

Empirical Properties of Inflation Expectations and the Zero Lower Bound*

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Abstract

Survey data on expectations shows that households have heterogeneous inflation expectations and households' inflation expectations respond sluggishly to realized shocks to future inflation. By contrast, in models with a zero lower bound currently used for monetary and fiscal policy analysis, households' inflation expectations are neither heterogeneous nor sticky. This paper therefore solves a New Keynesian model with a zero lower bound and dispersed information on the side of households. The main properties of the model with heterogeneous and sticky inflation expectations are as follows: (1) the deflationary spiral in bad states of the world is less severe than under perfect information, (2) communication by the central bank about the current state of the economy (even without any change in current or future policy) affects consumption, and the sign of this effect depends on whether or not the zero lower bound binds, (3) a commitment by the central bank to increase future inflation can reduce current consumption, (4) the government spending multiplier can be negative, and (5) shocks to uncertainty can have first-order effects.

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1 Introduction

Shortly after U.S. GDP contracted sharply in the fall of 2008 the zero lower bound on the federal funds rate became binding. In the following years, the Congress passed a major fiscal package and the Federal Reserve started using announcements about the future path of the policy rate (“forward guidance”) as one of its main policy tools. These policy actions are frequently justified by three common theoretical results in the New Keynesian literature on the zero lower bound: (1) even a small negative shock can have large adverse effects when the zero lower bound binds, (2) the government spending multiplier is larger when the zero lower bound binds, and (3) a commitment by the central bank to future inflation stimulates current consumption.

It is important to recognize that in a New Keynesian model households’ inflation expectations play a crucial role. Consumption is given by the Euler equation for a nominal asset. The economy eventually returns to the deterministic steady state after the shocks that have been emphasized in the literature on the zero lower bound. Solving the Euler equation forward then implies that current consumption is determined by the sum of current and future real interest rates. Hence, current consumption depends on current and future nominal interest rates and inflation expectations. So long as the nominal interest rate is at zero, the only variable that remains are inflation expectations. Therefore, most of the propagation of shocks at the zero lower bound comes from movements in inflation expectations and monetary and fiscal policy mainly act on consumption by changing inflation expectations.

It therefore seems desirable to model inflation expectations in a way that is consistent with survey data on inflation expectations. One pervasive feature of survey data on inflation expectations is heterogeneity: different households have different inflation expectations. See, for example, Armantier et al. (2011) for the inflation expectations survey of consumers conducted by the Federal Reserve Bank of New York. Another feature of survey data on inflation expectations is that inflation expectations respond sluggishly to realized shocks to future inflation. See Coibion and Gorodnichenko (2012).

In contrast, in any model with perfect information and rational expectations, all agents have the same expectation of aggregate inflation and this expectation responds instantly and one-for-one to realized shocks to future inflation. Since all agents have the same information and the same perceived law of motion of the economy (which equals the actual law of motion of the economy),

all agents have the same expectation of aggregate inflation. Furthermore, since agents know the exact realization of the shock and have correct beliefs about how the shock affects the conditional mean of future inflation, the expectation of future inflation responds instantly and one-for-one to realized shocks to future inflation.

Motivated by the importance of inflation expectations in New Keynesian models with a binding zero lower bound and the tension between empirical and model properties of inflation expectations, this paper solves a New Keynesian model with dispersed information on the side of households. The assumption that households have imperfect information yields the sluggish response of inflation expectations. The assumption that households have different pieces of information yields the heterogeneity in inflation expectations.

The main properties of the model with sticky and heterogeneous inflation expectations are as follows. First, the deflationary spiral in bad states of the world is less severe than under perfect information. Second, communication by the central bank about the state of the economy (even without any change in current or future policy) affects consumption, and the sign of this effect depends on whether or not the zero lower bound binds. Third, a commitment by the central bank to increase future inflation has smaller desirable effects than under perfect information and can even reduce current consumption. Fourth, the government spending multiplier is smaller than under perfect information and can even be negative.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 states the optimality conditions of households and firms. Section 4 contains closed-form solutions of the model under perfect information and under a simple version of dispersed information. Section 5 presents the main results on monetary policy and fiscal policy. Section 6 contains the solution of the model under a more sophisticated version of dispersed information.

2 Model

The economy consists of households, firms, and a government. The government consists of a fiscal authority and a monetary authority. The monetary authority controls the nominal interest rate, but cannot lower the net nominal interest rate below zero. Following Eggertsson and Woodford (2003) and Christiano, Eichenbaum and Rebelo (2011) and many others, I study the effect of a temporary

change in the households' desire to save. The difference to these papers is that households do not have perfect information.

Households. The economy is populated by a continuum of households of mass one. Households are indexed by $i \in [0, 1]$. The preferences of household i are given by

$$E_0^i \left[\sum_{t=0}^{\infty} \beta^t e^{\xi_{i,t}} \left(\frac{C_{i,t}^{1-\gamma} - 1}{1-\gamma} - \frac{L_{i,t}^{1+\psi}}{1+\psi} \right) \right],$$

where $C_{i,t}$ and $L_{i,t}$ are consumption and labor supply of household i in period t . Here E_0^i is the expectation operator conditioned on information of household i in period zero and the variable $\xi_{i,t}$ is a preference shock. The parameters satisfy: $\beta \in (0, 1)$, $\gamma > 0$, and $\psi \geq 0$.

