer this question, we now estimate the economy across the complete area of the parameter space as in Bianchi and Nicolò (2017)[11]. Their procedure can be implemented without knowing the boundaries between the four dynamic regions (see Figure 2). During the mode-finding process, the estimation can get stuck in a region without crossing the boundaries, even though, in theory, the Metropolis-Hastings algorithm can explore the entire parameter space. Therefore, our implemented estimation strategy was to set different initial values (in all regions) while leaving the priors the same. Once a region-specific (i.e. local) mode was found, the Markov chain Monte Carlo procedure was run at each mode. The priors are such that indeterminacy has the least prior probability so to err on the right side. To do this, we adjust the prior of the elasticity of collateral γ, which is now gamma-distributed centered at 0.5 with standard deviation 0.8. The prior for the parameter φ^* is uniformly distributed within [0,2]. In line with Bianchi and Nicolò (2017), we follow the approach proposed in Farmer et al. (2015) and construe the forecast error of output η_t^y as a belief shock with variance σ_η^2 and allow the expectation errors to be correlated with the fundamental shocks (Table 5 reports the equivalence of this setup to our benchmark model). The log data densities in Table 12 suggest that U.S. data strongly favours the indeterminacy model over versions of the economy in which animal spirits do not play a role.

7 Concluding remarks

This paper has presented evidence on the sources of U.S. aggregate fluctuations over the period 1955 to 2014. We perform a Bayesian estimation of a financial accelerator model which features an indeterminacy of rational expectations equilibria. Indeterminacy in the model is linked to the empirically observed countercyclical movement of financial market tightness. The interaction of time-varying collateral constraints and a countercyclical markup brings about an upwardly sloping wage-hours-locus and aggregate fluctuations can be driven by changes to people's animal spirits. The artificial economy is driven both by fundamental shocks as well as by animal spirits and its estimation supports the view that people's animal spirits play a significant role for the U.S. business cycle. Moreover, data favours the indeterminacy model over versions of the economy in which sunspots do not play a role. Variance decompositions suggest that animal spirits are behind around forty percent of output growth variations and they explain an even larger portion of fluctuations in invest-

[11] The Appendix explains their methodology in more detail.

ment spending. Technology shocks and financial frictions shocks are significantly less important and they explain no more than twenty percent of the oscillations in aggregate real economic activity. The 2007-2009 recession appears to have been chiefly caused by adverse confidence shocks.

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8 Appendix

The (not-to-be-published) Appendix sets out the complete model, a discussion of the typical animal spirits cycle, conducts more robustness checks, and it lists the data sources and definitions. We begin with collecting the model's equations.

8.1 Model equations and equilibrium dynamics

The first-order conditions for the household's optimization problems are

$$\varphi N_t^{\eta} = \frac{1}{C_t - \Delta_t} W_t$$
$$r_t = A_t \delta_0 u_t^{\nu}$$

and

$$\frac{A_t}{C_t - \Delta_t} = \beta E_t \left[\frac{1}{C_{t+1} - \Delta_{t+1}} (r_{t+1} u_{t+1} + A_{t+1}(1 - \delta_{t+1})) \right].$$

In the model, output, consumption, and real wage fluctuate around the same stochastic growth trend $X_t^Y = X_t A_t^{\alpha/(\alpha-1)}$, the growth rate of which is $\mu_t^y \equiv X_t^Y / X_{t-1}^Y = \mu_t^x (\mu_t^a)^{\frac{\alpha}{\alpha-1}}$. The trend in capital stock, which is also the trend in investment equals $X_t^K = X_t^Y / A_t$, the growth rate of which is $\mu_t^k \equiv X_t^K / X_{t-1}^K = \mu_t^x (\mu_t^a)^{\frac{1}{\alpha-1}}$. Besides, the government expenditure fluctuates around its own trend X_t^G. There is no growth trend in hours, utilization and marginal cost. We first derive the detrended dynamic equilibrium equations and then log-linearly approximate them around the deterministic steady state. Let $y_t = Y_t / X_t^Y$, $c_t = C_t / X_t^Y$, $w_t = W_t / X_t^Y$, $i_t = I_t / X_t^K$, $k_t = K_t / X_{t-1}^K$, $g_t = G_t / X_t^G$, and $y_t / \bar{y}$ approximately equal to $Y_t / \bar{Y}_t$, where $\bar{y}$ represents the steady state of detrended output. The log-linearized system is summarized by

