Product Scope and Economic Fluctuations*

Oscar Pavlov†
Queensland University of Technology

Mark Weder‡
The University of Adelaide

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Abstract

Recent empirical evidence suggests that product creation is procyclical and it occurs largely within existing firms. Motivated by these findings, the current paper investigates the role of intra-firm product scope adjustments in a general equilibrium economy with oligopolistic producers. It shows that the multi-product nature of firms makes the economy significantly more susceptible to sunspot equilibria. The estimated indeterminate model generates artificial business cycles that closely resemble empirically observed fluctuations.

Keywords: Multi-product firms, product scope, business cycles, indeterminacy, sunspot equilibria, markups.

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†Corresponding author. School of Economics and Finance, Queensland University of Technology, GPO Box 2434, Brisbane QLD 4001, Australia. E-mail address: oscar.pavlov@qut.edu.au. Ph:+61 7 3138 2740. Fax:+61 7 3138 1500.

‡School of Economics, The University of Adelaide, Adelaide SA 5005, Australia. E-mail address: mark.weder@adelaide.edu.au.

1 Introduction

This paper develops a model of the business cycle in which product creation and firms dynamics generate *soi-disant* sunspot equilibria which ultimately drive movements in real output. Specifically, it builds on recent empirical work that suggests that a large portion of firms are multi-product producers. Bernard, Redding and Schott (2010), for example, report that close to half of US manufacturing firms produce in multiple 5-digit SIC industries. The importance of this finding becomes apparent once noticing that these firms account for about 90 percent of total sales. Broda and Weinstein (2010) arrive at similar conclusions. In particular, they document that over 90 percent of product creation and destruction occurs within firms (i.e. as firms adjust their product scopes). This alludes that the contribution to aggregate output from product scope variations is at least as important as that from net business formation.

The current paper picks up on these observations by laying out an artificial economy that generates procyclical product creation within firms, net business formation and countercyclical markups, while also giving rise to endogenous business cycles. Specifically, we investigate the role of product scope adjustments and entry and exit in a general equilibrium economy with oligopolistic intermediate goods firms. Endogenous net product creation (in particular via changes in firms' product scopes) creates sunspot equilibria at very realistic situations. To demonstrate this, we estimate the indeterminate model and show that both belief (sunspot) and fundamental shocks generate artificial business cycles that resemble empirically observed fluctuations.

Our artificial economy borrows from Feenstra and Ma (2009) and Minniti and Turino (2013) who introduce multi-product firms into a general equilibrium model with imperfect competition and endogenous entry and exit of firms. They consider a saddle path stable model, and as such, only study fundamental shocks. In contrast, the current paper looks at indeterminate equilibria where belief shocks can drive the business cycle. Moreover, we separate the elasticity of substitution parameters from the variety effects (aka taste for variety, increasing returns to specialization) in the production of final goods, which makes the theoretical mechanisms in our paper far more transparent.

Indeterminacy arises because net business formation and product scope adjustments act as labor demand shifters in the presence of product variety effects. These effects represent the idea that a larger number of differentiated products leads to efficiency gains and thus net product creation gives rise to an endogenous efficiency wedge. Furthermore, the oligopolistic market structure leads to countercyclical markups that act as an additional shifter of production possibilities. Intuitively, if people feel optimistic about the future path of income, the wealth effect causes the labor supply curve to shift in. Consumption thus rises and, for a given wage, so would leisure as it is also a normal good. Yet, net product creation via the entry of new firms and within-firm product scope adjustments, as well as falling markups, lead to an outward shift of the labor demand curve. If product variety effects and the elasticities of net product entry and markups are sufficiently large, then employment and output also rise, thus allowing the initial expectation to become self-fulfilling.[1]

Other closely related work includes Jaimovich (2007) who shows that procyclical net business formation can lead to indeterminacy via the generation of countercyclical markups. Pavlov and Weder (2012) investigate the role of variety effects in generating sunspot equilibria. Both of these papers feature mono-product firms and hence do not consider intra-firm product scope adjustments, which (according to the empirical evidence mentioned previously) contributes more to aggregate output than entry and exit. Furthermore, while most of the indeterminacy literature simulates calibrated models by sunspot shocks only, we use Bayesian methods to estimate the indeterminate model with fundamental disturbances to preferences and technology.

The rest of this paper proceeds as follows. Section 2 lays out the model. Section 3 analyzes the local dynamics. Variable capital utilization is introduced in Section 4. The indeterminate model is estimated in Section 5. Section 6 concludes.

2 Model

The economy consists of intermediate good firms who are large relative to the size of the market and are able to choose how many products to produce. These goods are differentiated and hence bring about market power for these firms. The commodities are bought by competitive firms that weld them together into the final good that can be consumed or, by adding to the capital stock, invested.

[1] For this to occur, the reduced form aggregate labor demand curve (i.e. the wage-hours locus) must be upwardly sloping and steeper than the agents' labor supply curve.

2.1 Final goods

Final output, Y_t, is produced under perfect competition using the range of intermediate inputs supplied by M_t multi-product firms indexed i – each supplying $N_t(i)$ varieties of goods. Accordingly, the final good is constructed via two nested CES aggregators. The first encompasses the varieties from an individual firm i that, when put together, compose

$$Y_t(i) = N_t(i)^{1+\tau}\left(\frac{1}{N_t(i)}\int_0^{N_t(i)} y_t(i,j)^{\frac{\gamma-1}{\gamma}}dj\right)^{\frac{\gamma}{\gamma-1}} \qquad \tau > 0, \gamma > 1. \tag{1}$$

Here $y_t(i,j)$ is the amount of the unique intermediate good j produced by firm i. Parameters τ and γ stand for the intra-firm variety effect and the elasticity of substitution between goods, respectively. The firm-composite goods are then welded together to yield the final output

$$Y_t = M_t^{1+\omega}\left(\frac{1}{M_t}\int_0^{M_t} Y_t(i)^{\frac{\theta-1}{\theta}}di\right)^{\frac{\theta}{\theta-1}} \qquad \omega \geq 0, \theta > 1. \tag{2}$$

The parameter ω is the inter-firm variety effect and θ is elasticity of substitution between the firms' composite goods. Variety effects are separated from the elasticity of substitution as there is no *a priori* reason for such a strong link between them.[2] Moreover, the separation allows to clearly distinguish the variety effect and its impacts from that of imperfect competition. As we will see later, the intra-firm variety effect is crucial for firms to produce more than a single product. Lastly, Feenstra and Ma (2009) develop a related framework in which they assume $\theta = \gamma$. However, Broda and Weinstein's (2010) work suggest that these parameters are not equal, accordingly we will calibrate the model following their findings.

The profit maximization problem yields

$$y_t(i,j) = \left(\frac{P_t(i)}{P_t}\right)^{-\theta}\left(\frac{p_t(i,j)}{P_t(i)}\right)^{-\gamma} M_t^{\omega(\theta-1)-1}N_t(i)^{\tau(\gamma-1)-1}Y_t \tag{3}$$

where

$$P_t(i) = N_t(i)^{\frac{1}{\gamma-1}-\tau}\left(\int_0^{N_t(i)} p_t(i,j)^{1-\gamma}dj\right)^{\frac{1}{1-\gamma}} \tag{4}$$

is the price index for firm i's goods and the aggregate price index satisfies

$$P_t = M_t^{\frac{1}{\theta-1}-\omega}\left(\int_0^{M_t} P_t(i)^{1-\theta}di\right)^{\frac{1}{1-\theta}}. \tag{5}$$

[2] Benassy (1996).

