Equilibrium Asset Pricing in Directed Networks
with Mutually Exciting Jumps

Nicole Branger*   Patrick Konermann*   Christoph Meinerding**   Christian Schlag**

First version: May 2014

This version: November 30, 2015

ABSTRACT

We analyze the implications of network structures for equilibrium asset prices, where networks are represented via mutually exciting jump processes for dividends. Our approach provides a flexible and tractable framework for asset pricing when the direction of shock propagation is relevant. We document that this directedness matters, e.g., for return volatilities, and taking it into account can lead to a substantially different interpretation of empirical findings, e.g., concerning a flight-to-quality effect. The model generates the positive centrality premium documented empirically in Ahern (2013), and we show that this premium is distinctly different from a standard CAPM-like premium for market risk.

Keywords: Asset pricing, general equilibrium, recursive preferences, dynamic networks, mutually exciting processes, directed shocks

JEL: G01, G12, D85

Equilibrium Asset Pricing in Directed Networks
with Mutually Exciting Jumps

First version: May 2014

This version: November 30, 2015

ABSTRACT

We analyze the implications of network structures for equilibrium asset prices, where networks are represented via mutually exciting jump processes for dividends. Our approach provides a flexible and tractable framework for asset pricing when the direction of shock propagation is relevant. We document that this directedness matters, e.g., for return volatilities, and taking it into account can lead to a substantially different interpretation of empirical findings, e.g., concerning a flight-to-quality effect. The model generates the positive centrality premium documented empirically in Ahern (2013), and we show that this premium is distinctly different from a standard CAPM-like premium for market risk.

Keywords: Asset pricing, general equilibrium, recursive preferences, dynamic networks, mutually exciting processes, directed shocks

JEL: G01, G12, D85

1. Introduction

The notion of an economy as a network of more or less tightly linked units has recently become popular in the finance and economics literature, especially in the aftermath of the financial and economic crisis. In this paper we analyze the implications of different network structures for equilibrium asset prices and returns. One special aspect that we focus on is the fact that links in a network can have a direction, i.e., that it can make a difference if the link is from node i to node j or the other way around. We show that this directedness has first-order implications for key asset pricing quantities like valuation ratios, expected returns, and return volatilities.

A simple example for such a directed link is a customer-supplier relationship between two firms. Shocks to the profitability of a large customer will certainly have a negative impact on the small supplier and, among other things, increase its default risk, while a shock to the small supplier is much less likely to have significant consequences for the large customer. Another important aspect of this kind of shock propagation is that it has a time dimension and does not happen in one single instant. In the above example the riskiness of the supplier increases after the initial shock, implying a higher likelihood of negative shocks in the future, and this in turn might negatively affect other firms linked to the supplier.

These two features, i.e. the fact that shock propagation can have a direction and that the spreading of shocks through the system takes time, cannot be captured by a pure diffusion/correlation framework (due to its inherent symmetry) or by contemporaneous jumps. We therefore propose a model featuring multiple economic units ('firms')[1] whose dividends are linked via self-exciting and mutually exciting jump processes. With such processes, a downward jump in the cash flow of one firm then increases the probability of subsequent negative shocks to its own cash flow as well as to those of other firms. The direction and magnitude of this shock propagation characterizes the network structure of our economy, which thus manifests itself only indirectly via the dynamics of jump intensities as state variables, but not directly in the level of cash flows. Aggregate consumption, a key quantity in any equilibrium model, is driven by the sum of all the individual jumps, but a given jump always affects the cash flows of only one firm at a time. Our model is general and flexible in the sense that it can represent arbitrary network structures. Nevertheless, it still remains tractable with at least semi-closed form expressions for all equilibrium quantities, since it belongs to the exponentially affine class, for which there is a well-developed solution theory based on the work by Eraker and Shaliastovich (2008).

---

[1]We will use terms like 'node', 'firm', and 'asset' interchangeably.

The representative agent in our model has recursive preferences of the Epstein-Zin type.[2] These preferences are a key ingredient, since they generate premia for the risk of higher future jump intensities of the dividends. The representative agent's preference for early resolution of uncertainty, i.e., the fact that she cares about the risk associated with future values of the state variables, implies that the price-dividend ratios of all assets will react to a jump in one individual dividend, and it is the structure of the network which determines the direction and the magnitude of this reaction.

Our first set of results concerns the important issue of how *network centrality*, i.e., the relative importance of the individual assets within the network, affects the cross-section of expected returns. Using data on customer-supplier relationships between different industries, Ahern (2013) documents a positive market price of centrality, i.e., more central assets earn higher expected returns. We provide an equilibrium foundation for this empirical finding. In addition to explaining this key result, our model makes predictions for the relation between the cross-sectional dispersion of the centrality measure across the nodes in a network and the associated centrality premium. We find that this premium is the lower the higher the cross-sectional dispersion of the centrality measure.

Our model provides a controlled environment, so that we can address an important question raised by Ahern (2013), namely whether the positive centrality premium indeed represents the premium for a separate risk factor or whether it is just a CAPM-type market risk premium 'in disguise'. To this end we regress model-implied local expected excess returns on the theoretical market and centrality betas. As a result of this exercise we find that both market and centrality beta command significantly positive risk premia, so that the market and the centrality portfolio can be regarded as separate risk factors.

The analysis in Ahern (2013) focuses on undirected networks. One of the particular advantages of our model is its ability to represent directed links, and we show that direction has first-order implications for equilibrium asset prices and returns. We consider two stylized cases, the 'star network' and the 'reverse star network'. In the star network, shocks to the node at the center of the star are propagated to all the 'outer' nodes, but not vice versa. For the reverse star network the propagation of shocks is exactly the other way around. We show that, in line with intuition, the sign of the centrality premium is independent of the direction of the links, i.e., the most central asset has the highest expected excess return in both networks. However, direction does matter for quantities like return volatilities, market betas, or Treynor ratios. We also document that, contrary to intuition, the sign of the

---

[2]The asset pricing literature has largely documented the importance of such preferences in order to generate sizeable risk premia, but also to solve many other asset pricing puzzles. See Bansal and Yaron (2004) as the most important source.

centrality premium depends on the direction of the links if we assume CRRA preferences as, e.g., in Buraschi and Porchia (2013).

Our final result concerns the 'flight-to-quality' effect. Flight to quality means that in a crisis certain assets become valuable as safe havens (see, e.g., Baele, Bekaert, Inghelbrecht, and Wei (2015)). The equilibrium mechanism in our model nicely reproduces this phenomenon. We study two networks in particular, the 'directed ring network', where the shock in the dividend of a given asset has an impact only on its own jump intensity and on the one of the next asset in the ring, and the 'undirected ring network', where all the individual links in the ring go both ways. We find that the further away an asset from the original source of a shock, the less negative the reaction of its price-dividend ratio. For assets far enough away, the price actually even goes up, resulting exactly in the flight-to-quality effect. We show that direction is important here as well, since it creates the notion of distance between two nodes. In the directed ring network with ten assets, asset 1 and asset 10 are very far apart, and the price of asset 10 goes up in response to a shock in the dividend of asset 1. This equilibrium price response however seems counterintuitive to an econometrician using an undirected model where these two assets are very close.

In summary, our model featuring mutually and self-exciting jumps together with recursive utility is flexible, but still tractable, and provides a theoretical explanation for empirical results with respect to network centrality. It allows a precise analysis of the equilibrium impact of directed shock propagation in a multi-asset economy, which is an important step beyond the existing literature.

The paper by Eraker and Shaliastovich (2008) provides the methodological framework for our model with recursive utility and state variables like stochastic jump intensities. Besides, our project is related to a couple of other strands of literature. There is an increasing literature about asset pricing models with time-varying jump intensities. Aït-Sahalia, Cacho-Diaz, and Laeven (2014) are the first to discuss the role of mutually exciting jumps in finance applications, in particular when it comes to capturing stylized facts like jump clustering in equity prices. Aït-Sahalia, Laeven, and Pelizzon (2014) find evidence of self-excitation and asymmetric mutual excitation in the CDS market. Benzoni, Collin-Dufresne, Goldstein, and Helwege (2014) analyze defaultable bonds which are subject to contagion risk in a general equilibrium model with a hidden state of nature and a representative investor with fragile beliefs. Wachter (2013) focuses on the equity premium and the equity volatility in an economy with a stochastic intensity for rare consumption disasters, but does so in a model with only one endowment stream which obviously does not lend itself to any network applications. Nowotny (2011) investigates a similar setup with self-exciting processes. Branger, Kraft, and

Meinerding (2014) show that such dynamics can endogenously evolve in a framework with learning about latent disaster intensities.

A group of papers deals with the implications of network structures for asset prices and returns. The approaches differ with respect to which quantities are linked in a network, how these linkages are modeled (static vs. dynamic, directed vs. undirected, etc.) and which particular aspect of the data they are supposed to explain. The paper by Buraschi and Porchia (2013) is the closest to ours since it also focuses on cash flow networks. But as we are going to show below, their assumptions in terms of preferences and dividend dynamics may produce somewhat counterintuitive results. Ahern (2013) empirically analyzes sectoral input-output linkages and their implications for the cross-section of industry returns. Diebold and Yilmaz (2014) discuss several connectedness measures based on return variance decompositions. Aobdia, Caskey, and Ozel (2014) focus on the aspect of cross-industry predictability of returns and link this to network analysis. Billio, Getmansky, Gray, Lo, Merton, and Pelizzon (2014); Billio, Getmansky, Lo, and Pelizzon (2012) estimate network linkages directly from market data like stock returns and CDS spreads of the banking and insurance sector by identifying Granger-causal relations. They draw conclusions about systemic risk in the financial sector. Kelly, Lustig, and Nieuwerburgh (2013) propose a model in which the growth rates of firm sizes are linked in order to analyze the joint evolution of the cross-sectional firm size and firm volatility distribution. Colla and Mele (2010), Ozsoylev and Walden (2011), Han and Yang (2013), and Walden (2013) explore the implications of information networks in theoretical equilibrium asset pricing models with many agents where information flows from one agent to the other. Naturally, they put a particular emphasis on the trading behavior generated by such interaction among agents. Ozsoylev, Walden, Yavuz, and Bildik (2014) provide an empirical analysis of such trader networks. Herskovic (2014) analyzes networks of input-output relations in a multi-sector production economy model and deduces implications for cross-sectional asset pricing from this production network. Richmond (2015) applies a similar logic to a multi-country model and international asset pricing.

Our paper is also linked to a broader literature about microeconomic foundations of macroeconomic effects, which, however, does not put a special emphasis on asset prices and returns. Elliott, Golub, and Jackson (2014) consider a random graph model of financial intermediaries. The key result of their analysis is that the banking system is most susceptible to financial cascades when integration is intermediate and organizations are partly diversified. In related studies, Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012); Acemoglu, Ozdaglar, and Tahbaz-Salehi (2013, 2015), focus on the propagation of shocks in basically static one- or two-period networks. Gabaix (2011) studies the implications of the granularity of microeconomic networks for aggregate fluctuations in a dynamic macroeconomic model.

Acemoglu, Akcigit, and Kerr (2015) show that input-output network relations can be a driver of macroeconomic fluctuations, in particular in an international context.

2. Model

2.1. Consumption and dividends

We assume a Lucas endowment economy. Log aggregate consumption $y \equiv \ln Y$ follows

$$dy_t = \mu \, dt + \sum_{i=1}^{n} K_i \, dN_{i,t},$$

where μ is the constant expected growth rate and the N_i $(i = 1, \ldots, n)$ are self- and mutually exciting jump processes with constant jump sizes K_i.[3] Their stochastic jump intensities $\ell_{i,t}$ have dynamics

$$d\ell_{i,t} = \kappa_i \left(\bar{\ell}_i - \ell_{i,t} \right) dt + \sum_{j=1}^{n} \beta_{i,j} \, dN_{j,t}, \tag{1}$$

so that the coefficient $\beta_{i,j}$ represents the discrete change in ℓ_i induced by a jump in N_j. The $\beta_{i,j}$, collected in what we call the 'beta matrix', completely determine the structure of the given network and will play a key role in our analysis. To preclude negative intensities we assume $\beta_{i,j} \geq 0$ for all pairs (i, j).

There are n firms in the economy, indexed by i, with the following dynamics for log dividends y_i

$$dy_{i,t} = \mu_i \, dt + L_i \, dN_{i,t} \qquad (i = 1, \ldots, n). \tag{2}$$

Note that we do not explicitly link aggregate consumption to the sum of dividends, but model dividends as claims on certain risk factors in the consumption process. This is similar to the assumptions underlying the pricing of dividend claims in models like Bansal and Yaron (2004) or Backus, Chernov, and Martin (2011).

The recent literature about the propagation of shocks in firm networks with papers like Elliott, Golub, and Jackson (2014) and Acemoglu, Ozdaglar, and Tahbaz-Salehi (2013) pro-

---

[3]We do not include diffusion terms in the dynamics of aggregate consumption for parsimony. One could of course generalize the model to generate additional types of diffusive risk premia, e.g., by making the expected consumption growth rate time-varying as in Bansal and Yaron (2004). We abstract from such extensions to keep the model tractable and to focus on the jump components in all our analyses.

vides a number of arguments in support of our specification. According to the authors, Elliott, Golub, and Jackson (2014) "model contagions and cascades of failures among organizations linked through a network of financial interdependencies" and "identify how the network propagates discontinuous changes in asset values triggered by failures (e.g., bankruptcies, defaults, and other insolvencies)."[4] Acemoglu, Ozdaglar, and Tahbaz-Salehi (2013) argue that "the economy's input-output structure can fundamentally reshape the distribution of aggregate output, increasing the likelihood of large downturns from infinitesimal to substantial."[5] Furthermore, as shown in Branger, Kraft, and Meinerding (2014), when investors only have partial information about shock propagation in the fundamentals of the economy, mutually exciting processes can actually arise endogenously due to investor learning.

In addition to these arguments one can make a strong point in favor of mutually exciting processes as the most natural choice when it comes to the analysis of network effects in equilibrium asset pricing. First of all, mutually exciting processes allow for the analysis of both directed and undirected networks whereas pure diffusion models are restricted to symmetric, i.e., undirected, network structures. Second, within the class of jump processes, mutually exciting jumps are very tractable because they are affine so that we can solve for the equilibrium in our economy in semi-closed form applying the log-linearization procedure laid out in Eraker and Shaliastovich (2008). Third, as argued by Aït-Sahalia, Cacho-Diaz, and Laeven (2014), "unless exogenous state variables are extremely autocorrelated and jump themselves, it is not possible using exogenous state variables in intensities to [match] the empirical observations of jump clusters."[6] Fourth, the structure of the jump processes in our model is fundamentally different, both in a time series and in a cross-sectional dimension, from, for instance, contemporaneous jumps in many assets. The serial dependence of shocks resulting from mutual excitation is particularly relevant for asset pricing where the timing and sequentiality of shocks plays a prominent role.

