Does Ambiguity about Volatility Matter Empirically?

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This version: January 19, 2016

Abstract

It does. Depending on the forecast horizon, a one standard deviation increase in our measure for ambiguity about consumption volatility predicts a significant increase in average excess equity returns varying between 200 and 600 basis points annualized. The ambiguity measure we propose is easily obtained from the cross-section of analysts' forecasts for aggregate output growth and represents a simple proxy for latent factors in consumption-based asset pricing models. We estimate a version of the long-run risks model, where the investor is concerned about a potential misspecification of the variance dynamics. Since the usually latent state variables are now observable, we can perform the estimation just based on fundamental cash flow data, without the use of asset pricing information. The model produces return predictability patterns via the variance premium, which are in line with the data.

Keywords: Ambiguity, ambiguous volatility, asset pricing, long run risks

JEL: G12, E44, D81

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This paper was previously circulated under the title "Ambiguous Long Run Risks". The authors would like to thank Antje Berndt, Ric Colacito, David Dicks, Steffen Hitzemann, Christoph Meinerding, Jacob Sagi, Karl Schmedders, Michael Semenischev, Mark Trede, Ole Wilms, participants of the Arne Ryde Workshop in Financial Economics 2014 in Lund, the 18th SGF conference 2015 in Zurich, the 14th Colloquium on Financial Markets 2015 in Cologne, the 22nd Annual Meetings of the German Finance Association 2015 in Leipzig and seminar participants at Münster (WWU), Frankfurt (Goethe), Raleigh (NCSU), Zurich (UZH), Chapel Hill (Kenan-Flagler), and Duke (Fuqua) for valuable comments and suggestions. We also thank Yud Izhakian for sharing his data with us.

1 Introduction

This paper studies the link between ambiguity about macroeconomic volatility and asset prices. An economic decision problem exhibits ambiguity when agents are uncertain about the *distribution* of future states of the world, i.e., about the exact structure of the data generating process. This is different from risk, where this distribution is assumed to be known, although, of course, it is still unknown, which state of the world will occur next period.

Following the seminal work of Andersen et al. (2000), the impact of ambiguity on asset prices has been analyzed in a large number of papers, e.g., Collard et al. (2011), Jahan-Parvar and Liu (2014), Ju and Miao (2012), and Miao et al. (2012). One thing that all of these papers have in common is that they focus on ambiguity about expected growth rates.[1]

In an important step forward Epstein and Ji (2013) have recently proposed a theoretical (continuous-time) model featuring an investor who is concerned about ambiguity with respect to volatility, but conclude (p. 1774)

> *A question that remains to be answered more broadly and thoroughly is "does ambiguity about volatility [...] matter empirically?"*

In this paper we show that it does. To investigate the question, we construct a measure of ambiguity about volatility based on simple descriptive statistics for the forecasts of aggregate output growth collected in the Survey of Professional Forecasters (SPF). More precisely, we use interval forecasts to extract the individual forecaster's assessment of future macroeconomic volatility and then take the cross-sectional dispersion of these volatility forecasts as a proxy for ambiguity about consumption growth volatility.

Regressing future excess returns on the CRSP stock market index on this measure yields significantly positive coefficients for various forecast horizons. More precisely, a one standard

[1]The literature on model uncertainty and its implications for asset markets is reviewed by Epstein and Schneider (2010), Etner et al. (2012), and Guidolin and Rinaldi (2013).

deviation increase in our measure for ambiguity about macroeconomic volatility predicts a significant increase in average excess equity returns varying between 200 and 600 basis points annualized, depending on the forecast horizon. We interpret this as strong evidence for a substantial premium for ambiguity about volatility.

Forecast dispersion is a widely used ambiguity measure. Examples are Anderson et al. (2009), Andrei and Hasler (2014), Buraschi and Jiltsov (2006), Drechsler (2013), and Ulrich (2013). However, all these papers use dispersion in point forecasts. Engelberg et al. (2009) criticize this practice on various grounds. They point out that it is not clear whether forecasters report means, medians, modes, or any other characteristics of their subjective distributions when asked for a point forecast. When different forecasters report different characteristics, disagreement in point forecasts is an inconsistent measure for ambiguity. The authors moreover argue that 'even if all forecasters make their predictions in the same way [...] point predictions provide no information about the uncertainty that forecasters feel'. As suggested by Engelberg et al. (2009), we use interval forecasts to come up with consistent measures and to get our hands on the risk assessments of the individual analysts.

Our analysis shows that our measure for ambiguity about volatility is different from other recently proposed measures of uncertainty, like the uncertainty index proposed by Jurado et al. (2015), the ambiguity measure by Brenner and Izhakian (2011), 'vol-of-vol' as analyzed by Baltussen et al. (2012) or the uncertainty measures used by Bloom (2009). The pairwise correlations between any of these quantities and our ambiguity measure are low, and including them in our predictive regressions does not eliminate the predictive power of ambiguity about volatility.

Our empirical findings are consistent with the results generated by a discrete-time general equilibrium asset pricing model featuring a representative agent with recursive preferences, who is concerned about a potential misspecification of volatility. We opt for a discrete time setting, because the solution of the model appears much more straightforward than in the continuous-time setup considered by Epstein and Ji (2013, 2014) and because the frequency of the data we use to estimate the model is low.

Structurally our model is an extension of the long-run risks model introduced by Bansal and Yaron (2004), with the innovation that the conditional variance σ_t^2 of consumption growth is uncertain, i.e., the representative investor perceives it as ambiguous. Our model features a state variable representing the volatility level implied by the reference model, i.e., by the model considered the most likely by the investor. A further state variable then describes the time-varying magnitude of potential deviations from this reference volatility.

To estimate the dynamics of consumption, dividends, and the state variables we only rely on data about fundamental cash flows and quantities derived from the SPF, in particular the ambiguity measure mentioned above. This means that we do not make use of any information about asset prices, and by doing so we make sure that the cash flow part of the model properly represents the *time-series dynamics* of fundamentals (as opposed to just static moments). Especially with respect to the persistence of the state variables this approach yields estimates which are substantially different (in this case lower) than the usual parameter values produced by model calibrations as in, e.g., Bansal and Yaron (2004) and Bansal et al. (2012). The difficulty to reliably detect such a highly persistent process in the data is well-known (see e.g. Constantinides and Ghosh (2011)).

This less pronounced persistence directly implies that the model has a hard time generating return predictability for long forecast horizons, but on the other hand it nicely matches the properties of shorter term predictive regressions using the variance premium as a forecasting variable. From a technical perspective, the pronounced predictability observed here is caused by the joint dependence of the variance premium and the equity premium on the level of ambiguity about volatility in the model.

Regarding investor preferences we use the recursive smooth ambiguity model proposed by Klibanoff et al. (2005, 2009) to model the investor's attitudes towards risk and ambiguity. This approach allows a clear separation of ambiguity itself from attitudes towards ambiguity, which is difficult in other models such as the *maxmin*-model of Gilboa and Schmeidler (1989).

The preference parameters of this model have been estimated by Thimme and Völkert (2014), and we use their estimates as a guideline for the analysis of the asset pricing implications of our model.

In terms of dynamics our approach might look similar to other asset pricing models with sophisticated volatility structures, like those suggested by Bollerslev et al. (2012), Bollerslev et al. (2009), Jin (2013), and Zhou and Zhu (2014). The key difference is, however, that the state variables in these models are assumed to be perfectly observable, while we explicitly consider the situation that the investor faces ambiguity.

One may, of course, argue that volatility *can* indeed be observed, e.g. from high frequency stock return data, so that the dynamics of the volatility process can be estimated rather precisely. Carr and Lee (2009), however, point out that '... noise in the data generates noise in the estimate, raising doubts that a modeler can correctly select any parametric stochastic process from the menu of consistent alternatives'.[2] A large menu of such basically consistent alternatives may then lead to pronounced ambiguity about volatility, if the different alternatives imply rather different volatility levels.

The remainder of this paper is organized as follows. In Section 2 we explain the construction of uncertainty measures based on SPF data. We study the explanatory power of these measures for returns and volatilities in model-free regressions in Section 3. In Section 4, we introduce our asset pricing model. In Section 5, we estimate and calibrate the model and study its implications for asset prices. Section 6 concludes.

[2]Carr and Lee (2009), p. 325.

2 A measure of ambiguity about volatility

We assume throughout the paper that growth in log aggregate endowment, Δc_{t+1}, is (conditionally) Gaussian with conditional mean x_t and conditional variance σ_t^2, i.e.,

$$\Delta c_{t+1} = x_t + \sigma_t \varepsilon_{t+1}^c,$$

where the shocks ε_t^c are i.i.d. standard normal. We will now describe our empirical proxies for the state variables x and σ^2 as well as for the amount of possible ambiguity about x and especially σ^2.

To obtain our proxies we rely on SPF data. The SPF, which is conducted at a quarterly frequency by the Philadelphia Fed, contains individual responses by each analyst about the probability that growth in the gross domestic product (GDP) will realize in a certain interval.[3] It is exactly this structure of the forecast data with subjective interval probabilities, which enables us to construct an empirical measure for ambiguity about volatility (and other quantities which will later serve as state variables in our model).

Note that, strictly speaking, we would like to use forecasts of real consumption growth instead of real GDP growth. The SPF, however, only provides point forecasts of consumption growth but no interval probabilities as in the case of GDP growth. In the literature it is, however, common to use GDP as a proxy for consumption, see, e.g., Bansal and Shaliastovich (2010), Ulrich (2011, 2012), and Colacito et al. (2015). Furthermore, a comparison of the analysts' individual estimates for mean GDP growth with their assessment of mean consumption growth shows that both quantities are closely related. Figure 1 presents a scatter plot of all individual pairs of consumption and GDP forecasts for our sample period. As one can see, the points are rather close to the 45-degree line, and a regression of consumption on GDP forecasts yields an R^2 of close to 80 percent, so that GDP forecasts represent a good proxy for consumption

[3] The forecasted growth rate is annual average GDA in the next calender year divided by annual average GDA in this year minus 1. See Zarnowitz and Braun (1993) for details concerning the SPF. See also Engelberg et al. (2009) for a detailed analysis of the relation between analysts' interval forecasts and point forecasts.

predictions. Finally, from a more theoretical point of view, in endowment models like the one we are going to present below in Section 4, the representative investor has to instantly consume all the exogenous endowment, hence there is no difference between consumption and output anyway.

As mentioned above the SPF contains the subjective probabilities which analysts assign to prespecified intervals for the growth rate of GDP. Let J_t be the number of analysts featured in the survey published at time t. Each analyst is asked for her assessment of the probabilities that GDP growth falls in the intervals $I_1 \equiv (l_0, l_1)$, $I_2 \equiv (l_1, l_2)$ up to $I_{M_t+1} \equiv (l_{M_t}, l_{M_t+1})$ with $l_0 \equiv -\infty$, $l_{M_t+1} \equiv \infty$ and fixed width Δl for the interior intervals I_k for $k = 2, \ldots, M_t$.

The probabilities recorded for analyst j at time t are denoted by $P_t^j = (p_{1,t}^j, \ldots, p_{M_t+1,t}^j)$, where $p_{k,t}^j$ is the analyst's subjective probability that GDP growth falls in interval I_k ($k = 1, \ldots, M_t + 1$). Given these probability assessments we compute analyst j's subjective trend (i.e., expected) growth rate $x_{j,t}$ and growth volatility $\sigma_{j,t}^2$ as the mean and the variance of the normal distribution which most closely approximates the analyst's interval forecasts. More precisely, for every j and t we maximize the (log) likelihood function

$$\mathcal{L} = \sum_{i=1}^{M_t+1} \left(p_{i,t}^j \log \left[\Phi\left(\frac{l_i - x_{j,t}}{\sqrt{\sigma_{j,t}^2}} \right) - \Phi\left(\frac{l_{i-1} - x_{j,t}}{\sqrt{\sigma_{j,t}^2}} \right) \right] \right) \tag{1}$$

with respect to $x_{j,t}$ and $\sigma_{j,t}^2$. Here $\Phi(x)$ denotes the cumulative distribution function of the standard normal distribution.[4]

Given the estimates $x_{j,t}$ and $\sigma_{j,t}^2$ for $j = 1, \ldots, J_t$ we then compute our proxies for uncertainty about trend growth and volatility as the (cross-sectional) average squared deviations

[4]Alternatively, one could take the midpoint of I_k to represent the interval and then compute the means $x_{j,t}$ and the variances $\sigma_{j,t}^2$ as simple descriptive statistics without assuming a certain distribution. When we proceed like this, our results are basically left unchanged.

Vx_t and $V\sigma_t^2$ from the (cross-sectional) averages Ex_t and $E\sigma_t^2$, i.e.,[5]

$$Ex_t = \frac{1}{J_t}\sum_{j=1}^{J_t} x_{j,t}, \qquad Vx_t = \frac{1}{J_t - 1}\sum_{j=1}^{J_t} \left(x_{j,t} - Ex_t\right)^2,$$
$$E\sigma_t^2 = \frac{1}{J_t}\sum_{j=1}^{J_t} \sigma_{j,t}^2, \qquad V\sigma_t^2 = \frac{1}{J_t - 1}\sum_{j=1}^{J_t} \left(\sigma_{j,t}^2 - E\sigma_t^2\right)^2.$$

We interpret the cross-sectional averages Ex_t and $E\sigma_t^2$ as a representation of the reference model, which from a theoretical point of view is the parametrization that the investor considers most likely at time t.

