Discretionary Monetary Policy in the Calvo Model*

Willem Van Zandweghe[†] Alexander L. Wolman[‡]

September 12, 2008

Abstract

We study discretionary equilibrium in the Calvo pricing model for a monetary authority that chooses the money supply. Unlike the two-period Taylor model, policy does not accommodate predetermined prices in a way that inevitably leads to multiple private-sector equilibria. For the examples we compute, we find a unique equilibrium characterized by a steady state inflation rate that exceeds five percent.

JEL codes: E31; E52

Keywords: time-consistent monetary policy, relative price distortion

*We have benefited from discussions with Andreas Hornstein, Bob King, Per Krusell and Pierre Sarte, and from the feedback of brown bag lunch participants at the Richmond Fed. The views expressed in this paper are those of the authors alone. They are not the views of the Federal Reserve Bank of Kansas City, the Federal Reserve Bank of Richmond or the Federal Reserve System.

[†]Federal Reserve Bank of Kansas City, willem.vanzandweghe@kc.frb.org.

[‡]Federal Reserve Bank of Richmond, alexander.wolman@rich.frb.org.

1 Introduction

Over the last ten years the New Keynesian framework has become predominant in the world of applied monetary policy analysis. There is not a unique definition of the New Keynesian framework, but it is commonly characterized by linear models with strong forward-looking elements and which can be rationalized as approximations to micro-founded dynamic equilibrium models. The most common source of nominal rigidity in this framework is the Calvo (1983) pricing model as described by Yun (1996). Yun's version has price indexation, while the version in King and Wolman (1996) has no indexation. This paper investigates discretionary monetary policy in the Calvo model without indexation.

In one respect, there is a vast literature studying aspects of discretionary policy in New Keynesian models with Calvo pricing. But the typical practice in this literature, exemplified by Clarida, Gali and Gertler (1999) and Woodford (2003), has been to work with linear models.[1] In contrast, our approach is to solve for discretionary equilibrium of the underlying nonlinear model using a projection method. The solution is only approximate, but the approximation can be made arbitrarily accurate by increasing the fineness of the grid used to define the economy's state. Because there is only one state variable we are able to solve the model quite accurately (in principle).

A small number of papers have studied discretionary equilibrium in full-blown nonlinear sticky price models, but not with Calvo pricing. A common theme in those papers is multiple equilibria. Khan, King and Wolman (2001) and King and Wolman (2004) show that in Taylor-style models with prices set for three and two periods respectively, multiple equilibria naturally arise under discretionary policy. This finding points to the possibility of expectations traps, in which monetary policy can lead inflation expectations to become self-fulfilling.[2]

[1] An exception is Yun (2005), who conducts a nonlinear analysis of the Calvo model. But in his model a fiscal instrument for offsetting the markup distortion also eliminates the time-consistency problem. Adam and Billi (2007) take into account the nonlinearity arising from the zero bound on nominal interest rates.

[2] Siu (2008) extends King and Wolman's (2004) analysis by incorporating elements of state-dependent pricing and shows that Markov-perfect discretionary equilibrium is unique. Those papers assume that monetary policy is conducted with a money supply instrument. In contrast, Dotsey and Hornstein (2007) show that with an interest rate instrument there is a unique Markov-perfect discretionary equilibrium in a Taylor model with two-period pricing.

One of our central concerns is whether the Calvo model also generates multiple discretionary equilibria. For the examples we look at, the central element generating multiplicity in the Taylor model is not present in the Calvo model. The policy problem of choosing the money supply to maximize welfare involves a tradeoff between the markup and a relative price distortion. In the Taylor model, discretionary policy in the next period will adjust the money supply proportionally with the price set by firms in the present period. For price-setting firms in the current period, the expectation of this future policy response generates a convex best-response function with multiple fixed points. In the Calvo model, the future policymaker's trade-off is influenced by the inherited relative price distortion. The larger that distortion, the less the future money supply accommodates increases in the current price level. This generates a concave price-setting best response function with a unique fixed point. The complementarity necessary for generating multiple equilibria is broken because (i) the future policymaker chooses not to accommodate arbitrarily high prices set by firms today as these raise the relative price distortion, and (ii) given the existence of many cohorts of predetermined prices, arbitrarily high prices set by firms today have only a small effect on the current price level.

We find that discretionary equilibrium involves a steady state inflation rate of greater than five percent under a standard calibration. This suggests that the zero inflation approximation is inappropriate in the absence of a fiscal scheme to eliminate the monopoly distortion. Furthermore, the presence of an endogenous state variable leads to a gradual transition of inflation if the inherited relative price distortion deviates from its steady state value. In the approximation around zero inflation the state variable is absent and inflation jumps immediately to its steady state. In Yun's (2005) analysis of optimal monetary policy in a nonlinear Calvo model, the elimination of the monopoly distortion via fiscal policy implies that in the steady state inflation is zero and the relative price distortion also vanishes. The transition dynamics is similarly affected by the state variable.

The paper proceeds as follows. Section 2 contains a description of the Calvo and Taylor models – the latter with two-period pricing. At several points in the paper we compare the Calvo and Taylor models, and there is little cost to laying out a general framework that encompasses both models. Section 3 concerns the state variables, showing that in the Taylor model there are no real state variables whereas in the Calvo model the relevant state can be summarized by the past relative price distortion. Section 4 defines a discretionary equilibrium in the Calvo model. Section 5 presents our numerical results, and section 6 compares those results to the Taylor model, emphasizing the issue of (lack of) multiplicity. Section 7 concludes.

2 Two Sticky Price Models

We describe two models in this section. Common elements are a representative household that values consumption and dislikes supplying labor, a competitive labor market, a continuum of monopolistically competitive firms producing goods for which households have constant elasticity of substitution preferences, a constant velocity money demand equation, and a monetary authority that chooses the money supply. The models differ in their assumptions about price setting, which is treated as exogenous. In the *Calvo* model, each firm faces a constant probability of price adjustment. In the *Taylor* model each firm sets its price for two periods. In both models we assume the distribution of price adjustment is stationary, i.e. the fraction of firms adjusting is constant over time.

2.1 Households

There is a large number of identical, infinitely lived households. They act as price-takers in labor and product markets, and they own shares in the economy's monopolistically competitive goods-producing firms. Households' preferences over consumption (c_t) and labor input (n_t) are given by

$$\sum_{j=0}^{\infty} \beta^j \left[\ln(c_{t+j}) + \chi\left(1 - n_{t+j}\right)\right], \quad \beta \in (0,1)\,,$$

where consumption is taken to be the Dixit-Stiglitz aggregate of a continuum of differentiated goods

$$c_t = \left[\int_0^1 c_t(z)^{\frac{\varepsilon-1}{\varepsilon}} dz\right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad \varepsilon > 1. \tag{2.1}$$

The consumer's flow budget constraint is

$$P_t w_t n_t + R_{t-1} B_{t-1} \geq P_t c_t + B_t + \int_0^1 d_t\left(z\right) dz,$$

where w_t is the real wage, R_t is the one-period gross nominal interest rate, B_t is the quantity of one-period nominal bonds purchased in period t, $d_t(z)$ is the dividend paid by firm z, and P_t is the nominal price of a unit of consumption. The aggregator (2.1) implies that the price index P_t is given by

$$P_t = \left[\int_0^1 P_t(z)^{1-\varepsilon} dz\right]^{\frac{1}{1-\varepsilon}}.$$

From the consumer's intratemporal problem, we have the efficiency condition

$$\chi = \frac{w_t}{c_t}, \tag{2.2}$$

and from the intertemporal problem we have

$$\frac{1}{c_t} = \beta R_t \left(\frac{1}{c_{t+1}} \cdot \frac{1}{\pi_{t+1}}\right),$$

where we have introduced the variable π_t to denote the gross inflation rate between periods $t-1$ and t. That is

$$\pi_t \equiv \frac{P_t}{P_{t-1}}.$$

We assume that households hold money equal to the quantity of nominal consumption:

$$M_t = P_t c_t. \tag{2.3}$$

Finally, the aggregator (2.1) implies the demand functions for each good

$$c_t(z) = \left(\frac{P_t(z)}{P_t}\right)^{-\varepsilon} c_t, \tag{2.4}$$

where $P_t(z)$ is the price of good z.

