Higher-Order Income Risk over the Business Cycle: A Parametric Approach*

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Abstract

We develop a novel parametric approach to estimate the relationship between idiosyncratic and aggregate labor income risk. We derive closed form expressions for the variance and skewness of shocks, and achieve identification in a Generalized Method of Moments (GMM) framework. We apply our method to German and US individual and household data. For both Germany and the US, we find that the variance of permanent shocks to gross labor earnings of males increases in recessions, while at the same time the skewness becomes more negative: this is in line with negative log earnings realizations becoming more likely than positive ones. Considering German household gross labor earnings, we find insurance against transitory but not against permanent shocks. Finally, the German tax and transfer system provides insurance against both shocks; after taxes and transfers the cyclicality of household labor earnings risk is gone. Future versions of this paper will also contain results for households in the US economy.

Keywords: Labor Income Risk, Business Cycle, GMM Estimation, Skewness

J.E.L. classification codes: D31, E24, E32, H31, J31

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1 Introduction

Understanding how individual incomes vary over the business cycle is of major importance for economic policy, e.g., for the design of stabilization policies. Capturing the intuitive notion that downside labor income risk of workers is increasing in a recession with an income process featuring a countercyclical variance might, however, be misleading. Such a process implies that during a recession the probabilities of both, an income drop as well as a rise of income, are higher. The latter implication seems wrong. In order to allow for the possibility of higher downside risk along with constant (or lower) upside chances during a recession, one must take the third moment of the distribution, the skewness, into account. An income process with countercyclical left-skewness of individual income risk implies that in a recession the probability of a drop in income is higher—as also implied by a countercyclical variance—and that the probability of an increase of income is unchanged (or smaller)—unlike implied by a countercyclical variance.

In this paper we address this matter by developing a novel parametric approach to estimate the relationship between idiosyncratic labor income risk over the business cycle. We analyze the cyclicality of the distribution of idiosyncratic labor income shocks, i.e., shocks to income conditional on observable characteristics such as age and education. We do so by first adopting the standard approach to decompose labor income into a deterministic and a stochastic component. The stochastic component in turn is composed of a fixed effect as well as a persistent and a transitory shock to income. The distributions of these two components are parameterized by the respective variance and skewness. In addition, the moments of the persistent shock are assumed to be contingent on the aggregate state of the economy, i.e., whether the economy is in a recession or in a boom.

Our parametric estimation procedure allows for identification of all these moments of the shock distribution. Specifically, we derive closed form expressions for the (state contingent) variance and skewness and base identification on a standard Generalized Method of Moments (GMM) approach. To achieve this, we extend the influential method of Storesletten, Telmer, and Yaron (2004) (STY) who estimate an income process with a state-contingent variance of the persistent income shock. They find that the variance is higher in recessions which has been labelled a *counter-cyclical variance* effect.[1] STY base identification of the state contingent

[1] This terminology has been introduced in the macroeconomic asset pricing literature,

variance on the observation that persistent shocks accumulate over an individual's life-cycle such that the distribution of labor incomes observed for a given cohort widens as this cohort ages. This implies that cohorts that experienced different macroeconomic histories will feature different cross-sectional age-specific variances of labor incomes if the variance of income shocks varies over the business cycle.

We extend their framework and analyze how the skewness of the innovation accumulates when a cohort ages, using the same idea for identification (i.e., our identification is based on the notion that the accumulated skewness differs across cohorts if these cohorts experienced different macroeconomic histories). As a measure for skewness we use the third central moment of a distribution. Importantly, we do not base identification on the standard measure of skewness, which is the third centralized moment normalized by the variance of the distribution. Since we avoid this normalization, there is no interference between our estimates of the variance and the skewness of earnings shocks.

We apply our empirical approach to labor incomes from the German Socio-Economic Panel (SOEP) and to the US Panel Study of Income Dynamics (PSID). We base our estimation on gross labor earnings of males aged 25 to 60 [2], as well as on two measures of household level labor incomes. The first is based on gross labor income of household head and spouse, the second on post government net labor income.

Our results for Germany establish three important insights on labor income risk. First, the variance of log labor earnings shocks of males is countercyclical. Hence, the variance of log earnings is higher in recessions than it is in booms. The increase in the variance of log earnings in recessions is due to an increase of the left skewness: negative log labor earnings realizations are more likely in recessions than positive ones. Second, there is insurance against transitory income shocks at the household level, but not against permanent shocks. Relative to male earnings, the variance of transitory income shocks decreases but the moments for permanent shocks are (almost) unchanged. Third, the German tax and transfer system insures against both transitory and permanent earnings shocks. For post government earnings (after taxes and transfers) the distribtuion of transitory shocks is further compressed relative to pre government earnings and the cyclicality of earnings

see Mankiw (1986), Constantinides and Duffie (1996), Storesletten, Telmer, and Yaron (2007).

[2]For Germany, we focus on males that currently live in West Germany and did not immigrate after age 10.

shocks is gone.

For males, the results for the US economy mirror the ones for the German labor market. Future versions of this paper will also contain results for household level income in the US.

On the empirical side, several studies analyze patterns of residual income inequality over time and over the lifecycle. Examples for the US are Moffitt and Gottschalk (2011) and Heathcote, Perri, and Violante (2010), who document the development of residual inequality over the past three decades. Trends in income inequality in Germany are studied, e.g., by Dustmann, Ludsteck, and Schönberg (2009) using administrative data and Fuchs-Schündeln, Krueger, and Sommer (2010) using data from the German Socioeconomic Panel (SOEP). For Germany, Bayer and Juessen (2012) document a slightly procyclical variance of wage risk. In contrast to us, they focus on wages.

Recently, Guvenen, Ozkan, and Song (2013) stress the importance for estimating higher order moments of income processes. Using an extensive administrative dataset from US social security records they challenge the evidence by STY that cyclicality is solely in terms of the variance. Their findings instead suggest that the left-skewness of individual income risk increases in a recession, whereas the variance does not change. This motivates our approach. Methodologically, we differ from Guvenen, Ozkan, and Song (2013) in that we superimpose more parametric structure, as in STY. Hence, our approach is well suited for typically easily available smaller data sets.

In follow-up work to Guvenen, Ozkan, and Song (2013), Guvenen, Karahan, Ozkan, and Song (2016) show that most individuals experience very small earnings changes and a considerable number of workers very large ones. Hence, the kurtosis of labor earnings is much higher than the conventional assumption of log normality implies. Given the relatively small sample size of the SOEP, we do not estimate the kurtosis (and how it varies over the cycle). It is, however, straightforward to extend our empirical approach by additional moments for the kurtosis. To achieve independence of the variance, this should again be based on the fourth non-standardized moment of the distribution. Also, notice that our estimates of the variance and skewness (and how these moments vary over the cycle) are not affected by omitting the kurtosis.[3]

[3]In a similar line of research, Busch, Domeij, Guvenen, and Madera (2016) conduct a non-parametric analysis of earnings risk in Germany, Sweden and the US. For Germany, they find

All the aforementioned papers on earnings risk have in common that using the estimates in macroeconomic models requires a two-step procedure. As a first step, the estimation is carried out. In a second step, the estimates are approximated, cf., e.g., Guvenen, Karahan, Ozkan, and Song (2016) and McKay (2016). De Nardi, Fella, and Paz-Pardo (2016) suggest to avoid this by directly estimating a Markov process on the data.

An important difference between our paper and these recent papers on higher moment income risk[4] is that we adopt the tradition in the labor/consumption literature to distinguish between transitory and permanent shocks to income (Deaton 1992). This distinction is crucial to understand the disjuncture between consumption and earnings distribution and to study how households are insured against permanent and transitory shocks (Blundell, Pistaferri, and Preston 2008; Kaplan and Violante 2010). Our application establishes such insurance within the household and through the government. These findings share similarities with those of Blundell, Graber, and Mogstad (2014) who use Norwegian data, without looking at higher moments though.