Finally, in addition to all the fundamental advantages in terms of modeling and computation of the equilibrium, mutually exciting processes offer a very natural representation of the structure of a network. Each coefficient $\beta_{j,i}$ indicates a directed link with a certain strength from asset i to asset j, so that altogether the beta matrix can easily be translated into a network graph. Equations (1) and (2) formalize how the beta matrix gives rise to a dynamic shock propagation mechanism by which negative shocks to one dividend stream can spread across the economy. When $\beta_{j,i}$ is positive, a downward jump in dividend i immediately increases the jump intensity of dividend j by $\beta_{j,i}$. Once the increased intensity ℓ_j

---

[4]cp. the abstract of Elliott, Golub, and Jackson (2014)

[5]cp. the abstract of Acemoglu, Ozdaglar, and Tahbaz-Salehi (2013)

[6]cp. Aït-Sahalia, Cacho-Diaz, and Laeven (2014), page 6.

indeed leads to a jump in dividend j and there is a nonzero coefficient $\beta_{k,j}$, the initial shock is passed on to asset k and can in this way be propagated through the whole network. Note that our specification also allows for 'feedback loops'. Suppose that, for instance, there is another directed link from k back to i. Then there is a positive probability that an initial shock in dividend i is transmitted through the network and eventually reaches asset i again. Nevertheless, each jump only affects one dividend directly, so that network connectivity is captured exclusively via linkages in the dynamics of the state variables.

Our specification ensures that the vector $X = (y, \ell_1, \ldots, \ell_n, y_1, \ldots, y_n)'$ follows an affine jump process.[7] The joint process (N, ℓ) is Markov.[8] When we analyze the model quantitatively we will assume that all state variables (i.e., intensities) are at their respective long-run means. Due to the presence of the mutually exciting jump terms these long-run means $\bar{\bar{\ell}}_i$, i.e., the unconditional expectations, are not equal to the respective mean reversion levels $\bar{\ell}_i$, as it would be the case, e.g., for a standard square-root process. Instead, the $\bar{\bar{\ell}}_i$ are the solution to the following system of equations:

$$\bar{\bar{\ell}}_i = \frac{\kappa_i \bar{\ell}_i + \sum_{j \neq i} \beta_{i,j} \bar{\bar{\ell}}_j}{\kappa_i - \beta_{i,i}} \qquad (i = 1, \ldots, n). \tag{3}$$

We assume $\kappa_i > \beta_{i,i}$ for $i = 1, \ldots, n$ to ensure that all the $\bar{\bar{\ell}}_i$ are positive.

2.2. Representative agent

Our economy is populated by a representative agent with an infinite planning horizon. As discussed above network connectivity has no direct effect on the level of cash flows, but only on the state variables. To endogenously generate a risk premium for network connectivity we therefore assume that the agent has recursive preferences as in Bansal and Yaron (2004) and Eraker and Shaliastovich (2008).

To see why there is indeed such a nonzero premium, consider for a moment the Epstein-Zin utility function in discrete time:

$$U_t = \left[\left(1 - e^{-\delta} \right) Y_t^{\frac{1-\gamma}{\theta}} + e^{-\delta} \left(\mathbb{E}_t \left[U_{t+1}^{1-\gamma} \right] \right)^{\frac{1}{\theta}} \right]^{\frac{\theta}{1-\gamma}},$$

where δ is the subjective time preference rate, γ is the coefficient of relative risk aversion, ψ denotes the elasticity of intertemporal substitution (EIS), and $\theta \equiv \frac{1-\gamma}{1-\frac{1}{\psi}}$. CRRA preferences

---

[7]See Appendix A for details.

[8]See, e.g., Aït-Sahalia, Cacho-Diaz, and Laeven (2014) for details about mutually exciting processes, in particular conditions for the stationarity of the model.

are a special case with $\gamma = \frac{1}{\psi}$ and consequently $\theta = 1$.

Using the transformation $V_t = \frac{U_t^{1-\frac{1}{\psi}}}{1-\frac{1}{\psi}}$, Colacito and Croce (2013) derive the following approximation:

$$V_t \approx \left(1 - e^{-\delta}\right) \frac{Y_t^{1-\frac{1}{\psi}}}{1-\frac{1}{\psi}} + e^{-\delta} \mathbb{E}_t\left[V_{t+1}\right] - Var_t\left[V_{t+1}\right] \frac{(1-\theta)\, e^{-\delta}}{2\, \mathbb{E}_t\left[V_{t+1}\right]}. \tag{4}$$

When $\theta \neq 1$ the last term related to the variance of continuation utility does not vanish as it would for CRRA. Variations in state variables give rise to variations in continuation utility and thus affect total utility.

In our numerical analysis we will assume $\gamma = 10$, $\psi = 1.5$, and $\delta = 0.02$, so that the representative agent has a preference for early resolution of uncertainty, since $\gamma > \frac{1}{\psi}$ and $\theta < 1$. The approximation (4) shows that in this case the agent not only considers future expected utility $\mathbb{E}_t\left[V_{t+1}\right]$, but also exhibits an aversion against future utility risk $Var_t[V_{t+1}]$, i.e., total utility decreases with increasing $Var_t[V_{t+1}]$. Variation in state variables, i.e., in the intensities ℓ_i, does not affect the *level* of cash flows directly, but has an impact on the distribution of the continuation utility V_{t+1} and thus on $Var_t\left[V_{t+1}\right]$. This implies an extra risk premium in equilibrium, which can in our case be interpreted as a premium for the degree of connectivity in the network.

2.3. Pricing kernel and equilibrium

The derivation of the model solution closely follows Eraker and Shaliastovich (2008).[9] They show that the continuous-time dynamics of the pricing kernel M can be consistently defined in the following way:

$$d\ln M_t = -\delta\,\theta\,dt - (1-\theta)\,d\ln R_t - \frac{\theta}{\psi}\,dy_t.$$

The return on the consumption claim R_t satisfies the following continuous-time version of the Euler equation

$$0 = \frac{1}{dt}\,\mathbb{E}\left[\frac{d\left(e^{\ln M_t + \ln R_t}\right)}{e^{\ln M_t + \ln R_t}}\right]$$

---

[9]Details are presented in Appendix A.

and follows from the dynamics of the log wealth-consumption ratio v and aggregate consumption.

We employ the usual affine guess for the log wealth-consumption ratio v_t, i.e., we assume $v_t = A + B' \ell_t$ with $B = (B_1, \ldots, B_n)'$ and $\ell_t = (\ell_{1,t}, \ldots, \ell_{n,t})'$. We also use the Campbell-Shiller log-linear approximation $d \ln R_t = k_0 \, dt + k_1 \, dv_t - (1 - k_1) \, v_t \, dt + dy_t$ with linearizing constants k_0 and $0 < k_1 < 1$. We solve for the coefficients A and B as well as the linearizing constants numerically.

The dynamics of the pricing kernel are

$$\frac{dM_t}{M_t} = -r_t \, dt - \sum_{i=1}^{n} \mathrm{MPJR}_i \, dN_{i,t},$$

where r_t is the equilibrium risk-free rate.

The (in general negative) market prices of jump risk are given by

$$\mathrm{MPJR}_i = 1 - \exp\left\{ -\gamma \, K_i + k_1 \, (\theta - 1) \left[\sum_{j=1}^{n} B_j \, \beta_{j,i} \right] \right\} \tag{5}$$

and quantify the impact of jumps on the investor's total wealth. The exponential term is a product of two factors. The first one, $\exp\{-\gamma \, K_i\}$, represents the compensation for the immediate cash flow shock caused by the jump in dividend i. The second one with the remaining exponents is the compensation for the risk caused by variations in the state variables. It depends on the impact of the intensities ℓ_i on the equilibrium wealth-consumption ratio, represented by the components of the vector B. In the special case of CRRA utility ($\theta = 1$) this second term vanishes, implying that state variable risk is not priced.

In general, the components of B are all negative, i.e., the wealth-consumption ratio is decreasing in each of the jump intensities. For our parametrization with $\psi > 1$, this is due to the fact that the substitution effect dominates the income effect, so that the investor consumes more and saves less in bad times with high jump intensities.[10] With $k_1 > 0$, $\beta_{i,j} \geq 0$, and $\theta < 1$ we thus have $k_1 \, (\theta - 1) \left[\sum_{j=1}^{n} B_j \, \beta_{j,i} \right] > 0$, which increases (in absolute terms) the negative market price for jump risk relative to the CRRA case with $\theta = 1$.

The expression above nicely illustrates how the market prices of risk depend on the network topology represented by the beta matrix. First of all, the market price of risk for jumps in dividend i is the larger (in absolute terms), the stronger the network linkages from

---

[10]Note that with CRRA preferences the B_i would generally be positive, leading to higher price-to-fundamentals ratios in bad times.

asset i to other assets, i.e., the larger the coefficients $\beta_{j,i}$. When asset i is very central in the sense that its shocks have the potential to spread out to many other assets because many $\beta_{j,i}$ are nonzero, the market price for its shocks, MPJR_i, will be large. The same is true when asset i is linked to only a few assets, but these links are very strong, i.e., when the corresponding $\beta_{j,i}$ are large.

Moreover, for given $\beta_{j,i}$, the market price of risk for N_i also depends on the coefficients $B_1, \ldots, B_n$. These coefficients quantify the relative importance of the assets (or, equivalently, the intensities) for the wealth-consumption ratio. Suppose that $\beta_{j,i} \neq 0$ and the coefficient B_j is large (in absolute terms). This can be the case, for instance, if asset j is a very central and important asset. Then, economically speaking, asset i inherits centrality from asset j in the sense that the market price of risk for its own jumps is larger simply because it is linked to the central asset j. Again note that effects like these are not present in a model with CRRA preferences, since there we have $\theta = 1$.

In analogy to the return on the consumption claim, the returns $R_{i,t}$ on the individual dividend claims satisfy the continuous-time Euler equations

$$0 = \frac{1}{dt} \mathbb{E}\left[\frac{d\left(e^{\ln M_t + \ln R_{i,t}}\right)}{e^{\ln M_t + \ln R_{i,t}}}\right].$$

To compute the expected excess return on asset i, we proceed as in the case of the consumption claim, i.e., we employ an affine guess for the log price-dividend ratio of asset i, $v_{i,t} = A_i + C_i' \ell_t$ with $C_i = (C_{i,1}, \ldots, C_{i,n})'$, and use the Campbell-Shiller approximation $d\ln R_{i,t} = k_{i,0}\, dt + k_{i,1}\, dv_{i,t} - (1 - k_{i,1})\, v_{i,t}\, dt + dy_{i,t}$ with linearization constants $k_{i,0}$ and $k_{i,1}$. Again we solve for the coefficients A_i and $C_{i,j}$ $(j = 1, \ldots, n)$ as well as for the linearization constants $k_{i,0}$ and $k_{i,1}$ numerically.

The return on the i-th individual dividend claim is then given by

$$dR_{i,t} = \ldots dt + \sum_{j=1}^{n} \text{JEXP}_{i,j}\, dN_{j,t}$$

with the jump exposures

$$\text{JEXP}_{i,j} = \begin{cases} \exp\left(L_i + k_{i,1} \sum_{k=1}^{n} C_{i,k} \beta_{k,i}\right) - 1 & i = j \\ \exp\left(k_{i,1} \sum_{k=1}^{n} C_{i,k} \beta_{k,j}\right) - 1 & i \neq j \end{cases}. \tag{6}$$

The exponential term in the exposure of asset i to jumps in its own dividend, $\text{JEXP}_{i,i}$, has two components. First, the price change due to the immediate cash flow shock in the

dividend is represented via the jump size L_i. By assumption this component is only present in the exposure of asset i to jumps in dividend i, because N_i exclusively affects the level of y_i. The special feature of our (and also any other) model with recursive utility is the second component, $\exp(k_{i,1} \sum_{k=1}^{n} C_{i,k} \beta_{k,i})$, which quantifies the price-dividend ratio effect of the shock. For $i \neq j$, the exposure $\text{JEXP}_{i,j}$ only consists of this price-dividend effect.

The reaction of the price-dividend ratio of asset i to a jump in dividend j depends on two ingredients: (i) the impact of that jump on the intensities $\ell_1, \ldots, \ell_n$, represented by $\beta_{1,j}, \ldots, \beta_{n,j}$, and (ii) the impact of each of the state variables on the price-dividend ratio of asset i, measured by the (usually nonzero) coefficients $C_{i,1}, \ldots, C_{i,n}$. In particular, a jump in dividend j can lead to a nonzero return in asset i even if the two assets are not linked, i.e., even if $\beta_{i,j} = 0$. To see this, suppose $\beta_{k,j} > 0$ for some arbitrary k. Then $C_{i,k} \beta_{k,j} \neq 0$ so that $\text{JEXP}_{i,j} \neq 0$. More precisely, as soon as a jump in dividend j affects at least one of the state variables $\ell_1, \ldots, \ell_n$, this jump leads to equilibrium price reactions of *all* assets in the economy.

Finally, the local expected excess return of asset i can be written as follows:

$$\frac{1}{dt} \mathbb{E}\left[dR_{i,t}\right] - r_t \;=\; \sum_{j=1}^{n} \ell_{j,t} \, \text{MPJR}_j \, \text{JEXP}_{i,j}. \tag{7}$$

The premium which asset i earns for its loading on the jump risk of dividend j is calculated as the product of the jump intensity, the market price of risk for this jump, and the corresponding jump exposure.

3. Quantitative Analysis of the Model

3.1. Centrality premium

Our first set of results concerns the relation between the cross-section of expected excess returns and the degree of connectivity of an asset within the network, referred to in the literature as *network centrality*. In his recent empirical study Ahern (2013) finds that more central assets earn higher average returns, which implies a positive market price of centrality. He focuses on undirected networks characterized by symmetric beta matrices, and so we also restrict the analysis in this section to this type of networks. The impact of the direction of shocks will be discussed in Section 3.2.