2.2 Firms

Each firm z produces using a technology that is linear in labor, the only input, with a constant level of productivity that is normalized to unity:

$$y_t(z) = n_t(z),$$

where $y_t(z)$ is firm's $z's$ output in period t. The nominal profits in period t of a firm charging price X_t are

$$d(X_t; P_t, c_t, w_t) = X_t \left(\frac{X_t}{P_t}\right)^{-\varepsilon} c_t - P_t w_t \left(\frac{X_t}{P_t}\right)^{-\varepsilon} c_t.$$

When a firm adjusts its price, it chooses X_t to maximize the present discounted value of profits, which we denote V_t. In the Calvo model, each firm adjusts its price with constant probability η in any period, whereas in the Taylor model each firm adjusts its price every two periods. In both models, we assume that there is a stationary distribution of firms according to time since last price adjustment. This assumption means that in the Calvo model in any period t a fraction η of firms adjusts their prices, whereas in the Taylor model in any period t one half of the firms adjust their prices.

In the Calvo model the value of a firm upon adjustment is given by

$$V_t = \max_{X_t} \left\{ \sum_{j=0}^{\infty} Q_{t,t+j} (1-\eta)^j d(X_t; P_{t+j}, c_{t+j}, w_{t+j}) \right\}, \tag{2.5}$$

where $Q_{t,t+j}$ is the j-period ahead discount factor for nominal cash flows. With households owning firms, $Q_{t,t+j}$ is determined by the sequence of one-period nominal interest rates,[3]

$$Q_{t,t+j} = \frac{1}{\prod_{k=1}^{j} R_{t-1+k}} = \beta^j \left(\frac{P_t}{P_{t+j}}\right) \left(\frac{c_t}{c_{t+j}}\right).$$

The factor $(1-\eta)^j$ is the probability that a price set in period t will remain in effect in period $t+j$. Note that V_t is the present value of profits associated with charging the price X_t. When the firm has the opportunity to readjust, it will reoptimize, and thus those states are not relevant for determining the optimal price. The optimal price is determined by differentiating (2.5) with respect to X_t. We will denote the profit-maximizing value of X_t by $P_{0,t}$ and we will denote by $p_{0,t}$ the price $P_{0,t}$ normalized by the previous period's price level, which serves as an index of the predetermined prices in period t:

$$p_{0,t} = \frac{P_{0,t}}{P_{t-1}}.$$

[3] We adopt the convention that $\prod_{k=1}^{0} R_{t-1+k} = 1$.

Thus, we write the first order condition from (2.5) as,

$$\frac{P_{0,t}}{P_t} = \frac{p_{0,t}}{\pi_t} = \left(\frac{\varepsilon}{\varepsilon-1}\right) \frac{\sum_{j=0}^{\infty} (1-\eta)^j \beta^j \left(\frac{P_{t+j}}{P_t}\right)^{\varepsilon} w_{t+j}}{\sum_{j=0}^{\infty} (1-\eta)^j \beta^j \left(\frac{P_{t+j}}{P_t}\right)^{\varepsilon-1}}. \tag{2.6}$$

In the Taylor model, the value of a firm upon adjustment is given by

$$\tilde{V}_t = \max_{X_t} \left\{ d\left(X_t; P_t, c_t, w_t\right) + Q_{t,t+1} d\left(X_t; P_{t+1}, c_{t+1}, w_{t+1}\right) \right\},$$

and the optimal price satisfies the first order condition,

$$\frac{P_{0,t}}{P_t} = \left(\frac{\varepsilon}{\varepsilon-1}\right) \cdot \frac{w_t + \beta \left(\frac{P_{t+1}}{P_t}\right)^{\varepsilon} w_{t+1}}{1 + \beta \left(\frac{P_{t+1}}{P_t}\right)^{\varepsilon-1}}.$$

Whereas in the Calvo model the index of predetermined prices was given by P_{t-1}, in the Taylor model there is just one predetermined price, $P_{0,t-1}$. Normalizing the optimal price and the price index by $P_{0,t-1}$ and using the definitions $\tilde{p}_{0,t} \equiv P_{0,t}/P_{0,t-1}$ and $p_t \equiv P_t/P_{0,t-1}$, we have

$$\frac{\tilde{p}_{0,t}}{p_t} = \left(\frac{\varepsilon}{\varepsilon-1}\right) \cdot \frac{w_t + \beta \left(\frac{p_{t+1}}{p_t}\tilde{p}_{0,t}\right)^{\varepsilon} w_{t+1}}{1 + \beta \left(\frac{p_{t+1}}{p_t}\tilde{p}_{0,t}\right)^{\varepsilon-1}}. \tag{2.7}$$

With the constant elasticity aggregator (2.1) a firm's optimal markup of price over marginal cost is constant and equal to $\varepsilon/\left(\varepsilon-1\right)$. Because the firm cannot adjust its price each period, if the wage (here equal to marginal cost) or the inflation rate are not constant then the firm's markup will vary over time. The optimal pricing equations (2.6) and (2.7) indicate that the firm chooses a constant markup over an appropriately defined weighted average of current and future marginal cost. Note that economy-wide average markup is simply the inverse of the real wage, because firm-level productivity is assumed constant and equal to one.

2.3 Market clearing

Goods market clearing requires that the consumption demand for each individual good is equal to the output of that good:

$$c_t\left(z\right) = n_t\left(z\right) \; \forall z, \tag{2.8}$$

and labor market clearing requires that the labor input into the production of all goods equal the supply of labor by households:

$$\int_0^1 n_t(z)dz = n_t.$$

In the Calvo model, the labor market clearing condition is

$$n_t = \sum_{j=0}^{\infty} \eta \left(1-\eta\right)^j n_{j,t},$$

where $n_{j,t}$ is the labor input employed in period t by a firm that set its price in period $t-j$. Combining this expression with the goods market clearing condition (2.8), then using the demand curves (2.4) for each good, and dividing the expression by the consumption aggregator yields

$$\frac{n_t}{c_t} = \sum_{j=0}^{\infty} \eta \left(1-\eta\right)^j \left(\frac{P_{0,t-j}}{P_t}\right)^{-\varepsilon},$$

which can be written recursively as

$$\frac{n_t}{c_t} = \eta \pi_t^{\varepsilon} \left(p_{0,t}^{-\varepsilon} + \left(\frac{1-\eta}{\eta}\right) \frac{n_{t-1}}{c_{t-1}} \right). \tag{2.9}$$

Analogously, in the Taylor model the market clearing conditions give us

$$\begin{aligned} n_t &= \frac{1}{2} \sum_{j=0}^{1} n_{j,t} \\ \frac{n_t}{c_t} &= \frac{1}{2} p_t^{\varepsilon} \left(\left(\tilde{p}_{0,t}\right)^{-\varepsilon} + 1 \right). \end{aligned} \tag{2.10}$$

It is instructive to compare the market clearing conditions in the Calvo and Taylor models, (2.9) and (2.10). In both models, the average product of labor (or its inverse) is related to relative prices. In the Calvo model however, the lagged average product of labor also shows up in this expression. This will be the source of the single real state variable in the Calvo model.