The remainder of this paper is structured as follows. Section 2 presents our empirical approach, discusses the moment conditions used to identify the parameters of the earnings process and provides intuition for identification. Section 3 describes the application of our approach to German earnings data from the SOEP and the PSID. We start by describing the data and by defining business cycles and move on to illustrate how variance and skewness at different ages depend on histories, i.e., the number of recessions a cohort has worked through. We then present our main estimation results. Finally, Section 4 concludes.

2 Empirical Approach

2.1 Overview

The individual income process is specified in a way that allows us to separately identify cyclicality in the variance and skewness of innovations to the persistent component. Our identification strategy is an extension of the approach proposed

find qualitatively similar results as we do in this paper, not distinguishing between transitory and persistent shocks.

[4]In addition to our focus on how idiosyncratic risk varies over the business cycle, which only some of the higher moment income risk papers share.

by Storesletten, Telmer, and Yaron (2004). The basic idea is to exploit how the distribution of persistent idiosyncratic shocks accumulates over time: if the income process is persistent, then, as a cohort ages, the cross-sectional income distribution at any age, can be characterized by the sequence of shocks experienced by the cohort's members. If the variance of the innovation depends on the aggregate state of the economy, then the cross-sectional income variance at a certain age differs between two cohorts if these cohorts went through different macroeconomic histories. Storesletten, Telmer, and Yaron (2004) allow for two states of the variance—one in contractions and one in expansions—and classify each year as either an expansion or a contraction.

We extend their framework by analyzing how the skewness of the innovation accumulates when a cohort ages. As a measure for skewness we use the third central moment of a distribution. Given our specification of the income process, we derive closed-form expressions for the variance and skewness of income and develop a Generalized Method of Moments (GMM) estimator to identify all parameters of interest. In addition to variance and skewness, we consider the covariance and a measure of coskewness in our construction of moment conditions. These moments allow us to separately identify variance and skewness of transitory and permanent shocks and of the fixed component, as will be discussed below.

The key advantage of using central moments in the estimation is that we can remain agnostic about the exact distribution of the stochastic components in the estimated earnings process. However, measurement of central moments could be problematic given the available sample size, because the measures are sensitive to outliers. Percentile based measures are more robust. However, were we to use percentile based moments, we would have to estimate the process using a Method of Simulated Moments approach—and therefore take a stand on density functions. In order to evaluate the importance of potential small sample biases, we compare the age profiles of the applied central moments to the profiles of the percentile based counterparts to these moments. We find that qualitatively the age profile is the same, which encourages our choice of central moments.

2.2 The Income Process

We impose the following income process, which is commonly used in the literature. The (log) income of household i of age h in year t is

$$y_{ith} = f\left(\mathbf{X}_{ith}, Y_t\right) + \tilde{y}_{ith}, \tag{1}$$

where $f\left(\mathbf{X}_{ith}, Y_t\right)$ is the *deterministic* part of income, i.e., the part that can be explained by observable individual and aggregate characteristics, $\mathbf{X}_{ith}$ and Y_t, respectively, and $\tilde{y}_{ith}$ is the unexplained part of income that is assumed to be orthogonal to $f\left(\mathbf{X}_{ith}, Y_t\right)$. We consider age, education, and the household size as elements of $\mathbf{X}_{ith}$. More specifically, the deterministic component $f\left(\mathbf{X}_{ith}, Y_t\right)$ is a linear combination of a cubic in age h and the log of household size, $hhsize_{ith}$. The aggregate effects are captured by a time-varying intercept and the education premium is allowed to vary over time following a quadratic function:

$$f\left(\mathbf{X}_{ith}, Y_t\right) = \beta_{0t} + f_h\left(h\right) + I_{e_{it}=c} f_{EP}(EP) + \beta^{size} \log\left(hhsize_{ith}\right) \tag{2}$$

where $f_h\left(h\right) = \beta_1^{age} h + \beta_2^{age} h^2 + \beta_3^{age} h^3$ and $f_{EP}(EP) = \beta_0^{EP} + \beta_1^{EP} t + \beta_2^{EP} t^2$.

Residual income $\tilde{y}_{ith}$ is the main object of interest in the analysis. We model $\tilde{y}_{ith}$ as the sum of three components: a fixed effect χ_i, a persistent component z_{ith}, and an iid transitory shock ε_{ith}. The persistent component is modeled as an AR(1) process with innovation η_{ith}.

$$\tilde{y}_{ith} = \chi_i + z_{ith} + \varepsilon_{ith}, \text{ where } \varepsilon_{ith} \underset{iid}{\sim} F_\varepsilon\left(0, m_2^\varepsilon, m_3^\varepsilon\right), \ \chi_i \underset{iid}{\sim} F_\chi\left(0, m_2^\chi, m_3^\chi\right) \tag{3a}$$

$$z_{ith} = \rho z_{it-1h-1} + \eta_{ith}, \text{ where } \eta_{ith} \underset{iid}{\sim} F_\eta\left(0, m_2^\eta\left(s\left(t\right)\right), m_3^\eta\left(s\left(t\right)\right)\right), \tag{3b}$$

where $F_{\chi/\varepsilon/\eta}\left(\cdot\right)$ denotes the density functions of χ, ε_{ith} and η_{ith}, respectively. The fixed effect and both shocks are mean zero and $m_2^{\chi/\varepsilon/\eta}$ and $m_3^{\chi/\varepsilon/\eta}$ are the second and third moments, respectively. The second and third moments of the persistent shock are allowed to depend on the aggregate state of the economy in period t, denoted by $s\left(t\right)$.

2.3 The GMM Approach

The common approach in estimating (1) is to perform the estimation in two steps, where the first step estimation yields residuals and the second step fits the stochastic process (3) to cross-sectional moments of the distribution of residual (log) income. The imposed process implies the following moments of the distribution of residual income at age h in year t, derived in Appendix A.2:

$$m_2\left(\tilde{y}_{ith};\theta\right) = m_2^{\chi} + m_2^{\varepsilon} + \textstyle\sum_{j=0}^{h-1} \rho^{2j} m_2^{\eta}\left(s\left(t-j\right)\right) \tag{4a}$$

$$m_3\left(\tilde{y}_{ith};\theta\right) = m_3^{\chi} + m_3^{\varepsilon} + \textstyle\sum_{j=0}^{h-1} \rho^{3j} m_3^{\eta}\left(s\left(t-j\right)\right) \tag{4b}$$

$$cov\left(\tilde{y}_{ith}, \tilde{y}_{it+1h+1};\theta\right) = m_2^{\chi} + \rho \textstyle\sum_{j=0}^{h-1} \rho^{2j} m_2^{\eta}\left(s\left(t-j\right)\right) \tag{4c}$$

$$csk\left(\tilde{y}_{ith}, \tilde{y}_{it+1h+1};\theta\right) = m_3^{\chi} + \rho \textstyle\sum_{j=0}^{h-1} \rho^{2j} m_3^{\eta}\left(s\left(t-j\right)\right), \tag{4d}$$

where $\theta = \left(m_2^{\chi}, m_3^{\chi}, m_2^{\varepsilon}, m_3^{\varepsilon}, m_2^{\eta,E}, m_2^{\eta,C}, m_3^{\eta,E}, m_3^{\eta,C}\right)$ is a vector collecting the 8 second-stage parameters. $m_2\left(\tilde{y}_{ith};\theta\right)$ and $m_3\left(\tilde{y}_{ith};\theta\right)$ denote the second and third central moment; $cov\left(\tilde{y}_{ith}, \tilde{y}_{it+1h+1};\theta\right)$ and $csk\left(\tilde{y}_{ith}, \tilde{y}_{it+1h+1};\theta\right)$ denote the covariance and a measure of coskewness between $\tilde{y}_{ith}$ and $\tilde{y}_{it+1h+1}$. Coskewness is measured here as the covariance between $\tilde{y}_{ith}^2$ and $\tilde{y}_{it+1h+1}$, i.e., $csk\left(\tilde{y}_{ith}, \tilde{y}_{it+1h+1};\theta\right) \equiv cov\left(\tilde{y}_{ith}^2, \tilde{y}_{it+1h+1};\theta\right)$. The two covariance terms allow to separately identify the moments of χ and ε.