In period zero, each household is hit by a preference shock. The preference shock in period zero has N possible realizations: $\xi_{i,0} \in \{\xi_1, \xi_2, \dots, \xi_N\}$ with $\xi_1 < \xi_2 < \dots < \xi_N$. Different households may experience different realizations of the preference shock: a mass λ_1 of households experiences realization ξ_1 , a mass λ_2 of households experiences realization ξ_2 , and so on. Let $\lambda = (\lambda_1, \dots, \lambda_N)$ with $\sum_{n=1}^N \lambda_n = 1$ denote the cross-sectional distribution of preference shocks in period zero. In the following periods, the variable $\xi_{i,t}$ remains constant with probability μ and returns permanently to its normal value of zero with probability $1 - \mu$. The return to the normal value of zero occurs at the same time for all households. This specification of the preference shock is a generalization of the specification in Eggertsson and Woodford (2003) and Christiano, Eichenbaum and Rebelo (2011): The preference shock has N possible realizations and there may be heterogeneity across households.

The economy is in one of two aggregate states in period zero, called good state and bad state. These two aggregate states differ in terms of the cross-sectional distribution of preference shocks. Let λ^{good} and λ^{bad} denote the cross-sectional distribution of preference shocks in the good state and in the bad state. I assume that the cross-sectional mean of the preference shock is lower in the bad state and negative in both states

$$\sum_{n=1}^N \lambda_n^{bad} \xi_n < \sum_{n=1}^N \lambda_n^{good} \xi_n < 0.$$

Let $\theta \in (0, 1)$ denote the probability of the bad state. Think of the good state as a severe recession and the bad state as the worst recession in a century.

Households can save or borrow by holding (positive or negative amounts of) nominal government bonds. Let $B_{i,t}$ denote the bond holdings of household i between periods t and $t+1$. The evolution

of the household's bond holdings is given by the household's flow budget constraint

$$B_{i,t} = R_{t-1}B_{i,t-1} + W_tL_{i,t} + D_{i,t} - P_tC_{i,t} + Z_{i,t},$$

where R_{t-1} denotes the gross nominal interest rate on bond holdings between periods $t - 1$ and t , W_t is the nominal wage rate in period t , and $D_{i,t}$ denotes the difference between dividends received by the household in period t and nominal lump-sum taxes paid by the household in period t . The term $P_tC_{i,t}$ is the household's consumption expenditure, where P_t is the price of the final good in period t . The household can save or borrow at any point in time, that is, bond holdings can be positive or negative, but the household is not allowed to run a Ponzi scheme. For simplicity, I assume that all households have the same initial bond holdings in period minus one. For simplicity, I also assume that households can trade state-contingent claims with one another in period minus one (i.e., when all households are still identical). Each household is hit by a preference shock in period zero. The state-contingent claims are settled in period $T > 0$ when preference shocks revert back to their normal value of zero. A state-contingent claim specifies a payment to the household who purchased the claim that is contingent on the individual history of the household and the aggregate history of the economy (i.e., $\xi_{i,0}$, λ and T). The term $Z_{i,t}$ in the flow budget constraint is the net transfer associated with these state-contingent claims. This term therefore equals zero in all periods apart from period T . The fact that agents can trade these state-contingent claims in period minus one implies that in equilibrium all households will have the same post-transfer wealth in period T . This simplifies the analysis because one does not have to keep track of the dynamics of the wealth distribution in periods $0 \leq t < T$. A similar assumption is made in Lucas (1990), Lorenzoni (2010), and Curdia and Woodford (2009).

I entertain three different assumptions about the information sets of households. First, I assume that households have perfect information: in every period, all households know the complete history of the economy up to and including the current period. The assumption of perfect information is made in *all* of the existing literature on the zero lower bound. The assumption of perfect information in combination with the assumption of rational expectations implies that all households have the same inflation expectation and households' inflation expectation responds instantaneously to realized shocks to future inflation. In contrast, survey data on inflation expectations shows that households have heterogeneous inflation expectations and inflation expectations respond sluggishly to realized shocks to future inflation. Second, I assume that households observe in period zero their

individual state but not the aggregate state: in period zero, households observe their own preference shock but not the cross-sectional distribution of preference shocks. Households learn what the cross-sectional distribution of preference shocks has been when preference shocks revert back to their normal values. I initially assume that households observe no endogenous variables. This assumption is obviously extreme, but allows me to solve this model with imperfect information analytically. From the closed-form solution it is easy to see how heterogeneity of beliefs and sluggishness of beliefs affects equilibrium outcomes and the effectiveness of monetary and fiscal policy. Third, I introduce information diffusion into the model with imperfect information. Namely, I assume that in every period $t \geq 0$ a constant fraction of households update their information sets. Households that update their information sets are randomly selected and learn the true aggregate state of the economy perfectly. This way knowledge about the true aggregate state of the economy slowly diffuses in the society, as in Gabaix and Laibson (2002) and Mankiw and Reis (2002, 2006).