2.4 Monetary Authority

The monetary authority chooses the money supply, M_t. In a discretionary equilibrium the money supply will be chosen each period to maximize present value welfare.

In both the Calvo and Taylor models, we assume the sequence of actions within a period is as follows:

1. Predetermined prices ($P_{0,t-j}$, $j > 0$) are known at the beginning of the period.

2. The monetary authority chooses the money supply.

3. Firms that adjust in the current period set their prices, and simultaneously all other period-t variables are determined.

3 State Variables

Because we are interested in studying Markov-perfect equilibria (MPE) with discretionary monetary policy, it is important to establish what are the relevant state variables in the Taylor and Calvo models. It is clear that in the Taylor model there is one predetermined nominal price ($P_{0,t-1}$), whereas in the Calvo model there are an infinite number of predetermined nominal prices ($P_{0,t-j}$, $j = 1, 2, ...$). However, for the MPE, a state variable is relevant only if it affects the monetary authority's set of feasible real outcomes. We now show that according to this criterion there is a single state variable in the Calvo model and there are no state variables in the Taylor model.

3.1 Calvo model

In the Calvo model the equations describing private sector equilibrium are as follows. The labor supply equation (2.2) and money demand equation (2.3) are unchanged, although we write the money demand equation normalizing by the lagged price level:

$$m_t \equiv \frac{M_t}{P_{t-1}} = \pi_t c_t. \tag{3.1}$$

The price index is an infinite sum,

$$P_t = \left(\sum_{j=0}^{\infty} \eta \left(1-\eta\right)^j P_{0,t-j}^{1-\varepsilon} \right)^{1/(1-\varepsilon)},$$

but it can be simplified, first writing it recursively,

$$P_t = \left(\eta P_{0,t}^{1-\varepsilon} + \left(1-\eta\right) P_{t-1}^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}},$$

and then dividing by the lagged price level:

$$\pi_t = \left(\eta p_{0,t}^{1-\varepsilon} + \left(1-\eta\right) \right)^{\frac{1}{1-\varepsilon}}. \tag{3.2}$$

No predetermined variables show up in this transformed version of the price index.

The optimal pricing condition and the market clearing condition start out as infinite sums. The optimal pricing condition (2.6) can be written recursively by defining two new variables, N_t and D_t :

$$p_{0,t} = \left(\frac{\varepsilon}{\varepsilon - 1} \right) \frac{N_t}{D_t} \tag{3.3}$$

$$N_t = \pi_t^{\varepsilon} \left(w_t + \beta \left(1-\eta\right) N_{t+1} \right) \tag{3.4}$$

$$D_t = \pi_t^{\varepsilon - 1} \left(1 + \beta \left(1-\eta\right) D_{t+1} \right). \tag{3.5}$$

The market clearing condition has already been written recursively in (2.9).

The equations (2.2), (2.9), (3.1)−(3.5) and future policy implicitly determine the values for w_t, c_t, n_t, π_t, $p_{0,t}$, N_t and D_t attainable by the current period's monetary authority (choosing m_t). Predetermined variables appear in the private sector equilibrium conditions. Specifically, lagged n and c appear in the market clearing condition (2.9). Because it is only the ratio of those lagged variables that matters, we can reduce the number of state variables to one by defining a new variable Δ which will serve as the single state variable:

$$\Delta_t \equiv n_{t-1}/c_{t-1}. \tag{3.6}$$

We will sometimes refer to Δ_t as the lagged relative price distortion. The market clearing condition can now be written as

$$\Delta_{t+1} = \eta \pi_t^{\varepsilon} \left(p_{0,t}^{-\varepsilon} + \left(\frac{1-\eta}{\eta} \right) \Delta_t \right). \tag{3.7}$$

In an MPE of the Calvo model, the normalized money supply will be a function of the state, Δ_t.

3.2 Taylor model

In the Taylor model an analogous set of five equations characterizes private sector equilibrium. Labor supply is given by equation (2.2). Money demand (2.3) is normalized by the lagged optimal price instead of by the lagged price level

$$\tilde{m}_t \equiv \frac{M_t}{P_{0,t-1}} = p_t c_t. \tag{3.8}$$

We eliminate the predetermined variable from the price index,

$$P_t = \left(\frac{1}{2} P_{0,t}^{1-\varepsilon} + \frac{1}{2} P_{0,t-1}^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}},$$

by dividing by the lagged optimal price:

$$p_t = \left(\frac{1}{2} \tilde{p}_{0,t}^{1-\varepsilon} + \frac{1}{2} \right)^{\frac{1}{1-\varepsilon}}. \tag{3.9}$$

Optimal pricing is given by (2.7) and market clearing satisfies (2.10).

The five equations (2.2), (2.7), (2.10), (3.8) and (3.9), together with the behavior of future policymakers, implicitly define the set of feasible values for w_t, c_t, n_t, p_t and $\tilde{p}_{0,t}$ attainable by the current period's monetary authority. The current period monetary authority chooses the money supply, or equivalently $\tilde{m}_t$, the money supply normalized by the predetermined price $P_{0,t-1}$. This normalization is incorporated in (3.8). Unlike the Calvo model, no predetermined variables appear in the private sector equilibrium conditions, thus no state variables constrain the monetary authority in an MPE. The lagged optimal price $P_{0,t-1}$ matters for the levels of nominal variables, but is irrelevant for the determination of real allocations. Of course, one could imagine some monetary policy rules that would make real allocations depend on $P_{0,t-1}$ by making $\tilde{m}_t$ depend on $P_{0,t-1}$, but that will not occur in an MPE, because it would mean introducing an extraneous state variable. In an MPE of the Taylor model, the normalized money supply $\tilde{m}_t$ will be constant.

4 Discretionary Equilibrium in Calvo Model

Discretionary equilibrium is a mapping from the state to the (normalized) money supply such that, if private agents and the current-period monetary authority believe that all future periods will be described by a stationary equilibrium governed by that mapping, then the current period monetary authority chooses the same mapping from the state to the money supply.

4.1 Equilibrium for arbitrary monetary policy

As a preliminary to studying discretionary equilibrium, it is useful to consider stationary equilibrium for arbitrary monetary policy – that is, for arbitrary functions $m = \Gamma(\Delta)$. To describe equilibrium for arbitrary policy we use recursive notation, eliminating time subscripts and using a prime to denote a variable in the next period. The nine variables which need to be determined in equilibrium are N, D, p_0, π, Δ', w, c, m and n, and the nine equations are (i and ii) the laws of motion for N (3.4) and for D (3.5); (iii) the optimal pricing condition (3.3); (iv) the price index (3.2); (v) the market clearing condition or law of motion for the relative price distortion (3.7); (vi) the labor supply (2.2); (vii) the money demand (3.1); (viii) the policy rule $m = \Gamma(\Delta)$; and (ix) the definition of the relative price distortion (3.6).