Before implementing the second stage estimator, we impose more structure on the time dependency of $F_{\eta}\left(\cdot\right)$. Variance and skewness of the persistent innovation η_{it} are modelled as two state processes: $m_2^{\eta}\left(\cdot\right)$ and $m_3^{\eta}\left(\cdot\right)$ take on two possible values each, depending on the aggregate state $s\left(t\right)$, which is either an expansion or a contraction. To this end, define the indicator variable $I_{t=\text{expansion}}$ to be equal to 1 if year t is an expansion (denoted by E) and to be equal to 0 if year t is a contraction (denoted by C). We then have:

$$m_2^{\eta}\left(s\left(t\right)\right) = I_{s(t)=E} m_2^{\eta,E} + \left(1 - I_{s(t)=E}\right) m_2^{\eta,C} \tag{5a}$$

$$m_3^{\eta}\left(s\left(t\right)\right) = I_{s(t)=E} m_3^{\eta,E} + \left(1 - I_{s(t)=E}\right) m_3^{\eta,C} \tag{5b}$$

Small sample size can lead to central moments being measured imprecisely. We therefore calculate moments for $H_g < H$ age groups. Mean independence of shocks implies for the theoretical counterparts that:

$$
\begin{aligned}
m_k\left(\tilde{y}_{ith_g};\theta\right) &= \frac{1}{\sum_{h\in h_g} N_{h,t}} \sum\nolimits_{h\in h_g} N_{h,t} m_k\left(\tilde{y}_{ith};\theta\right) \text{ for } k=2,3 \\
cov\left(\tilde{y}_{ith},\tilde{y}_{it+1h+1};\theta\right)_{h\in h_g} &= \frac{1}{\sum_{h\in h_g} N_{h,t}} \sum\nolimits_{h\in h_g} N_{h,t} cov\left(\tilde{y}_{ith},\tilde{y}_{it+1h+1};\theta\right) \\
csk\left(\tilde{y}_{ith},\tilde{y}_{it+1h+1};\theta\right)_{h\in h_g} &= \frac{1}{\sum_{h\in h_g} N_{h,t}} \sum\nolimits_{h\in h_g} N_{h,t} csk\left(\tilde{y}_{ith},\tilde{y}_{it+1h+1};\theta\right).
\end{aligned}
$$

Given H_g age groups and T years of observations we obtain $H_g \times T$ cross-sectional measures of variance and skewness each, and $H_g \times (T-1)$ estimates of covariance and coskewness, i.e., $2 \times H_g \times T + 2 \times H_g \times (T-1)$ empirical moments. The moment conditions employed in the GMM estimation read as follows:

$$E\left[m_2\left(\tilde{y}_{ith_g}\right) - m_2\left(\tilde{y}_{ith_g};\theta\right)\right] = 0 \tag{6a}$$

$$E\left[m_3\left(\tilde{y}_{ith_g}\right) - m_3\left(\tilde{y}_{ith_g};\theta\right)\right] = 0 \tag{6b}$$

$$E\left[cov\left(\tilde{y}_{ith},\tilde{y}_{it+1h+1}\right)_{h\in h_g} - cov\left(\tilde{y}_{ith},\tilde{y}_{it+1h+1};\theta\right)_{h\in h_g}\right] = 0 \tag{6c}$$

$$E\left[csk\left(\tilde{y}_{ith},\tilde{y}_{it+1h+1}\right)_{h\in h_g} - csk\left(\tilde{y}_{ith},\tilde{y}_{it+1h+1};\theta\right)_{h\in h_g}\right] = 0, \tag{6d}$$

where the first term in each line is the empirically calculated moment, e.g., the variance of residual earnings in year 2000 of workers in age group 2. The second term in each line is the theoretical counterpart implied by a specific combination of parameters in θ. We define 7 age groups: 25-29, 30-34, 35-39, 40-44, 45-49, 50-54, 55-60. In the estimation we impose that each worker i enters the labor market at age 25 in some year t, draws a fixed effect χ_i from the distribution $F_\chi\left(0, m_2^\chi, m_3^\chi\right)$, which does not vary over time, and draws the first realizations of transitory and permanent shocks, ε_{it} and η_{it}, from the distributions $F_\varepsilon\left(0, m_2^\varepsilon, m_3^\varepsilon\right)$ and $F_\eta\left(0, m_2^\eta\left(s\left(t\right)\right), m_3^\eta\left(s\left(t\right)\right)\right)$. Given a classification of years as expansions or contractions, we can then use (6) together with (4) to estimate the parameters of the income process.

2.4 Identification

The use of cross-sectional moments for identification allows to exploit macroeconomic information that predates the micro panel, thereby incorporating more business cycles in the analysis than covered by the sample, as pointed out and elaborated on by STY.

In order to understand how identification works, consider the persistent component of the income process, cf. equation (3b): the variances of the innovations accumulate as a cohort ages, as can be seen from the theoretical moment in equation (4a). If the innovation variance is higher in contractionary years, then a cohort that lived through more contractions as it reaches a given age will have a higher income variance at that age than a cohort that lived through fewer contractions has at the same age.

Our extension of STY is based the insight that a similar accumulation holds for the skewness, as seen in equation (4b). If the probability of a large negative/positive income shock would be higher/lower for the average worker during a macroeconomic contraction, then the skewness of the shock in a contractionary period would be smaller (more negative) than in an expansion, i.e., $m_3^{\eta,C} < m_3^{\eta,E}$. Comparing again two cohorts when they reach a certain age, this would imply a more negative cross-sectional skewness for the cohort that worked through more recessions.

As seen in (4a), the sum $(m_2^{\chi} + m_2^{\varepsilon})$ is identified as the intercept of the variance profile over age. The same holds for $(m_3^{\chi} + m_3^{\varepsilon})$ in (4b), which is identified via the age profile of skewness. Considering the sum in (4a), we see that the magnitude of the increase of the cross-sectional variance over age identifies the variance of persistent shocks. The difference between $m_2^{\eta,C}$ and $m_2^{\eta,E}$ is identified by the difference of the cross-sectional variance of different cohorts of the same age. Likewise, the difference between $m_3^{\eta,C}$ and $m_3^{\eta,E}$ is identified by the difference of the cross-sectional skewness of different cohorts.

The last piece of information for identification comes from considering the expressions for variance and covariance in equations (4c) and (4a). It becomes immediately apparent that the difference between the two expressions identifies m_2^{χ} separately from m_2^{ε}. Likewise, the difference between the expressions for skewness and coskewness, equations (4b) and (4d), identifies m_3^{χ} separately from m_3^{ε}.

3 Application: Earnings Risk of German and US Households

The current version of the paper only contains results for Germany. US results are preliminary and we present only those for male earnigns here: future versions will also cover US household level results.