Vx is often referred to as *forecast dispersion* and is widely used as a general representation of ambiguity, e.g., in Andrei and Hasler (2014), Bansal and Shaliastovich (2010), Buraschi and Jiltsov (2006), Drechsler (2013), and Ulrich (2012).[6] As discussed above in the introduction we especially focus on ambiguity about volatility, and analogous to the argument in favor of Vx as an ambiguity measure about trend growth we propose $V\sigma^2$ as a measure for ambiguity about volatility. In more detail, we assume that each analyst represents one specific economic model, which is justified by Patton and Timmermann (2010) who find that analysts disagree because they use different models for forecasting. So the set of subjective (normal) distributions $\{\mathcal{N}(x_{j,t}, \sigma_{j,t}^2)\}$ $(j = 1, \ldots, J_t)$ can be considered a reasonable approximation of the set of possible models the investor faces at time t, and accordingly ambiguity about volatility can be approximated by the cross-sectional variation in analysts' individual volatility assessments.

We construct the above measures from SPF data for the period from 1992:Q1 to 2014:Q4. SPF data are basically available from the fourth quarter of 1968 on, but we discard the data until 1991, since they do not seem reliable for our purposes. Especially the period which the forecasts referred to were not clearly identified in the surveys. In addition to that, from 1968

[5] Our empirical approach could easily be extended to incorporate skewness as in Colacito et al. (2015). Given that our main interest is in ambiguity about volatility, we restrict the analysis to the first two cross-sectional moments of analysts' forecasts.

[6] In a paper on the link between inflation surveys and bond risk premia D'Amico and Orphanides (2014) refer to the analogue of Vx as *disagreement*, whereas they call $E\sigma^2$ *uncertainty*.

to 1991 analysts reported their assessments of growth first in nominal (until the second quarter of 1981) and then in real (until the end of 1991) GNP. Even when one considers GNP a close enough proxy for GDP, this would still leave the problem of having to use inflation forecasts to convert nominal quantities to real ones.[7] Engelberg et al. (2009), who also discard all data from before 1992, note that the Fed changed the number of intervals from six to ten in 1992, which may lead to inconsistencies when a longer time series is used. Finally, additional analyses (not shown) suggest a structural break in the time series of our SPF-based measure in the early 1990's, similar to the one reported by Lettau and Van Nieuwerburgh (2008) for price dividend ratios. Hence, since the data appear to be nonstationary for the longer sample, we only use the time series from 1992 onwards.

Before the SPF measures can actually be used for our empirical analysis they have to be processed in a final step. The analysts' forecasts are with respect to annual-average over annual-average output growth. This means that more and more information about the average becomes available over the course of a year time with an accompanying reduction in the overall statistical uncertainty associated with the forecast made in later parts of the year. This makes it seem appropriate to seasonally adjust the time series, which we do via the X-12-ARIMA procedure. Furthermore, output growth is naturally related to population growth, and to account for this effect we normalize all growth rates by the 12-month moving average growth of US population.

Figure 2 then presents plots of the seasonally adjusted and standardized per capita time series for Ex, $E\sigma^2$, Vx, and $V\sigma^2$. Our proxy Ex for trend consumption growth shows a clearly cyclical behavior. During the recessions in 2001 and 2009 analysts obviously predicted much lower consumption growth rates than over the rest of the sample. Our measure for ambiguity about trend consumption growth Vx spikes in particular during the 2009 financial crisis. Interestingly, the pure risk measure $E\sigma^2$ remains low during NBER recessions, which indicates that uncertainty is indeed different from risk in the data. Ambiguity about volatility $V\sigma^2$ spikes in periods of high expected volatility.

[7] Bansal and Shaliastovich (2010) suggest to proceed like this. In our situation one would have to rely on the rather strong assumption of independence between inflation and GDP growth to justify this approach.

Descriptive statistics for consumption and dividend growth as well as for the measures derived from SPF data are presented in Table 1. Although the numbers are not directly comparable, it is nevertheless interesting to note that the mean of Ex is close to the unconditional mean of log consumption growth. The unconditional standard deviation of consumption growth is $4.50 \cdot 10^{-3}$, and this number squared ($2.03 \cdot 10^{-5}$) is close to the average of $E\sigma^2$.

While the correlation between Ex and $E\sigma^2$ is quite small in absolute value, Ex and Vx exhibit pronounced negative comovement with a time series correlation of -0.54. Moreover, we find a large positive correlation between $E\sigma^2$ and $V\sigma^2$ (0.63). This makes sense intuitively: Ambiguity about expected growth is high during economic downturns while ambiguity about volatility is high if volatility itself is expected to be high. All other pairwise correlations are small.

Our proxies do not seem to vary due to variation in the number of analysts featured in the different surveys, since the correlations between the time series of the number of analysts and the cross-sectional moments derived from the SPF are low.

In Table 2 we compare our SPF-based risk and ambiguity measures to other measures of ambiguity and uncertainty, recently proposed in the literature. For example, Baltussen et al. (2012) suggest 'vol of vol', the variance of a stock's implied volatility (normalized by the mean of the same variable) over a certain period as a measure of ambiguity. They perform their analyses on the individual stock level, but one can easily apply the idea to the market as a whole by computing the variance and the mean of the squared VIX index over the given quarter. Jurado et al. (2015) compose an uncertainty index from a vector autoregressive model for a set of fundamental macroeconomic variables, and Bloom (2009) suggests the cross-sectional standard deviations of firm profits and of stock returns as uncertainty measures. Finally, there is also the 'expected ambiguity measure' as described by Brenner and Izhakian (2011).

The only correlations in Table 2 which are somewhat more pronounced are those between Ex and 'vol of vol', the firm profits based uncertainty measure, and the uncertainty index of Jurado et al. (2015). The negative correlations indicate that these uncertainty measures are countercyclical. The measure suggested by Jurado et al. (2015) is moreover positively correlated with Vx and also positively correlated (but less so) with our risk measure $E\sigma^2$ and ambiguity about volatility $V\sigma^2$. The remaining correlations are rather moderate, so that our SPF-based quantities are clearly not just simple transformations of measures previously suggested in the literature. Especially $V\sigma^2$ appears to be largely unrelated to any of them and thus seems to represent a new dimension of ambiguity.

3 Time-series regressions

In this section we analyze whether the four measures constructed from the SPF have explanatory power for cash flow dynamics and asset pricing quantities in contemporaneous and predictive regressions. We do this in a model-free fashion, i.e., we simply regress the quantity of interest (future excess returns or return volatilities) on our SPF-based measures Ex, $E\sigma^2$, Vx, and $V\sigma^2$ without imposing any model-induced restrictions.

We normalize and standardize our SPF-based measures to have a mean equal to zero and a standard deviation equal to one. The coefficient of a variable in a regression can then be readily interpreted as the change in the dependent variable implied by a one standard deviation change in the regressor. Appendix C provides an overview of the data.

The most important empirical results in our paper are those for the relation between annualized future excess returns and the SPF-based risk and ambiguity measures today, with a special focus on ambiguity about volatility $V\sigma^2$. Table 3 presents the results for excess return forecast horizons ranging from one to eight quarters. For every prediction horizon we show the results of two regressions with the excess return as the dependent variable, one with the full set of regressors $\{Ex, Vx, E\sigma^2, V\sigma^2\}$, and one with $V\sigma^2$ as the only right-hand side variable.

The results clearly indicate that from the set of SPF-based measures $V\sigma^2$ is the most relevant predictor for future excess returns. It is significant as the only regressor in five out of eight cases, and together with the other variables in seven out of eight. To get a feel for the implications of the results, consider the regression of the excess return over the next six months $r_{d,t+6} - r_{f,t}$ on the current values of the SPF-based measures and on $V\sigma^2$ alone. The coefficient of 4.19 in the latter regression means that excess returns increase on average by 419 basis points when the regressor goes up by one standard deviation, implying a sizable premium for ambiguity about volatility. Apparently, investors require a high compensation for holding equity in periods when ambiguity about volatility $V\sigma^2$ is high. When excess returns are regressed on the complete set of SPF predictors the coefficient for $V\sigma^2$ even exceeds 600 basis points for horizons of six and nine months. When the forecast horizon is increased, the coefficient for $V\sigma^2$ tends to decrease slightly, but for $\tau = 24$ months it is still close to 200 basis points and significant. There are only few cases when other SPF variables come out as significant in the predictive regressions, so that overall $V\sigma^2$ is clearly the dominant force.[8]

Figure 3 presents the coefficients of $V\sigma^2$ in the predictive regressions (together with the 90% confidence bands) graphically. It becomes obvious especially from the regressions including the full set of SPF-based measures that ambiguity about volatility has predictive power for horizons extending even beyond 24 months. As our sample is rather short the significance of $V\sigma^2$ as the only regressor is not so pronounced, but the coefficient pattern is very similar to the setup with the other variables included. The figures thus provide evidence in favor of the predictive ability of ambiguity about volatility.

To find out if the predictive power of ambiguity about volatility is robust, we include in the regressions several other variables, which have been shown to either have predictive power for excess equity returns or to be related to economic uncertainty. The results are presented in Table 4 for returns over six and twelve months. The first variables we consider are the price-dividend ratio and the variance premium of the aggregate stock market, which are well-known return predictors. Adding them as controls does not alter our results qualitatively: The coefficient of $V\sigma^2$ stays significantly positive. Moreover, the $\bar{R}^2$ increases from 14.29% (not reported) to 16.39% over the 6-month horizon, and from 19.19% to 21.26% over the 12-month

[8]We also run regressions on Vx alone and find that coefficients in these regressions are all insignificant.

horizon when $V\sigma^2$ and the other SPF proxies are added to the price-dividend ratio and the variance premium.

A widely-used macroeconomic predictor variable is *cay*, introduced by Lettau and Ludvigson (2001), which is supposed to approximate fluctuations in the consumption-to-wealth ratio. Including *cay* as a control variable does not alter the results. We also use a version of *cay* suggested by Bianchi et al. (2015), and the results are similar to those for *cay*.

Next, we add the five uncertainty measures, discussed in Section 2, to the regression. Ambiguity about volatility stays a significant predictor of excess returns, even in case of vol-of-vol. This shows that ambiguity about volatility is different from anticipated fluctuations in volatility, i.e. volatility risk, which is likely measured by vol-of-vol. In contrast to our measure Vx of macroeconomic ambiguity about trend growth, the ambiguity measure by Brenner and Izhakian (2011), which is based on high frequency stock return data, predicts returns with a positive coefficient. However, the coefficient of $V\sigma^2$ stays stable, significant and high.

We include the cross-sectional skewness of trend growth forecasts, as suggested by Colacito et al. (2015), and also the skewness of variance assessments. Both skewness measures are not significant when used together with our proxies in the regression. [9]

A concern could be that our results are driven by the time-varying number of analysts in the SPF. Including the number of analysts as a control variable, however, does not have an impact on the coefficients. We also exclude the years 2008 and 2009 from our sample to check if the recent financial crisis drives the results. Moreover, we exclude extreme outliers, i.e. we discard the 5 highest and 5 lowest trend and volatility assessments in each period when constructing our measure. For both alternative sets of time series, the coefficient of $V\sigma^2$ stays significant.

Finally, we use an alternative uncertainty measure. Instead of the cross-sectional variances we use the differences between the 90% and the 10% quantiles in the empirical distributions of

[9] As suggested by Colacito et al. (2015) we also run our regressions using a skewness measure based on quantile differences as suggested by Bowley (1920). The results (not reported) remain unchanged .

trend growth and variance assessments. The resulting proxy for ambiguity about volatility is less strong in predicting returns over the short horizon (the coefficient is high and positive but insignificant), but again predicts excess returns over the 1 year horizon.

The results of the predictive regressions with return volatility over the next three months as the dependent variable are presented in Table 5. When used in the regression together with the other variables, $E\sigma^2$ is a significant predictor for future realized variance, which is in line with intuition. This also holds if we regress on $E\sigma^2$ alone (not reported). The effect vanishes for longer horizons, which was to be expected since the persistence of $E\sigma^2$ is rather low.

The insignificance of $V\sigma^2$ in these regressions underscores the difference between volatility and ambiguity about volatility. Ambiguity may have an impact on the *level* of prices but does not have a great impact on their *volatility*. In particular, it is plausible to assume that cash-flows are unaffected by ambiguity about volatility.

Table 6 presents the results of regressions of consumption and dividend growth over the next one to four quarters on today's SPF measures. Ex predicts consumption growth with a positive coefficient, which is what one would assume intuitively. The insignificance of $V\sigma^2$ in these regressions shows that it is not a cash-flow channel through which ambiguity about volatility affects future excess returns.