$$\hat{y}_t = \alpha \hat{k}_t + \alpha \hat{u}_t - \alpha \hat{\mu}_t^k + (1-\alpha) \hat{N}_t$$
$$\hat{y}_t = [1 - \frac{\alpha \phi (\mu^k - 1 + \delta)}{\delta (1+\nu)} - \frac{G}{Y}] \hat{c}_t + \frac{\alpha \phi (\mu^k - 1 + \delta)}{\delta (1+\nu)} \hat{\imath}_t + \frac{G}{Y} (\hat{a}_t^g + \hat{g}_t)$$
$$\hat{y}_t = (1+\eta) \hat{N}_t + \hat{c}_t - \hat{\Delta}_t - \hat{\phi}_t$$
$$\hat{y}_t = (1+\nu) \hat{u}_t + \hat{k}_t - \hat{\phi}_t - \hat{\mu}_t^k$$
$$\hat{k}_{t+1} = \frac{(1-\delta)}{\mu^k} (\hat{k}_t - \hat{\mu}_t^k) + \frac{(\mu^k - 1 + \delta)}{\mu^k} \hat{\imath}_t - \frac{\delta (1+\nu)}{\mu^k} \hat{u}_t$$
$$\hat{c}_{t+1} = \hat{c}_t - \hat{\Delta}_t - [1 - \frac{\beta \delta (1+\nu)}{\mu^k}] \hat{\mu}_{t+1}^k + \hat{\Delta}_{t+1} + \frac{\beta \delta (1+\nu)}{\mu^k} (\hat{y}_{t+1} - \hat{k}_{t+1} + \hat{\phi}_{t+1} - \hat{u}_{t+1})$$

and

$$\hat{\phi}_t = \gamma \hat{y}_t + \hat{\theta}_t.$$

In these equations, variables without time subscripts refer to steady state values while the hatted variables denote percent deviations from their corresponding steady-state, e.g., $\hat{y}_t \equiv \log(y_t / \bar{y})$.

8.2 A Burns-Mitchell analysis of animal spirits

We employ a classical method of business cycle analysis developed by Burns and Mitchell (1946) and Adelman and Adelman (1959) to evaluate the belief shock driven model in terms of whether it mimics the cyclical behavior of post war U.S. data.[12] A brief description of the idea follows. The business cycle series consist of a sequence of reference cycles, measured trough-to-trough by convention. We use NBER dates to determine the peak of the reference cycle for both U.S. and artificially generated data. Our sample series includes eight complete trough-peak-trough cycles beginning in 1958:II and ending with the lower turning point in 2009:II. No prior filtering or detrending of the data has been undertaken that is we do not detrend the model output to allow for the presence of long-run technological progress and bring it in line with empirical data. Each complete reference cycle is divided into nine stages (I to IX). Stage I is the initial trough; stage V is the reference peak, and stage IX is the terminal trough. The expansion phase (stages I to V) is divided into three substages (II, III, and IV) of equal length (excluding time contained in stages I and V). The contraction phase (stages V to IX) is measured in an analogous fashion. Next, each observation in the cycle is expressed as a percentage of the cycle mean called cycle relatives. Mean cycle relatives per stage are averaged across all reference cycles to yield a graphical summary of an average business cycle in the nine-point-plot of Figure 10. The plot provides a visual impression of both the simulated data and the U.S. data.

Concretely, Figure 7 displays the average behavior, in cycle relatives, over the nine stages of the business cycle for per capita real GDP and the artificial equivalent when the model is counterfactually driven by belief shocks only. Stage I coincides with the initial trough, stage V with the peak, and stage IX corresponds to the terminal trough. The similar general shape of the two series demonstrates that artificial series matches well postwar U.S. cycles. The per capita real GDP exhibits a distinct procyclical pattern, rising during expansions and falling during contractions. Both series peak in the same stage.

[12] King and Plosser (1994) for a concise summary of the Burns-Mitchell procedure as well as its implementation in a general equilibrium context.

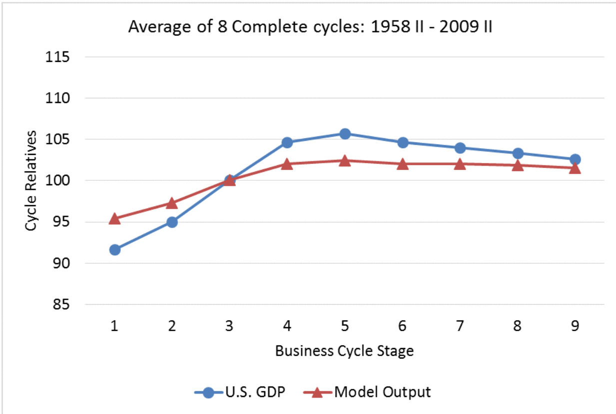

Figure 7: Nine-point graph for U.S. GDP and counterfactually belief driven output