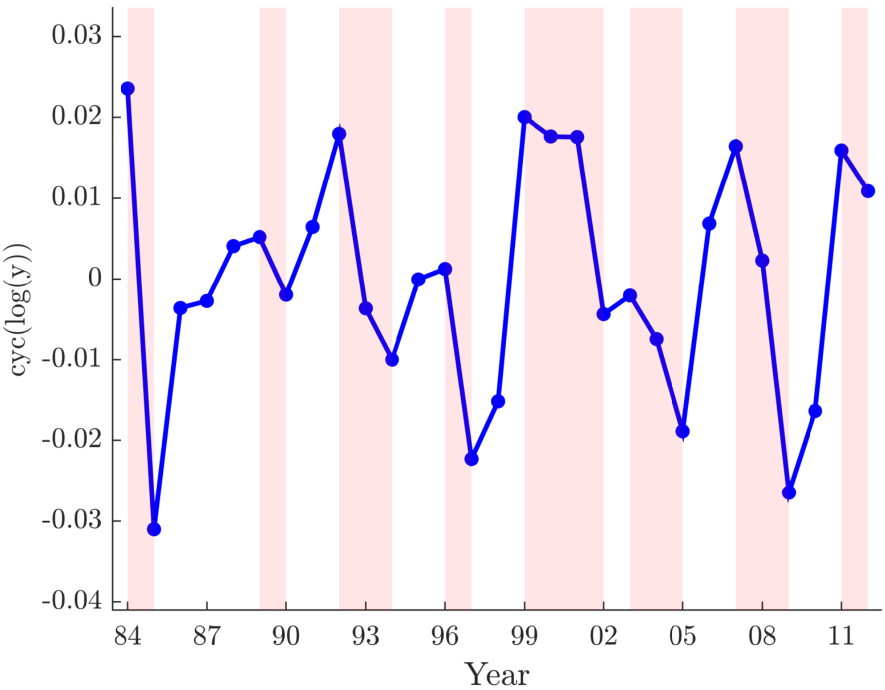
3.1 Data and Sample Selection

The Socio-Economic Panel (SOEP) is a survey based panel study covering the years 1984 to 2013. It was initiated with about 10,000 individuals in 5,000 households in 1984 and covers about 18,000 (10,500) individuals (households) in 2013. We define household level income variables as follows. Household labor income before taxes is calculated as the sum of head and spouse annual labor income. Labor income is the sum of income from first and second jobs, 13th and 14th monthly salaries, Christmas and vacation bonuses, profit sharing and other bonuses. 50% of income from self-employment is assigned to labor income.

Post-government income is defined as household labor income plus transfers minus taxes. Transfers are aggregated over all household members and include pensions (old age; disability; widows; orphans; or other), maternity benefit, student grants, unemployment benefits, unemployment assistance (before 2005), subsistence allowance, child allowance, unemployment benefit II (since 2005). Tax estimates are provided in the SOEP at the household level. All nominal values are deflated with the CPI.

We exclude SOEP subsamples D and G, which oversample immigrants and high income households, respectively, and apply the following sample selection criteria. We select males between 25 and 60 years of age, that currently live in West Germany, and did not immigrate after age 10. Labor earnings needs to be above a constant threshold, which is defined as the income from working 520 hours for three year 2000 Euros. A household is in the household sample if it is comprised of at least 2 adults. The age restriction applies to the household head and the income threshold needs to be exceeded by the minimum of male labor earnings and household post-government income. For the US analysis we use data from the Panel Study of Income Dynamics (PSID), which interviews households annualy from 1968 to 1997 and every other year since then. The representative

Figure 1: Business Cycle Dating Based on Mean Earnings



Note: Figure shows the cyclical component of HP-filtered (mean) male earnings in the SOEP; smoothing parameter for the HP-filter is 6.25.

core sample consists of about 2,000 households in each wave. We follow the same selection criteria as for the SOEP. As measure of male labor income we use "wages and salary", which is consistently measured over the sample period.

3.2 Defining Business Cycles

In order to implement the estimator we need to classify years as contractions or expansions. For Germany, we initiate the classification with peak and trough dates from ECRI, which is based on NBER methodology. Given the sluggish synchronization of labor market outcomes with the macroeconomic indicators that ECRI takes into account, we expand the dating based on mean earnings of males in the SOEP, as shown in Figure 1. For the classification of years in the pre-sample period, we use the unemployment rate, which during the sample time is highly correlated with male earnings. For the US analysis, we base our definition on NBER peaks and trough data.

Given the dating of peaks and troughs, we characterize years as expansions and contractions as follows. A year is classified as a contraction if: (i) it completely is in a contractionary period which is defined as the time from peak to trough, (ii) if the peak is in the first half of the year and the contraction continues into the next year, (iii) if a contraction started earlier and the trough is in the second half of the year. All years that are not classified as contraction are classified as expansions.

3.3 Intuition behind the Estimator

This section uses information from the data to discuss the intuition behind the estimator, thereby relating back to our discussion on identification in Subsection 2.4. Figure 2 shows the variance, m_2, of the cross-sectional distribution of male labor earnings for different age groups as a function of the share of contractionary years a cohort lived through. For each age group, the higher the share of contractions a cohort went through, the more dispersed is the cross-sectional earnings distribution. Recall that this is an implication of the earnings process if $m_2^{\eta,C} > m_2^{\eta,E}$, i.e., if the variance of permanent shocks is countercyclical, cf. equation (4a). The increasing variance in the share of recessions therefore identifies $m_2^{\eta,C}$ and $m_2^{\eta,E}$.

Similarly, Figure 3 shows the third central moment of the cross-sectional earnings distribution as a function of the share of contractions. For each age group, we observe a clear downward-sloping pattern, which is implied by the earnings process if $m_3^{\eta,C} < m_3^{\eta,E}$, i.e., if the skewness is procyclical, cf. equation (4b). The decreasing skewness in the share of recessions therefore identifies $m_3^{\eta,C}$ and $m_3^{\eta,E}$.

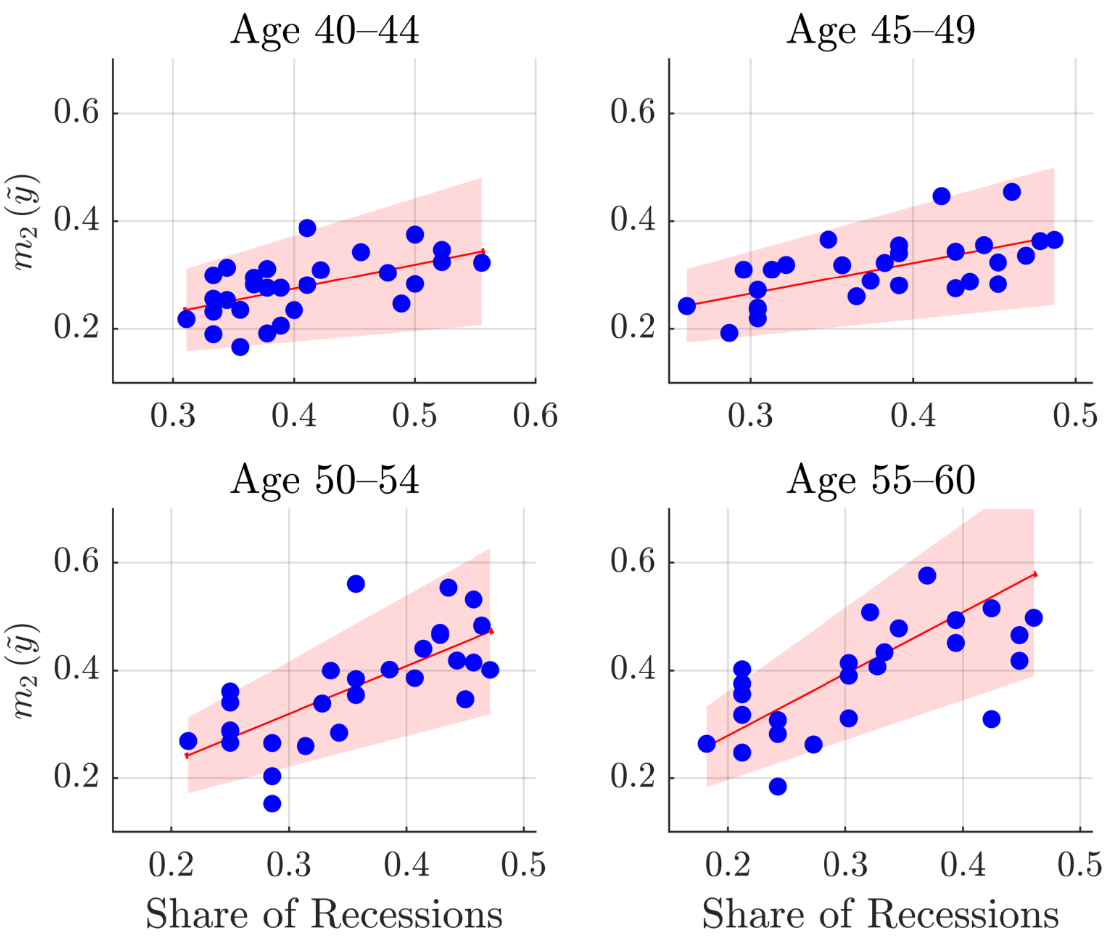
Fitting a linear regression in each figure suggests a statistically significant difference between the distribution of permanent shocks in contractions and expansions.[5]

This is indeed what we estimate below in Subsection 3.4.