Firms. The final good is produced by competitive firms using the technology

$$Y_t = \left(\int_0^1 Y_{j,t}^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}}.$$

Here Y_t denotes output of the final good and $Y_{j,t}$ denotes input of intermediate good j . The parameter $\theta > 1$ is the elasticity of substitution between intermediate goods. Final good firms have perfect information and fully flexible prices. Profit maximization of firms producing final goods implies the following demand function for intermediate good j

$$Y_{j,t} = \left(\frac{P_{j,t}}{P_t} \right)^{-\theta} Y_t,$$

where $P_{j,t}$ is the price of intermediate good j and P_t is the price of the final good. Furthermore, the zero profit condition of firms producing final goods implies

$$P_t = \left(\int_0^1 P_{j,t}^{1-\theta} dj \right)^{\frac{1}{1-\theta}}.$$

The intermediate good j is produced by a monopolist using the technology

$$Y_{j,t} = L_{j,t},$$

where $Y_{j,t}$ is output and $L_{j,t}$ is labor input of this monopolist. Monopolists producing intermediate goods have perfect information, but they are subject to a price-setting friction as in Calvo (1983).

Each monopolist can optimize its price with probability $1 - \alpha$ in any given period. With probability α the monopolist producing good j sets the price

$$P_{j,t} = P_{j,t-1}.$$

How firms value profit in different states of the world is determined by the ownership structure. I assume that each monopolist is owned by a single household and that the monopolist takes the marginal utility of consumption of this household as given because the household also owns many other firms.

Monetary policy. At some point in the past, the monetary authority announced that it would follow the rule

$$R_t = \max \left\{ 1, R\Pi_t^\phi \right\}.$$

Here $R = (1/\beta)$ is the nominal interest rate in the non-stochastic steady state with zero inflation, $\Pi_t = (P_t/P_{t-1})$ denotes the inflation rate, and $\phi > 1$ is a parameter. This rule says the following. The monetary authority follows a Taylor rule as long as the Taylor rule implies a non-negative net nominal interest rate. The monetary authority sets the net nominal interest rate to zero otherwise. In period zero, the monetary authority can announce a new rule for the periods $t \geq 0$. More precisely, the monetary authority can announce a new value for R after observing the aggregate state perfectly in period zero.

Fiscal policy. The fiscal authority can purchase units of the final good and finance the purchases with current or future lump-sum taxes. The government budget constraint in period t reads

$$T_t + B_t = R_{t-1}B_{t-1} + P_tG_t.$$

The government has to finance maturing nominal government bonds and any purchases of the final good, denoted G_t . The government can collect lump-sum taxes, denoted T_t , or issue new bonds. In period zero, the fiscal authority announces a fiscal policy for the periods $t \geq 0$ after observing the aggregate state perfectly in period zero.

3 Households' and firms' optimality conditions

Households. The consumption Euler equation in any period $t \geq 0$ reads

$$C_{i,t}^{-\gamma} = E_t^i \left[\beta \frac{e^{\xi_{i,t+1}}}{e^{\xi_{i,t}}} \frac{R_t}{\Pi_{t+1}} C_{i,t+1}^{-\gamma} \right].$$

The first-order condition for optimal labor supply in any period $t \geq 0$ reads

$$E_t^i \left[C_{i,t}^{-\gamma} \frac{W_t}{P_t} - L_{i,t}^\psi \right] = 0.$$

Let T denote the period in which the variable $\xi_{i,t}$ returns permanently to its normal value of one.

In period T the household has quasi-linear preferences. The first-order condition for $Z_{i,T}$ reads

$$C_{i,T}^{-\gamma} = \varphi.$$

To simplify exposition, I assume from now on that the parameter φ equals the marginal utility of consumption in the non-stochastic steady state. Consumption in period T then equals consumption in the non-stochastic steady state.

Log-linearizing the last three equations around the non-stochastic steady state yields

$$c_{i,t} = E_t^i \left[-\frac{1}{\gamma} (\xi_{i,t+1} - \xi_{i,t} + r_t - \pi_{t+1}) + c_{i,t+1} \right], \quad (1)$$

$$E_t^i [w_t - p_t] = \gamma c_{i,t} + \psi l_{i,t}, \quad (2)$$

and

$$c_{i,T} = 0, \quad (3)$$

where small letters denote log-deviations from the non-stochastic steady state with zero inflation. For example, $c_{i,t}$ denotes the log-deviation of consumption of household i in period t from the consumption of the household in the zero-inflation non-stochastic steady state.

Firms. An intermediate good firm j that can adjust its price in period t and is owned by household i sets the price

$$X_{j,t}^i = \arg \max_{P_{j,t} \in \mathbb{R}_{++}} E_t \left[\sum_{s=t}^{\infty} (\alpha\beta)^{s-t} \left(\frac{e^{\xi_{i,s}} C_{i,s}^{-\gamma} P_t}{e^{\xi_{i,t}} C_{i,t}^{-\gamma} P_s} \right) (P_{j,t} - W_s) \left(\frac{P_{j,t}}{P_s} \right)^{-\theta} Y_s \right].$$

Log-linearizing the first-order condition for the adjustment price yields

$$x_{j,t}^i = (1 - \alpha\beta) E_t \left[\sum_{s=t}^{\infty} (\alpha\beta)^{s-t} w_s \right].$$

