A Goldilocks Theory of Fiscal Policy*

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June 2021

Preliminary

Abstract

When an economy is close to the zero lower bound on nominal interest rates, governments face a trade-off: excessively conservative fiscal policy risks persistently low output but aggressive fiscal expansion raises sustainability concerns. This study builds a framework of dynamic fiscal policy, showing that there exists a *Goldilocks zone* in which deficits are permanent but not too high, the nominal interest rate on government debt (R) is lower than the economy's growth rate (G), government debt levels can be substantial, and deficits allow the economy to overcome weak demand to achieve potential output. The size of the Goldilocks zone can be estimated using empirically observed moments in the data, which suggest for the United States that government debt to GDP ratios can reach a maximum of about 220% in the Goldilocks zone, but the maximum permanent government deficit is only about 2% of GDP. In the model, R and G are endogenous to fiscal policy, which disciplines the ability of a government to boost deficits even if $R < G$. The Goldilocks zone is fragile: it can vanish in the face of a decline in potential GDP growth, a rise in aggregate demand, or a decline in income inequality.

*We are grateful to Gabriel Chodorow-Reich, Arvind Krishnamurthy, Hanno Lustig, Klaus Masuch, Ricardo Reis, Larry Summers, Annette Vissing-Jorgensen, and seminar participants at Queens University, Georgetown University, Harvard, Stanford GSB, and Berkeley Haas. Laurenz DeRosa, Jan Ertl, Pranav Garg, and Keelan Beirne provided excellent research assistance. Straub appreciates support from the Molly and Domenic Ferrante Award. Contact info: Mian: (609) 258 6718, atif@princeton.edu; Straub: (617) 496 9188, ludwigstraub@fas.harvard.edu; Sufi: (773) 702 6148, amir.sufi@chicagobooth.edu

1 Introduction

Advanced economies in recent years have been characterized by sustained proximity to the zero lower bound on nominal interest rates, which has led some economists to argue that persistent fiscal deficits may be necessary to generate demand.[1] At the same time, government debt to GDP ratios are at historical highs, leading to concerns about fiscal sustainability going forward if deficits are not reduced.[2] The tension between these views has triggered an active debate on fiscal policy with important consequences.

This study proposes a tractable model of fiscal policy that encapsulates the trade-off highlighted in this debate. The model adds two main features to an otherwise standard deterministic representative-agent economy. First, it introduces a preference for government debt. Such a preference generates a convenience premium on government debt, and the convenience premium in the model, as in the data, declines as the level of government debt outstanding rises (e.g., Krishnamurthy and Vissing-Jorgensen 2012). Second, the model assumes a zero lower bound on nominal interest rates, which allows for the possibility that persistently weak demand reduces output, inflation, and the nominal growth rate of the economy. The government's fiscal policy in the model can add to aggregate demand through borrowing, but the policy must satisfy the government budget constraint.

A key insight of the model is that fiscal policy can be excessively conservative, pushing the economy toward the ZLB with depressed demand. Or fiscal policy can be too aggressive, raising fiscal sustainability concerns. There is a *Goldilocks zone* for fiscal policy in between; this zone represents the set of equilibrium debt and deficit levels in which the government is able to run deficits that are permanent but not too high; the nominal interest rate on government debt R is lower than the economy's nominal growth rate G; government debt outstanding can be large; and deficits allow the economy to overcome weak demand in order to achieve its potential.

An analysis of the Goldilocks zone helps to provide answers to a number of central questions facing governments today. Does fiscal consolidation threaten to push an economy toward depressed output at the ZLB? How large can deficits and debt levels be before departing the Goldilocks zone? Is there a "free lunch" in which a government can raise

[1] This idea can be traced back to Abba Lerner's "Functional Finance" view, but has recently been articulated by Blanchard, Tashiro, et al. [2019] in their discussion of policy in Japan: "under current forecasts about the rest of the Japanese economy, primary deficits may be needed for a long time ... they are probably the best tool for maintaining demand and output at potential."

[2] Among others, the work by Jiang, Lustig, Van Nieuwerburgh, and Xiaolan [2019, 2020a] has raised questions on how the pricing of government debt can be consistent with the evolution and riskiness of revenues and expenditures going forward.

deficits to a higher permanent level without ever having to raise taxes? What underlying fundamentals of an economy explain the size of the Goldilocks zone, and what changes to an economy can lead the zone to shrink? Finally, what are the implications of the Goldilocks zone for tax policy?

The model addresses the growing body of research seeking to understand the implications for fiscal policy when the nominal interest rate on government debt is lower than the nominal growth rate of the economy, $R < G$. This condition is of central interest because, as Blanchard [2019] demonstrates, it holds the promise of raising deficits permanently or temporarily, without ever having to raise taxes. Indeed, we confirm below that assuming that R and G are exogenous allows a government to sustain arbitrarily large deficits indefinitely as long as $R < G$. We call a temporary or permanent expansion in deficits a "free lunch" policy if it is sustainable without a subsequent increase in taxes.

In our model, R and G are endogenous to fiscal policy, and this has important implications for whether a free lunch policy is possible. Boosting deficits leads to high debt levels, which raise interest rates because the convenience premium on government debt declines when the market is saturated. In contrast, excessively conservative fiscal policy leads to weak aggregate demand and nominal interest rates being trapped at the zero lower bound, depressing both real output and nominal growth G. As a result, the condition $R < G$ can be violated either because debt levels are high and convenience premia low, or because debt is low with R pinned at zero and G falling below zero due to deflation. The endogeneity of R and G to government debt levels implies that a simple examination of the difference between the two at any given point in time is insufficient to understand whether a free lunch policy is possible or not.

There is a maximum deficit possible in the Goldilocks zone, and the government debt level associated with that maximum deficit is crucial in determining whether a free lunch policy is available. If government debt outstanding is below this threshold, a free lunch policy is available that allows modest fiscal expansions without ever having to raise taxes. For example, a free lunch policy always exists for an economy which is at the ZLB. This helps capture the intuition that governments can safely boost deficits when interest rates on government debt are close to zero. If debt levels are above this threshold debt level, no free lunch policy is available, even if R is below G. In this region of the Goldilocks zone, an increase in deficits has to be met with higher subsequent taxes or reduced deficits. An economy can be in the Goldilocks zone with $R < G$ and yet be unable to pursue a free lunch policy.

The tractability of the model allows for closed form solutions for key determinants of the Goldilocks zone, such as the maximum permanent deficit possible and the largest amount of government debt for which R remains below G. An advantage of the model is that the estimation of these important objects is tied directly to empirical evidence from the macro-finance literature. More specifically, the maximum deficit and debt levels possible in the Goldilocks zone depend crucially on the sensitivity of the convenience yield, or more generally of R minus G, to government debt levels.

Using estimates from this literature, we conduct a simple calibration which suggests that the United States' Goldilocks zone has a maximum government debt to GDP ratio of 220%, at which point R rises above G; and a maximum permanent primary deficit of 2% of GDP obtained at the threshold debt level of 110% of GDP. The calibration can easily be done for any country for which an estimate of the sensitivity of convenience yields to government debt levels is available, and we provide a range of estimates for Germany, Italy, and Japan. A general finding from the calibration is that an economy is able to reach government debt to GDP ratios that are higher than where most advanced economies currently are before R rises above G, but the maximum possible permanent deficit level is small relative to the average deficits being run over the past 15 years.

How can the Goldilocks zone be so large? And what changes in the economic environment can make it disappear? These questions are of heightened importance given that government debt to GDP ratios have been high in many advanced economies, exceeding 100% in Belgium (102%), the United States (107%), Portugal (124%), Italy (134%), and Japan (237%).[3]

The model shows that the Goldilocks zone is especially large in the presence of weak aggregate demand. The model makes it clear that aggressive fiscal policy allows governments to treat the symptoms associated with weak aggregate demand. However, this also implies that the Goldilocks zone can disappear if sustainable aggregate demand returns. When debt levels are elevated at that point, governments may have to raise taxes by considerable amounts. The model also highlights that the Goldilocks zone can disappear if the trend growth rate falls, for example due to demographics or slower productivity growth. In general, while the Goldilocks zone may exist, it is fragile and can vanish with changes in the underlying economy.

High income inequality is also an important factor increasing the size of the Goldilocks zone, which we show by introducing hand-to-mouth agents into the model. High income inequality allows the government to sustain high debt levels because higher income individuals in the model, as in the data, hold the majority of the government debt outstanding

[3] These are averages from 2015 to 2019.

in the economy. Hence demand for government debt is higher when income inequality is higher, thereby lowering the interest rate on government debt. The flipside is also true: if inequality declines, the Goldilocks zone will shrink and potentially disappear, forcing the government to run potentially large surpluses. This shows the danger of relying on aggressive fiscal policy to address challenges to the economy coming from the secular long-run rise in income inequality. Policies that attempt to reduce inequality in the longer run may conflict with current policies that boost government deficits, in the sense that an ultimate decline in inequality will lead to a rise in interest rates forcing the government to cut deficits in order to pay back the debt that has been accumulated.

The extended model with hand-to-mouth agents also allows for an assessment of tax policy. An important implication of the model is that redistributive tax policy can make it harder to finance large deficits. When the government taxes savers, it is taxing the main holders of government debt, thereby lowering demand for government debt and raising the interest rate. This presents a challenge: a government cannot simultaneously eliminate inequality through redistribution and sustain large debt levels at low interest rates. The model also has implications for the implementation of financial repression and the effects of tax policy when an economy faces a slowdown in trend growth.

Related Literature. This study is part of a growing body of theoretical research that has emerged around two important facts on government debt. The first fact is that the nominal interest rate on government debt is lower than the nominal growth rate on average, $R < G$ (Feldstein 1976, Bohn 1991, Ball, Elmendorf, and Mankiw 1998, Blanchard 2019, Mehrotra and Sergeyev 2020). The second, and much more recent fact, is that the demand curve for government debt slopes down empirically, that is, the interest rate on government debt rises in the volume of government debt (Krishnamurthy and Vissing-Jorgensen 2012, Greenwood and Vayanos 2014, Greenwood, Hanson, and Stein 2015). This fact is attributed to “convenience benefits” of government debt, related to regulatory requirements, premia for liquidity, and premia for safety.

The literature has explored several ways to explain one or both of these facts. Bohn [1995] suggests that $R < G$ can naturally occur in complete markets economies with aggregate risk. Due to Barro [1974] - Ricardian equivalence, the model suggests that government debt neither affects R, nor can the government run a permanent deficit in each state of the world. Jiang et al. [2019] demonstrate that this approach cannot explain the valuation of US government debt. A potential way out is the recent work by Barro [2020] relying on the risk of rare disasters (Barro and Ursua 2008) that may not have realized yet for the US economy. However, the Barro [2020] model is inconsistent with the second fact above.[4]

The perhaps largest literature on $R < G$ is based on OLG models, going back to Samuelson [1958] and Diamond [1965]. An early motivation of this literature has been to understand when $R < G$ is a sign of dynamic inefficiency (Abel, Mankiw, Summers, and Zeckhauser 1989, Blanchard and Weil 2001), as well as when a possibility for a "free lunch" policy exists (Blanchard and Weil 2001, Blanchard 2019), whereby deficits can be increased today without future tax increases, in any state of the world. Such a policy was found to be more likely to exist when the economy is inefficient, and when capital is not crowded out. When a free lunch policy does not exist, deficits resemble a "gamble" (Ball et al. 1998).[5]

The above facts have also been approached using liquidity premia. Woodford [1990] illustrates how liquidity demand by producers or consumers can lead to $R < G$. Angeletos, Collard, and Dellas [2020] microfound a convenience yield function based on liquidity needs to revisit the optimality of the Barro [1979] tax smoothing results (see also Canzoneri, Cumby, and Diba 2016, Bhandari, Evans, Golosov, and Sargent 2017, Azzimonti and Yared 2019). The closest paper to ours among this class of models is Reis [2021]. The paper microfounds liquidity and safety premia of government debt in an economy with heterogeneous savers and producers hit by idiosyncratic productivity shocks. The paper shows that this economy leads to a "bubble premium" on public debt, defined as the gap between the marginal product of capital and R, which can be used to sustain permanent primary surpluses if $R < G$. Reis [2021] derives several sharp implications of the model, such as that more government spending reduces R in the model, while greater inequality increases R. Our analysis complements that of Reis [2021] by exploring a different microfoundation for why $R < G$, which is shown to lead to different predictions using a phase diagram analysis. Moreover, instead of modeling capital and investment, we study the interaction of $R < G$ with the ZLB.

Mehrotra and Sergeyev [2020] share with our paper the assumption of a convenience utility function $v(b)$ over government debt in a model with aggregate risk, and allowing for default risk. Different from Mehrotra and Sergeyev [2020], our focus is on a deterministic model, which we show can be tractably analyzed using phase diagrams for arbitrary $v(b)$. We also allow for a ZLB constraint, which we show meaningfully interacts with important

[4]We present a model in Section 7.2 that introduces default risk into the Barro [2020] model and shows that it endogenously generates a "safety premium" for government debt as long as no disaster occurs.

[5]Note that in our model, capital would be irrelevant for whether a "free lunch" policy is feasible or not (which is why we do not include it in our model). This is because the (de-trended) steady-state return on assets other than government is always equal to the discount rate of agents, irrespective of the steady-state level of government debt. Therefore, steady-state capital would be independent of the debt level, too.

comparative statics (e.g. that of falling growth rates).

Our model is based on the assumption that monetary policy remains active in stabilizing inflation and economic activity whenever it can. A recent branch of the literature explores deviations from this assumption. Brunnermeier, Merkel, and Sannikov [2020] derive a Laffer curve for the rate of inflation in a model with liquidity needs among producers. Sims [2019] argues that fiscal policy should, in general, use this "inflation tax" to generate seignorage-like revenue and reduce distortionary taxes (different from Chari and Kehoe 1999). The deficit-debt schedule that we derive, and on which our phase diagram is based, may seem similar to the inflation Laffer curve, but is quite distinct. In our schedule, debt is on the horizontal axis instead of inflation. With a binding ZLB, we find a positive relationship between inflation and debt levels, while this branch of the literature generally finds a negative one. With interest rates above the ZLB, inflation is independent of debt in our economy.