A stationary equilibrium can be expressed as two functions of the endogenous state variable Δ. The two functions $N(\Delta)$ and $D(\Delta)$ must satisfy the two functional equations

$$N(\Delta) = \pi^{\varepsilon}\left[w + \beta(1-\eta) N(\Delta')\right], \tag{4.1}$$
$$D(\Delta) = \pi^{\varepsilon-1}\left[1 + \beta(1-\eta) D(\Delta')\right]. \tag{4.2}$$

The other variables can be substituted out as functions of N, D and Δ:

$$p_0 = \left(\frac{\varepsilon}{\varepsilon-1}\right)\cdot\frac{N(\Delta)}{D(\Delta)}, \tag{4.3}$$
$$\pi = \left(\eta p_0^{1-\varepsilon} + (1-\eta)\right)^{1/(1-\varepsilon)} \tag{4.4}$$
$$\Delta' = \pi^{\varepsilon}\left(\eta p_0^{-\varepsilon} + (1-\eta)\Delta\right), \tag{4.5}$$
$$w = \chi c, \tag{4.6}$$
$$c = m/\pi, \tag{4.7}$$
$$m = \Gamma(\Delta), \tag{4.8}$$
$$n = \Delta' c. \tag{4.9}$$

4.2 Discretionary equilibrium defined

A discretionary equilibrium is a mapping from the state to the money supply, $\Gamma^*(\Delta)$, such that, if the current-period policymaker and the current-period private agents take as given that all future periods will be described by a stationary equilibrium governed by $\Gamma^*(\Delta)$, then the current period monetary authority chooses $m = \Gamma^*(\Delta)$ for every Δ.

More formally, a discretionary equilibrium is a policy function $\Gamma^*(\Delta)$ and a value function $v^*(\Delta)$ that satisfy

$$\begin{aligned} v^*(\Delta) &= \max_m \{\ln c + \chi(1-n) + \beta v(\Delta')\} \\ \Gamma^*(\Delta) &= \arg\max_m \{\ln c + \chi(1-n) + \beta v(\Delta')\} \end{aligned} \tag{4.10}$$

when $v() = v^*()$. The maximization is subject to the price index (4.4), market clearing (4.5), labor supply (4.6), money demand (4.7), the definition of the state variable (4.9), and optimal pricing by adjusting firms,

$$\begin{aligned} p_0 &= \left(\frac{\varepsilon}{\varepsilon-1}\right) \cdot \frac{\widetilde{N}}{\widetilde{D}} \\ \widetilde{N} &= \pi^{\varepsilon}\left[\chi\frac{m}{\pi} + \beta(1-\eta)N(\Delta')\right] \\ \widetilde{D} &= \pi^{\varepsilon-1}\left[1 + \beta(1-\eta)D(\Delta')\right], \end{aligned}$$

where the functions $N(\Delta)$ and $D(\Delta)$ satisfy (4.1) and (4.2) in the stationary equilibrium associated with $\Gamma^*(\Delta)$.

4.3 Computing a discretionary equilibrium

Our computational method involves selecting a grid of N points for the state variable ($\Delta_n, n = 1, 2, ..., N$) and then searching for values of v_n^* and Γ_n^*, $n = 1, ..., N$ that solve (4.10) at the grid points Δ_n. The value function and the expressions for N and D require evaluating functions away from the grid points, and we use linear interpolation to do this. As an initial guess we use the discretionary equilibrium for the static model – the final period of a finite horizon model – and then solve the optimization problem (4.10). If the value function and policy function that solve the optimization problem are identical to our guesses, then we have found an equilibrium. If not, we update the starting values by pushing out our initial guess one period into the future, and assuming the one-period ahead policy and value functions are given by our "temporary equilibrium."

5 Properties of Discretionary Equilibrium

There are three levels to a complete description of a discretionary equilibrium. At the highest level is the equilibrium transition function for the state variable ($\Delta'(\Delta)$), and the associated policy function ($m = \Gamma^*(\Delta)$), value function ($v^*(\Delta)$) and equilibrium functions for all other endogenous variables as functions of the state. Next is the objective function for the policymaker: for a given value of the state variable, how does welfare vary with the policy instrument m, and what are the trade-offs that drive the shape of the objective function? Finally, for given values of the state variable and the policy instrument, what is the nature of the private sector equilibrium? In this section we will concentrate on the first two levels, equilibrium as a function of the state and the policymaker's objective function. Comparing the Calvo and Taylor results in section 6 will take us into the details of the private sector equilibrium.

The presence of a state variable in the Calvo model means that our results are primarily numerical. Unless otherwise stated we use the following parameterization: $\varepsilon = 10, \beta = 0.99, \eta = 0.5, \chi = 4.5$. Some of these parameters are typical values used in the applied monetary policy literature. With $\varepsilon = 10$ the steady state markup is approximately 11 percent at low rates of inflation. Prices adjust with probability $\eta = 0.5$, which means that the expected duration of a price is two quarters. With $\chi = 4.5$, agents spend about twenty percent of their unit time endowment working. With $\beta = 0.99$ in our quarterly model the annualized real interest rate is 4.1 percent.

5.1 Equilibrium as a function of the state

Figure 1.A plots the transition function for the state variable as well as the function mapping from the state to the inflation rate in a discretionary equilibrium. The first thing to note about the figure is that there is a unique steady state inflation rate of approximately 5.4 percent annually. Two natural benchmarks against which to compare the steady state of the discretionary equilibrium are the inflation rate with highest steady state welfare and the inflation rate in the long run under optimal policy with commitment. Following King and Wolman (1999), we refer to these benchmarks as the golden rule and the modified golden rule respectively. For our parameterization, the golden rule inflation rate is just barely positive (less than one tenth of a percent) and the modified golden rule inflation rate is zero. There is no

widespread agreement on the value of η in the monetary policy literature. Figure 2 therefore displays the steady state annual inflation rate for higher degrees of price stickiness, ranging from our baseline value of $\eta = 0.5$ to $\eta = 0.3$, in which case a firm's price remains unchanged for three and one third quarters. In that case the steady state inflation rate takes it highest value of 10.5 percent.[4]

A second feature of Figure 1.A is that the dynamics for the state variable display monotonic convergence to the steady state. This means that a policymaker inheriting a relative price distortion that is large (small) relative to steady state finds it optimal to bequeath a smaller (larger) relative price distortion to her successor. Together with the monotone downward-sloping equilibrium function for inflation, this implies that the inflation dynamics in the transition from a steady state with a larger (smaller) relative price distortion and inflation rate involves an initial decrease (increase) of inflation and a subsequent gradual rise (fall) to the steady state.[5]

Figure 1.B displays the policy variable m and welfare as function of the state variable in the discretionary equilibrium (m is indicated on the left scale and welfare on the right scale). Both functions are downward sloping. Intuition for the welfare function's downward slope is straightforward. From (3.6) we have that the current relative price distortion represents average productivity. But the current relative price distortion is also a summary statistic for the dispersion in relative prices. The higher is the lagged relative price distortion, the higher is the inherited dispersion in relative prices, and through (3.7) this contributes to a higher dispersion in current relative prices. Higher dispersion in current relative prices in turn hurts current productivity, reducing welfare.