Our model provides an equilibrium explanation for the main finding of Ahern (2013). To show this, we analyze the cross-section of expected excess returns in an economy in which

assets differ in their eigenvector centrality. This centrality measure has recently been suggested in a number of papers besides Ahern (2013), e.g., in Ahern and Hardford (2014) and Ozsoylev, Walden, Yavuz, and Bildik (2014). The general idea behind the concept of eigenvector centrality is that the centrality of a node depends on the centrality of its neighbors, so that a node is supposed to be central when it has many neighbors, important neighbors, or both.

As the name already indicates, eigenvector centrality is related to the eigenvalues and eigenvectors of the matrix characterizing the network, i.e., the beta matrix. Formally, let $\varphi_1, \ldots, \varphi_n$ denote the eigenvectors of β, sorted in descending order by their absolute values, and $\alpha \in \mathbb{R}^{n \times n}$ (with generic element $\alpha_{i,j}$) the so-called centrality matrix containing the associated eigenvectors as columns. Then the eigenvector centralities of the network nodes are given by the eigenvector associated with the principal eigenvalue φ_1, i.e., by the first column of α with elements $\alpha_{i,1}$ $(i = 1, \ldots, n)$.

We construct a beta matrix such that it represents an economy where all assets in the network exhibit a different eigenvector centrality. From linear algebra we know that we can represent the matrix β as $\beta = \alpha \varphi \alpha^{-1}$ where φ is the diagonal matrix containing the eigenvalues of β. We choose $\varphi_1 = 0.4$ and $\varphi_j = 0$ for $j = 2, \ldots, 10$. For the centrality vector, i.e., the principal eigenvector $\alpha_1 = (\alpha_{1,1}, \ldots, \alpha_{10,1})$ corresponding to φ_1, we choose the components as $\alpha_{10,1} = 0.01$ and then with step size s, $\alpha_{i-1,1} = \alpha_{i,1} + s$ for $i = 3, \ldots, 10$. Finally, $\alpha_{1,1}$ is chosen such that the vector has unit length. With our benchmark step size of $s = 0.04$, this results in $\alpha_1 = (0.8026, 0.33, 0.29, \ldots, 0.05, 0.01)'$. The remaining eigenvectors are chosen such that the beta matrix is symmetric, i.e., the network is undirected.[11] The left graph of Figure 1 depicts the corresponding network graphically.

The remaining model parameters are reported in Table 1. For the following exercises the parameters of the consumption process are $\mu = 0.05$ and $K_1 = \ldots = K_{10} = -0.01$. The dividend parameters – except for the beta matrix – are identical across assets with $\mu_1 = \ldots = \mu_{10} = 0.05$ and $L_1 = \ldots = L_{10} = -0.10$. Finally, we set the mean-reversion level of the intensities to $\bar{\ell}_1 = \ldots = \bar{\ell}_{10} = 0.1$ and the mean reversion speeds to $\kappa_1 = \ldots = \kappa_{10} = 0.8$, i.e., shocks have a half-life of about 0.9 years.

The right graph in Figure 1 shows the local expected excess returns of the assets 1 to 10 as a function of their eigenvector centrality. These expected excess returns defined in Equation (7) are strongly increasing in network centrality, i.e., more central assets earn higher expected excess returns, so that there is indeed a centrality premium. Our model thus

---

[11]A sufficient condition for a symmetric beta matrix is that its eigenvectors form an orthonormal basis of $\mathbb{R}^n$, i.e., the eigenvector matrix is an orthogonal matrix. Further details are given in Appendix B.

provides strong theoretical support for this empirical finding presented in Ahern (2013).

Furthermore, the centrality premium is not simply a CAPM-type market risk premium. To show this we perform a cross-sectional regression of local expected excess returns on the sensitivities of the assets to the market and the centrality portfolio, where the market portfolio is represented by the consumption claim, and the return on the centrality portfolio is given by the return difference between the most central asset 1 and the least central asset 10. Since we expect the coefficients in these regressions to depend on the state, we simulate the model to generate 10,000 realizations of the vector of intensities ℓ. For given values of these state variables the local betas of the ten assets are available in closed form. We then regress the local expected excess returns on the two local betas individually and jointly.

Table 2 reports the median slope coefficients as well as the 2.5% and 97.5% quantiles. The coefficients for market and centrality beta are positive and significant both in the uni-variate and the multivariate case.[12] This indicates that the centrality portfolio is a separate risk factor in addition to the traditional CAPM-type market portfolio.

Besides reproducing the key result in Ahern (2013) our model can also be used to derive new testable hypotheses related to the centrality premia in networks, which differ with respect to the cross-sectional dispersion of centrality. In our model we can directly control this dispersion via the step size s. We compute the centrality premium for different networks, characterized by step sizes s ranging from 0.02 to 0.055. The dispersion of centrality is inversely related to s, i.e., the larger s the more evenly spread the centrality values and the lower both their standard deviation across the ten assets and the centrality of the most central asset (which is always asset 1).

The main result is that the more disperse the eigenvector centrality in a network, the smaller the centrality premium. This can be seen from Figure 2 where we plot expected excess returns as a function of eigenvector centrality for the economies differing with respect to the step size s and thus the cross-sectional dispersion of centrality. The line is the flatter and thus the centrality premium is the lower, the more disperse centrality is across the ten assets.

To see where this effect comes from, we decompose the expected excess return into the premia for self- and for mutually exciting jumps

$$\frac{1}{dt}\mathbb{E}\left[dR_{i,t}\right] - r_t = \ell_{i,t}\,\mathrm{MPJR}_i\,\mathrm{JEXP}_{i,i} + \sum_{j \neq i}^{n} \ell_{j,t}\,\mathrm{MPJR}_j\,\mathrm{JEXP}_{i,j}. \tag{8}$$

---

[12]The 2.5% quantile for the centrality beta in the multivariate case is low at first glance. However the regression coefficients are positive for all 10,000 regressions that we perform, i.e. the regression coefficient is statistically significant at any possible level.

Table 3 presents this decomposition for assets 1 and 2 for the two economies characterized by the most and the least pronounced dispersion of centrality, i.e., by $s = 0.02$ and $s = 0.055$, respectively. The self-exciting part always represents the largest fraction of the total risk premium. The main reason for this is that the exposure to jumps in its own dividend, $\mathrm{JEXP}_{i,i}$, is always the largest exposure for any asset i, since it is the only one to contain the immediate cash flow component.

Given this, the market price of risk of its own jumps MPJR_i is the most important determinant of asset i's risk premium. Figure 3 shows the market prices of risk, MPJR_i, as a function of eigenvector centrality. Clearly the differences in the slopes in Figure 2 are mirrored in Figure 3, so that differences in the centrality premium across different economies are mainly driven by the market prices of jump risk.

In equilibrium market prices of risk have to be related to the riskiness of aggregate consumption. We simulate the economies for s equal to 0.02 and 0.055, respectively, and estimate the unconditional moments of the consumption jump intensity $\sum_{i=1}^{n} \ell_i$. From Table 4 one can see that both mean and standard deviation are significantly higher for the economy with a lower cross-sectional dispersion of centrality. In such an economy, more assets are relatively central, so that an initial shock will be propagated much more through the economy. With a higher cross-sectional dispersion of centrality the most central asset can excite the nine other assets, but due to the relatively weaker links of these other assets to each other the initial shock is hardly propagated any further. Related to this, Table 3 shows that the ratio of the risk premium for mutually exciting jumps to the total varies substantially with the dispersion of centrality. When this dispersion is high, mutually exciting jumps account for about 8% of the total premium for the central asset, compared to about 32% for low dispersion.

Finally, a Monte Carlo simulation of the economies characterized by step sizes between 0.02 and 0.055 shows that, consistent with the findings above, the returns of more central assets are more volatile (see Figure 4). Comparing across economies we note that for high centrality dispersion the most central asset can exhibit values for its return volatility which significantly exceed those for the other assets. More pronounced cross-sectional dispersion of centrality thus also translates into more pronounced differences in the second moments of returns between the most central and the other assets.

14

3.2. Directed shocks

As already discussed, our model allows us to study explicitly whether and to what degree directedness of a network affects the structural properties of risk premia. Technically speaking, a network where direction matters is characterized by an asymmetric beta matrix.

We study two stylized directed networks. The first is the 'star network' which is also the one analyzed by Buraschi and Porchia (2013). It is presented in the left graph of Figure 5. Shocks to asset 1 are propagated to all other assets in the economy, but asset 1 does not receive any shocks from other assets. For the other assets the situation is obviously exactly reversed. In terms of the beta matrix the star network structure implies that all elements except for the first column are equal to zero. For our example we set $\beta_{1,1} = 0$ and $\beta_{i,1} = 0.3$ for $i = 2, \ldots, 10$, which means that we exclusively focus on the impact of asset 1 on the other assets and shut down self-exciting jumps. All other model parameters are chosen as in Section 3.1. While from a technical point of view the concept of network centrality via eigenvectors can be guaranteed to work only for symmetric beta matrices with at least one nonzero eigenvalue,[13] one would intuitively still consider an asset central when many links originate from this asset, like asset 1 in this case. This interpretation of centrality in a directed network is also in line with the approach in Buraschi and Porchia (2013).

We also analyze the 'reverse star network' shown in the middle graph of Figure 5. It is obtained from the star network by simply reversing the direction of all the links, which formally corresponds to a transposition of the beta matrix. Asset 1 is now receiving shocks from all other assets 2 to 10, while these are immune to shocks from any other asset. In analogy to the previous case, asset 1 would nevertheless be considered central in the sense that it is exposed to all consumption risk factors and therefore to a large amount of systematic risk. Finally, for the sake of completeness, we also look at the 'undirected star network' presented in the right graph of Figure 5. This network is obtained by taking the average of the beta matrices representing the star and the reverse star network. Also in this case asset 1 is obviously central relative to assets 2 to 10.

Table 5 presents the local risk premia as defined in Equation (7), return volatilities, Sharpe ratios, market betas, and Treynor ratios for all assets, conditional on the jump intensities being equal to their unconditional means, i.e., $\ell_i = \bar{\bar{\ell}}_i$.[14] We see that the central asset 1 has the highest risk premium in all three types of networks, which at first glance might

---

[13]The symmetry of a real matrix guarantees that all eigenvalues are real. The converse is not true.

[14]Conditional on $\ell_i = \bar{\bar{\ell}}_i$, the local return volatility of asset i is given by $\sqrt{\sum_{j=1}^{n} \bar{\bar{\ell}}_j \left(\text{JEXP}_{i,j} \right)^2}$. The return on the consumption claim is $dR_t = \ldots dt + \sum_{i=1}^{n} \text{JEXP}_{y,i} \, dN_{i,t}$ with the exposure to jumps in dividend i given by $\text{JEXP}_{y,i} = \exp \left(K_i + k_1 \sum_{k=1}^{n} B_k \beta_{k,i} \right) - 1$. From this we can deduce the volatility of the return on the consumption claim and the covariance between the return on asset i and the return on consumption claim,

be interpreted as evidence that the direction of shocks does not matter. In fact, it does matter significantly, as indicated by the other quantities. In the star network the central asset has the least volatile return, whereas we observe the exact opposite in the reverse star and the undirected star network. Similarly the central asset exhibits the smallest beta relative to the consumption claim in the star network (where all betas are pretty similar), but has a much higher beta than the other assets in the other two networks. This impact of the direction of shocks also carries over to the Treynor ratio, where the central asset exhibits the highest value in the star and the undirected star network, but the lowest in the reverse star network. In addition to the discussion of the undirected case in Section 3.1, these results show that also in directed networks centrality and market beta are fundamentally different concepts.

To explain these results, we analyze the properties of jump intensities, jump exposures, and market prices of jump risk in the three networks in more detail. Table 6 shows that the market prices of jump risk are in general higher for shock-spreading assets (i.e., asset 1 in the star network and assets 2 to 10 in the reverse star network). In the undirected network all assets are of course shock-spreading, but asset 1 obviously more so than the others. The unconditional means of the jump intensities behave exactly the other way round in that they are higher for shock-receiving assets, which is not surprising. Jump exposures also exhibit the expected pattern. Shock-receiving assets react negatively to shocks in shock-spreading assets, and this result again highlights that direction matters.

Overall, the fact that a shock originates from asset i and then impacts asset j and not the other way around is relevant for all the individual components of risk premia. The ranking of assets with respect to their risk premia can however be the same for different shock directions. This also explains why return volatilities and market betas are not monotonically related to risk premia in the star network, since volatilities and betas only depend on the jump intensities and the jump exposures, but not on the market prices of risk.

Besides the direction of shocks also the preference specification is important for the equilibrium results delivered by the model. We employ recursive Epstein-Zin preferences, whereas Buraschi and Porchia (2013) consider the restricted case of CRRA. They show that in their model, like in ours, the central asset commands the highest risk premium in the star network.

To see if this type of centrality premium is robust with respect to preferences we restrict our model to the case of CRRA utility by setting $\psi = \frac{1}{\gamma}$. The right column in Table 7 shows

which are equal to $\sqrt{\sum_{j=1}^{n} \bar{\bar{\ell}}_j \left(\text{JEXP}_{y,j} \right)^2}$ and $\sum_{j=1}^{n} \bar{\bar{\ell}}_j \text{JEXP}_{y,j} \text{JEXP}_{i,j}$, respectively. From the covariance and the variance of the return on the consumption claim we readily obtain the market beta of asset i as $\frac{\sum_{j=1}^{n} \bar{\ell}_j \text{JEXP}_{y,j} \text{JEXP}_{i,j}}{\sum_{j=1}^{n} \bar{\ell}_j \left(\text{JEXP}_{y,j} \right)^2}$.

the risk premia for this case. As one can see, the structure of risk premia changes dramatically. They all become negative, and the central asset 1 even exhibits the lowest expected excess return.

This apparent divergence between our results and the findings in Buraschi and Porchia (2013) can be reconciled by looking at the specification of dividend dynamics in the two models. In Buraschi and Porchia (2013) the expected dividend growth rates depend on the state of the economy and are higher in bad states. Together with the fact that in CRRA models valuation ratios tend to go up when the state of the economy worsens, this leads to the pattern in expected excess returns in the Buraschi and Porchia (2013) model. In the reverse star network with CRRA preferences asset 1 exhibits the highest risk premia. This ranking is consistent with Buraschi and Porchia (2013), however, in their metric, asset 1 would no longer be central.