Finally, this study is closely related to the burgeoning literature on the sources and implications of safe asset demand (e.g., Caballero, Farhi, and Gourinchas 2008, Caballero and Farhi 2018, and Farhi and Maggiori 2018). In their model of the international monetary system, Farhi and Maggiori [2018] explore an equilibrium in which there is large demand for debt issued by a hegemon government. When this is met by too much issuance, default risk emerges. Farhi and Maggiori [2018] explain how a zero lower bound constraint can make this a more likely outcome.

2 Model

We begin with a stylized model that we extend in later sections. The model runs in continuous time and is deterministic.[6] It involves two actors, a government and a representative household. The government issues government debt, spends, and raises lump-sum taxes. The representative agent consumes and draws convenience benefits from holding government debt.

Throughout, we denote by R_t the nominal interest rate on government debt and by $G_t \equiv \gamma + \pi_t$ the nominal growth rate, which is equal to real trend growth γ plus inflation π_t. $G^* \equiv \gamma + \pi^*$ corresponds to nominal trend growth, when inflation is at its target π^*.

To save on notation, we will conduct our analysis entirely in the context of a model that

[6]Aggregate risk is useful to explain the gap between R and G in the data, but it will not affect how R changes in government debt b, e.g. see the contributions by Bohn [1995] and Barro [2020]. This is why we focus on a deterministic model for the majority of the paper. In Section 7.2 we extend the Barro [2020] model with default risk and show that it can micro-found the convenience utility assumed here.

was de-trended with the nominal growth rate. Potential output y^* in the de-trended model is thus constant, and we normalize it to one, $y^* \equiv 1$. Any quantities, such as the level of government debt b_t are thus to be understood as government debt relative to potential GDP. Moreover, we refer to $R_t - G_t$ as the "de-trended rate of return" on government debt, as it is the return R_t net of the re-investment that is necessary to keep government debt stable relative to potential GDP.

Households. The economy is populated by a representative household choosing paths of consumption c_t and government debt holdings b_t in order to maximize

$$\max_{\{c_t, b_t\}} \int_0^\infty e^{-\rho t} \{\log c_t + v(b_t)\} \, dt \tag{1}$$

subject to the budget constraint

$$c_t + \dot{b}_t \leq (R_t - G_t) b_t + w_t n_t - \tau_t. \tag{2}$$

The objective (1) involves flow utility from consumption $\log c_t$ and a utility $v(b_t)$ from holding government debt (relative to potential GDP). The latter captures safety and liquidity benefits that have been used extensively and are well documented in the literature (e.g. Sidrauski 1967, Krishnamurthy and Vissing-Jorgensen 2012). In line with this literature, we assume that the utility over government debt is twice differentiable, increasing and concave, $v' \geq 0, v'' \leq 0$.[7] Flow utility is discounted using a discount rate ρ. The discount rate pins down the steady state return on assets other than government debt, which, as we derive below, is given by $\rho + G^*$.

The budget constraint (2) involves labor income $w_t n_t$ and lump-sum taxes τ_t. Labor income derives from agents selling their labor endowment n_t. We assume that agents wish to sell an endowment of 1 but may be unable to do so due to a standard downward nominal wage rigidity (which happens at the ZLB in our model). In particular, the path of nominal wages W_t satisfies

$$\frac{\dot{W}_t}{W_t} \geq \pi^* - \kappa(1 - n_t). \tag{3}$$

This implies that whenever labor demand is falling short of the unit labor endowment, wage inflation will fall short of π^*. The lower labor demand is, the lower wage inflation will be, just like in a standard Phillips curve. $\kappa \geq 0$ parameterizes the slope of the Phillips curve.

[7] We also assume that the range of v' is given by $[0, \infty)$ or $(0, \infty)$ and that $v'' < 0$ whenever $v' > 0$.

Representative firm. We assume that labor is used by a representative firm with linear production technology $y_t = n_t$. The firm charges flexible prices, pinning down the real wage $w_t = 1$. Price inflation π_t in our de-trended model is equal to wage inflation and therefore determined by (3).

Government. The government sets fiscal and monetary policy. Fiscal policy consists of paths $\{x, b_t, \tau_t\}$ of government spending x, government debt b_t and taxes τ_t, subject to the flow budget constraint

$$x + (R_t - G_t)\, b_t \leq \dot{b}_t + \tau_t \tag{4}$$

The primary deficit is given by

$$z_t \equiv x - \tau_t \tag{5}$$

We assume taxes adjust to ensure that z_t follows a given fiscal rule $z_t = \mathcal{Z}(b_t)$. Typically, $\mathcal{Z}(b)$ is downward-sloping in debt b, corresponding to a lower deficits or greater surplus with higher debt levels.

Government debt b_t is short-term and real in our baseline model. We relax both of these assumptions in Section 7.1. Government spending $x \geq 0$ is assumed to be constant for now. Our analysis below is parallel to one in which government spending is allowed to vary while taxes are kept fixed (see also Section 6).

Monetary policy is dominant in our model and successfully implements the natural allocation whenever feasible. In particular, we denote by $\{R_t^*\}$ the path of the nominal natural interest rate, which would materialize in the absence of nominal rigidities in our model, assuming inflation is constant at its target π^*. We assume that the actual nominal interest rate then follows

$$R_t = \max\{0, R_t^*\}. \tag{6}$$

In particular, whenever the natural interest rate is positive, R_t tracks the natural interest rate R_t^*, and the economy is at potential, $y_t = n_t = 1$. When the natural rate is negative, however, R_t is constrained to be equal to zero by the ZLB. In that case, we will find that the economy falls below potential, $y_t = n_t < 1$.[8]

Equilibrium. We define equilibrium in our model as follows.

Definition 1. Given an initial level of debt b_0 and a fiscal rule $\mathcal{Z}(\cdot)$, a (competitive) equilibrium consists of a tuple $\{c_t, y_t, n_t, b_t, R_t, G_t, \pi_t, \tau_t, z_t, w_t\}$, such that: (a) $\{c_t, b_t\}$ maximizes

[8] This is similar to the rationing equilibria in Barro and Grossman [1971], Malinvaud [1977], and Benassy [1986].

the household objective (1) subject to (2); (b) the deficit $\{z_t\}$ follows the fiscal rule $\mathcal{Z}$ and taxes are in line with (5); (c) debt evolves in line with the flow budget constraint (4) and remains bounded; (d) monetary policy sets the nominal rate R_t in line with the rule (6); (e) inflation π_t is determined by the Phillips curve (3); (f) output y_t is given by $y_t = n_t$ and the real wage is $w_t = 1$; (g) the goods market clears $c_t + x = y_t$. A *steady state equilibrium* is an equilibrium in which all quantities and prices are constant.

Stability at the ZLB. One concern with our analysis may be that, when the economy is at the ZLB, the nominal rate is fixed at zero, $R_t = 0$. In textbook New-Keynesian (NK) models, this leads to indeterminacy and thus multiplicity of bounded equilibria (Benhabib, Schmitt-Grohé, and Uribe 2001, Cochrane 2017). Our model differs from a textbook NK model in that it involves utility over government debt, which can lead to stability at the ZLB (Michaillat and Saez 2019). Following a similar logic, one can easily show that equilibria in our model are locally determinate whenever the Phillips curve is not too steep,

$$\kappa < \frac{\rho + G^*}{1 - x}. \tag{7}$$

We assume that this condition is satisfied for the remainder of our analysis.

Features of government debt in the model The model follows the extensive body of research arguing that government debt directly enters the utility of those who hold it, which explains why government debt has low yields relative to similar assets (e.g., Krishnamurthy and Vissing-Jorgensen 2012, Caballero, Farhi, and Gourinchas 2017). The underlying logic of this assumption is government debt has certain benefits that lead it to be valued above and beyond future cash flows. These benefits can be due to primitive factors such as household demand for liquidity and safety, institutional factors such as regulatory requirements facing financial institutions, or international factors such as the demand for dollar-denominated assets.[9] Such an assumption generates a convenience yield that helps

[9]The demonstration of such a convenience yield on U.S. government debt is the subject of a large body of research including studies by Krishnamurthy and Vissing-Jorgensen [2012], Vandeweyer [2019], Van Binsbergen, Diamond, and Grotteria [2019], Koijen and Yogo [2020], Mota [2020]. There is a considerable range of estimates given in the literature for the size of the convenience yield based on the fact that comparable assets such as AAA rated bonds may also have a convenience yield. Mota [2020] estimates an average convenience yield of U.S. Treasuries of 130 basis points. Studies focused on demand for dollar-denominated debt estimate a convenience yield of U.S. Treasuries of 200 to 215 basis points (e.g., Jiang, Krishnamurthy, and Lustig [2021], Koijen and Yogo [2020]). Del Negro, Giannone, Giannoni, and Tambalotti [2017] estimated that rising convenience yields may have been an important factor behind the recent decline of the riskless rate in the US.

considerably in explaining the pricing of government debt. For example, of all the potential solutions to the "U.S. public debt valuation puzzle" considered by Jiang et al. [2019], the assumption of a convenience yield is the only one that makes a serious dent in the puzzle. There is a large empirical literature arguing that the pricing advantage of government debt declines in the amount of government debt outstanding, which is a main feature of our model. This literature is discussed in detail below in Section 4.2.

An alternative reason for low yields on government debt is that they provide a hedge against aggregate risk (Bohn 1995), and specifically disaster risk (Barro 2020). These papers demonstrate that permanent deficits cannot be sustained in every state of the world, including after disaster shocks. Building on these insights, we provide a microfoundation of $v(b)$ as "safety premium" in a model with disaster risk and default in Section 7.2 below. In this microfoundation, disasters occur in different sizes. Debt b is "safe" if it is repaid after disasters of (almost) all sizes, carrying a large premium. Greater b increases the range of disasters for which debt is defaulted on, reducing the premium. We show that the dynamics of this economy *before* the disaster occurs are isomorphic to those implied by a model with an exogenous convenience utility $v(b)$.

3 The Goldilocks zone

We begin our equilibrium analysis by studying steady state equilibria.

3.1 Steady state equilibria

Our model admits a set of steady state equilibria, indexed by the level of steady state debt $b \geq 0$. For each b, one can find a primary deficit z such that $\dot{b} = 0$ and the economy remains steady at that level of debt b. We distinguish two cases, according to whether the economy is above or at the zero lower bound (ZLB).

Above the ZLB. When the economy is above the ZLB, monetary policy implements the natural allocation. This implies that the interest rate is equal to the natural rate, $R_t = R_t^*$, output and employment are at potential, $y_t = n_t = 1$, inflation is at its target, $\pi_t = \pi^*$, and the nominal growth rate is equal to nominal trend growth, $G_t = G^*$.

To see how the natural rate is determined, consider the household's Euler equation

$$\frac{\dot{c}_t}{c_t} = R_t^* - G^* - \rho + v'(b_t)c_t \tag{8}$$

Here, $v'(b_t)$ enters as it is the marginal convenience utility from saving one more unit in government bonds. It enters with the opposite sign as the discount rate ρ and therefore effectively makes the household more patient when saving in government bonds.

In a steady state, consumption is constant and equal to $1 - x$ by goods market clearing. This lets us solve (8) for the natural interest rate,

$$R^*(b) = \rho + G^* - \underbrace{v'(b) \cdot (1 - x)}_{\text{convenience yield}}. \tag{9}$$

This expression for the natural interest rate on government debt is intuitive. The natural rate is equal to $\rho + G^*$, which would be the steady state return on any non-convenience-bearing assets, minus the steady state convenience yield $v'(b) \cdot (1 - x)$. The expression already suggests how R^* moves with debt. As v is a concave utility function, R^* weakly increases in government debt b.

At ZLB. For low levels of government debt b, the natural rate $R^*(b)$ may be negative and therefore unattainable for monetary policy due to the ZLB. In this region of the state space, output falls short of potential and labor is rationed. Using the goods market clearing condition, output is given by

$$y_t = c_t + x \tag{10}$$

where consumption follows an Euler equation again,

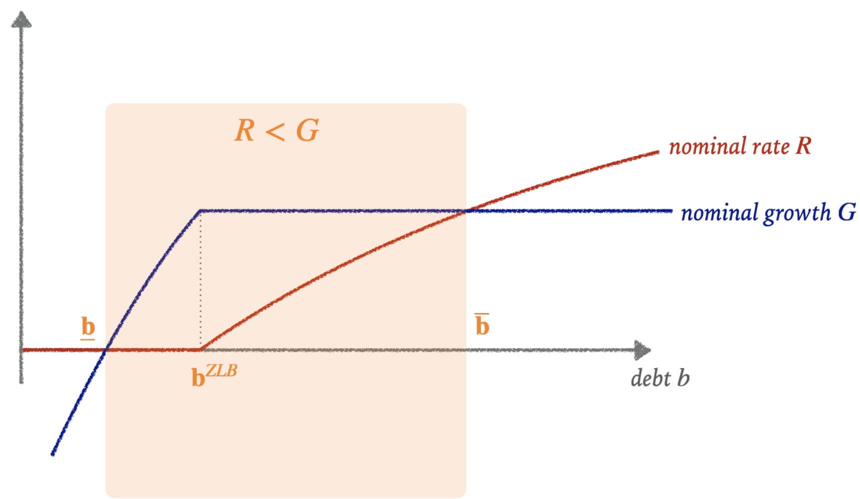
$$\frac{\dot{c}_t}{c_t} = \underbrace{0 - (G^* - \kappa (1 - y_t))}_{R_t - G_t} - \rho + v'(b_t) c_t \tag{11}$$

but this time involving a nominal interest rate of zero, $R_t = 0$, and an endogenous nominal growth rate $G_t = \gamma + \pi_t$ with inflation determined by the Phillips curve (3). Solving the system of (10) and (11) at a steady state with $\dot{c}_t = 0$, we find

$$G(b) = G^* - \frac{\kappa}{v'(b) - \kappa} \left(-R^*(b)\right). \tag{12}$$

At the ZLB, the interest rate is exogenous, $R = 0$, but the nominal growth rate G is endogenous to the debt level. It lies below nominal trend growth G^*, and by more so the more negative $R^*(b)$ is. As we show in the appendix, the fraction $\frac{\kappa}{v'(b) - \kappa}$ is well defined with $v'(b) > \kappa$ in expression (12) due to our assumption (7).