It is less straightforward to understand the downward sloping policy function, $m = \Gamma^* (\Delta)$. At a superficial level, it seems consistent with the state transition function for $\Gamma^* ()$ to be downward sloping: if equilibrium involves the relative price distortion declining from a high level, then a large lagged relative price distortion ought to be met with a relatively low normalized money supply, so that newly adjusting firms do not exacerbate the relative

[4]The solutions underlying Figure 2 were generated with varying fineness of the grid for the state variable. We are still working on solutions based on a finer grid.

[5]Yun's (2005) analysis displays similar transition dynamics of inflation. But in his model, the steady state inflation rate under optimal policy is zero, so the transition from a steady state with positive inflation inevitably involves a period of deflation.

price distortion.[6] However, in order to develop the intuition for $\Gamma^*(\Delta)$ more fully it is necessary to examine the nature of the policy problem in equilibrium.

5.2 Policymaker's objective function

Figure 3 displays the policymaker's objective function (Panel A) and the current period component of the objective function (Panel B) for two values of the state variable (1 and 1.005). Both panels display functions that are concave, and the unique maximum is achieved with lower values of m for the higher value of the state. We do not plot the future component of value, $\beta v(\Delta')$ in (4.10), but it is decreasing in m for all values of Δ. From Figure 3 then, the fact that m is a decreasing function of Δ seems to be associated with the state variable's influence on the current utility component of welfare. As discussed in King and Wolman (1999, 2004), real effects of monetary policy in models such as this one work through the relative price distortion (Δ') and the average markup of price over marginal cost ($1/w$ here). Thus, to understand why the current component of the objective function is maximized with a lower m the higher is Δ requires us to look at the behavior of these two distortions.

Figure 4 plots the markup distortion (Panel A) and the relative price distortion (Panel B) as a function of m for the same two values of the state variable. In both cases higher values of m correspond to a lower markup and a higher relative price distortion. This feature is the essential short-run policy trade-off in the Calvo or Taylor model: a higher money supply will bring down the markup at the cost of increasing the relative price distortion. From Figures 3 and 4 it is apparent that as the state variable increases, the trade-off shifts in favor of the relative price distortion. That is, the policymaker chooses lower m at higher values of Δ because the decrease in the markup that would come from holding m fixed at higher Δ is more than offset by welfare costs of a higher relative price distortion Δ'.

What is the intuition for increased sensitivity of the relative price distortion to m at higher levels of inherited relative price dispersion (Δ)? Although we cannot explicitly solve for the relationship between the relative price distortion and the money supply, we can study the relationship between the

[6] Note that this intuition relies on the inflation rate being generally positive: if the inflation rate were generally negative, then reducing the relative price distortion over time would mean making firms set relatively high prices.

relative price distortion and the relative price chosen by adjusting firms. Assuming (correctly) a positive relationship between equilibrium p_0 and m, this relationship is informative for understanding why the relative price distortion can be viewed as driving the shape of the policy function.

Combining the market clearing condition (4.5) with the transformed price index (4.4), we have

$$\Delta' = \frac{\eta p_0^{-\varepsilon} + (1-\eta)\,\Delta}{\left((1-\eta) + \eta p_0^{1-\varepsilon}\right)^{\varepsilon/(\varepsilon-1)}}. \tag{5.1}$$

From this expression it follows that the sensitivity of the relative price distortion to the relative price of adjusters is increasing in the state:

$$\frac{\partial^2 \Delta'}{\partial p_0 \partial \Delta} = \frac{\varepsilon\eta\,(1-\eta)\,p_0^{-\varepsilon}}{\left((1-\eta) + \eta p_0^{1-\varepsilon}\right)^{1+[\varepsilon/(\varepsilon-1)]}} > 0. \tag{5.2}$$

Figure 5 illustrates this relationship: the current relative price distortion is a locally convex function of the relative price set by adjusting firms.[7] If there is no inherited relative price dispersion ($\Delta = 1$) then the relative price distortion is minimized at $p_0 = 1$, whereas for higher inherited dispersion the relative price distortion is minimized at a lower value of p_0. As (5.2) states, higher Δ also corresponds to a steeper relative price distortion with respect to Δ. Summarizing our argument then: as the state variable increases, the current policymaker would incur increasing welfare losses due to relative price distortions if she did not react by choosing m so that price setters set a *lower* relative price. We have not plotted the relationship between m and p_0, but in a discretionary equilibrium it is positive and nearly linear. So this reasoning leads to a policy that sets m as a decreasing function of Δ.

6 Properties of Private Sector Equilibrium

Our discussion thus far has involved explaining the nature of the policy problem in equilibrium and how the state variable shifts the policy problem and thus its solution. In this section we turn to the private sector equilibrium.

[7]The relative price distortion as a function of p_0 becomes flat and thus concave at high values of p_0; for high enough p_0 customers have negligible demand for the goods sold by adjusters, and additional price increases have no effect on the relative price distortion.

Understanding the private sector equilibrium is central to understanding why we do not find multiple equilibria, in contrast to what King and Wolman (2004) find for the Taylor model.

6.1 Background results from Taylor model

We have seen above that in a discretionary equilibrium of the Taylor model $\tilde{m}$ will be constant. King and Wolman (2004) use a price-setting firm's best response function to study discretionary equilibria in the Taylor model. The best response function describes an individual firm's optimal price as a function of the price set by other adjusting firms. That function is the optimal pricing condition (2.7) rewritten so that the right hand side is in terms of current and future $\tilde{m}$ and current and future $\tilde{p}_0$. To derive the best response function, first use the labor supply equation (2.2) to write the real wage in terms of consumption in the optimal pricing condition. Next use the money demand equation (3.8) to eliminate consumption. And finally use the price index (3.9) to write p as a function of $\tilde{p}_0$, resulting in the best response function,

$$\tilde{p}_0 = \left(\frac{\varepsilon\chi}{\varepsilon - 1}\right) \cdot \left(\left(1-\theta'\right)\tilde{m} + \theta'\tilde{m}'\tilde{p}_0\right), \tag{6.1}$$

where

$$\theta' \equiv \frac{\beta\left(\frac{p\left(\tilde{p}_0'\right)}{p(\tilde{p}_0)}\tilde{p}_0\right)^{\varepsilon-1}}{1+\beta\left(\frac{p\left(\tilde{p}_0'\right)}{p(\tilde{p}_0)}\tilde{p}_0\right)^{\varepsilon-1}} \tag{6.2}$$

Generically, for any value of $\tilde{m}$, King and Wolman show that for fixed $\tilde{m}'$ and $\tilde{p}_0'$ the best response function is monotonically increasing and strictly convex with two fixed points or no fixed points. The presence of two fixed points for arbitrary $\tilde{m}$ means that there are multiple discretionary equilibria, indexed by the distribution over the two fixed points of the best response function (these fixed points vary with the distribution). In a discretionary equilibrium there are endogenous fluctuations over these two fixed points.

King and Wolman stress that the complementarity necessary for multiple fixed points is associated with the fact that under discretion, the policymaker in the next period is certain to raise the nominal money supply proportionally with the price set by firms in the current period. An individual firm in the current period responds positively to the price set by other firms, in order to avoid being stuck with high demand and a nominal price that is low relative to nominal costs in the future. In the Taylor model, this effect is relatively weak at low values of p_0 and relatively strong at high values of p_0. Another way to view the complementarity is between future policy and expected future policy: if firms expect a higher nominal money supply in the future, they will set a higher price today, and the future policymaker will accommodate with a higher money supply.

