Output Gap and Inflation with Persistent Demand Shocks*

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EU Commission

May 2020

Abstract: The combination of low inflation and low interest rates poses challenges for macroeconomic analysis: Inflation in the euro area (EA) has remained markedly below the ECB's price stability target. Yet, measures of economic slack indicate that economy operated at its potential before the pandemic shock. Using a standard New-Keynesian model, we show under which conditions such decoupling of the output gap and inflation can arise. Our focus is on a secular stagnation environment, characterised by a long-lasting liquidity trap and persistent demand shocks. To solve the model with a secular zero lower bound, we provide a new solution algorithm.

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Introduction

Since around 2014, the euro area (EA) is recovering from the financial and sovereign debt crisis. Standard estimates of the output gap point towards a closing between 2017, and 18[1]. However, core inflation remains low and fluctuates narrowly around 1%. The discussion on low inflation started around 2014. With signs of a recovery in the EA becoming stronger, it was noticed that standard inflation forecasting models over-predicted inflation and also inflation expectations remained subdued (see Conti et al. (2014) and Landau (2014)). The phenomenon that inflation stays low despite an ongoing recovery has been labelled the "missing inflation puzzle" (Constâncio, 2015). Recent work by the ECB has systematically explored various explanations based on Phillips curve regressions. Bobeica and Sokol (2019) look at various alternative demand indicators, measures of inflation expectations and indicators of (foreign) supply shocks and conclude that there is robust evidence in favour of an over-prediction of core inflation based on 550 alternative Phillips curve specifications. Moretti et al. (2019) extend the analysis to non-linear Phillips curves and systematically test for the presence of changes in the slope of the Phillips curve. Neither do they find lower slope coefficients in recent samples nor significant coefficients for non-linear terms in the Phillips curve. They also consider the standard set of explanatory variables and find that inflation expectations are important for predicting low inflation.

An analysis of the decoupling between economic activity and inflation based on Phillips curve regressions misses the interaction between aggregate supply and aggregate demand. In addition, it does not consider the changed monetary policy environment i.e. both inflation expectations and measures of economic slack are not exogenous variables. This paper therefore goes beyond single Phillips curve regressions and asks under which conditions the new-Keynesian (NK) model can generate decoupling between the output gap and inflation. In particular, we explore a change in trend inflation, due to persistent demand shocks, which lower the riskless rate persistently to levels, which constrains monetary policy at the zero lower bound for a longer period.

[1] The German Council of Economic Advisors (SVR, 2019) compared output gap estimates of international organisations (based on a production function (PF) methodology) with output gap estimates using an HP-Filter, a Hamilton Filter and a factor model for several large EA member states. All estimates point towards a closing of the output gap.

A declining trend of the riskless interest rate plays an important role in the debate about secular stagnation (see Summers, 2016, Rachel and Smith, 2017). Proponents of the secular stagnation hypothesis relate this phenomenon mostly to high savings rates, e.g. due to anticipated demographic pressures and flight to safety behaviour. The latter may have pushed down real rates further in the 2009/12 double dip recession. Thus the EA could have entered a ‘low for long’ interest rate environment with constrained nominal rates for a number of years if not decades similar to Japan, which is in such a situation already for about 20 years.

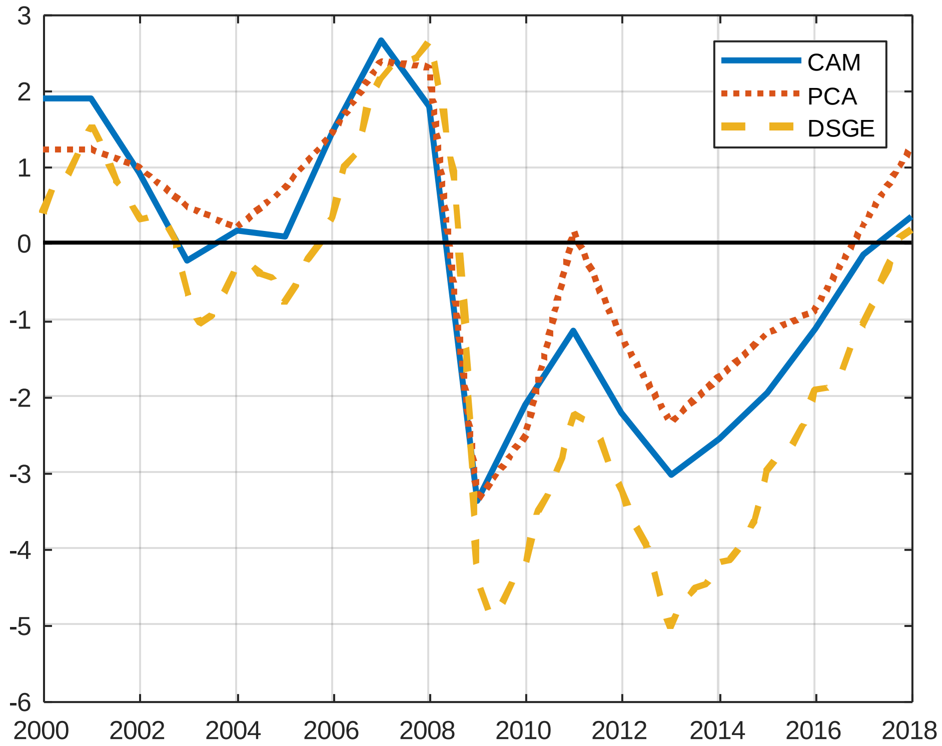
The relationship between inflation and the output gap at the zero lower bound has been studied intensively (see Lubik and Schorfheide (2003) and Cochrane (2017). In this paper, we want to add to this discussion by looking at a more persistent demand shock and explore a small set of equilibria besides the standard full rational expectations (RE) equilibrium and the no inflation jump equilibrium advocated by Cochrane. As emphasised by Cochrane, ‘equilibrium selection can be an empirical project as well as a theoretical one’, and we find that both equilibria do not fit the stylised facts about output gap and inflation very well. We also go beyond the existing literature and look at the robustness of results by comparing output gap-inflation nexus with a standard forward-looking Phillips curve and a hybrid Phillips curve in an NK model.

This paper is structured as follows. Section 1 presents some empirical evidence about the output gap and core inflation in the EA, documenting the degree in which there has been a decoupling in recent years. Section 2 presents the simple NK model and analyses the link between inflation and the output gap with persistent demand shocks with different assumptions about the degree in which inflation expectations are forward-looking in the Phillips curve. Section 3 extends the analysis to the ZLB regime and presents our solution algorithm.

1. Empirical evidence:

This section provides some evidence about output gap estimates and the evolution of core inflation. As can be seen from Figure 1.1, after the 2009 financial crisis, the EA economy exhibited a negative output gap for nearly 10 years. This episode is accompanied by low (core) inflation which does not show signs of increasing despite the evidence on the recent closing of the output gap.

Figure 1.1: CAM output gap versus PCA and DSGE estimates for the EA (2000-2018)



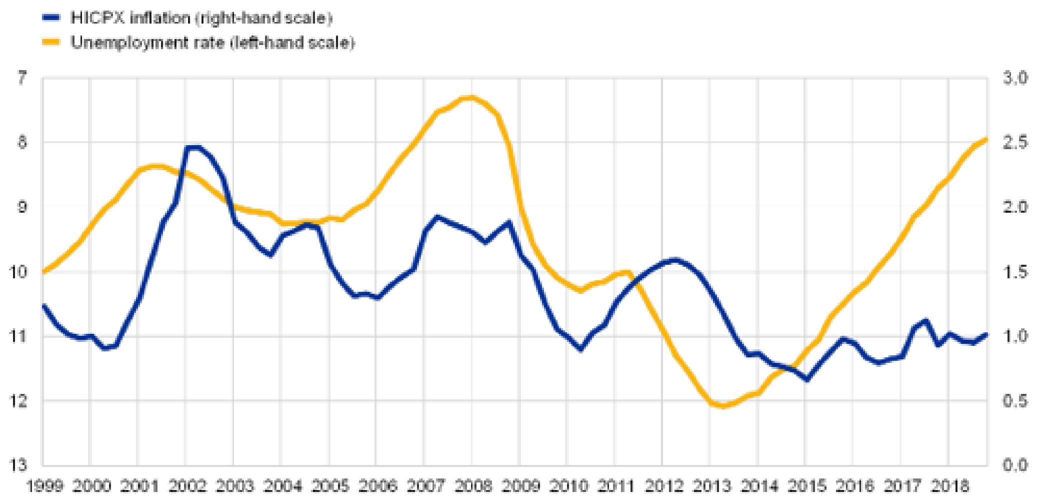
Notes: The PCA output gap is calculated using only the first component and rescaled to the standard deviation of the Commonly Agreed Methodology output gap. The DSGE output gap is based on the model-based production function. The vertical axis shows the output gap in percent.