In words, the demand for each variety depends negatively on its price, positively on the aggregate price index P_t, and positively (negatively) on the firm price index $P_t(i)$ if $\gamma > \theta$ ($\gamma < \theta$).

2.2 Intermediate good firms

The problem of the intermediate good firm is solved in the first stage, by firms choosing their optimal product scopes. In the second stage, they set their pricing rules by acting as Bertrand competitors in the product market.[3] Each period, the entry and exit of firms is determined by a zero profit condition.

Intermediate goods are produced using capital, $k_t(i,j)$, and labor, $h_t(i,j)$, supplied on perfectly competitive factor markets. The production technology is Cobb-Douglas and involves two fixed costs. The variety-level fixed cost, ϕ, applies once a variety is added to the production line. It restricts the amount of varieties a firm will produce and at the same time implies that it is only profitable to produce multiple products if the intra-firm variety effect is operating. The firm-level fixed cost, ϕ_f, provides economies of scope. It determines the number of active firms. Hence,

$$\int_0^{N_t(i)} y_t(i,j)dj = \int_0^{N_t(i)} \left[k_t(i,j)^{\alpha} h_t(i,j)^{1-\alpha} - \phi\right] dj - \phi_f. \tag{6}$$

Each firm sets prices to maximizes profits

$$\pi_t(i) = \int_0^{N_t(i)} p_t(i,j)y_t(i,j) - w_t h_t(i,j) - r_t k_t(i,j)dj \tag{7}$$

where w_t and r_t are the labor and capital rental rates. Following Yang and Heijdra (1993), intermediate good firms are large enough to take the aggregate price index into consideration when making their pricing decision. Appendix A.2 shows that firm i charges the same price, $p_t(i)$, for all of its varieties. Then, the optimal markup, $\mu_t(i) = p_t(i)/mc_t$ becomes

$$\mu_t(i) = \frac{\theta[1-\epsilon_t(i)]}{\theta[1-\epsilon_t(i)]-1}$$

where mc_t is the marginal cost of producing an additional variety, and $\epsilon_t(i)$ is firm i's market share:

$$\epsilon_t(i) \equiv \frac{P_t(i)Y_t(i)}{P_tY_t} = \frac{N_t(i)^{-\tau(1-\theta)}p_t(i)^{1-\theta}}{\int_0^{M_t} N_t(i)^{-\tau(1-\theta)}p_t(i)^{1-\theta}di}.$$

[3] This is a subgame perfect Nash equilibrium concept.

This share increases in $N_t(i)$. Without the intra-firm variety effect, τ, the market share would not depend on the product scope and profits would be decreasing in $N_t(i)$ because of the variety-level fixed cost ϕ.

Firms determine their optimal number of products by maximizing profits with respect to $N_t(i)$ by taking into account the effect on its own and other firms' pricing decisions (see Appendix A.3). The first-order condition is

$$\theta P_t Y_t \left(\frac{p_t(i) - mc_t}{p_t(i)}\right)^2 \frac{\partial \epsilon_t(i)}{\partial N_t(i)} + Y_t \epsilon_t(i) \left(\frac{p_t(i) - mc_t}{p_t(i)}\right) \frac{\partial P_t}{\partial N_t(i)} = mc_t \phi. \quad (8)$$

With the presence of the intra-firm variety effect, introducing a new product increases the firm's market share and profits (the first term on the left-hand side). There is an additional impact of the product scope on the aggregate price index. Specifically, a higher product scope reduces the aggregate price index, $\partial P_t / \partial N_t(i) < 0$, which from (3) leads to a lower demand for firm i's products. The right-hand side represents the cost of producing one more variety.

2.3 Symmetric equilibrium

In the symmetric equilibrium, each firm produces the same number of varieties, $N_t(i) = N_t$, charges the same price, $p_t(i) = p_t$, and has the same market share $\epsilon_t(i) = 1/M_t$. Let us designate the final good as the numeraire, $P_t = 1$, and therefore from (4) and (5), the price of a variety is determined by the variety effects

$$p_t = N_t^{\tau} M_t^{\omega}.$$

Using the above, (1) and (2), output per variety is

$$y_t = \frac{Y_t}{p_t N_t M_t}. \quad (9)$$

The markup simplifies to

$$\mu_t = \frac{\theta(M_t - 1)}{\theta(M_t - 1) - M_t}. \quad (10)$$

Note that firm entry decreases the markup. It is this mechanism that renders the markup countercyclical. An increase in the firm's product scope raises the firm's own price and reduces the prices of other firms. To reduce price competition, firms under-expand their product scopes in comparison to the case of monopolistic competition where such strategic linkages are absent.

The extent of this under-expansion can be seen by substituting $\partial\epsilon_t(i)/\partial N_t(i)$ and $\partial P_t/\partial N_t(i)$ into (8) and rearranging:

$$y_t(\mu_t - 1)\tau(\theta-1)\left[\frac{(M_t-1)(\theta+(1-\theta)M_t)}{\theta(M_t-1)+M_t^2(1-\theta)} - \frac{1}{M_t(\theta-1)}\right] = \phi$$

The term in the square brackets is less than one and is increasing in M_t: the strategic effect of the product scope decision becomes less important as the number of firms increases and this gives an incentive to introduce new varieties. When M_t becomes very large this term approaches unity, as would be the case under monopolistic competition. Likewise, the markup converges to it's monopolistic competition level of $\theta/(\theta-1)$.[4] Intuitively, as the number of firms grows, the impact on the market share of adding an additional variety becomes smaller, which has then a smaller impact on the price of the variety. Further rearrangement yields the product scope

$$N_t = \frac{\tau Y_t}{\phi p_t}\left[\frac{(\theta-1)(M_t-1)}{\theta(1-M_t)+M_t^2(\theta-1)} + \frac{1}{M_t[M_t(1-\theta)+\theta]}\right].$$

Using (6), (9) and the zero profit condition determines M_t as

$$M_t = \frac{(\mu_t-1)K_t^\alpha H_t^{1-\alpha}}{\mu_t(N_t\phi+\phi_f)} \tag{11}$$

where $K_t = M_t N_t k_t$ and $H_t = M_t N_t k_t$. To obtain aggregate output, substitute (6) in (9), then use (11) to simplify:

$$Y_t = \frac{p_t}{\mu_t} K_t^\alpha H_t^{1-\alpha}$$

where p_t/μ_t is the endogenous efficiency wedge. Finally, the equilibrium real wage and rental rate are given by

$$w_t = (1-\alpha)\frac{Y_t}{H_t} \qquad \text{and} \qquad r_t = \alpha\frac{Y_t}{K_t}.$$

2.4 People

There is a nonatomic measure-one space of agents each characterized by lifetime utility

$$U = \int_0^\infty e^{-\rho t} u(C_t, H_t)dt \qquad\qquad \rho > 0.$$

[4] These results imply that the oligopolistic competition setup is identical to monopolistic competition in the limit where M_t approaches infinity.