Finally, Table 7 reports the results for contemporaneous regressions of various asset pricing quantities on the SPF-based measures. The price dividend ratio is high in periods of high expected consumption growth Ex, and higher values of $E\sigma^2$ come together with lower interest rates. When $V\sigma^2$ is the only regressor, it is also significantly related to the real interest rate. These results intuitively make sense, since increases in expected volatility or ambiguity about volatility are likely to strengthen the investors' precautionary savings motive, leading to a lower interest rate. The variance premium is significantly higher in periods of high ambiguity about volatility. This suggests a positive link between uncertainty about macroeconomic volatility, as measured by $V\sigma^2$, and uncertainty about volatility of future stock returns, as quantified by the variance premium. Notably, time variation in the variance premium might not be due to variation in volatility risk alone, so that our analysis suggests an additional new channel compared to the standard view in the literature (see e.g. Bollerslev et al. (2009)). If a part of the equity premium is a premium for ambiguity, as suggested by the results described above, it is also natural to assume that a part of the variance premium is a compensation for ambiguity about volatility, especially in light of the link between $E\sigma^2$ and future stock market volatility, shown in Table 5 above.

Summing up, our SPF-based measures exhibit very plausible properties: Ex predicts cash flows, while $E\sigma^2$ predicts return volatility. Moreover, the price-dividend ratio covaries with expected trend growth Ex, whereas there is a precautionary savings motive related to expected volatility $E\sigma^2$. The information content of the two ambiguity measures is different: While ambiguity about trend growth has only very little or no explanatory power in the time series for any of the considered quantities, ambiguity about volatility explains the variance premium and predicts excess returns. For horizons ranging from two to eight quarters the ambiguity premium in excess equity returns are positive and both statistically and economically significant.

4 A model with ambiguity about volatility

To rationalize our empirical findings we now present an equilibrium model, more precisely a version of the long-run risks model featuring ambiguity about consumption growth volatility. We keep the model parsimonious by leaving out ambiguity about trend consumption growth. The reason is that Vx did not turn out to be very important in the regressions in Section 3, but the model can easily be generalized to include also this feature.

4.1 Endowment

The representative investor is endowed with an exogenous stream of a perishable consumption good and prices a claim on all future dividends. Growth rates of aggregate consumption and aggregate dividends are conditionally lognormal:

$$\Delta c_{t+1} = \mu_c + x_t + \tilde{\sigma}_t \varepsilon^c_{t+1}, \tag{2}$$

$$\Delta d_{t+1} = \mu_d + \varphi_x x_t + \tilde{\sigma}_t(\pi_d \varepsilon^c_{t+1} + \varphi_\sigma \varepsilon^d_{t+1}), \tag{3}$$

where ε^c and ε^d are i.i.d. sequences of standard normal variables. Dividends are represented as levered consumption, with φ_x and $(\pi_d^2 + \varphi_\sigma^2)$ greater than one. Furthermore, consumption and dividend growth are locally correlated through the common components x and ε^c. In our estimation in Section 5.1 it turns out that the parameter π_d is very close to zero such that correlation between the cash flows stems solely from the fact that they both load on x.

To model ambiguity about volatility we assume that the investor is uncertain about the volatility $\tilde{\sigma}_t$ at time t. She entertains a non-degenerate model set, whose elements can be indexed by the realizations σ_t of the random variable $\tilde{\sigma}_t$. Several ways to model ambiguity (and attitudes towards ambiguity) have been suggested in the literature. With maxmin expected utility as suggested, e.g. by Gilboa and Schmeidler (1989) and Epstein and Schneider (2003), the investor would consider a set of possible σ's, e.g. an interval $[\underline{\sigma}, \overline{\sigma}]$, but would base her decisions only on the worst case, that is $\overline{\sigma}$ if she is risk-averse. We in turn apply the smooth model proposed by Klibanoff et al. (2005), in which the investor does not only consider the worst case but a weighted average of alternative scenarios. The weights depend on the investor's ambiguity attitude. For this purpose we have to model the investor's subjective probability distribution on the set of candidate σ's, in addition to the set itself.

We assume that this distribution is conditionally Gaussian with dynamics

$$\tilde{\sigma}_t^2 = v_t + \sqrt{q_t}\, \varepsilon^\sigma_t,$$

where ε^σ_t is standard normal and independent of shocks to consumption and dividends. v_t is called *reference volatility*, and it characterizes the most likely model from the investor's point of view. Given the above specification, there is a continuum of models that all yield the same growth rate $\mu_c + x_t$ of consumption but different volatility levels. The magnitude of possible deviations from that reference is driven by q_t, which quantifies the time-varying ambiguity about consumption growth volatility.

The state variables x, v, and q exhibit the following dynamics:

$$x_{t+1} = \rho_x x_t + \sqrt{\pi_v v_t + \pi_q q_t}\, \varepsilon^x_{t+1} \tag{4}$$

$$v_{t+1} = \bar{v} + \rho_v(v_t - \bar{v}) + \sigma_v\, \varepsilon^v_{t+1} \tag{5}$$

$$q_{t+1} = \bar{q} + \rho_q(q_t - \bar{q}) + \sigma_q\, \varepsilon^q_{t+1} \tag{6}$$

where ε^x, ε^v, and ε^q are again standard normal, independent of each other and of all previously introduced shocks. The state vector $s_t = (x_t, v_t, q_t)'$ represents perceived moments of consumption growth and volatility. Uncertainty about the future growth rate could in general be considered a separate kind of uncertainty and modeled as an additional state variable. However, to keep the model parsimonious, we tie uncertainty about x_{t+1} to v_t and q_t.

The long run risks model of Bansal et al. (2012) (BKY) is the special case of our model, in which q is identically equal to zero (and consequently $\bar{q} = \sigma_q = 0$), which means that the investor always perfectly trusts the reference model represented by v_t.

4.2 Preferences

The representative investor in our model has recursive preferences as developed by Epstein and Zin (1989) and Kreps and Porteus (1978). Future consumption paths $C = (C_t)_{t=0,1,\ldots}$ are evaluated with respect to the value function

$$V_t(C) = \left[\left(1 - e^{-\delta}\right) C_t^{1-\rho} + e^{-\delta}(\mathcal{R}_t(V_{t+1}(C)))^{1-\rho}\right]^{\frac{1}{1-\rho}},$$

where δ and ρ denote the investor's subjective discount rate and the reciprocal of her elasticity of intertemporal substitution (EIS), respectively. The uncertainty aggregator $\mathcal{R}$ accounts for risk and ambiguity in the continuation value $V_{t+1}(C)$ of future consumption. Here we use the specification suggested by Klibanoff et al. (2009), i.e.,

$$\mathcal{R}_t(z) = v^{-1}\left(\mathbb{E}_{s_t}\left[v\left(u^{-1}\left(\mathbb{E}_{\sigma_t}\left[u(z)\right]\right)\right)\right]\right),$$

where u and v are utility functions (e.g., of the CRRA type).

The operator $\mathbb{E}_{s_t}[\,\cdot\,] := \mathbb{E}[\,\cdot\,|s_t]$ denotes expectations conditional on state s_t with $s_t = (x_t, v_t, q_t)'$ and $\mathbb{E}_{\sigma_t}[\,\cdot\,] := \mathbb{E}[\,\cdot\,|\tilde{\sigma}_t^2, s_{t+1}]$ denotes expectations conditional on $\tilde{\sigma}_t^2$ and state s_{t+1}.[10]

The curvature of the utility function u characterizes the investor's risk attitude. The certainty equivalent $u^{-1}\left(\mathbb{E}_{\sigma_t}\left[u(z)\right]\right)$ of z is conditional on full information about the distribution of z. As long as the volatility $\tilde{\sigma}_t$ is ambiguous, $u^{-1}\left(\mathbb{E}_{\sigma_t}\left[u(z)\right]\right)$ is a random variable, and the investor considers expected utility of certainty equivalents conditional on the available information s_t about the model set.

The curvature of the composite function $v \circ u^{-1}$ determines the investor's ambiguity attitude. She appreciates a large variation across expected utilities $\mathbb{E}_{\sigma_t}[u(z)]$ when $v \circ u^{-1}$ is convex, while she is ambiguity-averse if $v \circ u^{-1}$ is concave. We choose $u(x) = \frac{x^{1-\gamma}}{1-\gamma}$ and $v(x) = \frac{x^{1-\eta}}{1-\eta}$ with uncertainty attitude parameters γ and η. The investor is risk averse whenever u is concave, i.e. $\gamma > 0$, and ambiguity averse whenever $v \circ u^{-1}$ is concave, which is equivalent to $\eta > \gamma$.

A smooth ambiguity investor prices any claim on a future dividend stream $(D_{i,\tau})_{\tau \geq t}$, such that the return $R_{i,t+1}$ on this claim in the next period satisfies

$$1 = \mathbb{E}_{s_t}\left[\xi_{t,t+1} R_{i,t+1}\right], \tag{7}$$

where $\xi_{t,t+1}$ denotes the stochastic discount factor (SDF). As shown by Hayashi and Miao (2011)

[10] Our definition of $\mathbb{E}_{\sigma_t}[\,\cdot\,]$ implies that the future state of the economy is ambiguous, not risky. Technically this implies that $\mathbb{E}_{\sigma_t}[\varepsilon_t^\sigma] = \varepsilon_t^\sigma$, $\mathbb{E}_{\sigma_t}[\varepsilon_{t+1}^i] = \varepsilon_{t+1}^i$ for $i \in \{x, v, q\}$ and $\mathbb{E}_{\sigma_t}[\varepsilon_{t+1}^i] = 0$ for $i \in \{c, d\}$. This choice is somewhat arbitrary and the model can easily be solved under the assumption $\mathbb{E}_{\sigma_t}[\,\cdot\,] := \mathbb{E}[\,\cdot\,|\tilde{\sigma}_t^2]$ (that means $\mathbb{E}_{\sigma_t}[\varepsilon_{t+1}^i] = 0$ for $i \in \{c, d, x, v, q\}$) as well.

the SDF of a smooth ambiguity investor is given by the expression

$$\xi_{t,t+1} = e^{-\delta\theta_1} \left(\frac{C_{t+1}}{C_t}\right)^{-\rho\theta_1} R_{w,t+1}^{\theta_1 - 1} \left(\mathbb{E}_{\sigma_t}\left[e^{-\delta\theta_1} \left(\frac{C_{t+1}}{C_t}\right)^{-\rho\theta_1} R_{w,t+1}^{\theta_1}\right]\right)^{\theta_2 - 1} \tag{8}$$

where $\theta_1 = \frac{1-\gamma}{1-\rho}$, $\theta_2 = \frac{1-\eta}{1-\gamma}$, and $R_{w,t+1}$ denotes the return on the claim on aggregate consumption.

The pricing kernel in Equation (8) simplifies to the standard Epstein and Zin (1989) stochastic discount factor in two special cases. When there is no ambiguity about σ_t and s_{t+1}, i.e. $\mathbb{E}_{s_t}[\mathbb{E}_{\sigma_t}[\,\cdot\,]] = \mathbb{E}_{\sigma_t}[\,\cdot\,]$, Equations (7) and (8) together imply

$$1 = \mathbb{E}_{\sigma_t}\left[e^{-\delta\theta_1} \left(\frac{C_{t+1}}{C_t}\right)^{-\rho\theta_1} R_{w,t+1}^{\theta_1 - 1} R_{i,t+1}\right] \left(\mathbb{E}_{\sigma_t}\left[e^{-\delta\theta_1} \left(\frac{C_{t+1}}{C_t}\right)^{-\rho\theta_1} R_{w,t+1}^{\theta_1}\right]\right)^{\theta_2 - 1}.$$

In the absence of ambiguity the second expectation in parenthesis is equal to one, since it is the Euler equation of the return on the consumption claim, which yields

$$\xi_{t,t+1} = e^{-\delta\theta_1} \left(\frac{C_{t+1}}{C_t}\right)^{-\rho\theta_1} R_{w,t+1}^{\theta_1 - 1}, \tag{9}$$

i.e., the Epstein and Zin (1989) pricing kernel. Under ambiguity neutrality, i.e., when $\gamma = \eta$ and consequently $\theta_2 = 1$, Equation (8) also simplifies to Equation (9).

4.3 Model solution

As in Bansal and Yaron (2004), we use the return approximation of Campbell and Shiller (1988) and impose affine linear guesses for the valuation ratios of the consumption and dividend claim to find approximate solutions for the asset pricing quantities of interest.[11] The log wealth-consumption ratio z and the log price-dividend ratio z_d of the dividend claim are thus given as $z_t = A + B's_t$ and $z_{d,t} = A_d + B_d' s_t$ respectively, with the state $s_t = (x_t, v_t, q_t)'$ and the coefficients A, B, A_d, and B_d as shown in Appendix A.

[11] The solution technique is demonstrated in detail by Eraker and Shaliastovich (2008) and Drechsler and Yaron (2011).