Figure 1: Nominal interest rates and nominal growth rates across steady state debt levels



3.2 Steady state deficits

Taken together, the previous two sections can be summarized by Figure 1. To explain it, we go from high to low levels of debt. For high levels of debt, the economy is above the ZLB and the interest rate R on government debt lies above the growth rate G. As we reduce debt levels, R falls below G at some $b = \overline{\mathbf{b}}$ and hits the ZLB at some $b = \mathbf{b}^{ZLB}$. At the ZLB, G starts falling with lower b, as inflation undershoots its target. Eventually, nominal growth falls below zero due to deflation at $b = \underline{\mathbf{b}}$. Left of $\underline{\mathbf{b}}$, R lies above G again.

We can characterize the thresholds in closed form.

Proposition 1. *Define*

1. $\underline{\mathbf{b}}$ *by*

$$v'(\underline{\mathbf{b}})(1-x) = \frac{1-x}{1-x-G^*/\kappa} \cdot \rho$$

if $1-x > G^*/\kappa$. *Else set* $\underline{\mathbf{b}}$ *equal to the lower bound of the domain of* v'.

2. $\mathbf{b}^{ZLB}$ *by*

$$v'(\mathbf{b}^{ZLB})(1-x) = \rho + G^*$$

3. $\overline{\mathbf{b}}$ *by*

$$v'(\overline{\mathbf{b}})(1-x) = \rho \tag{13}$$

Then, $\underline{\mathbf{b}} < \mathbf{b}^{ZLB} < \overline{\mathbf{b}}$. *Moreover,* $R(b) > G(b)$ *iff* $b < \underline{\mathbf{b}}$ *or* $b > \overline{\mathbf{b}}$, *and* $R(b) = 0$ *iff* $b < \mathbf{b}^{ZLB}$.

Proposition 1 characterizes the three thresholds that split up the state space into the four regions shown in Figure 1 according to the relative positions of $R(b)$ and $G(b)$. One noteworthy implication of the equation for the upper bound (13) is that the size of $\overline{\mathbf{b}}$ can be large, and is in no meaningful way constrained by existing household or private wealth of agents. In fact, with $\rho \to 0$, $\overline{\mathbf{b}}$ would diverge to infinity, allowing the government to run permanent deficits even for very large debt levels. In this limit, private wealth relative to potential GDP would become unboundedly large. This distinguishes our analysis from that in Reis [2021], who finds that the level of government debt relative to GDP is bounded above by the level of private wealth to GDP. There is no such bound in our economy.

An immediate corollary to the proposition is that the gap between the two, $G(b) - R(b)$, is hump-shaped and positive over the interval $(\underline{\mathbf{b}}, \overline{\mathbf{b}})$. It peaks right at the point at which the economy hits the ZLB, $\mathbf{b}^{ZLB}$. This has an important implication for the level of the primary deficit $z(b)$ that the government is required to choose for the economy to be in a steady state equilibrium at b,

$$z(b) = (G(b) - R(b))\, b.$$

Indeed, the primary deficit is also positive over the interval $(\underline{\mathbf{b}}, \overline{\mathbf{b}})$. Since $G(b) - R(b)$ increases for b left of $\mathbf{b}^{ZLB}$, the peak of $z(b)$ must lie weakly to the right of $\mathbf{b}^{ZLB}$. To characterize the shape of $z(b)$ more formally, we introduce notation to characterize the semi-elasticity of the convenience yield,

$$\varphi\left(b\right) \equiv -(1-x)\frac{\partial v'(b)}{\partial \log b} = -(1-x)v''(b)b$$

We then have the following result.

Proposition 2. *The steady state primary deficit is positive over the interval $(\underline{\mathbf{b}}, \overline{\mathbf{b}})$ and given by*

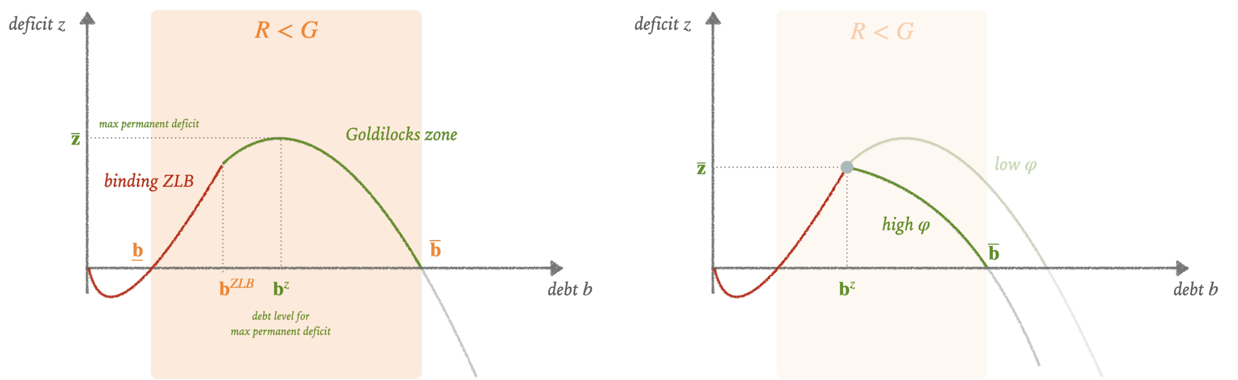
$$z(b) = \left(v'(b)\cdot(1-x) - \rho\right) b + \frac{v'(b)}{v'(b)-\kappa}\left(R^*(b)\right)^- b \tag{14}$$

- *When the interest rate is at the ZLB, $b < \mathbf{b}^{ZLB}$, the deficit $z(b)$ strictly increases in debt.*
- *It peaks at $\mathbf{b}^{ZLB}$ if $\varphi\left(\mathbf{b}^{ZLB}\right) > G^*/(G^*+\rho)$. In this case, $\mathbf{b}^z = \mathbf{b}^{ZLB}$.*
- *It peaks at some $\mathbf{b}^z > \mathbf{b}^{ZLB}$ if $\varphi\left(\mathbf{b}^{ZLB}\right) < G^*/(G^*+\rho)$. In this case $\mathbf{b}^z$ satisfies*

$$v'(\mathbf{b}^z)(1-x) = \rho + \varphi(\mathbf{b}^z) \tag{15}$$

Figure 2: Steady state primary deficits $z(b)$

(a) Goldilocks zone **(b)** Goldilocks zone with higher elasticity φ



At the peak, the maximum deficit is given by

$$\overline{\mathbf{z}} = \varphi(\mathbf{b}^z)\mathbf{b}^z \tag{16}$$

Proposition 2 characterizes the shape of steady state primary deficits $z(b)$ as a function of the debt level b. We sketch the shape in Figure 2. The function $z(b)$ increases when the ZLB is binding. It stops increasing as soon as the ZLB stops binding if the elasticity of the convenience yield $\varphi\left(\mathbf{b}^{ZLB}\right)$ is higher than $G^*/(G^*+\rho)$. This ratio corresponds to the ratio of nominal trend growth to the nominal return on assets other than government debt. With an elasticity below this ratio, primary deficits keep increasing further, above the ZLB, until they peak at some debt level $\mathbf{b}^z$. As shown in (15) this debt level is greater the smaller the elasticity of the convenience yield $\varphi\left(\mathbf{b}^z\right)$ is. The maximum deficit is given by $\overline{\mathbf{z}}$. Equations (15) and (16) are implicit equations in general. We solve them below explicitly for special functional forms of the convenience utility $v(b)$.

However, we already note here that in our model, there is no force that sets a "hard limit" as to how large the maximum permanent deficit $\overline{\mathbf{z}}$ may be. In fact, as we let the convenience utility $v(b)$ approximate a linear function of government debt, not only does $\overline{\mathbf{b}}$ diverge to infinity, so does $\mathbf{b}^z$ and $\overline{\mathbf{z}}$ with it. In this limit, therefore, there exist steady states with debt levels and deficits that are many multiples of potential GDP. In practice, realistic values for the elasticity φ may prevent such large deficits and debt levels. This will be one motivation behind our measurement exercise in Section 4.

3.3 The Goldilocks zone

The steady states with debt levels between $\mathbf{b}^{ZLB}$ and $\overline{\mathbf{b}}$ in Figure 2 are situated such that the interest rate R lies above zero—and hence the economy is above the ZLB—but is not so large that the economy needs to run a primary surplus, i.e. $R < G$. We refer to this set of steady states as the *Goldilocks zone*.

The Goldilocks zone represents a region of the state space in which the economy successfully avoids the ZLB on the one side and fiscally unsustainable deficits on the other. The latter presumes that the economy is able to sustain a steady state with $R \leq G$, or in other words, is able to close any primary deficit. In that case, the Goldilocks zone corresponds to a "conservative" region of the state space that achieves fiscal sustainability. Alternatively, if a maximum amount of tax revenue $\overline{\tau}$ is known, that could be used to derive an upper bound of the primary surplus $x - \overline{\tau}$. That upper bound could then be used to describe the upper limit of the Goldilocks zone, in lieu of $\overline{\mathbf{b}}$. We consider an endogenous upper bound that is emerging from welfare considerations in Section 7.

3.4 When is rising debt a "free lunch"?

One idea that has garnered considerable attention in the literature surrounding $R < G$ (see, e.g., Blanchard 2019) is that the condition seemingly allows economies to run larger deficits temporarily, and then simply "grow out" of the resulting increased debt levels without a need to raise taxes. We refer to this idea as the "free lunch" property of higher deficits. A stronger version of the "free lunch" idea is that *permanent* increases in deficits do not require tax increases going forward, even if they lead to permanently greater (non-explosive) debt levels.

Both versions of the free lunch idea can easily be derived from the government budget constraint (4), under the assumption of a constant interest rate R and a constant growth rate $G > R$. Then,

$$\dot{b}_t = -(G - R)\, b_t + z \tag{17}$$

describes a stable differential equation for debt b. This implies that temporary increases in deficits of arbitrary magnitude, leading to greater debt levels, can always be grown out of over time. Also, a permanent increase in deficits by some Δz simply raises steady state debt levels by $\Delta z / (G - R)$, with no need for a reduction in deficits, i.e. an increase in taxes, at any point. Both versions of the free lunch property are satisfied with exogenous R and G in (17). This is clearly a stylized example but it captures one, if not *the* most, important reason why fiscal policy in a world with $R < G$ is thought to be so different from fiscal policy with $R > G$.

We next investigate the extent to which the free lunch idea holds true in our model. What distinguishes our model from the stylized analysis in (17), is that both G and R are endogenous to the debt level. To understand the dynamics of b_t, it is crucial to incorporate this endogeneity. To do so, we first describe the behavior of the debt level for a general exogenous path z_t of primary deficits. Then, we feed in the specific paths for deficits that correspond to the two versions of the free lunch property. We separate again the case at the ZLB from the case above the ZLB.

Transitions above the ZLB. Above the ZLB, even along transitions, consumption remains constant at $1-x$. Thus, the natural rate $R^*(b_t)$ is still given by (9),

$$R^*(b_t) = \rho + G^* - v'(b_t) \cdot (1-x).$$

Therefore, the dynamics of the debt level simplify follow

$$\dot{b}_t = -\left(G^* - R^*(b_t)\right) b_t + z_t \tag{18}$$

for an exogenous path of deficits z_t.[10] Notably, the dynamics of debt are perfectly backward looking, despite households being forward looking with rational expectations. This stems from the fact that consumption is constant even along transitions due to the goods market clearing condition, pinning down the natural interest rate in each instant.

Transitions at the ZLB. At the ZLB, the situation is more complex as consumption is no longer constant. In this case, the economy is described by a system of two differential equations, the Euler equation

$$\frac{\dot{c}_t}{c_t} = -\left(G^* - \kappa\left(1-x-c_t\right)\right) - \rho + v'(b_t) c_t \tag{19}$$

in addition to the government budget constraint

$$\dot{b}_t = -\left(G^* - \kappa\left(1-x-c_t\right)\right) b_t + z_t. \tag{20}$$

Representing transitions in the deficit-debt diagram. A useful diagram to study the effects of temporary or permanent changes in deficits is the deficit-debt diagram. In

[10] If deficits followed a fiscal rule $z_t = \mathcal{Z}(b_t)$ instead, one would simply have to replace z_t in (18).

Figure 3: Transitions when changing the deficit

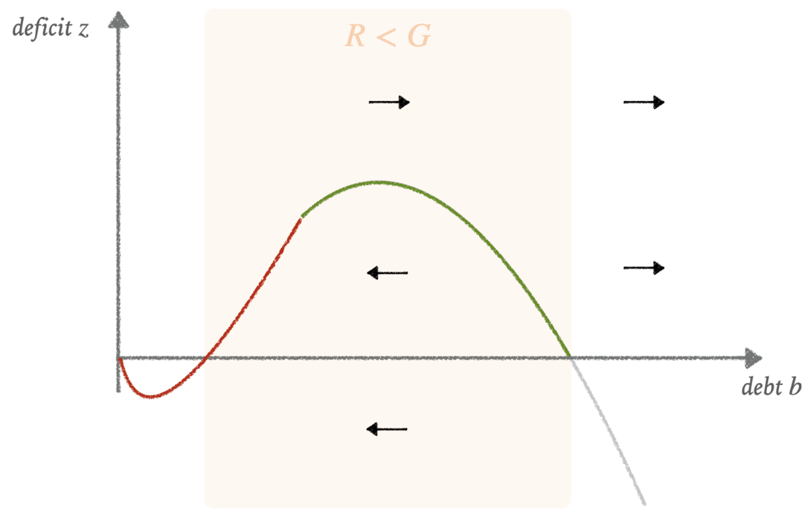


Figure 3 we indicate with arrows the direction the economy travels in when deficits are moved above or below the steady state locus.