Finally, recall from our discussion in Subsection 2.4 how the moments of the transitory income shock, m_2^{ε} and m_3^{ε} can be identified given the equations in (4). To illustrate identification of these two terms, we compute the difference between variance and covariance in each cross-section and take the average over all years. This suggests that $m_2^{\varepsilon} \approx 0.0815$, cf. equations (4a) and (4c). Similarly, calculating the difference between skewness and coskewness in each cross-section and taking the average over all years suggests that $m_3^{\varepsilon} \approx -0.1083$. These values are indeed

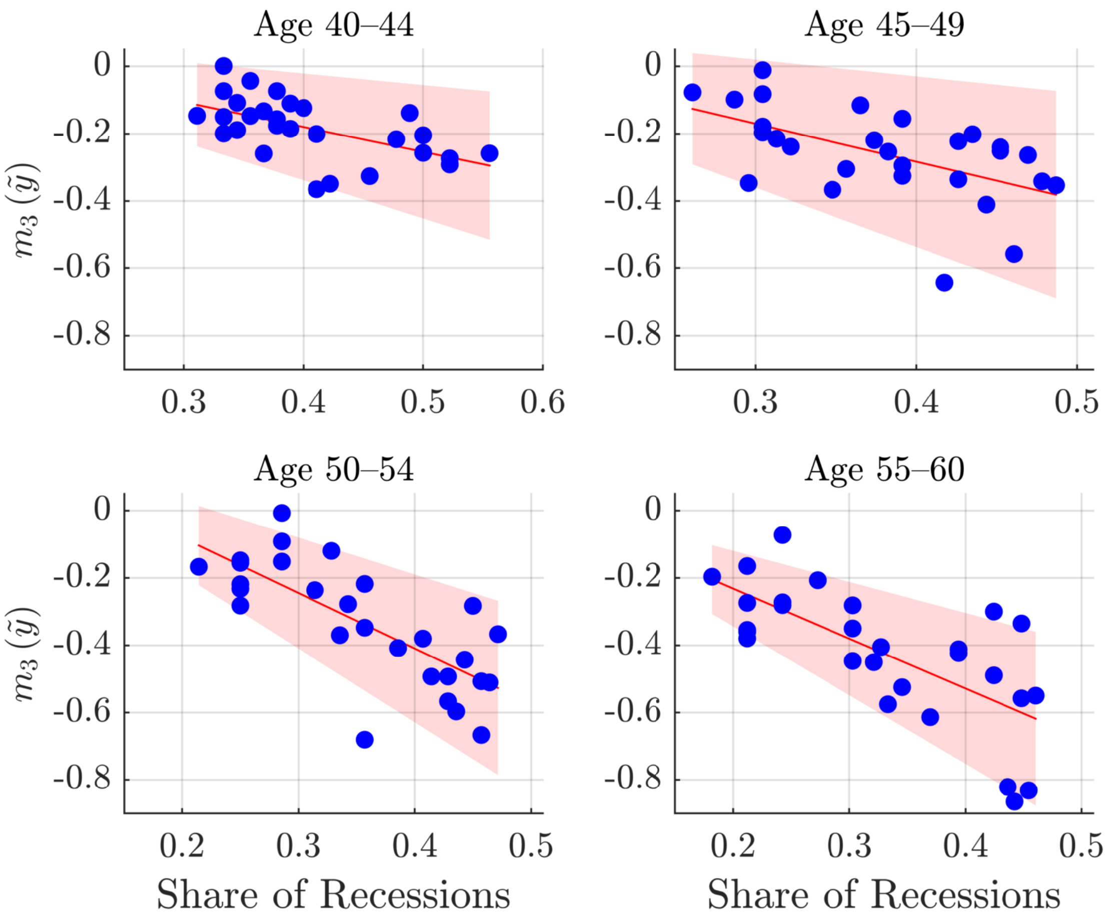
[5] Appendix A.1 contains corresponding results for the US and shows the coefficient estimates of the linear regressions.

Figure 2: Intuition: Cross-Sectional Second Moment in SOEP



Note: The x-axis of each figure shows the share of contractions in all years a cohort went through at a certain age. The y-axis shows the second central moments for different cohorts at different ages.

Figure 3: Intuition: Cross-Sectional Third Moment in SOEP



Note: The x-axis of each figure shows the share of contractions in all years a cohort went through at a certain age. The y-axis shows the third central moments for different cohorts at different ages.

very close to what the estimation yields in Subsection 3.4 to which we turn next.

3.4 Estimation Results: Cyclical Income Risk

Estimating income processes, we started with a specification where we estimated the persistence of the persistent income shock process, ρ. It turned out that ρ was not significantly different from 1. We therefore restrict $\rho = 1$, hence η is a permanent shock and z is the permanent income component.

Our main estimates under this parametric restriction are summarized in Table 1, showing the point estimates of all parameters along with the 5^{th} and 95^{th} percentile of 250 bootstrap draws for three different specifications: male earnings, household pre government (before taxes and transfers) and household post government (after taxes and transfers) earnings. In all models, each moment in (6a) to (6d) is weighted equally, reflecting insights of (Altonji and Segal 1996), who show that the identity weighting matrix dominates the asymptotically optimal weighting matrix in small samples. We apply a block bootstrap procedure in which we resample individuals—thereby preserving the autocorrelation structure of the original sample.

As a first observation, notice that the moments of the fixed effect, m_2^{χ} and m_3^{χ}, are very imprecisely estimated in all models. We therefore put no emphasis on the interpretation of these estimates. One reason for this imprecision might be that the fixed effect estimates take up cohort effects that are otherwise missing from our specification of the income process.

More important is the interpretation of the variance and skewness terms for the transitory and permanent earnings shocks which yield a number of interesting insights when moving across the different models. We start with the results for male earnings which are shown in the first panel of Table 1. Observe that the central moments estimated for the transitory shock, m_2^{ε} and m_3^{ε}, are at 0.0718 and -0.1, respectively, thereby coming very close to what we estimate in our illustration in the preceding Subsection 3.3. Accordingly, transitory income shocks are left-skewed: negative shock realizations have more weight than positive ones.

