Primary Market Design in the Presence of When-Issued Markets*

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JEL Classifications: G32

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1 Introduction

In this paper we analyze mechanisms for selling unseasoned securities in primary markets. These mechanisms are sets of rules that map reports from potential buyers into prices and allocations. The rules effectively specify a game in which investors participate by submitting reports (often called bids). The design of pricing mechanisms has been studied in the context of Treasury auctions and initial public offerings (IPOs) of equity securities.[1] The distinguishing feature of our analysis is that we consider the mechanism participants' reports as one, but not necessarily the only source information that can be mapped into prices and allocations. Another potential source of such information is when-issued trading of the securities, i.e., trading of forward contracts with a delivery date after the securities are issued. Such trading regularly takes place in a number of primary markets, such as US and European Treasury markets and European IPO markets.

Our analysis is motivated by the observation that, even though when-issued trading may reveal information about the value of securities issues, such information is not always used for pricing the issues. In many markets when-issued trading continues after bids are no longer being accepted by the seller, but before the securities are priced and allocated. Such post-bid when-issued trading may thus reveal relevant information beyond what is contained in the bids, but in US Treasury issues such information is not used to determine either prices or allocations. Instead, the pricing and allocation of Treasury issues is fully determined by auction bids.[2] There are, however, other primary markets where when-issued trading does affect the pricing of securities issues. IPOs are commonly priced by means of "bookbuilding" mechanisms in which underwriters (acting on behalf of the issuers) elicit "indications of interest" from investors. There are no strict pricing rules, and the underwriters can exercise discretion in order to condition their pricing decisions both on the indications of interest and on information revealed through when-issued trading.[3] Cornelli, Ljungqvist and Goldreich (2006) and Aussenegg, Pichler and Stomper (2006)

[1]See Back and Zender (1993), Benveniste and Spindt (1989), Biais, Bossaerts and Rochet (2002), and Maksimovic and Pichler (2006) for a few references.

[2]Any when-issued trading that takes place prior to the auction can of course affect the bids and so affect the auction outcome. But, any new information that is revealed through post-auction when-issued trading is precluded from affecting the auction outcome. See the following section for a discussion of the relative importance of post-auction when-issued trading.

[3]Even in IPOs that use auctions the allocation rule is typically well specified, but the pricing rule is not. Thus, even in auctioned IPOs information other than the bids can affect the issue price. See www.wrhambrecht.com for a description of their OpenIPO auction.

find evidence that the pricing of European IPOs takes into account information contained in when-issued market prices.

Our objective in this article is to both explain why we observe such differences in mechanisms for pricing unseasoned securities, and to analyze general advantages and disadvantages of allowing the pricing to depend on when-issued market prices. We consider a generic model of a securities issue, and compare pricing mechanisms of two types. "Constrained" mechanisms are similar to Treasury auctions in that the issuer commits to set the price and allocations based only on the participants' bids. The participants on their part commit to accept the price and allocations, as long as the securities are not overpriced relative to the bids. "Unconstrained" mechanisms are similar to methods used for pricing IPOs: no commitments are made by any of the players, and the issue price and allocations can depend both on the participants' bids and on information that is revealed through when-issued trading. For both types of mechanism we restrict the design of our mechanism by assuming that prices must be uniform. This assumption matches a key institutional feature of these markets and allows us to focus on our main question.

After determining the optimal mechanism of each type, we next determine which type of mechanism, constrained or unconstrained, is optimal. Our mechanism design problem thus consists of three parts: a choice of mechanism type (constrained or unconstrained) and optimal pricing and allocation rules, given that type.

A key difference between Treasury security issues and IPOs is that active when-issued trading accompanies virtually all Treasury issues, but only a fraction of IPOs.[4] We incorporate this difference into our model by including as one of our model parameters the probability that when-issued trading will take place for a given issue.[5] We show that this probability is the key determinant in choosing between a constrained and unconstrained mechanism type. If the probability that when-issued trading will take place is sufficiently low, then an unconstrained mechanism type is unequivocally optimal. Otherwise, a constrained mechanism type may be optimal; in this case the choice of mechanism type depends on other parameters in addition to the probability of when-issued trading.

Our analysis proceeds in several steps. We first show that the probability with which when-issued trading will take place is a key determinant of the structure of the optimal

[4]See the following section for a description of existing markets and empirical evidence.

[5]For much of our analysis we assume that this probability is exogenously given. We ease this assumption and allow the probability to be endogenous in a later section.

allocation rule in unconstrained mechanisms. If this probability is sufficiently low, then the optimal allocation rule satisfies the standard "implementability" condition: investors' allocations are nondecreasing in their bids.[6] If, however, the probability of when-issued trading is sufficiently high, then larger allocations must be made to investors whose reports are more consistent with information revealed through the trading, if it takes place. In this case, allocations may be decreasing in bids. A different result is obtained for constrained mechanisms, i.e., mechanisms where the issue price and allocations cannot depend on information revealed through when-issued trading. For constrained mechanisms, optimal allocations are always nondecreasing in investors' bids.

The intuition for the structural differences between constrained and unconstrained pricing mechanisms follows from the effect that the possibility of when-issued trading has on investors' incentives to truthfully report their private information. In a constrained mechanism, the participants know that their bids will determine the primary market price. In an unconstrained mechanism, the participants have incentives to free-ride on price discovery in when-issued trading. If such trading is predicted to take place with a sufficiently high probability, then investors may decide not to condition their bidding on their own private information. They may instead try to simply maximize their expected allocations since they know that the issue will be (at least weakly) underpriced, relative to the when-issued market. This incentive to free-ride is eliminated by conditioning allocations on the extent to which investors' bids are consistent with the prices that are subsequently observed in when-issued trading, if such trading takes place.

We next consider the optimal choice of mechanism type: constrained or unconstrained. When making this choice, the issuer faces a trade-off. On the one hand, an unconstrained pricing mechanism benefits the issuer because when-issued trading is a cost-free source of information. On the other hand, the use of such information can reduce informed investors' incentives to truthfully report their own information (as discussed above). We show that the optimal resolution of this trade-off depends on the probability that when-issued trading will take place. If this probability is sufficiently low, then the use of an unconstrained mechanism is unequivocally optimal. If instead, when-issued trading is very likely to take place, then a constrained mechanism may be optimal. That is, the issuer may be best off pre-committing not to use any information that may be revealed by such

[6] See Myerson (1981), Krishna (2002), Maskin and Riley (1989) and Bennouri and Falconieri (2008) for discussions of this condition. We derive in the paper the precise cut-off value for the probability.

trading. These results are consistent with the stylized facts mentioned above. Constrained mechanisms are indeed used in Treasury markets, where when-issued trading regularly takes place, but not in IPO markets, where when-issued trading occurs less regularly.

Throughout most of the paper, in the interest of tractability, we assume that the probability of when-issued trading is exogenous. There is, however, evidence that when-issued trading occurs endogenously in IPO markets:[7] such trading never starts before the publication of offering prospectuses that contain information which has been elicited from prospective investors, and may not start at all if there is insufficient information. In an extension of our analysis, we show that the possibility of endogenous market failure provides additional incentives for investors to truthfully report their private information. We thus find a further rationale for the unconstrained mechanism type to be optimal in IPO markets.

This paper extends the existing literature on the design of mechanisms for pricing financial securities. Previous contributions to this literature are Biais, Bossaerts and Rochet (2002), Maksimovic and Pichler (2006), and a growing literature that shows how the optimal design of mechanisms depends on an issuer's ability to allocate to non-strategic agents who do not submit pricing relevant reports.[8] Effects of when-issued trading of IPOs have been analyzed by Cornelli, Ljungqvist and Goldreich (2006), while Viswanathan and Wang (2000) and Nyborg and Strebulaev (2004) analyze effects of when-issued trading of Treasury issues.[9] None of these papers explains the differences between IPO markets and Treasury markets that motivate our analysis. In terms of this research agenda, our paper is most closely related to Habib and Ziegler (2007) who also explain observed differences in the types of mechanisms used to issue different types of securities. Habib and Ziegler show that it is optimal to price some types of security issues in a way such that investors do not have incentives to gather private information before bidding. We make a complementary point. We consider investors who are serendipitously informed and point out that pricing an issue conditional on information revealed through when-issued trading reduces investors' incentives to condition their bids on private information. Our paper differs from all of the above papers in that we consider an outside source of

[7]See Cornelli, Ljungqvist and Goldreich (2006) and Aussenegg, Pichler and Stomper (2006).

[8]See Wang and Zender (2002), Maksimovic and Pichler (2006) and Bennouri and Falconieri (2008).

[9]Bikchandani and Huang (1993), Simon (1994) and Nyborg and Sundaresan (1996) also discuss concerns regarding the interaction of the when-issued market and auctions for U.S. Treasury securities. Simon (1994) presents evidence that participants in the auction and the post-auction when-issued market possess private information regarding demand for securities.

information (in addition to the prospective buyers) and we ask the question of whether the seller should optimally incorporate this information into the pricing and allocation rules.

The paper is organized as follows. In the next section, we provide a brief description of some existing primary markets and the mechanisms used to price and allocate financial securities. In the third section, we present the basic model and a "benchmark mechanism", which is optimal if the probability of when-issued trading is zero. Our main results appear in Section 4, in which we determine the optimal mechanism for eliciting information from investors, assuming that when-issued trading takes place with an exogenously given probability. In Section 5 we allow this probability to be endogenously determined. In Section 6 we provide concluding remarks.

2 A selective survey of when-issued trading and pricing mechanisms in financial markets

In this section, we briefly survey the structure of some primary financial markets, with a focus on when-issued trading. When-issued markets are forward markets for trading in not-yet-issued securities. The forward contracts represent commitments to trade when, and if, a security is issued. Net selling in these markets is, by definition, short selling.

When-issued trading of Treasury securities: Bikhchandani and Huang (1993) and Nyborg and Sundaresan (1996) describe institutional features of the primary market for U.S. Treasury securities, including the market for when-issued trading of Treasuries. The contracts in this market specify physical delivery of the underlying security. Trading of these contracts starts on the date of the announcement of a Treasury auction and continues after the auction takes place, up until the issue date.[10] The issue price and the allocations to auction participants are determined by the bids in the auction.[11] The pricing of issues can depend on information revealed by when-issued trading, but only to the extent that this information is contained in the auction bids. The bidding however regularly closes before the end of when-issued trading.

It is generally accepted that the fundamental valuation of government securities typ-

[10] When-issued trading of Treasury securities was prohibited prior to 1970 and again between 1977 and 1981. Restrictions on when-issued trading were removed at the suggestion of the dealers who argued that such trading "would facilitate price discovery and new-issue distribution." Garbade (2004), p43.

[11] See Garbade and Ingher (2005) for more details.

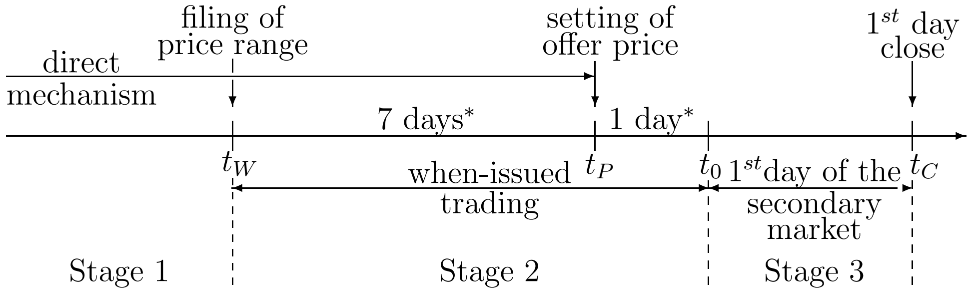

Figure 1: The German IPO Pricing Process
Source: Aussenegg, et al. (2006)

ically does not involve significant amounts of private information. However, the price of any given issue can be strongly affected by demand, and information about demand may be privately held by auction participants. Hortaçsu and Sareen (2005), using data from Canadian Treasury auctions, provide evidence that this is indeed the case. Both Nyborg and Sundaresan (1996) and Hortaçsu and Sareen (2005) provide evidence that when-issued trading prior to the auction is not very liquid. Thus, it appears that any significant information that is revealed by when-issued trading is revealed after the auction closes.

When-issued trading of IPO shares: When-issued trading also occurs in IPO markets, but (for regulatory reasons) mostly outside the U.S.[12] Cornelli, Goldreich and Ljungqvist (2006) report that most German and Italian IPOs trade in when-issued markets while when-issued trading is less common in other European countries (such as France or Sweden). The market with the highest incidence of when-issued trading is the German IPO market. This market has been separately analyzed in Löffler, Panther and Theissen (2005) and Aussenegg, Pichler and Stomper (2006).