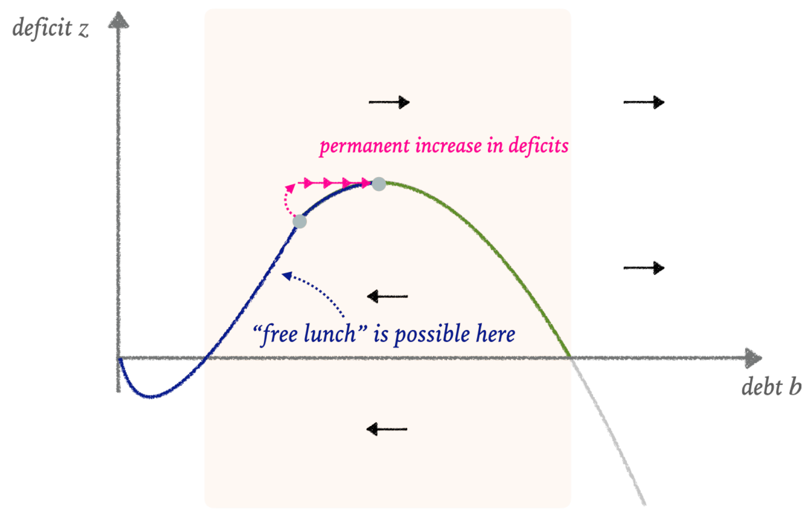
The behavior is intuitive. When deficits are raised above the steady state locus, debt grows, until either the steady state locus is hit, or until, at some point in the future, the deficit is reduced again down to the steady state locus.When deficits are reduced below the steady state locus, debt falls over time.

Mathematically, this behavior follows immediately from (18) in the case where the economy is above the ZLB and the evolution of debt is purely backward looking. In the case where the economy is at the ZLB, the behavior follows directly from a phase diagram analysis of (19) and (20).

The free lunch region in our model. The directions in Figure 3 allows us to see the region of the state space in which the government can obtain a "free lunch". Indeed, any steady state on the increasing part to the left of the peak at $\mathbf{b}^z$ allows for some form of a free lunch. For example, starting at any of these steady states, a permanent increase in the deficit to any value below or equal to $\bar{\mathbf{z}}$ can be sustained indefinitely. If the deficit increase is temporary, it can exceed $\bar{\mathbf{z}}$, as long as it is reduced back to $\bar{\mathbf{z}}$ or below in time. We show example transitions along these lines in Figure 4.

An important implication of this behavior of our model is that a free lunch policy is always available when the interest rate is at the ZLB and $b < \mathbf{b}^{ZLB}$, since the deficit-debt locus always increases in the ZLB region. This is a result that is often informally made by

Figure 4: Free lunch (or not)



advocates of "Modern Monetary Theory" (e.g. Kelton 2020). Deficit-financed fiscal stimulus can always be used to ensure that an economy at the ZLB returns to full employment, and the resulting increased level of debt need not be repaid by greater tax levels.

However, while the diagram in Figure 4 illustrates how a "free lunch" policy is indeed possible, it also makes the limits of such a policy very clear. For example, if deficits are increased by too much and / or for too long, a free lunch cannot obtained.

More fundamentally, a free lunch policy cannot work if the initial debt level already exceeds $\mathbf{b}^z$, that is, the initial steady state lies on the downward-sloping branch of the deficit-debt locus in Figure 4.[11] In this case, any deficit increases, however temporary, must ultimately be met by reduced deficits, or even, surpluses. In other words, taxes must increase. Crucially, this logic applies despite the fact that the economy may display $R < G$ throughout.

How is this possible? The aspect of our theory that is responsible for this result is the endogeneity of interest rates $R^*(b)$ to the debt level. As the debt level increases, the convenience yield of government debt falls, raising the interest cost on all (infra-marginal) outstanding debt positions. This can undo the positive effect of a greater debt position on the government budget constraint when $R < G$ that we highlighted at the beginning of this section. In fact, as Figure 4 illustrates, this precisely happens for debt levels greater than $\mathbf{b}^z$.

[11]Strictly speaking, there could be multiple local maxima of $z(b)$ in our model. The condition for the absence of a free lunch policy is that there be no steady state with a greater debt level and a greater or equal deficit z.

This analysis shows that, if the level of debt is larger than $\mathbf{b}^z$, the economics behind the financing of fiscal deficits are entirely conventional: greater debt must be repaid by raising taxes. Whether $R < G$ or $R > G$ is irrelevant for this question when debt is above $\mathbf{b}^z$.

4 Measuring the Goldilocks zone

We have shown in the previous section that the shape of the deficit-debt locus pins down several key quantities that help us understand the abilities and limits of fiscal policy. In this section, we set out to calibrate the model to illustrate the key factors determining the deficit-debt locus, and to offer an attempt to measure the locus as accurately as possible. Along the way, we focus on three specific quantities that are determined by the deficit-debt locus: the maximum permanent deficit $\overline{\mathbf{z}}$, the associated debt level $\mathbf{b}^z$, beyond which a free lunch policy ceases to be feasible, and the level of debt $\overline{\mathbf{b}}$ at which the interest rate R rises above the growth rate G.

The crucial object to calibrate is the shape of the convenience yield $v'(b)(1-x)$. We view the fact that $v'(b)(1-x)$ is a crucial object in our model as a promising feature because there is already a large body of research that seeks to estimate this derivative. We discuss this literature in detail below.

Our calibration strategy for $v'(b)$ proceeds in two steps. We first assume a parametric family of functional forms for $v'(b)$ and then determine the parameters that match a given steady state with debt b_0 (the US economy in 2019) as well as estimates of the local (semi-)elasticity of the convenience yield $\varphi(b_0)$. We henceforth abbreviate $\varphi(b_0)$ by φ. Since matching the elasticity only provides accuracy in a neighborhood of b_0, we provide analyses of robustness with respect to the functional forms below.

4.1 Functional forms for the convenience yield

We consider two functional forms for $v'(b)$ that the empirical literature has documented a good empirical fit of (e.g., Krishnamurthy and Vissing-Jorgensen 2012, Presbitero and Wiriadinata 2020). The first is a linear specification, which will be our baseline,

$$\text{linear:} \quad v'(b)\,(1-x) = v'(b_0)\,(1-x) - \varphi \frac{b-b_0}{b_0}. \tag{21}$$

The second is a log-linear specification,

$$\text{log-linear:} \quad v'(b)\,(1-x) = v'(b_0)\,(1-x) - \varphi \log \frac{b}{b_0}. \tag{22}$$

In each case, the intercept is determined by the initial steady state (assumed to be above the ZLB), for which the Euler equation pins down the convenience yield $v'(b_0)\,(1-x)$ as

$$v'(b_0)\,(1-x) = \rho + G^* - R_0. \tag{23}$$

For both cases, we can explicitly solve the three main quantities of interest.

Proposition 3. *For the linear specification* (21)*, we have*

$$\frac{\overline{\mathbf{b}}}{b_0} = 1 + \frac{1}{\varphi}\,(G^* - R_0)$$

$$\frac{\mathbf{b}^z}{b_0} = \begin{cases} \frac{1}{2}\frac{\overline{\mathbf{b}}}{b_0} & \text{if } \varphi < G^* + R_0 \\ 1 - \frac{1}{\varphi}R_0 & \text{if } \varphi \geq G^* + R_0 \end{cases}$$

$$\overline{\mathbf{z}} = \begin{cases} b_0 \frac{\varphi}{4}\left(1 + \frac{1}{\varphi}\,(G^* - R_0)\right)^2 & \text{if } \varphi < G^* + R_0 \\ b_0\left(1 - \frac{1}{\varphi}R_0\right)G^* & \text{if } \varphi \geq G^* + R_0 \end{cases}$$

For the log-linear specification, we have

$$\log \frac{\overline{\mathbf{b}}}{b_0} = \frac{1}{\varphi}\,(G^* - R_0)$$

$$\log \frac{\mathbf{b}^z}{b_0} = \begin{cases} \log \frac{\overline{\mathbf{b}}}{b_0} - 1 & \text{if } \varphi < G^* \\ -\frac{1}{\varphi}R_0 & \text{if } \varphi \geq G^* \end{cases}$$

$$\overline{\mathbf{z}} = \begin{cases} \varphi \cdot \mathbf{b}^z & \text{if } \varphi < G^* \\ G^* \cdot \mathbf{b}^z & \text{if } \varphi \geq G^* \end{cases}.$$

These are simple expressions that allow us to translate empirical estimates of φ directly into the three objects of interest. Interestingly, the objects are pinned down by only four statistics: the elasticity φ, the initial debt level b_0, nominal trend growth G^*, and the initial interest rate R_0.[12] Neither government spending x nor the discount rate ρ are relevant. The

[12]The ZLB threshold can also be computed in closed form based on these statistics. We find $\mathbf{b}^{ZLB}/b_0 = 1 - \varphi^{-1}R_0$ for the linear specification and $\log\left(\mathbf{b}^{ZLB}/b_0\right) = -\varphi^{-1}R_0$ for the log-linear one.

expressions allow for simple tests, for example, to determine if an economy is still in the free lunch region, $b_0 < \mathbf{b}^z$, or already past it, $b_0 > \mathbf{b}^z$.

Corollary 1. *The economy is past the free lunch region if $R_0 < G^* - \varphi$. This expression holds for any functional form of the convenience yield $v'(b)(1-x)$ with φ being the semi-elasticity of the convenience yield with respect to debt at b_0.*

The elasticity φ takes a crucial role in the formulas we introduced in this section, which is why we measure it next.

4.2 Measuring the elasticity φ

Given the importance of the elasticity φ to the determination of the shape of the deficit-debt locus, we discuss the measurement of this parameter at length in this section. There are different ways to estimate the elasticity φ that are equivalent within the context of the model. Expanding (23), we can write

$$\varphi = -\frac{\partial\left(\rho + G - R\right)}{\partial \log b} = -b_0 \frac{\partial\left(\rho + G - R\right)}{\partial b} \tag{24}$$

Alternative ways to obtain φ are given by

$$\varphi = -\frac{\partial\left(G - R\right)}{\partial \log b} = -b_0 \frac{\partial\left(G - R\right)}{\partial b} \tag{25}$$

because, in the model, ρ is independent of b. As both (24) and (25) are valid ways to obtain φ, we will compare estimates across these specifications.

The key derivative terms in equations (24) and (25) have been estimated in the literature, and we summarize these estimates in Table 1.[13] For equation (24), Krishnamurthy and Vissing-Jorgensen [2012] focus on estimates of $\frac{\partial(\rho+G-R)}{\partial \log b}$. This derivative measures how the difference between the rate of return on government debt R and the return on other assets $\rho + G$ varies with a change in the log government debt to GDP ratio. Krishnamurthy and Vissing-Jorgensen [2012] use the yield spread difference between corporate bonds rated Baa and 10-year Treasuries as the measure of $\rho + G - R$, and they show a semi-elasticity of -0.013 to -0.017, depending on the sample. This implies that a 10 percent increase in debt to GDP leads to a 13 to 17 basis point decline in the convenience yield. Alternatively, one

[13] A detailed explanation of the exact specifications used from the existing literature to construct Table 1 is in Appendix A. We thank Sam Hanson, Andrea Presbitero, Quentin Vandeweyer, and Ursula Wiriadinata for helpful discussions.

can use the Krishnamurthy and Vissing-Jorgensen [2012] estimates to measure $b_0 \frac{\partial(\rho+G-R)}{\partial b}$, which gives estimates between -0.011 and -0.018 when using the average debt to GDP ratio over the relevant sample period for b_0. Using short-term T-bills and more high frequency data, Greenwood et al. [2015] also find estimates for $b_0 \frac{\partial(\rho+G-R)}{\partial b}$ in this range, around -0.014.

There is also a literature estimating the key derivative in equation (25), which is $b_0 \frac{\partial(G-R)}{\partial b}$. In particular, the recent study by Presbitero and Wiriadinata [2020] estimate this derivative in a sample of 56 countries from 1950 to 2019. They provide estimates of $\frac{\partial(G-R)}{\partial b}$ for 17 advanced economies and for the full sample. After multiplying these estimates by b_0, which is the average debt to GDP ratio in each of the respective samples, the implied estimates of $b_0 \frac{\partial(G-R)}{\partial b}$ are -0.014. For this study, we replicated the Presbitero and Wiriadinata [2020] results for the 17 advanced economies and also for the Group of 7 (G7) countries, and the coefficient estimate ranges are also reported in Table 1. The appendix shows the full results from the regressions. The estimates of interest are robust to the inclusion of both time and country fixed effects. Overall, most of the estimates across the different samples and the two different objects fit between -0.010 and -0.025.

An alternative technique to estimate $\frac{\partial(G-R)}{\partial \log b}$ is an analysis of the 2021 Georgia Senate run-off elections that took place on January 5th in the United States. Ex-ante, there was about an even probability of the two Democrat candidates winning their elections as there was that at least one of the two winning candidates was Republican. In the event of a Democrat win, Democrats would obtain the Senate majority, and would likely pass the $1.9 billion deficit-financed stimulus package already proposed by President-elect Biden. This was unlikely to happen otherwise. As shown in Figure 13 in Appendix A, the wins by both Democrats in Georgia led to a significant persistent increase in real 10 year Treasury yields of about 8 basis points. The effect is concentrated right after the election. It is unlikely that the election was associated with a change in long-term growth prospects; as a result, we interpret the evidence as suggesting that the prospect of the $1.9 trillion stimulus package, approximately corresponding to 7.4% of outstanding debt, led to a persistent 8 basis point reduction in $G - R$. As this the Democrat win was anticipated with 50% likelihood, this gives

$$\frac{\partial (G-R)}{\partial \log b} = -0.022.$$

The natural experiment yields an effect of government debt on $G - R$ that is in the same range as the estimates from the existing literature. Please see the Appendix A for details on this calculation.