The log-linearized adjustment price is the same for all adjusting firms and is independent of who owns firms. Therefore, one can drop the subscript j and the superscript i . The last equation can be stated in recursive form as

$$x_t = (1 - \alpha\beta) w_t + \alpha\beta E_t [x_{t+1}].$$

Log-linearizing the equation for the price of the final good presented in Section 2 and using the fact that (i) adjusting firms are selected randomly and (ii) the log-linearized adjustment price is the same for all adjusting firms yields

$$p_t = \int_0^1 p_{j,t} dj = \alpha p_{t-1} + (1 - \alpha) x_t.$$

Using the last equation to substitute for the adjustment prices x_t and x_{t+1} in the previous equation and rearranging yields a standard New Keynesian Phillips curve for our incomplete markets economy

$$\pi_t = \frac{(1 - \alpha)(1 - \alpha\beta)}{\alpha} (w_t - p_t) + \beta E_t [\pi_{t+1}]. \quad (4)$$

4 Households' inflation expectations and equilibrium outcomes at the zero lower bound

This section derives the equilibrium under two benchmark information structures. Under perfect information, a household observes in period zero the own preference shock and the cross-sectional distribution of preference shocks. Under dispersed information, a household observes in period zero only the own preference shock. In this section, the monetary and the fiscal authority cannot make announcements and there is no learning from endogenous variables to exhibit the properties of the model very clearly. In the following sections, I introduce announcements and learning from endogenous variables.

4.1 Perfect information benchmark

Let us first derive the equilibrium under perfect information as a benchmark. Let us begin by rewriting the New Keynesian Phillips curve (4). When households have perfect information, equation (2) implies

$$w_t - p_t = \gamma c_{i,t} + \psi l_{i,t}.$$

Integrating across households and using labor market clearing, the production function of intermediate good j , and the demand function for intermediate good j yields

$$w_t - p_t = (\gamma + \psi) c_t, \quad (5)$$

where c_t denotes aggregate consumption of the final good in period t . Using the last equation to substitute for the real wage rate in the New Keynesian Phillips curve (4) gives

$$\pi_t = \kappa c_t + \beta E_t [\pi_{t+1}], \quad (6)$$

where

$$\kappa = \frac{(1 - \alpha)(1 - \alpha\beta)}{\alpha} (\gamma + \psi).$$

Following Eggertsson and Woodford (2003) and Christiano, Eichenbaum and Rebelo (2011) and many others, I consider equilibria of the following form: endogenous variables take on the same value in all periods $0 \leq t < T$ and endogenous variables take on the same value in all periods $t \geq T$. Equations (3) and (6) then imply that in all periods $t \geq T$ we have

$$c_{i,t} = c_t = \pi_t = 0.$$

What happens in periods $0 \leq t < T$ depends on the realization of the aggregate state in period zero. Let $c_{i,s}$, c_s , π_s , and r_s denote consumption of household i , aggregate consumption, inflation, and the nominal interest rate in periods $0 \leq t < T$ when the aggregate state is s , where $s \in \{good, bad\}$.

It is easy to solve for the endogenous variables in periods $0 \leq t < T$ as a function of the aggregate state s . Since households have perfect information and all variables remain at their current values with probability μ and revert to their steady-state values with probability $1 - \mu$, the consumption Euler equation (1) reduces to

$$c_{i,s} = \frac{-\frac{1}{\gamma}}{1 - \mu} [-(1 - \mu) \xi_{i,0} + r_s - \mu\pi_s].$$

Let $\bar{\xi}_s$ denote the cross-sectional mean of the preference shock in state s . Aggregate consumption in any period $0 \leq t < T$ equals

$$c_s = \frac{-\frac{1}{\gamma}}{1 - \mu} [-(1 - \mu) \bar{\xi}_s + r_s - \mu\pi_s]. \quad (7)$$

Furthermore, the New Keynesian Phillips curve (6) can be expressed as

$$\pi_s = \frac{\kappa}{1 - \beta\mu} c_s, \quad (8)$$

and the monetary policy rule can be written as

$$\ln(R_s) = \max\{0, \ln(R) + \phi\pi_s\}. \quad (9)$$

If the zero bound on the nominal interest rate is binding in state s , substituting equations (8)-(9) into equation (7) yields¹

$$c_s = \frac{\frac{1}{\gamma}\bar{\xi}_s + \frac{1}{1-\mu}\ln(R)}{1 - \frac{1}{1-\mu}\frac{\mu\kappa}{1-\beta\mu}}. \quad (10)$$

If the zero bound is not binding in state s , substituting equations (8)-(9) into equation (7) yields

$$c_s = \frac{\frac{1}{\gamma}\bar{\xi}_s}{1 + \frac{1}{1-\mu}\frac{(\phi-\mu)\kappa}{1-\beta\mu}}. \quad (11)$$

Furthermore, the zero bound is binding if and only if²

$$\pi_s < -\frac{1}{\phi}\ln(R),$$

or equivalently

$$\bar{\xi}_s < -\frac{1}{\phi}\ln(R) \frac{1 + \frac{1}{1-\mu}\frac{(\phi-\mu)\kappa}{1-\beta\mu}}{\frac{\kappa}{1-\beta\mu}\frac{1}{\gamma}}. \quad (12)$$

An important insight in the existing literature on the zero bound is that if the zero bound is binding, the drop in consumption can be arbitrarily large. Formally, the denominator in equation (10) is a difference between two positive numbers and can be arbitrarily small in absolute value.