ρ denotes the subjective discount rate and period utility, $u(.,.)$, is separable in consumption, C_t, and hours worked, H_t. It takes on the functional form

$$u(C_t, H_t) = \ln C_t - \upsilon \frac{H_t^{1+\chi}}{1+\chi} \qquad \upsilon > 0, \chi \geq 0$$

where χ is the inverse of the Frisch labor supply elasticity. Logarithmic utility is the only additive-separable form consistent with balanced growth. The agents own the capital stock and sell labor as well as capital services. Any generated profits, Π_t, flow back to them. Let X_t denote investment, then the budget is constrained by

$$w_t H_t + r_t K_t + \Pi_t \geq X_t + C_t$$

where investment is added to the capital stock such that:

$$\dot{K}_t = X_t - \delta K_t \qquad 0 < \delta < 1.$$

Time derivatives are denoted by dots and δ stands for the constant rate of physical depreciation of the capital stock. Optimality implies

$$\upsilon H_t^{\chi} = \frac{w_t}{C_t} \tag{12}$$

$$\frac{\dot{C}_t}{C_t} = r_t - \delta - \rho \tag{13}$$

Equation (12) describes the agents' leisure-consumption trade-off, while (13) is the intertemporal Euler equation. In addition a transversality condition holds.

3 Dynamics

This section analyzes the local dynamical properties of the artificial economy. We log-linearize the equilibrium conditions and arrange the dynamical system to

$$\begin{bmatrix} \dot{K}_t/K_t \\ \dot{C}_t/C_t \end{bmatrix} = \mathbf{J} \begin{bmatrix} \hat{K}_t \\ \hat{C}_t \end{bmatrix}.$$

Hatted variables denote percent deviations from their steady-state values and $\mathbf{J}$ is the 2×2 Jacobian matrix of partial derivatives. Note that C_t is a non-predetermined variable and that K_t is predetermined. Hence, indeterminacy requires that the two roots of $\mathbf{J}$ to be negative, or simply Det$\mathbf{J} > \mathbf{0} >Tr\mathbf{J}$. We calibrate standard parameters as $\alpha = 0.3$, $\rho = 0.01$, $\delta = 0.025$ and $\chi = 0$.

3.1 Mono-product model

To better illustrate the contribution of the product scope decision on indeterminacy, we first consider the case of mono-product firms where such a decision is absent. Figure 1 presents the stability zones, assuming that the variety effect depends on the elasticity of substitution between intermediate goods: $\omega = 1/(\theta - 1)$. As can be seen, the minimum markup allowing for indeterminacy is $\mu = 1/(1-\alpha) = 1.429$, where the variety effect is $1/(\theta - 1) = 0.429$. This exactly corresponds to the result reported in Pavlov and Weder (2012) for a mono-product model with monopolistic competition. Why is this the case? Note that from (10), the steady state number of firms is

$$M = 1 + \frac{\mu}{\mu(\theta - 1) - \theta}.$$

As θ approaches $\mu/(\mu - 1)$, the number of firms approaches infinity: the steady state markup and local dynamics converge to the case of monopolistic competition. This implies that the steady state markup must be higher under oligopolistic competition for indeterminacy. On the other hand, the required variety effect drops considerably with higher values of θ. We do not consider situations where $\theta < \mu/(\mu - 1)$ because this implies that $M < 0$.

A higher steady state markup and greater substitutability between differentiated goods (and hence a lower variety effect) imply a lower number of firms and a more elastic markup over the business cycle. Note that a lower variety effect makes indeterminacy more difficult to obtain, while a more elastic markup leads to the opposite outcome. Therefore, the dashed stability line in the figure is upwardly sloping because the lower variety effect (via higher θ) needs to be offset by a higher markup elasticity (via higher μ). Yet, the line eventually becomes downwardly sloping because the gain from the higher markup elasticity starts to dominate the influence of the lower variety effect on the endogenous efficiency wedge as goods become closer substitutes.

3.2 Multi-product model

Figure 2 presents the numerical indeterminacy region for the multi-product model with $\omega = \tau = 1/(\theta - 1) = 1/(\gamma - 1)$. Once again, the model converges to the one with monopolistic competition along the $\theta = \mu/(\mu - 1)$ line. This is because both markups and product scopes are constant over the business cycle (see Appendix A.4). Recall that under oligopolistic competition, the entry of new competitors reduces existing firms' market shares and encourages them to expand their product scopes. This additional channel of product creation reduces the minimum steady state markup, for example, at $\theta = \gamma = 14$, a markup of $\mu = 1.3$ is enough for indeterminacy.[5] At this point, the variety effect is only $\omega = \tau = 0.077$ (compared to the required 0.429 under monopolistic competition).

We now consider the case where $\theta \neq \gamma$. Figure 3 plots the numerical indeterminacy region with an intermediate steady state markup of $\mu = 1.3$. Indeterminacy is easier to obtain when $\theta > \gamma$, which is the case where each firm produces less closely related products. If instead $\gamma > \theta$ (as suggested by Broda and Weinstein, 2010) then a firm produces goods that are close substitutes to each other.

4 Capital utilization

While we have shown that the possibility of sunspot equilibria increases when firms are able to choose their product scopes, the inter firm elasticity of substitution, θ, needs to be sufficiently high for indeterminacy to be possible under realistic levels of market power. This section addresses this issue by augmenting the multi-product model by endogenous capital utilization.

Each intermediate good firm i now operates the production technology

$$\int_0^{N_t(i)} y_t(i,j)dj = \int_0^{N_t(i)} \left[U_t^\alpha k_t(i,j)^\alpha h_t(i,j)^{1-\alpha} - \phi\right] dj - \phi_f$$

where U_t stands for the utilization rate of capital set by its owners. The aggregate production function in the symmetric equilibrium is thus

$$Y_t = \frac{p_t}{\mu_t}(U_t K_t)^\alpha H_t^{1-\alpha}.$$

Capital accumulation follows

$$\dot{K}_t = X_t - \delta_t K_t = X_t - \frac{1}{\varrho} U_t^\varrho K_t \qquad \varrho > 1$$

and the optimal rate of capital utilization is

$$r_t = U_t^{\varrho - 1}.$$

The calibration remains as in the previous section and $\varrho = 1.4$.[6] Indeterminacy becomes more likely due to agents increasing the intensity of capital

[5] It can be shown that for very high values of θ, the markup required for indeterminacy is as low as 1.05, albeit in only a small region.

[6] See Wen (1998).

utilization as the marginal product of capital increases from higher employment, which increases the demand for labor. As Figures 4 and 5 demonstrate, the introduction of variable capital utilization significantly reduces the level of market power and the inter firm elasticity of substitution required for indeterminacy.

5 Simulations

We have shown that intra-firm product creation can generate indeterminacy under more plausible situations. Although this can be considered as progress, it would be rendered void if the model is unable to replicate the basic business cycle facts. This is done next by using U.S. quarterly data to estimate the indeterminate model (see Appendix A.5 for the data sources).