Table 1: How does government debt to GDP affect convenience yield and $G - R$?

Study	Countries	Sample	Object	Estimated φ
Convenience yield: $\rho + G - R$				
Krishnamurthy and Vissing-Jorgensen [2012]	USA	1926-2008	$b_0 \frac{\partial(\rho+G-R)}{\partial b}$	-0.011
Krishnamurthy and Vissing-Jorgensen [2012]	USA	1969-2008	$b_0 \frac{\partial(\rho+G-R)}{\partial b}$	-0.018
Krishnamurthy and Vissing-Jorgensen [2012]	USA	1926-2008	$\frac{\partial(\rho+G-R)}{\partial \log b}$	-0.013
Krishnamurthy and Vissing-Jorgensen [2012]	USA	1969-2008	$\frac{\partial(\rho+G-R)}{\partial \log b}$	-0.017
Greenwood et al. [2015]	USA	1983-2007	$b_0 \frac{\partial(\rho+G-R)}{\partial b}$	-0.014
Growth minus Interest Rate: $G - R$				
Presbitero and Wiriadinata [2020]	17 AEs	1950-2019	$b_0 \frac{\partial(G-R)}{\partial b}$	-0.014
Presbitero and Wiriadinata [2020]	31 AEs & 25 EMs	1950-2019	$b_0 \frac{\partial(G-R)}{\partial b}$	-0.013
This paper	17 AEs	1950-2019	$\frac{\partial(G-R)}{\partial \log b}$	-0.015 to -0.031
This paper	G7	1950-2019	$\frac{\partial(G-R)}{\partial \log b}$	-0.020 to -0.028
This paper	USA, Senate	Jan 2021	$\frac{\partial(G-R)}{\partial \log b}$	-0.022
Negative Real Interest Rate: $-R$				
Laubach [2009]	USA	1976-2006	$b_0 \frac{\partial(\pi-R)}{\partial b}$	-0.015 to -0.022*
Engen and Hubbard [2004]	USA	1976-2003	$b_0 \frac{\partial(\pi-R)}{\partial b}$	-0.015*

Notes. This table summarizes estimates from the existing literature of the effect of government debt to GDP ratios on convenience yields (upper panel) and $G - R$ (lower panel). All of the details on the exact specifications used are in the appendix. Further details on the country-year panel regressions done in this study and the evaluation of the Georgia Senate election results of January 2021 are also in the appendix.

* Estimates in Laubach [2009] and Engen and Hubbard [2004] are stated in terms of $\frac{\partial(-R)}{\partial b}$. To obtain $b_0 \frac{\partial(-R)}{\partial b}$, estimates were multiplied by $b_0 = 0.5$, the average level of total federal debt to GDP over the sample period.

Table 2: Baseline calibration

Parameter	Description	Chosen to match target	Value
b_0	initial debt to GDP	2019 US federal debt to GDP	100%
x	gov. spending to GDP	2019 gov outlays to GDP	20%
R_0	initial nominal rate	fall 2019 5-year Treasury yield	1.5%
G^*	nominal trend growth	CBO long-term growth projection	3.5%
ρ	discount rate	return on private wealth other than gov. debt	0.03
κ	slope of the wage Phillips curve	standard value	0.03

Finally, Laubach [2009] and Engen and Hubbard [2004] estimate the effect of government debt to GDP on real interest rates, finding effects in the range 0.03 to 0.044. The average level of government debt (total federal debt) to GDP over their sample period was about 50%. Together, this gives an estimate of φ of $b_0 \frac{\partial(G-R)}{\partial b} \approx -0.015$ to -0.022 under the assumption that the real growth rate is unaffected by government debt.

Overall, while there is some variation, most of the implied elasticity estimates φ lie in the range 0.011 – 0.025. We pick the average estimate $\varphi = 0.017$ as our baseline parameter and explore robustness to $\varphi = 0.012$ and $\varphi = 0.022$ below.

4.3 Calibrating other parameters to the US

We calibrate the remaining model parameters to match the US economy in the fall of 2019, before the pandemic recession of 2020/21. We set the initial debt level to $b_0 = 100\%$ of GDP, assume government spending of $x = 20\%$. We set the initial nominal rate to $R_0 = 1.5\%$ in line with nominal interest rates in the fall of 2019.[14] We set the nominal trend growth rate to $G^* = 3.5\%$. In line with $G^* - R_0 = 2\%$, the US was indeed running a primary deficit of about 2% before the pandemic. We set the discount rate to $\rho = 3\%$, in line with about a $\rho + G^* = 6.5\%$ rate of return on private wealth other than government debt. Finally, we set the slope of the Phillips curve to a standard value, $\kappa = 0.03$. This parameter is not crucial for our analysis. We collect all our baseline parameters in Table 2.

Figure 5: Calibrated deficit-debt diagram

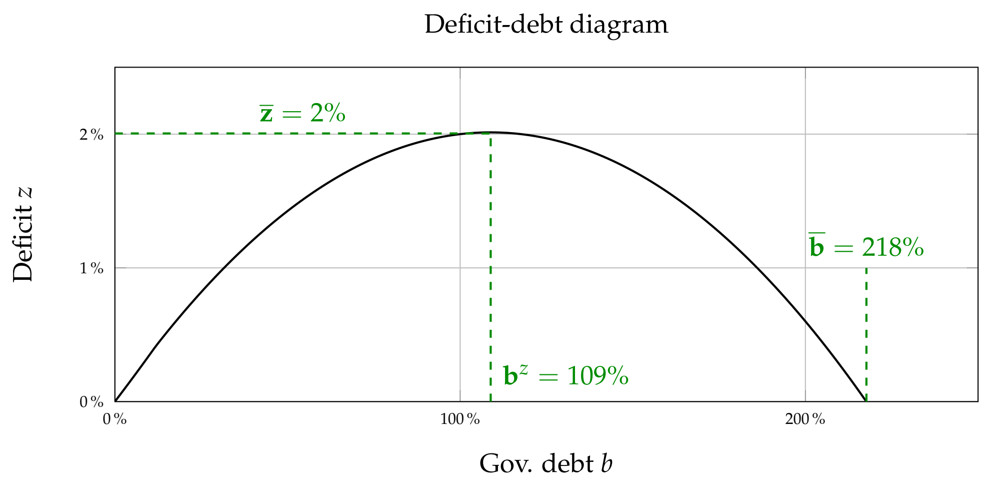
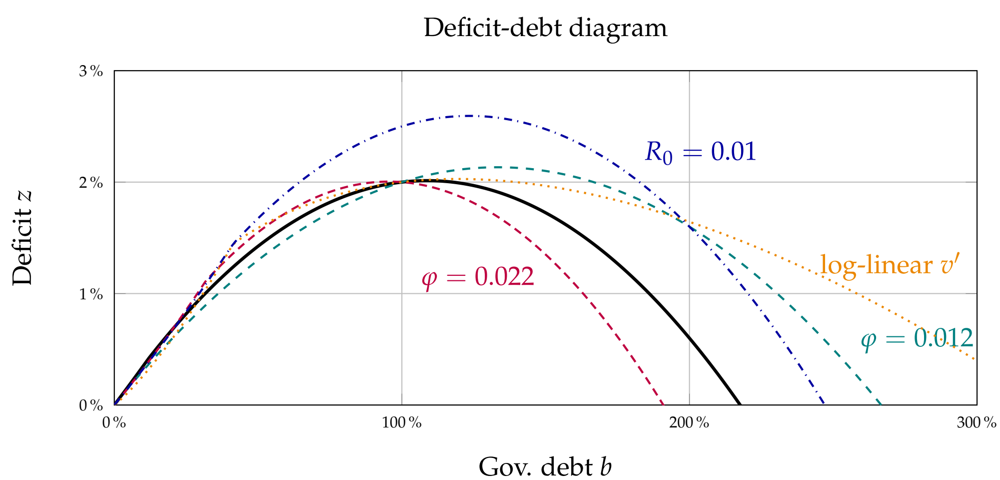


Figure 6: Robustness in deficit-debt diagram



4.4 Results for the US

Figure 5 shows the deficit-debt diagram that is implied by our calibration, as well as the three main quantities we set out to measure. The calibration suggests that the permanent primary deficit that the United States can run is $\overline{\mathbf{z}} = 2.0\%$ of GDP; it can run that at a debt level of $\mathbf{b}^z = 109\%$ of GDP; and the debt level at which R rises above G is $\overline{\mathbf{b}} = 218\%$ of GDP. These numbers are interesting against the backdrop that US government debt currently stands at around 126% as of the fourth quarter of 2020.[15] According to this calibration, any increase in debt above 109% of GDP would have to be met by increases in taxes going forward (even if R remains below G). Moreover, note that even if debt remained stable at 109%, this would only allow the US government to run a 2% permanent deficit. As of March 2021, the average projected annual primary fiscal deficit to GDP ratio by the CBO for 2021 to 2030 is 3.3%. For 2021 to 2050, it is 3.9%. If the large projected deficit of 2021 is excluded, then the averages are 2.7% and 3.7%, respectively.

Clearly the implied numbers by our calibration should not be taken as gospel. They instead illustrate how our stylized model can be put to work to parse through recent US data. We show robustness across alternative assumptions about the convenience yield in Figure 6. In particular, we show deficit debt diagrams for smaller or greater elasticities φ; for the log-linear functional form (22); and for a reduced pre-Covid natural interest rate $R_0 = 1\%$. Shifting G^* and R_0 in parallel (e.g. $G^* = 4\%$, $R_0 = 2\%$) does not affect the deficit-debt schedule. Also, as we discussed above, neither ρ nor the level of government spending x affects the deficit-debt schedule conditional on calibrating R_0. Across the alternatives shown in Figure 6, the maximum deficit $\overline{\mathbf{z}}$ remains relatively robust around 2-2.5%. Among alternatives with the linear functional form (21), the debt level $\mathbf{b}^z$ at which $\overline{\mathbf{z}}$ is attained varies in the range 100% to 130%. The debt level at which R crosses G is most uncertain, with estimates varying between just below 200% to just above 300%.

[14] The effective federal funds rate was 1.55%, the 5-year Treasury yield was about 1.65%, the 10-year yield just above that.

[15] The current government debt to GDP ratio represents all government debt outstanding as measured by the Financial Accounts of the Federal Reserve scaled by nominal GDP as measured in the National Income and Product Accounts. As of the end of 2020, the U.S. Treasury had a large cash balance at the Federal Reserve, which should be netted out. Using an unveiling process as in Mian, Straub, and Sufi [2020], the U.S. Government (other than the Federal Reserve) held 14% of GDP in claims on government bonds, primarily through this large cash balance at the Federal Reserve. If this position is netted out, then the outstanding government debt to GDP ratio held by entities other than the U.S. Government as of the end of 2020 was 112%. It is not appropriate to net out other holdings of the Federal Reserve, as these other holdings are financed by the private banking system through reserves.

Table 3: Calibration to other countries

Calibration parameters		Values by country			
Parameter	Description	USA	Japan	Italy	Germany
φ	semi-elasticity of conv. yield	$\varphi \in \{0.012, 0.017, 0.022\}$			
b_0	initial debt to GDP	100%	225%	135%	60%
R_0	initial nominal rate	1.5%	0%	0%	0%
G^*	nominal trend growth	3.5%	0.5%	0.7%	2.7%

Model statistics		Values by country			
Statistic	Description	USA	Japan	Italy	Germany
$\overline{\mathbf{b}}$	upper bound of Goldilocks zone	218%	291%	191%	155%
$\overline{\mathbf{z}}$	highest permanent deficit	2.0%	1.1%	0.9%	1.7%
$\mathbf{b}^z$	upper bound of free lunch region	109%	225%	135%	78%

4.5 Comparison across countries

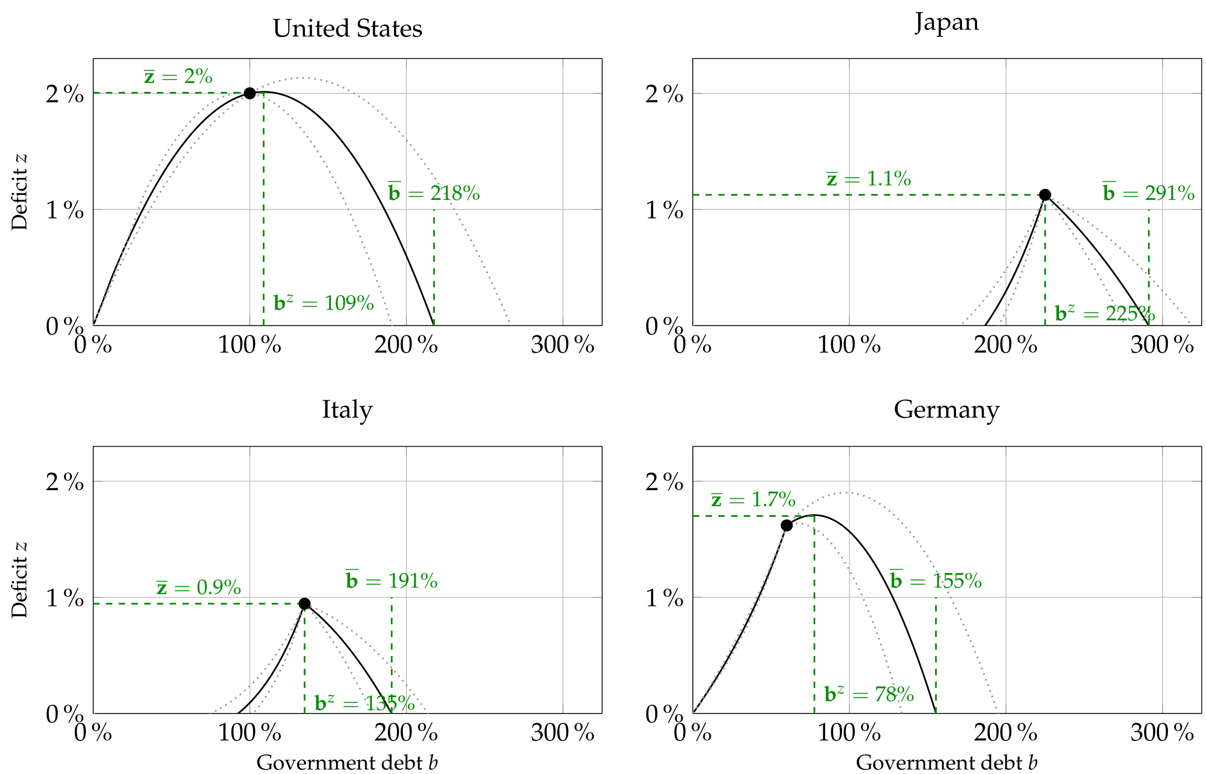
Our main calibration is meant to capture the pre-Covid US economy. In this subsection we explore simple calibrations to other countries, namely Japan, Italy and Germany.