To sum up, in our model more central assets earn higher risk premia in different types of networks and independent of the direction of links. Both shock-spreading and shock-receiving assets carry a positive risk premium. However, decomposing the risk premia into their components reveals that return volatilities, market betas, and risk premia do not always line up in directed networks. Moreover, the result that central assets always earn the highest overall risk premium would be significantly harder to obtain in a model with simpler preferences.

3.3. Flight to Quality

In crises one often observes that most assets exhibit negative returns, but that there are also some, whose price actually goes up, despite the fact that the economy as a whole has become significantly riskier. This is sometimes referred to as a 'flight to quality', i.e., there is a strong demand for safer assets which then makes the prices of these assets go up.[15]

We will now show that our model provides an equilibrium foundation for this effect. To this end, we study two stylized networks. The 'directed ring network' is characterized by a structure where shocks in asset i are propagated only to the next asset $i + 1$. Technically this means that $\beta_{i,j} \neq 0$ only for $i = j + 1$ and, to close the ring, for $(i, j) = (1, 10)$. In detail, we set $\beta_{2,1} = \beta_{3,2} = \ldots = \beta_{10,9} = \beta_{1,10} = 0.4$. Again we shut down self-exciting jumps by setting $\beta_{i,i} = 0$ for $i = 1, \ldots, 10$. All the other model parameters are kept at the previously used values. The 'undirected ring network' is derived from the directed version by making all the links symmetric, i.e., we set $\beta_{1,2} = \beta_{2,1} = \beta_{3,2} = \beta_{2,3} = \ldots = 0.2$. The two networks are presented graphically in Figure 6.

---

[15]See, e.g., Baele, Bekaert, Inghelbrecht, and Wei (2015).

First consider the directed ring network. Table 8 reports the jump exposures $\text{JEXP}_{i,1}$ for $i = 1, \ldots, 10$, which are computed according to Equation (6). All asset prices react to a jump in dividend 1, and this happens despite the fact that it is only the jump intensity of asset 2 which is directly affected by this jump. In equilibrium all coefficients $C_{i,k}$ are generally nonzero, i.e., the price-dividend ratios of all assets i are affected by a change in any of the state variables ℓ_k.

The jump exposure is negative and largest in absolute value for asset 1 itself, which is not surprising, since this asset suffers from a large negative cash flow effect. For assets 2 to 5 the jump exposure $\text{JEXP}_{i,1}$ is still negative, so they represent examples for contagion in the ring network. Assets 6 to 10 on the other hand exhibit positive exposures to jumps in dividend 1. This is an example for the 'flight-to-quality' effect described above. The dividend of asset 1 has experienced a downward jump, the levels of all other dividends have remained unchanged, *all* assets have become and will become riskier in the future due to the propagation of the shock in dividend 1 through the ring network. Nevertheless, the prices of *some* assets go up, and the reason is that they are far enough away from the source of the shock and thus provide a hedge.

An interesting feature of the equilibrium price reactions to a jump in dividend 1 is that (at least for the parametrization in this example) the total realized return on asset 1 is *less* negative than if caused by the cash flow shock alone. If the price-dividend ratio of asset 1 did not change, the return would be equal to $e^{-0.10} - 1 = -0.095$, but the total effect as shown in Table 8 is -0.085, so that the log price-dividend ratio v_1 has actually gone *up* after the jump. The reason is that this shock is directly propagated only to asset 2 due to $\beta_{2,1}$, but then asset 1 is furthest away from the most likely source of the next shock in the directed ring network.

We now perform the same type of exercise in the undirected ring network. As shown by the results presented in Table 9 essentially the same mechanism as before is at work here. The difference is that now, with the initial shock in asset 1 spreading in two directions, asset 6 becomes the 'safe haven' in this economy, since it is furthest away from the source of the shock.

Viewed through the lens of our model, these results are almost obvious. However, from a more general standpoint, this result has important consequences for how we think about the results of empirical studies applying network theory to stock returns like Ahern (2013), Herskovic (2014), or Chen (2014). Many papers in this strand of the literature employ undirected measures of connectedness. For example, although the basic data for the analysis in Ahern (2013) are directed industry input-output tables, the links between industries are

made symmetric, before they are used in the estimation process. Herskovic (2014) proposes a production economy model which takes direction into account. In his empirical analysis he constructs factors like network sparsity or network concentration from input-output tables and uses the loadings on these factors in cross-sectional regressions. However, he does not study whether changes in the direction of links have an effect on the cross-section of returns via network sparsity or network concentration factor loadings.

The empirical approaches in these papers are thus implicitly based on the assumption that the direction of shocks does not matter, while our model shows that it certainly can. Imagine the data are actually generated by the directed ring network, but the econometrician neglects the directional information in the test of the model. As shown above, asset 10 will exhibit a positive return in response to a jump in the dividend of asset 1 in this case. Without taking the direction of the shock into account in the empirical analysis, assets 1 and 10 seem closely linked, but then the price increase in asset 10 would appear strongly counterintuitive. Obviously when the econometric model explicitly allows for directed shocks, the returns on assets 1 and 10 would be perfectly in line with theory.

4. Conclusion

Networks have recently received considerable attention in the economics and finance literature. With this paper we provide an equilibrium foundation for asset pricing when the direction of the connections between the individual nodes in a network is relevant.

Our setup is very general, featuring a representative investor with recursive utility and dynamics of firms' cash flows driven by self- and mutually exciting jump processes. This means that a shock to the cash flow of a given firm can increase its own jump intensity as well as those of other firms in the network. It is this type of process which allows to model the direction of shock propagation together with its time dimension in a still parsimonious and tractable setup, and the mechanics of our model are substantially different from what would be generated by contemporaneous jumps or a diffusion-based model.

In line with recently documented empirical findings our model generates a positive market price of centrality, i.e., more central assets earn higher risk premia. Beyond this our model also shows that the exact network topology has an influence on the amount of this market price of centrality, which turns out to be a decreasing function of the centrality dispersion in the network. Using cross-sectional regressions with data simulated from the model, we also document that network centrality is a separate risk factor in addition to CAPM-type market risk.

We further provide an equilibrium justification for the flight-to-quality effect by showing that in equilibrium the price effects of a jump in a given dividend are less negative for assets 'further away' from the source of the shock, and can even become positive. The latter is a remarkable outcome, since the economy as a whole and all dividend streams become riskier after the initial shock. In the context of this analysis we highlight the importance of taking the direction of shock propagation into account when analyzing a network. Ignoring the potential asymmetry of links may lead to a severe mis-interpretation of empirical findings.

The issue of direction is also important when we look at the impact of preferences. With recursive utility we obtain plausible results for a variety of network structures in the sense that assets which are either shock-spreading or shock-receiving in a pronounced fashion command high premia. With CRRA preferences these results cannot be obtained with more standard dividend dynamics.

APPENDIX

A. Model solution

Our equilibrium solution follows Eraker and Shaliastovich (2008). The vector $X \equiv (y, \ell_1, \ldots, \ell_n, y_1, \ldots, y_n)'$ follows the affine jump process

$$dX_t \; = \; \mu\left(X_t\right) dt + \xi_t \, dN_t$$

where we use the following notation:

- $\mu(X_t) = \mathcal{M} + \mathcal{K} \, X_t$

  with $\mathcal{M} = \begin{pmatrix} \mu_c \\ \kappa_1 \, \bar{\ell}_1 \\ \vdots \\ \kappa_n \, \bar{\ell}_n \\ \mu_1 \\ \vdots \\ \mu_n \end{pmatrix}$ and $\mathcal{K} = \begin{pmatrix} 0 & 0 & \ldots & 0 & \ldots & 0 \\ 0 & -\kappa_1 & \ldots & 0 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & -\kappa_n & \ldots & 0 \\ 0 & 0 & \ldots & 0 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & 0 & \ldots & 0 \end{pmatrix}$,

- $\ell_t = l_0 + l_1 \, X_t$

  with $l_0 = \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix}$ and $l_1 = \begin{pmatrix} 0 & 1 & \ldots & 0 & 0 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & 1 & 0 & \ldots & 0 \end{pmatrix}$,

- $\xi_t = \begin{pmatrix} \xi_{1,t}, & \ldots, & \xi_{n,t} \end{pmatrix} = \begin{pmatrix} K_1 & \ldots & K_n \\ \beta_{1,1} & \ldots & \beta_{1,n} \\ \vdots & \ddots & \vdots \\ \beta_{n,1} & \ldots & \beta_{n,n} \\ L_1 & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & L_n \end{pmatrix}$,

- jump transform $\varrho\left(u\right) = \mathbb{E}\left[\begin{pmatrix} e^{u' \, \xi_{1,t}}, & \ldots, & e^{u' \, \xi_{n,t}} \end{pmatrix}\right]' = \begin{pmatrix} e^{u' \, \xi_{1,t}}, & \ldots, & e^{u' \, \xi_{n,t}} \end{pmatrix}'.$

We define $dy_t = \delta_y' \, dX_t$, $d\ell_{1,t} = \delta_{\ell,1}' \, dX_t$, $\ldots$, $d\ell_{n,t} = \delta_{\ell,n}' \, dX_t$, $dy_{1,t} = \delta_{y,1}' \, dX_t$, $\ldots$, $dy_{n,t} = \delta_{y,n}', \, dX_t$ where $\delta_{(\cdot)}$ represents an appropriate selection vector. The discrete-time Euler equation for the claim on aggregate consumption is $\mathbb{E}_t\left[M_{t+\Delta t} \, R_{t+\Delta t}\right] = 1$. The continuous-time version of the Euler

equation can be written as

$$0 = \frac{1}{dt} \mathbb{E} \left[\frac{d \left(e^{\ln M_t + \ln R_t} \right)}{e^{\ln M_t + \ln R_t}} \right]. \tag{A.1}$$

The logarithm of the pricing kernel has dynamics

$$d \ln M_t = -\delta \theta \, dt - (1 - \theta) \, d \ln R_t - \frac{\theta}{\psi} \, dy_t.$$

We apply the usual affine conjecture for the log wealth-consumption ratio

$$
\begin{aligned}
v_t &= A + B' X_t = A + \begin{pmatrix} 0, & B_1, & \ldots, & B_n, & 0, & \ldots, & 0 \end{pmatrix} X_t \\
&= A + \begin{pmatrix} B_1, & \ldots, & B_n \end{pmatrix} l_t,
\end{aligned}
$$

and use the Campbell-Shiller approximation for the return on the consumption claim

$$d \ln R_t = k_0 \, dt + k_1 \, dv_t - (1 - k_1) \, v_t \, dt + dy_t.$$

Combining the Campbell-Shiller approximation, the affine guess for v_t, and the dynamics of the log pricing kernel, we get

$$
\begin{aligned}
\frac{d \left(e^{\ln M_t + \ln R_t} \right)}{e^{\ln M_t + \ln R_t}} &= \left\{ -\delta \theta + \theta k_0 - \theta (1 - k_1) \left(A + B' X_t \right) + \chi_y' \left(\mathcal{M} + \mathcal{K} X_t \right) \right\} dt \\
&\quad + \left\{ e^{\chi_y' \xi_t} - \mathbb{1} \right\} dN_t,
\end{aligned}
$$

where

$$\chi_y = \theta \left[\left(1 - \frac{1}{\psi} \right) \delta_y + k_1 B \right] = \begin{pmatrix} -\theta \left(\frac{1}{\psi} - 1 \right), & \theta k_1 B_1, & \ldots, & \theta k_1 B_n, & 0, & \ldots, & 0 \end{pmatrix}'$$

and $\mathbb{1}$ is a vector of ones with length n. We plug this expression into the Euler equation (A.1) to get a system of equations for A_C and B_C:

$$0 = \theta \left[-\delta + k_0 - (1 - k_1) \, A \right] + \mathcal{M}' \chi_y + l_0' \left[\varrho (\chi_y) - \mathbb{1} \right] \tag{A.2}$$

$$0 = \mathcal{K}' \chi_y - \theta (1 - k_1) \, B + l_1' \left[\varrho (\chi_y) - \mathbb{1} \right]. \tag{A.3}$$

We have two additional equations for the loglinearization constants k_0 and k_1:

$$0 = -k_0 - \ln k_1 + (1 - k_1) \left[A + B' \mu_X \right] \tag{A.4}$$

$$0 = A + B' \mu_X - \ln (k_1) + \ln (1 - k_1) \tag{A.5}$$

22

where μ_X is a vector with i-th component $\mathbb{E}[X_i]$ if that number exists and 0 otherwise. According to Aït-Sahalia, Cacho-Diaz, and Laeven (2014), the n unconditional expectations $\mathbb{E}[dN_i] = \bar{\ell}_i \, dt$ satisfy the system of n linear equations

$$\bar{\bar{\ell}}_i = \frac{\kappa_i \, \bar{\ell}_i + \sum_{j=1, j \neq i}^{n} \beta_{i,j} \, \bar{\bar{\ell}}_j}{\kappa_i - \beta_{i,i}}.$$

We solve the four equations (A.2), (A.3), (A.4) and (A.5) via iteration. We start with $k_1 = \delta$, then compute k_0, A and B. Then we compute k_1 again and iterate forward until the system converges.