The model employed here is a discrete time economy with capital utilization and labor augmenting technological progress, Γ_t, which grows at the constant rate $1 + g = 1.004566$. There are two fundamental shocks. First, aggregate total factor productivity, z_t, affects all firms equally and follows the process

$$\log z_t = \psi_z \log z_{t-1} + \varepsilon_t^z, \qquad 0 \leq \psi_z < 1$$

where ε_t^z is an i.i.d. disturbance with variance σ_z^2. This implies that aggregate output is given by

$$Y_t = \frac{z_t p_t}{\mu_t}(U_t K_t)^{\alpha}(\Gamma_t H_t)^{1-\alpha}.$$

Second, preference shocks take the form

$$u(C_t, H_t) = \ln(C_t - \Delta_t) - \upsilon \frac{H_t^{1+\chi}}{1+\chi}$$

where a positive shock to Δ_t increases the marginal utility of consumption and leads to an urge to consume. It follows a similar process with a persistence parameter ψ_Δ and an i.i.d. disturbance ε_t^Δ with variance σ_Δ^2. The model is then detrended and log-linearized around the steady state.

It is now well known that under indeterminacy, the economy's response to shocks is not uniquely determined and that sunspots propagate fundamental disturbances (see Lubik and Schorfheide, 2003 and 2004). We follow Farmer, Khramov, and Nicolò (2014) in dealing with such loose expectation errors. Specifically, we reclassify the expectation error to output, η_t^Y, as a new exogenous shock:

$$\hat{Y}_t = E_{t-1}\hat{Y}_t + \eta_t^Y.$$

Understanding that fundamental shocks have an effect on output on impact, we go a step further by breaking down the expectation error into fundamental and non-fundamental components

$$\eta_t^Y = \Omega_z \varepsilon_t^z + \Omega_\Delta \varepsilon_t^\Delta + \varepsilon_t^s$$

where parameters Ω_z and Ω_Δ determine the effect of technology and preferences shocks on output and ε_t^s is an i.i.d. sunspot shock that is independent of fundamentals with variance σ_s^2.

The model is then estimated via Bayesian methods using the real per capita growth rates of output, consumption, investment and hours worked as observables. The measurement equation is thus

$$\begin{bmatrix} \ln Y_t - \ln Y_{t-1} \\ \ln C_t - \ln C_{t-1} \\ \ln X_t - \ln X_{t-1} \\ \ln H_t - \ln H_{t-1} \end{bmatrix} = \begin{bmatrix} \hat{Y}_t - \hat{Y}_{t-1} \\ \hat{C}_t - \hat{C}_{t-1} \\ \hat{X}_t - \hat{X}_{t-1} \\ \hat{H}_t - \hat{H}_{t-1} \end{bmatrix} + \begin{bmatrix} g \\ g \\ g \\ 0 \end{bmatrix} + \begin{bmatrix} \varepsilon_t^{m.e.} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

where $\varepsilon_t^{m.e.}$ is the measurement error on output growth. The standard parameters remain as in the previous sections: $\alpha = 0.3$, $\delta = 0.025$, $\chi = 0$ and the discount factor is $\beta = (1+\rho)^{-1} = 0.99$. Following the estimates reported in Broda and Weinstein (2010), we set $\theta = 7.5$ and $\gamma = 11.5$. Table 1 presents the prior and posterior distributions for the estimated model parameters. We assume a gamma distribution for the steady state markup, μ, with a lower limit of 1.154 to keep the steady state number of firms, M, above unity. The mean is centered around the middle of value-added markup estimates for the U.S. (see Jaimovich, 2007). Although it is likely that output responds positively to technology and preference shocks, we assume a wide uniform distribution for parameters Ω_z and Ω_Δ. The measurement error is restricted to account for not more than ten percent of output growth. We use the Metropolis-Hastings algorithm to obtain 100,000 draws from the posterior mean and adjust the scale in the jumping distribution to achieve a 30 percent acceptance rate. The estimated markup, although higher than the prior mean, is still well inside the empirically plausible range. Expectation error parameters Ω_z and Ω_Δ imply that output responds positively to both technology and preference shocks.

Table 1						
Prior distribution for model parameters					Posterior distribution	
Name	Range	Density	Mean	Std. Dev.	Mean	90% Interval
μ	[1.154,$+\infty$]	Gamma	1.3	0.05	1.3749	[1.3532,1.3977]
ψ_z	[0,1)	Beta	0.9	0.05	0.9956	[0.9929,0.9985]
ψ_Δ	[0,1)	Beta	0.9	0.05	0.9923	[0.9867,0.9980]
σ_s	R^+	Inverse Gamma	0.001	Inf	0.0070	[0.0064,0.0075]
σ_z	R^+	Inverse Gamma	0.001	Inf	0.0047	[0.0044,0.0050]
σ_Δ	R^+	Inverse Gamma	0.001	Inf	0.0072	[0.0066,0.0076]
$\sigma^{m.e.}$	$[0, 0.005]$	Uniform	0.003	0.0014	0.0050	[0.0050,0.0050]
Ω_z	[-3,3]	Uniform	0	1.7321	1.6472	[1.3828,1.9163]
Ω_Δ	[-3,3]	Uniform	0	1.7321	1.3680	[1.1937,1.5428]

Table 2 presents the second moments of the U.S. data and of the estimated artificial economy. The model overpredicts output and consumption volatility as well as their correlation, yet, it does a far better job in matching the moments of hours worked and investment.

Table 2				
Business Cycle Dynamics				
	Data		Model	
x	σ_x	$\rho(x, \ln(Y_t/Y_{t-1}))$	σ_x	$\rho(x, \ln(Y_t/Y_{t-1}))$
$\ln(Y_t/Y_{t-1})$	0.98	1	1.63	1
$\ln(C_t/C_{t-1})$	0.57	0.52	1.28	0.85
$\ln(X_t/X_{t-1})$	2.43	0.67	3.57	0.79
$\ln(H_t/H_{t-1})$	0.93	0.74	1.04	0.79
		$\rho(x, Y)$		$\rho(x, Y)$
Y_t	1.69	1	1.93	1
C_t	0.89	0.78	1.61	0.89
X_t	4.99	0.79	4.45	0.82
H_t	1.98	0.88	1.32	0.83

σ_Y denotes the standard deviation of output and $\rho(x, Y)$ is the correlation of variable x and output. The last four variables have been HP filtered.

The impulse response functions based on the mean parameter estimates can be seen in Figures 6, 7 and 8. Both net product creation and net business formation positively comove with output, with the former being more volatile than the latter.

6 Conclusion

Previous studies have shown that procyclical product creation via entry and exit of mono-product firms can be an important source of sunspot equilibria. Yet, recent empirical evidence suggests that product creation occurs largely within existing firms. Motivated by these findings, the current paper investigates the role of intra-firm product scope adjustments in a general equilibrium economy with oligopolistic producers. It shows that the multi-product nature of firms makes the economy significantly more susceptible to sunspot equilibria. The estimated indeterminate model driven by both belief and fundamental disturbances generates artificial business cycles that closely resemble empirically observed fluctuations.