Figure 1 presents a time-line of the IPO pricing process in the German market. There are three stages. During Stage 1 (prior to the posting of the price range), underwriters gather information that is at least partially released when they file the preliminary offering prospectus. When-issued trading opens at time t_W, i.e., after the prospectus has been filed. This aspect of the timeline is typical of when-issued trading in European IPO markets.

[12]U.S. Regulation M, Rule 105, which became effective on March 4, 1997, prohibits the covering of short positions in IPO shares that were created within the last five days before pricing, with allocations received in the IPO. This prohibition effectively prevents when-issued trading. In addition to this rule, there are also restrictions on trading in unregistered shares.

The trading never opens before the filing of the preliminary offering prospectus.[13] As in U.S. Treasury markets, when-issued trading continues up to the issue date.

The empirical evidence suggests that when-issued markets are informative, and that these markets generate information that affects the IPO offering prices. Löffler, Panther and Theissen (2005) find that the final prices in the when-issued market are unbiased predictors of opening prices in the secondary market. Aussenegg, Pichler and Stomper (2006) and Cornelli, Goldreich and Ljungqvist (2006) present evidence that information revealed through when-issued trading is incorporated into the pricing of bookbuilt issues. The latter evidence is consistent with the word on the street. According to one of the largest market makers in the German when-issued market for IPO shares: "By observing when-issued trading, the underwriter can gauge the market's interest in an IPO."[14] Moreover, underwriters can respond to information revealed through when-issued trading since IPOs are not priced according to stringent rules. Underwriters are free to base their pricing decisions on both information that they obtain directly from investors, and information revealed through when-issued trading.

3 The Benchmark: No When-Issued Trading

In this section we present the basic model and we determine the optimal selling mechanism without when-issued trading. We begin by describing the information structure and formalizing a standard rationale for eliciting valuation information from selected investors: minimizing the underpricing ("lemon's discount") required in order to sell the issue to investors who face adverse selection risk. A list of variables and their definitions is given in Table 2 in the Appendix.

3.1 The Basic Model

An issuer wishes to maximize expected proceeds from selling a fixed number of securities. Potential investors are risk neutral, and so are willing to purchase securities as long as the expected return is nonnegative. $\tilde{V}$ is the unknown secondary market value of the *total* offering: per security value times the number of securities sold. An investor who

[13]We thank Gary Beechener of Tullett & Tokyo Liberty (securities) Ltd. for providing this information.

[14]This quote was taken from the website of Schnigge AG, ://www.schnigge.de/info/service/pre-ipo-trading.html. The original quote was in German: "Der Emissionsführer kann auf Grund der Handelstätigkeit im Handel per Erscheinen das Interesse des Marktes an der Neuemission messen."

purchases securities obtains a fraction of this value. $\tilde{V}$ is given by:

$$\tilde{V} = v_0 + \tilde{s}w, \tag{1}$$

where v_0 is the prior expected value of $\tilde{V}$ and w is a positive parameter that is strictly smaller than v_0. $\tilde{s}$ is a random variable that can take on one of two realizations, $s \in \{-1, 1\}$. The prior probability that $s = 1$ is $\pi_0 = 1/2$.

A fraction α ($0 < \alpha < 1/2$) of all potential investors are privately informed. We assume that these investors are serendipitously informed. Our focus is thus on how this information is used, rather than on the acquisition of the information. Each informed investor has observed a noisy signal of $\tilde{s}$. The signal of investor i is a random variable $\tilde{\varsigma}_i$ that can take on one of two realizations, $\varsigma_i \in \{-1, 1\}$. Conditional on the realization of $\tilde{s}$, the signals $\tilde{\varsigma}_i$ and $\tilde{\varsigma}_j$ of any two informed investors i and j are independent of each other and identically distributed. With probability $q > 1/2$, any given informed investor has correctly observed the realization of $\tilde{s}$. For an investor who sees a positive signal, the probability that $s = 1$ is q and the probability that $s = -1$ is $1 - q$, so that the expected value of $\tilde{s}$ is $q - (1 - q) = 2q - 1 > 0$. For an investor who sees a negative signal, the expected value of $\tilde{s}$ is $1 - 2q < 0$. On average, a fraction $q\alpha$ of investors will have correctly observed the realization of $\tilde{s}$ and $(1 - q)\alpha$ will have observed $-\tilde{s}$. Model (1) and the parameters v_0, w, α and q are all common knowledge.

We do not distinguish between the issuer and any intermediary, such as an underwriter, who may assist in the security issuance process. We effectively treat these individuals as a single agent, and in what follows we consistently use the term issuer to refer to this agent. The securities offering is a "public" offering. This means that shares may be allocated to the general public, typically referred to as "retail investors". Retail investors include both informed and uninformed investors, as described above. To ensure that a retail allocation can be successfully placed, the issue may have to be priced at a discount so that uninformed investors are willing to purchase securities. The issue price, p_I, must be less than the expected value of the issue: $p_I \leq E[\tilde{V}] - u_{AS}$, where $u_{AS} > 0$ is the underpricing required to compensate uninformed investors for adverse selection risk that they bear due to the presence of informed investors in the market.[15]

[15]Fewer informed investors place orders when an issue is overpriced ($s = -1$) than when it is underpriced ($s = 1$). Thus, uninformed investors are more likely to receive allocations if the issue is overpriced than if it is underpriced. If the issue were priced at its expected value, $E[\tilde{V}]$, the uninformed investors' expected return would be negative, and they would refuse to participate.

Our model of expected underpricing due to adverse selection risk is a simplified version of the model in Rock (1986). Underpricing is a function of the information asymmetry that remains between informed and uninformed investors, conditional on the issue price. This quantity is summarized by the parameter π_p, which is the probability that $s = 1$, given all of the information that is used in the primary market pricing process. In our model, this information is summarized by the issue price as a sufficient statistic. The level of underpricing due to adverse selection risk, derived in the Appendix, is:

$$u_{AS} = \frac{q - 2\pi_p(1-\pi_p) - (2\pi_p - 1)^2 q}{1 - ((2\pi_p - 1)q + 1 - \pi_p)\alpha}\, \alpha w \,. \tag{2}$$

If π_p is equal to the prior, $\pi_0 = 1/2$, then no new information has been incorporated into the issue price. If $\pi_p = 0$ or $\pi_p = 1$, then perfect information about the realization of $\tilde{s}$ has been incorporated into the issue price, and as can be easily verified from equation (2), $u_{AS} = 0$. This makes sense because incorporating perfect information into the price fully eliminates the informational asymmetry among investors. We show in the Appendix that u_{AS} is decreasing in $|\pi_p - \pi_0|$ which is a measure of the extent to which the pricing process has ameliorated the informational asymmetry. In the remainder of the paper we refer to u_{AS} as underpricing due to "residual adverse selection risk". The term "residual" is used because this is adverse selection risk that remains after some information has been elicited from a set of informed investors, and used to price the issue. We take the issuer's desire to minimize u_{AS} as the rationale for the use of a mechanism to elicit information prior to pricing the issue.

3.2 The Benchmark Mechanism

We assume that the issuer can identify a number of "regular" investors who are privately informed.[16] In Treasury issues the regular investors are the primary dealers; in equity issues they are typically institutional investors. The issuer can elicit the regular investors' private information through a mechanism, and then allocate securities to these investors. The issuer may also allocate securities to retail investors from whom the issuer does not attempt to elicit information. The offering is, however, restricted to be uniform price, so

[16]This assumption is not inconsistent with our assumption of an adverse selection risk when selling to retail investors. Such risk would not occur if the issuer could bar all informed investors from participating in retail sales. We assume that the issuer cannot verify that all participating retail investors are uninformed.

the issuer cannot price discriminate between regular and retail investors.[17]

In this section we determine the *benchmark mechanism.* This is the optimal selling mechanism in the absence of when-issued trading. We invoke the Revelation Principle and only consider direct mechanisms, that is mechanisms in which investors submit reports of their signals.[18] In order to obtain our results, some of which depend on our assumption that informed investors have positively, but not perfectly, correlated signals, we need at least two polled investors. To keep the model as simple as possible we assume that information is elicited from exactly two informed investors.[19] As discussed, securities may be allocated to these two polled investors and to a number of retail investors, a fraction α of whom are informed. The optimal mechanism will be characterized in terms of optimal allocation and pricing policies.

We start the analysis by specifying the conditional expected value of the issue, given the polled investors' reports. There are three possible outcomes: either both polled investors report positive information, both report negative information, or one reports positive and the other negative information. We represent these outcomes with the pair $(a, b) \in \{(++), (+-), (-+), (--)\}$, where $(++)$ indicates that both polled investors reported positive information. Information gathered through the mechanism can also be represented with a simple sufficient statistic: the number of reported positive signals minus the number of reported negative signals. We denote this difference with the parameter z, where $z \in \{2, 0, -2\}$. If the polled investors' reports are the only sources of information for pricing the issue, then $\pi_p = \pi(z)$:[20]

$$\pi(2) = 1 - \pi(-2) = \frac{q^2}{1 - 2q(1-q)} \qquad \text{and} \qquad \pi(0) = \pi_0 = 1/2. \tag{3}$$

A zero value of z indicates that the investors' reports are contradictory ($(+-)$ or $(-+)$) and, hence, overall uninformative. Otherwise: $\pi(2) > 1/2 > \pi(-2)$.

[17] Our "optimal" selling mechanism is thus optimal within the class of uniform-price selling mechanisms. We make this restriction because it is realistic and it allows us to focus our attention on the question of whether and how to incorporate information from when-issued trading.

[18] In a direct mechanism potential buyers directly report their types (private information), rather than bids which are functions of their types. Sufficient for the Revelation Principle to hold is that each buyer's type be the only relevant information that determines any message that the buyer might submit. See Krishna (2002), p.62.

[19] See Maksimovic and Pichler (2006) for a study in which the optimal number of polled investors is determined in a mechanism that is similar to our benchmark.

[20] See the Appendix for the derivation of $\pi(z)$. In Section 4, the value of π_p may also depend on information generated by when-issued trading.

The expected value of the issue, given the reports of the polled investors, is

$$E[\tilde{V}|z] = v_0 + \frac{(z/2)(2q-1)w}{1-2q(1-q)} \ . \tag{4}$$

We next specify the incentive problem that the issuer must resolve in order to obtain information from the polled investors. Any polled investor who has observed a positive signal will want to report negative information instead so as to induce the issuer to set a low price. Following directly from equation (4) we see that, by falsely reporting a negative signal, the investor can decrease $E[\tilde{V}|z]$ by an amount w_L:

$$w_L \equiv \text{“impact of a lie”} = \frac{(2q-1)w}{1-2q(1-q)} \ < \ w \ . \tag{5}$$

Falsely reporting a positive signal will increase $E[\tilde{V}|z]$ by an amount w_L, but this is typically not a concern, because buyers wish to pay lower, not higher, prices. It is important to note that w_L is the impact on the expected value, $E[\tilde{V}|z]$, but not necessarily on the issue price. The issuer may “underprice” the issue, either to ensure participation or to elicit truthful information from polled investors. We write the issue price as: $p_I = E[\tilde{V}|z] - u^{ab}$, where u^{ab} is the expected level of underpricing, given that one polled investor reported a and the other reported b.

We now characterize the constrained optimization problem that determines the benchmark mechanism, i.e., the optimal mechanism when there is no when-issued trading (subscript N). We have assumed that a fixed quantity of securities will be issued, so maximizing issue proceeds is equivalent to minimizing underpricing, and we can write the issuer's objective as:

$$\begin{aligned} \min \ \ E[u^{ab}] \ &= \ \frac{1}{2}(1-2q(1-q))u^{++} + 2q(1-q)u^{+-} + \frac{1}{2}(1-2q(1-q))u^{--} \ , && (6) \\ &= \ Er_N^{+} + Er_N^{-} + Er_N^{R} && (7) \end{aligned}$$

$$\begin{aligned} \text{where} \quad Er_N^{+} \ &= \ \text{expected return to a polled investor who sees and reports } + \\ &= \ (1-2q(1-q))u^{++}h^{++} + 2q(1-q)u^{+-}h^{+-} \ , && (8) \\ Er_N^{-} \ &= \ \text{expected return to a polled investor who sees and reports } - \\ &= \ (1-2q(1-q))u^{--}h^{--} + 2q(1-q)u^{+-}h^{-+} \ , && (9) \\ Er_N^{R} \ &= \ \text{expected return to retail investors (investors not polled)} \\ &= \ \frac{1}{2}(1-2q(1-q))u^{++}(1-2h^{++}) \ + \frac{1}{2}(1-2q(1-q))u^{--}(1-2h^{--}) \\ &\quad + 2q(1-q)u^{+-}(1-h^{+-}-h^{-+}) \ , && (10) \end{aligned}$$

and where h^{ab} is the fraction of the offering that is allocated to a polled investor who reports a while the other reports b. The objective function is minimized by choosing the values of u^{ab} and h^{ab} subject to the following constraints.