To understand why the fall in consumption can be so large, I propose the following decomposition. The consumption Euler equation (7) can be written as

$$c_s = \frac{1}{\gamma}\bar{\xi}_s - \frac{1}{1-\mu}r_s + \frac{1}{1-\mu}\mu\pi_s.$$

Aggregate consumption in state s equals the sum of three terms. The first term is the direct effect of the preference shock on consumption. The second term is the effect of the current nominal interest rate on consumption. The third term is the effect of expected inflation on consumption. Substituting the equilibrium nominal interest rate when the zero lower bound is binding (i.e., $r_s \equiv \ln(R_s) - \ln(R) = -\ln(R)$) and equilibrium inflation when the zero lower bound is binding

¹Following common practice in the literature on the zero bound, I assume that parameters are such that the denominator in equation (10) is positive. For the other case, see Mertens and Ravn (2012).

²More precisely, if condition (12) is satisfied, the unique equilibrium is given by equation (10); whereas if condition (12) is not satisfied, the unique equilibrium is given by equation (11).

into the last equation yields

$$c_s = \frac{1}{\gamma} \bar{\xi}_s + \frac{1}{1-\mu} \ln(R) + \frac{1}{1-\mu} \underbrace{\mu \frac{\kappa}{1-\beta\mu} \frac{\frac{1}{\gamma} \bar{\xi}_s + \frac{1}{1-\mu} \ln(R)}{1 - \frac{1}{1-\mu} \frac{\mu\kappa}{1-\beta\mu}}}_{\text{expected inflation}}.$$

The second term is positive because the monetary authority can lower the nominal interest rate to some extent relative to its steady-state value. The first term is negative and simply reflects the direct effect of the preference shock on consumption. The third term is also negative and reflects the indirect effect of the preference shock on consumption acting through inflation expectations. The reason why the fall in consumption can be arbitrarily large for a given size of the shock is the third term. The model of this subsection predicts that the fall in consumption can be arbitrarily large for a given size of the shock *because* the drop in inflation expectations can be arbitrarily large for a given size of the shock. All the amplification of the shock comes through inflation expectations! Hence, how we model inflation expectations is absolutely crucial for results concerning dynamics at the zero lower bound.

4.2 Dispersed information

For ease of exposition, I assume that the equilibrium real wage rate is still given by equation (5). An information-based micro-foundation for this assumption is that the household consists of a shopper and a worker. The shopper makes consumption decisions under dispersed information. The worker makes labor supply decisions under perfect information. I consider again equilibria of the following form: endogenous variables take on the same value in all periods $0 \leq t < T$ and endogenous variables take on the same value in all periods $t \geq T$.

The consumption Euler equation (1) reads

$$c_i = \frac{-\frac{1}{\gamma}}{1-\mu} [-(1-\mu) \xi_{i,0} + E^i[r_S - \mu\pi_S]].$$

Let $\bar{E}_s[r_S - \mu\pi_S] = \int_0^1 E^i[r_S - \mu\pi_S] di$ denote the average expectation of the real interest rate in state s . Furthermore, let $\bar{\xi}_s$ denote the cross-sectional mean of the preference shock in state s .

Aggregate consumption in any period $0 \leq t < T$ in state s equals

$$c_s = \frac{-\frac{1}{\gamma}}{1-\mu} [-(1-\mu) \bar{\xi}_s + \bar{E}_s[r_S - \mu\pi_S]]. \quad (13)$$

Let \bar{p}_s^{good} denote the average probability that households assign to being in the good state when the economy is actually in state s . Let \bar{p}_s^{bad} denote the average probability that households assign to being in the bad state when the economy is actually in state s . The average expectation of the real interest rate in state s equals

$$\bar{E}_s [r_S - \mu\pi_S] = \bar{p}_s^{good} (r_{good} - \mu\pi_{good}) + \bar{p}_s^{bad} (r_{bad} - \mu\pi_{bad}). \quad (14)$$

Under perfect information, households are absolutely certain about the current state of the economy. Therefore, $\bar{p}_{good}^{good} = \bar{p}_{bad}^{bad} = 1$, and the average expectation of the real interest rate when the economy is actually in the good state equals

$$\bar{E}_{good} [r_S - \mu\pi_S] = r_{good} - \mu\pi_{good},$$

and the average expectation of the real interest rate when the economy is actually in the bad state equals

$$\bar{E}_{bad} [r_S - \mu\pi_S] = r_{bad} - \mu\pi_{bad}.$$

By contrast, under imperfect information, households assign some probability to the wrong state and this changes the average expectation of the real interest rate. I am now going to derive the implications for dynamics at the zero lower bound and the effect of different policies at the zero lower bound.

The New Keynesian Phillips curve and the monetary policy rule are again given by equations (8) and (9). An important difference to the previous subsection is that the outcome in the good state now affects the outcome in the bad state (and vice versa) through equation (14). For these reason, we now have to distinguish three cases: (i) the zero lower bound is binding in both states, (ii) the zero lower bound is binding in no state, and (iii) the zero lower bound is binding in the bad state but not in the good state.