Let $r_{f,t}$ be the log return on a risk-free bond from t to $t+1$. It is also affine in s_t, i.e., $r_{f,t} = A_f + B_f' s_t$ and one obtains

$$
\begin{aligned}
r_t^f \;=\; & \left[\delta - \frac{1}{2}(1-\theta_1\theta_2)k_1^2\left(B_2^2\sigma_v^2 + B_3^2\sigma_q^2\right) + \rho\mu_c\right] \\
& + \rho x_t \\
& - \frac{1}{2}\Big((1-\theta_1\theta_2)k_1^2\pi_v B_1^2 + \rho(\gamma-1) + \gamma\Big)v_t \\
& - \frac{1}{2}\Big((1-\theta_1\theta_2)k_1^2\pi_q B_1^2 + \frac{1}{4}\left((\gamma-\eta+\gamma\eta)^2 - (\rho-\eta)(1-\gamma)^2(1-\eta)\right)\Big)q_t. \qquad (10)
\end{aligned}
$$

As shown by Lucas (1978) and Bansal and Yaron (2004), interest rates are related to consumption growth x_t via the inverse EIS ρ. Furthermore, we find precautionary savings terms proportional to the volatility level v_t and to ambiguity about volatility q_t, where the coefficient in front of q_t is increasing in γ and η, i.e., in risk aversion and ambiguity aversion.

Let $r_{d,t+1}$ denote the log return on the dividend claim from t to $t+1$. The conditionally expected excess return, i.e. the equity premium is then given by

$$
\begin{aligned}
& \mathbb{E}_{s_t}[r_{d,t+1}] - r_{f,t} = \\
& \quad \zeta(B_2 B_{d,2}\sigma_v^2 + B_3 B_{d,3}\sigma_q^2) - \frac{1}{2}k_{1,d}^2(B_{d,2}^2\sigma_v^2 + B_{d,3}^2\sigma_q^2) \\
& \quad + \left[\zeta B_1 B_{d,1}\pi_v - \frac{1}{2}k_{1,d}^2 B_{d,1}^2\pi_v + \gamma\pi_d - \frac{1}{2}(\pi_d^2 + \varphi_\sigma^2)\right] v_t \\
& \quad + \Bigg[\zeta B_1 B_{d,1}\pi_q - \frac{1}{2}k_{1,d}^2 B_{d,1}^2\pi_q \\
& \qquad + \frac{\eta(\gamma-1)+\gamma}{2}\left(\frac{1}{2}(\pi_d^2+\varphi_\sigma^2) - \gamma\pi_d\right) - \frac{1}{2}\left(\frac{1}{2}(\pi_d^2+\varphi_\sigma^2) - \gamma\pi_d\right)^2\Bigg] q_t, \qquad (11)
\end{aligned}
$$

where $\zeta = (1-\theta_1\theta_2)k_1 k_{1,d}$. All quadratic terms with a factor $-\frac{1}{2}$ in front on the right-hand side of (11) are simply Jensen corrections. The terms featuring $(1-\theta_1\theta_2)k_1 k_{1,d}$ as a factor represent *long-run* premia for fluctuations in the state variables x_t, v_t, and q_t, all of which ultimately affect consumption growth. These premia are proportional to the variances of the state variables, which, as one can see from the specification in Equations (4) to (6), are constant in the case of v_t and q_t and equal to $\pi_v v_t + \pi_q q_t$ in the case of x_t. These long run premia highlight the key characteristic of our model, since it features not only the usual premia for long-run *risk* represented by x and v (as in Bansal and Yaron (2004)), but also a compensation for long-run *ambiguity*, represented by q. In our empirical analysis in Section 3, we found a large and positive premium for ambiguity about volatility while a risk premium could not be detected.

Given $\pi_d = 0$, i.e., a zero local correlation between consumption and dividend innovations as indicated by our parameter estimates (see Section 5.1), Equation (11) simplifies to

$$\begin{aligned} \mathbb{E}_{s_t}[r_{d,t+1}] - r_{f,t} &= \zeta(B_2 B_{d,2}\sigma_v^2 + B_3 B_{d,3}\sigma_q^2) - \frac{1}{2}k_{1,d}^2(B_{d,2}^2\sigma_v^2 + B_{d,3}^2\sigma_q^2) \\ &+ \left[\zeta B_1 B_{d,1}\pi_v - \frac{1}{2}k_{1,d}^2 B_{d,1}^2\pi_v - \frac{1}{2}\varphi_\sigma^2\right] v_t \\ &+ \left[\zeta B_1 B_{d,1}\pi_q - \frac{1}{2}k_{1,d}^2 B_{d,1}^2\pi_q + \frac{1}{4}(\eta(\gamma-1)+\gamma)\varphi_\sigma^2 - \frac{1}{8}\varphi_\sigma^4\right] q_t. \end{aligned}$$

The term $\frac{1}{4}(\eta(\gamma-1)+\gamma)\varphi_\sigma^2 q_t$ is the short-run premium for ambiguity about volatility. It is increasing in the investor's risk and ambiguity aversion γ and η as well as in the dividend leverage parameter φ_σ.

The local return variance for the dividend claim is given by

$$\mathbb{V}_{s_t}[r_{d,t+1}] = k_{1,d}^2 B_{d,2}^2\sigma_v^2 + k_{1,d}^2 B_{d,3}^2\sigma_q^2 + \big(k_{1,d}^2 B_{d,1}^2\pi_v + \pi_d^2 + \varphi_\sigma^2\big)v_t + \big(k_{1,d}^2 B_{d,1}^2\pi_q\big)q_t. \tag{12}$$

An important quantity in the context of an equilibrium asset pricing model is the variance premium *vp*. In a discrete-time model like ours it is defined at time t as the difference between the risk neutral and physical expectations of the return variance from t to $t+2$.[12] Its computation

[12] In a continuous-time model this premium would be equal to the difference between the risk-neutral and the physical expectation of the integrated variance from t to $t+\tau$. In our setup, setting $\tau > 2$ would lead to further terms not reported below, which are structurally identical to the terms in Equation (13).

is shown in detail in Appendix B. One obtains

$$
\begin{aligned}
vp_t &= \mathbb{E}^Q_{s_t}\left[V_{\sigma_t}(r_{d,t+1}+r_{d,t+2})\right] - \mathbb{E}^P_{s_t}\left[V_{\sigma_t}(r_{d,t+1}+r_{d,t+2})\right] \\
&= (\theta_1\theta_2-1)k_1B_2\big(k_{1,d}^2B_{d,1}^2\pi_v+\pi_d^2+\varphi_\sigma^2\big)\sigma_v^2+(\theta_1\theta_2-1)k_1B_3\big(k_{1,d}^2B_{d,1}^2\pi_q\big)\sigma_q^2 \\
&\quad +\frac{1}{2}(\eta(\gamma-1)+\gamma)(\pi_d^2+\varphi_\sigma^2)q_t. \qquad (13)
\end{aligned}
$$

The first two terms of the right-hand side are structurally similar to the variance premium in other models such as in Bollerslev et al. (2009), who find that the variance premium is proportional to the variance of the conditional return variance. The third term is special to our model and is related to uncertainty about the return variance in the period from t to $t+1$. This term is absent in standard long run risks models, since σ_t (and thus the return variance over the next time step) is known. In our model this additional term is proportional to the amount of ambiguity about volatility q_t and increases in the investor's risk and ambiguity aversion coefficients γ and η. This makes sense intuitively, since at time t, the investor faces a variety of possible realizations of $\tilde{\sigma}_t^2$, and thus a variety of corresponding return variances. The more these return variances differ from each other, i.e., the larger q_t, the higher this part of the variance premium.

5 Quantitative analysis of the model

In this section, we first describe the estimation of our model via GMM. We then look at the unconditional asset pricing moments generated by the model and the properties of predictive regressions in the model relative to the data.

5.1 Estimation

We use our SPF-based measures Ex, $E\sigma^2$, and $V\sigma^2$ as proxies for the state variables x, v, and q, relying on the assumption that the set of analysts' subjective distributions presented in

Section 2, is a good approximation of the representative investor's model set. This renders a complex and potentially error-prone recovery of the state variables from cash-flow or even asset pricing data unnecessary.

The cash flow and state variable dynamics in our model are represented by the system

$$\begin{aligned} \Delta c_{t+1} &= \mu_c + x_t + \pi_c \tilde{\sigma}_t \varepsilon^c_{t+1}, \\ \Delta d_{t+1} &= \mu_d + \varphi_x x_t + \pi_d \tilde{\sigma}_t \varepsilon^c_{t+1} + \varphi_\sigma \tilde{\sigma}_t \varepsilon^d_{t+1}, \\ x_{t+1} &= \rho_x x_t + \sqrt{\pi_v v_t + \pi_q q_t} \varepsilon^x_{t+1} \\ v_{t+1} &= \bar{v} + \rho_v (v_t - \bar{v}) + \sigma_v \varepsilon^v_{t+1} \\ q_{t+1} &= \bar{q} + \rho_q (q_t - \bar{q}) + \sigma_q \varepsilon^q_{t+1} \end{aligned} \tag{14}$$

Note that shocks to log consumption growth are scaled by the factor π_c, which we introduce to account for the low level of our time series of expected variance $E\sigma^2$.

We estimate the vector of model parameters

$$\theta = (\mu_c, \pi_c, \mu_d, \varphi_x, \pi_d, \varphi_\sigma, \rho_x, \tilde{\pi}, \bar{v}, \rho_v, \sigma_v, \bar{q}, \rho_q, \sigma_q)'$$

with $\tilde{\pi} \in \{\pi_q, \pi_v\}$ via GMM. The parameters π_v and π_q cannot be identified separately, since they only appear together in the conditional volatility of x. They are, however, individually important for model-based asset pricing quantities like the equity premium (see Section 4). We therefore estimate restricted versions of the model, in which either π_q or π_v is constrained to equal zero. These restrictions do not affect any other parameter of the model in the estimation.

The moment conditions we use are the four conditional expectations[13] and five conditional variances arising from Equations (14), together with the covariance between consumption and dividend growth and the first-order autocovariances of dividend growth and the three state variables. The parameters are exactly identified. Details concerning the moment conditions are presented in Appendix D.

[13] We demean Ex and separately estimate the unconditional mean growth rate μ_c.

We would like to emphasize that the estimation of the model is exclusively based on cash flows (consumption and dividends) and our SPF-based measures Ex, $E\sigma^2$, and $V\sigma^2$. Using asset prices in the estimation would imply the severe risk in a model like ours that the parameter estimates for cash flows and state variables are ultimately only chosen to fit asset pricing moments, as pointed out by Nakamura et al. (2012). The drawback of such a clean procedure is that the model may have a hard time matching a wide range of asset pricing moments simultaneously. We analyze the asset pricing implications of the estimated parameters in Section 5.2.

The point estimates together with standard errors are reported in Table 8. The scaling factor π_c is estimated around 1.8 which indicates that our SPF-based volatility measure $E\sigma^2$ is somewhat lower than realized consumption volatility. In the data the evidence in favor of positive local covariation between consumption and dividends does not appear very strong, since π_d is estimated to be essentially zero.

The difference between estimation based on cash flow data and calibration based mainly on asset pricing data becomes clear when we look at the persistence coefficient of expected consumption growth variance v_t. Although of course for a different sample period, the fact that BKY obtain an estimate for ρ_v of 0.997, as compared to our point estimate of 0.23 (with a standard error of only 0.08), again highlights the tendency of calibrations to produce extremely persistent dynamics for state variables. The calibration of BKY implies a half-life of shocks to σ^2 of 57.7 years. Obviously, it is hard to detect such a component in a sample of only 23 years. However, we do not take a stand on whether this component exists or not. We decided not to include such a 'very-long-run' component in our model, but we are aware that this may come with some drawbacks for model-implied unconditional asset pricing moments.

The estimate for the long-run mean $\bar{v}$ is much lower than in BKY, by a factor of about 10. The first order autocorrelation of trend consumption growth x_t is estimated at 0.83, which is close to values in other papers like BKY. Finally, the dynamics of our ambiguity measure q_t exhibit significant persistence with a half life of shocks equal to about two thirds of a year.

Overall, q is rather small numerically, so that both the long-run mean and the volatility are very small as well.

5.2 Unconditional asset pricing moments

We use the point estimates obtained in Section 5.1 to parametrize our model and subsequently generate asset pricing moments. We follow the literature and assume that the investor's decision interval is monthly, i.e., we convert our quarterly point estimates to monthly parameters.

To compute model-implied asset pricing moments we set the preference parameters regarding intertemporal fluctuations as in BKY, i.e., the investor is impatient with a subjective time discount rate of $\delta = -\log(0.9989)$ and has an EIS greater than one ($\rho = \frac{1}{1.5}$). In terms of the investor's attitude towards uncertainty we consider two alternative specifications. In the first, the investor is assumed to be ambiguity neutral and risk averse ($\gamma = \eta = 10$), which corresponds to the settings in BKY. In the second, she is mildly risk averse ($\gamma = 2$) and ambiguity averse ($\eta = 24$). These values are in line with the estimates in Thimme and Völkert (2014).

We draw 10,000 paths of 123 years of monthly data each and discard the first 100 years on each path to keep the impact of the initial conditions on the results small. The monthly data from the simulation are then aggregated to annual.

Table 9 reports the medians of the simulated values, along with 90% confidence bounds. Since the length of our simulated economies matches the length of the sample period of only 23 years, confidence bands are rather wide.[14]

In the tables we also show the empirical counterparts of the model-implied values. Note that the numbers from the data for our sample differ from what is reported in most studies, since our analysis is based on the relatively short and recent period from 1992 to 2014. In particular, interest rates are low, while price dividend ratios are high and less volatile compared to earlier samples.

[14]As usual the bands become narrower when we extend the simulated sample.