Participation of polled investors: We assume that polled investors will accept allocations if and only if securities are not overpriced, conditional on the information that has been reported in the mechanism:

$$u^{ab} \geq 0 \qquad \forall u^{ab} \in \{u^{++}, u^{+-}, u^{--}\}\,. \qquad (PC_N - I)$$

This participation constraint is "ex-post" in that polled investors can choose after observing the outcome of the mechanism to refuse an allocation of shares if the above constraint is not satisfied.[21]

Incentive compatibility: Investors will truthfully report their information, as long as the following incentive compatibility constraints are satisfied:

$$Er_N^+ \geq (1-2q(1-q))\left(u^{+-}+w_L\right)h^{-+} + 2q(1-q)\left(u^{--}+w_L\right)h^{--}\,, \qquad (IC_N^+)$$

$$Er_N^- \geq (1-2q(1-q))\left(u^{+-}-w_L\right)h^{+-} + 2q(1-q)\left(u^{++}-w_L\right)h^{++}\,. \qquad (IC_N^-)$$

In writing the IC constraints we implicitly assume that a polled investor will not refuse an allocation, as long as $(PC_N - I)$ is satisfied.[22] Thus, sending a false positive report exposes an investor to the risk of receiving an over-priced allocation. For this reason, (IC_N^-) will not be binding. This is one aspect of the mechanism design problem that may change in the presence of when-issued trading.

Participation of retail investors: We assume that the issue is public and so must be priced such that retail investors are willing to participate.[23] In order to ensure sufficient retail participation the issue must be priced to compensate uninformed investors for adverse

[21] This constraint, which is essentially a limited liability constraint, may prevent the issuer from extracting all of the surplus from polled investors, in contrast to the result in Crémer and McLean (1985).

[22] In Treasury auctions and in some IPO markets investors are legally bound to purchase allocations. In U.S. IPOs investors are typically not legally bound to purchase allocations, but polled investors and intermediaries who assist in issuing securities are typically engaged in repeated interactions. There are thus reputational reasons for not refusing an allocation.

[23] By assuming that $h_R > 0$ we essentially rule out a private placement, i.e., a sale of the entire issue to the polled investors. We make this assumption because it is realistic and because it affords us with an intuitive benchmark in which underpricing is driven entirely by underpricing due to adverse selection risk. Easing this constraint would not substantively change our results.

selection risk:[24]

$$u^{++} \geq u_{AS}(2),\ \ u^{+-} \geq u_{AS}(0),\ \ u^{--} \geq u_{AS}(-2)\,. \qquad (PC_N - R)$$

u_{AS}, introduced in Section 3.1, is the expected underpricing due to adverse selection risk. We use the notation $u_{AS}(z)$ to indicate that this expected underpricing is a function of the information obtained through the mechanism. The values for $u_{AS}(0)$, $u_{AS}(2)$ and $u_{AS}(-2)$ are stated in the Appendix, where we also show that $w_L/3 > u_{AS}(0) > u_{AS}(2) > u_{AS}(-2)$.

Allocation constraints: The issue must be fully allocated, and cannot be over-allocated. h_R is the allocation to retail investors.

$$2h^{--},\ 2h^{++},\ h^{+-} + h^{-+} \leq 1 - h_R < 1 \qquad \text{and} \qquad h^{ab} \geq 0\,. \qquad (AC_N)$$

The following proposition characterizes the solution to this optimization problem.[25]

Proposition 1. The Benchmark Mechanism. *Without when-issued trading the optimal mechanism has the following characteristics:*

1. *Investors who report positive information receive the largest possible allocation. Investors who report negative information receive no allocation.*

2. *Positive expected underpricing is required in each state, due to residual adverse selection risk. The incentive compatibility constraints (IC) are nonbinding, and so no further underpricing is needed in order to induce truthful reporting.*

3. *The expected underpricing is:*

$$Eu_N \;=\; 2q(1-q)u_{AS}(0) + (1 - 2q(1-q))\left(u_{AS}(2) + u_{AS}(-2)\right)/2 \qquad (11)$$

The benchmark mechanism has three key characteristics. First, the allocation policy in the benchmark mechanism is non-decreasing. That is, more securities are allocated to investors who report more positive information about the security value. Second, a direct implication of parts 1 and 2 of Proposition 1 is that a polled investor who has observed positive information expects to earn strictly positive informational rents, while

[24] Polled investors do not face an adverse selection risk, because their allocations are based only on the reported information, not on any information that may still reside with other investors. For this reason, the constraint $(PC_N - I)$, requires only nonnegative underpricing.

[25] This proposition reproduces results from Maksimovic and Pichler (2006). We present these results here in proposition form so that the reader can more easily compare them to our results in the following sections.

an investor who has observed negative information expects to earn zero rents. Third, underpricing in the benchmark mechanism is not determined by the need to pay polled investors informational rents, but rather by the discount that the uninformed investors require in order to bear adverse selection risk (as discussed at the end of Section 3.1).

4 Optimal Selling Mechanisms with When-Issued Trading

We assume from this point forward that when-issued trading is permitted. Permitting when-issued trading does not, however, ensure that such trading will indeed take place. Primary markets differ substantially in the incidence of when-issued trading. In Treasury markets a complete failure of when-issued trading is virtually unheard of, but such trading frequently fails in IPO markets. An explanation for this difference between IPO and Treasury markets is that it is often quite hard to determine the value of IPO shares based only on publicly available information. The issuers of IPO shares are often relatively small firms with short track records. When-issued trading of these shares is therefore easily stifled by the presence of insiders poised to profit from trading with lesser-informed counterparties.[26]

We capture the possible failure of when-issued trading by assuming that such trading takes place with probability γ, where $\gamma \in [0, 1]$. For now, we assume that this probability is exogenously given.[27] The sequence of events is similar to that illustrated in Figure 1. This sequence is typical for IPO markets where when-issued trading does not open until after information has been obtained through a mechanism. When-issued trading of Treasuries typically does open prior to the auction mechanism, and then continues after the bidding in the auction has closed. But, as pointed out by Nyborg and Sundaresan (1996) and Hortaçsu and Sareen (2005), pre-auction trading is often illiquid or even unobservable.[28] In addition, we expect that any information that is revealed by pre-auction trading will be incorporated into the bidders' prior expectations. For the design of selling mechanisms

[26]Renneboog and Spaenjers (2008) report that firm size is indeed a significant determinant of the incidence of when-issued trading in the Dutch market. Findings of Cornelli, Goldreich and Ljungqvist (2006) and Dorn (2009) suggest that problems of market failure in the when-issued trading of IPOs are sometimes overcome due to the presence of overly optimistic traders.

[27]In Section 5 we model this probability as an endogenous outcome.

[28]Hortaçsu and Sareen (2005) report that they couldn't observe a single trade in the pre-auction when-issued market for Canadian treasuries during their sample period in the years 2001 - 2003.

the interesting question is whether to incorporate into the pricing and allocation rules information that is revealed *after* mechanism participants (bidders) have submitted their reports, but before the securities have been issued. We therefore focus on when-issued trading that takes place after the mechanism, but before the securities are issued.

The sequence of events in our model is as follows. First, the issuer elicits information directly from polled investors. After that, publicly observable when-issued trading opens, or doesn't open. If the trading takes place, then it fully reveals all privately held information; i.e., the value of $\tilde{s}$ is revealed.[29]

4.1 No direct mechanism.

We begin the analysis of this section by defining a second benchmark: the case where the issue is priced without using a direct mechanism for gathering information.[30] Without a direct mechanism, the issue is priced based on publicly available information. If when-issued trading reveals the value of the issue, the price can be set equal to this value. Otherwise, the issue is priced according to prior information. In the latter case, the issue must be priced at $v_0 - u_{AS}(0)$ in order to ensure retail participation. Thus, the expected underpricing is: $(1-\gamma)u_{AS}(0)$.

4.2 Two types of direct mechanisms

The benchmark mechanism, described in Section 3.2, is comprised of two components: a pricing rule (u^{ab}) and an allocation rule (h^{ab}). These two rules map polled investors' reports into prices and allocations. If when-issued trading is permitted, then the selling mechanism is comprised of three components: a specification of whether information from when-issued trading may be incorporated into the issue price and allocations, and pricing and allocation rules that are consistent with this specification. If the issuer specifies that the mechanism will be "constrained" (type C), then any investors who want to buy securities in the primary market must submit binding bids, and the investors' bids fully

[29]While the assumption of full revelation is a simplification, it is consistent with evidence about both Treasury and IPO markets. There is a broad consensus that publicly observable when-issued trading contributes to price discovery in Treasury markets. See, for example, Nyborg and Sundaresan (1996). Löffler, Panther and Theissen (2005) find that the when-issued market prices are unbiased predictors of the prices at which IPO shares trade on the first day of secondary market trading.

[30]If a mechanism is used but the pricing and allocation rules do not provide incentives for participants to truthfully report their information, then the mechanism is uninformative. From an information gathering perspective, this is equivalent to having no direct mechanism.

determine the pricing and the allocation of the issue. Post-bidding when-issued trading may reveal information that is not contained in the bids, but such information cannot be used as an input of the pricing and allocation rules. In this respect, constrained mechanisms resemble Treasury auctions. If the issuer specifies that the mechanism will be "unconstrained" (type U), then investors' bids are *not* immediately binding. The pricing and allocation of issues may depend not only on the bids, but also on information revealed through when-issued trading. Unconstrained mechanisms are thus similar to what is observed in IPOs.

We first determine the optimal type U mechanism and the optimal type C mechanism. We then determine under what conditions each mechanism type is optimal.

4.3 Optimal Unconstrained Mechanisms

4.3.1 Unconstrained mechanisms with certain when-issued trading

In order to develop our intuition we first determine the unconstrained mechanism for the case in which when-issued trading occurs with probability one, i.e., $\gamma = 1$. This case is clearly unrealistic in that if $\gamma = 1$, then the issuer should forego using a direct mechanism altogether and simply price the issue based on information from when-issued trading. By examining this case, however, we are able to clearly illustrate the effect that when-issued trading has on the direct mechanism. In the following subsection we solve the more reasonable case of $\gamma < 1$.

Our focus is on the design of the selling mechanism. We have thus chosen not to model the microstructure of the when-issued market, but our assumptions regarding this market are consistent with a model such as Kyle (1985). We assume that polled investors' reports are made public. Any polled investor who misreports her private information will be able to trade on the private information that her report was incorrect. Trading in the when-issued market may take place continuously over a fixed period of time. Other traders may also have information that is correlated with that of the informed trader. We assume that, if when-issued trading opens, then by the end of the trading period all privately held information is revealed. Given the assumptions of our model, this means that the value of $\tilde{s}$ is revealed.[31] However, consistent with the Kyle model, because this information is not revealed instantaneously the informed trader (the polled investor who has misreported) is

[31] We could weaken this assumption and allow for only partial revelation. This would, however, increase the computational complexity of the model without adding any intuition.

able to profit on her information. In the analysis that follows we need only assume that her expected trading profits, as a result of misreporting, are strictly positive.[32]

Our problem here is to determine the "optimal incentive compatible" mechanism, that is, the lowest cost mechanism that induces polled investors to truthfully report their information. As discussed above, such a mechanism will not be generally optimal when $\gamma = 1$.[33] For this reason Proposition 2 (on the following page) presents the IC (incentive compatible) unconstrained mechanism, rather than the optimal unconstrained mechanism. We present Proposition 2 as a stepping stone to the results of the following section.

We can now summarize the ways in which when-issued trading affects the unconstrained incentive-compatible mechanism design. First, any polled investor who misreports her private information forgoes the opportunity to profit from informed trading in the when-issued market. Second, in an unconstrained mechanism neither the retail investors nor the polled investors commit to accept allocations until after information has been revealed by the when-issued market. Thus, the participation constraints specify that there must be no overpricing conditional on information from both the mechanism and the when-issued market. Third, any pricing errors that could result from polled investors misreporting will be corrected by price discovery in the when-issued market. As a consequence, the term w_L (the impact of a single polled investors' lie) does not appear in the incentive compatibility constraints. Finally, because the polled investors' signals are correlated with information that is revealed through when-issued trading, the issuer can discipline polled investors by conditioning the mechanism outcome on information from this market. But, the use of this disciplinary tool is limited by the issuer's inability to compel investors to accept overpriced allocations. (This is the "limited liability" constraint that prevents the issuer from extracting all surplus from the polled investors.)