Just as before, our calibration requires four objects—the elasticity φ, the initial debt level b_0, nominal trend growth G^*, and the initial interest rate R_0.[16] We work with the same elasticity as before, $\varphi = 0.017$, but allow as robustness check $\varphi \in \{0.012, 0.022\}$. Our choices for the other parameters are listed in Table 3.

We begin by computing the three numbers that we focused on before, the upper bound $\overline{\mathbf{b}}$ of the Goldilocks zone, the highest permanent deficit $\overline{\mathbf{z}}$, and the upper bound $\mathbf{b}^z$ of the free lunch region (the level at which $\overline{\mathbf{z}}$ is attained). We do so for $\varphi = 0.017$ and the linear specification (21). The results are shown in Table 3. Several observations are noteworthy. First, Italy and Japan are similar in several regards. Both economies are at the ZLB, carry high debt levels but have anemic nominal growth. Their permanent deficits $\overline{\mathbf{z}}$ are small, about half the US level. They also have no "free lunch" left, as $\mathbf{b}^z$ coincides with b_0. Germany, while also at the ZLB, is quite different. Its debt level is far smaller, but due to significantly faster growth its permanent deficit is almost as high as that of the US. Among the four countries, Germany is farthest away from the upper bound of the free lunch region,

[16]While the ZLB threshold is only a function of those four objects, the size of the deficits in the region where the ZLB is binding also depend on κ and x. We keep the same calibration as before for those.

Figure 7: Deficit-debt diagrams across countries

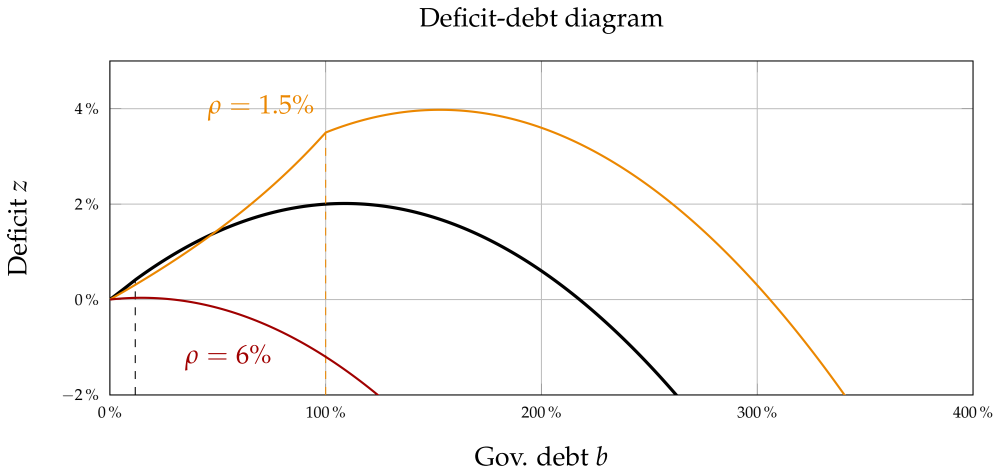


namely 18% of GDP (relative to its pre-Covid debt level). This indicates that the German government may not have to raise taxes to pay for the fiscal expansion during the Covid pandemic. This findings are graphically displayed in Figure 7, including robustness to $\varphi = 0.012$ and $\varphi = 0.022$ for each country (dashed lines).

5 What determines the size of the Goldilocks zone?

Across most advanced economies, the Goldilocks zone appears to be quite large; government debt to GDP ratios are high, and yet government deficits continue to be substantial. An examination of the underlying parameters of the model sheds light on how this situation continues to persist. Furthermore, the model highlights challenges to governments running large deficits while experiencing a decline in trend growth due to population aging or lower productivity growth. We focus in particular on three factors that are relevant for the Goldilocks zone: aggregate demand (as captured by discount rates), trend growth rates, and inequality.

Figure 8: Shifts in discount rates



5.1 Aggregate demand

We capture movements in aggregate demand by shifts in the discount rate ρ. A greater discount rate means agents are more impatient and would like to spend more and save less, which raises R^*. Vice versa, a smaller discount rate means agents are more patient and would like to spend less and save more, which lowers R^*.[17]

Figure 8 plots the Goldilocks zone for ρ's equal to 1.5%, 3% (our baseline), and 6%. For the case $\rho = 1.5\%$ (orange line), we see that the Goldilocks zone generally expanded, as R^* is reduced and $G^* - R^*$ grows. However, the lower ρ also leads there to be a tight ZLB constraint, with $\mathbf{b}^{ZLB}$ at 100% (dashed orange line). We confirm this in the following proposition.

Proposition 4. *A reduction in the discount rate* ρ

- *increases permanent deficits in the Goldilocks zone.*
- *shrinks permanent deficits at the ZLB.*
- *raises the ZLB threshold* $\mathbf{b}^{ZLB}$ *according to* $v'(\mathbf{b}^{ZLB}) = \frac{\rho + G^*}{1-x}$.

The opposite holds for an increase in ρ.

One way to interpret a reduction in ρ is that during recessions such as the Great Recession or the Covid recession, agents have a greater desire to save. This raises $\mathbf{b}^{ZLB}$,

[17]The discount factor ρ does affect the Goldilocks zone in this experiment, as we do not calibrate R_0 and G^* here, different from our calibration in the previous section.

possibly above b_0, pushing the economy into the ZLB region. Deficit-financed fiscal stimulus can mitigate the recession and move the economy to debt levels above $\mathbf{b}^{ZLB}$. Just like before, we see that there is always a Goldilocks zone at the ZLB (as long as $G^* > 0$). This also illustrates that negative shocks to aggregate demand are unlikely to cause fiscal sustainability concerns.

The case $\rho = 6\%$ corresponds to a situation with less desire to save and a greater R^*. In fact, in this example, R^* is sufficiently high for any level of debt b that it consistently lies above the trend growth rate G^*. The economy needs to run a primary surplus, there is no Goldilocks zone. This can be thought of as the state of the US economy in the 1980s or early 1990s. One can say that the Goldilocks zone that the United States can currently afford is a "symptom" of low secular demand / low secular interest rates R^*.

5.2 Trend growth

A reduction in nominal trend growth G^*—whether caused by a productivity growth slowdown, falling inflation expectations, or declining population growth—seems like it may tighten the Goldilocks zone by moving G^* closer to R. But this is not obvious as slower growth rates lead to a greater desire for saving by households, pushing R^* down alongside G^*. With log preferences over consumption as in (1), R^* falls one for one with G^*, as in (9), leaving $G^* - R^*$ unchanged. This is why, in our model, growth does not affect steady state deficits in the Goldilocks zone. This implies that as long as the economy is above the ZLB, the deficit-debt schedule is unchanged. However, slowing growth does affect the ZLB threshold and deficits at the ZLB, as the following proposition demonstrates.

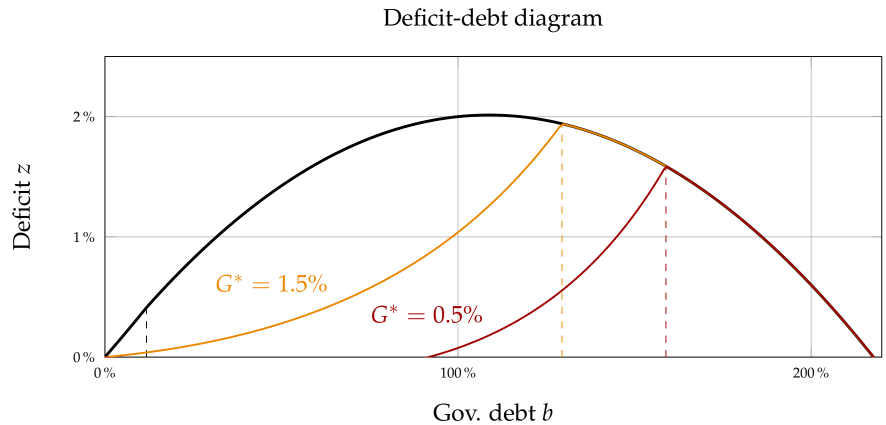
Proposition 5. *A slowdown in nominal trend growth G^**

- *leaves permanent deficits in the Goldilocks zone unchanged.*
- *shrinks permanent deficits at the ZLB.*
- *raises the ZLB threshold $\mathbf{b}^{ZLB}$ according to $v'(\mathbf{b}^{ZLB}) = \frac{\rho+G^*}{1-x}$.*

The proposition follows directly from the results in Section 3. We illustrate it in Figure 9. Above the ZLB, the economy can continue running the same primary deficits as before, even as growth slows. However, as soon as R^* falls sufficiently to cross zero, the economy is at the ZLB, where output is no longer at potential, and the permanent primary deficit actually falls in G^*.

In Figure 9, this can be seen as the orange and red lines showing tighter and tighter ZLB thresholds $\mathbf{b}^{ZLB}$ (vertical dashed lines) that force policymakers to run greater deficits

Figure 9: Shifts in trend growth



temporarily so as to move the economy out of the ZLB region. Ultimately, the economy ends up with much greater levels of government debt and possibly smaller steady state deficits. At a stylized level, this seems to capture Japan's experience over the past three decades.

5.3 The role of inequality

Inequality is relevant for fiscal sustainability, as it is mainly richer households that, directly or indirectly, own government debt. Figure 10 shows that the top 10% of the wealth distribution in the United States holds almost 70% of the government debt outstanding held by the U.S. household sector. Furthermore, the bottom 50% of the wealth distribution holds almost no government debt at all. This implies that the willingness or ability of richer households to save is a primary factor in the determination of R^*. To speak to these issues, we extend our model to allow for two types of agents.

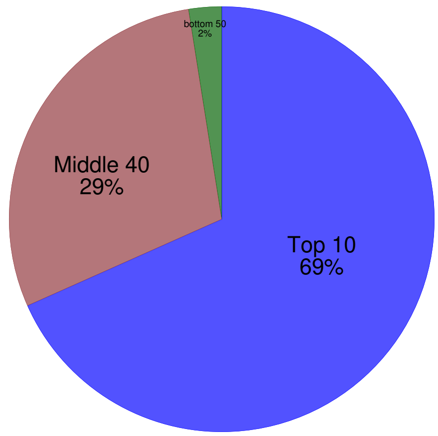
Savers (or *bondholder*s) earn a share $\omega^s \in (0, 1)$ of labor income and behave just like the representative agent did above. In particular, savers maximize utility $\int_0^\infty e^{-\rho t} \{\log c_t^s + v(b_t)\} dt$ subject to the budget constraint

$$c_t^s + \dot{b}_t \leq (R_t - G_t) b_t + \omega^s w_t n_t - \tau_t^s. \tag{26}$$

Hand-to-mouth agents earn a share $\omega^h = 1 - \omega^s$ of labor income and have no access to financial markets, that is,

$$c_t^h = \omega^h w_t n_t - \tau_t^h.$$

Figure 10: Holdings of U.S. Government Debt across the Household Wealth Distribution, as of 2020



Notes. This figure plots the fraction of U.S. government debt held by households across the wealth distribution. These are fractions held after unveiling the financial sector. Please see Mian et al. [2020] for more details on the unveiling process. The unveiling process that produces this figure uses the Distributional Financial Accounts of the Federal Reserve to measure the share of assets owned by each wealth group. U.S. households hold almost 60% of U.S. government debt, with the rest of the world holding most of the remaining U.S. government debt.

Here, τ_t^h may be negative in order to capture transfers to hand-to-mouth agents. Observe that, above the ZLB, this model can be thought of as a reinterpretation of our representative-agent model, in which the government spends on behalf of hand-to-mouth agents.

We calibrate this model by identifying savers with the top 10% and setting $\omega^s = 50\% = \omega^h$. For now, we set $\tau_t^h = 0$, which we relax in Section 6 below. We keep the other parameters unchanged.