Figure 1.2: Core inflation

Notes: Blue line: Annual rate of change of the Harmonised Index of Consumer Prices (HICP) excluding energy, food, alcohol and tobacco (EA 19). Source: Eurostat (prc_hicp_manr TOT_X_NRG_FOOD). The red line indicates the two percent annual inflation.

The inflation puzzle can also be seen by comparing movements of the unemployment rate with inflation as shown inn Figure 1.3.

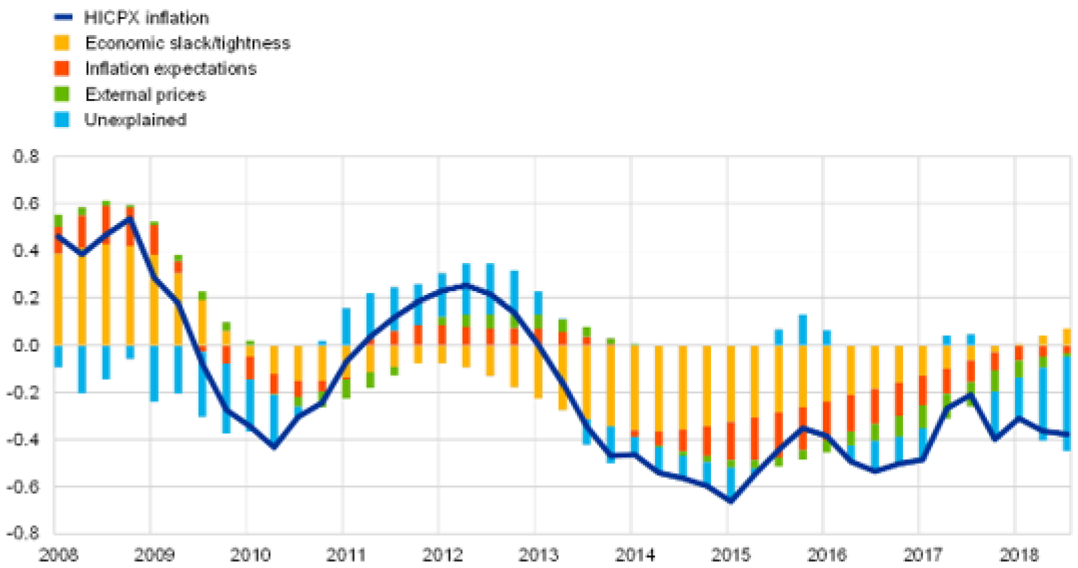
Figure 1.3: Unemployment and HICP inflation



Annual percentage change (right hand scale), percentages (left hand scale, inverted)
Source: Bobeika and Sokol (2019)

Another way to look at the inflation puzzle is to look at predictions based on Phillips curves

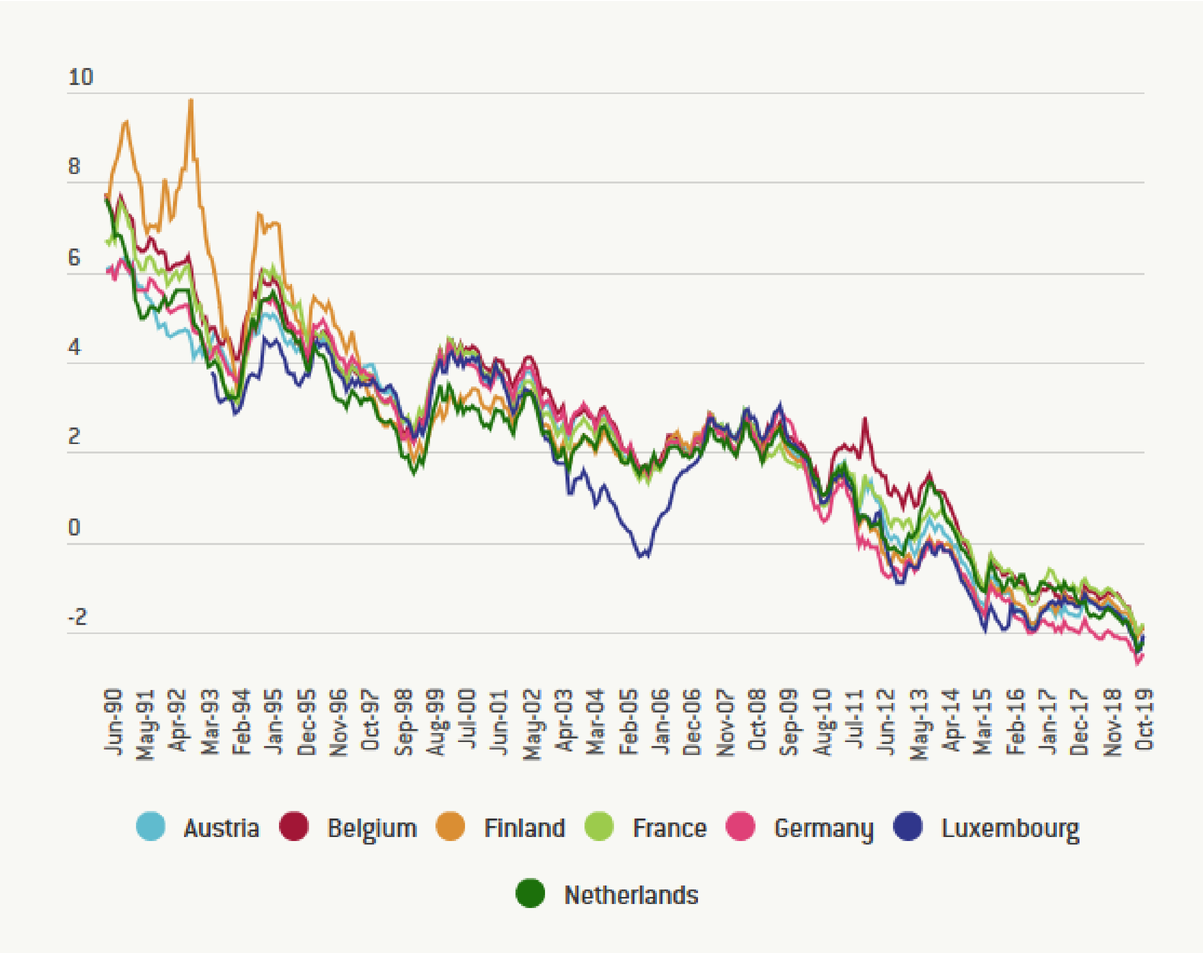
Figure 1.4: Phillips curve based decomposition of inflation



Annual percentage changes and percentage point contributions; all values in terms of deviations from their averages since 1999
Source: Bobeika and Sokol (2019)

The current situation differs from previous downturns and recoveries by the fact that the nominal interest rate is stuck at the ZLB. In our view, constrained monetary policy reflects both the severity of the financial and sovereign debt crisis in the EA (see Kollmann et al. (2016) for an identification of these shocks) but also coincides with a secular savings trend which has resulted in negative risk-free rates in the EA since the beginning of 2015 (see Figure 1.5). A major hypothesis underlying our analysis in this paper is that this trend will not reverse and interest rates will stay low for a number of years if not decades.

Figure 1.5: Real 10 year government bond yields



Note: 10-year government bond yield deflated by 10-year ahead expected inflation; the latter is derived from the IMF World Economic Outlook projections.