The new incentive compatibility constraints are:

$$Er^{+}_{U1} \geq q^2 u^{+-}_{+} h^{-+}_{w} + (1-q)^2 u^{+-}_{-} h^{-+}_{c} + q(1-q) u^{--}_{w} h^{--}_{w} + q(1-q) u^{--}_{c} h^{--}_{c} + \Psi \quad (IC^{+}_{U1})$$

$$Er^{-}_{U1} \geq q^2 u^{+-}_{-} h^{+-}_{w} + (1-q)^2 u^{+-}_{+} h^{+-}_{c} + q(1-q) u^{++}_{w} h^{++}_{w} + q(1-q) u^{++}_{c} h^{++}_{c} + \Psi \quad (IC^{-}_{U1})$$

The above constraints differ from (IC^{+}_{N}) and (IC^{-}_{N}) in a number of ways. First, we have

[32]Such a polled investor could also make positive expected trading profits in the post-IPO market, if the when-issued market does not open. This fact does not affect the results of this section. We did, however, ignore such a possibility when calculating the benchmark mechanism in the previous section. In a later section when we compare constrained and unconstrained mechanisms we implicitly assume that expected trading profits are higher if the when-issued market does open, than if it does not.

[33]Because no mechanism is needed to obtain the information.

added subscripts to the underpricing and allocation variables to indicate information learned from when-issued trading. A subscript of c (w) indicates that a polled investor's report is correct (wrong), relative to information from when-issued trading. A subscript of $+$ ($-$) is used for the underpricing variable when the polled investors disagree with each other and the market indicates that $+$ ($-$) is the correct report. Second, as discussed above, the variable w_L, impact of a lie, no longer appears. Third, Ψ represents the expected trading profits that may be obtained after misreporting. We assume that the expected trading profit is the same for investors who misreport positive and negative information. Finally, Er^a_{U1} represents the a priori expected rents to a polled investor who sees and truthfully reports a, given that $\gamma = 1$:

$$Er^+_{U1} = q^2 u_c^{++} h_c^{++} + (1-q)^2 u_w^{++} h_w^{++} + q(1-q) u_+^{+-} h_c^{+-} + q(1-q) u_-^{+-} h_w^{+-} \quad (12)$$

$$Er^-_{U1} = q^2 u_c^{--} h_c^{--} + (1-q)^2 u_w^{--} h_w^{--} + q(1-q) u_+^{+-} h_w^{-+} + q(1-q) u_-^{+-} h_c^{-+} \quad (13)$$

The following proposition describes the optimal *incentive-compatible* (IC) pricing and allocation rules. These rules are optimal in the sense that they result in the highest expected proceeds, given that an unconstrained mechanism type has been specified (U) and that the polled investors have incentives to truthfully report their private information. The proposition is written so that it can be contrasted directly with Proposition 1 for the benchmark mechanism. The intuition for the results is discussed following the proposition.

Proposition 2. The IC Unconstrained mechanism if $\gamma = 1$. *If when-issued trading takes place with probability one, $\gamma = 1$, then the optimal unconstrained incentive-compatible pricing and allocation rules have the following characteristics:*

1. *Investors whose reports turn out to be consistent with information revealed by the when-issued market receive the largest possible allocation, regardless of whether their information is positive or negative.*
   Investors whose reports are contradicted by the market receive no allocation.

2. *Both incentive compatibility constraints are strictly binding. Rents must be paid in order to induce truthful reporting, both to polled investors with positive and with negative information.*

3. *The expected underpricing is:*

$$Eu_{U1} = \frac{2q\Psi}{(2q-1)(1-h_R)} \quad (14)$$

The results of Proposition 2 are quite different from those of Proposition 1. In the benchmark mechanism, the incentive compatibility constraints are satisfied through the use of a "carrot" and a "stick". The carrot, the possibility of receiving an underpriced allocation after truthfully reporting positive information, eliminates investors' incentives to shade their bids. The stick, the possibility of receiving an overpriced allocation after falsely reporting positive information, eliminates investors' incentives to pad their bids. If when-issued trading is certain to reveal the value of the issue, and the mechanism design is unconstrained, so that it allows information from this market to be incorporated into the issue price, then the stick disappears. The polled investors have incentives to "free-ride" on price discovery in when-issued trading. In addition, the when-issued market affords trading opportunities to polled investors who misreport. As a result, both (IC_{U1}^{+}) and (IC_{U1}^{-}) are strictly binding and incentive compatibility requires the issuer to pay informational rents both to polled investors who see positive information and to those who see negative information. The most striking contrast between the two propositions is in the the allocation rules. In Proposition 2 the investors who receive the largest allocations are not necessarily those who report the highest valuations. The mechanism in Proposition 2 is in this way very different from a standard auction.

Proposition 2 is useful in that it illustrates how when-issued trading can alter the mechanism design. It, however, makes no sense to pay informational rents to polled investors if the when-issued market is certain to reveal the information. In the following section we expand on Proposition 2 to determine the optimal unconstrained mechanism under the more realistic assumption that when-issued trading may not open: $\gamma \in [0, 1)$.

4.3.2 Unconstrained mechanisms with uncertain when-issued trading

For $\gamma \in (0, 1)$, the optimal unconstrained mechanism must satisfy incentive-compatibility constraints that are weighted averages (weighted by γ and $1 - \gamma$) of the incentive compatibility constraints in Section 4.3.1 (where $\gamma = 1$) and in the benchmark case (where $\gamma = 0$). We denote the resulting incentive compatibility constraints as IC_U^{+} and IC_U^{-}. These constraints, and the participation constraints are stated in the Appendix (in the proof of Proposition 3).

Propositions 1 and 2 indicated that the incentive compatibility constraints are slack in the absence of when-issued trading ($\gamma = 0$), and binding if such trading is certain to take place ($\gamma = 1$). Those results are special cases of the following proposition.

Proposition 3. *If the mechanism is unconstrained, then there exist values,* $\underline{\gamma}^+$ *and* $\underline{\gamma}^-$ *such that* $0 < \underline{\gamma}^+ < \underline{\gamma}^- < 1$*, and:*

1. $\forall \gamma \in [0, \underline{\gamma}^+]$ (IC_U^+) *is nonbinding, and* $\forall \gamma \in (\underline{\gamma}^+, 1]$ (IC_U^+) *is strictly binding.*
2. $\forall \gamma \in [0, \underline{\gamma}^-]$ (IC_U^-) *is nonbinding, and* $\forall \gamma \in (\underline{\gamma}^-, 1]$ (IC_U^-) *is strictly binding.*
3. $$\underline{\gamma}^+ = \frac{A}{\hat{\Psi} + A} \qquad \text{and} \qquad \underline{\gamma}^- = \frac{B}{\hat{\Psi} + B} \tag{15}$$

   where $\hat{\Psi} = \Psi/(1 - h_R)$, $A = 2q(1-q)u_{AS}(0) + (1 - 2q(1-q))u_{AS}(2)/2$ *and* $B = q(1 - q(1-q))w_L - (2q-1)\left(qu_{AS}(0) + (1-q)u_{AS}(2)/2\right) > A$.

The values $\underline{\gamma}^+$ and $\underline{\gamma}^-$ define three distinct parameter regions in which the incentive-compatible mechanism design problem is subject to different sets of binding constraints. The following proposition describes the mechanism and expected underpricing in each of the three parameter regions. Proposition 4 is written so that it can be compared directly to Propositions 1 and 2.

Proposition 4. The optimal Unconstrained mechanism, $\gamma \in [\mathbf{0}, \mathbf{1})$**.**

1. *If when-issued trading takes place, then investors whose reports are consistent (inconsistent) with information revealed by the market receive the largest possible (zero) allocations.*
   If when-issued trading does not take place, then investors who reported positive (negative) signals receive positive (zero) allocations.
2. *For all* $\gamma \in [0, 1)$*, investors who have observed positive signals expect informational rents that are strictly positive and strictly higher than the expected informational rents of investors who have observed negative signals.*
   If $\gamma \leq \underline{\gamma}^-$*, then investors who have observed negative signals expect zero rents.*
3. *The expected underpricing depends on the value of* γ*:*
   If $\gamma \in [0, \underline{\gamma}^+]$*, then* $Eu_U = (1-\gamma)Eu_N$.
   If $\gamma \in (\underline{\gamma}^+, \underline{\gamma}^-]$*, then* $Eu_U = (1-\gamma)Eu_N + \gamma\hat{\Psi} - (1-\gamma)A > (1-\gamma)Eu_N$.
   If $\gamma \in (\underline{\gamma}^-, 1)$*, then* $Eu_U = (1-\gamma)Eu_N + \frac{\gamma 2q\hat{\Psi} - (1-\gamma)q(A+C)}{(2q-1)} > (1-\gamma)Eu_N$.
   where $\hat{\Psi} = \Psi/(1-h_R)$, $A = 2q(1-q)u_{AS}(0) + (1-2q(1-q))u_{AS}(2)/2 > 0$ *and* $C \equiv (1-2q(1-q))\left(w_L - u_{AS}(0)\right) + q(1-q)(w_L - u_{AS}(2))$.

Part 1 of Proposition 4 shows that the optimal allocation policy specifies allocations that are conditional on the occurrence and the outcome of price discovery in when-issued trading. If the trading takes place, then investors are "rewarded" with an allocation if their reports turn out to be consistent with information revealed through the trading. If the trading doesn't take place, then investors are "rewarded" for reporting positive signals, like in the benchmark mechanism.

Part 2 of the proposition indicates that investors' informational rents depend on the a priori probability with which when-issued trading takes place. Investors with positive private information generally receive higher informational rents than those with negative private information. If $\gamma \leq \underline{\gamma}^{-}$, then the latter investors receive zero rents, like in the benchmark mechanism.

Part 3 of Proposition 4 shows how the possibility of when-issued trading affects the expected underpricing of the issue, relative to the benchmark case. An unambiguous decrease in expected underpricing occurs only if it is sufficiently unlikely that the trading will actually take place, i.e., if $\gamma \in (0, \underline{\gamma}^{+}]$. In contrast, if $\gamma > \underline{\gamma}^{-}$, then the expected underpricing may exceed that in the benchmark case, for the following reasons: the polled investors will truthfully report their private information only if they are compensated for the expected trading profits thus lost, and if they are given incentives to abstain from free-riding on price discovery in when-issued trading. Since these "hurdles" are increasing in γ, so too are the informational rents that investors require to disclose their private information.

4.4 Optimal Constrained Mechanisms

We now consider the class of constrained mechanisms, inspired by the rules in Treasury auctions: investors commit to accept allocations when they submit their reports (bids), and the pricing and allocation rules are functions only of the polled investors' reports. The constrained mechanism design problem shares features with the benchmark case and the unconstrained mechanism design problem. Like in the benchmark case, polled investors may receive overpriced allocations if they falsely report positive information. The similarities with the unconstrained mechanism design problem are due to the trading opportunities that the investors forego in the when-issued market if they truthfully disclose their private information.

Proposition 5 characterizes the optimal constrained pricing and allocation rules. The incentive compatibility constraints for this problem (presented in the proof of Proposition 5) are a cross between those for the unconstrained problem and the benchmark case. The participation constraints are the same as in the benchmark case.

Proposition 5. The optimal Constrained mechanism, $\gamma \in [\mathbf{0}, \mathbf{1}]$.
If $\Psi \leq (3/8)(1 - h_R)w_L$,[34] *then:*

1. *Investors who report positive information receive the largest possible allocation. Investors who report negative information receive no allocation.*

2. *Positive expected underpricing is required, due to residual adverse selection risk and the possibility of trading profits. If expected trading profits are high enough, then the incentive compatibility (IC) constraint is binding for investors with positive information. The IC constraint is nonbinding for investors with negative information.*

3. *The expected underpricing is:*
   $Eu_C \;=\; Eu_N + \max\big[0,\;\; \gamma\hat{\Psi} - A\big]$
   where $\hat{\Psi} = \Psi/(1 - h_R)$ *and* $A = 2q(1-q)u_{AS}(0) + (1 - 2q(1-q))u_{AS}(2)/2 > 0$.

Comparing the optimal constrained (C) mechanism with the optimal unconstrained (U) mechanism reveals that the former mechanism type (C) avoids a problem that can make it costly to obtain truthful reports from the polled investors using an unconstrained mechanism, i.e., the investors' incentive to free-ride on price discovery in the when-issued market. With a constrained mechanism, the polled investors know that the pricing of the issue depends solely on their bids. For reasonable parameter values, it is therefore always incentive compatible for the investors to truthfully report negative private information about the value of the issue. Since the investors need not be rewarded for reporting negative signals, the optimal allocation rule is similar to that of a "standard" auction: investors who report higher valuations receive larger allocations. In addition, polled investors earn zero expected informational rents if they have negative information about the value of the issue.