What happens to the Goldilocks zone when inequality increases? It is now the Euler equation of savers that determines the steady state natural interest rate,

$$\frac{\dot{c}_t^s}{c_t^s} = R_t^* - G^* - \rho + v'(b_t)c_t^s$$

where, above the ZLB, $c_t^s = 1 - \omega^h - x$. We thus obtain the natural interest rate R^* as

$$R^*(b_t) = \rho + G^* - v'(b_t)\left(1 - \omega^h - x\right)$$

Different from before, the income distribution now directly influences the convenience yield. Following the same steps as in Section 3, we obtain:

Proposition 6. *Greater inequality ($\omega^s \uparrow$, $\omega^h \downarrow$ with $\omega^s + \omega^h$ unchanged)*

- *expands permanent deficits in the Goldilocks zone (above the ZLB), where*

$$z(b) = \left(v'(b_t)\left(1 - \omega^h - x\right) - \rho\right) b.$$

- *shrinks permanent deficits at the ZLB, where*

$$z(b) = \frac{v'(b)}{v'(b) - \kappa} G^* b + \frac{\kappa}{v'(b) - \kappa}\left(\rho - v'(b)\left(1 - \omega^h - x\right)\right) b.$$

- *raises the ZLB threshold* $\mathbf{b}^{ZLB}$ *according to* $v'(\mathbf{b}^{ZLB}) = \frac{\rho + G^*}{1 - \omega^h - x}$.

The results are reversed with lower inequality.

Proposition 6 shows that inequality is tightly connected to the Goldilocks zone. On the one hand, it increases fiscal space above the ZLB, allowing the government to run greater permanent deficits. This is because inequality puts more resources into the hands of savers, increasing their ability and willingness to hold larger quantities of government debt. But this very fact also implies that, at the ZLB, greater inequality tightens the Goldilocks zone, as it reduces demand, inflation, and ultimately nominal growth G. Figure 11 illustrates these findings.

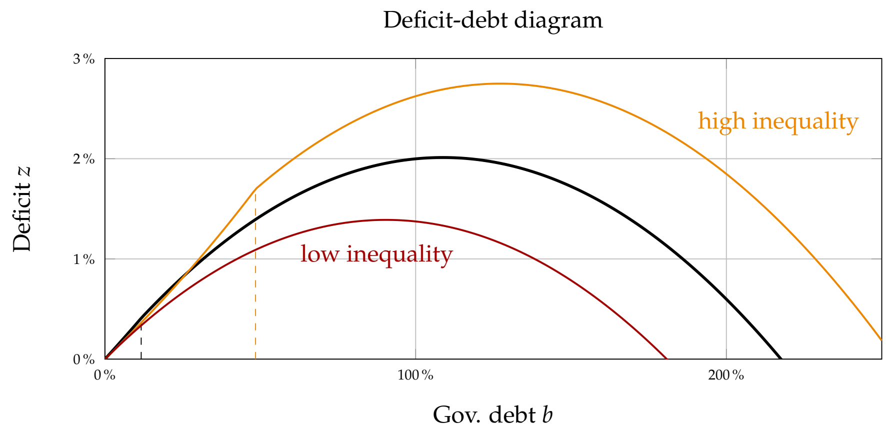
The model provides intuition behind the observation that rising income inequality has been accompanied by rising fiscal deficits and government debt levels in many advanced economies. Rising income inequality allows governments to borrow more cheaply from savers. However, accumulating high government debt in response to a rise in income inequality raises challenges going forward. For example, if the government implements policies that reduce inequality, it will need to raise taxes to cover the higher debt costs associated with the high debt levels accumulated when inequality was high. It also presents challenges to tax policy, which we discuss in the next section.

6 The role of tax policy

The fact that inequality, and in particular the economic position of savers, matters for the size of the Goldilocks zone has important implications for tax policy, which we explore next. We continue to use the model with two types of agents, as introduced in section 5.3.

Tax instruments. We begin by introducing the tax instruments. In addition to separate taxes on savers and hand-to-mouth agents, we also allow for consumption taxes τ_t^c (which

Figure 11: Rising income inequality



are equivalent to value added taxes in our model) and capital income taxes τ_t^b. Savers thus maximize their utility function subject to the modified budget constraint

$$\left(1+\tau_t^c\right) c_t^s + \dot{b}_t \leq \left(\left(1-\tau_t^b\right) R_t - G_t\right) b_t + \omega^s w_t n_t - \tau_t^s \tag{27}$$

while the budget constraint of hand-to-mouth agents is given by

$$\left(1+\tau_t^c\right) c_t^h = \omega^h w_t n_t - \tau_t^h. \tag{28}$$

Together, this gives us four tax instruments: regressive income taxes τ_t^h on the hand-to-mouth, progressive income taxes τ_t^s on savers, consumption taxes τ_t^c, and capital income taxes τ_t^b. In the presence of capital income taxes, before-tax and after-tax returns on government bonds differ. Throughout this section we follow the convention that R_t^* denotes the after-tax return on government bonds. We do so because (a) it is the after-tax return that matters for the ZLB and (b) the government effectively pays the after-tax return on its debt once tax revenue from capital income taxes is netted out.

Taxes and the Goldilocks zone. With this convention, and the four tax instruments, savers' Euler equation now reads

$$\frac{\dot{c}_t^s}{c_t^s} = R_t^* - G^* - \rho + v'(b_t)\left(1+\tau_t^c\right) c_t^s \tag{29}$$

Goods market clearing requires that $c_t^s + c_t^h + x = 1$, where (28) implies that $c_t^h = \frac{\omega^h - \tau_t^h}{1+\tau_t^c}$ in the natural allocation. With constant tax rates, the natural interest rate can be obtained by rearranging (29),

$$R^*(b_t) = \rho + G^* - v'(b_t)\left((1+\tau^c)(1-x) - \omega^h + \tau^h\right). \tag{30}$$

The steady state primary deficit is then given by

$$z(b) = \begin{cases} \frac{v'(b)}{v'(b)-\kappa} G^* b + \frac{\kappa}{v'(b)-\kappa}\left(\rho - v'(b)\left((1+\tau^c)(1-x) - \omega^h + \tau^h\right)\right) b & \text{at the ZLB} \\ \left(v'(b)\left((1+\tau^c)(1-x) - \omega^h + \tau^h\right) - \rho\right) b & \text{above the ZLB} \end{cases} \tag{31}$$

Our main result in this section uses (31) to characterize the effect of the four taxes on the shape of the Goldilocks zone.

Proposition 7. *The tax instruments affect the deficit-debt schedule as follows.*

- *Increased regressive income taxes τ_t^h and consumption taxes τ_t^c expand the Goldilocks zone the most, by $v'(b)b$ per tax dollar. They reduce steady state deficits at the ZLB, and raise the ZLB threshold $\mathbf{b}^{ZLB}$ in line with $v'(\mathbf{b}^{ZLB}) = \frac{\rho + G^*}{(1+\tau^c)(1-x)-\omega^h+\tau^h}$.*
- *Increased capital income taxes leave the Goldilocks zone unchanged.*

Proposition 7 studies the effects of raising taxes on the deficit-debt schedule. To interpret the results, we bear in mind that a change in permanent deficits is, by construction, accommodated by a change in the tax on savers τ^s, which is why τ^s itself mechanically has no effect on the Goldilocks zone and is excluded from Proposition 7.

The first part of Proposition 7 studies the effects of raising income taxes on the hand-to-mouth and consumption taxes (using the proceeds to reduce τ^s). Both kinds of taxes are regressive, reducing aggregate demand and natural interest rates R^* in (30). Above the ZLB, this relaxes the government budget constraint and allows for greater permanent primary deficits. However, it also tightens the ZLB, and reduces primary deficits at the ZLB. The second part of Proposition 7 shows that increased capital income taxes are irrelevant for fiscal policy. In fact, they do not even raise any revenue in our model, as the before-tax return on government debt immediately adjusts upwards to keep the after-tax return constant. Two caveats are in place here. First, with longer-duration debt or large surprise taxes at date $t = 0$, some initial expropriation occurs, which can be used for a one-time reduction in government debt. Second, the only source of capital income in our model is interest income from government debt. If other types of capital income, such as dividends, were present, the capital income tax would adopt some of the properties of a tax on savers' income.

There are three immediate implications of Proposition 7, which we now go through in detail.

Implication 1: Redistribution narrows fiscal space. The first implication is that redistributive policies narrow "fiscal space" in the sense of reduced permanent deficits. Taxing savers and giving it to the hand-to-mouth means taxing the buyer of government debt. This reduces the demand for government debt and pushes up R^*, requiring a greater reduction in the deficit. Vice versa, a more regressive tax response limits the required correction in the deficit. This highlights an important dilemma. Remaining in the Goldilocks zone, especially when a "free lunch" policy is to be successful, necessitates that savers have enough resources to purchase bonds at high prices, i.e. at low R^*. One cannot simultaneously eliminate inequality—by redistributing from rich bondholders to poorer hand-to-mouth agents—and sustain a large debt level at low interest rates. The model suggests that a government that chooses to use aggressive fiscal policy in response to a rise in income inequality should recognize that large debt levels make any subsequent effort at redistribution more costly from a fiscal perspective.

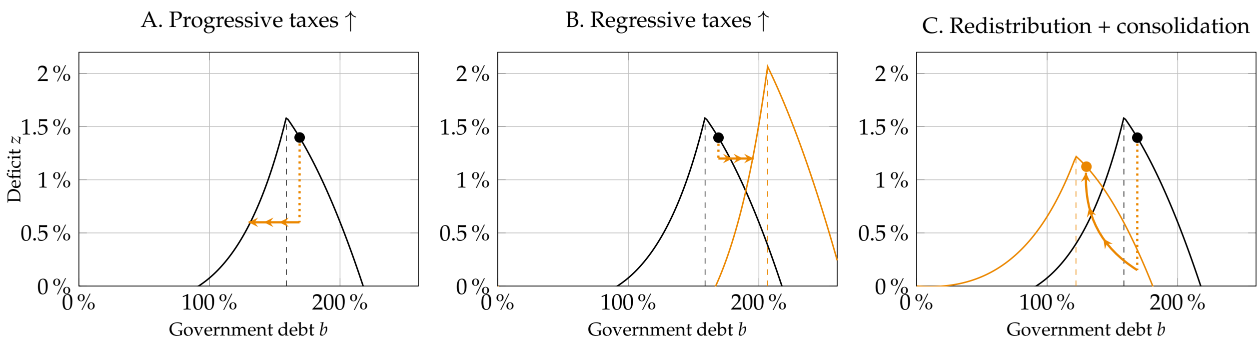
Taking this logic one step further establishes that regressive taxation (e.g. consumption taxes) is able to finance a greater level of government debt than progressive taxation, holding fixed the overall tax burden. Governments with sufficiently large debt levels and interest rates R near or above G may thus be forced to resort to such regressive taxation.[18]

Implication 2: Handling a growth slowdown. As we saw in Section 5.2, a slowdown in growth, causing high levels of government debt in a very narrow Goldilocks zone, poses a conundrum to policymakers. On the one hand, debt levels are high and R is close to G, raising concerns about fiscal sustainability. On the other, R is close to the ZLB, making it imperative to avoid reductions in the natural interest rate.

This conundrum leads to counterintuitive results. For example, it may seem that regressive tax policies, such as consumption taxes, relax the Goldilocks zone by reducing R^* and allow for fiscal consolidation. However, regressive tax policies also tighten the ZLB constraint, requiring increased deficits and debt levels as remedy. This is best seen in the equation for $\mathbf{b}^{ZLB}$ in Proposition 7, $v'(\mathbf{b}^{ZLB}) = \frac{\rho+G^*}{(1+\tau^c)(1-x)-\omega^h+\tau^h}$. Increased τ^c requires increased debt $\mathbf{b}^{ZLB}$ to stay out of the ZLB. Other simple alternative policies do not work either. For example, lowering deficits by reducing taxes on the rich is contractionary as

[18] This argument can, in principle, be taken even further. Any policy instrument that discourages demand reduces natural interest rates R^* and thus has beneficial effects on the government's interest expenses.

Figure 12: Fiscal consolidation during a growth slowdown



well (albeit by less), and hence pushes the economy into the ZLB as well. We illustrate these policies as paths (A) and (B) in Figure 12.

How can the conundrum be resolved? What the above discussion shows is that strategies that reduce the natural interest rate R^* are doomed to fail, as they further tighten the ZLB. This is despite the fact that such strategies allow for greater fiscal space above the ZLB. Instead, strategies that raise R^* are more promising, when coupled with deficit reductions. An example of such a strategy is increased taxation of savers that is partly used to redistribute to the hand-to-mouth, and partly used to reduce deficits. Path (C) of Figure 12 shows the effect of such a strategy. Increased R^* implies a thinner Goldilocks zone, yet one with a ZLB threshold further to the left. This gives the economy room to move to the left with deficit reductions, without hitting the ZLB.[19]

Implication 3: Financial repression. Our third implication concerns financial repression. Financial repression can be modeled as the government imposing a lower bound on the required bond holdings of the saver, $b_t \geq \underline{b}$, thereby allowing it to reduce the interest rate it pays on government debt, from the market rate R_t to some $R_t - \xi_t$, where ξ_t measures the extent of financial repression. Modeled this way, financial repression corresponds to nothing other than a tax on bondholders. When large debt positions are financed this way, a significant amount of repression is necessary. Since it acts like a tax, it reduces the resources of savers and hence reduces their demand for bonds, requiring even more stringent financial repression.

[19] In practice, when debt levels are very high, even such a strategy may not be without dangers. For example, when R^* increases too far, say beyond G, fiscal sustainability may be at risk as the government may have to shift towards running primary surpluses.

7 Extensions

Our baseline model made several simplifying assumptions. We now explore two extensions that relax those.

7.1 Long-term bonds

The first extension relaxes our assumption of short-term debt. In particular, we now assume that debt is both nominal and long-term. We model this with exponentially declining coupons, as in Hatchondo and Martinez [2009]. We denote by $\tilde{b}_t$ the nominal principal outstanding, normalized by (nominal) potential GDP as before, and assume each pays a fixed coupon of 1. The price of a bond is then given by

$$q_t = \int_t^\infty e^{-\int_t^s (R_u + \lambda) du} ds$$

We continue to denote by $b_t \equiv q_t \tilde{b}_t$ the present value of government debt, and assume it is b_t that enters agents' utility. One can show that the government's and representative agent's budget constraints, (4) and (2), are unchanged. The only difference in this model, relative to our baseline in Section 2, is that there is a date-0 revaluation effect, changing the value of government debt relative to the previous steady state value b,

$$\frac{b_0}{b} = (R + \lambda) \int_t^\infty e^{-\int_t^s (R_u + \lambda) du} ds \tag{32}$$

The sole factor determining the strength of the revaluation is the nominal interest rate. In particular, revaluation only depends on inflation through the nominal interest rate.