8.3 More robustness checks

Justiniano, Primiceri and Tambalotti (2011) push for shocks that affect the production of installed capital from investment goods or the transformation of savings into the future capital input. This is an alternative way of how to model exogenous financial frictions. The idea of shocks to the marginal efficiency to investment (MEI) goes back to Greenwood, Hercowitz and Huffman (1988) who formulate the ideas as

$$K_{t+1} = (1 - \delta_t)K_t + \nu_t I_t$$

where we abstract from adjustment costs to not mess with the indeterminacy properties of the artificial economy. The shock ν_t affects the marginal efficiency of capital and it follows the process

$$\ln \nu_t = \rho_v \ln \nu_{t-1} + \varepsilon_t^{\nu}.$$

The shock are likely a "proxy for more fundamental disturbances to the functioning of the financial sector." (Justiniano, Primiceri and Tambalotti, 2011). Again, we add a measurement error to the spread equation (to impose discipline on the inference of the MEI shock). Table 13 shows, in line with our previous findings, that the animal spirits shocks remain the most prominent drivers of the U.S. business cycle.

Table 14 shows the posterior means of the parameters in the model with transitory technology productivity. The estimated parameters are similar with that of the baseline estimation.

8.4 Bianchi and Nicolò (2017)

We briefly set out the methodology that we apply. It closely follows Bianchi and Nicolò (2017) and it does not require to know the (analytical solution) of the boundaries of the determinacy region. The parameters of the loglinearized benchmark model are contained in the vector

$$\mathbf{\Theta} = [\alpha, \phi, \mu^y, \mu^a, \mu^k, \delta, \nu, \eta, \beta, \gamma, G/Y, \rho_x, \rho_a, \rho_\Delta, \rho_g, \rho_\theta, \sigma_x, \sigma_a, \sigma_\Delta, \sigma_g, \sigma_\theta].$$

The linear rational expectations (LRE) model can be rewritten in the canonical form

$$\mathbf{\Gamma}_0(\mathbf{\Theta})\mathbf{s}_t = \mathbf{\Gamma}_1(\mathbf{\Theta})\mathbf{s}_{t-1} + \mathbf{\Psi}(\mathbf{\Theta})\boldsymbol{\varepsilon}_t + \mathbf{\Pi}(\mathbf{\Theta})\boldsymbol{\eta}_t, \tag{3}$$

where

$$\mathbf{s}_t = [\hat{y}_t, \hat{c}_t, \hat{\imath}_t, \hat{N}_t, \hat{k}_t, \hat{u}_t, \hat{\phi}_t, E_t[\hat{y}_{t+1}], E_t[\hat{c}_{t+1}], E_t[\hat{\phi}_{t+1}], E_t[\hat{u}_{t+1}], \hat{\mu}^y_t, \hat{\mu}^a_t, \hat{\mu}^k_t, \hat{g}_t, \hat{a}^g_t, \hat{\Delta}_t, \hat{\theta}_t]'$$

is a vector of endogenous variables, $\varepsilon_t = [\varepsilon^x_t, \varepsilon^a_t, \varepsilon^\Delta_t, \varepsilon^g_t, \varepsilon^\theta_t]'$ is a vector of exogenous shocks, and $\eta_t = [\eta^y_t, \eta^c_t, \eta^\phi_t, \eta^u_t]'$ collects the one-step ahead forecast errors for the expectational variables of the system. Since our model can generate at most one degree of indeterminacy, Bianchi and Nicolò suggest to append the original linear rational expectations model (3) with the autoregressive process

$$\omega_t = \varphi^* \omega_{t-1} + \upsilon_t - \eta_{f,t} \tag{4}$$

where υ_t is the sunspot shock and $\eta_{f,t}$ can be any element of the forecast errors vector η_t. We choose $\eta_{f,t} = \eta^y_t$. The variable φ^* belongs to the interval (-1,1) when the model is determinate or it is outside the unit circle under indeterminacy. Under determinacy the Blanchard-Kahn condition is satisfied and the absolute value of φ^* is inside the unit circle since the number of explosive roots of the original LRE model in (3) already equals the number of expectational variables in the model. Then the autoregressive process ω_t does not affect the solution for the endogenous variables s_t. On the other hand, under indeterminacy the Blanchard-Kahn condition is not satisfied. The system is characterized by one degree of indeterminacy and it is necessary to introduce another explosive root to fulfill the Blanchard-Kahn condition – the absolute value of φ^* falls outside the unit circle. Denoting the newly-defined vector of endogenous variables $\hat{s}_t \equiv (s_t, \omega_t)'$ and the vector of exogenous shocks $\hat{\varepsilon}_t \equiv (\varepsilon_t, \upsilon_t)'$, then the system (3) and (4) can be condensed into