The unconstrained mechanism type is, however, superior to the constrained type on another account: polled investors can be motivated to reveal their private information by giving them allocations if their reports turn out to be consistent with information revealed through when-issued trading. In the next section, we examine the trade-off between mechanism types more extensively and derive conditions that describe this trade-off.

[34] As can be seen in the proof of the proposition, we impose this restriction so that we can focus on what we consider to be the most reasonable case, rather than presenting two different cases. To get an intuitive sense of the restriction: If a polled investor misreports *and* then receives the entire nonretail allocation *and* then is able to trade this entire allocation at the expected value that is determined by the investor's lie, then this polled investor will obtain a trading profit of $(1 - h_R)w_L$. This is the highest possible outcome. Any expected trading profit that is consistent with both our model and existing models of market microstructure will result in expected trading profits that are a fraction of $(1 - h_R)w_L$.

Mechanism type	Probability market opens	Expected underpricing
no direct mechanism	$\gamma \in [0, 1]$	$(1-\gamma)u_{AS}(0)$
unconstrained (U)	$\gamma \in [0, \underline{\gamma}^+]$	$(1-\gamma)Eu_N$
unconstrained (U)	$\gamma \in (\underline{\gamma}^+, \underline{\gamma}^-]$	$(1-\gamma)Eu_N + \gamma\hat{\Psi} - (1-\gamma)A$
unconstrained (U)	$\gamma \in (\underline{\gamma}^-, 1)$	$(1-\gamma)Eu_N + \frac{\gamma 2q\hat{\Psi} - (1-\gamma)q(A+C)}{(2q-1)}$
constrained (C)	$\gamma \in [0, 1]$	$Eu_N + \max[0, \ \gamma\hat{\Psi} - A]$

Table 1: Expected Underpricing for three different specifications: no direct mechanism, an unconstrained direct mechanism, and a constrained direct mechanism

4.5 The Optimal Mechanism

We now consider which type of mechanism is overall optimal by comparing the optimal constrained and the optimal unconstrained mechanisms. Table 1 presents a summary of the expected underpricing for the two types of mechanisms. Proposition 6 shows that the optimal choice of mechanism type depends on the probability γ with which when-issued trading takes place.

Proposition 6: The optimal selling mechanism.

1. *If $\gamma \leq \underline{\gamma}^-$, then the optimal unconstrained direct mechanism, U, results in higher expected issue proceeds than the optimal constrained direct mechanism, C.*

2. *If $\gamma > \underline{\gamma}^-$, then there exist parameter values such that the optimal constrained direct mechanism, C, results in strictly higher expected issue proceeds than the optimal unconstrained direct mechanism, U. In the limit as $\gamma \to 1$, $w_L/4 < \hat{\Psi} \leq 3w_L/8$ is a sufficient condition such the optimal C-type mechanism results in strictly higher issue proceeds than the optimal U-type mechanism.*

Proposition 6 tells us that if the probability of when-issued trading is low enough, $\gamma \leq \underline{\gamma}^-$, then the unconstrained mechanism is unambiguously optimal. That is, the issuer should employ a mechanism that allows information from the when-issued market to affect the issue price. If, however, $\gamma > \underline{\gamma}^-$, then the results are not as clear cut. In this case the optimal choice between the two mechanism types depends on the relative values of expected trading profits (Ψ), the expected cost to the issuer of pricing the issue without any when-issued trading information (Eu_N), and the probability that informed investors have correct information (q). If, for example, γ is high and the expected trading profits are high, then the constrained mechanism type is optimal.

These results are consistent with observed phenomena. When-issued trading is permitted in both European IPOs and in Treasury issues, but the likelihood that such trading will actually take place is much higher for Treasury issues than for IPOs. Our results thus predict that IPO mechanisms should be unconstrained, consistent with evidence described in Section 2 of this paper. To understand the use of constrained mechanisms in Treasury issues, we consider the case in which $\gamma > \gamma^{-}$. The latter condition is not in itself sufficient for the optimal mechanism to be of the constrained type. As is indicated in Proposition 6, a constrained mechanism is optimal if both γ and the expected trading profit, Ψ, that a polled investor can earn by misrepresenting her private information are sufficiently high. Thus, our results are consistent with observed practice in Treasury markets if expected trading profits are sufficiently high in these markets. Hortaçsu and Sareen (2005) provide evidence that bidders in government treasury auctions have private information regarding demand that has been submitted only to the individual bidder. As such, these bidders are monopolists, or near monopolists, with regard to their private information, resulting in higher expected trading profits from this information.[35] The use of constrained mechanisms for Treasury issues can thus be explained by the combination of the high incidence of when-issued trading and the nature of the private information in Treasury auctions.

Our analysis also implies that if the Treasury were to change its policy and use an unconstrained mechanism, instead of a constrained mechanism, then the allocation policy that is currently used in Treasury auctions (bidders whose bids are above the clearing price receive allocations) may not be optimal. This is because the bidders would have incentives to "free-ride" on price discovery in the when-issued market. Rather than conditioning their bids on their private information, they would simply bid in a way that maximizes their expected allocations.

The focus of our work has been on the design of the optimal pricing mechanism, but our analysis also provides insight into the value of permitting when-issued trading. These results are summarized in the following corollary.

Corollary 1. *If $\gamma \leq \underline{\gamma}^{+}$, then the expected issue proceeds are higher than in the absence of when-issued trading. If $\gamma > \underline{\gamma}^{+}$, then the expected issue proceeds may be lower than in the absence of when-issued trading.*

Corollary 1 and Proposition 6 present some results that seem counterintuitive. Allowing

[35] Compare, for example, Kyle (1985) and Holden and Subrahmanyam (1992).

when-issued trading is most clearly beneficial to the issuer when there is a sufficiently low probability ($\gamma \leq \underline{\gamma}^+$) that such trading will actually take place. And, if the probability of when-issued trading is higher than this, then even though the trading, if it opens, will generate information about the value of the issue, it may be optimal for the issuer to precommit to ignore such information when setting the issue price, i.e., to specify a constrained mechanism.

5 Endogenous When-Issued Trading

We have up to this point ignored the possibility that the opening of when-issued trading may depend on the mechanism itself. A natural extension of our analysis is suggested by an institutional regularity in European IPO markets. In these markets, when-issued trading never starts before the issuer releases a preliminary offering prospectus that includes a price range. It is our understanding that this price range is based on information that the underwriters obtain in the course of discussions with regular investors. Thus, it appears that these investors report information that then allows when-issued trading to open. This idea certainly makes sense if traders in the when-issued market fear losing from trading with insiders, and the insiders' informational advantage is reduced once the price range is published.[36] It is, however, at odds with an assumption in our analysis up to this point: that when-issued trading takes place with an exogenous probability.[37]

In this section we assume that the probability of when-issued trading is endogenous and depends on the outcome of the mechanism. We assume that trading will take place if and only if the mechanism is informative, i.e., if and only if $z \in \{2, -2\}$, where z is the sum of the polled investors' reports. If the polled investors submit conflicting reports, then the mechanism is uninformative ($z = 0$) and when-issued trading will not take place. As long as both polled investors truthfully report, then the probability that when-issued trading takes place is $1 - 2q(1 - q)$ which is greater than $1/2$. If one polled investor lies, then this probability is $2q(1 - q)$ which is less than $1/2$. Thus, misreporting lowers the probability that when-issued trading will take place.[38]

[36] In an earlier version of this paper we included a simple market microstructure model that was based on that of Glosten and Milgrom (1985) and that illustrated this point. We have removed this model as it is not central to the mechanism design problem.

[37] The assumption that when-issued trading opens with an exogenously-given high probability does seem consistent with Treasury issues, but not with IPOs.

[38] We also considered relaxing the assumption that only two investors are polled in the mechanism. With

Proposition 7. *If the opening of when-issued trading depends on information obtained in the mechanism, then*

1. *In the unconstrained mechanism, U, IC^- is nonbinding; IC^+ may be binding.*

2. *The optimal mechanism is the optimal unconstrained direct mechanism.*

Proposition 7 indicates that the optimal mechanism design and the expected proceeds in the case with an endogenous probability of market opening are qualitatively the same as in the case where $\gamma \leq \underline{\gamma}^-$: the constrained mechanism is not optimal. As discussed above, the constrained mechanism involves a commitment on the part of the issuer to ignore when-issued trading as a source of information when pricing the issue. Such a commitment discourages investors from free-riding on information discovery in when-issued trading. If, however, the opening of when-issued trading depends on the investors' reports, then free-riding is no longer a concern. Lying to take advantage of informed trading opportunities is self-defeating, because doing so lessens the probability that such opportunities will be available, and may also increase the likelihood of being awarded an overpriced allocation. As a result, the cost to the issuer of inducing truth-telling is low enough that the unconstrained pricing mechanism is optimal for all parameter values. That is, it is optimal for the issuer to incorporate information both from the direct mechanism and from when-issued trading when pricing the securities.

These results are consistent with what we observe in European IPOs: unconstrained mechanisms are employed and information from the mechanisms seems to be necessary for when-issued trading to open. It is only in the case such that when-issued trading is very likely to open, regardless of the polled investors' reports, that a constrained mechanism may be optimal.

6 Conclusion

We analyze mechanisms for pricing unseasoned securities in a setting in which price discovery may occur in when-issued trading. We thus extend the mechanism design problem

only two polled investors, our model exaggerates the impact that each has on the informativeness of the mechanism, and, thus, on the probability of when-issued trading. However, the model also exaggerates the impact (and thus personal benefit) that each can have by misreporting. Since these two effects are counteracting, it does not seem that a model with more than two polled investors would yield results that are qualitatively different from those in Proposition 7.

in order to allow for pricing and allocation rules based on both information contained in investors' bids, and information revealed through when-issued trading.

Our results provide an explanation of why pricing mechanisms for new issues of Treasury securities differ in a key qualitative aspect from mechanisms for pricing unseasoned corporate equity securities. Treasury auctions are designed so that the issue price depends only on the bids that are submitted in the auction, and not on information from outside sources, such as post-auction when-issued trading. In contrast, the most commonly used method for issuing equity securities, bookbuilding, allows the pricing of such issues to depend both on participants' indications of interest and on information revealed through when-issued trading.

We show that the key explanatory variable for this difference in primary market design is the likelihood that when-issued trading will actually take place. In Treasury markets it is virtually certain that when-issued trading will occur after the close of bidding in the auction. In equity markets such trading sometimes fails to open, and empirical evidence suggests that the reasons are common causes of market failure, such as a lack of publicly available information about the value of equity issues.

Our analysis reveals general advantages and disadvantages of allowing the pricing of a securities issue to depend on when-issued market prices. We show that, if polled investors (those who are invited to submit bids or report indications of interest) are almost certain that when-issued trading will take place, then these investors have incentives to "free-ride" on the price discovery that occurs during the trading and to conceal their private information about the value of an issue. To avoid such free-riding, it may be optimal to price a securities issue based just on the information contained in the investors' reports. This result provides a rationale for pricing U.S. Treasury issues solely on the basis of the auction bids. If, however, the likelihood of when-issued trading is relatively low, then there is no need to discourage free-riding. As a consequence, it is optimal to allow information from such trading, if it opens, to affect the primary market pricing and allocation of issues. In the latter case, the optimal mechanism incorporates two sources of information: information that investors report within the mechanism and information revealed through when-issued trading. This result is consistent with evidence that both types of information indeed determine the pricing of European IPOs.

While our analysis has been inspired by existing institutions, such as those for pricing unseasoned Treasury and equity securities, we expect that our results are more generally applicable. For example, there was much discussion about the pricing of non-traded financial assets during the 2008 financial market crisis. Suggestions for pricing mechanisms included auctions or reverse auctions (in which participants submit offers to sell), i.e., "constrained" mechanisms for pricing securities based solely on information contained in the participants' reports. Our analysis suggests that the optimality of such a policy depends on the liquidity of the financial markets that may contribute to price discovery.

Appendix

Random variables:

$\tilde{V}$ = secondary market value $\in \{v_0 + w, v_0 - w\}$

$\tilde{s} \equiv \frac{\tilde{V}-v_0}{w} \in \{-1, 1\}$

$\tilde{\varsigma}_i$ = informed trader i's signal of $\tilde{s}$.