Equation (32) has two implications. First, long-term nominal bonds neither affect the size of the Goldilocks zone nor the "free lunch" region. While a temporary increase in deficits z_t necessarily leads to a reduction in the real value of outstanding government debt—due to the expectation of rising debt levels and interest rates—this reduction can never outweigh the increased deficits. Debt will eventually always rise above the previous steady state level.[20] However, long-term nominal bonds do help slow down the increase in debt after a given deficit policy.

The second implication of (32) is that, as long interest rates are zero at the ZLB, $R_t = R = 0$, this effect is muted. In other words, even if deficits are raised and inflation increases,

[20]This follows from a simple proof by contradiction. Imagine debt b_t was to remain below the initial steady state level b. Then, R_t must remain below R and (32) prescribes an increase in b_0, not a reduction, which contradicts our hypothesis.

as long as the nominal rate stays put, b_0 will not be revalued.

In sum, we find that the government cannot "pull itself up by its bootstraps" and devalue its government debt by spending more.

7.2 Aggregate risk and safety premia

Throughout, we have assumed that $v(b)$ captures exogenous convenience benefits. These benefits are typically thought of as stemming from either liquidity or safety. Many micro-foundations exist for liquidity (e.g. Lagos and Wright 2005), and they have been shown to reduce to a $v(b)$ function (Angeletos et al. 2020). In this section, instead, we propose a model of safety premia, interpreting bonds as being safe if they are very likely pay out even in a big disaster.

Consider an economy like the one in Section 2, with two changes. First, there is no ad-hoc convenience utility function v. Second, there is a flow probability $\lambda > 0$ with which a disaster occurs. Conditional on the disaster occurring, it reduces potential output y^* from 1, our normalized pre-disaster value, to $\delta \in (0, 1)$, with probability $f(\delta)$, where $\int_0^1 f(\delta) d\delta = 1$. The only friction that we assume in this model is that the government can only raise tax revenue τ_t up to some fraction $\overline{\tau} + x$ of output.[21] If debt service requires greater taxes, we assume that the government defaults. For simplicity, we assume that default entails default costs (in the form of transfers to households, not resource costs) that are sufficiently large so that all bond wealth is lost.

We analyze this model in two steps. First, we analyze the economy after a disaster of size δ happened. Then, we study the economy before the disaster shock, and argue that it is largely isomorphic to our model in Section 2.

When a disaster of size δ materializes, the interest rate rises to $R = G^* + \rho$, as bonds lose their specialness. This requires the economy to run a primary surplus of $\rho b/\delta$ relative to GDP. Given the upper bound on taxes of $\overline{\tau} + x$, default occurs when output after the shock δ falls below $\underline{\delta} \equiv \rho b/\overline{\tau}$. We denote by $\tilde{V}_t(b; \delta)$ the utility of an individual agent with bond position b after shock δ realizes.

Before the disaster occurs, households now maximize utility

$$\rho V_t(b) \equiv \max_c \log c + \lambda \int_0^1 f(\delta) \left(\tilde{V}_t(b; \delta) - V_t(b) \right) d\delta + \dot{V}_t(b) + V_t'(b) \dot{b}_t \tag{33}$$

where $\dot{b}_t$ is given by the budget constraint (2). This formulation can be shown to imply a

[21] We include the share of government spending x here so that the government can always finance its spending. This is equivalent to a cap on the primary surplus of $\overline{\tau}$.

natural rate before the disaster that depends on b and is given by

$$R^*(b) = \rho + G^* + \lambda - \lambda \int_{\rho b/\overline{\tau}}^1 f(\delta)\delta^{-1} d\delta$$

This expression has exactly the same shape as (9), just with $\lambda - \lambda \int_{\rho b/\overline{\tau}}^1 f(\delta)\delta^{-1} d\delta$ instead of $v'(b)(1-x)$. As before, the convenience yield falls in b. In the special case where the density is equal to $f(\delta) = 2\delta$, we find that the convenience yield is given by

$$\lambda \int_{\rho b/\overline{\tau}}^1 f(\delta)\delta^{-1} d\delta - \lambda = \lambda - 2\frac{\rho\lambda}{\overline{\tau}} b$$

microfounding our affine-linear specification (21).

This illustrates that our analysis in (3) applies to a model with a microfounded convenience yield based on safety premia.

8 Conclusion

Government debt to GDP ratios in many advanced economies, including the United States, have entered into unchartered territory. At the same time, interest rates on government debt have been pinned against the zero lower bound. The situation has led to two types of concerns: first, there is a real threat of a long-run secular stagnation equilibrium with low demand and low output. Second, such high levels of debt threaten a future, potentially large, rise in taxes or cut in spending if interest rates rise. This study builds a model that helps characterize fiscal policy in the face of these two concerns.

The framework shows that there is a Goldilocks zone in which fiscal policy is neither too cold, threatening secular stagnation, nor too hot, threatening a subsequent rise in taxes and cut in spending. The Goldilocks zone can be quantified using empirical moments that are the subject of a large body of research. The framework shows that both nominal interest rates (R) and nominal growth rates (G) are endogenous to government debt levels, and it makes it clear that it is possible in some situations for the government to raise deficits without ever having to raise taxes of cut spending to pay for the deficits.

While we find that a government can sustain a large debt to GDP ratio in the Goldilocks zone, the maximum permanent deficit that can be sustained is in the range of 2%, which is lower than most projections of expected future deficits for the United States. Furthermore, the Goldilocks zone is fragile; a rise in aggregate demand, a decline in trend growth, or a decline in inequality can lead to a shrinkage, or even a vanishing, of the zone. The framework suggests that governments can try to live in the Goldilocks zone, but this is potentially dangerous when deficits are used to address longer-run structural problems such as rising inequality and an aging population.

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Appendix

A Details on estimation of φ

A.1 Further discussion of estimates from the literature

The estimates of $\frac{\partial(\rho+G-R)}{\partial \log b}$ from Krishnamurthy and Vissing-Jorgensen [2012] reported in Table 1above come from their Table 1, columns 4 and 5. The measure of the spread is the Baa corporate yield minus the Treasury bond yield, which they prefer because "Aaa bonds offer some convenience services of Treasuries and thus the Baa-Treasury spread is more appropriate for capturing the full effect of Treasury supply on the Treasury convenience yield." For the estimates of $b_0\frac{\partial(\rho+G-R)}{\partial b}$, we collected the same data as in Krishnamurthy and Vissing-Jorgensen [2012] and regressed the Baa minus Treasury spread on the level of the debt to GDP ratio. We multiply this coefficient $\frac{\partial(\rho+G-R)}{\partial b}$ (which is -0.027 and -0.048 for the long and short time periods, respectively) by the average level of the debt to GDP ratio b_0 (which is 0.42 and 0.36 for the long and short timer periods, respectively) to get the estimate.

The Greenwood et al. [2015] estimate is from column 1 of Panel B of their Table 1. The measure of the spread is the difference between the actual yield on an 2-week Treasury bill and the 2-week fitted yield, based on the fitted Treasury yield curve in Gürkaynak, Sack, and Swanson [2007]. The derivative is with respect to the amount of Treasury bills outstanding scaled by GDP. The implied estimate of $\frac{\partial(\rho+G-R)}{\partial b}$ is -0.167, which we then multiply by the average Treasury bill to GDP ratio b_0 (which is 0.084) to get the estimate. We use the estimate from Panel B which goes only through 2007 because of the endogeneity issues discussed by Greenwood et al. [2015] surrounding the Great Recession and financial crisis (see the last full paragraph on page 1689 of their article). The Vandeweyer [2019] regression estimate comes from column 2 of Table 4 of his study. The measure of the spread is the 3-month T-bill rate minus the 3-month General Collateral Repo rate, and this is regressed on the ratio of outstanding T-bills to GDP. The implied estimate of $\frac{\partial(\rho+G-R)}{\partial b}$ is -0.040, which we then multiply by the average Treasury bill to GDP ratio b_0 (which is 0.010) to get the estimate. We use column 2 of Table 4, as this regression controls for the Federal Funds rate as suggested by Nagel [2016]. The Vandeweyer [2019] natural experiment involves the 2016 money market reform which led to a large rise in demand for T-bills by money market funds. Money market funds increased their holdings of T-bills by $400 billion, which was about 20% of the stock outstanding. Vandeweyer [2019] uses a model-based counter-factual to show that this shock led to an 18 basis point reduction in yields on government debt, which gives $\frac{\partial(\rho+G-R)}{\partial \log b} = 0.009$. The estimate from Takaoka [2018] comes from Table 4, and the estimate from Jiang, Lustig, Van Nieuwerburgh, and Xiaolan [2020b] comes from Table 5, panel A, column 2. For the Jiang et al. [2020b] estimate of -0.01, we multiply by the average government debt to GDP ratio in their sample to get the final estimate of -0.008.

The estimates of $b_0 \frac{\partial(G-R)}{\partial b}$ come from Presbitero and Wiriadinata [2020], Table A3, column 1. The coefficients $\frac{\partial(G-R)}{\partial b}$come from that table (-0.027 for advanced economies, -0.024 for the full sample), and then these are multiplied by the average government debt to GDP ratiob_0 for the respective samples, which are 0.53 and 0.56 for the advanced economies and the full sample, respectively.

A.2 Regressions based on Presbitero and Wiriadinata [2020]

The other estimates from Table 1 come from our own data analysis using a data set constructed exactly as the one used by Presbitero and Wiriadinata [2020]. The associated regression table is Table 4.

Table 4: Results from regressions on Presbitero and Wiriadinata [2020] data

	Left hand side: G - R					
	(1)	(2)	(3)	(4)	(5)	(6)
Log(Gov Debt/GDP)	-0.024***	-0.031***	-0.015**	-0.025**	-0.028**	-0.020**
	(0.006)	(0.005)	(0.004)	(0.007)	(0.006)	(0.003)
Observations	1184	1184	1184	490	490	490
R^2	0.103	0.179	0.553	0.162	0.209	0.698
FE		Country	Country and Year		Country	Country and Year
Sample						

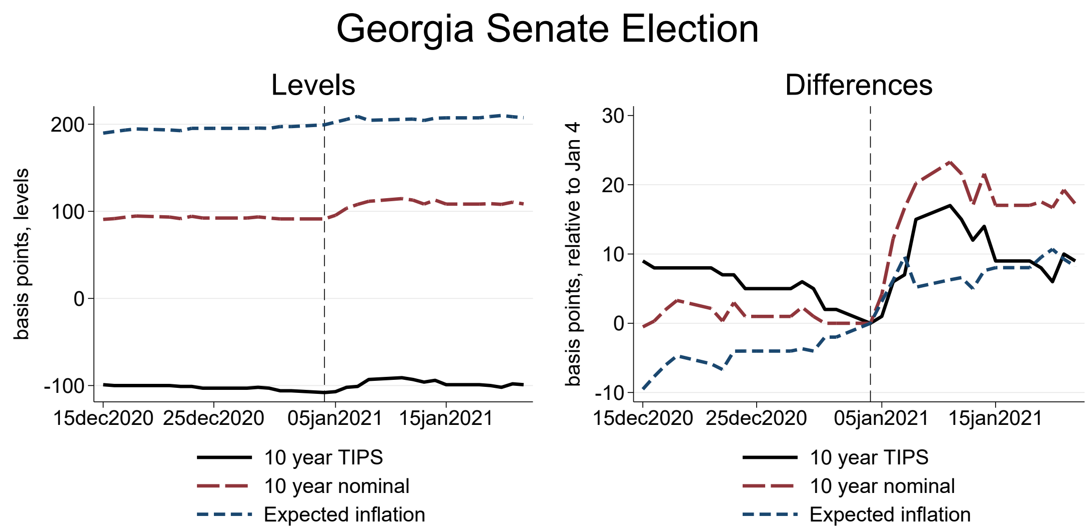
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.
Standard errors are heteroskedasticity-robust, clustered by country.

Note. This table presents coefficient estimates of $G - R$ on government debt to GDP ratios. The sample for the first three columns are the 17 advanced economies covered by the JST Macrohistory data base. The sample for columns 4 through 6 is G7 countries (Canada, France, Germany, Italy, Japan, United Kingdom, United States). The time period covered is 1950 to 2019. Please see Presbitero and Wiriadinata [2020] for more details.

A.3 Georgia Senate election

The Georgia Senate election of January 5, 2021 offers a unique opportunity to assess how markets perceive a sudden rise in expected government debt. On the eve of the election, trading at Electionbettingodds.com implied a 50.8% probability of the Republicans controlling the Senate, and a 49.1% probability of the Democrats controlling the Senate.

Figure 13: The change in real interest rates around the January 5th, 2021 Georgia run-off election.



It was widely reported in the press that President-Elect Biden's administration would propose a $1.9 trillion "American Rescue Plan" once the President-Elect took office. Our assumption in the calculation below is that the win by the two Democrats in the Georgia Senate election of January 5, 2021 increased the expected government debt by $2 trillion, which at the time was about 7.4% of total debt outstanding.

Figure 13 shows the effect on the 10 year nominal interest rate, the 10 year TIPS interest rate, and expected inflation. As it shows the victory by the Democrats in the Georgia Senate election led to a 15 basis point immediate reaction which then declined to an 8 basis point reaction after a week. Taken together, these numbers imply that a 3.7% rise in total government debt outstanding relative to prior expectations led to an 8 basis point decline in $G - R$, which gives an estimate of $\frac{\partial(G-R)}{\partial \log b}$ of -0.022. The data for these calculations come from Bloomberg.