Source: Bruegel

2. New Keynesian model

To keep the analysis analytically tractable, we use the simplest New Keynesian workhorse model. We follow Clarida et al. (1999)[2] and consider an aggregate demand equation and a price equation (New Keynesian Phillips curve). Since we are interested in the dynamic effects of demand shocks, we abstract from supply shocks. Thus, under flexible prices, our economy would always produce at a constant level of output y^{ss} and we define the output gap as the difference between actual output and its steady state y^{ss}. We also assume that in normal times, the CB adheres to an inflation target $\bar{\pi}$ and the model explains deviations of inflation from the target. In the ZLB regime, inflation will still be expressed as deviation from the inflation target. All variables are expressed as deviation from their steady state level.

$$y_t = (y_t^L - y^{ss}) \tag{1}$$

$$\pi_t = \pi_t^L - \pi^{ss} \tag{2}$$

$$i_t = i_t^L - i^{ss} \tag{3}$$

Variables with superscript L denote original (log) levels. The steady state is calculated for the model with the Taylor rule imposed ($\pi^{ss} = \bar{\pi}$, $i^{ss} = \rho + \bar{\pi}$).

The simplest NK model represents the economy by an aggregate demand schedule, which relates the expected change of the output gap to the expected real interest and the rate of time preference. The rate of time preference can be subject to shocks, which we will denote demand shocks. The second relationship is a Phillips curve, which expresses the deviation of current inflation from the target as a positive function of the output gap and expected future inflation. We allow for the presence of a hybrid Phillips curve. Because this is an empirically important case and we will show that the relationship between inflation and the output gap depends crucially on the relative importance of the lead and lagged inflation variable in the Phillips curve. Concerning monetary policy, we assume that the central bank (CB) sets the policy rate in each period as a function of the real rate and inflation target and responds aggressively (with $\tau > 1$) to any deviation of inflation from the target. As shown in Clarida et al. (1999), it is optimal for the CB to lower interest rates directly as a response to the negative demand shock. Monetary policy is subject to a ZLB constraint.

$$y_{t,t+1} = \sigma\big(i_t - \pi_{t,t+1} + s_t\big) + y_t \tag{4}$$

[2] See for example Woodford (1996) for a detailed derivation of the aggregate demand and supply equation.

$$\pi_t = s^f \beta \pi_{t,t+1} + (1 - s^f)\pi_{t-1} + \gamma y_t \quad (5)$$

$$i_t = Max(-(\rho + \bar{\pi}), \tau \pi_t) \quad (6)$$

Shock assumption: $$s_t = \rho^s s_{t-1} + e_t > 0 \quad (7)$$

Parameters:

$$\rho = 0.01;\ \ \sigma = 1.;\ \ \gamma = 0.125;\ 0 \le s^f \le 1\ \tau = 1.5;\ s^f = (1 \text{ or } 0.55);\ \rho^s = 0.975$$

In all our simulation experiments, we consider an annual frequency, with a rate of time preference of 1%, an interest elasticity of one, a slope of the Phillips curve equal to 0.125. The inflation coefficient in the Taylor rule is set to 1.5. We choose a persistence of the demand shock, implying a half-life of 25 years. The size of the savings shock is 2%. This allows the model to generate an adjustment path with a negative (risk-free) real interest rate in an order of magnitude as observed in the data.

Output gap and inflation with a Taylor rule

This model has a stable and determined solution if the Taylor principle holds. In this case, the transition matrix has two Eigenvalues larger than one, corresponding to two forward-looking variables y_t and π_t.The solution under the Taylor rule is given by

$$y_t^{TR} = \theta^y s_t = \frac{1}{\rho^s - 1 + \frac{(\rho^s - \tau)\gamma}{1 - \beta\rho^s}} s_t \quad (8)$$

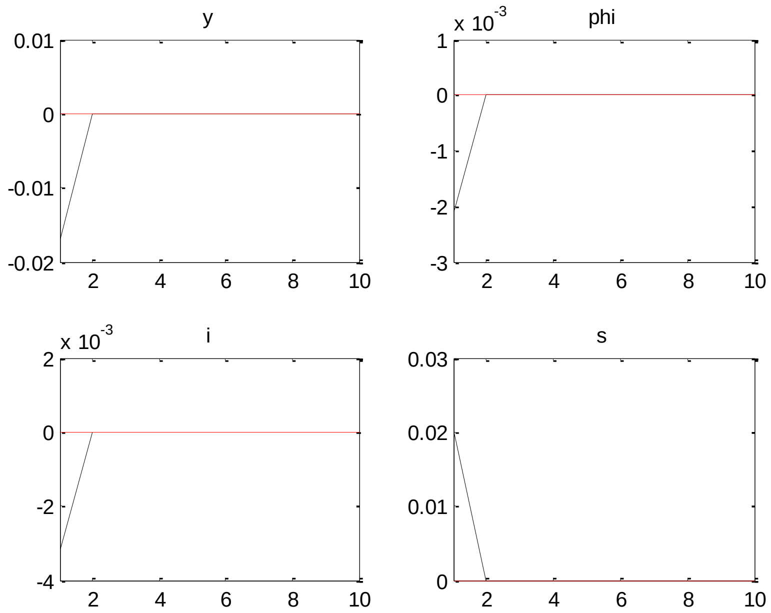
$$\pi_t^{TR} = \theta^\pi s_t = \frac{-\gamma}{1 + \gamma\tau - \rho^s - \rho^s\beta - \rho^s\gamma + \beta{\rho^s}^2} s_t \quad (9)$$

$$i_t^{TR} = \tau\theta^\pi s_t \quad (10)$$

and shows the complete co-movement between the output gap and inflation.

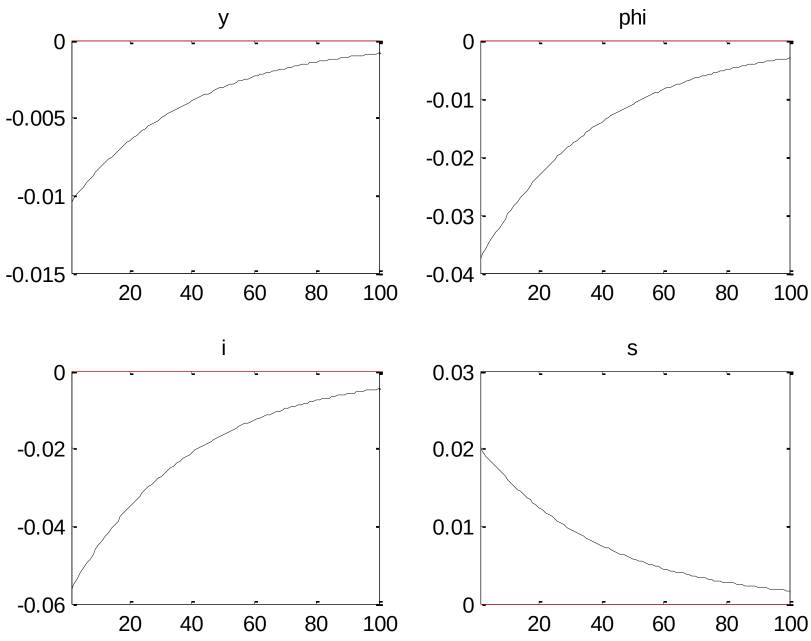
Since persistent demand shocks are non-standard, we first consider a white noise demand shock as a benchmark. For this shock, we see directly from equations (8) to (10) that the output gap will be negative and inflation will undershoot the target for one year. The percentage point (ppt) deviation of inflation from target is γ times the deviation of output from potential. As shown in Figure 2.1, output declines by about 1.5%, while inflation declines by close to 20Bp. A non-persistent demand shock of 2% does not violate the ZLB constraint.

Figure 2.1: Temporary Demand Shock -Forward-looking Phillips curve



A persistent demand shock of the same size but unchanged monetary policy response has a different impact effect.