Exogenous parameters: (The exogenous parameters are all common knowledge)

v_0 = prior expected value of $\tilde{V}$

π_0 = prior probability that $s = 1$

w = constant (See above for $\tilde{V}$)

q = probability that $\varsigma_i = s$, i.e., that an informed investor has correct information, $1/2 < q < 1$

α = fraction of investors who are informed. $0 < \alpha < 1/2$

γ =probability that when-issued trading opens

h_R = minimum fraction of the offering that must be allocated to retail investors

Other variables:

N: used as a subscript to indicate No when-issued trading

T: used as a subscript to indicate when-issued Trading takes place and trading information is incorporated into price and allocations

δ = zero if mechanism precludes when-issued trading information from price and allocations; one, otherwise

p_I = issue price

π_p = probability that $s = 1$, given all info known by issuer at time of setting price

z = sum of signals reported by polled investors

$\pi(z)$ = probability that $s = 1$, given z

$u_{AS}(z)$ = expected underpricing due to residual adverse selection risk

w_L = impact of a lie on the expected value of the security

u^{ab} = underpricing, given one polled investor reports a and one reports b

h^{ab} = fraction allocated to polled investor who reports a when other reports b

u^{ab}_x = underpricing, given the polled investors reports ab and the market reveals x

h^{ab}_x = allocation to investor who reports a when other reports b & the market reveals x

Ψ = expected trading profit for polled investor who misreports $\leq (3/8)(1 - h_R)w_L$

$\hat{\Psi} = \Psi/(1 - h_R)$

Table 2: Notation

Underpricing due to adverse selection risk.

$$E[\tilde{V}|\pi_p] = v_0 + (2\pi_p - 1)w \tag{16}$$

Informed investor i has observed a signal of $\tilde{s}$: $\tilde{\varsigma}_i \in \{-1, 1\}$.

$$prob\{s=1|\pi_p, \varsigma_i = 1\} = \frac{q\pi_p}{q\pi_p + (1-q)(1-\pi_p)} > \pi_p,$$
$$prob\{s=1|\pi_p, \varsigma_i = -1\} = \frac{(1-q)\pi_p}{(1-q)\pi_p + q(1-\pi_p)} < \pi_p.$$

Given these probabilities, an informed investor values the issue as follows:

$$E[\tilde{V}|\pi_p, \varsigma_i = 1] = v_0 + \frac{q\pi_p - (1-q)(1-\pi_p)}{q\pi_p + (1-q)(1-\pi_p)} w > E[\tilde{V}|\pi_p] \quad (17)$$
$$E[\tilde{V}|\pi_p, \varsigma_i = -1] = v_0 + \frac{(1-q)\pi_p - q(1-\pi_p)}{(1-q)\pi_p + q(1-\pi_p)} w < E[\tilde{V}|\pi_p] \quad (18)$$

Investors arrive randomly in the retail market. Allocations are given on a first-come first-served basis until the issue is sold. An investor who "participates in the offering" joins the queue for an allocation. If $s = 1$, then on average a fraction q of the informed investors will participate; if $s = -1$, then on average a fraction $1 - q$ will participate. Table 3 presents the expected value and the expected relative allocations to each group of investors (informed and uninformed), for each realization of $\tilde{s}$. The table is written assuming that $p_I > E[\tilde{V}|\pi_p, \varsigma_i = -1]$, so that investors who have observed negative signals do not participate; this is checked below.[39] Because $q > 1/2$, the uninformed will on average receive more securities if the value of these securities is low ($s = -1$).

Realization of $\tilde{s}$	$s=-1$	$s=1$
Probability of this realization	$1-\pi_p$	π_p
Expected secondary market value $\tilde{V}$	$v_0 - w$	$v_0 + w$
Allocation to informed investors	$\frac{(1-q)\alpha}{1-q\alpha}$	$\frac{q\alpha}{1-(1-q)\alpha}$
Allocation to uninformed investors	$\frac{1-\alpha}{1-q\alpha}$	$\frac{1-\alpha}{1-(1-q)\alpha}$

Table 3: Expected Value and Allocations

Uninformed investors will participate in the offering only if their expected return is nonnegative. When underpricing is minimized, this expected return is zero:

$$0 = (1-\pi_p)(v_0 - w - p_I)\frac{1-\alpha}{1-q\alpha} + \pi_p(v_0 + w - p_I)\frac{1-\alpha}{1-(1-q)\alpha}, \quad (19)$$

Solving equation (19) for p_I yields:

$$p_I = v_0 + \left(\frac{2\pi_p - 1 - (\pi_p + q - 1)\alpha}{1 - ((2\pi_p - 1)q + 1 - \pi_p)\alpha}\right) w. \quad (20)$$

[39]The number of investors who reveal their information through a mechanism is small relative to the total number of informed investors. Thus, the fraction of informed investors is not affected by information gathering.

The above expression is $> E[\tilde{V}|\pi_p, \varsigma_i = -1]$, so those who have observed negative signals do not participate. The expected underpricing due to adverse selection risk is:

$$u_{AS}(\pi_p) \;=\; E[\tilde{V}|\pi_p] - p_I \;=\; \frac{q - 2\pi_p(1-\pi_p) - (2\pi_p - 1)^2 q}{1 - ((2\pi_p - 1)q + 1 - \pi_p)\alpha}\,\alpha w. \tag{21}$$

Define the following variable:

$$\bar{u}_{AS} \equiv \frac{u_{AS}(\pi)}{w\alpha} = \frac{q - 2\pi(1-\pi) - (2\pi-1)^2 q}{1 - ((2\pi-1)q + 1 - \pi)\alpha}$$

$$\frac{\partial \bar{u}_{AS}}{\partial \pi} =$$

$$(2q-1)\left(\frac{2(1-2\pi)(1-(1-q)\alpha - (2q-1)\pi\alpha) + \alpha(q - 2\pi(1-\pi) - (2\pi-1)^2 q)}{(1-((2\pi-1)q+1-\pi)\alpha)^2}\right)$$

$$= (2q-1)\left(\frac{2(1-2\pi) - 2(1-\pi)^2\alpha + (1-2\pi)^2 q\alpha + q\alpha}{(1-((2\pi-1)q+1-\pi)\alpha)^2}\right) \tag{22}$$

The denominator of (22) is strictly positive. The numerator is strictly decreasing in π and strictly positive if $\pi = 1/2$. Thus, (22) is strictly positive $\forall \pi \leq 1/2 \Longrightarrow$

a) For all $\pi_p \leq 1/2$, u_{AS} is strictly increasing in π_p.

The numerator of (22) is strictly negative if $\pi = (1-\alpha)/2 + \alpha q$. (This result can be obtained by setting $q = 1/2 + \varepsilon$, where $0 < \varepsilon < 1/2$.) Thus, (22) is negative $\forall \pi \geq (1-\alpha)/2 + \alpha q$. In addition,

$$\bar{u}_{AS}\Big|_{z=0} = \frac{2q-1}{2-\alpha} \;>\; \bar{u}_{AS}\Big|_{z=1} = \frac{(2q-1)2q(1-q)}{1-((2q-1)q+1-q)\alpha} \quad\Longrightarrow$$

b) for all $\pi_p \geq q$, u_{AS} is strictly decreasing in π_p, and $u_{AS}(\pi_p) < u_{AS}(\pi_0)$.

Points a) and b) together imply that, if the issue is priced after obtaining information that is at least as good as that of one informed investor, then underpricing due to adverse selection risk will be lower than without the information. And, as more information is learned, underpricing due to adverse selection risk decreases.

Derivation of $\pi(z)$: We know that

$$\pi(0) = 1/2\,, \qquad \pi(1) = q\,, \qquad \text{and} \qquad \pi(-1) = 1 - q = 1 - \pi(1)\,.$$

We can define $\pi(z)$ as a function of all signals obtained, except i's signal, together with i's signal ς_i. For $z \geq 1$, we let $\varsigma_i = 1$:

$$\pi(z) \;=\; \frac{prob\{\varsigma_i = 1|s=1\}\pi(z-1)}{prob\{\varsigma_i = 1|s=1\}\pi(z-1) + prob\{\varsigma_i = 1|s=-1\}(1-\pi(z-1))}$$

$$\;=\; \frac{q\pi(z-1)}{q\pi(z-1) + (1-q)(1-\pi(z-1))} \tag{23}$$

For $z \leq -1$, we let $\varsigma_i = -1$:

$$\begin{aligned}\pi(z) &= \frac{prob\{\varsigma_i = -1|s = 1\}\pi(z+1)}{prob\{\varsigma_i = -1|s = 1\}\pi(z+1) + prob\{\varsigma_i = -1|s = -1\}(1-\pi(z+1))} \\ &= \frac{(1-q)\pi(z+1)}{(1-q)\pi(z+1) + q(1-\pi(z+1))} \qquad (24)\end{aligned}$$

Using these equations and the above values for $\pi(1)$ and $\pi(-1)$, we obtain equation (3).

Underpricing due to <u>residual</u> adverse selection risk. From equations (2) and (3):

$$\frac{u_{AS}(0)}{w\alpha} = \frac{2q-1}{2-\alpha} \qquad (25)$$

$$\begin{aligned}\frac{u_{AS}(2)}{w\alpha} &= \frac{q - \frac{2q^2(1-q)^2}{(q^2+(1-q)^2)^2} - \left(\frac{2q-1}{q^2+(1-q)^2}\right)^2 q}{1 - \left(\frac{2q-1}{q^2+(1-q)^2}\right) q\alpha - \left(\frac{(1-q)^2}{q^2+(1-q)^2}\right)\alpha} \\ &= \frac{(2q-1)2q^2(1-q)^2}{(q^2+(1-q)^2)\left(q^2+(1-q)^2 - \alpha(1-3q(1-q))\right)} \qquad (26)\end{aligned}$$

$$\begin{aligned}\frac{u_{AS}(-2)}{w\alpha} &= \frac{q - \frac{2q^2(1-q)^2}{(q^2+(1-q)^2)^2} - \left(\frac{2q-1}{q^2+(1-q)^2}\right)^2 q}{1 + \left(\frac{2q-1}{q^2+(1-q)^2}\right) q\alpha - \left(\frac{q^2}{q^2+(1-q)^2}\right)\alpha} \\ &= \frac{(2q-1)2q^2(1-q)^2}{(q^2+(1-q)^2)\left(q^2+(1-q)^2 - \alpha q(1-q)\right)} \qquad (27)\end{aligned}$$

$q > 1/2 \implies 1 - 3q(1-q) > q(1-q) \implies u_{AS}(2) > u_{AS}(-2)$.

$u_{AS}(0)$ is strictly increasing in q. When q is close to 1/2, $\partial u_{AS}(2)/\partial q$ and $\partial u_{AS}(-2)/\partial q$ are positive; when q is close to one, $\partial u_{AS}(2)/\partial q$ and $\partial u_{AS}(-2)/\partial q$ are negative.

From equations (25) and (5):

$$u_{AS}(0) = \left(\frac{\alpha}{2-\alpha}\right)(1-2q(1-q))w_L < \frac{w_L}{3}. \qquad (28)$$

Proof of Proposition 1. The benchmark mechanism. Rearranging the incentive compatibility constraints, (IC_N^+) and (IC_N^-):

$$\begin{aligned}&(1-2q(1-q))\left(u^{++}h^{++} - (w_L + u^{+-})h^{-+}\right) + \\ &\qquad 2q(1-q)\left(u^{+-}h^{+-} - (w_L + u^{--})h^{--}\right) \geq 0 \qquad (29)\\ &(1-2q(1-q))\left(u^{--}h^{--} + (w_L - u^{+-})h^{+-}\right) + \\ &\qquad 2q(1-q)\left(u^{+-}h^{-+} + (w_L - u^{++})h^{++}\right) \geq 0 \qquad (30)\end{aligned}$$

It is optimal to set $h^{+-} = 2h^{++} = 1 - h_R$, $h^{-+} = 0$, and $h^{--} = 0$.