$$\hat{\mathbf{\Gamma}}_0\hat{\mathbf{s}}_t = \hat{\mathbf{\Gamma}}_1\hat{\mathbf{s}}_{t-1} + \hat{\mathbf{\Psi}}\hat{\boldsymbol{\varepsilon}}_t + \hat{\mathbf{\Pi}}\boldsymbol{\eta}_t,$$

where

$$\hat{\mathbf{\Gamma}}_0 \equiv \begin{bmatrix} \mathbf{\Gamma}_0(\mathbf{\Theta}) & 0 \\ 0 & 1 \end{bmatrix}, \ \hat{\mathbf{\Gamma}}_1 \equiv \begin{bmatrix} \mathbf{\Gamma}_1(\mathbf{\Theta}) & 0 \\ 0 & \varphi^* \end{bmatrix}$$

and

$$\hat{\mathbf{\Psi}} \equiv \begin{bmatrix} \mathbf{\Psi}(\mathbf{\Theta}) & 0 \\ 0 & 1 \end{bmatrix}, \ \hat{\mathbf{\Pi}} \equiv \begin{bmatrix} \mathbf{\Pi}_n(\mathbf{\Theta}) & \mathbf{\Pi}_f(\mathbf{\Theta}) \\ 0 & -1 \end{bmatrix}.$$

The matrix $\Pi(\Theta)$ in (3) is partitioned as $\Pi(\Theta) = [\Pi_n(\Theta) \quad \Pi_f(\Theta)]$ without loss of generality. Figure 2 shows that our model has two determinacy regions and one indeterminacy region. During the mode-finding process, or MCMC, we encountered situations in which the estimation got stuck in one of these regions without crossing the boundaries, even though, in theory, the Metropolis-Hastings algorithm can explore the entire parameter space. Therefore, our estimation strategy sets different initial values (for each of the regions) while leaving the priors the same. Once we find for each region a (local) mode, we start running the MCMC at each mode. Finally, we compare the three log data densities generated for each region. To start with, the prior probability of determinacy or indeterminacy is set. The priors are such that indeterminacy has the least prior probability so to err on the right side. All priors are as before but the prior for the elasticity of collateral γ is gamma-distributed with mean 0.5 and standard deviation 0.8 and the prior for the parameter φ^* follows a uniform distribution within [0,2]. Then, the prior probability for indeterminacy is 20 percent (a remaining prior probability falls to the source region, however, any draws from this regime are discarded). Following Bianchi and Nicolò (2017) the forecast error of output η_t^y is the belief shock with variance σ_η^2, that is, the expectation error is correlated with the fundamental shocks and these correlations are estimated as in Farmer, Kharamov and Nicolò (2015).

8.5 Data description

This appendix is to describe the details of the source and construction of the data used in estimation. The sample period covers the first quarter of 1955 through the fourth quarter of 2014:

1. Real Gross Domestic Product. Billions of Chained 2009 Dollars, Seasonally Adjusted Annual Rate. Source: Bureau of Economic Analysis, NIPA Table 1.1.6.

2. Gross Domestic Product. Billions of Dollars, Seasonally Adjusted Annual Rate. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

3. Personal Consumption Expenditures, Nondurable Goods. Billions of Dollars, Seasonally Adjusted Annual Rate. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

4. Personal Consumption Expenditures, Services. Billions of Dollars, Seasonally Adjusted Annual Rate. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

5. Gross Private Domestic Investment, Fixed Investment, Residential. Billions of Dollars, Seasonally Adjusted Annual Rate. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

6. Gross Private Domestic Investment, Fixed Investment, Nonresidential. Billions of Dollars, Seasonally Adjusted Annual Rate. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

7. Government Consumption Expenditure. Billions of Dollars, Seasonally Adjusted Annual Rate. Source: Bureau of Economic Analysis, NIPA Table 3.9.5.

8. Government Gross Investment. Billions of Dollars, Seasonally Adjusted Annual Rate. Source: Bureau of Economic Analysis, NIPA Table 3.9.5.

9. Nonfarm Business Hours. Index 2009=100, Seasonally Adjusted. Source: Bureau of Labor Statistics, Series Id: PRS85006033.