We consider four versions of our model. The first three feature an ambiguity averse investor with three different specifications for fundamental dynamics. The first case is given by the model without ambiguity about volatility ($q_t \equiv 0$, $\bar{q} = \sigma_q = 0$), which structurally corresponds to the approach proposed by BKY. Next, the restriction $\pi_q = 0$ generates a model with ambiguity about volatility, which, however, does not impact the volatility of expected consumption growth. In the third case, with $\pi_v = 0$, this volatility is driven exclusively by the amount of ambiguity q_t. To get a feel for the impact of the ambiguity attitude, we compare this third version of the model to an otherwise identical setup with an ambiguity-neutral investor.

The most important finding from the analysis is that the model with ambiguity aversion and a long-run ambiguity premium due to $\pi_q > 0$ is the one that delivers a high equity and a high variance premium, structurally similar to what we observe in the data. Compared to the specification with $\pi_v > 0$ (and consequently $\pi_q = 0$) this version generates a higher persistence in the variance of the expected growth rate x, since q is more persistent than v and more volatile relative to its mean (see Table 8).

The results for the case of ambiguity neutrality, but with $\pi_q > 0$ furthermore show that it is not enough just to have an impact of q on x, it is also necessary that the investor is actually ambiguity averse. Under ambiguity neutrality there is also a premium for fluctuations in q when $\pi_q > 0$, but it is too small to generate a large enough total equity premium. Only when the impact of q on x and ambiguity aversion come together, there is a substantial long run ambiguity premium which generates high expected returns on the dividend claim. In fact, the average return generated by the model is even larger than in the data. This is a direct consequence of the model being estimated only on cash flow data, but without the use of asset pricing information.

Figure 4 shows decompositions of the equity premium stated in Equation (11) into a constant part, a risk premium, and a premium for ambiguity about volatility. With $\pi_q = 0$ and $\pi_v > 0$ the ambiguity premium is close to zero. If $q \not\equiv 0$, the unconditional ambiguity premium is positive but negligible. Accordingly, setting $\pi_v = 0$ leads to a negligible risk premium. While the long-run risk premium is exactly equal to zero, the short-run risk premium is even negative but very close to zero. This version of the model with $\pi_v = 0$ and $\pi_q > 0$ is in line with our results in Section 3, where we found a high positive premium for ambiguity about volatility, but no significant risk premium. The negative short-run risk premium is a result of our estimate of the local correlation between consumption and dividend innovations π_d being close to zero. Thus short-run innovations in dividend growth are not priced.

Due to the high volatility of the expected growth rate of consumption x_t the return volatility is high in all specifications considered, and all of the versions of the model also match the low persistence of stock market excess returns in the data. Also concerning the real risk-free rate our model with a long-run ambiguity premium and ambiguity aversion is generally closest to the data with respect to the point estimate, although the confidence bands in all specifications contain a negative risk-free rates. Interest rate volatility and persistence are about the same for all models.

The price dividend ratio is not precisely matched by any of the models. Interestingly, the model with ambiguity premium and ambiguity aversion produces price-dividend ratios which are on average lower than in the data, while we observe the opposite for the other three specifications. In contrast, the models perform very well with respect to the volatility of the price-dividend ratio. This is an important feature of our model(s), since, e.g., Beeler and Campbell (2012) consider the low volatility of the price dividend ratio as a major shortcoming of the BKY model.

As discussed above in Section 4 the variance premium is constant without ambiguity about volatility. Even if ambiguity about volatility is included the magnitude of the time-varying part of the variance premium is generally very small.[15] However, the *level* of the variance premium for the specification with $\pi_q > 0$ and ambiguity aversion is strikingly high. The average variance premium of 17.19%, reported in column 1 refers to the measure of Bollerslev et al. (2009). There is, however, a debate about appropriate ways to approximate the $\mathbb{P}$-expectation of return

[15]Note that the variance premium could easily be made more volatile by allowing for time-varying volatility levels σ_v and σ_q of v_t and q_t, for example by introducing square root processes (see e.g. Zhou and Zhu (2014)).

variances. Bekaert and Hoerova (2014) and Drechsler and Yaron (2011) report much lower values for the average variance premium, and their estimates are well in line with the value produced by our model. This shows that it is possible to generate a sizable variance premium also without jumps, so that our model offers an alternative to the approaches suggested by Todorov (2010) and Drechsler and Yaron (2011).

Overall we find that our model is reasonably well able to explain asset pricing moments. This is even more remarkable given that we estimate the model only based on observable data on cash flows and state variables and chose the preference parameters as suggested in the literature. Ambiguity about volatility, in combination with ambiguity aversion, leads to high and volatile excess returns, low interest rates, and a sizable variance premium.

5.3 Return predictability

In the data we observe that price dividend ratios predict excess returns over long horizons of several years. The rationale in the long run risks model of Bansal and Yaron (2004) is that the price dividend ratio decreases with positive innovations in volatility σ_t. At the same time, risk premia are proportional to σ_t^2. The key to explain predictability is then that σ_t^2 is very persistent such that once risk premia are high, they are likely to remain high for very long periods of time. As mentioned in Section 5.1, our model does not feature such a highly persistent uncertainty measure, and hence it is 'by construction' not able to generate return predictability over long horizons of several years.

Since the uncertainty measures v_t and q_t in our model are less persistent, their most pronounced predictive power should be observed over shorter horizons up to one year. Such short horizon predictability has been documented in the literature when the variance premium is used as a predictor (see, among others, Bollerslev et al. (2009), Bollerslev et al. (2011), and Drechsler and Yaron (2011)). Motivated by this we run regressions of the form

$$r_{d,t+h} - r_{f,t} = \alpha(h) + \beta(h)\, vp_t + \varepsilon_{t+h}, \tag{15}$$

where vp_t denotes the variance premium at time t, and h is the prediction horizon measured in months. We annualize all returns before running regression (15).

The results are presented in Table 10. In the data the coefficients are significant at the 10% level for horizons up to 12 months and decreasing in magnitude with the forecast horizon. As is known from the literature the R^2 peaks at a horizon of three months. Again the model with $\pi_q > 0$ and ambiguity aversion matches the properties of the data best. It is the only version of the model which produces significantly positive slope coefficients and values for R^2 very close to the data. The model-implied betas are somewhat large, which is due to the fact that the model tends to generate equity returns which are higher than in the data, as described in Section 5.2. In this specification, there is a large positive long run ambiguity premium in periods of high ambiguity about volatility q_t. In these periods, the variance premium is also higher and, due to the persistence in ambiguity about volatility, predicts high excess returns in the following periods. As the persistence of q_t is moderate (compared to, e.g., in BKY) the predictive power is highest for short horizons, with the largest value of the R^2 for the three month horizon. Here the model-implied R^2 is also very close to the empirical value.

The median slope coefficients for the case of ambiguity neutrality are closer to the empirically observed values, but none of them is significantly different from zero. With $\pi_q = 0$ there is only a short run ambiguity premium, since q does not have an impact on the volatility of x, and the variance premium cannot predict future excess returns.

Overall, the findings from this prediction exercise, together with the empirical results from Section 3, strongly suggest that there is a substantial positive premium for ambiguity about macroeconomic volatility.

6 Conclusion

We propose a measure for ambiguity about consumption growth volatility which can be computed easily from publicly available data as the cross-sectional variance of professional forecasters' variance predictions. This measure predicts excess returns and explains time-variation in the variance premium. These findings are consistent with a general equilibrium asset pricing model with long-run risks. The key feature of the model, besides ambiguity aversion in general, is the existence of a long run ambiguity premium.

Interestingly, our proxy for ambiguity about trend growth does not appear as powerful in explaining asset pricing quantities as ambiguity about volatility. Ambiguity about trend growth has been commonly used in the literature as a proxy for ambiguity in general, but our results seem to indicate that ambiguity about volatility is at least as important as ambiguity about the trend growth rate.

Our paper represents a first step towards a better understanding of the role of ambiguity about volatility in an asset pricing context. The scope of analysis is certainly not limited to equity, but can be extended to other asset classes like, e.g., bonds or derivatives. Furthermore, different, possibly richer, dynamics of the uncertainty processes may yield further insights concerning the importance of ambiguous volatility. For example, it could be interesting to analyze the impact of large positive innovations (jumps) to ambiguous volatility in the course of extreme events such as the recent financial crisis.

A Model solution

We impose an affine guess for the log wealth-consumption ratio

$$z_t = A + B' s_t = A + B_1 x_t + B_2 v_t + B_3 q_t, \tag{A.1}$$

and approximate the log return on the claim to aggregate consumption with the linearization of Campbell and Shiller (1988)

$$r_{w,t+1} = k_0 + k_1 z_{t+1} - z_t + \Delta c_{t+1}, \tag{A.2}$$

where k_0 and k_1 are linearizing constants. It holds that $k_0 = \log(1 - e^{\bar{z}}) - k_1 \bar{z}$ and $k_1 = \frac{e^{\bar{z}}}{1+e^{\bar{z}}}$, where $\bar{z}$ is the long run mean of the log wealth-consumption ratio. Introducing Equations (A.1), (A.2), and the pricing kernel in Equation (8) into the Euler equation $1 = \mathbb{E}_{s_t}[\xi_{t,t+1} R_{w,t+1}]$ and solving for the coefficients yields

$$\begin{aligned}
A &= \frac{1}{1-k_1}\left(-\delta + k_0 + (1-\rho)\mu_c + k_1 B_2 (1-\rho_v)\bar{v} + k_1 B_3 (1-\rho_q)\bar{q} + \frac{1}{2}\theta_1\theta_2 k_1^2 (B_2^2 \sigma_v^2 + B_3^2 \sigma_q^2)\right), \\
B_1 &= \frac{1-\rho}{1-k_1\rho_x} \\
B_2 &= \frac{1}{2(1-k_1\rho_v)}\big(\theta_1\theta_2 k_1^2 B_1^2 \pi_v + (1-\rho)(1-\gamma)\big) \\
B_3 &= \frac{1}{2(1-k_1\rho_q)}\big(\theta_1\theta_2 k_1^2 B_1^2 \pi_q + \frac{1}{4}(1-\rho)(1-\gamma)^2(1-\eta)\big)
\end{aligned}$$

With these coefficients at hand we calculate coefficients for the log risk-free interest rate $r_{f,t} = -\log \mathbb{E}_{s_t}\mathbb{E}_{\sigma_t}[\xi_{t,t+1}]$ which are given in Equations (10). The price dividend ratio $z_{d,t} = A_d + B_d' Y_t$ can be calculated similar to the wealth-consumption ratio. Its constant coefficient is

$$\begin{aligned}
A_d = \frac{1}{1-k_{1,d}}\Big(& -\delta + k_{0,d} + \mu_d - \rho\mu_c + k_{1,d} B_{d,2}(1-\rho_v)\bar{v} + k_{1,d} B_{d,3}(1-\rho_q)\bar{q} \\
&+ \frac{1}{2}(1-\theta_1\theta_2)k_1^2(B_2^2\sigma_v^2 + B_3^2\sigma_q^2) - (1-\theta_1\theta_2)k_1 k_{1,d}(B_2 B_{d,2}\sigma_v^2 + B_3 B_{d,3}\sigma_q^2) + \frac{1}{2}k_{1,d}^2(B_{d,2}^2\sigma_v^2 + B_{d,3}^2\sigma_q^2)\Big)
\end{aligned}$$

while B_d is given by

$$\begin{aligned}
B_{d,1} =& \frac{\varphi_x - \rho}{1-k_{1,d}\rho_x} \\
B_{d,2} =& \frac{1}{2(1-k_{1,d}\rho_v)}\Big((1-\theta_1\theta_2)k_1^2 B_1^2 \pi_v - 2(1-\theta_1\theta_2)k_1 k_{1,d} B_1 B_{d,1}\pi_v + k_{1,d}^2 B_{d,1}^2 \pi_v \\
&+ (\pi_d - \gamma)^2 + (\gamma-\rho)(1-\gamma) + \varphi_\sigma^2\Big) \\
B_{d,3} =& \frac{1}{2(1-k_{1,d}\rho_q)}\Big((1-\theta_1\theta_2)k_1^2 B_1^2 \pi_q - 2(1-\theta_1\theta_2)k_1 k_{1,d} B_1 B_{d,1}\pi_q + k_{1,d}^2 B_{d,1}^2 \pi_q \\
&+ \frac{1}{4}\big(\big[(\pi_d-\gamma)^2 + (\gamma-\eta)(1-\gamma) + \varphi_\sigma^2\big]^2 + (\eta-\rho)(1-\gamma)^2(1-\eta)\big)\Big)
\end{aligned}$$

Introducing these coefficients into Equation (A.2) yields a representation of the return on the dividend claim from which the conditional equity premium and the return volatility can easily be calculated. The formulae can be found in Equations (11) and (12).