Next, notice that the variance of the permanent income shock features a strong countercyclicality—$m_2^{\eta,C} = 0.018$ and $m_2^{\eta,E} = 0.0005$ with the difference being highly significant. Our estimates of the skewness show that this countercyclicality of the variance is due to a procyclical skewness—$m_3^{\eta,C} = -0.0243$ and $m_3^{\eta,E} =$

Table 1: Estimation Results: Germany (SOEP)

	Males	HH Pre	HH Post
ρ	1	1	1
	imposed	*imposed*	*imposed*
m_2^{χ}	.1180	.0927	.0661
	$(.0661; .1631)$	$(.0630; .1213)$	$(.0437; .0865)$
m_2^{ε}	.0718	.0534	.0427
	$(.0651; .0793)$	$(.0479; .0585)$	$(.0380; .0461)$
m_3^{χ}	$-.0219$	.0199	$-.0149$
	$(-0.0838; 0.0350)$	$(-.0193; .0672)$	$(-.0540; -.0149)$
m_3^{ε}	$-.1000$	$-.0613$	$-.0471$
	$(-.1081; -.0853)$	$(-.0700; -.0517)$	$(-.0530; -.0395)$
$m_2^{\eta,E}$	.0005	.0022	.0050
	$(.0005; .0010)$	$(.0005; .0044)$	$(.0036; .0064)$
$m_2^{\eta,C}$	.0181	.0157	.0024
	$(.0108; .0245)$	$(.0082; .0176)$	$(.0005; .0051)$
$m_3^{\eta,E}$	.0037	$-.0014$	$-.0025$
	$(.0002; .0073)$	$(-.0043; .0007)$	$(-.0045; -.0004)$
$m_3^{\eta,C}$	$-.0243$	$-.0189$	$-.0055$
	$(-.0369; -.0170)$	$(-.0231; -.0107)$	$(-.0070; -.0023)$

Note: Table shows estimated moments for the three specifications described in the text. Parantheses show the 5^{th} and 95^{th} percentile of 250 bootstrap draws.

0.0037, with both estimates significantly different from zero and from each other. Accordingly, in contractions negative log earnings realizations are more likely than positive ones. In expansions, while our estimates suggest a positive skewness, the point estimate is very small. Hence, the distribution of permanent shocks is estimated to be almost symmetric in expansions.

Moving from male earnings to household pre government earnings—shown in the second panel of Table 1—we notice that there is insurance against shocks at the household level. Both the variance and the skewness of transitory earnings shocks decrease (in absolute terms) relative to male earnings. However, the estimates also show that there is no insurance against permanent shocks at the household level. The estimates of the variance and skewness in both contractions and expansions are not statistically different from what we find for male earnings. Hence, also for household pre government earnings, negative log earnings realizations are more likely than positive ones in contractions and the distribution of permanent shocks is estimated to be almost symmetrical in expansions.[6]

Finally, when considering household post government earnings—shown in the third panel of Table 1—, both variance and skewness of transitory income shocks decrease further.[7] Also, the cyclicality of permanent shocks is gone. The variance in expansions is not statistically different from what we estimate for pre government earnings, but the variance in contractions decreases strongly and is no longer statistically different from the variance in expansions. Likewise, the skewness in contractions of permanent shocks decreases strongly when moving from pre to post government earnings and is statistically indifferent from the skewness in expansions. Now, the point estimates of the skewness in both states is small so that the distribution of permanent shocks is almost symmetric in contractions and expansions.

Table 2 shows the corresponding results for the PSID. The estimates for male earnings display the same characteristics as for Germany. [TBC]

[6] The estimate of the skewness in expansions is now statistically indifferent from zero, but the point estimate for males was also very small.

[7] The confidence intervals for the skewness of transitory shocks overlap slightly between pre and post government household earnings.

Table 2: Estimation Results: USA (PSID)

	Males	HH Pre	HH Post
ρ	.9596	.9628	.9666
	(.9475; .9730)	(.9467; .9821)	(.9527; .9852)
m_2^{χ}	.2136	.2057	.1934
	(.1963; .2329)	(.1811; .2424)	(.1688; .2222)
m_2^{ε}	.1113	.1077	.0996
	(.1063; .1165)	(.1004; .1155)	(.0923; .1065)
m_3^{χ}	$-.1463$	$-.1283$	$-.1135$
	$(-.1858; -.1031)$	$(-.1756; -.0907)$	$(-.1526; -.0748)$
m_3^{ε}	$-.1522$	$-.1675$	$-.1377$
	$(-.1599; -.1403)$	$(-.1768; -.1530)$	$(-.1468; -.1208)$
$m_2^{\eta,E}$	.0019	.0118	.0095
	(.0005; .0029)	(.0048; .0171)	(.0036; .0134)
$m_2^{\eta,C}$	.0342	.0322	.0243
	(.0249; .0417)	(.0158; .0419)	(.0113; .0343)
$m_3^{\eta,E}$	.0205	$-.0044$	$-.0022$
	(.0109; .0297)	$(-.0091; .0024)$	$(-.0066; .0048)$
$m_3^{\eta,C}$	$-.0321$	$-.0347$	$-.0193$
	$(-.0459; -.0159)$	$(-.0543; -.0036)$	$(-.0349; -.0015)$

Note: Table shows estimated moments for the three specifications described in the text. Parantheses show the 5^{th} and 95^{th} percentile of 250 bootstrap draws.

4 Conclusion

This paper develops a new parametric estimator of higher moment income risk. We show how to use information on pre-data economic booms and recessions to identify how the variance and the skewness vary over the business cycle. We implement this by a Generalized Method of Moments estimator.

We apply our method to earnings data for Germany and the US.

We show that permanent income shocks to male earnings exhibit strong countercyclicality, whereby the higher variance of male earnings in recessions is due to the fact that negative log income realizations are more likely in recessions than positive ones. We also establish that there is insurance against transitory earnings shocks at the household level and against transitory and permanent income shocks through the German tax and transfer system. In addition, according to our estimates, the cyclicality of earnings risk is gone for household post government earnings.

In this paper, we focus on the second and third moment of transitory and permanent shocks to the earnings distribution. Recent work by Guvenen, Karahan, Ozkan, and Song (2016) emphasizes the importance of the fourth moment, the kurtosis. It is straightforward to extend our method to including higher moments. For reasons of data limitations (we apply our method to a relatively small panel data set, the German Socioeconomic Panel, SOEP; and the Panel Study of Income Dynamics, PSID), we have not approached this. However, because we employ non-standardized moments, our estimates of the skewness are independent of our estimates of the variance. Likewise, omitting the kurtosis does not affect our estimates for variance and skewness of earnings shocks.

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A Appendix

A.1 Intuition

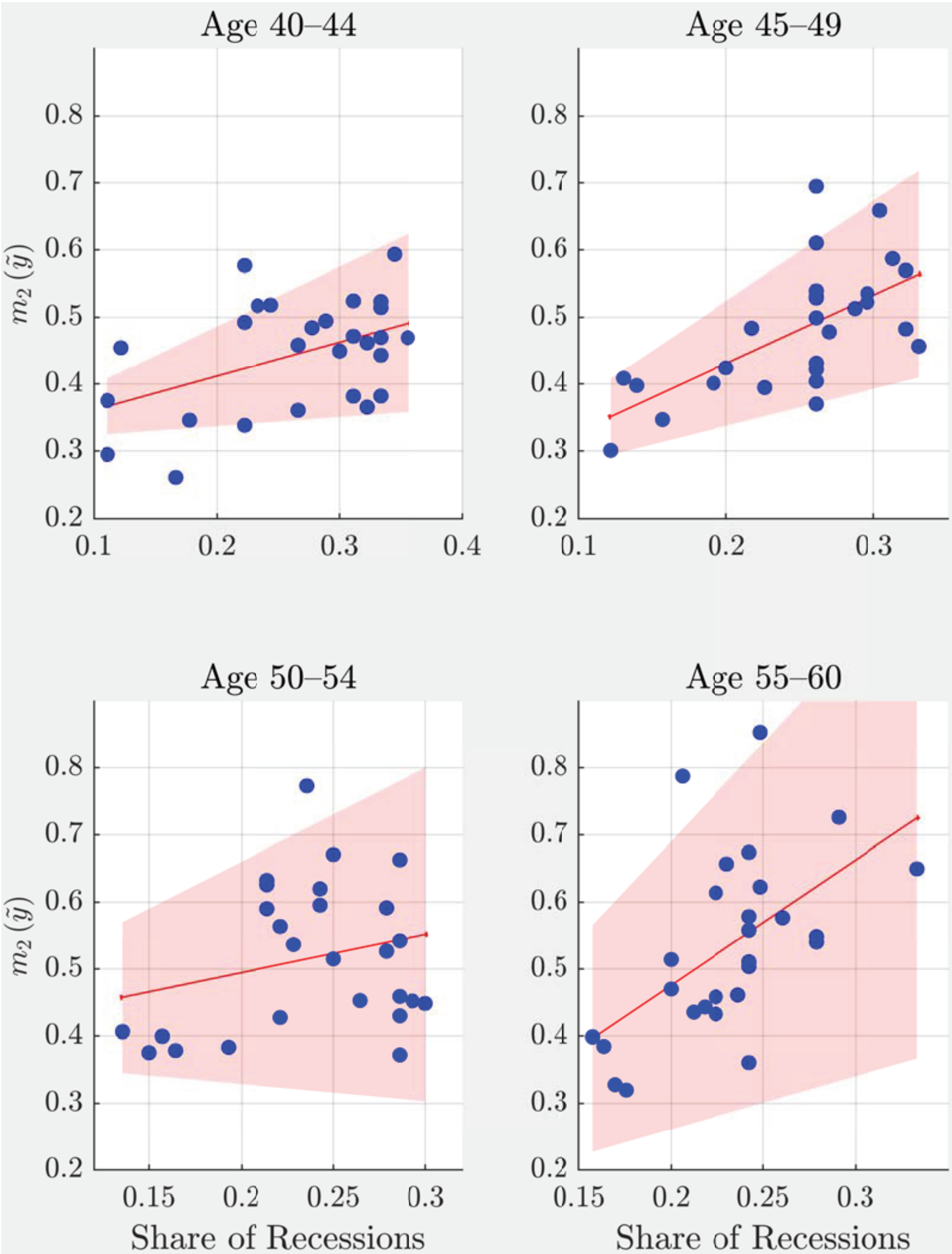
Figures A.1 and A.2 are the analogues of Figures 2 and 3 for the US. Table A.1 shows the coefficients of the fitted lines in these figures