The pricing kernel has dynamics

$$\frac{dM_t}{M_t} = -r_t \, dt - [\mathbb{1} - \varrho(-\lambda)]' \, dN_t$$

with

$$\lambda = \gamma \, \delta_y + (1 - \theta) \, k_1 \, B = (\gamma, \quad (1 - \theta) \, k_1 \, B_1, \quad \dots, \quad (1 - \theta) \, k_1 \, B_n, \quad 0, \quad \dots, \quad 0)'$$

so that we can immediately read off the risk-free rate and the market prices of risk. The risk-free rate equals

$$\begin{aligned}
r_t &= \Phi_0 + \Phi_1' \, X_t \\
\Phi_1 &= (1 - \theta) \, (k_1 - 1) \, B + \mathcal{K}' \, \lambda - l_1' \, [\varrho(-\lambda) - \mathbb{1}] \\
\Phi_0 &= \theta \, \delta + (\theta - 1) \, \big[\ln k_1 + (k_1 - 1) \, B' \, \mu_X\big] + \mathcal{M}' \, \lambda - l_0' \, [\varrho(-\lambda) - \mathbb{1}].
\end{aligned}$$

The market prices of jump risk are given by

$$[\mathbb{1} - \varrho(-\lambda)] = \begin{pmatrix} \text{MPJR}_1 \\ \vdots \\ \text{MPJR}_n \end{pmatrix} = \begin{pmatrix} 1 - \exp\left(-\gamma \, K_1 + k_1 \, (\theta - 1) \, [B_1 \, \beta_{1,1} + \dots + B_n \, \beta_{n,1}]\right) \\ \vdots \\ 1 - \exp\left(-\gamma \, K_n + k_1 \, (\theta - 1) \, [B_1 \, \beta_{1,n} + \dots + B_n \, \beta_{n,n}]\right) \end{pmatrix}.$$

The return on the consumption claim is given by

$$dR_t = \{\dots\} \, dt + \{\varrho(\delta_y + k_1 \, B) - \mathbb{1}\} \, dN_t$$

with jump exposure

$$\varrho(\delta_y + k_1 \, B) - \mathbb{1} = \begin{pmatrix} \text{JEXP}_{y,1} \\ \vdots \\ \text{JEXP}_{y,n} \end{pmatrix} = \begin{pmatrix} \exp\left(K_1 + k_1 \, [B_1 \, \beta_{1,1} + \dots + B_n \, \beta_{n,1}]\right) - 1 \\ \vdots \\ \exp\left(K_n + k_1 \, [B_1 \, \beta_{1,n} + \dots + B_n \, \beta_{n,n}]\right) - 1 \end{pmatrix}.$$

To obtain the expected excess returns on the dividend claims, we follow the same approach as for the consumption claim. The continuous-time Euler equation reads

$$0 = \frac{1}{dt} \mathbb{E} \left[\frac{d \left(e^{\ln M_t + \ln R_{i,t}} \right)}{e^{\ln M_t + \ln R_{i,t}}} \right].$$

Applying the Campbell-Shiller approximation

$$d \ln R_{i,t} = k_{i,0}\, dt + k_{i,1}\, dv_{i,t} - (1 - k_{i,1})\, v_{i,t}\, dt + dy_{i,t}$$

and the usual affine guess for the log price-dividend ratio

$$v_{i,t} = A_i + C_i' X_t = A_i + \left(\; 0, \;\; C_{i,1}, \;\; \ldots, \;\; C_{i,n}, \;\; 0, \;\; \ldots, \;\; 0 \; \right) X_t$$
$$= A_i + \left(\; C_{i,1}, \;\; \ldots, \;\; C_{i,n} \; \right) \ell_t,$$

we arrive at

$$\frac{d \left(e^{\ln M_t + \ln R_{i,t}} \right)}{e^{\ln M_t + \ln R_{i,t}}} = \left\{ -\delta\,\theta - (1 - \theta) \left[k_0 - (1 - k_1) \left(A + B' X_t \right) \right] + k_{i,0} \right.$$
$$\left. - (1 - k_{i,1}) \left[A_i + C_i' X_t \right] + \chi_{y,i}' \left(\mathcal{M} + \mathcal{K} X_t \right) \right\} dt$$
$$+ \left\{ e^{\chi_{y,i}' \xi_t} - \mathbb{1} \right\} dN_t,$$

where $\chi_{y,i} = k_{i,1}\, C_i + \delta_{y,i} - \lambda$. Plugging this expression into the Euler equation, we get a system of equations for A_i and C_i:

$$0 = -\theta\,\delta + (1 - \theta) \left[\ln k_1 - (1 - k_1)\, B'\, \mu_X \right] - \ln k_{i,1} + (1 - k_{i,1})\, C_i'\, \mu_X$$
$$+ \mathcal{M}'\, \chi_{y,i} + l_0' \left[\varrho \left(\chi_{y,i} \right) - \mathbb{1} \right]$$
$$0 = \mathcal{K}'\, \chi_{y,i} + (1 - \theta)\, (1 - k_1)\, B - (1 - k_{i,1})\, C_i + l_1' \left[\varrho \left(\chi_{y,i} \right) - \mathbb{1} \right].$$

The two additional equations for the loglinearization constants $k_{i,0}$ and $k_{i,1}$ read

$$0 = -k_{i,0} - \ln k_{i,1} + (1 - k_{i,1}) \left(A_i + C_i'\, \mu_X \right)$$
$$0 = A_i + C_i'\, \mu_X - \ln k_{i,1} + \ln (1 - k_{i,1}).$$

The return of the individual dividend claim i is then given by

$$dR_{i,t} = \left\{ -\ln k_{i,1} + (1 - k_{i,1})\, C_i' \left(\mu_X - X_t \right) + \left[\delta_i + k_{i,1}\, C_i \right]' \left(\mathcal{M} + \mathcal{K} X_t \right) \right\} dt$$
$$+ \left\{ \varrho \left(\delta_{y,i} + k_{i,1}\, C_i \right) - \mathbb{1} \right\} dN_t.$$

The jump exposure of the return is thus given by

$$
[\varrho\,(\delta_{y,i} + k_{i,1}\,C_i) - \mathbb{1}] \;=\; \begin{pmatrix} \mathrm{JEXP}_{i,1} \\ \vdots \\ \mathrm{JEXP}_{i,i} \\ \vdots \\ \mathrm{JEXP}_{i,n} \end{pmatrix} \;=\; \begin{pmatrix} \exp\left(k_{i,1}\,[C_{i,1}\,\beta_{1,1} + \ldots + C_{i,n}\,\beta_{n,1}]\right) - 1 \\ \vdots \\ \exp\left(L_i + k_{i,1}\,[C_{i,1}\,\beta_{1,i} + \ldots + C_{i,n}\,\beta_{n,i}]\right) - 1 \\ \vdots \\ \exp\left(k_{i,1}\,[C_{i,1}\,\beta_{1,n} + \ldots + C_{i,n}\,\beta_{n,n}]\right) - 1 \end{pmatrix}.
$$

The expected return on the i-th dividend claim can then be written as

$$
\begin{aligned}
\frac{1}{dt}\,\mathbb{E}\,[dR_{i,t}] \;=\;& -\ln k_{i,1} + (1 - k_{i,1})\,C_i'\,(\mu_X - X_t) + [\delta_i + k_{i,1}\,C_i]'\,(\mathcal{M} + \mathcal{K}\,X_t) \\
& + [\varrho\,(\delta_{y,i} + k_{i,1}\,C_i) - \mathbb{1}]\,(l_0 + l_1\,X_t).
\end{aligned}
$$

The expected excess return is given by

$$
\frac{1}{dt}\,\mathbb{E}\,[dR_{i,t}] - r_t \;=\; (l_0 + l_1\,X_t)'\,[\varrho\,(\chi_{y,i} + \lambda) + \varrho\,(-\lambda) - \varrho\,(\chi_{y,i}) - \mathbb{1}]
$$

which can be reformulated as

$$
\frac{1}{dt}\,\mathbb{E}\,[dR_{i,t}] - r_t \;=\; \sum_{j=1}^{n} \ell_{j,t}\,\mathrm{MPJR}_j\,\mathrm{JEXP}_{i,j}.
$$

B. Setting up the beta matrix

In the following, we describe how we operationalize the concept of eigenvector centrality to determine the beta matrix

$$
\beta \;=\; \begin{pmatrix} \beta_{1,1} & \cdots & \beta_{1,n} \\ \vdots & \ddots & \vdots \\ \beta_{n,1} & \cdots & \beta_{n,n} \end{pmatrix}.
$$

W.l.o.g., we assume that the eigenvalues $\varphi_1, \ldots, \varphi_n$ of β are sorted according to their absolute size, i.e., φ_1 is the principal eigenvalue. The eigenvectors for these eigenvalues are collected in the matrix

$$
\alpha \;=\; \begin{pmatrix} \alpha_{1,1} & \cdots & \alpha_{1,n} \\ \vdots & \ddots & \vdots \\ \alpha_{n,1} & \cdots & \alpha_{n,n} \end{pmatrix}
$$

so that we have the usual diagonalization

$$
\beta \;=\; \alpha \begin{pmatrix} \varphi_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \varphi_n \end{pmatrix} \alpha^{-1}.
$$

W.l.o.g. we assume that the eigenvectors in α are normalized to have length 1.

The β matrix is supposed to be non-negative because there is no economic interpretation for a negative β_{ij} in our model. The Perron-Frobenius theorem then says that there exists an eigenvector with only non-negative components. The Perron-Frobenius theorem also says that the non-negative eigenvector is associated with the absolutely largest eigenvalue (called the spectral radius) which is also non-negative. The eigenvectors related to all other eigenvalues must contain negative entries.[16]

Besides, the beta matrix is supposed to be symmetric in the benchmark case. Simple linear algebra implies that the matrix β is symmetric if the eigenvector matrix is an orthogonal matrix (i.e. $\alpha\alpha' = \mathbf{1}$ or $\alpha' = \alpha^{-1}$):

$$
\left(\alpha \cdot \mathrm{diag}(\varphi_1,\ldots,\varphi_n)\cdot \alpha^{-1}\right)' \;=\; \left(\alpha^{-1}\right)' \cdot \mathrm{diag}(\varphi_1,\ldots,\varphi_n)' \cdot \alpha' \;=\; \alpha \cdot \mathrm{diag}(\varphi_1,\ldots,\varphi_n)\cdot \alpha^{-1}.
$$

Taking these two facts into account, we construct the beta matrix using the following algorithm. We assume that $\varphi_2 = \ldots = \varphi_n$, i.e., the eigenvalues other than φ_1 are equal and then construct the eigenvectors as follows. We assume that $\alpha_{1,1}^2 + \ldots + \alpha_{n,1}^2 = 1$. This implies that the first eigenvector is normalized to length 1. Moreover, we assume that all entries $\alpha_{i,1}$ in the first column are positive because of the Perron-Frobenius theorem. The remaining elements of the matrix α are then chosen such that the matrix becomes an orthogonal matrix, i.e. the columns of the matrix are mutually orthogonal and normalized to length 1. In a first step, we choose the vectors such that they are all mutually orthogonal:

$$
\begin{pmatrix}
\alpha_{1,1} & 1 & 1 & 1 & \cdots & 1 \\
\alpha_{2,1} & -\frac{\alpha_{1,1}}{\alpha_{2,1}} & -\frac{1}{\alpha_{2,2}} & -\frac{1}{\alpha_{2,2}} & \cdots & -\frac{1}{\alpha_{2,2}} \\
\alpha_{3,1} & 0 & -\frac{\alpha_{1,1}+\alpha_{2,1}\alpha_{2,3}}{\alpha_{3,1}} & -\frac{1+\alpha_{2,4}^2}{\alpha_{3,3}} & \cdots & -\frac{1+\alpha_{2,n}^2}{\alpha_{3,3}} \\
\alpha_{4,1} & 0 & 0 & -\frac{\alpha_{1,1}+\alpha_{2,1}\alpha_{2,4}+\alpha_{3,1}\alpha_{3,4}}{\alpha_{4,1}} & \cdots & -\frac{1+\alpha_{2,n}^2+\alpha_{3,n}^2}{\alpha_{4,4}} \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
\alpha_{n,1} & 0 & 0 & 0 & \cdots & -\frac{\alpha_{1,1}+\alpha_{2,1}\alpha_{2,n}+\alpha_{3,1}\alpha_{3,n}+\ldots+\alpha_{n-1,1}\alpha_{n-1,n}}{\alpha_{n,1}}
\end{pmatrix}
$$

In a second step, we scale every column by its norm so that all eigenvectors have length 1. The eigenvalues and eigenvectors uniquely determine the beta matrix. We have thus reduced the choice

---

[16]Moreover, the Perron-Frobenius theorem says that the principle eigenvalue is simple if the matrix is irreducible and that there is no other eigenvalue with the same absolute value if the matrix is aperiodic. The matrix constructed below is irreducible and aperiodic.

of the beta matrix to the choice of two eigenvalues φ_1 and φ_2 and one eigenvector which contains the eigenvector centrality of each node.

REFERENCES

ACEMOGLU, D., U. AKCIGIT, AND W. KERR (2015): "Networks and the Macroeconomy: An Empirical Exploration," Working Paper.

ACEMOGLU, D., V. CARVALHO, A. OZDAGLAR, AND A. TAHBAZ-SALEHI (2012): "The Network Origins of Aggregate Fluctuations," *Econometrica*, 80, 1977–2016.

ACEMOGLU, D., A. OZDAGLAR, AND A. TAHBAZ-SALEHI (2013): "The Network Origins of Large Economic Downturns," Working Paper.

——— (2015): "Systemic Risk and Stability in Financial Networks," *American Economic Review*, 105, 564–608.

AHERN, K. (2013): "Network Centrality and the Cross-Section of Expected Returns," Working Paper.

AHERN, K., AND J. HARDFORD (2014): "The Importance of Industry Links in Merger Waves," *Journal of Finance*, 69, 527–576.

AÏT-SAHALIA, Y., J. CACHO-DIAZ, AND R. LAEVEN (2014): "Modeling Financial Contagion Using Mutually Exciting Jump Processes," *Journal of Financial Economics*, forthcoming.

AÏT-SAHALIA, Y., R. LAEVEN, AND L. PELIZZON (2014): "Mutual Excitation in Eurozone Sovereign CDS," *Journal of Econometrics*, forthcoming.

AOBDIA, D., J. CASKEY, AND N. B. OZEL (2014): "Inter-industry network structure and the cross-predictability of earnings and stock returns," *Review of Accounting Studies*, forthcoming.

BACKUS, D., M. CHERNOV, AND I. MARTIN (2011): "Disasters Implied by Equity Index Options," *Journal of Finance*, 66, 1969–2012.

BAELE, L., G. BEKAERT, K. INGHELBRECHT, AND M. WEI (2015): "Flights to Safety," Working Paper.

BANSAL, R., AND A. YARON (2004): "Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles," *Journal of Finance*, 59, 1481–1509.

BENZONI, L., P. COLLIN-DUFRESNE, R. S. GOLDSTEIN, AND J. HELWEGE (2014): "Modeling Contagion via the Updating of Fragile Beliefs," *Review of Financial Studies*, forthcoming.

BILLIO, M., M. GETMANSKY, D. GRAY, A. LO, R. MERTON, and L. PELIZZON (2014): "Sovereign, Bank, and Insurance Credit Spreads: Connectedness and System Networks," Working Paper.

BILLIO, M., M. GETMANSKY, A. LO, and L. PELIZZON (2012): "Econometric measures of connectedness and systemic risk in the finance and insurance sectors," *Journal of Financial Economics*, 104, 535–559.

BRANGER, N., H. KRAFT, and C. MEINERDING (2014): "Partial Information about Contagion Risk, Self-Exciting Processes and Portfolio Optimization," *Journal of Economic Dynamics and Control*, 39, 18–36.