B Computation of the variance premium

The variance premium is

$$vp_t = \mathbb{E}^Q_{s_t}\left[V_{\sigma_t}(r_{t+1}+r_{t+2})\right] - \mathbb{E}^P_{s_t}\left[V_{\sigma_t}(r_{t+1}+r_{t+2})\right]. \tag{B.1}$$

Using $V_{\sigma_t}(x) = V_{\sigma_t}(\mathbb{E}_{s_{t+1}}[x]) + \mathbb{E}_{\sigma_t}[V_{s_{t+1}}(\mathbb{E}_{\sigma_{t+1}}[x])] + \mathbb{E}_{\sigma_t}[\mathbb{E}_{s_{t+1}}[V_{\sigma_{t+1}}(x)]]$ yields

$$\begin{aligned} V_{\sigma_t}(r_{t+1}+r_{t+2}) = {} & (k_{1,d}B_{d,2}\sigma_v)^2 + (k_{1,d}B_{d,3}\sigma_q)^2 + ((k_{1,d}B_{d,1})^2\pi_v + \pi_d^2 + \varphi_\sigma^2)v_{t+1} \\ & +(k_{1,d}B_{d,1})^2\pi_q q_{t+1} + (\pi_d^2 + \varphi_\sigma^2)\sigma_t^2 \end{aligned} \tag{B.2}$$

with $\sigma_t^2 = v_t + \sqrt{q_t}\varepsilon_t^\sigma$ and, thus,

$$\begin{aligned} \mathbb{E}^P_{s_t}[V_{\sigma_t}(r_{t+1}+r_{t+2})] = {} & (k_{1,d}B_{d,2}\sigma_v)^2 + (k_{1,d}B_{d,3}\sigma_q)^2 \\ & +((k_{1,d}B_{d,1})^2\pi_v + \pi_d^2 + \varphi_\sigma^2)(1-\rho_v)\bar{v} + (k_{1,d}B_{d,1})^2\pi_q(1-\rho_q)\bar{q} \\ & +[((k_{1,d}B_{d,1})^2\pi_v + \pi_d^2 + \varphi_\sigma^2)\rho_v + (\pi_d^2 + \varphi_\sigma^2)]v_t + (k_{1,d}B_{d,1})^2\pi_q\rho_q q_t. \end{aligned}$$

The log pricing kernel $m_{t,t+1} = \log(\xi_{t,t+1})$ is

$$m_{t,t+1} = \mathbb{E}_{s_t}[m_{t,t+1}] - \Lambda_c\varepsilon_{t+1}^c - \Lambda_\sigma\varepsilon_{t+1}^\sigma - \Lambda_x\varepsilon_{t+1}^x - \Lambda_v\varepsilon_{t+1}^v - \Lambda_q\varepsilon_{t+1}^q$$

where

$$\begin{aligned} \Lambda_c &= \gamma\sigma_t \\ \Lambda_\sigma &= \frac{1}{2}(\eta-\gamma)(1-\gamma)\sqrt{q_t} \\ \Lambda_x &= (1-\theta_1\theta_2)k_1B_1\sqrt{\pi_v v_t + \pi_q q_t} \\ \Lambda_v &= (1-\theta_1\theta_2)k_1B_2\sigma_v \\ \Lambda_q &= (1-\theta_1\theta_2)k_1B_3\sigma_q \end{aligned}$$

To calculate $\mathbb{E}^Q_{s_t}\left[V_{\sigma_t}(r_{t+1}+r_{t+2})\right]$ we use that $V_{\sigma_t}(r_{t+1}+r_{t+2})$ is normal conditional on s_t, so that

$$\begin{aligned} & \mathbb{E}^Q_{s_t}\left[V_{\sigma_t}(r_{t+1}+r_{t+2})\right] + \frac{1}{2}V_{s_t}\left[V_{\sigma_t}(r_{t+1}+r_{t+2})\right] = \log\mathbb{E}^Q_{s_t}\left[\exp(V_{\sigma_t}(r_{t+1}+r_{t+2}))\right] \\ & \quad = \log\mathbb{E}^P_{s_t}\left[\exp(m_{t,t+1} + V_{\sigma_t}(r_{t+1}+r_{t+2}))\right] - \log\mathbb{E}^P_{s_t}\left[\exp(m_{t,t+1})\right] \end{aligned} \tag{B.3}$$

Introducing (B.2) gives

$$\begin{aligned} & \log\mathbb{E}^P_{s_t}\left[\exp(m_{t,t+1} + V_{\sigma_t}(r_{t+1}+r_{t+2}))\right] \\ & \quad = (k_{1,d}B_{d,2}\sigma_v)^2 + (k_{1,d}B_{d,3}\sigma_q)^2 + \mathbb{E}_{s_t}[m_{t,t+1}] + ((k_{1,d}B_{d,1})^2\pi_v + \pi_d^2 + \varphi_\sigma^2)(1-\rho_v)\bar{v} \\ & \quad + (k_{1,d}B_{d,1})^2\pi_q(1-\rho_q)\bar{q} + [\pi_d^2 + \varphi_\sigma^2 + \frac{1}{2}\gamma^2 + ((k_{1,d}B_{d,1})^2\pi_v + \pi_d + \varphi_\sigma^2)\rho_v]v_t \\ & \quad + (k_{1,d}B_{d,1})^2\pi_q\rho_q q_t + \frac{1}{2}[(\pi_d^2 + \varphi_\sigma^2)\sqrt{q_t} + \frac{1}{2}(\eta(\gamma-1)+\gamma)\sqrt{q_t}]^2 + \frac{1}{2}\Lambda_x^2 \\ & \quad + \frac{1}{2}[((k_{1,d}B_{d,1})^2\pi_v + (\pi_d^2 + \varphi_\sigma^2))\sigma_v - \Lambda_v]^2 + \frac{1}{2}[(k_{1,d}B_{d,1})^2\pi_q\sigma_q - \Lambda_q]^2 \end{aligned} \tag{B.4}$$

According to Equation (B.3), we have to subtract $\frac{1}{2}V_{s_t}\left[V_{\sigma_t}(r_{t+1}+r_{t+2})\right]$ and $\log\mathbb{E}^P_{s_t}\left[\exp(m_{t,t+1})\right]$ from the expression in Equation (B.4). These terms are

$$V_{s_t}\left[V_{\sigma_t}(r_{t+1}+r_{t+2})\right] = (\pi_d^2+\varphi_\sigma^2)q_t + ((k_{1,d}B_{d,1})^2\pi_v + \pi_d^2+\varphi_\sigma^2)^2\sigma_v^2 + ((k_{1,d}B_{d,1})^2\pi_q)^2\sigma_q^2$$

and

$$\log \mathbb{E}_{s_t}^P\left[\exp(m_{t,t+1})\right] = \mathbb{E}_{s_t}^P[m_{t,t+1}] + \frac{1}{2}\gamma^2 v_t + \frac{1}{2}\left[\frac{1}{2}(\eta(\gamma-1)+\gamma)\right]^2 q_t + \frac{1}{2}\Lambda_x^2 + \frac{1}{2}\Lambda_v^2 + \frac{1}{2}\Lambda_q^2$$

Putting the pieces together yields formula (13) for the variance premium in Section 4.3.

C Data

We use quarterly data from the first quarter of 1992 to the fourth quarter of 2014.

Analysts' forecasts: We use data from the table *Individual PRGDP*, which can be downloaded from the sites of the Philadelphia Fed (`https://www.philadelphiafed.org/research-and-data/real-time-center/survey-of-professional-forecasters/data-files`).

Consumption: We use data from NIPA Table 2.3.5 released by the Bureau of Economic Analysis (`www.bea.gov/iTable/index_nipa.cfm`). The data is seasonally adjusted at annual rates. We only use nondurables and services and transform to 2014 U.S. dollars by adjusting with the Consumer Price Index (CPI). We obtain the CPI from the Bureau of Labor Statistics (`www.bls.gov/cpi`). We divide by a one year moving average of U.S. population to calculate real per capita consumption. We use a one-year moving average due to the strong seasonality in U.S. population growth. Data about U.S. population is from NIPA Table 7.1. For the predictive regressions in Section 3 we use consumption growth in the quarters that followed the quarter in which the respective surveys were published.

Dividends: Dividends are taken from the homepage of Robert Shiller at Yale (`www.econ.yale.edu/~shiller/data.htm`). It comprehends real dividends of all firms that are listed in the S&P Composite Index. To calculate real growth rates we subtract log dividends of the preceding month from log dividends of the current month. For the predictive regressions in Section 3 we add log growth rates in the months following the month in which the respective survey was published.

Price-dividend ratio: We use the price dividend ratio from the same table as the dividends, i.e. from Robert Shiller's homepage (`www.econ.yale.edu/~shiller/data.htm`). For the regressions in Section 3 we use log price dividend ratios from February, May, August, and November, i.e. the months in which the respective surveys were published.

Risk-free rate: The risk-free rate is proxied by the 3-month secondary market Treasury bill rate taken from the H.15 release of the Federal Reserve Board of Governors (`http://www.federalreserve.gov/releases/h15/data.htm`). To calculate real rates, we proceed as described in Beeler and Campbell (2012) and Constantinides and Ghosh (2011), i.e., we regress the ex post real yield on a 3-month Treasury bill on the three-month nominal yield and the realized growth in the CPI and use the fitted value as ex ante real rate. For the regressions in Sections 3 we use the rates from February, May, August, and November, i.e., from the months in which the surveys were published.

Variance premium: The data are taken from Hao Zhou's webpage (series VRP, file `VRPtable.txt`, `https://sites.google.com/site/haozhouspersonalhomepage/`). For the regressions in Section 3 we use variance premia from February, May, August, and November, i.e., from the months in which the surveys were published.

Stock returns: Monthly excess returns are taken from Kenneth French's homepage (`http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`). They are based on the CRSP value-weighted stock return index. For the predictive regressions in Sections 3 we add log excess returns of the months following the month in which the respective survey was published.

Return volatility: We use daily excess returns from French's homepage (`http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`), based on the CRSP value-weighted stock return index. To calculate realized variance, we follow Anderson et al. (2009) and employ the formula

$$\hat{\sigma_t^2} = \sqrt{\frac{1}{n-1}\left(\sum_{t=1}^{n}(r_t - \bar{r})^2 + 2\sum_{t=2}^{n}(r_{t-1} - \bar{r})(r_t - \bar{r})\right)},$$

i.e., we take potential serial correlation in daily returns into account. $\bar{r}$ denotes the mean of the n daily returns considered. For the regressions in Section 3 we use daily returns from the 66 trading days after the end of the month in which the respective surveys were published.

D Moments used in GMM estimation

We estimate the following two vectors of parameters:

$$\begin{aligned} \vartheta_1 &= (\mu_c, \pi_c, \mu_d, \varphi_x, \pi_d, \varphi_\sigma, \rho_x, \pi_v, \bar{v}, \rho_v, \sigma_v, \bar{q}, \rho_q, \sigma_q)' \\ \vartheta_2 &= (\mu_c, \pi_c, \mu_d, \varphi_x, \pi_d, \varphi_\sigma, \rho_x, \pi_q, \bar{v}, \rho_v, \sigma_v, \bar{q}, \rho_q, \sigma_q)'. \end{aligned}$$

As explained in Section 5.1, we constrain π_q to be zero when estimating ϑ_1 and we constrain π_v to be zero when estimating ϑ_2. We use the following moment conditions:

1. $0 = \Delta c_{t+1} - \mu_c$ (unconditional mean of consumption growth)
2. $0 = (\Delta c_{t+1} - \mu_c)^2 - \frac{\pi_v \bar{v} + \pi_q \bar{q}}{1-\rho_x^2} - \pi_c^2 \bar{v}$ (unconditional variance of consumption growth)
3. $0 = \Delta d_{t+1} - \mu_d$ (unconditional mean dividend growth)
4. $0 = (\Delta d_{t+1} - \mu_d)^2 - \varphi_x^2 \frac{\pi_v \bar{v} + \pi_q \bar{q}}{1-\rho_x^2} - (\pi_d^2 + \varphi_\sigma^2)\bar{v}$ (unconditional mean consumption growth)
5. $0 = (\Delta d_t - \mu_d)(\Delta d_{t+1} - \mu_d) - \varphi_x^2 \rho_x \frac{\pi_v \bar{v} + \pi_q \bar{q}}{1-\rho_x^2}$ (autocovariance of dividend growth)
6. $0 = (\Delta d_{t+1} - \mu_d)(\Delta c_{t+1} - \mu_c) - \varphi_x \frac{\pi_v \bar{v} + \pi_q \bar{q}}{1-\rho_x^2} - \pi_c \pi_d \bar{v}$ (covariance of consumption and dividend growth)
7. $0 = x_{t+1}^2 - \frac{\pi_v \bar{v} + \pi_q \bar{q}}{1-\rho_x^2}$ (unconditional variance of expected consumption growth)
8. $0 = x_{t+1} x_t - \rho_x \frac{\pi_v \bar{v} + \pi_q \bar{q}}{1-\rho_x^2}$ (autocovariance of expected consumption growth)
9. $0 = v_{t+1} - \bar{v}$ (unconditional mean of variance of expected consumption growth)
10. $0 = (v_{t+1} - \bar{v})^2 - \frac{\sigma_v^2}{1-\rho_v^2}$ (unconditional variance of variance of expected consumption growth)
11. $0 = (v_{t+1} - \bar{v})(v_t - \bar{v}) - \rho_v \frac{\sigma_v^2}{1-\rho_v^2}$ (autocovariance of variance of expected consumption growth)
12. $0 = q_{t+1} - \bar{q}$ (mean ambiguity)
13. $0 = (q_{t+1} - \bar{q})^2 - \frac{\sigma_q^2}{1-\rho_q^2}$ (variance of ambiguity)
14. $0 = (q_{t+1} - \bar{q})(q_t - \bar{q}) - \rho_q \frac{\sigma_q^2}{1-\rho_q^2}$ (autocovariance of ambiguity)