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A Appendix

A.1 Price elasticity of demand

This Appendix derives the demand elasticities of an intermediate good with respect to changes in its own price and the price of other goods produced by the same firm. Taking logs of (3) we obtain

$$\begin{aligned}\ln y_t(i,j) &= -\gamma \ln p_t(i,j) - (\theta-\gamma)\ln P_t(i) + \theta \ln P_t + \ln Y_t \\ &\quad +[\tau(\gamma-1)-1]\ln N_t(i) + [\omega(\theta-1)-1]\ln M_t.\end{aligned}$$

From (4)

$$\frac{\partial \ln P_t(i)}{\partial \ln p_t(i,j)} = \left(\frac{p_t(i,j)}{P_t(i)}\right)^{1-\gamma} N_t(i)^{\tau(\gamma-1)-1}.$$

Then from (5)

$$\frac{\partial \ln P_t}{\partial \ln p_t(i,j)} = \left(\frac{p_t(i,j)}{P_t(i)}\right)^{1-\gamma} N_t(i)^{\tau(\gamma-1)-1} \left(\frac{P_t(i)}{P_t}\right)^{1-\theta} M_t^{\omega(\theta-1)-1}.$$

Then the price elasticity of demand is

$$\begin{aligned}\frac{\partial \ln y_t(i,k)}{\partial \ln p_t(i,j)} &= \underbrace{-\gamma}_{\text{absent for } k\neq j} - (\theta-\gamma)\left(\frac{p_t(i,j)}{P_t(i)}\right)^{1-\gamma} N_t(i)^{\tau(\gamma-1)-1} \\ &\quad +\theta \left(\frac{p_t(i,j)}{P_t(i)}\right)^{1-\gamma} N_t(i)^{\tau(\gamma-1)-1}\left(\frac{P_t(i)}{P_t}\right)^{1-\theta} M_t^{\omega(\theta-1)-1}\end{aligned} \tag{A.1}$$

Note that under monopolistic competition, firms are too small to influence the aggregate price index, P_t, and hence the last term in (A.1) would be absent.

A.2 Markups

This Appendix derives the optimal markups of intermediate good firms. Firm i maximizes profit (7) subject to the constraint (6):

$$\begin{aligned}\mathcal{L} &= \int_0^{N_t(i)} p_t(i,j)y_t(i,j) - w_t h_t(i,j) - r_t k_t(i,j)dj \\ &\quad +\Lambda_t \left(\int_0^{N_t(i)} \left[z_t k_t(i,j)^\alpha h_t(i,j)^{1-\alpha} - \phi\right] dj - \phi_f - \int_0^{N_t(i)} y_t(i,j)dj\right).\end{aligned}$$

Optimality gives

$$\frac{\partial \mathcal{L}}{\partial p_t(i,j)} = y_t(i,j) + \int_0^{N_t(i)} [p_t(i,j) - \Lambda_t] \frac{\partial y_t(i,j)}{\partial p_t(i,j)} dj = 0 \tag{A.2}$$

$$\frac{\partial \mathcal{L}}{\partial h_t(i,j)} = -w_t + \Lambda_t (1-\alpha) z_t k_t(i,j)^{\alpha} h_t(i,j)^{-\alpha} = 0 \tag{A.3}$$

$$\frac{\partial \mathcal{L}}{\partial k_t(i,j)} = -r_t + \Lambda_t \alpha z_t k_t(i,j)^{\alpha-1} h_t(i,j)^{1-\alpha} = 0. \tag{A.4}$$

The Lagrange multiplier, Λ_t, is obtained by combining (A.3) and (A.4) and amounts to the marginal cost, mc_t, of producing one more variety:

$$mc_t \equiv \Lambda_t = \frac{w_t^{1-\alpha} r_t^{\alpha}}{z_t (1-\alpha)^{1-\alpha} \alpha^{\alpha}}.$$

Hence, the costs of production are

$$\int_0^{N_t(i)} w_t h_t(i,j) + r_t k_t(i,j) dj = mc_t \left(\int_0^{N_t(i)} [y_t(i,j) + \phi] dj + \phi_f \right)$$

and profits are

$$\pi_t(i) = \int_0^{N_t(i)} y_t(i,j) [p_t(i,j) - mc_t] dj - mc_t \left[N_t(i)\phi + \phi_f \right]. \tag{A.5}$$

Substituting (A.1) into (A.2) and some algebra yields

$$y_t(i,j) - \gamma \frac{y_t(i,j)}{p_t(i,j)} [p_t(i,j) - mc_t] = \int_0^{N_t(i)} \frac{y_t(i,k)}{p_t(i,j)} [p_t(i,k) - mc_t] \, dk$$
$$\times \left(\frac{p_t(i,j)}{P_t(i)} \right)^{1-\gamma} N_t(i)^{\tau(\gamma-1)-1} \left[\theta - \gamma + \theta \left(\frac{P_t(i)}{P_t} \right)^{1-\theta} M_t^{\omega(\theta-1)-1} \right].$$

Substituting (3) for $y_t(i,j)$, the above equation simplifies to

$$P_t Y_t \left(\frac{P_t(i)}{P_t} \right)^{1-\theta} M_t^{\omega(\theta-1)-1} \left[1 - \gamma \frac{p_t(i,j) - mc_t}{p_t(i,j)} \right] =$$
$$\int_0^{N_t(i)} y_t(i,k) \, [p_t(i,k) - mc_t] \, dk \left[\theta - \gamma - \theta \left(\frac{P_t(i)}{P_t} \right)^{1-\theta} M_t^{\omega(\theta-1)-1} \right].$$

As the second part of this equation is the same for all $j \in [0, N_t(i)]$, this implies that firm i will charge the same price for all of its varieties. Hence, $p_t(i,j) = p_t(i,k) = p_t(i)$ and the equation simplifies to

$$1 - \gamma \frac{p_t(i) - mc_t}{p_t(i)} = \tag{A.6}$$
$$N_t(i)^{\tau(\gamma-1)} \left(\frac{p_t(i)}{P_t(i)}\right)^{1-\gamma} \frac{p_t(i) - mc_t}{p_t(i)} \left[\theta - \gamma - \theta \left(\frac{P_t(i)}{P_t}\right)^{1-\theta} M_t^{\omega(\theta-1)-1}\right].$$

To solve for firm i's markup, first note from (4) that $P_t(i) = N_t(i)^{-\tau} p_t(i)$. Then using this together with (1), (3) and (5), we can express firm i's market share, $\epsilon_t(i) \equiv P_t(i)Y_t(i)/(P_tY_t)$, as

$$\epsilon_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{1-\theta} M_t^{\omega(\theta-1)-1} = \frac{N_t(i)^{-\tau(1-\theta)} p_t(i)^{1-\theta}}{\int_0^{M_t} N_t(i)^{-\tau(1-\theta)} p_t(i)^{1-\theta} di}. \tag{A.7}$$