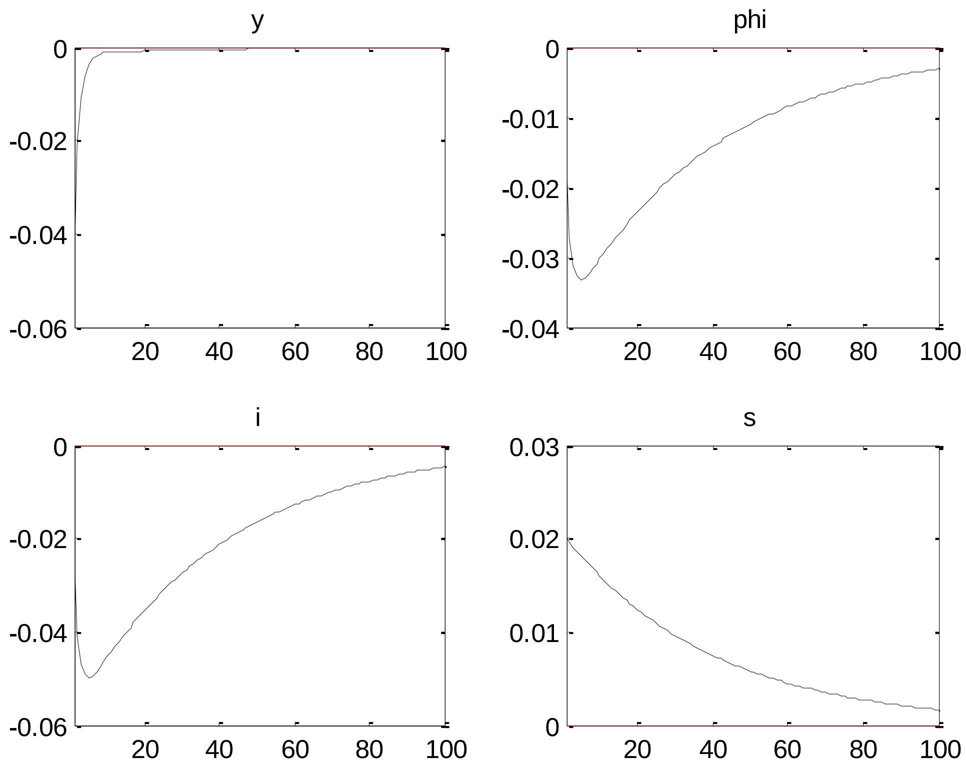
Figure 2.2a: Forward-looking Phillips curve



Assuming that the ZLB does not constrain the Taylor rule, the CB lowers the policy rate by nearly 600Bp. This reaction limits the decline of GDP to -1%, but inflation declines by 350Bp. Both GDP and inflation recover at a low pace. The output gap is extremely persistent and does not close before 100 years. It takes inflation more than 100 years to reach the target. The nominal interest rate becomes negative.

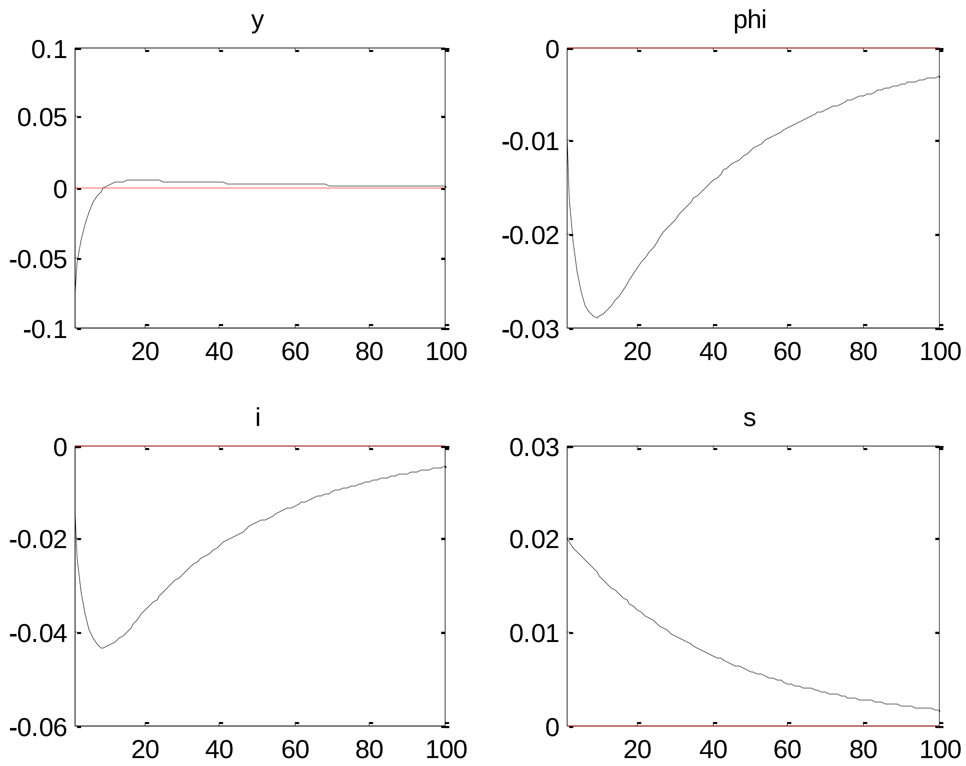
Why does the persistent demand shock have different effects on output and inflation than the temporary shock? Because of nominal frictions, firms adjust prices stronger to persistent shocks than to more temporary shocks.

Figure 2.2: Hybrid Phillips curve



With a hybrid Phillips curve, the co-movement between inflation and the output gap becomes weaker. In particular, the output gap closes rapidly in the first five years but remains marginally negative afterwards, while inflation is still low and approaches the inflation target very slowly. The output gap closes quickly after inflation picks up from a low level. This output gap inflation dynamics comes closer to the accelerationist view to which we turn next.

Figure 2.3: Backward-looking Phillips curve



The decoupling is more pronounced with a purely Backward-looking Phillips curve. Now the output gap becomes positive after inflation has reached a trough.

This section has shown that in combination with demand shocks, co-movement between the output gap and inflation only occurs under the pure forward-looking Phillips curve, while more backward-looking variants can generate decoupling where the output gap closes before inflation is reaching the inflation target. However, these simulations fail to be consistent with the data on two points. First, in all three variants, the ZLB constraint is violated by a demand shock with this persistence and size and second the adjustment of inflation shows a strong upward trend after the initial trough, while observed core inflation remain persistently at a lower level.

3. Solution with a ZLB constraint

With the ZLB constraint, the Taylor rule is not effective. Thus, the transition matrix has only one Eigenvalue larger than one for two jumping variables, yielding indeterminacy. We make the indeterminacy explicit by introducing expected inflation conditioned on t, π_t^e, i. e. by introducing a state variable (see Lubik and Schorfheide (2003) or Farmer and Khramov (2015)) and by linking inflation to expected inflation via a inflation forecasting error equation. Farmer and Khramov argue as follows: 'the rationale for our procedure is based on the notion that agents situated in an environment with multiple rational expectations equilibria must still choose to act. And to act rationally, they must form some forecast of the future and, therefore, we can model the process of expectations formation by specifying how the forecast errors co-vary with the other fundamentals'. Here we restrict the forecast error to be perfectly correlated with the demand shock (which is the only shock).

$$\pi_t^e = \pi_{t,t+1} \quad (11)$$

$$\pi_t = \pi_{t-1}^e - \omega(.)e_t \quad (12)$$

Equation (12) differs from characterising a fully rational forecast error by a free parameter ω which is not constrained by underlying structural parameters of the model. However, we can determine the value of ω corresponding to the full RE solution. We restrict the analysis of solutions to cases characterised by different values of ω, including the value implied by the full RE solution. Also Cochrane's 'no inflation jump' solution is a special case with $\omega = 0$. Now, the dynamic equations of the model can be rewritten in terms of y_t and π_t^e

$$\pi_{t-1}^e - \omega(.)e_t = \beta\pi_t^e + \gamma y_t \quad (13)$$

$$y_{t.t+1} = (-(\rho + \bar{\pi}) - \pi_t^e + s_t) + y_t \quad (14)$$

This model has one predetermined variable and one non-predetermined (jumping) variable which is required by the determinacy condition.

For this small model, we can find an analytical solution with an endogenous switch point under full rational expectations (i.e. no indeterminacy at the ZLB) and under indeterminacy. In the first case, the underlying structural parameters determine $\omega(.)$, while in the latter case ω remains exogenous. For the model with a hybrid Phillips curve, which has a state variables under the Taylor rule (TR) regime, we resort to a numerical solution procedure (see Appendix A).

Full RE solution vs. multiple indeterminate solutions

We now discuss differences between the two solutions using the simple NK model.