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Table 1: **Descriptive statistics**

		mean	std	$AC(1)$	skewness	kurtosis
Cash flow growth	Δc	3.59×10^{-3}	4.50×10^{-3}	0.6309	-0.6034	1.1057
	Δd	7.21×10^{-3}	2.22×10^{-2}	0.8011	-1.2209	2.9577
Asset pricing moments	r_f	-0.0095	0.3943	0.9300	-0.0713	-1.2897
	$p-d$	5.3439	0.2631	0.9479	-0.0458	-0.4673
	$E(r_d-r_f)$	1.6315	8.3253	0.0984	-1.3072	4.5375
	$\sigma(r_d-r_f)$	1.0062	0.5641	0.6136	2.6699	11.0108
	vp	16.5372	20.1864	-0.0091	3.3959	16.6133
SPF-based measures	Ex	3.84×10^{-3}	3.07×10^{-3}	0.8243	-1.6554	4.7447
	$E\sigma^2$	1.44×10^{-5}	2.85×10^{-6}	0.2441	0.2459	1.2030
	Vx	1.29×10^{-6}	1.10×10^{-6}	0.6413	3.3615	15.0737
	$V\sigma^2$	3.91×10^{-10}	2.90×10^{-10}	0.3550	2.6156	11.2304
Correlations		Ex	$E\sigma^2$	Vx	$V\sigma^2$	# analysts
	Ex	1.0000	-0.1431	-0.5408	-0.1070	-0.1655
	$E\sigma^2$		1.0000	0.1010	0.6309	0.1270
	Vx			1.0000	0.1706	-0.0229
	$V\sigma^2$				1.0000	0.1193

This table shows descriptive statistics for fundamental cash flow growth, asset pricing moments, and the time series Ex_t, $E\sigma_t^2$, Vx_t, and $V\sigma_t^2$ derived from SPF data (see Section 2). Δc (Δd) denotes the growth in log consumption (dividends), $r_d - r_f$ is the excess return on the CRSP index with r_f as the risk-free rate. All moments are on a quarterly basis. Growth rates and returns are in percent, the variance premium in percent squared. The sample period is from 1992:Q1 to 2014:Q4. *AC(1)* denotes first order autocorrelation. The data for cash flows and asset pricing moments are described in Appendix C.

Table 2: **Correlations of SPF-based measures with uncertainty measures**

Source	Baltussen et al. (2012)	Bloom (2009)	Bloom (2009)	Jurado et al. (2015)	Brenner and Izhakian (2011)
Variable	Vol of vol	Std(firm profits)	Std(stock returns)	Uncertainty index	Ambiguity measure
Ex	-0.2095	-0.2531	-0.0909	-0.4896	0.0591
$E\sigma^2$	0.1303	0.0710	-0.0320	0.2884	0.0286
Vx	0.1020	0.0752	0.1623	0.4348	-0.2166
$V\sigma^2$	0.0694	0.1009	-0.0926	0.2671	-0.0961

The table shows correlations for our SPF-based measures described in Section 2 and uncertainty measures from the literature. 'Vol of vol' denotes the variance of the squared volatility index VIX^2 over a given quarter normalized by the average of VIX^2 as described in Baltussen et al. (2012). Following Bloom (2009), 'std(profit growth)' is the cross-sectional standard deviation of normalized profit growth computed as $(profit_t - profit_{t-1})/(0.5 \cdot (sales_t + sales_{t-1}))$. The data are taken from Compustat for firms with more than 150 quarters of data. Also analogous to Bloom (2009), 'std(stock returns)' is the cross-sectional standard deviation of stock returns for firms with at least 500 months of data in the CRSP files. The uncertainty index developed in Jurado et al. (2015) for a horizon of $h = 3$ months is taken from Ludvigson's website (`http://www.econ.nyu.edu/user/ludvigsons/data.htm`). For all these variables the sample period is from 1992:Q1 to 2014:Q4. The 'ambiguity measure' is the empirical proxy for expected ambiguity as described in Brenner and Izhakian (2011). This time series was kindly provided by Yud Izhakian and covers the sample 1996:Q1 to 2014:Q4.

Table 3: **Predictive regressions for excess returns**

	Ex_t	$E\sigma_t^2$	Vx_t	$V\sigma_t^2$	$\overline{R^2}$
$r_{d,t+3} - r_{f,t}$	-6.12	-1.82	**-4.91**	4.45	-0.92
	[-1.54]	[-0.46]	[-1.72]	[0.91]	
$r_{d,t+3} - r_{f,t}$				3.12	-0.22
				[0.95]	
$r_{d,t+6} - r_{f,t}$	-5.21	-2.38	**-6.35**	**6.23**	4.06
	[-1.55]	[-0.85]	[-2.39]	[2.17]	
$r_{d,t+6} - r_{f,t}$				**4.19**	1.82
				[2.07]	
$r_{d,t+9} - r_{f,t}$	-4.54	-3.99	**-5.75**	**6.02**	5.82
	[-1.42]	[-1.62]	[-1.92]	[2.99]	
$r_{d,t+9} - r_{f,t}$				**3.01**	1.12
				[1.75]	
$r_{d,t+12} - r_{f,t}$	-3.42	-3.13	-3.54	**4.76**	2.71
	[-1.11]	[-1.41]	[-1.09]	[2.67]	
$r_{d,t+12} - r_{f,t}$				2.55	1.02
				[1.48]	
$r_{d,t+15} - r_{f,t}$	-3.32	-3.12	-2.25	**3.88**	1.80
	[-1.16]	[-1.62]	[-0.77]	[2.56]	
$r_{d,t+15} - r_{f,t}$				1.88	0.29
				[1.48]	
$r_{d,t+18} - r_{f,t}$	-3.86	-2.74	-2.28	**3.89**	4.09
	[-1.55]	[-1.38]	[-0.99]	[2.38]	
$r_{d,t+18} - r_{f,t}$				**2.17**	1.08
				[1.82]	
$r_{d,t+21} - r_{f,t}$	**-4.36**	-2.02	-2.59	**3.44**	5.99
	[-1.85]	[-1.22]	[-1.20]	[2.46]	
$r_{d,t+21} - r_{f,t}$				**2.19**	1.42
				[1.91]	
$r_{d,t+24} - r_{f,t}$	**-4.41**	-1.83	-2.26	**2.99**	6.45
	[-1.87]	[-1.16]	[-1.02]	[2.16]	
$r_{d,t+24} - r_{f,t}$				**1.94**	1.07
				[1.83]	

This table presents results of regressions of excess returns on the CRSP market index (annualized and in percent) on the SPF-based measures Ex, $E\sigma_t^2$, Vx_t, and $V\sigma_t^2$ presented in Section 2. $r_{d,t+\tau} - r_f$ represents the excess return over the next τ months. A detailed description of the data can be found in Appendix C. The numbers shown are the estimated coefficients together with their Newey and West (1987) t-statistics (in brackets). Coefficients in bold face are significant at a level of 10% or lower. The last column presents the adjusted R^2.

Table 4: **Predictive regressions - robustness**

	Ex_t	$E\sigma_t^2$	Vx_t	$V\sigma_t^2$	Y_1	Y_2	$\overline{R^2}$
$Y_1 = p_t - d_t$, $Y_2 = vp_t$							
$r_{d,t+6} - r_{f,t}$	-1.80	-1.64	**-5.01**	**5.64**	**-6.02**		7.35
	[-0.55]	[-0.60]	[-2.18]	[2.05]	[-2.14]		
$r_{d,t+12} - r_{f,t}$	0.81	-2.02	-1.89	**3.80**	**-7.34**		15.01
	[0.31]	[-0.92]	[-0.67]	[2.22]	[-2.83]		
$r_{d,t+6} - r_{f,t}$	-4.17	-3.15	**-6.42**	**6.36**		**5.80**	8.16
	[-1.20]	[-1.18]	[-2.21]	[2.19]		[2.10]	
$r_{d,t+12} - r_{f,t}$	-3.00	-3.57	-3.54	**4.97**		2.40	3.53
	[-0.92]	[-1.56]	[-1.05]	[2.76]		[1.13]	
$r_{d,t+6} - r_{f,t}$	1.67	-1.56	**-4.45**	**4.56**	**-9.15**	**8.47**	16.39
	[0.49]	[-0.69]	[-1.77]	[1.88]	[-3.79]	[3.37]	
$r_{d,t+12} - r_{f,t}$	2.88	-1.98	-1.56	**3.16**	**-9.22**	**5.08**	21.26
	[1.06]	[-0.93]	[-0.53]	[2.18]	[-3.78]	[2.90]	
$Y_1 = cay_t$							
$r_{d,t+6} - r_{f,t}$	-4.88	-1.82	**-6.14**	**5.94**	1.81		3.42
	[-1.38]	[-0.63]	[-2.25]	[2.05]	[0.77]		
$r_{d,t+12} - r_{f,t}$	-2.68	-2.11	-3.00	**4.21**	**3.88**		6.35
	[-0.84]	[-1.07]	[-0.89]	[2.41]	[1.94]		
$Y_1 = cay_t^{MS}$							
$r_{d,t+6} - r_{f,t}$	-5.09	-2.25	**-6.35**	**6.12**	0.36		2.91
	[-1.46]	[-0.80]	[-2.38]	[2.12]	[0.13]		
$r_{d,t+12} - r_{f,t}$	-1.70	-2.31	-3.07	**3.40**	**4.25**		6.48
	[-0.58]	[-1.15]	[-0.96]	[1.83]	[1.79]		
Y_1 = vol-of-vol (Baltussen et al. (2012))							
$r_{d,t+6} - r_{f,t}$	-4.73	-2.66	**-6.31**	**6.29**	**2.33**		3.84
	[-1.36]	[-0.94]	[-2.29]	[2.22]	[1.87]		
$r_{d,t+12} - r_{f,t}$	-2.84	-3.50	-3.51	**4.90**	**3.02**		4.47
	[-0.89]	[-1.54]	[-1.03]	[2.72]	[3.54]		
Y_1 = std. of firm profits (Bloom (2009))							
$r_{d,t+6} - r_{f,t}$	-5.12	-2.38	**-6.32**	**6.20**	0.29		2.96
	[-1.49]	[-0.85]	[-2.34]	[2.12]	[0.21]		
$r_{d,t+12} - r_{f,t}$	-3.32	-3.11	-3.52	**4.77**	0.44		1.62
	[-1.06]	[-1.40]	[-1.08]	[2.58]	[0.28]		

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Table 4 – continued from previous page

	Ex_t	$E\sigma_t^2$	Vx_t	$V\sigma_t^2$	Y_1	Y_2	$\overline{R^2}$
Y_1 = std. of stock returns (Bloom (2009))							
$r_{d,t+6} - r_{f,t}$	-5.22	-2.15	**-5.43**	**5.46**	**-5.03**		7.16
	[-1.66]	[-0.76]	[-3.18]	[1.96]	[-2.27]		
$r_{d,t+12} - r_{f,t}$	-3.46	-2.81	-2.75	**4.05**	**-4.61**		8.44
	[-1.35]	[-1.27]	[-1.19]	[2.27]	[-2.20]		
Y_1 = uncertainty index (Jurado et al. (2015))							
$r_{d,t+6} - r_{f,t}$	**-8.09**	-1.03	**-4.47**	**7.02**	**-8.54**		11.41
	[-2.48]	[-0.36]	[-2.14]	[2.78]	[-1.76]		
$r_{d,t+12} - r_{f,t}$	**-5.17**	-2.28	-2.47	**5.26**	-5.09		7.41
	[-1.81]	[-1.23]	[-0.87]	[3.15]	[-1.35]		
Y_1 = ambiguity measure (Brenner and Izhakian (2011))							
$r_{d,t+6} - r_{f,t}$	-5.80	-3.05	**-6.18**	**7.47**	**4.68**		11.99
	[-1.56]	[-0.94]	[-2.27]	[2.33]	[1.81]		
$r_{d,t+12} - r_{f,t}$	-3.75	-3.71	-3.01	**6.04**	**5.24**		15.54
	[-1.17]	[-1.47]	[-0.85]	[2.95]	[2.46]		
Y_1 = cross-sectional skewness of x, Y_2 = cross-sectional skewness of σ^2							
$r_{d,t+6} - r_{f,t}$	-3.53	-2.87	**-5.79**	**6.54**	3.38		4.52
	[-0.98]	[-1.04]	[-2.23]	[2.32]	[1.22]		
$r_{d,t+12} - r_{f,t}$	-3.20	-3.17	-3.49	**4.85**	0.49		1.62
	[-1.03]	[-1.44]	[-1.10]	[2.62]	[0.25]		
$r_{d,t+6} - r_{f,t}$	-5.30	**-4.80**	**-5.91**	**10.72**		-5.49	5.92
	[-1.52]	[-1.77]	[-2.06]	[3.75]		[-1.22]	
$r_{d,t+12} - r_{f,t}$	-3.52	**-5.00**	-3.21	**8.30**		-4.24	5.00
	[-1.08]	[-1.68]	[-0.93]	[2.97]		[-1.24]	
$r_{d,t+6} - r_{f,t}$	-3.27	**-5.68**	**-5.18**	**11.65**	4.11	-6.17	7.12
	[-0.85]	[-1.87]	[-1.80]	[3.48]	[1.29]	[-1.32]	
$r_{d,t+12} - r_{f,t}$	-3.03	**-5.16**	-3.05	**8.49**	0.97	-4.39	4.10
	[-0.89]	[-1.74]	[-0.90]	[2.99]	[0.50]	[-1.31]	
Y_1 = number of analysts in survey t							
$r_{d,t+6} - r_{f,t}$	**-5.64**	-2.29	**-6.66**	**6.39**	-1.45		3.46
	[-1.73]	[-0.80]	[-2.46]	[2.36]	[-0.53]		
$r_{d,t+12} - r_{f,t}$	-3.82	-3.06	-3.82	**4.96**	-1.49		2.26
	[-1.24]	[-1.45]	[-1.14]	[2.93]	[-0.48]		