Let us begin with the case where the zero bound on the nominal interest rate is binding in both states. Stating equation (13) for the good state and the bad state and substituting in equation (14), equation (8), and $\ln(R_{good}) = \ln(R_{bad}) = 0$ yields a system of two equations in the two unknowns c_{good} and c_{bad} . The solution can be stated as

$$c_{good} = \frac{\frac{1}{\gamma}\bar{c}_{good} + \frac{1}{1-\mu}\ln(R)}{1 - \frac{1}{1-\mu}\frac{\mu\kappa}{1-\beta\mu}} - \bar{p}_{good}^{bad} \frac{\frac{1}{1-\mu}\frac{\mu\kappa}{1-\beta\mu}}{1 - \frac{1}{1-\mu}\frac{\mu\kappa}{1-\beta\mu}} (c_{good} - c_{bad}), \quad (15)$$

$$c_{bad} = \frac{\frac{1}{\gamma}\bar{\xi}_{bad} + \frac{1}{1-\mu}\ln(R)}{1 - \frac{1}{1-\mu}\frac{\mu\kappa}{1-\beta\mu}} + \bar{p}_{bad}^{good} \frac{\frac{1}{1-\mu}\frac{\mu\kappa}{1-\beta\mu}}{1 - \frac{1}{1-\mu}\frac{\mu\kappa}{1-\beta\mu}} (c_{good} - c_{bad}), \quad (16)$$

and

$$c_{good} - c_{bad} = \frac{\frac{1}{\gamma}(\bar{\xi}_{good} - \bar{\xi}_{bad})}{1 - \frac{1}{1-\mu}\frac{\mu\kappa}{1-\beta\mu}} > 0. \quad (17)$$

$$1 + \left(\bar{p}_{good}^{bad} + \bar{p}_{bad}^{good} \right) \frac{\frac{1}{1-\mu}\frac{\mu\kappa}{1-\beta\mu}}{1 - \frac{1}{1-\mu}\frac{\mu\kappa}{1-\beta\mu}}$$

When the zero lower bound is binding in both states, dispersed information on the side of households increases consumption in the bad state and reduces consumption in the good state. To see this, compare equations (15)-(17) to equation (10). The reason is that in the bad state households are now assigning some probability to a state with a lower real interest rate. This reduces the fall in consumption and the fall in inflation in the bad state.

Let us turn to the case where the zero bound on the nominal interest rate is binding in no state. Stating equation (13) for the good state and the bad state, and substituting in equation (14), equation (8), and $\ln(R_{good}) = \ln(R) + \phi\pi_{good}$ as well as $\ln(R_{bad}) = \ln(R) + \phi\pi_{bad}$ yields a system of two equations in the two unknowns c_{good} and c_{bad} . The solution can be stated as

$$c_{good} = \frac{\frac{1}{\gamma}\bar{\xi}_{good}}{1 + \frac{1}{1-\mu}\frac{(\phi-\mu)\kappa}{1-\beta\mu}} + \bar{p}_{good}^{bad} \frac{\frac{1}{1-\mu}\frac{(\phi-\mu)\kappa}{1-\beta\mu}}{1 + \frac{1}{1-\mu}\frac{(\phi-\mu)\kappa}{1-\beta\mu}} (c_{good} - c_{bad}), \quad (18)$$

$$c_{bad} = \frac{\frac{1}{\gamma}\bar{\xi}_{bad}}{1 + \frac{1}{1-\mu}\frac{(\phi-\mu)\kappa}{1-\beta\mu}} - \bar{p}_{bad}^{good} \frac{\frac{1}{1-\mu}\frac{(\phi-\mu)\kappa}{1-\beta\mu}}{1 + \frac{1}{1-\mu}\frac{(\phi-\mu)\kappa}{1-\beta\mu}} (c_{good} - c_{bad}), \quad (19)$$

and

$$c_{good} - c_{bad} = \frac{\frac{1}{\gamma}(\bar{\xi}_{good} - \bar{\xi}_{bad})}{1 + \frac{1}{1-\mu}\frac{(\phi-\mu)\kappa}{1-\beta\mu}} > 0. \quad (20)$$

$$1 - \left(\bar{p}_{good}^{bad} + \bar{p}_{bad}^{good} \right) \frac{\frac{1}{1-\mu}\frac{(\phi-\mu)\kappa}{1-\beta\mu}}{1 + \frac{1}{1-\mu}\frac{(\phi-\mu)\kappa}{1-\beta\mu}}$$

When the zero lower bound is binding in no state, dispersed information on the side of households reduces consumption in the bad state and increases consumption in the good state. To see this, compare equations (18)-(20) to equation (11). The reason is that in the bad state households are now assigning some probability to a state with a higher real interest rate, due to the Taylor principle. This increases the fall in consumption and the fall in inflation in the bad state.

Finally, when the zero bound on the nominal interest rate is binding only in the bad state (both under dispersed information and under perfect information), one can solve again analytically for consumption in the good state, c_{good} , and consumption in the bad state, c_{bad} . However, the solution has a substantially more complicated structure than in the two cases presented above. To understand the effect of households' dispersed information on equilibrium consumption in this case, it is useful to state the outcome under dispersed information relative to the outcome under perfect information. Let c_s and c_s^* denote equilibrium consumption in state s under dispersed information and under perfect information, respectively. Let $r_s - \mu\pi_s$ and $r_s^* - \mu\pi_s^*$ denote the equilibrium real interest rate in state s under dispersed information and under perfect information, respectively. The consumption Euler equation (13) implies that

$$c_{good} - c_{good}^* = \frac{-\frac{1}{\gamma}}{1-\mu} [\bar{E}_{good} [r_S - \mu\pi_S] - (r_{good}^* - \mu\pi_{good}^*)], \quad (21)$$

and

$$c_{bad} - c_{bad}^* = \frac{-\frac{1}{\gamma}}{1-\mu} [\bar{E}_{bad} [r_S - \mu\pi_S] - (r_{bad}^* - \mu\pi_{bad}^*)]. \quad (22)$$