10. Relative Price of Investment Goods. Index 2009=1, Seasonally Adjusted.

Source: Federal Reserve Economic Data, Series Id: PIRIC.

11. Civilian Noninstitutional Population. 16 years and over, thousands. Source: Bureau of Labor Statistics, Series Id: LNU00000000Q.

12. Confidence: Business Tendency Survey for Manufacturing, Composite Indicators, OECD Indicator for the United States, Series Id: BSCICP03USM665S.

13. Total Factor Productivity. "A Quarterly, Utilization-Adjusted Series on Total Factor Productivity", retrieved from

http://www.frbsf.org/economicresearch/economists/john-fernald/.

14. Moody's Seasoned Baa Corporate Bond Yield, Not Seasonally Adjusted, Average of Daily Data, Percent. Source: Board of Governors of the Federal Reserve System.

15. Moody's Seasoned Aaa Corporate Bond Yield, Not Seasonally Adjusted, Average of Daily Data, Percent. Source: Board of Governors of the Federal Reserve System.

16. 10 Year Treasury Constant Maturity Rate, Not Seasonally Adjusted, Average of Daily Data, Percent. Source: Board of Governors of the Federal Reserve System.

17. Effective Federal Funds Rate, Not Seasonally Adjusted, Average of Daily Data, Percent. Source: Board of Governors of the Federal Reserve System.

18. GDP deflator= (2)/(1).

19. Real Per Capita Output, $Y_t = (1)/(11)$.

20. Real Per Capita Consumption, $C_t = [(3) + (4)]/(19)/(11)$.

21. Real Per Capita Investment, $I_t = [(5) + (6)]/(19)/(11)$.

22. Real Per Capita Government Expenditure, $G_t = [(7) + (8)]/(19)/(11)$.

23. Per Capita Hours Worked, $N_t = (9)/(11)$.

24. Credit spread $= (14) - (16)$.

Table 1: Calibration

Parameter	Values	Description
β	0.99	Subjective discount factor
α	1/3	Capital share
η	0	Labor supply elasticity parameter
λ	10	Elasticity of substitution between goods
δ	0.0333	Steady-state depreciation rate
u	1	Steady-state capacity utilization rate
G/Y	0.21	Steady-state government expenditure share of GDP
μ^y	1.0041	Steady-state gross per capita GDP growth rate
μ^a	0.9949	Steady-state gross growth rate of price of investment

Table 2: Estimation

	Prior distribution		Posterior distribution	
Estimated parameters	Range	Density [mean,std]	Mean	90% Interval
Steady-state marginal cost, ϕ	[0.83,0.90]	Beta [0.88,0.01]	0.833	[0.831,0.834]
Elasticity of collateral, γ	[0.160,0.607]	Uniform	0.322	[0.315,0.329]
Gov. trend smoothness, ψ_{yg}	[0,1)	Beta [0.5,0.2]	0.965	[0.953,0.977]
Scale parameter, x	R^+	IGam [44,Inf]	47.33	[44.28,50.46]
AR technology shock, ρ_x	[0,1)	Beta [0.5,0.2]	0.025	[0.008,0.041]
AR investment shock, ρ_a	[0,1)	Beta [0.5,0.2]	0.029	[0.013,0.045]
AR preference shock, ρ_Δ	[0,1)	Beta [0.5,0.2]	0.984	[0.981,0.988]
AR government shock, ρ_g	[0,1)	Beta [0.5,0.2]	0.986	[0.982,0.989]
AR collateral shock, ρ_θ	[0,1)	Beta [0.5,0.2]	0.992	[0.990,0.994]
Belief shock volatility, σ_b	R^+	IGam [0.1,Inf]	0.640	[0.610,0.660]
SE technology shock, σ_x	R^+	IGam [0.1,Inf]	0.690	[0.650,0.730]
SE investment shock, σ_a	R^+	IGam [0.1,Inf]	0.560	[0.530,0.600]
SE preference shock, σ_Δ	R^+	IGam [0.1,Inf]	0.390	[0.360,0.410]
SE government shock, σ_g	R^+	IGam [0.1,Inf]	0.940	[0.900,0.990]
SE collateral shocks, σ_θ	R^+	IGam [0.1,Inf]	0.130	[0.120,0.140]
SE measurement error, σ_y^{me}	[0,0.29]	Uniform	0.290	[0.290,0.290]
SE measurement error, σ_s^{me}	[0,27.42]	Uniform	27.28	[27.11,27.42]
Technology shock effect, Ω_x	[-3,3]	Uniform	-0.514	[-0.590,-0.438]
Investment shock effect, Ω_a	[-3,3]	Uniform	0.271	[0.176,0.367]
Preference shock effect, Ω_Δ	[-3,3]	Uniform	0.872	[0.756,0.994]
Government shock effect, Ω_g	[-3,3]	Uniform	0.256	[0.205,0.305]
Collateral shock effect, Ω_θ	[-3,3]	Uniform	0.999	[0.610,1.393]
Log data density		4064.98		