The full RE solution and the solution under indeterminacy differ by how the Taylor rule is imposed after the ZLB period. Under the full RE solution the Taylor rule is immediately applied by the CB at $t+\bar{J}$, which requires that agents anticipate a shift in the policy regime from an exogenous constant nominal interest rate to a (Taylor) rule-based system which is strictly enforced starting in period $t+\bar{J}$

$$\pi^{e}_{t+\bar{J}-1} = \pi^{TR}_{t+\bar{J}} \tag{15}$$

For the indeterminate solutions, we follow Cochrane and assume that the CB sticks to an ex-ante defined (and announced) path for the interest rate after the ZLB regime. Here we assume that the path for the policy rate after the ZLB is conditional on $s_{t+\bar{J}+k}$. More precisely we assume that the CB sets the nominal rate according to

$$i^{TR}_{t+\bar{J}+k} = \tau\theta^{\pi} s_{t+\bar{J}+k}, \qquad k = 0,1,\dots.. \tag{10'}$$

This equation corresponds to the unconstrained solution for the nominal interest rate. Though this solution is consistent with the Taylor rule, eq. 10' does not imply that agents anticipate that the CB will strictly apply the Taylor rule after $t+j$. Eq 10' only implies that the CB follows an interest rate rule contingent on $s_{t+\bar{J}+k}$ (and not on the deviation of inflation from the target). The only constraint implied by 10' is the prediction that the nominal interest rate will be set at the steady state level implied by the TR when s_t goes to zero. Thus, eq, 10' implies that the CB is not enforcing a short run inflation adjustment as implied by the Taylor but allows a smooth transition to the inflation target. The two alternative solutions have one thing in common, namely an identical path for the policy rate. For solving the NK model (without predetermined variables under the TR regime) and allowing for a switch point to the Taylor rule regime, we find it useful to apply a slightly modified Blanchard Kahn (1980) algorithm.

Full RE solution:

A first step is to transform the model into Jordan canonical form, with the two variables Y_{t-1} and Q_t, where Y_t is related to the eigenvalue smaller than one λ_1 and Q_t to the eigenvalue larger than one λ_2 . Q_t is a linear combination between y_t and π^{e}_{t-1} (see Kollmann and Zeugner, 2018). We define

$$Q_t^T = y_t - \left(\frac{\lambda_1 - 1/\beta}{-\gamma/\beta}\right)\pi_{t-1}^e \quad (16)$$

with the coefficient of y_t normalised to one. Under the assumption that the economy remains in a zero nominal interest rate regime forever Q_t^T has the following forward solution

$$Q_t^T = \left[\left(\frac{\lambda_1 - \frac{1}{\beta}}{-\frac{\gamma}{\beta}}\right)\left(-\frac{\omega}{\beta}\right) - \frac{\omega}{\beta}\right]\left(\frac{1}{\lambda_2}\right)e_t + \left(\frac{1}{\lambda_2}\right)\sum_{i=0}^{\infty}\left(\frac{1}{\lambda_2}\right)^{i+1}(\rho + \bar{\pi}) - \left(\frac{1}{\lambda_2}\right)\sum_{i=0}^{\infty}\left(\frac{1}{\lambda_2}\right)^{i+1} s_{t,t+i} \quad (17)$$

Except for boundedness, no specific restrictions are imposed on the exogenous variables on the RHS of eq.17. With an expected reversal to the active monetary policy regime in $t + \bar{J}$, the solution for Q_t^T takes the following form

$$Q_t^T = \left[\left(\frac{\lambda_1 - \frac{1}{\beta}}{-\frac{\gamma}{\beta}}\right)\left(-\frac{\omega}{\beta}\right) - \frac{\omega}{\beta}\right]\left(\frac{1}{\lambda_2}\right)e_t + \left(\frac{1}{\lambda_2}\right)\sum_{i=0}^{\bar{J}-1}\left(\frac{1}{\lambda_2}\right)^{i}(\rho + \bar{\pi}) - \left(\frac{1}{\lambda_2}\right)\sum_{i=0}^{\bar{J}-1}\left(\frac{1}{\lambda_2}\right)^{i} s_{t,t+i} + \left(\frac{1}{\lambda_2}\right)^{\bar{J}} Q_{t,t+\bar{J}}^{TR,Z}$$

(18)

With the terminal condition

$$Q_{t,t+\bar{J}}^{TR,Z} = y_{t,t+\bar{J}}^{TR} - \left(\frac{\lambda_1 - 1/\beta}{-\gamma/\beta}\right)\pi_{t,t+\bar{J}}^{TR} \quad (19)$$

Thus, the forward variable Q_t^T incorporates the expected switch in the monetary policy regime. Notice, at this point we take the switch point as given. Below we show how to determine the switch point endogenously. The state variable π_t^e can also be written as a function of Q_t^T

$$(1 - \lambda_1 L)\pi_t^e = -\frac{\omega}{\beta}e_t - \frac{\gamma}{\beta}Q_t^T \quad (20)$$

Under the full rational expectations solution, agents perfectly foresee the exit from the ZLB regime and anticipate that the CB invokes the Taylor rule from date $t + \bar{J}$. Therefore, inflation expectation in the last period under ZLB, $t + \bar{J} - 1$, takes into account that inflation will be determined by the Taylor rule in $t + \bar{J}$

$$\pi_{t+\bar{J}}^{TR} = \pi_{t+\bar{J}-1}^e$$

Combining the respective solutions of the inflations from the two regimes (eq. 9 and 20), yields

$$\pi_{t+\bar{J}-1}^e = -\frac{\omega}{\beta}e_t\lambda_1^{\bar{J}-1} - \frac{\gamma}{\beta}\sum_{k=0}^{\bar{J}-1}\lambda_1^{\bar{J}-1-k}Q_{t+k}, \quad (21)$$

One can derive the following expression for the ω parameter:

$$\frac{1}{\omega}=-\frac{1}{\pi_{t+\bar{J}}^{TR}\beta}e_t\lambda_1^{\bar{J}-1}\left(1-\frac{\gamma}{\beta}\frac{1}{\lambda_2}\left(\frac{\lambda_1-\frac{1}{\beta}}{-\frac{\gamma}{\beta}}+1\right)\right)$$

$$-\frac{\gamma}{\beta}\left(\frac{1}{\lambda_2-1}(\rho+\bar{\pi})\frac{1-\lambda_1^{\bar{J}}}{1-\lambda_1}-\frac{\lambda_1^{\bar{J}-1}}{\lambda_2-\rho^s}\frac{1-\left(\frac{\rho^s}{\lambda_1}\right)^{\bar{J}}}{1-\frac{\rho^s}{\lambda_1}}s_t\right.$$

$$\left.-\frac{\rho+\bar{\pi}}{\lambda_2-1}\frac{1}{\lambda_2-\lambda_1}\left(1-\left(\frac{\lambda_1}{\lambda_2}\right)^{\bar{J}}\right)\right)-\frac{\gamma}{\beta}\frac{(\rho^s)^{\bar{J}}}{\lambda_2-\rho^s}\frac{1}{\lambda_2-\lambda_1}\left(1-\left(\frac{\lambda_1}{\lambda_2}\right)^{\bar{J}}\right)s_t$$

$$-\frac{\gamma}{\beta}\frac{1}{\lambda_2-\lambda_1}\left(1-\left(\frac{\lambda_1}{\lambda_2}\right)^{\bar{J}}\right)Q_{t+j}^{TR,Z}$$

(21)

The full RE solution generates an initial jump of inflation and the output gap such that inflation expectations in the transition period between the ZLB and the TR regime are fully consistent. This consistency requirement eliminates indeterminacy.

Solution with indeterminacy under the ZLB (ω exogenous):

With indeterminacy, agents anticipate an ex ante path for the policy rate beyond the ZLB regime, i.e. they anticipate a path for the nominal interest rate which is only determined by exogenous variables. This yields the modified Q equation

$$Q_t^T=\left[\left(\frac{\lambda_1-\frac{1}{\beta}}{-\frac{\gamma}{\beta}}\right)\left(-\frac{\omega}{\beta}\right)-\frac{\omega}{\beta}\right]\left(\frac{1}{\lambda_2}\right)e_t-\left(\frac{1}{\lambda_2}\right)\sum_{i=0}^{\infty}\left(\frac{1}{\lambda_2}\right)^i i_{t,t+i}^{ex}-\left(\frac{1}{\lambda_2}\right)\sum_{i=0}^{\infty}\left(\frac{1}{\lambda_2}\right)^i s_{t,t+i} \qquad (22)$$

With

$$i_{t,t+i}^{ex}=\begin{cases}-(\rho+\bar{\pi}) & for\ j<\bar{J}\\ \tau\theta^{\pi}s_{t+\bar{J}} & for\ j\geq\bar{J}\end{cases}$$

The dynamic equation for π_t^e remains unaffected.