As long as $\tau > 0$, the price index $P_t(i)$ is decreasing in $N_t(i)$, and so increasing the product scope increases the firm's market share. Finally, the markup, $\mu_t(i) \equiv p_t(i)/mc_t$, can be found by rearranging (A.6):

$$\mu_t(i) = \frac{\theta[1 - \epsilon_t(i)]}{\theta[1 - \epsilon_t(i)] - 1} \tag{A.8}$$

A.3 Product scope

This Appendix derives the firms' optimal product scope. Substituting (3) into (A.5), then using (4) and (A.7), we rewrite profits as

$$\pi_t(i) = \left(\frac{p_t(i) - mc_t}{p_t(i)}\right) P_t Y_t \epsilon_t(i) - mc_t[N_t(i)\phi + \phi_f].$$

Firm i takes the number of firms and their product scopes as given and maximizes its profits with respect to $N_t(i)$ by taking account the effect of its product scope decision on its own and all other producers' pricing decisions. The first-order condition is

$$\frac{\partial \pi_t(i)}{\partial N_t(i)} = \theta P_t Y_t \left(\frac{p_t(i) - mc_t}{p_t(i)}\right)^2 \frac{\partial \epsilon_t(i)}{\partial N_t(i)} + Y_t \epsilon_t(i) \left(\frac{p_t(i) - mc_t}{p_t(i)}\right) \frac{\partial P_t}{\partial N_t(i)} - mc_t \phi = 0. \tag{A.9}$$

We now calculate $\partial \epsilon_t(i)/\partial N_t(i)$ and $\partial P_t/\partial N_t(i)$ and then substitute in (A.9) to obtain firm i's product scope. Differentiating (A.7) with respect to $N_t(i)$ yields

$$\frac{\partial \epsilon_t(i)}{\partial N_t(i)} = \tau(\theta - 1)\frac{\epsilon_t(i)}{N_t(i)} - (\theta - 1)\epsilon_t(i) \left[\frac{1}{p_t(i)} \frac{\partial p_t(i)}{\partial N_t(i)} - \frac{1}{P_t} \frac{\partial P_t}{\partial N_t(i)}\right]. \tag{A.10}$$

Note that the second term on the right hand side of (A.10) would not be present in the case of monopolistic competition. As we will see, $\partial p_t(i)/\partial N_t(i)$ and $\partial P_t/\partial N_t(i)$ are positive and negative, respectively; implying that firms contract their product scopes compared to the case of monopolistic competition. We rewrite the aggregate price index (5) as

$$P_t = M_t^{\frac{1}{\theta-1}-\omega}\left(\int_0^{M_t} N_t(k)^{-\tau(1-\theta)}p_t(k)^{1-\theta}dk\right)^{\frac{1}{1-\theta}}.$$

Then, after some algebra $\partial P_t/\partial N_t(i)$ can be expressed as

$$\frac{\partial P_t}{\partial N_t(i)} = P_t^{\theta}M_t^{\omega(\theta-1)-1}\left[\int_0^{M_t} N_t(k)^{-\tau(1-\theta)}p_t(k)^{-\theta}\frac{\partial p_t(k)}{\partial N_t(i)}dk - \tau N_t(i)^{-\tau(1-\theta)-1}p_t(i)^{1-\theta}\right]. \tag{A.11}$$

We now show that the first term in the square brackets is equal to zero. From (A.8)

$$\frac{p_t(k)}{p_t(k)-mc_t} = \theta - \theta\epsilon_t(k).$$

Then

$$\int_0^{M_t}\frac{p_t(k)}{p_t(k)-mc_t}dk = \theta M_t - \theta.$$

Differentiating with respect to $N_t(i)$ gives

$$\int_0^{M_t} -\frac{mc_t}{[p_t(k)-mc_t]^2}\frac{\partial p_t(k)}{\partial N_t(i)}dk = 0$$

which under symmetry collapses to

$$(M_t-1)\frac{\partial p_t(k)}{\partial N_t(i)} + \frac{\partial p_t(i)}{\partial N_t(i)} = 0.$$

Replacing $\partial p_t(k)/\partial N_t(i)$ in (A.11) with $-[\partial p_t(i)/\partial N_t(i)]/(M_t-1)$ and assuming symmetry, the first term in the square brackets drops out and some rearrangement yields

$$\frac{\partial P_t}{\partial N_t(i)} = -\tau P_t\frac{\epsilon_t(i)}{N_t(i)}.$$

An increase in the product scope therefore reduces the aggregate price index. Inserting this result in (A.10) gives

$$\frac{\partial \epsilon_t(i)}{\partial N_t(i)} = \tau(\theta-1)\frac{\epsilon_t(i)}{N_t(i)}[1-\epsilon_t(i)] - (\theta-1)\frac{\epsilon_t(i)}{p_t(i)}\frac{\partial p_t(i)}{\partial N_t(i)}. \tag{A.12}$$

The next step is to compute $\partial p_t(i)/\partial N_t(i)$. From (A.8) we obtain

$$\frac{\partial p_t(i)}{\partial N_t(i)} = \frac{\theta mc_t}{[1-\theta+\theta\epsilon_t(i)]^2}\frac{\partial \epsilon_t(i)}{\partial N_t(i)}.$$

Then using this in (A.12) and some simplification yields

$$\frac{\partial \epsilon_t(i)}{\partial N_t(i)} = \tau(\theta-1)\frac{\epsilon_t(i)}{N_t(i)}\frac{[1-\epsilon_t(i)]^2(\theta[1-\epsilon_t(i)]-1)}{\theta(1-\epsilon_t(i)[1-\epsilon_t(i)])-1}.$$

Here, $\partial\epsilon_t(i)/\partial N_t(i) > 0$ and hence $\partial p_t(i)/\partial N_t(i) > 0$. Inserting $\partial\epsilon_t(i)/\partial N_t(i)$ and $\partial P_t/\partial N_t(i)$ into (A.9), assuming symmetry where $\epsilon_t(i) = \epsilon_t = 1/M_t$, and some rearrangement gives

$$N_t = \frac{\tau P_t Y_t}{p_t\phi}\left[\frac{(\theta-1)(M_t-1)}{\theta(1-M_t)+M_t^2(\theta-1)} + \frac{1}{M_t[M_t(1-\theta)+\theta]}\right].$$

A.4 Monopolistic competition

In this Appendix we show that under monopolistic competition, the product scope and output per variety are constant over the business cycle. Moreover, this implies that the local dynamics and conditions for indeterminacy are identical to the mono-product model described in Pavlov and Weder (2012).

[To be completed]

A.5 Data Sources

This Appendix details the source and construction of the U.S. data used in Section 5. All data is quarterly and for the period 1948:I-2012:IV.

1. Gross Domestic Product. Seasonally adjusted at annual rates, billions of chained (2009) dollars. Source: Bureau of Economic Analysis, NIPA Table 1.1.6.

2. Gross Domestic Product. Seasonally adjusted at annual rates, billions of dollars. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

3. Personal Consumption Expenditures, Nondurable Goods. Seasonally adjusted at annual rates, billions of dollars. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

4. Personal Consumption Expenditures, Services. Seasonally adjusted at annual rates, billions of dollars. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

5. Gross Private Domestic Investment, Fixed Investment, Residential. Seasonally adjusted at annual rates, billions of dollars. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

6. Gross Private Domestic Investment, Fixed Investment, Nonresidential. Seasonally adjusted at annual rates, billions of dollars. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

7. Nonfarm Business Hours. Index 2009=100, seasonally adjusted. Source: Bureau of Labor Statistics, Series Id: PRS85006033.