Table 3: Business cycle dynamics (band-pass filtered)

	USA			Model		
x	σ_x	$\rho(x,Y)$	ACF	σ_x	$\rho(x,Y)$	ACF
Y_t	1.45	1	0.93	1.17	1	0.91
C_t	0.84	0.85	0.92	0.73	0.75	0.90
I_t	4.71	0.89	0.94	3.61	0.88	0.92
G_t	1.44	0.01	0.94	1.12	0.21	0.90
N_t	1.80	0.87	0.94	1.18	0.98	0.92

Table 4: Unconditional variance decomposition

Series/shocks	ε_t^b	ε_t^x	ε_t^a	ε_t^Δ	ε_t^g	ε_t^θ	$\varepsilon_{y,t}^{me}$	$\varepsilon_{s,t}^{me}$
$\ln(Y_t/Y_{t-1})$	43.43	11.17	5.72	15.70	9.93	6.71	7.33	0.00
$\ln(C_t/C_{t-1})$	6.18	40.42	2.76	39.84	1.96	8.82	0.00	0.00
$\ln(I_t/I_{t-1})$	66.53	2.41	7.06	9.34	7.09	7.57	0.00	0.00
$\ln(N_t/\bar{N})$	21.24	2.54	9.37	26.50	22.06	18.30	0.00	0.00
$\ln(G_t/G_{t-1})$	0.00	0.98	0.16	0.00	98.85	0.00	0.00	0.00
$\ln(A_t/A_{t-1})$	0.00	0.00	100	0.00	0.00	0.00	0.00	0.00
Credit spread	12.26	2.06	4.85	17.99	15.06	43.49	0.00	4.29

Table 5: Posterior distribution comparison

	Model with $\eta_t^y = \varepsilon_t^b$	
Parameter	Mean	90% Interval
ϕ	0.833	[0.831,0.834]
γ	0.322	[0.315,0.329]
ψ_{yg}	0.965	[0.954,0.977]
x	47.30	[44.18,50.35]
ρ_x	0.025	[0.008,0.042]
ρ_a	0.029	[0.014,0.045]
ρ_Δ	0.984	[0.981,0.988]
ρ_g	0.986	[0.982,0.989]
ρ_θ	0.992	[0.990,0.994]
σ_η	0.860	[0.820,0.900]
σ_x	0.690	[0.650,0.730]
σ_a	0.560	[0.520,0.600]
σ_Δ	0.390	[0.360,0.410]
σ_g	0.940	[0.900,0.990]
σ_θ	0.130	[0.120,0.140]
σ_y^{me}	0.290	[0.290,0.290]
σ_s^{me}	27.28	[27.11,27.42]
$\rho(x,\eta^y)$	-0.406	[-0.465,-0.349]
$\rho(a,\eta^y)$	0.172	[0.110,0.233]
$\rho(\Delta,\eta^y)$	0.388	[0.338,0.438]
$\rho(g,\eta^y)$	0.275	[0.226,0.327]
$\rho(\theta,\eta^y)$	0.151	[0.091,0.213]
Log data density	4066.02	

Table 6: Unconditional variance decomposition (Baa-Aaa spread)

Series/shocks	ε_t^b	ε_t^x	ε_t^a	ε_t^Δ	ε_t^g	ε_t^θ	$\varepsilon_{y,t}^{me}$	$\varepsilon_{s,t}^{me}$
$\ln(Y_t/Y_{t-1})$	45.46	11.34	5.34	15.63	9.12	6.31	6.80	0.00
$\ln(C_t/C_{t-1})$	6.67	41.08	2.65	38.98	1.84	8.78	0.00	0.00
$\ln(I_t/I_{t-1})$	68.22	2.32	6.45	9.04	6.24	7.73	0.00	0.00
$\ln(N_t/\bar{N})$	23.25	2.31	9.08	25.25	20.31	19.79	0.00	0.00
$\ln(G_t/G_{t-1})$	0.00	1.07	0.17	0.00	98.76	0.00	0.00	0.00
$\ln(A_t/A_{t-1})$	0.00	0.00	100	0.00	0.00	0.00	0.00	0.00
Credit spread	13.12	1.87	4.59	16.51	13.48	47.13	0.00	3.30