How to determine $\bar{J}$?

Economic agents anticipate that the CB will invoke the Taylor rule at date $t + \bar{J}$. This date is characterised by a state of demand $s_{t+\bar{J}}$, such that the policy rate can be set marginally above zero ($i^{L}_{t+\bar{J}} \geq 0$)[3].

When monetary policy is unconstrained, there is a linear relationship between i^{TR}_{t+j} and s_{t+j} (eq. 10), and the Taylor rule yields an interest rate of zero when

$$i^{TR}_{t+\bar{J}} = -(\rho + \bar{\pi}) = \tau\theta^{\pi} s_{t+\bar{J}} = \tau\theta^{\pi}\rho^{\bar{J}} s_t \qquad (21)$$

This equation can be solved for $\bar{J}$

$$(\bar{J})\log(\rho^{s}) = \log\left(\frac{-(\rho+\bar{\pi})}{\tau\theta^{\pi} s_t}\right) \qquad (22)$$

Following these steps provides an analytical solution for the NK model when there is a temporary ZLB constraint.

In the following scenarios we assume that the natural rate of interest declines by 2% persistently (and turns negative). Similar to Werning and Cochrane , the CB is setting interest rates to zero until $t + \bar{J}$ and then increases the interest rate gradually as the natural rate increases[4].

Replicating the full rational expectations solution

Figure 3.1 shows the adjustment path of output, inflation and the nominal interest rate for a persistent demand shock. The value of ω implied by the smoothness condition is 1.9485e+06 and implies a strong jump of output and inflation after agents learn about the persistent nature of the demand shock[5].

[3] We assume that between $t + \bar{J} - 1$ and $t + \bar{J}$ starts to bind marginally such that $i^{L}_{t+\bar{J}} \geq 0$.

[4] Werning and Cochrane consider a strong temporary reduction of the natural rate (5 periods). In contrast we consider a persistent decline which only recovers gradually.

[5] This solution is numerically identical to the OCCBIN solution.

Figure 3.1a: Adjustment path under full RE solution-global

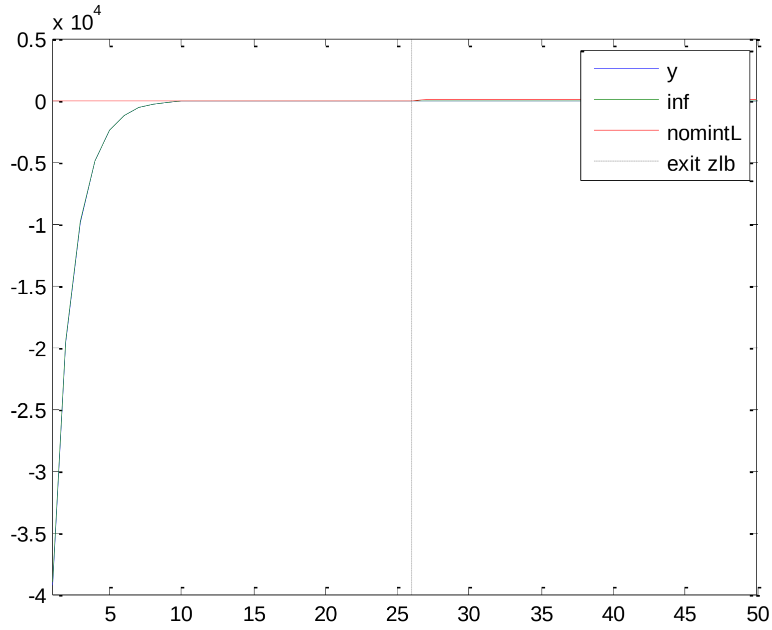
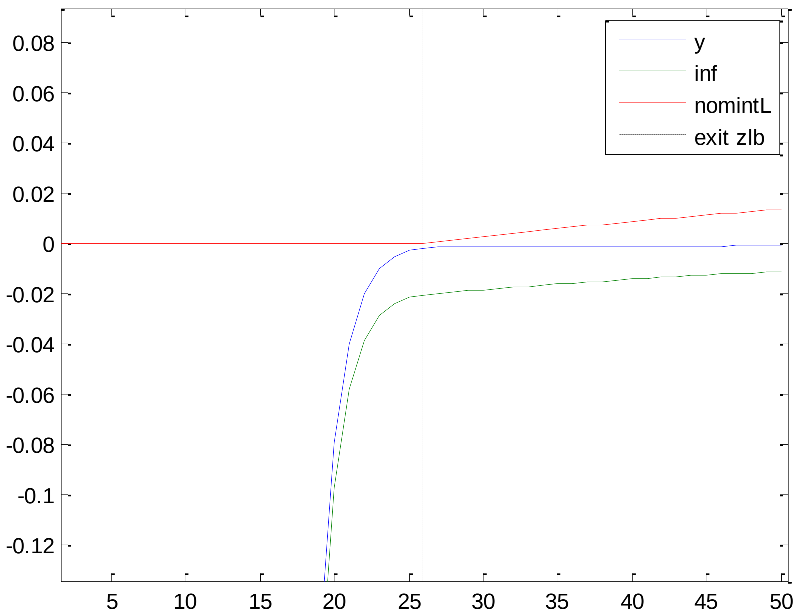


Figure 3.1b: Adjustment path under full RE solution-local



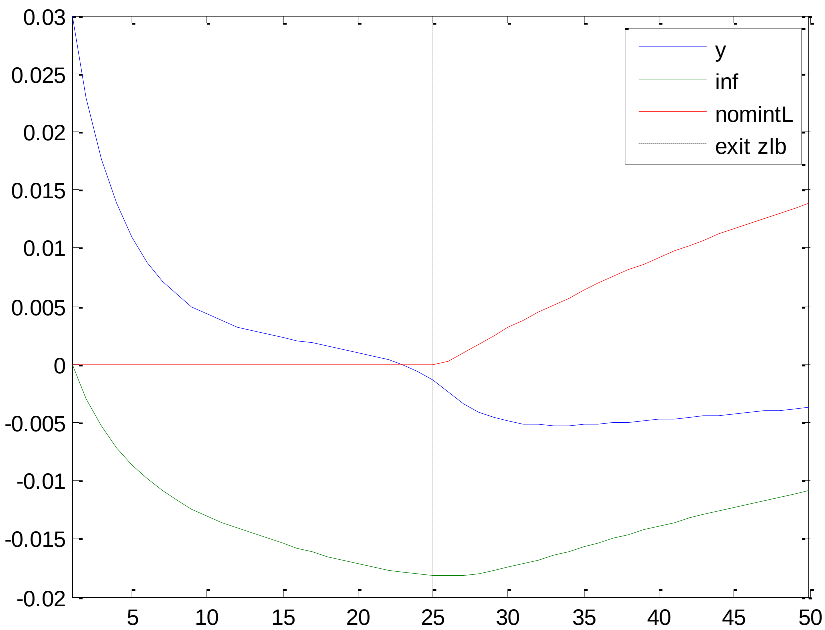
Under the full RE solution of a persistent negative demand shock with a ZLB, both the output gap and inflation become strongly negative and both drop by unrealistic magnitudes. The persistence of the demand shock implies that the economy is stuck at the ZLB for about 25 years. Unlike in the economy without ZLB (see Figure 2.1) where the output gap was hardly affected, the constrained monetary policy cannot avoid a fall of output of a similar magnitude as inflation. Both drop excessively. Only a few periods before the transition to the Taylor regime is inflation moving to zero (2% below target) which is the rate at which the CB starts to apply the Taylor rule. Inflation approaches zero from below.

The take away from this exercise is that quantitatively both the output gap and inflation move much closer together compared to the situation with unconstrained monetary policy. However, the magnitude of the response of both inflation and the output gap is unrealistically excessive.

Adjustment with smooth inflation when the shock occurs (no inflation jump equilibrium)

Cochrane (2017) advocated a solution, which constrains the jump of inflation to zero. In case of a temporary shock, we apply his equilibrium concept to the case of a persistent demand shock.