Deducting the last equation from the previous equation and using equation (14) for the average expectation of the real interest rate in state s yields

$$(c_{good} - c_{bad}) - (c_{good}^* - c_{bad}^*) = \frac{-\frac{1}{\gamma}}{1-\mu} \left[\left[1 - \left(\bar{p}_{good}^{bad} + \bar{p}_{bad}^{good} \right) \right] [(r_{good} - \mu\pi_{good}) - (r_{bad} - \mu\pi_{bad})] - \left(r_{good}^* - \mu\pi_{good}^* \right) + (r_{bad}^* - \mu\pi_{bad}^*) \right].$$

Finally, using the New Keynesian Phillips curve (8) and using the assumption that the zero lower bound is binding in the bad state but not in the good state (both under dispersed information and under perfect information) yields

$$\begin{aligned} & (c_{good} - c_{bad}) - (c_{good}^* - c_{bad}^*) \\ = & -\frac{\frac{1}{\gamma}}{1-\mu} \left[1 - \left(\bar{p}_{good}^{bad} + \bar{p}_{bad}^{good} \right) \right] \frac{\phi (\pi_{bad} - \pi_{bad}^*)}{1 + \frac{\frac{1}{\gamma}}{1-\mu} \left[1 - \left(\bar{p}_{good}^{bad} + \bar{p}_{bad}^{good} \right) \right] \frac{(\phi-\mu)\kappa}{1-\beta\mu}} \\ & + \frac{\frac{1}{\gamma} \left(\bar{p}_{good}^{bad} + \bar{p}_{bad}^{good} \right)}{1 + \frac{\frac{1}{\gamma}}{1-\mu} \left[1 - \left(\bar{p}_{good}^{bad} + \bar{p}_{bad}^{good} \right) \right] \frac{(\phi-\mu)\kappa}{1-\beta\mu}} [(r_{good}^* - \mu\pi_{good}^*) - (r_{bad}^* - \mu\pi_{bad}^*)]. \end{aligned} \quad (23)$$

The last equation can also be stated as

$$\begin{aligned}
& (c_{good} - c_{bad}) - (c_{good}^* - c_{bad}^*) \\
= & - \frac{\frac{1}{1-\mu} \left[1 - \left(\bar{p}_{good}^{bad} + \bar{p}_{bad}^{good} \right) \right] \phi \left(\pi_{good} - \pi_{good}^* \right)}{1 - \frac{1}{1-\mu} \left[1 - \left(\bar{p}_{good}^{bad} + \bar{p}_{bad}^{good} \right) \right] \frac{\mu\kappa}{1-\beta\mu}} \\
& + \frac{\frac{1}{1-\mu} \left(\bar{p}_{good}^{bad} + \bar{p}_{bad}^{good} \right)}{1 - \frac{1}{1-\mu} \left[1 - \left(\bar{p}_{good}^{bad} + \bar{p}_{bad}^{good} \right) \right] \frac{\mu\kappa}{1-\beta\mu}} \left[(r_{good}^* - \mu\pi_{good}^*) - (r_{bad}^* - \mu\pi_{bad}^*) \right]. \quad (24)
\end{aligned}$$

From equations (21)-(24) one can directly derive the following results. First, if the perfect-information equilibrium real interest rate in the good state equals the perfect-information equilibrium real interest rate in the bad state (i.e., $r_{good}^* - \mu\pi_{good}^* = r_{bad}^* - \mu\pi_{bad}^*$), then households' dispersed information has no effect on equilibrium consumption (i.e., $c_{good} = c_{good}^*$ and $c_{bad} = c_{bad}^*$).³ Second, if under perfect information the real interest rate in the good state exceeds the real rate in the bad state (i.e., $r_{good}^* - \mu\pi_{good}^* > r_{bad}^* - \mu\pi_{bad}^*$), then households' dispersed information pulls consumption in the two states apart (i.e., $c_{good} - c_{bad} > c_{good}^* - c_{bad}^*$) and reduces consumption in the bad state (i.e., $c_{bad} < c_{bad}^*$). Third, if under perfect information the real interest rate in the bad state exceeds the real rate in the good state (i.e., $r_{good}^* - \mu\pi_{good}^* < r_{bad}^* - \mu\pi_{bad}^*$), then households' dispersed information pushes consumption in the two states together (i.e., $c_{good} - c_{bad} < c_{good}^* - c_{bad}^*$) and increases consumption in the bad state (i.e., $c_{bad} > c_{bad}^*$).

4.3 The effect of central bank communication

Before studying monetary and fiscal policy, let us study the effect of central bank communication. Suppose the central bank announces the aggregate state in period zero and this makes households assign a larger probability to the correct state. Equations (15)-(17) and (18)-(20) immediately imply the following result. When the zero bound on the nominal interest rate is binding in both states, announcing the aggregate state reduces consumption in the bad state and increases consumption

³It is interesting to note several features of the case $r_{good}^* - \mu\pi_{good}^* = r_{bad}^* - \mu\pi_{bad}^*$. The perfect-information equilibrium real interest rate can be the same in the two states only if the zero lower bound is binding in the bad state but not in the good state. The equilibrium real interest rate is the same in the two states, but equilibrium inflation is not the same in the two states. Hence, under dispersed information, households have heterogeneous inflation expectations.

in the good state. The reason is that the central bank partially undoes the effect of dispersed information and dispersed information helps to keep consumption high in the bad state.