6.2 Comparing Calvo and Taylor

Our computational approach with the Calvo model has led to finding a single MPE, in which allocations are determined by fundamentals alone. In section 5 we discussed some of the properties of the equilibrium for one set of parameters. Although we have not proved that the equilibrium is unique, in the example studied in this paper we have found no evidence of multiplicity. This is in stark contrast to the Taylor model with two period price setting, in which King and Wolman (2004) *proved* the existence of multiple equilibria, with sunspot fluctuations in most equilibria. To understand why multiplicity is less apparent in the Calvo model, we turn to the object King and Wolman use to study multiplicity, the best response function for price-setting firms.[8]

As stated above, the best response function in the Taylor two-period-pricing model is upward sloping, strictly convex and generically has either two fixed points or no fixed points. In Figure 6 we plot a typical best response function in a discretionary equilibrium of the Calvo model, using the same parameters as above. It is concave with a unique fixed point. Equation (5.1) implies that for low values of p_0 the future state is decreasing in p_0, holding fixed the current state:

$$\frac{\partial \Delta'}{\partial p_0} = \frac{\varepsilon \eta (1-\eta) p_0^{-\varepsilon-1}}{\left((1-\eta) + \eta p_0^{1-\varepsilon}\right)^{1+[\varepsilon/(\varepsilon-1)]}} (\Delta p_0 - 1).$$

Therefore, given that the equilibrium policy function is decreasing, future m is increasing in p_0. Other things equal a higher future normalized money supply makes an individual firm want to set a higher price, so the best response function is increasing for low p_0. At high values of p_0 this relationship is reversed: increases in p_0 raise the future state, and the policymaker would

[8]We write "multiplicity is less apparent" because we have not proved uniqueness.

respond by reducing future m, which would reduce the firm's optimal price today.

Complementary intuition comes from imagining (counterfactually) that the future policymaker set a fixed m, raising the nominal money supply in proportion to the index of predetermined prices (the lagged price level). For low p_0, the price level is very sensitive to p_0 and hence future nominal money would rise steeply with p_0 with a corresponding effect on an individual firm's optimal price. For high p_0 the price level eventually becomes fixed with respect to further increases in p_0 and thus the future money supply does not respond, making an individual firm indifferent to the higher prices set by other firms.

Both of our attempts at intuition rely on the fact that there are many cohorts of firms with predetermined prices in the Calvo model. The first attempt relies on the presence of a real state variable in the Calvo model, whereas the second attempt relies on the fact that tomorrow's nominal state is not entirely determined by the actions of current period price setters. In the Taylor model with two-period pricing, there is no real state variable, and tomorrow's nominal state is identical to the price set by firms today.

7 Conclusion

The Calvo model linearized around a zero inflation steady state yields the New Keynesian Phillips curve, which has become the leading framework for applied monetary policy analysis. While there have been numerous analyses of discretionary monetary policy using the NKPC, little attention has been devoted to understanding discretionary equilibrium in the underlying (non-linear) Calvo model. This paper has aimed to further such understanding. We have found that the complementarity inherent in the Taylor model with two-period pricing (King and Wolman, 2004) does not arise in the Calvo model when firms set their prices for two periods on average. Discretionary equilibrium involves a steady state inflation rate of greater than five percent, suggesting that the zero inflation approximation is inappropriate in the absence of a fiscal scheme to eliminate the monopoly distortion.

References

[1] Adam, K. and R.M. Billi (2007), "Discretionary Monetary Policy and the Zero Lower Bound on Nominal Interest Rates", *Journal of Monetary Economics* 54, 728-752.

[2] Calvo, G.A. (1983), "Staggered Prices in a Utility-Maximizing Framework", *Journal of Monetary Economics* 12, 383-398.

[3] Clarida, R., J. Gali and M. Gertler (1999). "The Science of Monetary Policy: A New Keynesian Perspective", *Journal of Economic Literature* 37, 1661-1707.

[4] Dotsey, M. and A. Hornstein (2007). "Interest Rate Versus Money Supply Instruments: On the Implementation of Markov-Perfect Optimal Monetary Policy", Federal Reserve Bank of Philadelphia, Working Paper 07-27.

[5] Khan, A., R.G. King and A.L. Wolman (2001), "The Pitfalls of Discretionary Monetary Policy", Working Paper 01-16, Federal Reserve Bank of Philadelphia.

[6] King, R.G. and A.L. Wolman (2004), "Monetary Discretion, Pricing Complementarity, and Dynamic Multiple Equilibria", *Quarterly Journal of Economics* 119, 1513-1553.

[7] King, R.G. and A.L. Wolman (1999), "What Should the Monetary Authority Do When Prices Are Sticky?" in Monetary Policy Rules, J.B. Taylor ed., University of Chicago Press, 1999, 349-398.

[8] King, R.G. and A.L. Wolman (1996), "Inflation Targeting in a St. Louis Model of the 21st Century", Proceedings, Federal Reserve Bank of St. Louis, 83-107.

[9] Siu, H.E. (2008), "Time Consistent Monetary Policy with Endogenous Price Rigidity", *Journal of Economic Theory* 138, 184-210.

[10] Woodford, M., *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press, 2003.

[11] Yun, T. (2005), "Optimal Monetary Policy with Relative Price Distortions," *American Economic Review* 95, 89-108

[12] Yun, T. (1996), "Nominal Price Rigidity, Money Supply Endogeneity, and Business Cycles", *Journal of Monetary Economics* 37, 345-370.