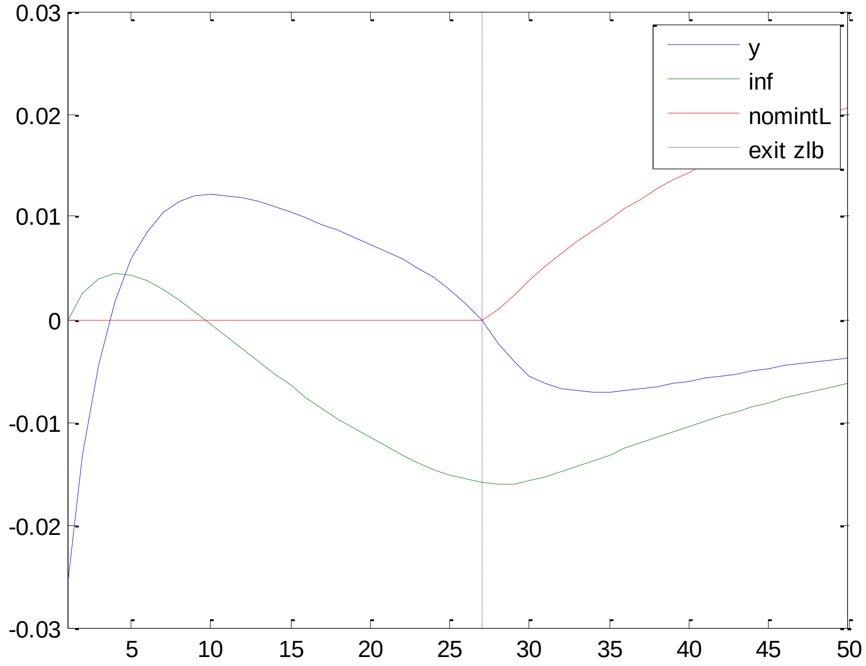
Figure 3.2a: Adjustment in the no inflation jump equilibrium (s=0.02, rhos=0.975)



A relatively small but persistent demand shock leads to deflation and a positive output gap. Given the size and duration of the negative demand shock, the CB sets nominal interest rates to zero, the lowest value attainable. However, since the smooth inflation adjustment condition constrains inflation, it only gradually declines. This effect lowers the real interest rate and temporarily boosts output. As inflation declines over time, the output gap closes from above. Imposing what Cochrane calls the 'no inflation jump equilibrium' leads to a strong decoupling between inflation and the output gap. The output gap remains positive throughout the ZLB episode, while inflation declines continuously towards zero, consistent with the expected inflation at the switch point to the Taylor rule regime.

This adjustment path of output and inflation differs from the Cochrane solution, because our shock has a smaller size but is more persistent. Especially the initial size of the demand shock is important, increasing the size generates a stronger demand shortfall and reduces the output gap below zero. Similar to Cochrane, decoupling occurs in this case as well since inflation rises initially.

Figure 3.2b: Adjustment in the no inflation jump equilibrium (s=0.045, rhos=0.95)



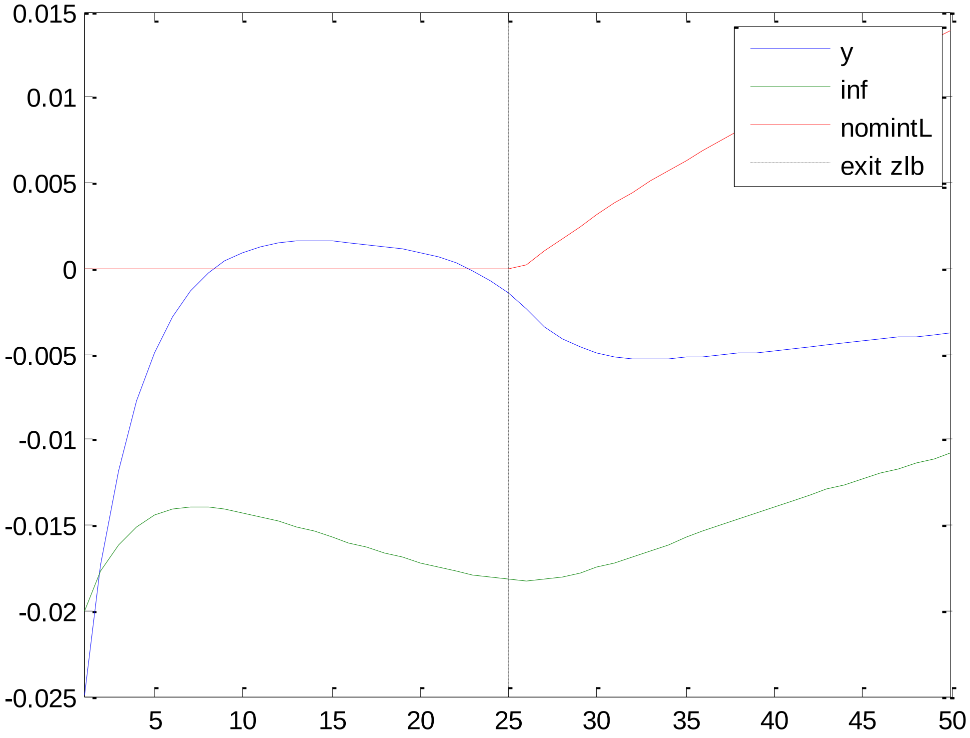
Towards a more realistic adjustment path

The problem with the no inflation jump solution (this applies even more to the stable backward solution proposed by Cochrane) is that it either does not replicate the low inflation episode in the EA since 2009, or the negative output gap.

The no inflation jump equilibrium also does not seem plausible if we take into account that agents would use previous experiences (with unconstrained monetary policy) from which they can infer that inflation jumps downwards to a negative demand shock. Though initially agents may not be familiar with the new ZLB environment, it seems reasonable to assume that agents also expect a downward jump of inflation to such a demand shock. Based on this reasoning, we choose a value of $\omega > 0$ which implies a downward jump of inflation. With $\omega = 1$, the model can come closer to replicating inflation (see Figure 3.3). Now the model generates a negative output gap and decline of inflation. Initially there is co-movement. However, decoupling occurs after a few periods: The output gap closes, while inflation does not reach the target. Instead inflation starts to deviate again from the target and converges to zero inflation, which is the expected inflation at which the CB starts to invoke the Taylor rule after the negative demand shock has become sufficiently small.

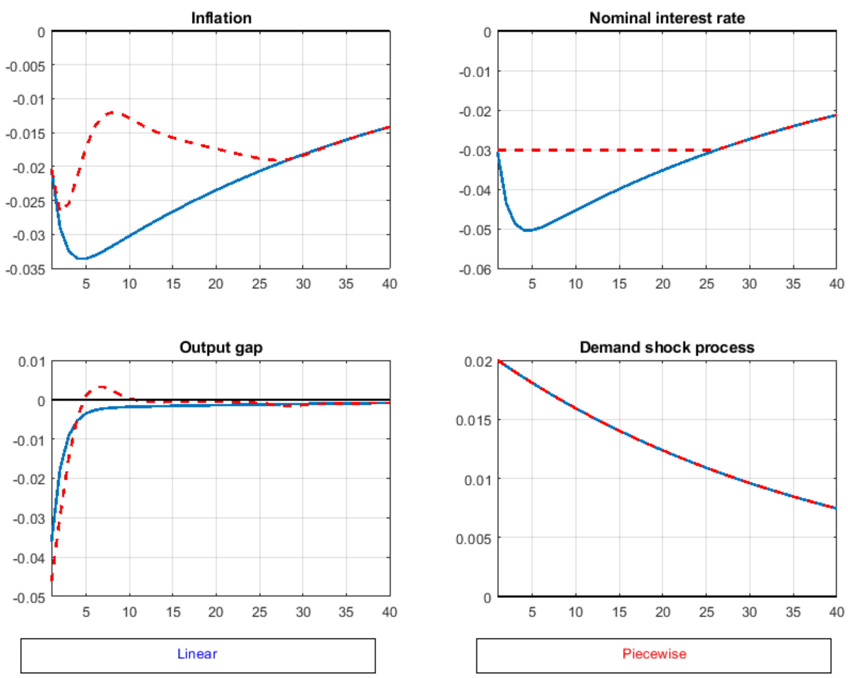
Short and medium-term dynamics can be distinguished. Initially, both output and inflation drop associated with an increase in the real interest rate. However, with inflation rising the real interest rate declines and the output gap closes. The output gap closure is also driven by the expectation of a turnaround in inflation. With sufficiently decelerating inflation, the output gap becomes (slightly) positive. The economic intuition for this is as follows. Producers, which expect inflation to decline (gradually) over time, will reduce prices/mark up already in the current period to avoid price adjustment costs. This generates demand and leads to a (small) positive output gap.