By contrast, when the zero bound on the nominal interest rate is binding in no state, announcing the aggregate state increases consumption in the bad state and decreases consumption in the good state. The reason is again that the central bank partially undoes the effect of dispersed information, but now dispersed information amplifies the fall in consumption in the bad state.

Hence, central bank communication (without a change in current or future policy) affects consumption and the sign of this effect depends on whether or not the zero lower bound is binding. An announcement that increases consumption when the zero lower bound is not binding reduces consumption when the zero lower bound is binding.

5 Monetary policy and fiscal policy

Announcing a new monetary policy or a new fiscal policy in period zero has two effects. First, the policy has the usual effect that is also present in a model with perfect information. Second, the policy has a new effect that is only present in a model with imperfect information on the side of private agents: The policy announcement can make private agents assign a higher probability to the correct state. This second effect can dominate and therefore a commitment to future inflation can reduce consumption today and the government spending multiplier can be negative.

6 Information diffusion

The model can also be solved with information diffusion, as in Mankiw and Reis (2003). As before, all households observe their own preference shocks in period zero and learn what the aggregate state has been in period T . In addition, in every period a constant fraction of randomly selected households updates their information to perfect information.

The equations characterizing equilibrium can be written as a linear difference equation

$$A_t x_t = b + B x_{t+1}, \tag{25}$$

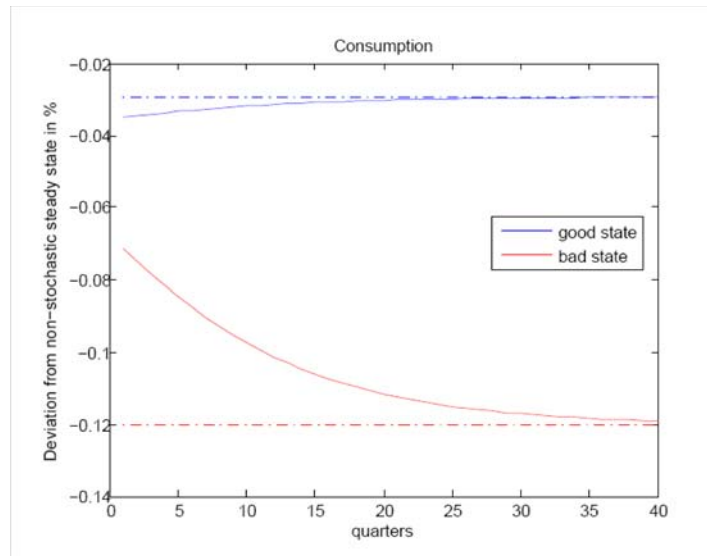
where

$$x_t = \begin{pmatrix} c_{good,t}^{hi} \\ c_{bad,t}^{hi} \\ c_{good,t}^{li} \\ c_{bad,t}^{li} \\ c_t^{hu} \\ c_t^{lu} \\ c_{good,t} \\ c_{bad,t} \\ \pi_{good,t} \\ \pi_{bad,t} \end{pmatrix}. \quad (26)$$

There are four types of households in any given period $0 \leq t < T$, because a household has had a high or a low preference shock in period zero and has perfect or imperfect information in period t . In the following, a household with a high preference shock in period zero and perfect information in period t is called a “high, informed” household in period t , a household with a high preference shock in period zero and imperfect information in period t is called a “high, uninformed” household in period t , and so on. The first and second equation in (25) are the consumption Euler equation of a “high, informed” household in the good state and the bad state, respectively. The third and fourth equation in (25) are the consumption Euler equation of a “low, informed” household in the good state and the bad state, respectively. The fifth and six equation in (25) are the consumption Euler equation of a “high, uninformed” household and a “low, uninformed” household, respectively. The next two equations characterize aggregate consumption in the good state and the bad state, respectively, by aggregating across households. Since the fraction of households with perfect information changes over time, the coefficients in these two equations are a function of t . For this reason, the matrix A_t is a function of t . Finally, the last two equations in (25) are the New Keynesian Phillips curve in the good state and the bad state, respectively.

The following figure shows the solution of the model for the following parameter values. The preference parameters are $\beta = 0.99$ and $\gamma = 1$. The preference shock parameters are $\xi_L = -0.085$, $\xi_H = -0.05$ and $\mu = 0.8$. The fraction of high types in the good state and the bad state equals $\lambda^{good} = 0.8$ and $\lambda^{bad} = 0.2$, respectively. The unconditional probability of the bad state is $\theta = 0.1$.

That is, the bad state is an unusual event. The slope of the Phillips curve equals $\kappa = 0.04$ and the coefficient on inflation in the Taylor rule equals $\phi = 1.5$. These are standard values. Finally, in every period $0 \leq t < T$ 10 percent of households update their information to perfect information. For these parameter values, the zero lower bound binds in both states under perfect information and also under slow information diffusion. The dashed lines in the following figure show equilibrium aggregate consumption under perfect information in the good state and in the bad state. The solid lines show equilibrium aggregate consumption in the case of slow information diffusion. The sluggish response of households' inflation expectations keeps consumption high in the bad state. Communicating the aggregate state perfectly to households (without any change in current or future policy) would reduce aggregate consumption in the bad state because it would lower the average inflation expectation of households in the bad state.



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