BURASCHI, A., and P. PORCHIA (2013): "Dynamic Networks and Asset Pricing," Working Paper.

CHEN, Y. (2014): "Network Risk and Cross-Section of Expected Stock Returns," Working Paper.

COLACITO, R., and M. CROCE (2013): "International Asset Pricing with Recursive Preferences," *Journal of Finance*, 68, 2651–2686.

COLLA, P., and A. MELE (2010): "Information Linkages and Correlated Trading," *Review of Financial Studies*, 23, 203–246.

DIEBOLD, F., and K. YILMAZ (2014): "On the Network Topology of Variance Decompositions: Measuring the Connectness of Financial Firms," *Journal of Econometrics*, 182, 119–134.

ELLIOTT, M., B. GOLUB, and M. JACKSON (2014): "Financial Networks and Contagion," *American Economic Review*, 104, 3115–3153.

ERAKER, B., and I. SHALIASTOVICH (2008): "An Equilibrium Guide to Designing Affine Pricing Models," *Mathematical Finance*, 18, 519–543.

GABAIX, X. (2011): "The Granular Origins of Aggregate Fluctuations," *Econometrica*, 79, 733–772.

HAN, B., and L. YANG (2013): "Social Networks, Information Acquisition, and Asset Prices," *Management Science*, 59, 1444–1457.

HERSKOVIC, B. (2014): "Networks in Production: Asset Pricing Implications," Working Paper.

KELLY, B., H. LUSTIG, AND S. V. NIEUWERBURGH (2013): "Firm Volatility in Granular Networks," Working Paper.

NOWOTNY, M. (2011): "Disaster Begets Crisis: The Role of Contagion in Financial Markets," Working Paper.

OZSOYLEV, H., AND J. WALDEN (2011): "Asset Pricing in Large Information Networks," *Journal of Economic Theory*, 146, 2252–2280.

OZSOYLEV, H., J. WALDEN, M. YAVUZ, AND R. BILDIK (2014): "Investor Networks in the Stock Market," *Review of Financial Studies*, 27, 1323–1366.

RICHMOND, R. (2015): "Trade Network Centrality and the Currency Carry Trade," Working Paper.

WACHTER, J. (2013): "Can time-varying risk of rare disasters explain aggregate stock market volatility?," *Journal of Finance*, 68, 987–1035.

WALDEN, J. (2013): "Trading Profits, and Volatility in a Dynamic Information Network Model," Working Paper.

Investors		
Relative risk aversion	γ	10
Intertemporal elasticity of substitution	ψ	1.5
Subjective discount rate	δ	0.02
Aggregate consumption		
Expected growth rate of log aggregate consumption	μ	0.05
Jump size of log aggregate consumption	$K_1 = \ldots = K_{10}$	-0.01
Dividends 1, ..., 10		
Expected growth rate of log dividends 1, ..., 10	$\mu_1 = \ldots = \mu_{10}$	0.05
Jump size of log dividends 1, ..., 10	$L_1 = \ldots = L_{10}$	-0.10
Stochastic jump intensities		
Mean reversion level	$\bar{\ell}_1 = \ldots = \bar{\ell}_{10}$	0.10
Mean reversion speed	$\kappa_1 = \ldots = \kappa_{10}$	0.80

Table 1: **Parameters**

The table reports the general parametrization of our economy. These parameters are held constant throughout the paper. Only the network matrix $(\beta_{i,j})_{i,j=1,\ldots,n}$ varies across Sections 3.1, 3.2, and 3.3, and is therefore discussed in each of these sections separately.

Regressors		
const.	market portfolio beta	centrality portfolio beta
-0.000842	0.004724	
[-0.0015,-0.0007]	[0.0032, 0.0110]	
0.001525		0.020714
[0.0013, 0.0026]		[0.0106, 0.0611]
-0.000522	0.004076	0.002874
[-0.0007,-0.0002]	[0.0032, 0.0075]	[0.0002, 0.0196]

Table 2:

Cross-sectional regressions of local expected returns on market and centrality betas

The table reports the results from cross-sectional regressions of local expected excess returns on local market betas and local centrality portfolio betas. Given the intensities and the equilibrium solution of the model, the local market beta of asset i at time t is given as $\frac{\sum_{j=1}^{n} \ell_{j,t} \, \text{JEXP}_{y,j} \, \text{JEXP}_{i,j}}{\sum_{j=1}^{n} \ell_{j,t} \, \text{JEXP}_{y,j}^2}$ with $\text{JEXP}_{y,i} = \exp\left(K_i + k_1 \sum_{k=1}^{n} B_k \, \beta_{k,i}\right) - 1$ and $\text{JEXP}_{i,j}$ as defined in Equation (6). The local return on the centrality portfolio is given by the difference between the local returns on the most and the least central asset (i.e., return on asset 1 minus return on asset 10). The exposure of the centrality portfolio to jump j is given by $\text{JEXP}_{\text{CEN},j} \equiv \text{JEXP}_{1,j} - \text{JEXP}_{10,j}$, which implies that the local centrality beta of asset i is equal to $\frac{\sum_{j=1}^{n} \ell_{j,t} \, \text{JEXP}_{\text{CEN},j} \, \text{JEXP}_{i,j}}{\sum_{j=1}^{n} \ell_{j,t} \, \text{JEXP}_{\text{CEN},j}^2}$. Given these betas, we perform cross-sectional regressions of the local expected excess returns of the ten assets from t to $t + dt$ given by Equation (7) on their betas at time t. Since the cross-sectional slope coefficients depend on the value of the state variables $\ell_1, \ldots, \ell_{10}$, we simulate the model to obtain a representative sample of state variable combinations. We simulate a daily time series (2,500,000 days) of the jump intensities of the ten assets based on the dynamics in (1). We then take every 250-th observation from this time series to have 10,000 realizations of the vector of state variables. The table reports the median coefficients for these regressions. The numbers is square brackets represent the 2.5% and 97.5% quantiles. The parameters are given in Section 3.1.

	self-exciting jumps	mutually exciting jumps	total
High cross-sectional dispersion of centrality ($s = 0.02$)			
Asset 1	0.0174	0.0015	0.0190
Asset 2	0.0018	0.0003	0.0021
Low cross-sectional dispersion of centrality ($s = 0.055$)			
Asset 1	0.0102	0.0048	0.0150
Asset 2	0.0064	0.0039	0.0103

Table 3:

Decomposition of expected excess returns for varying cross-sectional dispersion of centrality

The table reports the local expected excess return due to self-exciting and mutually-exciting jump risk, and the total risk premium as the sum of both components for assets 1 and 2. The results are shown for economies with a high ($s = 0.02$) and a low ($s = 0.055$) cross-sectional dispersion of centrality as described in Section 3.1. Expected excess returns are computed according to Equation (8). For the analysis the jump intensities $\ell_1, \ldots, \ell_{10}$ are set equal to their unconditional means $\bar{\bar{\ell}}_1, \ldots, \bar{\bar{\ell}}_{10}$ as defined in Equation (3). The parameters are given in Section 3.1.

	Mean	Std.dev.
High cross-sectional dispersion of centrality ($s = 0.02$)	1.3015	0.3517
Low cross-sectional dispersion of centrality ($s = 0.055$)	1.6917	0.5099

Table 4:

Moments of consumption jump intensity

The table reports the unconditional mean and standard deviation for the consumption jump intensity which is equal to the sum over all individual jump intensities. The results are shown for economies with a high ($s = 0.02$) and a low ($s = 0.055$) cross-sectional dispersion of centrality as described in Section 3.1. All quantities have been generated from a monthly Monte Carlo simulation over 80 years with monthly time steps ($\Delta t = 1/12$) and 10,000 paths. The parameters are given in Section 3.1.

	Exp. excess return	Return volatility	Sharpe ratio	Market beta	Treynor ratio
Star network					
Asset 1	0.0038	0.0228	0.1667	0.9173	0.0042
Asset 2,...,10	0.0021	0.0356	0.0590	0.9495	0.0022
Reverse star network					
Asset 1	0.0089	0.0708	0.1257	4.8407	0.0018
Asset 2,...,10	0.0010	0.0294	0.0340	0.5024	0.0020
Undirected star network					
Asset 1	0.0469	0.1014	0.4625	4.1641	0.0113
Asset 2,...,10	0.0034	0.0389	0.0874	0.3862	0.0088

Table 5:

Expected excess returns, return volatilities, Sharpe ratios, market betas, and Treynor ratios in different types of star networks

The table reports the local expected excess returns, return volatilities, Sharpe ratios, market betas, and Treynor ratios of the ten assets for different types of star networks. Expected excess returns are computed according to Equation (7), the return volatility of asset i is given by $\sqrt{\sum_{j=1}^{n} \bar{\bar{\ell}}_j \left(\text{JEXP}_{i,j}\right)^2}$, and the Sharpe ratio is the quotient of these two quantities. The market beta of asset i is as $\frac{\sum_{j=1}^{n} \bar{\bar{\ell}}_j \, \text{JEXP}_{y,j} \, \text{JEXP}_{i,j}}{\sum_{j=1}^{n} \bar{\bar{\ell}}_j \left(\text{JEXP}_{y,j}\right)^2}$ with $\text{JEXP}_{y,i} = \exp\left(K_i + k_1 \sum_{k=1}^{n} B_k \, \beta_{k,i}\right) - 1$ and $\text{JEXP}_{i,j}$ as defined in Equation (6). The Treynor ratio is the ratio of expected excess return and market beta. The star, reverse star, and undirected star networks are shown graphically in Figure 5. In all three networks assets 2 to 10 are identical, so that all quantities shown in the table are the same for all of them. For the analysis the jump intensities are assumed to be at their unconditional means, i.e., $\ell_i = \bar{\bar{\ell}}_i$ for $i = 1, \ldots, 10$. The remaining parameters are given in Table 1.

i	1	2	3,...,10
	Star network		
MPJR$_i$	-0.5314	-0.1052	-0.1052
$\bar{\bar{\ell}}_i$	0.1000	0.1375	0.1375
JEXP$_{1,i}$	-0.0721	-0.0000	-0.0000
JEXP$_{2,i}$	-0.0139	-0.0952	-0.0000
	Reverse star network		
MPJR$_i$	-0.1052	-0.1460	-0.1460
$\bar{\bar{\ell}}_i$	0.4375	0.1000	0.1000
JEXP$_{1,i}$	-0.0952	-0.0341	-0.0341
JEXP$_{2,i}$	0.0000	-0.0926	0.0028
	Undirected star network		
MPJR$_i$	-0.5950	-0.2299	-0.2299
$\bar{\bar{\ell}}_i$	0.3931	0.1737	0.1737
JEXP$_{1,i}$	-0.1444	-0.0365	-0.0365
JEXP$_{2,i}$	-0.0019	-0.0930	0.0024

Table 6:

Market prices of jump risk, jump intensities, and jump exposures in different types of star networks

The table reports the market prices of risk for jumps in dividend i, MPJR$_i$ given in Equation (5), the unconditional mean jump intensities $\bar{\bar{\ell}}_i$ as defined in Equation (3), and the exposures of asset 1 and 2 to a jump in dividend i, JEXP$_{1,i}$ and JEXP$_{2,i}$ computed according to Equation (6). The star, reverse star, and undirected star networks are shown graphically in Figure 5. The remaining parameters are given in Table 1.

	Expected excess return EZ	Expected excess return CRRA
Star network		
Asset 1	0.0038	-0.0015
Asset 2,...,10	0.0021	-0.0011
Reverse star network		
Asset 1	0.0089	0.0044
Asset 2,...,10	0.0010	-0.0015
Undirected star network		
Asset 1	0.0469	-0.0058
Asset 2,...,10	0.0034	-0.0133

Table 7:

Expected excess returns with EZ and CRRA preferences in different types of star networks

The table reports the expected excess returns with Epstein-Zin (EZ) preferences ($\gamma = 10$, $\psi = 1.5$) and with CRRA preferences ($\gamma = 10$, $\psi = \frac{1}{10}$) of the ten assets for different types of star networks. The risk premium for asset i with Epstein-Zin preferences is defined in Equation (7). The risk premium for asset i under CRRA utility is obtained from the model with Epstein-Zin preferences by setting $\psi = \frac{1}{\gamma}$. The star, reverse star, and undirected star networks are shown graphically in Figure 5. In all three networks assets 2 to 10 are identical, so that all quantities shown in the table are the same for all of them. For the analysis the jump intensities are assumed to be at their unconditional means, i.e., $\ell_i = \bar{\bar{\ell}}_i$ for $i = 1, \ldots, 10$. The remaining parameters are given in Table 1.

Asset i	1	2	3	4	5	6	7	8	9	10
	Relevant entries of the beta matrix									
$\beta_{i,1}$	0	0.4000	0	0	0	0	0	0	0	0
$\beta_{i,10}$	0.4000	0	0	0	0	0	0	0	0	0
	Coefficients of price-dividend ratios									
$C_{i,1}$	-0.1201	-0.0602	-0.0250	-0.0039	0.0089	0.0168	0.0216	0.0245	0.0264	0.0275
$C_{i,2}$	0.0275	-0.1201	-0.0602	-0.0250	-0.0039	0.0089	0.0168	0.0216	0.0245	0.0264
	Return exposures									
$\text{JEXP}_{i,1}$	-0.0853	-0.0464	-0.0236	-0.0099	-0.0015	0.0035	0.0067	0.0086	0.0098	0.0105
$\text{JEXP}_{i,10}$	-0.0464	-0.0236	-0.0099	-0.0015	0.0035	0.0067	0.0086	0.0098	0.0105	-0.0853

Table 8:

Return exposures (directed ring network)

The table reports the relevant entries of the beta matrix, the coefficients $C_{i,1}$ and $C_{i,2}$ in the expression for the log price-dividend ratio of asset i, $v_{i,t} = A_i + C_i' \ell_t$, and the return exposures $\text{JEXP}_{i,1}$ and $\text{JEXP}_{i,10}$ for asset i ($i = 1, \ldots, 10$). The jump exposures are computed according to Equation (6). For the special case of the directed ring network shown in Figure 6 the equations are

$$
\begin{aligned}
\text{JEXP}_{1,1} &= \exp\left(L_1 + k_{1,1}\, C_{1,2}\, \beta_{2,1}\right) - 1 \\
\text{JEXP}_{i,1} &= \exp\left(k_{i,1}\, C_{i,2}\, \beta_{2,1}\right) - 1 \qquad (i = 2, \ldots, 10) \\
\text{JEXP}_{i,10} &= \exp\left(k_{i,1}\, C_{i,1}\, \beta_{1,10}\right) - 1 \qquad (i = 1, \ldots, 9) \\
\text{JEXP}_{10,10} &= \exp\left(L_{10} + k_{10,1}\, C_{10,1}\, \beta_{1,10}\right) - 1.
\end{aligned}
$$

Furthermore, $L_1 = L_{10} = -0.10$ and $k_{1,1} = k_{2,1} = \ldots = k_{10,1} = 0.9899$. The remaining parameters are given in Table 1.