Figure 3.3: Adjustment with $\omega = 1$



Using a numerical solution algorithm for the hybrid case yields the following similar adjustment dynamics.

Figure 3.4: Adjustment with $\omega = 1$ (hybrid Philipps curve)



Conclusion

The combination of low inflation and low interest rates poses challenges for macroeconomic analysis: Inflation in the euro area (EA) has remained markedly below the ECB's price stability target. Yet, measures of economic slack indicate that economy operated at its potential before the pandemic shock. Using a standard New-Keynesian model, we show under which conditions such decoupling of the output gap and inflation can arise. Our focus is on a secular stagnation environment, characterised by a long-lasting liquidity trap and persistent demand shocks.

Besides the unique full RE solution we also allow for equilibria with self-fulfilling expectations which can arise under a ZLB regime. We find that both the full RE solution and the no inflation jump solution yield implausible results. Within the range of solutions offered by our parametrisation, we find an inflation forecast error process, which allows some initial response of inflation to a negative demand shock better matches the stylised facts of a decoupling between inflation and the output gap.

For the small NK model studied in this paper, we can provide a full analytical solution for the case in which the economy is temporary (but possibly over a long period) in a ZLB regime both for the full RE solution and all solutions characterised by exogenous choices of the forecast error equation.

However, more work is needed to empirically estimate this model and to identify the driving shocks. We believe that extending the estimation to models which allow for indeterminacy will make it easier to identify persistent shocks. Currently estimated full RE models tend to identify a sequence of non-persistent demand shocks, since this is the only way to reconcile an interest rate stuck at the ZLB for a longer period of time without generating strong negative inflation. Our analysis has also neglected supply shocks, in particular associated with a fall in the NAWRU which can be observed since 2013 in the Euro Area. This could could be an alternative explanation for low inflation.

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ANNEX A NUMERICAL SOLUTION TO GENERAL PROBLEM with smooth transition

We represent the baseline TR (linearised) rational expectation model in the ABC form:

$$A\, y_{t+1} + By_t + Cy_{t-1} + J\varepsilon_t = 0$$

with solution

$$y_t = Ty_{t-1} + R\varepsilon_t$$

Interest rate i_t is an element of y_t and we can apply the z-transform of the state space representation to express i_t only as a function of exogenous shock(s) and its own lags:

$$i_t^{TR} = \sum_j \frac{b_j(L)}{a(L)} \varepsilon_t^j$$

where $a(L)$ $b_j(L)$ are polynomials in the lag operator L, and $a(L)$ is the characteristic polynomial of the TR rational expectation model. Note that for each exogenous shock there will be a different polynomial at the numerator. This transfer function or ARMA representation of interest rate versus exogenous shocks is extremely convenient to obtain the smooth transition path under the constrained regime, since it allows to evaluate the future interest rate path as a stand-alone process.

To allow for smooth transition, the model is re-formulated by assuming interest rate being an exogenous process, following the path prescribed by the TF form (C.1), and where inflation becomes a backward looking variable in the expectation error form. Let define the model under constrained regime as the 'star' model:

$$A^*\, y_{t+1} + B^* y_t + C^* y_{t-1} + J\varepsilon_t + Ki_t^* = 0$$

where we augmented the shocks in the model by the exogenous variable, which follows the path

$$i_{t+j}^* = \max(-(\rho + \bar{\pi}), i_{t+j}^{TR}) \text{ for } j = 0, \dots, \infty$$

and where the Tylor rule is replaced by the expression $i_t = i_t^*$ in the 'star' model. Equation (C.1) also allows to determine $\bar{J}$, by checking periods where $i_{t+j}^{TR} < -(\rho + \bar{\pi})$.

The 'star' model has a unique saddle path solution given by:

$$y_t = T^*\, y_{t-1} + R^* \varepsilon_t + Qi_t^*$$

Let us define a future time period j^∞ above which $i_{t+j^\infty+s}^* \sim 0$ $(s > 0)$, i.e. interest rate path prescribed by TR is back to equilibrium. Hence, for j^∞+1 we can write

$$y_{t+j^\infty+1} = T^* y_{t+j^\infty}$$

We can replace this in the 'star' model

$$A^* T^* y_{t+j^\infty} + B^* y_{t+j^\infty} + C^* y_{t+j^\infty-1} + K i^*_{t+j^\infty} = 0$$

which leads to the backward form:

$$y_{t+j^\infty} = -inv(A^* T^* + B^*)C^* y_{t+j^\infty-1} - inv(A^* T^* + B^*)K i^*_{t+j^\infty} =$$

$$= T^* y_{t+j^\infty-1} + Q i^*_{t+j^\infty}$$

which can be backward propagated

$$A^* T^* y_{t+j^\infty-1} + A^* Q i^*_{t+j^\infty} + B^* y_{t+j^\infty-1} + C^* y_{t+j^\infty-2} + K i^*_{t+j^\infty-1} = 0$$

implying

$$y_{t+j^\infty-1} = T^* y_{t+j^\infty-2} + Q i^*_{t+j^\infty-1} - inv(A^* T^* + B^*)A^* Q i^*_{t+j^\infty}$$

$$y_{t+j} = T^* y_{t+j-1} + \sum_{s=j}^{j^\infty} (-1)^{(s-j)} \left(inv(A^* T^* + B^*)A^*\right)^{s-j} Q i^*_{t+s}$$

and so on until t. This provides a backward recursion that allows to solve, given i^*_{t+s} and ω, the forward path of all model variables.

Please note that here we assume that the only exogenous process which is not white noise is i^*_t, hence we can always consider $\varepsilon_{t+s} = 0$.

For illustration purposes we report here also the ABC and ABC* form of the NK model.

Model with hybrid Phillips curve and Taylor rule in matrix notation

$$\begin{bmatrix} -1 & 0 & 0 \\ -1 & 0 & -1 \\ -\beta s^P & 0 & 0 \end{bmatrix} \begin{bmatrix} \pi_{t.t+1} \\ \pi^e_{t.t+1} \\ y_{t.t+1} \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 \\ \tau & 0 & 1 \\ 1 & 0 & -\gamma \end{bmatrix} \begin{bmatrix} \pi_t \\ \pi^e_t \\ y_t \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -(1-s^P) & 0 & 0 \end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ \pi^e_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} s_t = 0$$

Model with hybrid Phillips curve and ZLB in matrix notation

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \pi_{t.t+1} \\ \pi^e_{t.t+1} \\ y_{t.t+1} \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 1 \\ 1 & -\beta s^P & -\gamma \end{bmatrix} \begin{bmatrix} \pi_t \\ \pi^e_t \\ y_t \end{bmatrix} + \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \\ -(1-s^P) & 0 & 0 \end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ \pi^e_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} \omega & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_t \\ i^*_t \\ s_t \end{bmatrix} = 0$$

Alternative formulation.

Expand the ABC* formulation augmented by the offline TR solution

$$\mathbb{A} y_{t+1} + \mathbb{B}\, y_t + \mathbb{C}\, y_{t-1} + \mathbb{J} \varepsilon_t = 0$$

where

$\mathbb{A} = \begin{bmatrix} A^* & 0 \\ 0 & 0 \end{bmatrix} \mathbb{B} = \begin{bmatrix} B^* & K^* \\ 0 & I \end{bmatrix} \mathbb{C} = \begin{bmatrix} C^* & 0 \\ 0 & -T \end{bmatrix} \mathbb{J} = \begin{bmatrix} J \\ -R \end{bmatrix}$

Where K^* is a matrix of zeros except for a (-1) in the diagonal corresponding to the i_t entry (i.e. for equation $i_t = i_t^*$).

When i_t^* hits the lower bound, the augmented system switches to

$$\mathbb{A}y_{t+1} + \mathbb{B}^* y_t + \mathbb{C}\, y_{t-1} + \mathbb{J}\varepsilon_t + \mathbb{D}^* = 0$$

$\mathbb{B}^* = \begin{bmatrix} B^* & 0 \\ 0 & I \end{bmatrix} \mathbb{C} = \begin{bmatrix} C^* & 0 \\ 0 & T \end{bmatrix} \mathbb{J} = \begin{bmatrix} J \\ R \end{bmatrix}$