Figure 1: Equilibrium as a function of the state

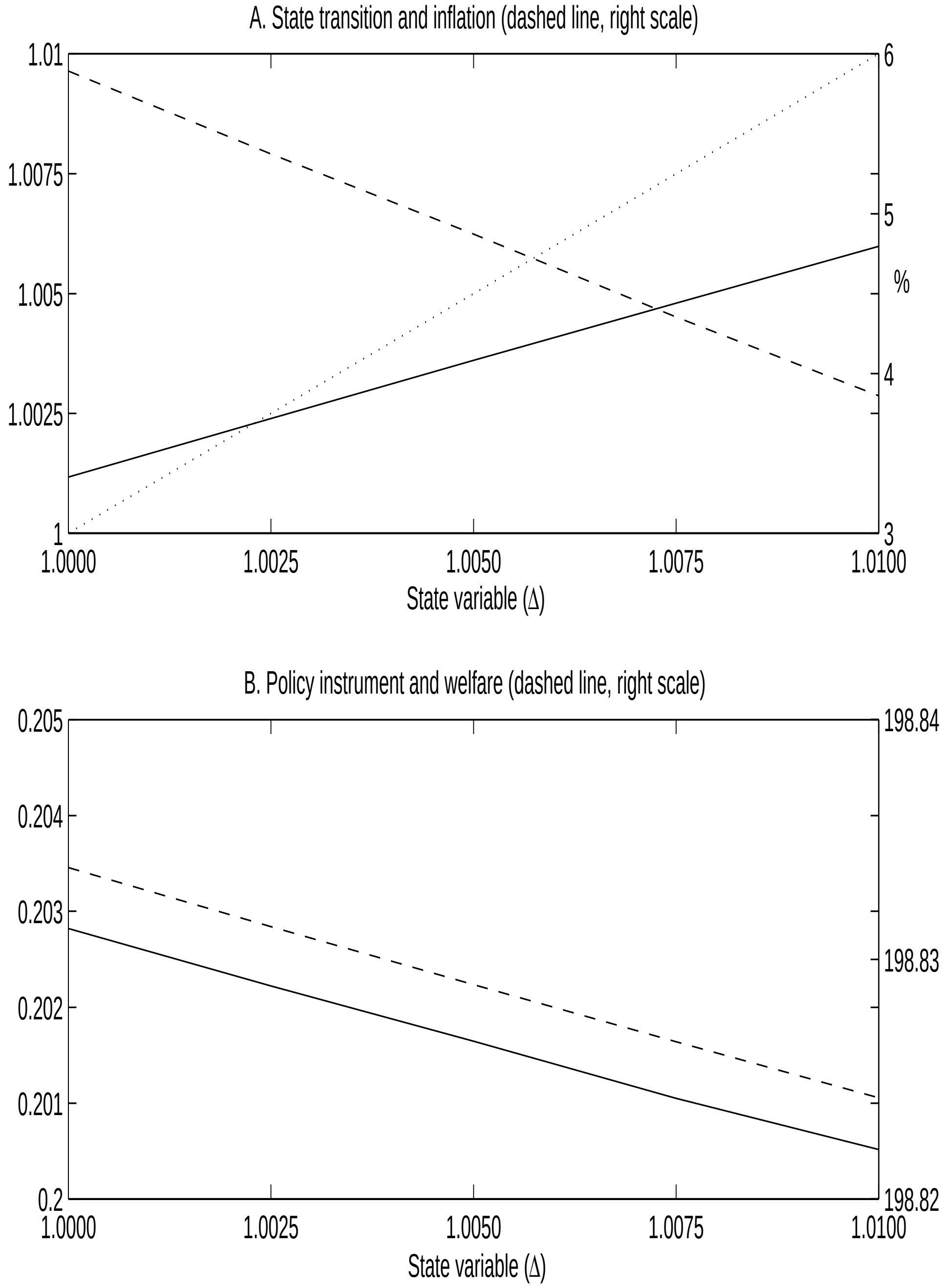


Figure 2: Steady state inflation rate

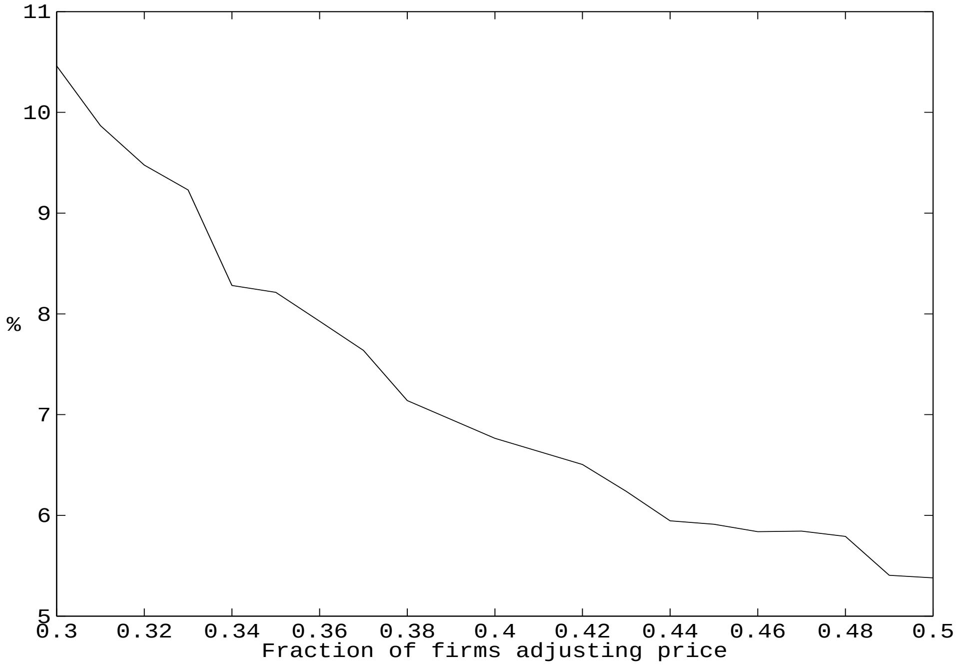
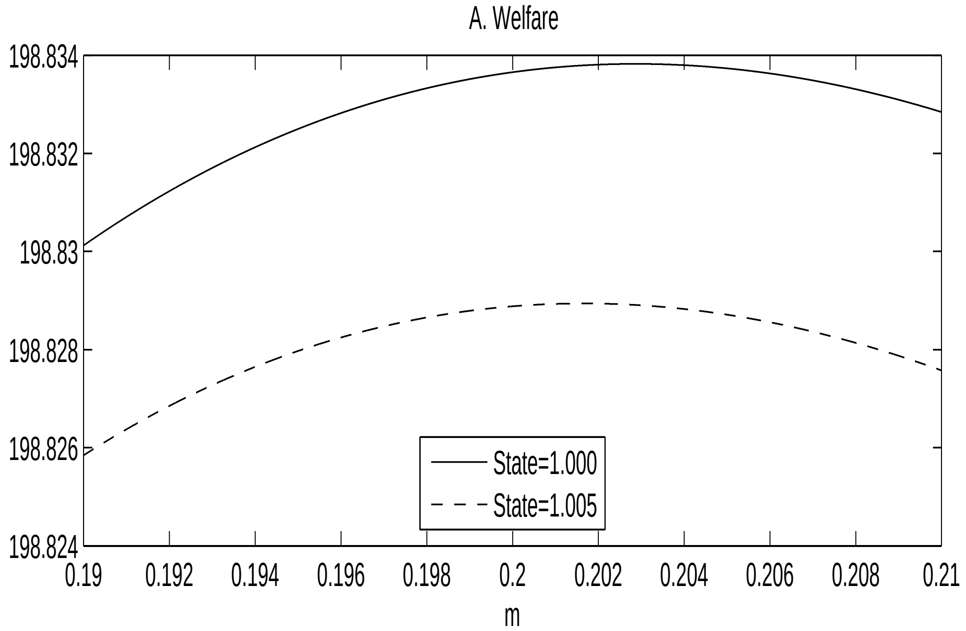