Table 7: Unconditional variance decomposition (Baa-FF spread)

Series/shocks	ε_t^b	ε_t^x	ε_t^a	ε_t^Δ	ε_t^g	ε_t^θ	$\varepsilon_{y,t}^{me}$	$\varepsilon_{s,t}^{me}$
$\ln(Y_t/Y_{t-1})$	42.35	12.38	6.10	17.45	9.40	4.97	7.34	0.00
$\ln(C_t/C_{t-1})$	5.93	43.61	3.01	39.50	1.86	6.09	0.00	0.00
$\ln(I_t/I_{t-1})$	65.43	2.62	7.51	10.04	7.00	7.40	0.00	0.00
$\ln(N_t/\bar{N})$	22.11	2.33	10.53	26.72	22.55	15.76	0.00	0.00
$\ln(G_t/G_{t-1})$	0.00	1.02	0.17	0.00	98.81	0.00	0.00	0.00
$\ln(A_t/A_{t-1})$	0.00	0.00	100	0.00	0.00	0.00	0.00	0.00
Credit spread	14.32	2.19	6.08	20.38	17.16	34.61	0.00	5.26

Table 8: Unconditional variance decomposition (Gilchrist and Zakrajšek' spread)

Series/shocks	ε_t^b	ε_t^x	ε_t^a	ε_t^Δ	ε_t^g	ε_t^θ	$\varepsilon_{y,t}^{me}$	$\varepsilon_{s,t}^{me}$
$\ln(Y_t/Y_{t-1})$	27.74	12.14	8.14	21.96	10.42	13.44	6.16	0.00
$\ln(C_t/C_{t-1})$	2.57	34.53	3.13	45.18	1.25	13.34	0.00	0.00
$\ln(I_t/I_{t-1})$	45.59	2.75	10.57	13.46	6.76	20.87	0.00	0.00
$\ln(N_t/\bar{N})$	11.05	1.13	10.02	30.71	14.06	33.02	0.00	0.00
$\ln(G_t/G_{t-1})$	0.00	0.77	0.16	0.00	99.07	0.00	0.00	0.00
$\ln(A_t/A_{t-1})$	0.00	0.00	100	0.00	0.00	0.00	0.00	0.00
Credit spread	5.24	0.77	4.17	17.59	7.72	61.05	0.00	3.46

Table 9: Unconditional variance decomposition (Fernald TFP)

Series/shocks	ε_t^b	ε_t^x	ε_t^a	ε_t^Δ	ε_t^g	ε_t^θ	$\varepsilon_{y,t}^{me}$	$\varepsilon_{s,t}^{me}$	$\varepsilon_{tfp,t}^{me}$
$\ln(Y_t/Y_{t-1})$	39.02	10.35	5.10	12.63	9.13	17.01	6.77	0.00	0.00
$\ln(C_t/C_{t-1})$	4.63	38.01	2.18	34.21	1.49	19.48	0.00	0.00	0.00
$\ln(I_t/I_{t-1})$	59.56	2.09	6.31	8.56	6.12	17.36	0.00	0.00	0.00
$\ln(N_t/\bar{N})$	16.00	2.35	7.14	21.74	16.70	36.07	0.00	0.00	0.00
$\ln(G_t/G_{t-1})$	0.00	1.08	0.15	0.00	98.76	0.00	0.00	0.00	0.00
$\ln(A_t/A_{t-1})$	0.00	0.00	100.00	0.00	0.00	0.00	0.00	0.00	0.00
Credit spread	6.54	1.34	2.61	10.38	8.13	67.28	0.00	3.71	0.00
$\ln(TFP_t/TFP_{t-1})$	0.00	92.29	0.00	0.00	0.00	0.00	0.00	0.00	7.71

Table 10: Unconditional variance decomposition (transitory TFP)

Series/shocks	ε_t^b	ε_t^z	ε_t^a	ε_t^Δ	ε_t^g	ε_t^θ	$\varepsilon_{y,t}^{me}$	$\varepsilon_{s,t}^{me}$
$\ln(Y_t/Y_{t-1})$	39.18	16.79	5.28	15.64	8.21	8.69	6.22	0.00
$\ln(C_t/C_{t-1})$	3.78	43.19	2.19	40.98	1.13	8.73	0.00	0.00
$\ln(I_t/I_{t-1})$	57.92	11.81	6.28	10.25	5.64	8.10	0.00	0.00
$\ln(N_t/\bar{N})$	16.08	17.47	8.35	26.25	15.10	16.75	0.00	0.00
$\ln(G_t/G_{t-1})$	0.00	0.00	0.22	0.00	99.78	0.00	0.00	0.00
$\ln(A_t/A_{t-1})$	0.00	0.00	100	0.00	0.00	0.00	0.00	0.00
Credit spread	5.63	41.34	2.59	10.61	6.17	30.37	0.00	3.30