Table A.1: Central Moments as Function of Share of Contractions

	Germany		US	
	$m_2(\tilde{y})$	$m_3(\tilde{y})$	$m_2(\tilde{y})$	$m_3(\tilde{y})$
Age 40–44	0.45	-3.65	0.51	0.12
	(3.54)	(-3.65)	(2.67)	(0.36)
Age 45–49	0.57	-1.13	1.02	-1.13
	(4.27)	(-3.50)	(4.27)	(-1.86)
Age 50–54	0.90	-1.65	0.57	0.85
	(5.37)	(-5.87)	(1.36)	(1.15)
Age 55–60	1.15	-1.48	1.87	-1.98
	(5.48)	(-5.19)	(3.41)	(-1.99)

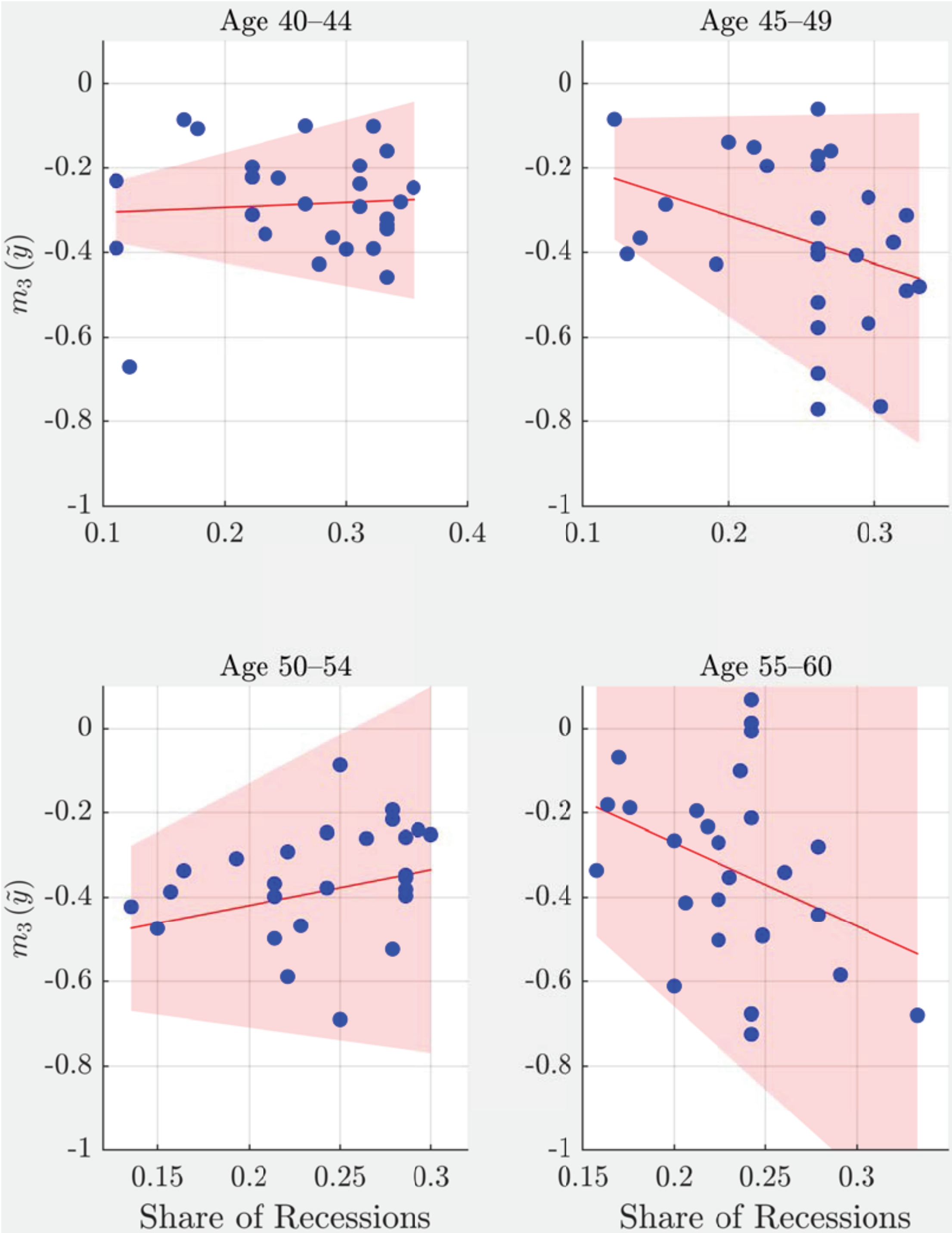
Note: Each column shows the slope coefficient of the respective fitted line in figures 2 and 3. T-statistics are in parantheses.

Figure A.1: Intuition: Cross-Sectional Second Moment in PSID



Note: The x-axis of each figure shows the share of contractions in all years a cohort went through at a certain age. The y-axis shows the second central moments for different cohorts at different ages.

Figure A.2: Intuition: Cross-Sectional Third Moment in PSID



Note: The x-axis of each figure shows the share of contractions in all years a cohort went through at a certain age. The y-axis shows the third central moments for different cohorts at different ages.