8. Civilian Noninstitutional Population. 16 years and over, thousands. Source: Bureau of Labor Statistics, Series Id: LNU00000000Q.

9. GDP Deflator = (2)/(1).

10. Real Per Capita Consumption, $C_t = [(3) + (4)]/(9)/(8)$.

11. Real Per Capita Investment, $X_t = [(5) + (6)]/(9)/(8)$.

12. Real Per Capita Output, $Y_t = (1)/(8)$.

13. Per Capita Hours Worked, $H_t = (7)/(8)$.

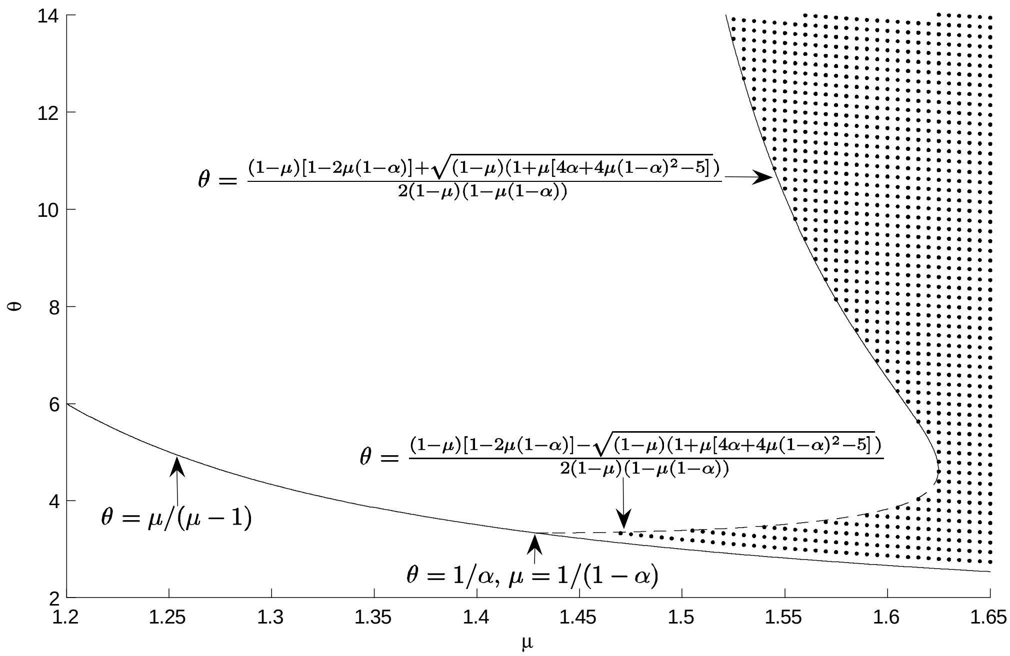

Figure 1: Mono-product model.

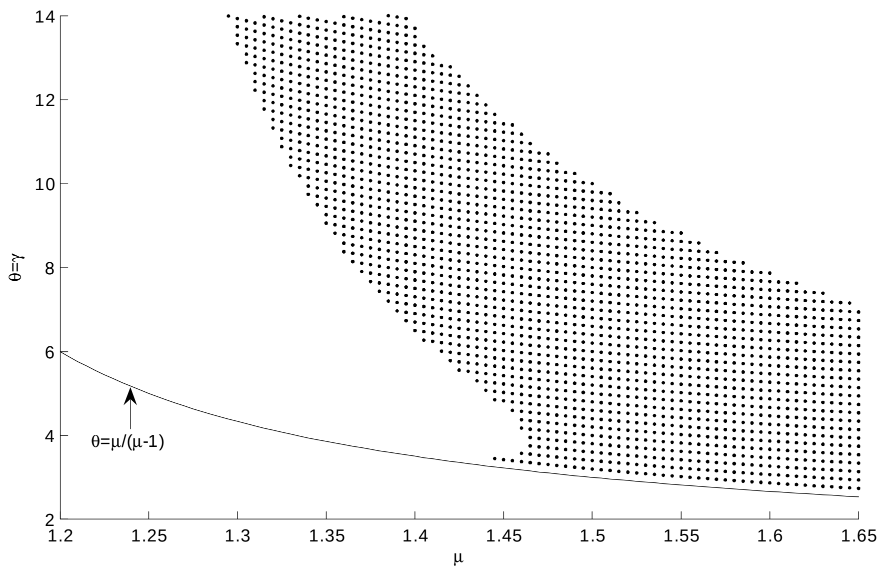

Figure 2: Multi-product model, $\theta = \gamma$.

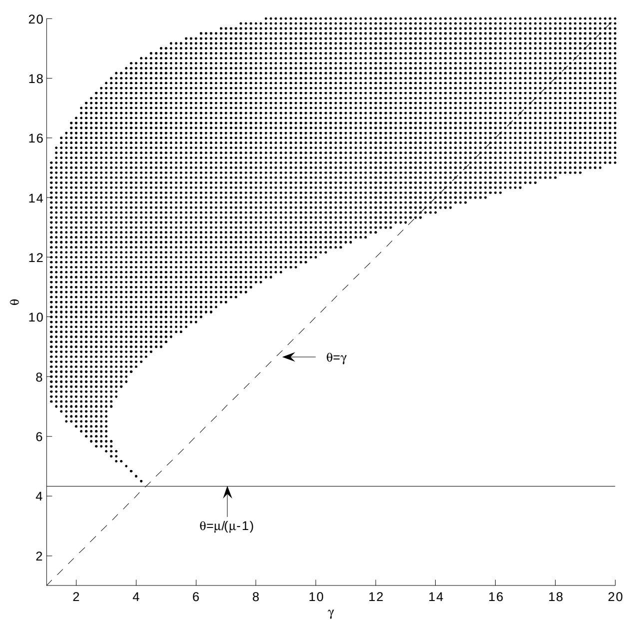

Figure 3: Multi-product model, $\mu = 1.3$.

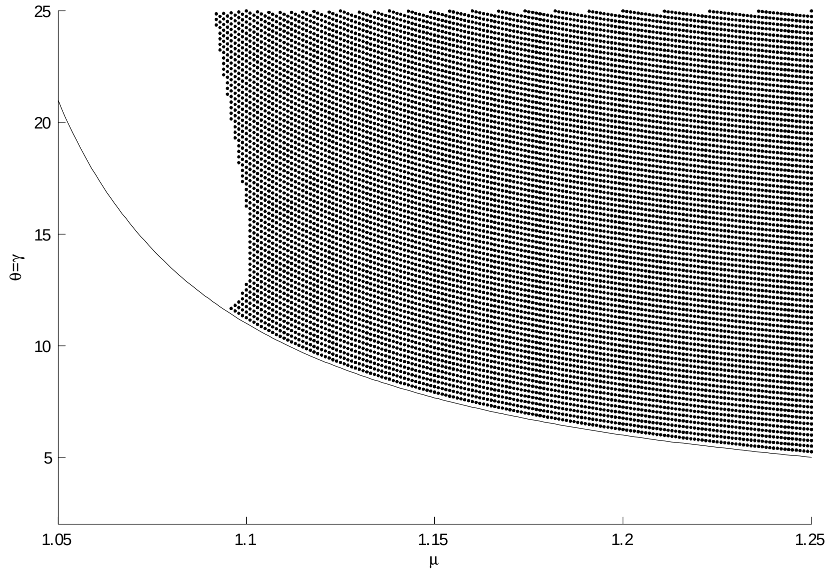

Figure 4: Multi-product model with endogenous capital utilization, $\theta = \gamma$.