Figure 3: Policymaker's objective function



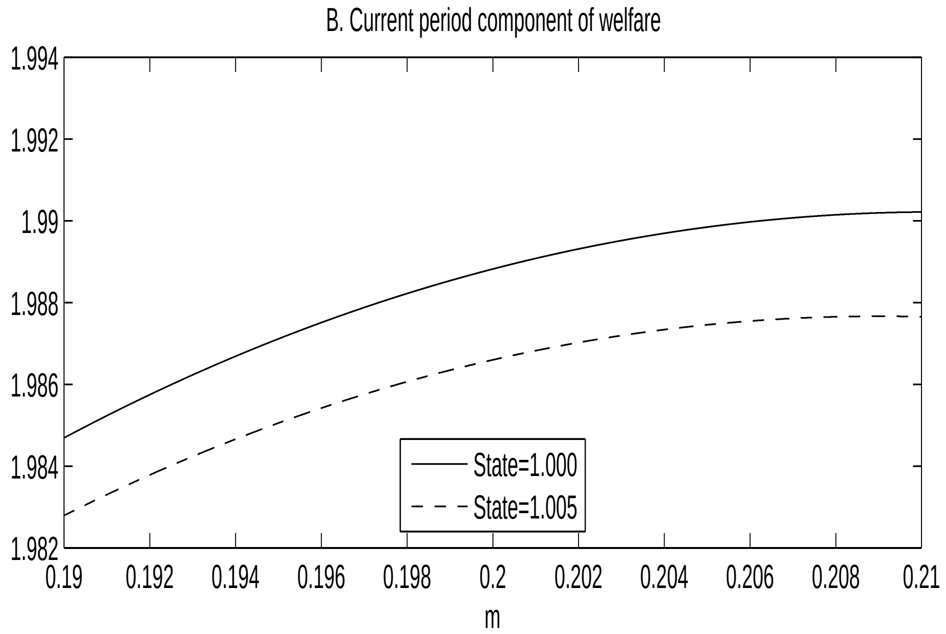
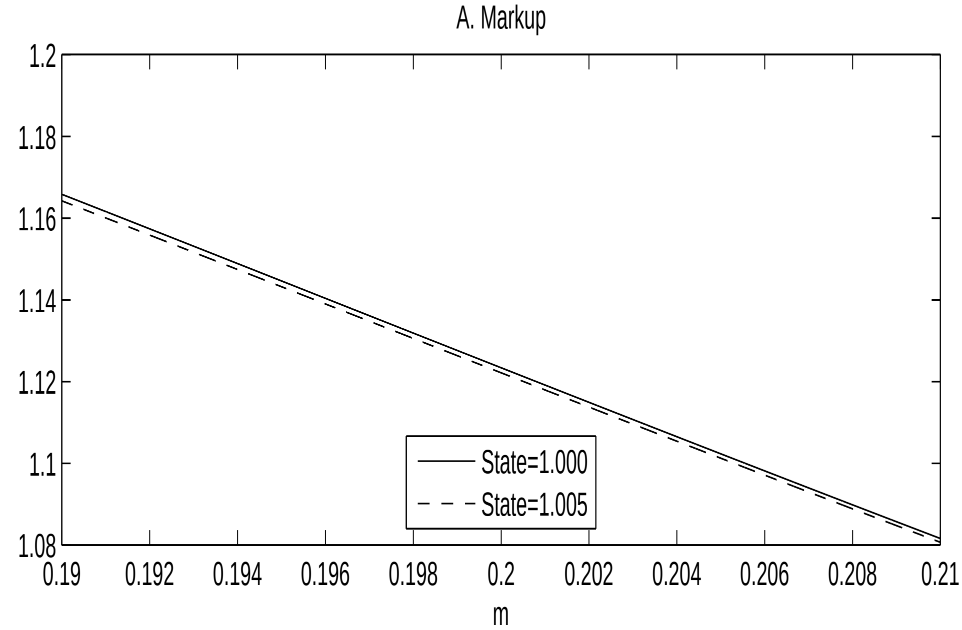


Figure 4: Distortions as functions of m



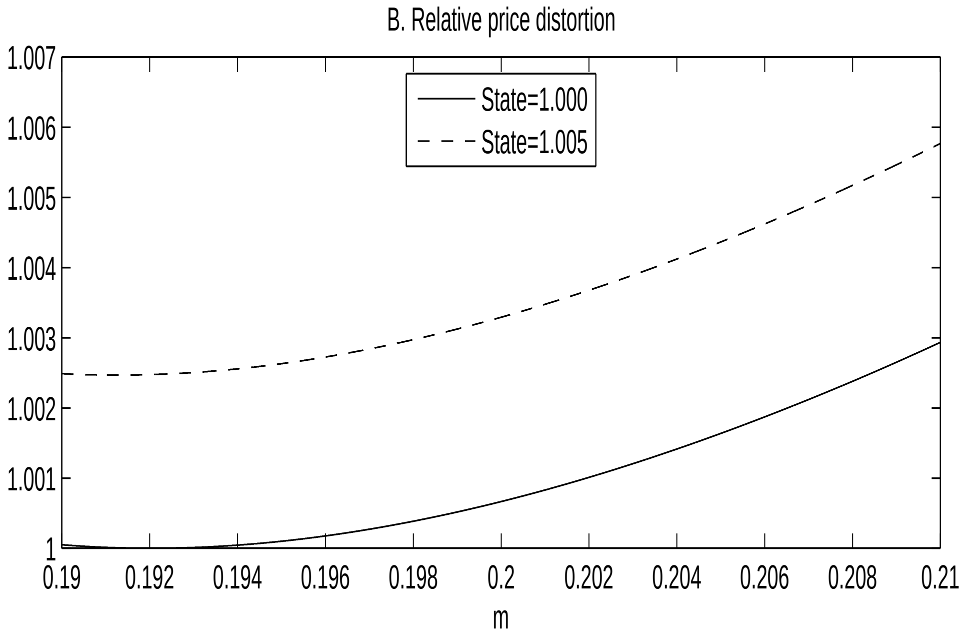


Figure 5: Relative price distortion as function of p_0

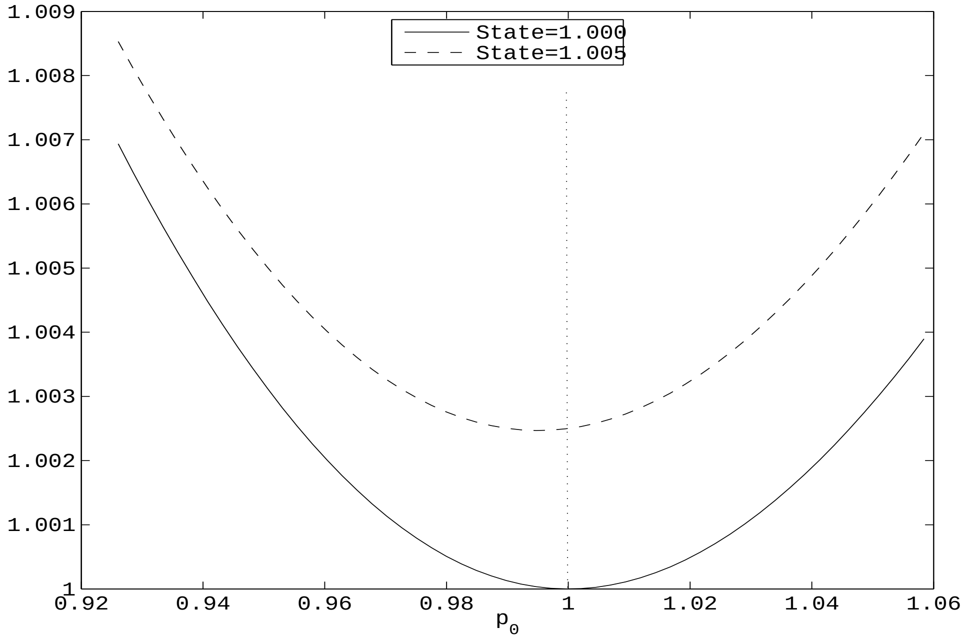


Figure 6: Pricing best response function: State = 1.0025, $m = 0.2022$