Asset i	1	2	3	4	5	6	7	8	9	10
				Relevant entries of the beta matrix						
$\beta_{i,1}$	0	0.2000	0	0	0	0	0	0	0	0.2000
$\beta_{i,10}$	0.2000	0	0	0	0	0	0	0	0.2000	0
				Coefficients of price-dividend ratios						
$C_{i,1}$	-0.1531	-0.0323	0.0081	0.0219	0.0265	0.0276	0.0265	0.0219	0.0081	-0.0323
$C_{i,2}$	-0.0323	-0.1531	-0.0323	0.0081	0.0219	0.0265	0.0276	0.0265	0.0219	0.0081
$C_{i,9}$	0.0081	0.0219	0.0265	0.0276	0.0265	0.0219	0.0081	-0.0323	-0.1531	-0.0323
$C_{i,10}$	-0.0323	0.0081	0.0219	0.0265	0.0276	0.0265	0.0219	0.0081	-0.0323	-0.1531
				Return exposures						
$\text{JEXP}_{i,1}$	-0.1067	-0.0283	-0.0021	0.0069	0.0099	0.0106	0.0099	0.0069	-0.0021	-0.0283
$\text{JEXP}_{i,10}$	-0.0283	-0.0021	0.0069	0.0099	0.0106	0.0099	0.0069	-0.0021	-0.0283	-0.1067

Table 9:

Return exposures (undirected ring network)

The table reports the relevant entries of the beta matrix, the coefficients $C_{i,j}$ in the expressions for the log price-dividend ratios of the ten assets, $v_{i,t} = A_i + C_i' \ell_t$, and the return exposures $\text{JEXP}_{i,1}$ and $\text{JEXP}_{i,10}$ for asset i ($i = 1, \ldots, 10$). The jump exposures are computed according to Equation (6). For the special case of the undirected ring network shown in Figure 6 the equations are

$$\text{JEXP}_{1,1} = \exp\left(L_1 + k_{1,1}\left[C_{1,2}\,\beta_{2,1} + C_{1,10}\,\beta_{10,1}\right]\right) - 1$$
$$\text{JEXP}_{i,1} = \exp\left(k_{i,1}\left[C_{i,2}\,\beta_{2,1} + C_{i,10}\,\beta_{10,1}\right]\right) - 1 \qquad (i = 2, \ldots, 10)$$
$$\text{JEXP}_{i,10} = \exp\left(k_{i,1}\left[C_{i,1}\,\beta_{1,10} + C_{i,9}\,\beta_{9,10}\right]\right) - 1 \qquad (i = 1, \ldots, 9)$$
$$\text{JEXP}_{10,10} = \exp\left(L_{10} + k_{10,1}\left[C_{10,1}\,\beta_{1,10} + C_{10,9}\,\beta_{9,10}\right]\right) - 1.$$

Furthermore, $L_1 = L_{10} = -0.10$ and $k_{1,1} = k_{2,1} = \ldots = k_{10,1} = 0.9905$. The remaining parameters are given in Table 1.
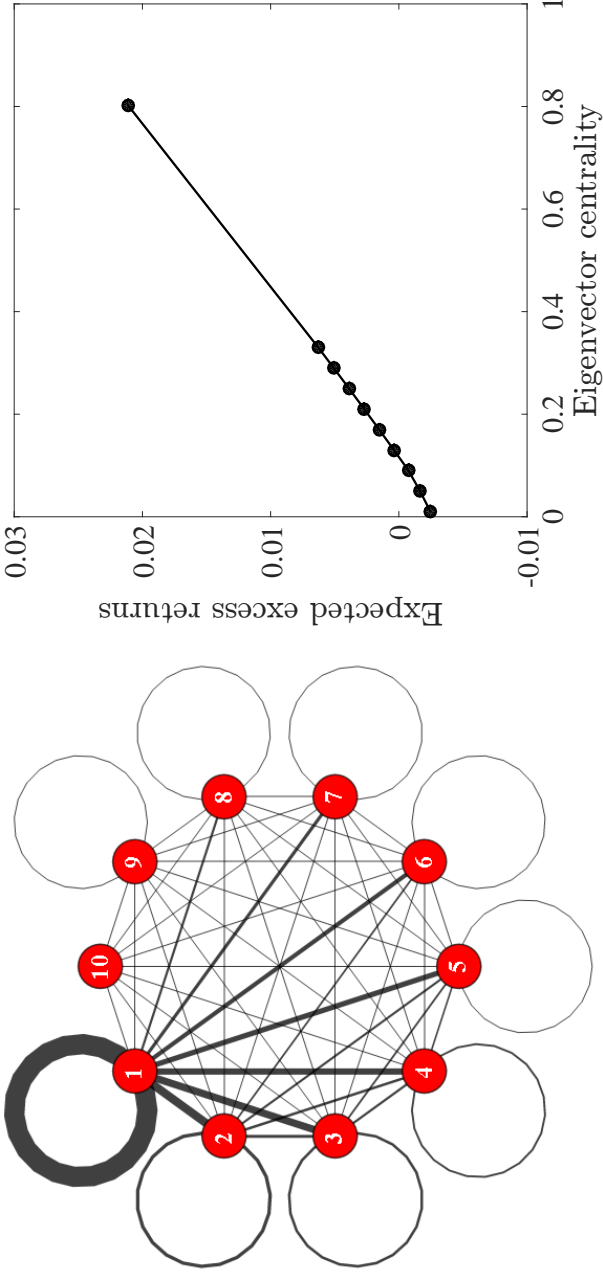
Figure 1:
Centrality premium

The left graph visualizes the network with the centrality vector $\alpha = (0.8026, 0.33, 0.29, \ldots, 0.05, 0.01)$. The beta matrix is symmetric, all links are undirected. The right graph shows the local expected excess return of the assets as a function of their eigenvector centrality. The jump intensities $\ell_1, \ldots, \ell_{10}$ are equal to their unconditional means $\bar{\bar{\ell}}_1, \ldots, \bar{\bar{\ell}}_{10}$ as defined in Equation (3). Expected excess returns are computed according to Equation (7). The parameters are given in Section 3.1.
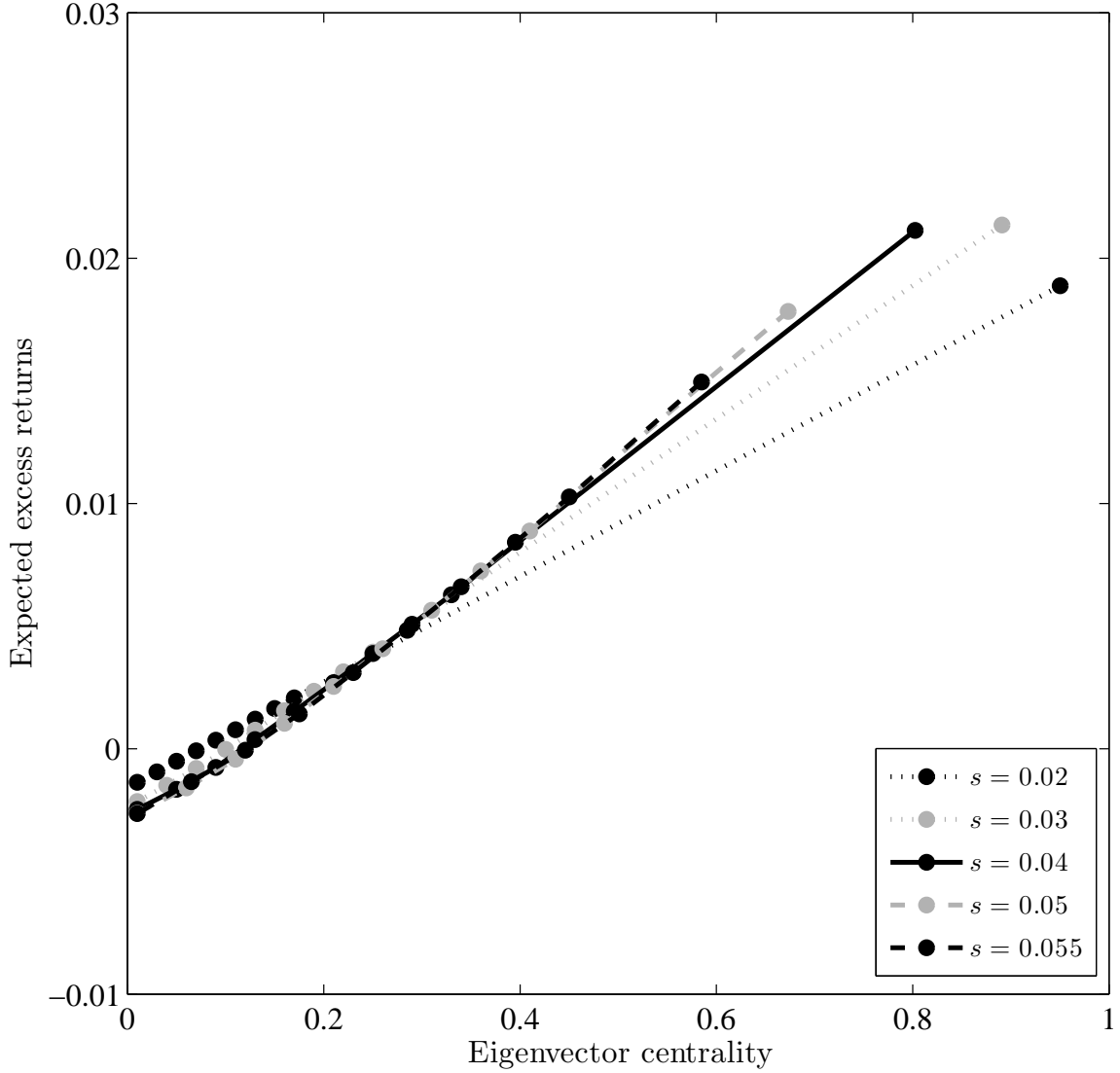
Figure 2:

Centrality premium for varying cross-sectional dispersion of centrality

The figure depicts local expected excess returns as a function of eigenvector centrality for economies with different cross-sectional dispersions of centrality (CSDC). To determine the eigenvector centralities of the ten assets in each economy, we set $\alpha_{10} = 0.01$ and $\alpha_{i-1} = \alpha_i + s$ for $i = 3, \ldots, 10$, where $s \in \{0.02, 0.03, 0.04, 0.05, 0.055\}$. α_1 is then determined such that the vector α has length 1. The black dashed line shows the results for $s = 0.02$ (highest CSDC), the gray dashed line those for $s = 0.03$, the black solid line those for $s = 0.04$, the gray dotted line those for $s = 0.05$ and the black dotted line those for $s = 0.055$ (lowest CSDC). Each dot represents the combination of eigenvector centrality and expected excess return for an asset which is computed according to Equation (7). The jump intensities $\ell_1, \ldots, \ell_{10}$ are equal to their unconditional means $\bar{\bar{\ell}}_1, \ldots, \bar{\bar{\ell}}_{10}$ as defined in Equation (3). The parameters are given in Section 3.1.
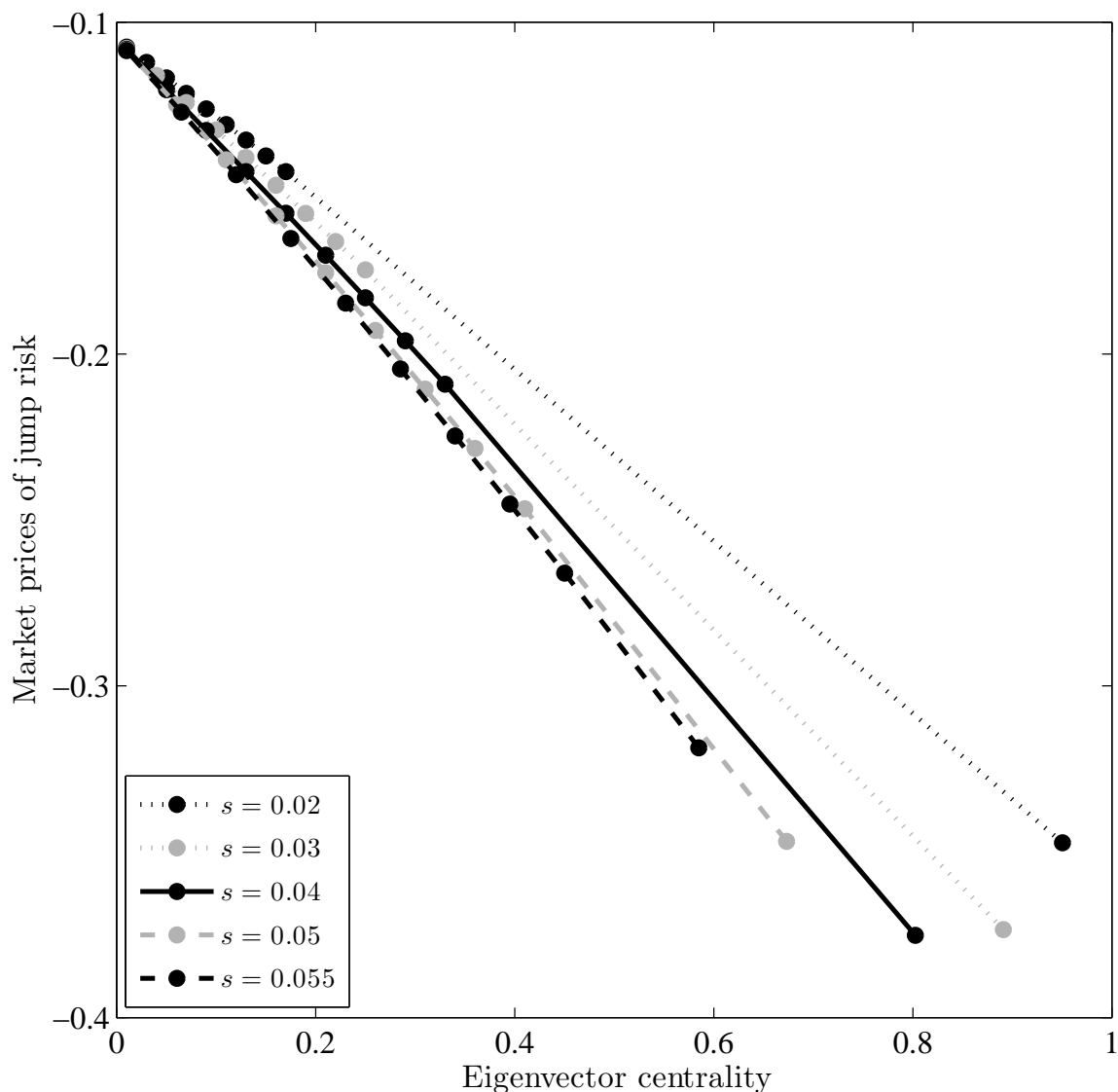
Figure 3:

Market prices of jump risk for varying cross-sectional dispersion of centrality

The figure depicts the market price of jump risk for assets 1 to 10 as a function of eigenvector centrality for economies with different cross-sectional dispersions of centrality (CSDC). To determine the eigenvector centralities of the ten assets in each economy, we set $\alpha_{10} = 0.01$ and $\alpha_{i-1} = \alpha_i + s$ for $i = 3, \ldots, 10$, where $s \in \{0.02, 0.03, 0.04, 0.05, 0.055\}$. α_1 is then determined such that the vector α has length 1. The black dashed line shows the results for $s = 0.02$ (highest CSDC), the gray dashed line those for $s = 0.03$, the black solid line those for $s = 0.04$, the gray dotted line those for $s = 0.05$ and the black dotted line those for $s = 0.055$ (lowest CSDC). Each dot represents the combination of eigenvector centrality and market price of jump risk for an asset which is computed according to Equation (5). The parameters are given in Section 3.1.
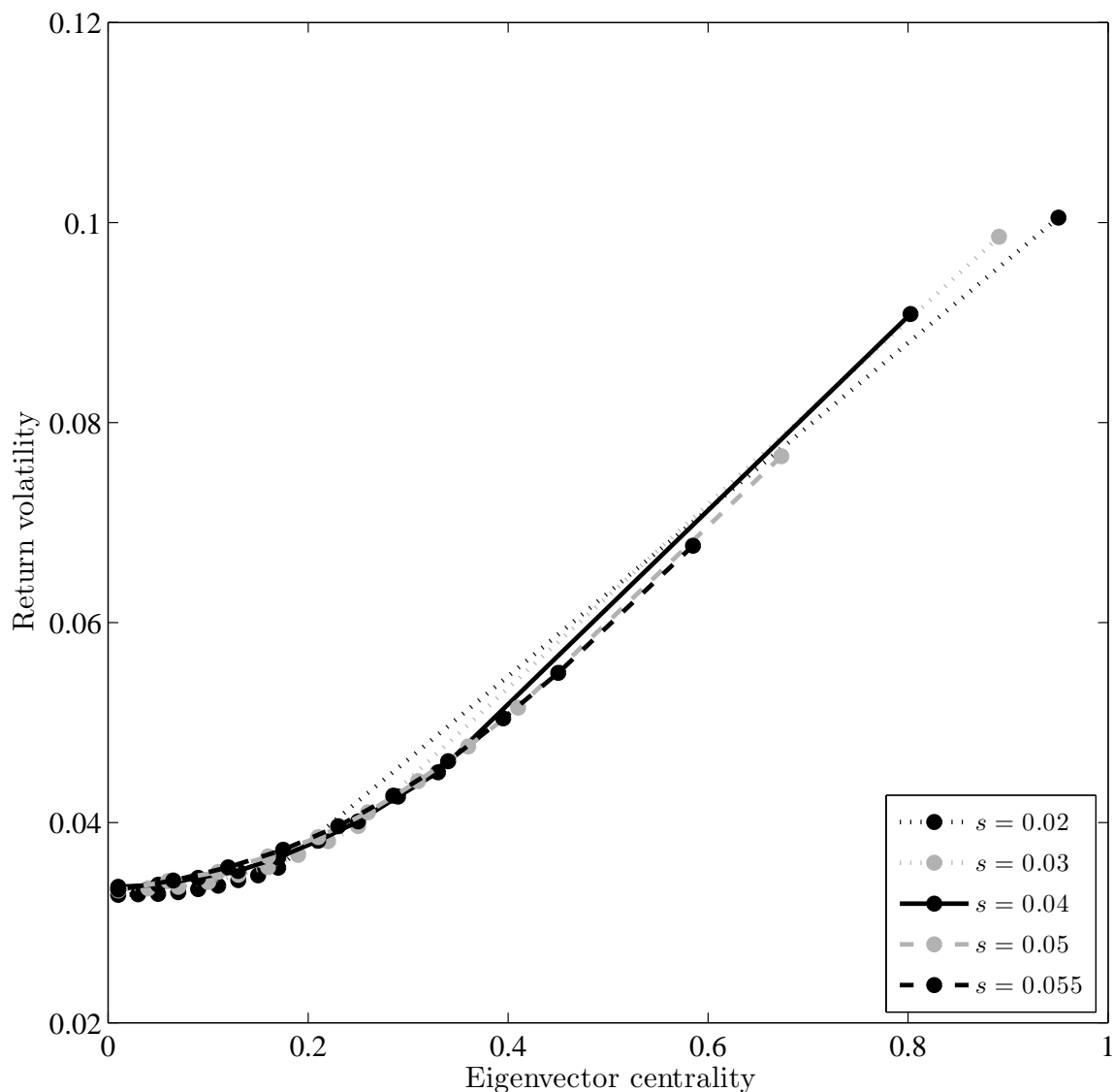
Figure 4:

Return volatility as a function of eigenvector centrality

The figure depicts the volatility of asset returns as a function of eigenvector centrality for economies with different cross-sectional dispersions of centrality (CSDC). The returns have been generated from a monthly Monte Carlo simulation over 80 years with monthly time steps ($\Delta t = 1/12$) and 10,000 paths. To determine the eigenvector centralities of the ten assets in each economy, we set $\alpha_{10} = 0.01$ and $\alpha_{i-1} = \alpha_i + s$ for $i = 3, \ldots, 10$, where $s \in \{0.02, 0.03, 0.04, 0.05, 0.055\}$. α_1 is then determined such that the vector α has length 1. The black dashed line shows the results for $s = 0.02$ (highest CSDC), the gray dashed line those for $s = 0.03$, the black solid line those for $s = 0.04$, the gray dotted line those for $s = 0.05$ and the black dotted line those for $s = 0.055$ (lowest CSDC). Each dot represents the combination of eigenvector centrality and return volatility for an asset. The parameters are given in Section 3.1.
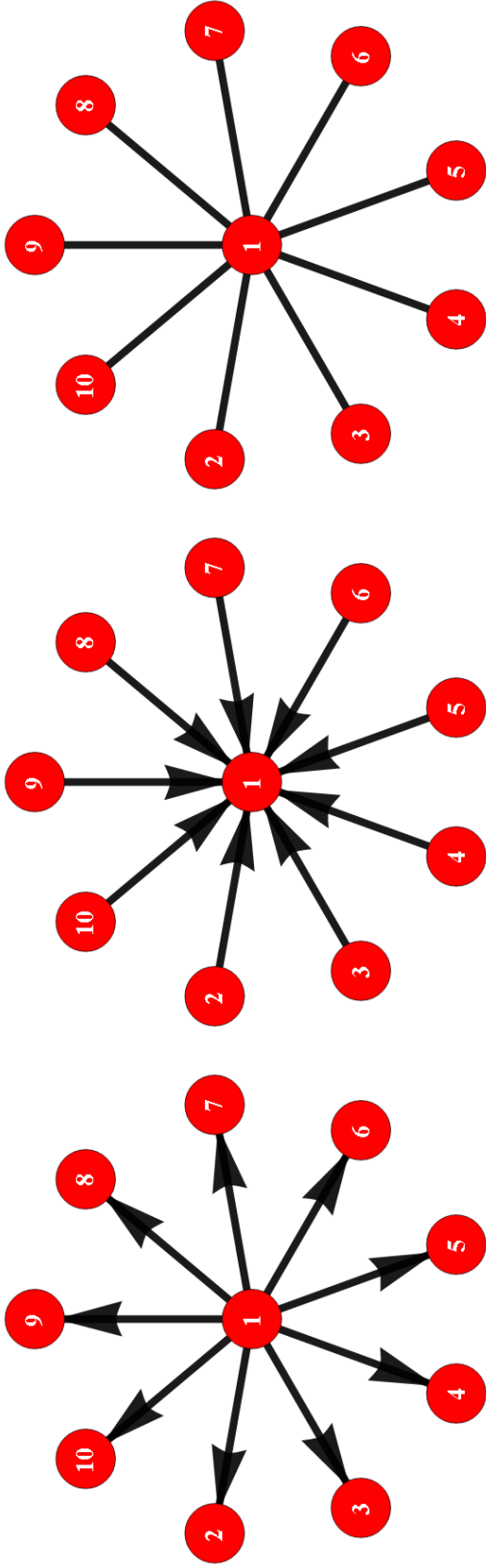
Figure 5:

Star network, reverse star network, and undirected star network

The figure depicts (from left to right) the star network, the reverse star network, and the undirected star network. A link (i.e., an arrow or a line) between two nodes represents the respective entry in the beta matrix (i.e., an arrow from node i to node j implies that asset j is impacted by jumps in asset i so that $\beta_{j,i} > 0$), and the thickness of the link corresponds to the size of the respective entry. The star network is characterized by $\beta_{i,1} = 0.3$ for $i = 2, \ldots, 10$, and $\beta_{i,j} = 0$ for all other combinations of i and j. For the reverse star network we set $\beta_{1,i} = 0.3$ for $i = 2, \ldots, 10$ and $\beta_{i,j} = 0$ otherwise. The undirected star network is obtained by averaging the beta matrices for the star and the reverse star network.
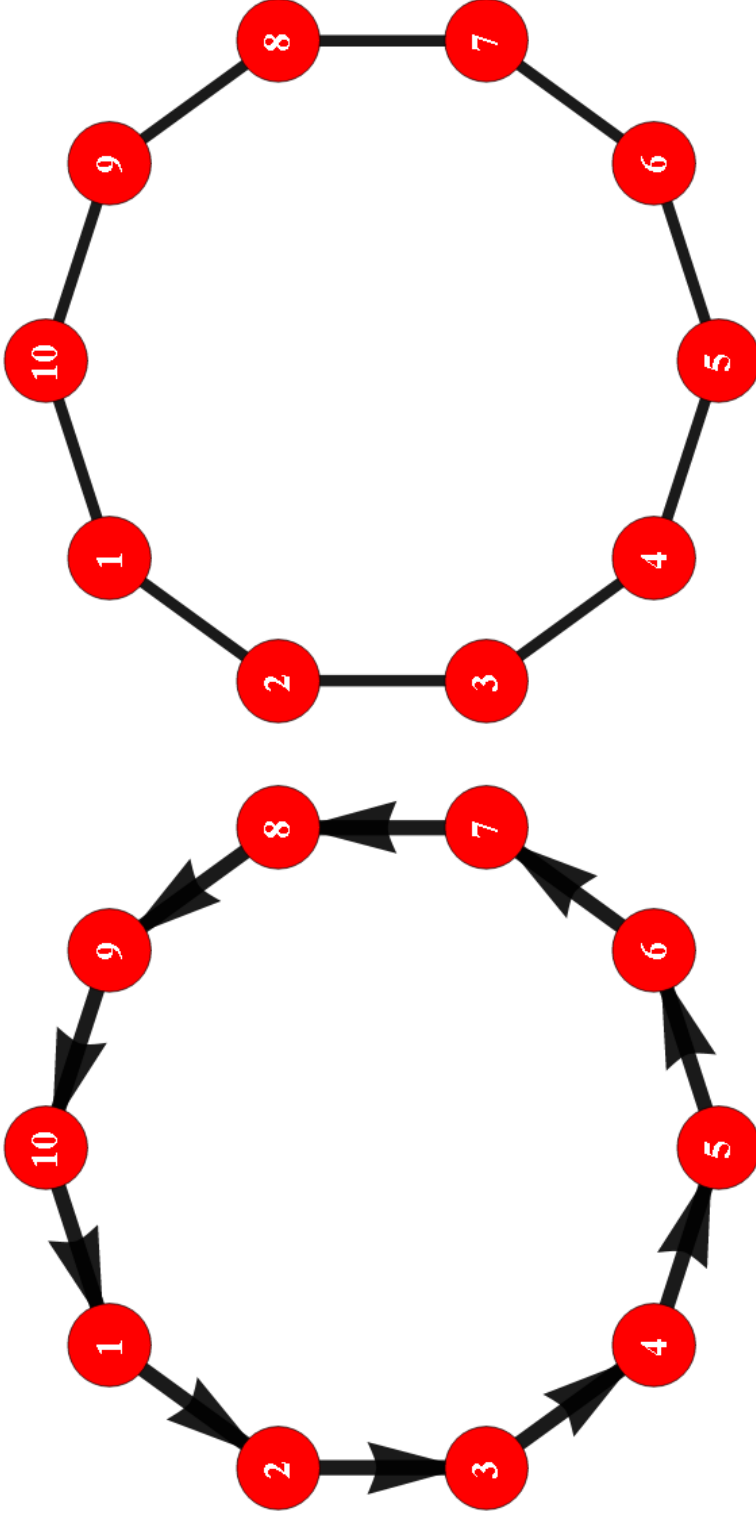
Figure 6:

Directed and undirected ring network

The figure depicts the graphs for the ring and the undirected ring network. A link (i.e., an arrow or a line) between two nodes represents the respective entry in the beta matrix (i.e., an arrow from node i to node j implies that asset j is impacted by jumps in asset i so that $\beta_{j,i} > 0$), and the thickness of the link corresponds to the size of the respective entry. The ring network is characterized by $\beta_{2,1} = \beta_{3,2} = \ldots = \beta_{10,9} = \beta_{1,10} = 0.4$ and $\beta_{i,j} = 0$ for all other combinations of i and j. For the undirected ring we set $\beta_{1,2} = \beta_{2,1} = \beta_{3,2} = \beta_{2,3} = \ldots = 0.2$ and $\beta_{i,j} = 0$ otherwise.