$(PC_N - R)$ is binding and the IC constraints are nonbinding for all values of z. Thus:

$u^{--} = u_{AS}(-2)$, $u^{+-} = u_{AS}(0)$ and $u^{++} = u_{AS}(2)$, and the expected underpricing is thus given by equation (11). ∎

Proof of Proposition 2. Because $q > 1-q$, it is optimal to set $u^{ab}h_w^{ab} = 0\,\forall$ pairs of (a,b). The IC constraints can thus be written:

$$q\left(qu_c^{++}h_c^{++} + (1-q)u_+^{+-}h_c^{+-}\right) - (1-q)\left((1-q)u_-^{+-}h_c^{-+} + qu_c^{--}h_c^{--}\right) \geq \Psi \quad (31)$$
$$q\left(qu_c^{--}h_c^{--} + (1-q)u_-^{+-}h_c^{-+}\right) - (1-q)\left((1-q)u_+^{+-}h_c^{+-} + qu_c^{++}h_c^{++}\right) \geq \Psi \quad (32)$$

Because $\Psi > 0$ and residual adverse selection risk is zero, the above are strictly binding:

$$qu_c^{++}h_c^{++} + (1-q)u_+^{+-}h_c^{+-} = qu_c^{--}h_c^{--} + (1-q)u_-^{+-}h_c^{-+} = \Psi/(2q-1)\,. \quad (33)$$

Equation (33) gives the expected rents for an investor who truthfully reports and turns out to be correct. Thus, the a priori expected rents for an investor who truthfully reports are as follows, regardless of whether the information is positive or negative:

$$Er_{U1}^{+} = Er_{U1}^{-} = q\Psi/(2q-1)\,. \quad (34)$$

In the optimal mechanism $h_c^{++} = h_c^{--} = (1-h_R)/2$ and $h_c^{+-} = h_c^{-+} = 1-h_R$. Combining this allocation rule with equation (33) and the result that $u_w^{ab} = 0$, we can calculate the expected underpricing:

$$Eu_{U1} = \frac{q^2}{2}\left(u_c^{++} + u_c^{--}\right) + q(1-q)\left(u_+^{+-} + u_-^{+-}\right) = \frac{2q\hat{\Psi}}{(2q-1)} \quad (35)$$

where $\hat{\Psi} = \Psi/(1-h_R)$. ∎

Proof of Proposition 3. $\gamma \in [0,1]$, $R = U$, and mechanism is IC.
Each constraint IC_U^a, $a \in \{+,-\}$, is the weighted average of the constraints (IC_N^a) and (IC_{U1}^a). More specifically, the LHS of (IC_U^+) is given by: $Er_U^+ = (1-\gamma)Er_N^+ + \gamma Er_{U1}^+$ and the RHS is $(1-\gamma)$ times the RHS of (IC_N^+) plus γ times the RHS of (IC_{U1}^+). The LHS of (IC_U^-) is given by: $Er_U^- = (1-\gamma)Er_N^- + \gamma Er_{U1}^-$ and the RHS is $(1-\gamma)$ times the RHS of (IC_N^-) plus γ times the RHS of (IC_{U1}^-).
We first solve the mechanism design problem as far as we can for a general value of γ. We then show the existence of and determine the values of $\underline{\gamma}^+$ and $\underline{\gamma}^-$.

As in Proposition 2, the issuer will optimally set $u^{ab}h_w^{ab} = 0\,\forall$ pairs of (a,b). We can thus write the IC constraints by combining equations (29) and (31) to obtain equation

(IC_U^+), and combining equations (30) and (32) to obtain equation (IC_U^-):

$$\begin{aligned}\gamma q^2 u_c^{++}h_c^{++} \;-\; \gamma(1-q)^2 u_-^{+-}h_c^{-+} \;+\; \gamma q(1-q)\left(u_+^{+-}h_c^{+-} - u_c^{--}h_c^{--}\right) \;+& \\ (1-\gamma)\left(1-2q(1-q)\right)\left(u^{++}h^{++} - \left(w_L + u^{+-}\right)h^{-+}\right) \;+& \\ 2(1-\gamma)q(1-q)\left(u^{+-}h^{+-} - \left(w_L + u^{--}\right)h^{--}\right) \quad &\geq \gamma\Psi \qquad (IC_U^+)\end{aligned}$$

$$\begin{aligned}\gamma q^2 u_c^{--}h_c^{--} \;-\; \gamma(1-q)^2 u_+^{+-}h_c^{+-} \;+\; \gamma q(1-q)\left(u_-^{+-}h_c^{-+} - u_c^{++}h_c^{++}\right) \;+& \\ (1-\gamma)\left(1-2q(1-q)\right)\left(u^{--}h^{--} - \left(u^{+-} - w_L\right)h^{+-}\right) \;+& \\ 2(1-\gamma)q(1-q)\left(u^{+-}h^{-+} - \left(u^{++} - w_L\right)h^{++}\right) \quad &\geq \gamma\Psi \qquad (IC_U^-)\end{aligned}$$

The participation constraints require that:

$$u_c^{aa},\; u_w^{aa},\; u_+^{+-},\; u_-^{+-} \;\geq 0, \quad u^{++} \geq u_{AS}(2),\;\; u^{--} \geq u_{AS}(-2),\;\; u^{+-} \geq u_{AS}(0) \tag{36}$$

(In pricing regime U participation must be ex post rational. I.e., investors observe all of the available information and the offer price, then they decide whether to invest.) The allocation constraints are the same as in the earlier problems. The objective is to minimize the expected value of underpricing. The optimal value of $u_w^{aa}, a \in \{+,-\}$ is 0, so the expected underpricing is:

$$\begin{aligned}&\gamma\left(q^2(u_c^{++} + u_c^{--})/2 + q(1-q)\left(u_+^{+-} + u_-^{+-}\right)\right) \;+ \\ &(1-\gamma)\left(\left(1-2q(1-q)\right)\left(u^{++} + u^{--}\right)/2 \;+\; 2q(1-q)u^{+-}\right)\end{aligned} \tag{37}$$

With the problem written as above we can state the following: i) It is optimal to give the largest possible allocations to investors whose reports are confirmed if the market opens, and to those who report + if the market doesn't open: $2h_c^{++} = 2h_c^{--} = h_c^{+-} = h_c^{-+} = 2h^{++} = h^{+-} = (1-h_R)$. And, to award the smallest possible allocation to those who report $-$ if the market doesn't open: $h^{-+} = h^{--} = 0$. ii) It's clearly optimal to set u^{--} to its lowest feasible value: $u^{--} = u_{AS}(-2)$.

At this point we define: $U^+ \equiv q u_c^{++}/2 + (1-q)u_+^{+-}$ and $U^- \equiv q u_c^{--}/2 + (1-q)u_-^{+-}$. ($qU^a(1-h_R)$ is the expected rent received by a polled investor who observes and reports a, if $\gamma = 1$.) We can now write the problem as follows:

Choose u_c^{++}, u_c^{--}, u_+^{+-}, u_-^{+-}, u^{++} and u^{+-} to:

$$\begin{aligned}&\min Eu_U = \gamma q\left(U^+ + U^-\right) + \\ &\quad (1-\gamma)\left(Eu_N \;+\; (1-2q(1-q))(u^{++} - u_{AS}(2))/2 \;+\; 2q(1-q)(u^{+-} - u_{AS}(0))\right)\end{aligned} \tag{38}$$

subject to:

$$u_c^{++}, u_c^{--}, u_+^{+-}, u_-^{+-} \geq 0, \quad u^{++} \geq u_{AS}(2),\ u^{+-} \geq u_{AS}(0) \tag{39}$$

$$\gamma\left(qU^+ - (1-q)U^-\right) + (1-\gamma)\Big((1-2q(1-q))\,u^{++}/2 \ + 2q(1-q)u^{+-}\Big) \geq \gamma\hat{\Psi} \tag{40}$$

$$\gamma\left(qU^- - (1-q)U^+\right) + (1-\gamma)\Big((1-2q(1-q))\left(w_L - u^{+-}\right) \ + q(1-q)\left(w_L - u^{++}\right)\Big) \geq \gamma\hat{\Psi} \tag{41}$$

where $\hat{\Psi} = \Psi/(1-h_R)$. Equation (39) is $(PC-R)$, (40) is (IC_U^+) and (41) is (IC_U^-). We know from Propositions 1 and 2 that if $\gamma = 0$, both (40) and (41) are nonbinding; and if $\gamma = 1$, both are binding. But, because $w_L - u_{AS}(0) > 2u_{AS}(0) > 2u_{AS}(2)$ and $1 - 2q(1-q) > 2q(1-q)$, (40) binds at a lower value of γ, than does (41).

Suppose that both (40) and (41) ((IC_U^+) and (IC_U^-)) are nonbinding. Then, all of the constraints in (39) are binding: $u^{++} = u_{AS}(2)$, $u^{+-} = u_{AS}(0)$, $u_c^{++} = u_+^{+-} = u_c^{--} = u_-^{+-} = 0$, and $U^+ = U^- = 0$. Putting these values into (40) we see that (IC_U^+) is satisfied with equality, but not strictly binding, when:

$$(1-\underline{\gamma}^+)Eu_N = \underline{\gamma}^+\hat{\Psi} + (1-\underline{\gamma}^+)\,(1-2q(1-q))\,u_{AS}(-2)/2 \implies$$

$$\underline{\gamma}^+ = \frac{A}{\hat{\Psi} + A} \tag{42}$$

where $A \equiv Eu_N - (1-2q(1-q))\,u_{AS}(-2)/2$.

If $\gamma \leq \underline{\gamma}^+$, then (IC_U^+) is nonbinding; if $\gamma > \underline{\gamma}^+$, then (IC_U^+) is strictly binding.

As stated above, if $\gamma \leq \underline{\gamma}^+$, then (41) is nonbinding.

Suppose now that (40) is binding and (41) is satisfied with equality, but not strictly binding. Then $U^- = 0$ and (40) and (41) can be written:

$$\gamma qU^+ \ + \ (1-\gamma)\Big((1-2q(1-q))\,u^{++}/2 \ + 2q(1-q)u^{+-}\Big) \ = \gamma\hat{\Psi} \tag{43}$$

$$-\gamma(1-q)U^+ \ + (1-\gamma)\Big((1-q(1-q))\,w_L - (1-2q(1-q))\,u^{+-} \ - q(1-q)u^{++}\Big) \ = \gamma\hat{\Psi} \tag{44}$$

To satisfy (43) the issuer must set U^+, u^{++} and/or u^{+-} strictly above what is required by (39), the participation constraint.

As long as γ is low enough so that (41) is strictly nonbinding, then in the optimal mechanism these parameters can be set to any values that satisfy equations (43) and (39).

(Note that the coefficients for these parameters in the objective function (38) are identical to those in (43).) However, when we take into account constraint (41), then it becomes optimal to set u^{++} and u^{+-} to their lowest feasible values, so that all rents are paid by setting a higher value of U^+. (This is simply because $q > 1/2$, so that $q/(1-q) > (1-2q(1-q))/(2q(1-q)) > (2q(1-q))/(1-2q(1-q))$.)
The constraints (43) and (44) are now written as:

$$\underline{\gamma}^- qU^+ = \underline{\gamma}^- \hat{\Psi} \; - \; (1-\underline{\gamma}^-)A \tag{45}$$

$$-\underline{\gamma}^-(1-q)U^+ = \underline{\gamma}^- \hat{\Psi} \; - \; (1-\underline{\gamma}^-)C \tag{46}$$

where $C \equiv (1-2q(1-q))\,(w_L - u_{AS}(0)) \; + \; q(1-q)(w_L - u_{AS}(2))$.
Subtracting (46) from (45):

$$\underline{\gamma}^- U^+ = \left(1-\underline{\gamma}^-\right)(C-A) \tag{47}$$

Putting (47) into (46) and solving for $\underline{\gamma}^-$:

$$\underline{\gamma}^- = \frac{qC+(1-q)A}{\hat{\Psi} \; + \; qC+(1-q)A} = \frac{B}{\hat{\Psi} \; + \; B} \tag{48}$$

where $B \equiv qC+(1-q)A = q(1-q(1-q))w_L - (2q-1)\,(qu_{AS}(0)+(1-q)u_{AS}(2)/2)$.
If $\gamma \leq \underline{\gamma}^-$, then (IC_U^-) is nonbinding; if $\gamma > \underline{\gamma}^-$, then (IC_U^-) is binding. We know that $C > A$. (Actually $C > 2A$.) Thus, $\underline{\gamma}^- > \underline{\gamma}^+$. ∎

Proof of Proposition 4. The proof for part 1 is given above in the proof of Proposition 3. The proof for part 2 follows directly from Proposition 1 and the proof of Proposition 3: Conditioned on the market not opening, those with positive signals expect positive informational rents and those with negative signals expect zero rents. qU^+ and qU^- are the respective rents expected by these investors, conditioned on the market opening. We've already shown that $U^+ = 0$ (> 0) if $\gamma \leq \underline{\gamma}^+$ ($\gamma > \underline{\gamma}^+$), and $U^- = 0$ if $\gamma \leq \underline{\gamma}^-$.
If $\gamma > \underline{\gamma}^-$, then we can replace (45) and (46) with

$$\gamma\left(qU^+ - (1-q)U^-\right) = \gamma\hat{\Psi} \; - \; (1-\gamma)A \tag{49}$$

$$\gamma\left(qU^- - (1-q)U^+\right) = \gamma\hat{\Psi} \; - \; (1-\gamma)C \tag{50}$$

Solving the above:

$$U^+ = \frac{\hat{\Psi}}{(2q-1)} - \frac{(1-\gamma)(qA+(1-q)C)}{\gamma(2q-1)} \tag{51}$$

$$U^- = \frac{\hat{\Psi}}{(2q-1)} - \frac{(1-\gamma)(qC+(1-q)A)}{\gamma(2q-1)} \tag{52}$$

Because $C > A$ and $q > 1-q$, $U^+ > U^-$.