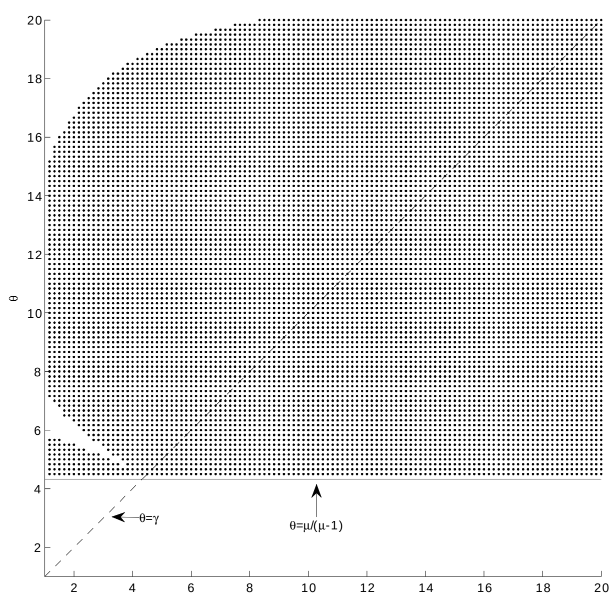

Figure 5: Multi-product model with endogenous capital utilization, $\mu = 1.3$.

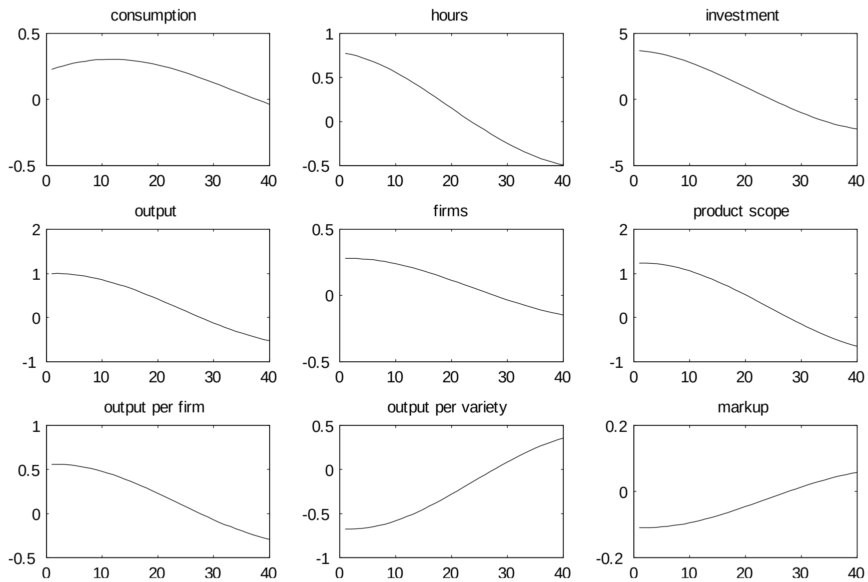

Figure 6: Impulse responses to a sunspot shock, ε_t^s.

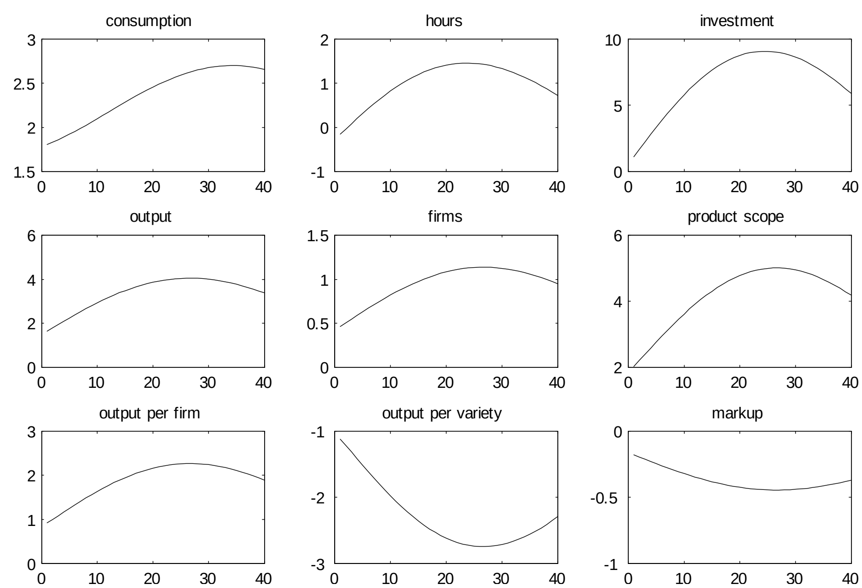

Figure 7: Impulse responses to a technology shock, ε_t^z.

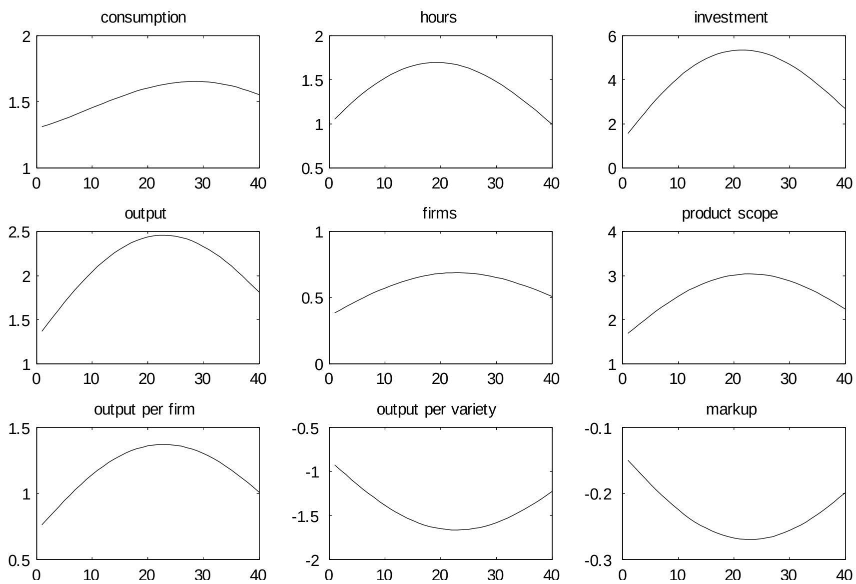

Figure 8: Impulse responses to a preference shock, ε_t^{Δ}.