Table 11: Model comparison

	Baseline: permanent TFP	Alternative: transitory TFP
Log data density	4064.98	3811.89

Table 12: Model comparison

	Determinacy 1	Determinacy 2	Indeterminacy
Model prior probability (in percent)	23	48	20
Log data density	3842.48	3474.90	4064.07
Model posterior probability	0	0	1

Table 13: Unconditional variance decomposition

Series/shocks	ε_t^b	ε_t^x	ε_t^a	ε_t^Δ	ε_t^g	ε_t^{MEI}	$\varepsilon_{y,t}^{me}$	$\varepsilon_{s,t}^{me}$
$\ln(Y_t/Y_{t-1})$	46.82	10.15	5.51	15.76	11.18	2.08	8.49	0.00
$\ln(C_t/C_{t-1})$	8.77	40.93	2.92	43.77	2.96	0.66	0.00	0.00
$\ln(I_t/I_{t-1})$	69.61	2.35	6.77	9.82	8.68	2.77	0.00	0.00
$\ln(N_t/\bar{N})$	25.57	3.62	10.02	31.30	27.17	2.31	0.00	0.00
$\ln(G_t/G_{t-1})$	0.00	0.75	0.13	0.00	99.12	0.00	0.00	0.00
$\ln(A_t/A_{t-1})$	0.00	0.00	100	0.00	0.00	0.00	0.00	0.00
Credit spread	0.00	0.00	0.00	0.00	0.00	99.95	0.00	0.05

Table 14: Estimation (transitory TFP)

	Prior distribution		Posterior distribution	
Estimated parameters	Range	Density [mean,std]	Mean	90% Interval
Steady-state marginal cost, ϕ	[0.83,0.90]	Beta [0.88,0.01]	0.832	[0.831,0.833]
Elasticity of collateral, γ	[0.160,0.607]	Uniform	0.296	[0.291,0.301]
Gov. trend smoothness, ψ_{yg}	[0,1)	Beta [0.5,0.2]	0.953	[0.932,0.975]
Scale parameter, x	R^+	IGam [44,Inf]	44.38	[42.62,46.24]
AR technology shock, ρ_z	[0,1)	Beta [0.5,0.2]	0.997	[0.996,0.998]
AR investment shock, ρ_a	[0,1)	Beta [0.5,0.2]	0.020	[0.008,0.032]
AR preference shock, ρ_Δ	[0,1)	Beta [0.5,0.2]	0.979	[0.974,0.983]
AR government shock, ρ_g	[0,1)	Beta [0.5,0.2]	0.981	[0.976,0.987]
AR collateral shock, ρ_θ	[0,1)	Beta [0.5,0.2]	0.992	[0.991,0.994]
Belief shock volatility, σ_b	R^+	IGam [0.1,Inf]	0.660	[0.640,0.690]
SE technology shock, σ_z	R^+	IGam [0.1,Inf]	0.320	[0.310,0.330]
SE investment shock, σ_a	R^+	IGam [0.1,Inf]	0.560	[0.530,0.600]
SE preference shock, σ_Δ	R^+	IGam [0.1,Inf]	0.470	[0.440,0.490]
SE government shock, σ_g	R^+	IGam [0.1,Inf]	0.940	[0.890,0.990]
SE collateral shocks, σ_θ	R^+	IGam [0.1,Inf]	0.140	[0.130,0.160]
SE measurement error, σ_y^{me}	[0,0.29]	Uniform	0.290	[0.290,0.290]
SE measurement error, σ_s^{me}	[0,27.42]	Uniform	27.29	[27.12,27.42]
Technology shock effect, Ω_z	[-3,3]	Uniform	1.054	[0.924,1.187]
Investment shock effect, Ω_a	[-3,3]	Uniform	0.277	[0.188,0.371]
Preference shock effect, Ω_Δ	[-3,3]	Uniform	0.729	[0.644,0.818]
Government shock effect, Ω_g	[-3,3]	Uniform	0.255	[0.203,0.305]
Collateral shock effect, Ω_θ	[-3,3]	Uniform	1.546	[1.186,1.931]