A.2 Derivation of Equation (4)

Rewrite the stochastic process (3) by recursive substitution as

$$\tilde{y}_{ith} = \chi_i + \epsilon_{ith} + \sum_{j=0}^{h-1} \rho^j \eta_{it-jh-j} \tag{7}$$

A.2.1 Variance

The variance is defined as

$$\begin{aligned} var(\tilde{y}_{ith}) &= E\left[(\tilde{y}_{ith} - E\left[\tilde{y}_{ith}\right])^2\right] \\ &= E\left[\tilde{y}_{ith}^2\right] \end{aligned}$$

because $E\left[\tilde{y}_{ith}\right] = 0$. Using (7) in the above we get, by our assumption of independence of the respective random variables,

$$var(\tilde{y}_{ith}) = m_2^{\chi} + m_2^{\epsilon} + \sum_{j=0}^{h-1} \rho^{2j} m_2^{\eta}(s(t-j)).$$

A.2.2 Skewness

The non-standardized skewness is defined as

$$\begin{aligned} skew(\tilde{y}_{ith}) &= E\left[(\tilde{y}_{ith} - E\left[\tilde{y}_{ith}\right])^3\right] \\ &= E\left[\tilde{y}_{ith}^3\right]. \end{aligned}$$

Using (7) in the above we get

$$skew(\tilde{y}_{ith}) = m_3^{\chi} + m_3^{\epsilon} + \rho \sum_{j=0}^{h-1} \rho^{3j} m_3^{\eta}(s(t-j)).$$

A.2.3 Covariance

The covariance is given as

$$\begin{aligned} cov(\tilde{y}_{ith}, \tilde{y}_{it+1h+1}) &= E\left[\left(\tilde{y}_{ith} - E\left[\tilde{y}_{ith}\right]\right)\left(\tilde{y}_{it+1h+1} - E\left[\tilde{y}_{it+1h+1}\right]\right)\right] \\ &= E\left[\tilde{y}_{ith}\tilde{y}_{it+1h+1}\right]. \end{aligned}$$

Using (7) in the above we get

$$
\begin{aligned}
cov(\tilde{y}_{ith}, \tilde{y}_{it+1h+1}) &= E\left[\left(\chi_i + \epsilon_{ith} + \sum_{j=0}^{h-1} \rho^j \eta_{it-jh-j}\right)\left(\chi_i + \epsilon_{it+1h+1} + \sum_{j=0}^{h} \rho^j \eta_{it+1-jh+1-j}\right)\right] \\
&= m_2^\chi + E\left[\left(\eta_{ith} + \rho\eta_{it-1h-1} + \rho^2\eta_{it-2h-2} + \ldots + \rho^{h-1}\eta_{it-(h-1)1}\right) \cdot \right. \\
&\quad \left. \left(\eta_{it+1h+1} + \rho\eta_{ith} + \rho^2\eta_{it-1h-1} + \rho^3\eta_{it-2h-2} + \ldots + \rho^h\eta_{it-(h-1)1}\right)\right] \\
&= m_2^\chi + \rho\left(m_2^\eta(s(t)) + \rho^2 m_2^\eta(s(t-1)) + \ldots + \rho^{2(h-1)} m_2(s(t-(h-1)))\right) \\
&= m_2^\chi + \rho \sum_{j=0}^{h-1} \rho^{2j} m_2^\eta(s(t-j)).
\end{aligned}
$$

For the application of our method to PSID data, we further need to look at the covariance between y_{ith} and $y_{it+2h+2}$, hence

$$
\begin{aligned}
cov(\tilde{y}_{ith}, \tilde{y}_{it+2h+2}) &= E\left[\tilde{y}_{ith}\tilde{y}_{it+2h+2}\right] \\
&= E\left[\left(\chi_i + \epsilon_{ith} + \sum_{j=0}^{h-1} \rho^j \eta_{it-jh-j}\right)\left(\chi_i + \epsilon_{it+2h+2} + \sum_{j=0}^{h+1} \rho^j \eta_{it+2-jh+2-j}\right)\right] \\
&= m_2^\chi + E\left[\left(\eta_{ith} + \rho\eta_{it-1h-1} + \rho^2\eta_{it-2h-2} + \ldots + \rho^{h-1}\eta_{it-(h-1)1}\right) \cdot \right. \\
&\quad \left. \left(\eta_{it+2h+2} + \rho\eta_{it+1h+1} + \rho^2\eta_{ith} + \rho^3\eta_{it-1h-1} + \ldots + \rho^{h+1}\eta_{it-(h-1)1}\right)\right] \\
&= m_2^\chi + \rho^2\left(m_2^\eta(s(t)) + \rho^2 m_2^\eta(s(t-1)) + \ldots + \rho^{2(h-1)} m_2(s(t-(h-1)))\right) \\
&= m_2^\chi + \rho^2 \sum_{j=0}^{h-1} \rho^{2j} m_2^\eta(s(t-j)).
\end{aligned}
$$

A.2.4 Coskewness

The coskewness is given as

$$
\begin{aligned}
csk(\tilde{y}_{ith}, \tilde{y}_{it+1h+1}) &= cov(\tilde{y}_{ith}^2, \tilde{y}_{it+1h+1}) \\
&= E\left[\tilde{y}_{ith}^2 \tilde{y}_{it+1h+1}\right].
\end{aligned}
$$

Using (7) in the above we get

$$csk(\tilde{y}_{ith}, \tilde{y}_{it+1h+1}) = E\left[\left(\chi_i + \epsilon_{ith} + \sum_{j=0}^{h-1} \rho^j \eta_{it-jh-j}\right)^2 \left(\chi_i + \epsilon_{it+1h+1} + \sum_{j=0}^{h} \rho^j \eta_{it+1-jh+1-j}\right)\right]$$
$$= E\left[\left(\chi_i + \epsilon_{ith} + \left(\eta_{ith} + \rho\eta_{it-1h-1} + \rho^2\eta_{it-2h-2} + \ldots + \rho^{h-1}\eta_{it-(h-1)1}\right)\right)^2 \cdot\right.$$
$$\left.\left(\chi_i + \epsilon_{it+1h+1} + \left(\eta_{it+1h+1} + \rho\eta_{ith} + \rho^2\eta_{it-1h-1} + \rho^3\eta_{it-2h-2} + \ldots + \rho^h\eta_{it-(h-1)1}\right)\right)\right].$$

Multiplying out terms and factoring in the expectations operator we notice that all terms that involve cross products with random variables to the power of one cancel out because of our independence assumption and because the mean of the respective random variable is zero. We can therefore rewrite the RHS of the above as

$$m_3^\chi + \rho E\left[\eta_{ith}^3 + \rho^3\eta_{it-1h-1}^3 + \rho^6\eta_{it-2h-2}^3 + \ldots + \rho^{3(h-1)}\eta_{it-(h-1)1}^3\right]$$
$$= m_3^\chi + \rho\sum_{j=0}^{h-1} \rho^{3j} m_3^\eta(s(t-j)).$$

For the PSID, we also need

$$csk(\tilde{y}_{ith}, \tilde{y}_{it+2h+2}) = E\left[\tilde{y}_{ith}^2 \tilde{y}_{it+2h+2}\right].$$

Using (7) in the above we get

$$csk(\tilde{y}_{ith}, \tilde{y}_{it+2h+2}) = E\left[\left(\chi_i + \epsilon_{ith} + \sum_{j=0}^{h-1} \rho^j \eta_{it-jh-j}\right)^2 \left(\chi_i + \epsilon_{it+2h+2} + \sum_{j=0}^{h+1} \rho^j \eta_{it+2-jh+2-j}\right)\right]$$
$$= E\left[\left(\chi_i + \epsilon_{ith} + \left(\eta_{ith} + \rho\eta_{it-1h-1} + \rho^2\eta_{it-2h-2} + \ldots + \rho^{h-1}\eta_{it-(h-1)1}\right)\right)^2 \cdot\right.$$
$$\left.\left(\chi_i + \epsilon_{it+2h+2} + \left(\eta_{it+2h+2} + \rho\eta_{it+1h+1} + \rho^2\eta_{ith} + \rho^3\eta_{it-1h-1} + \ldots + \rho^{h+1}\eta_{it-(h-1)1}\right)\right)\right].$$

Again multiplying out and factoring in the expectations operator we get that the RHS of the above rewrites as

$$m_3^\chi + \rho^2 E\left[\eta_{ith}^3 + \rho^3\eta_{it-1h-1}^3 + \rho^6\eta_{it-2h-2}^3 + \ldots + \rho^{3(h-1)}\eta_{it-(h-1)1}^3\right]$$
$$= m_3^\chi + \rho^2\sum_{j=0}^{h-1} \rho^{3j} m_3^\eta(s(t-j)).$$