The proof for part 3 also follows from the proof of Proposition 3:

If $\gamma \in [0, \underline{\gamma}^+]$, then neither IC constraint is binding and $(PC - R)$ is binding. Thus, $Eu_U(\gamma \leq \underline{\gamma}^+) = (1-\gamma)Eu_N$.

If $\gamma \in (\underline{\gamma}^+, \underline{\gamma}^-]$, then (40) is binding, but (41) is not, and expected underpricing is given by inserting (43) into (38):

$Eu_U = \gamma\hat{\Psi} + (1-\gamma)(1-2q(1-q))u_{AS}(-2)/2 = (1-\gamma)Eu_N + \gamma\hat{\Psi} - (1-\gamma)A.$

(Note: If $\gamma = \underline{\gamma}^+$, then $\gamma\hat{\Psi} = (1-\gamma)A$; if $\gamma > \underline{\gamma}^+$, then $\gamma\hat{\Psi} > (1-\gamma)A$.)

If $\gamma \in (\underline{\gamma}^-, 1]$, then both (40) and (41) are binding. Expected underpricing is determined by setting $u^{++} = u_{AS}(2)$ and $u^{+-} = u_{AS}(0)$, and by inserting (51) and (52) into (38):

$$
\begin{aligned}
Eu_U &= (1-\gamma)Eu_N + \gamma q(U^+ + U^-) \\
&= (1-\gamma)Eu_N + \frac{\gamma 2q\hat{\Psi}}{(2q-1)} - \frac{(1-\gamma)q(A+C)}{(2q-1)}. \qquad (53)
\end{aligned}
$$

From the proof of Proposition 3 we know that if $\gamma > \underline{\gamma}^-$, then $\gamma\hat{\Psi} > (1-\gamma)(qC+(1-q)A)$. It follows (because $C > A$ and $q > 1-q$) that if $\gamma > \underline{\gamma}^-$, then $\gamma\hat{\Psi} > (1-\gamma)((1-q)C+qA)$. Thus, if $\gamma > \underline{\gamma}^-$, then $\gamma 2q\hat{\Psi} > (1-\gamma)q(A+C)$. ∎

Proof of Proposition 5. The IC constraints are the same as in the benchmark case, (29) and (30), except that an investor who lies expects to make trading profits if the market opens:

$$
\begin{aligned}
&(1-2q(1-q))\left(u^{++}h^{++} - (w_L + u^{+-})h^{-+}\right) + \\
&\qquad 2q(1-q)\left(u^{+-}h^{+-} - (w_L + u^{--})h^{--}\right) \geq \gamma\Psi \qquad (54) \\
&(1-2q(1-q))\left(u^{--}h^{--} + (w_L - u^{+-})h^{+-}\right) + \\
&\qquad 2q(1-q)\left(u^{+-}h^{-+} + (w_L - u^{++})h^{++}\right) \geq \gamma\Psi \qquad (55)
\end{aligned}
$$

Because of the factor w_L, it is optimal to set $h^{-+} = 0$ and $2h^{++} = h^{+-} = (1-h_R)$. It is also optimal to set $u^{+-} = u_{AS}(0)$. The above constraints can thus be written:

$$
(1-2q(1-q))\frac{u^{++}}{2} + 2q(1-q)\left(u_{AS}(0) - \frac{(w_L + u^{--})h^{--}}{(1-h_R)}\right) \geq \gamma\hat{\Psi} \qquad (56)
$$

$$
(1-2q(1-q))\left(\frac{u^{--}h^{--}}{(1-h_R)} + w_L - u_{AS}(0)\right) + q(1-q)(w_L - u^{++}) \geq \gamma\hat{\Psi} \qquad (57)
$$

where $\hat{\Psi} = \Psi/(1-h_R)$. It is clear that (56) will bind before (57). We start by assuming that (57), (IC^-), is nonbinding. (We check this at the end of the proof.) It is thus optimal to set $h^{--} = 0$ and $u^{--} = u_{AS}(-2)$. If (56) is nonbinding, then expected underpricing is Eu_N, the same as in the benchmark case. If (56) is binding, then

$$u^{++} = \frac{2}{1-2q(1-q)}\left(\gamma\hat{\Psi} - 2q(1-q)u_{AS}(0)\right) . \tag{58}$$

The expected underpricing is thus:

$$\begin{aligned} Eu_C &= \max\left[Eu_N\,,\ \gamma\hat{\Psi} + (1-2q(1-q))\,u_{AS}(-2)/2\right] && (59) \\ &= Eu_N + \max\left[0,\ \gamma\hat{\Psi} - A\right] && (60) \end{aligned}$$

where A is defined in Proposition 3.

As a final step we check that (57) is nonbinding. Inserting (58) into (57):

$$\gamma\hat{\Psi} \leq (1-2q(1-q))(1-q(1-q))w_L - (1-4q(1-q))u_{AS}(0) \tag{61}$$

Noting that $u_{AS}(0) = (1-2q(1-q))\left(\frac{\alpha}{2-\alpha}\right)w_L$, (61) can be written:

$$\frac{\gamma\hat{\Psi}}{w_L} \leq (1-2q(1-q))\Big(1-q(1-q) - (1-4q(1-q))\alpha/(2-\alpha)\Big) \tag{62}$$

The RHS of (62) is $> 3/8$. (This can be seen by allowing q to go to its lower limit of 1/2.) Thus, $\Psi \leq (3/8)(1-h_R)w_L$ is sufficient to ensure that (57) is nonbinding. ∎

Proof of Proposition 6. This proposition follows from Propositions 4 and 5 (as summarized in Table 1), and:

Part 1: $Eu_C \geq Eu_N - A + \gamma\hat{\Psi} > (1-\gamma)(Eu_N - A) + \gamma\hat{\Psi}$.

Part 2: From the proof of Proposition 4, if $\gamma > \underline{\gamma}^-$, then $\gamma 2q\hat{\Psi} > (1-\gamma)q(A+C)$.

As $\gamma \to 1$, $Eu_U \to (2q/(2q-1))\hat{\Psi}$.

From equations (25) to (28), we know that $w_L > 3u_{AS}(0)$ and $u_{AS}(0) > u_{AS}(2) > u_{AS}(-2)$. Thus, $\hat{\Psi} > w_L/4$ (and $\hat{\Psi} \leq 3w_L/8$, as in Proposition 5) is sufficient so that, as $\gamma \to 1$, $Eu_C \to \hat{\Psi} + u_{AS}(-2)/2$.

$(2q/(2q-1))\hat{\Psi} - \hat{\Psi} - u_{AS}(-2)/2 > 0$ for $\hat{\Psi} > w_L/4$.

Thus, $w_L/4 < \hat{\Psi} \leq 3w_L/8$ is sufficient such that as $\gamma \to 1$, $Eu_C < Eu_U$. ∎

Proof of Corollary 1. Eu_N = expected underpricing with no when-issued trading $> (1-\gamma)Eu_N$. But, $\gamma\hat{\Psi} - (1-\gamma)A > 0$. ∎

Proof of Proposition 7. The market opens if the mechanism is $++$ or $--$, but doesn't if the outcome is $+-$. Thus, in the IC constraints for pricing regime U, w_L appears only in the outcome $+-$:

$$Er^{+} = q^2 u_c^{++} h_c^{++} + (1-q)^2 u_w^{++} h_w^{++} + 2q(1-q) u^{+-} h^{+-} \quad (63)$$

$$Er^{-} = q^2 u_c^{--} h_c^{--} + (1-q)^2 u_w^{--} h_w^{--} + 2q(1-q) u^{+-} h^{-+} \quad (64)$$

$$\begin{aligned} Er^{+} \geq\ & \left(q^2 + (1-q)^2\right)\left(u^{+-} + w_L\right) h^{-+} + \\ & q(1-q)\left(u_c^{--} h_c^{--} + u_w^{--} h_w^{--}\right) + 2q(1-q)\Psi_1 \qquad (IC^{+}) \\ Er^{-} \geq\ & \left(q^2 + (1-q)^2\right)\left(u^{+-} - w_L\right) h^{+-} + \\ & q(1-q)\left(u_c^{++} h_c^{++} + u_w^{++} h_w^{++}\right) + 2q(1-q)\Psi_1 \qquad (IC^{-}) \end{aligned}$$

Note on expected trading profits, Ψ_1: Suppose that polled investor i lies and the market still opens. This means that polled investor j reported a signal that agrees with i's report and thus disagrees with i's actual signal. Investor i does have private information that she can trade on, but this information is arguably of lower quality that in the case without endogenous market opening. We can assume that $0 < \Psi_1 < \Psi$.

As in the earlier proofs, it is optimal to set $u_w^{--} = h_w^{--} = 0$ and $h^{-+} = 0$. We thus write the problem as follows: The objective is to minimize:

$$Eu = q^2(u_c^{++} + u_c^{--})/2 + (1-q)^2 u_w^{++}/2 + 2q(1-q)u^{+-} \quad (65)$$

Subject to the following IC constraints:

$$q^2 u_c^{++} h_c^{++} + (1-q)^2 u_w^{++} h_w^{++} + 2q(1-q) u^{+-} h^{+-} - q(1-q) u_c^{--} h_c^{--} \geq 2q(1-q)\Psi_1 \quad (66)$$

$$q^2 u_c^{--} h_c^{--} - \left(q^2 + (1-q)^2\right)\left(u^{+-} - w_L\right) h^{+-} - q(1-q)\left(u_c^{++} h_c^{++} + u_w^{++} h_w^{++}\right) \geq 2q(1-q)\Psi_1 \quad (67)$$

and the following participation constraints:

$$u_c^{++},\ u_w^{++},\ u_c^{--} \geq 0, \quad u^{+-} \geq u_{AS}(0) \quad (68)$$

The allocation constraints are the same as in the earlier problems.

Because $q > 1/2$, the optimal solution calls for the maximum feasible allocation to polled investors whose reports are verified when the market does open: $2h_c^{++} = 2h_c^{--} = 1 - h_R$, and the minimum allocation those whose reports are contradicted: $h_w^{++} = 0$.

Part 1: Either (66) and (67) are both nonbinding, or (66) is binding and (67) is nonbinding, or both are binding. For the moment we'll assume that (67) is nonbinding; we'll check this below. It is thus optimal to set u^{+-} to its lowest feasible value of $u_{AS}(0)$. From equation (28) we know that $u_{AS}(0) < w_L/3$. Thus, it is optimal to set h^{+-} to the largest feasible amount: $h^{+-} = 1 - h_R$, and to set $u_c^{--} = 0$ and $qu_c^{++}/2 = \max\left[2(1-q)\left(\hat{\Psi}_1 - u_{AS}(0)\right), 0\right]$. This results in the following expected underpricing:

$$Eu_U = 2q(1-q)\ \max\left[u_{AS}(0), \hat{\Psi}_1\right]\ . \tag{69}$$

We now need to check if (67) is nonbinding. We know that (67) can only bind if (66) is also binding. Thus, necessary and sufficient for (**??**) to be nonbinding is:

$$\begin{aligned}(1-2q(1-q))\,(w_L - u_{AS}(0)) - 2(1-q)^2\left(\hat{\Psi}_1 - u_{AS}(0)\right) \;\geq\; 2q(1-q)\hat{\Psi}_1 \;\Longrightarrow \\ (1-2q(1-q))\,w_L - (2q-1)u_{AS}(0) \;\geq\; 2(1-q)\hat{\Psi}_1\end{aligned} \tag{70}$$

As stated above, $u_{AS}(0) < w_L/3$. So sufficient for (70) is:

$$\left(1 - 2q(1-q) - \frac{(2q-1)}{3}\right) w_L \;\geq\; 2(1-q)\hat{\Psi}_1 \tag{71}$$

Keeping in mind that $q > 1/2$, it is easy to show that $\hat{\Psi}_1 \leq w_L/2$ is sufficient for (71). Thus, for all reasonable parameter values, (67) is nonbinding.

Part 2: We now consider pricing regime C. This problem is identical to that of Proposition 5, except that after a lie the market opens with probability $2q(1-q)$. Following from equation (60):

$$Eu_C = Eu_N + \max\left[0,\ 2q(1-q)\hat{\Psi}_1 - A\right] \tag{72}$$

where A is defined in Proposition 3.

$Eu_N > 2q(1-q)u_{AS}$ and $A < Eu_N$, so $Eu_U < Eu_C$. ∎

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