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Table 4 – continued from previous page

	Ex_t	$E\sigma_t^2$	Vx_t	$V\sigma_t^2$	Y_1	Y_2	$\overline{R^2}$
Excluding the 2008/2009 crisis							
$r_{d,t+6} - r_{f,t}$	-1.75	-3.65	**-5.82**	**7.55**			14.34
	[-0.73]	[-1.39]	[-2.25]	[3.61]			
$r_{d,t+12} - r_{f,t}$	-1.69	-2.79	-2.62	**5.16**			4.55
	[-0.56]	[-1.57]	[-0.77]	[3.25]			
Excluding extreme outliers							
$r_{d,t+6} - r_{f,t}$	-4.26	**-8.65**	**-6.04**	**9.72**			8.12
	[-1.51]	[-1.86]	[-2.29]	[1.82]			
$r_{d,t+12} - r_{f,t}$	**-4.43**	**-5.29**	**-6.77**	**5.92**			13.21
	[-1.91]	[-1.90]	[-2.43]	[2.06]			
Y_1 = interquantile diff. in means, Y_2 = interquantile diff. in variances							
$r_{d,t+6} - r_{f,t}$	-4.36	-6.13			**-5.40**	8.83	3.31
	[-1.41]	[-1.34]			[-2.00]	[1.63]	
$r_{d,t+12} - r_{f,t}$	**-4.32**	**-5.69**			**-6.01**	**6.95**	9.01
	[-1.82]	[-2.01]			[-2.63]	[2.59]	

This table presents results of regressions of excess returns on the CRSP market index (annualized and in percent) on the SPF-based measures Ex, $E\sigma_t^2$, Vx_t, and $V\sigma_t^2$ presented in Section 2. In addition, we add controls to the regressions. These are in order: the log price-dividend ratio and the variance premium (see Appendix C), *cay* as in Lettau and Ludvigson (2001), Markov-switching *cay* as in Bianchi et al. (2015), the five uncertainty measures described in Table 2, the number of analysts featured in the SPF at time t, and the cross-sectional skewness of individual x and σ^2. We exclude the years 2008 and 2009 from the sample to control for the impact of the recent financial crisis. "Excluding extreme outliers" means that we discard the five highest and five lowest mean growth rate forecasts, as well as the five highest and five lowest variance forecasts before calculating the SPF-based measures. The interquantile difference is the difference between 90%- and 10%-quantiles in the cross-section of mean and variance forecasts. $r_{d,t+\tau} - r_f$ represents the excess return over the next τ months. A detailed description of the data can be found in Appendix C. The numbers shown are the estimated coefficients together with their Newey and West (1987) t-statistics (in brackets). Coefficients in bold face are significant at a level of 10% or lower. The last column presents the adjusted R^2.

Table 5: **Predictive regressions for return volatilities**

	Ex_t	$E\sigma_t^2$	Vx_t	$V\sigma_t^2$	$\overline{R^2}$
$\sigma(r_{d,t+3})$	-1.06	**2.09**	0.85	0.15	7.03
	[-0.70]	[2.07]	[0.83]	[0.12]	
$\sigma(r_{d,t+3})$				1.73	2.74
				[1.50]	
$\sigma(r_{d,t+6})$	-0.66	**1.83**	1.61	-0.64	5.81
	[-0.47]	[2.02]	[1.63]	[-0.79]	
$\sigma(r_{d,t+3})$				0.86	-0.04
				[0.99]	
$\sigma(r_{d,t+9})$	-0.62	**1.86**	1.68	-0.67	6.91
	[-0.48]	[1.80]	[1.57]	[-0.78]	
$\sigma(r_{d,t+9})$				0.86	0.02
				[0.88]	
$\sigma(r_{d,t+12})$	-0.75	1.75	1.21	-0.61	4.80
	[-0.57]	[1.66]	[1.19]	[-0.71]	
$\sigma(r_{d,t+12})$				0.76	-0.19
				[0.76]	

This table presents results of regressions of return volatility of the CRSP market index on the SPF-based measures Ex, $E\sigma_t^2$, Vx_t, and $V\sigma_t^2$ presented in Section 2. $\sigma(r_{d,t+\tau})$ represents the volatility in daily returns over the next $21 \cdot \tau$ trading days, annualized by multiplication with $\sqrt{252}$ and expressed in percent. A detailed description of the data can be found in Appendix C. The numbers shown are the estimated coefficients together with their Newey and West (1987) t-statistics (in brackets). Coefficients in bold face are significant at a level of 10% or lower. The last column presents the adjusted R^2.

Table 6: **Predictive regressions for cash flow growth**

	Ex_t	$E\sigma_t^2$	Vx_t	$V\sigma_t^2$	$\overline{R^2}$
Δc_{t+3}	**0.75**	**-0.39**	0.17	0.09	27.53
	[4.25]	[-2.71]	[1.48]	[0.65]	
Δc_{t+3}				-0.21	0.96
				[-1.03]	
Δc_{t+6}	**0.64**	**-0.36**	0.13	0.09	25.64
	[3.50]	[-2.70]	[0.99]	[0.54]	
Δc_{t+6}				-0.19	0.95
				[-0.88]	
Δc_{t+9}	**0.53**	**-0.35**	0.11	0.06	20.01
	[2.74]	[-2.41]	[0.83]	[0.36]	
Δc_{t+9}				-0.20	1.38
				[-0.96]	
Δc_{t+12}	**0.42**	**-0.36**	0.08	0.07	15.28
	[2.21]	[-2.34]	[0.57]	[0.42]	
Δc_{t+12}				-0.19	1.28
				[-0.95]	
Δd_{t+3}	2.71	0.68	**-3.70**	0.01	36.99
	[1.28]	[0.65]	[-2.25]	[0.02]	
Δd_{t+3}				-0.48	-0.82
				[-0.38]	
Δd_{t+6}	2.75	0.63	-2.96	0.27	31.59
	[1.19]	[0.70]	[-1.61]	[0.33]	
Δd_{t+6}				-0.13	-1.09
				[-0.10]	
Δd_{t+9}	2.62	0.33	-2.45	0.79	26.65
	[1.16]	[0.38]	[-1.28]	[1.01]	
Δd_{t+9}				0.29	-1.00
				[0.24]	
Δd_{t+12}	2.37	0.24	-2.17	1.01	22.97
	[1.16]	[0.30]	[-1.16]	[1.45]	
Δd_{t+12}				0.53	-0.66
				[0.49]	

This table presents results of regressions of log consumption growth and log dividend growth (both annualized and expressed in percent) on Ex_t, $E\sigma_t^2$, Vx_t, and $V\sigma_t^2$. A detailed description of the data can be found in Appendix C. The numbers shown are the estimated coefficients together with their Newey and West (1987) t-statistics (in brackets). Coefficients in bold face are significant at a level of 10% or lower. The last column presents the adjusted R^2.

Table 7: **Contemporaneous regressions for asset pricing quantities**

	Ex_t	$E\sigma_t^2$	Vx_t	$V\sigma_t^2$	$\overline{R^2}$
$p_t - d_t$	**0.15**	0.05	**0.06**	-0.04	22.32
	[3.94]	[1.43]	[2.66]	[-1.38]	
$p_t - d_t$				-0.02	-0.48
				[-0.68]	
$r_{f,t}$	0.64	**-1.47**	0.24	-0.36	11.63
	[0.74]	[-2.47]	[0.30]	[-0.74]	
$r_{f,t}$				**-1.31**	6.67
				[-2.63]	
vp_t	-1.65	4.09	1.04	-0.71	-0.56
	[-0.74]	[1.43]	[-1.00]	[-0.33]	
vp_t				**1.90**	-0.23
				[1.90]	

This table presents results of regressions of the log price dividend ratio $p_t - d_t$, the log of one plus the real risk-free rate $r_{f,t}$ (expressed in percent), and the variance premium vp_t (expressed in percent squared) on the SPF-based measures Ex_t, $E\sigma_t^2$, Vx_t, and $V\sigma_t^2$. A detailed description of the data can be found in Appendix C. The numbers shown are the estimated coefficients together with their Newey and West (1987) t-statistics (in brackets). Coefficients in bold face are significant at a level of 10% or lower. The last column presents the adjusted R^2.

Table 8: **Parameter estimates**

Aggregate consumption growth

Parameter	μ_c		π_c	
Estimate	3.68×10^{-3}		1.77	
	[7.41×10^{-4}]		[0.13]	

Aggregate dividend growth

Parameter	μ_d	ϕ_x	π_d	φ_σ
Estimate	7.05×10^{-3}	6.79	-0.03	3.51
	[4.62×10^{-3}]	[0.75]	[0.88]	[0.47]

Trend consumption growth

Parameter		ρ_x	π_v	π_q
Estimate		0.83	0.20	7.30×10^{3}
		[0.06]	[0.11]	[3.85×10^{3}]

Expected consumption growth variance

Parameter	$\bar{v}$	ρ_v	σ_v	
Estimate	1.44×10^{-5}	0.25	2.76×10^{-6}	
	[4.16×10^{-7}]	[0.08]	[2.21×10^{-7}]	

Ambiguity about consumption growth variance

Parameter	$\bar{q}$	ρ_q	σ_q	
Estimate	3.93×10^{-10}	0.36	2.70×10^{-10}	
	[4.22×10^{-11}]	[0.07]	[5.80×10^{-11}]	

The table shows the GMM parameter estimates for the cash flow and state variable dynamics in our model represented by the system (14). The estimate of π_v is conditional on the assumption $\pi_q = 0$ and the estimate of π_q is conditional on the assumption $\pi_v = 0$. The table thus presents results from two different estimations. Since the constraints $\pi_q = 0$ (resp. $\pi_v = 0$) do not influence the other parameter estimates, we show the results for these other paramters only once for the two estimations. The moments used for estimation are described in Appendix D. Standard errors adjusted for heteroskedasticity and autocorrelation according to Newey and West (1987) are shown in square brackets. The data on consumption and dividend growth are described in Appendix C.

Table 9: **Unconditional asset pricing moments**

Ambiguity attitude	Data	averse	averse	averse	neutral
		$q_t \equiv 0$	$\pi_q = 0,\ \pi_v > 0$	$\pi_q > 0,\ \pi_v = 0$	$\pi_q > 0,\ \pi_v = 0$
Equity premium					
mean	6.43	3.24	3.23	13.71	1.76
		[-10.91,17.36]	[-10.95,17.41]	[0.45,26.96]	[-13.58,17.01]
std	18.67	34.16	34.18	33.63	36.86
		[24.51,45.11]	[24.48,45.13]	[23.48,45.70]	[25.62,50.09]
$AC1$	0.05	-0.05	-0.05	-0.09	-0.05
		[-0.42,0.34]	[-0.42,0.34]	[-0.45,0.30]	[-0.41,0.33]
Risk-free rate					
mean	-0.10	1.09	1.09	0.72	1.61
		[-0.44,2.57]	[-0.44,2.57]	[-0.97,2.31]	[-0.05,3.20]
std	4.62	1.89	1.89	2.09	2.02
		[1.23,2.78]	[1.23,2.78]	[1.34,3.11]	[1.28,3.04]
$AC1$	0.74	0.52	0.52	0.48	0.51
		[0.15,0.76]	[0.15,0.76]	[0.11,0.74]	[0.14,0.76]
Price-dividend ratio					
mean	3.96	4.37	4.37	2.18	5.59
		[4.11,4.64]	[4.11,4.64]	[1.91,2.43]	[5.30,5.87]
std	0.27	0.35	0.35	0.35	0.38
		[0.23,0.51]	[0.24,0.51]	[0.23,0.51]	[0.25,0.55]
$AC1$	0.67	0.43	0.43	0.42	0.43
		[0.03,0.71]	[0.03,0.71]	[0.08,0.70]	[0.02,0.71]
Variance premium					
mean	17.19	0.66	0.66	8.39	1.79
		[0.66,0.66]	[0.66,0.66]	[8.39,8.39]	[1.79,1.80]
std	20.41	0.00	0.00	0.00	0.00
		[0.00,0.00]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]
$AC1$	0.29	-	0.65	0.65	0.65
			[0.54,0.73]	[0.54,0.73]	[0.54,0.73]

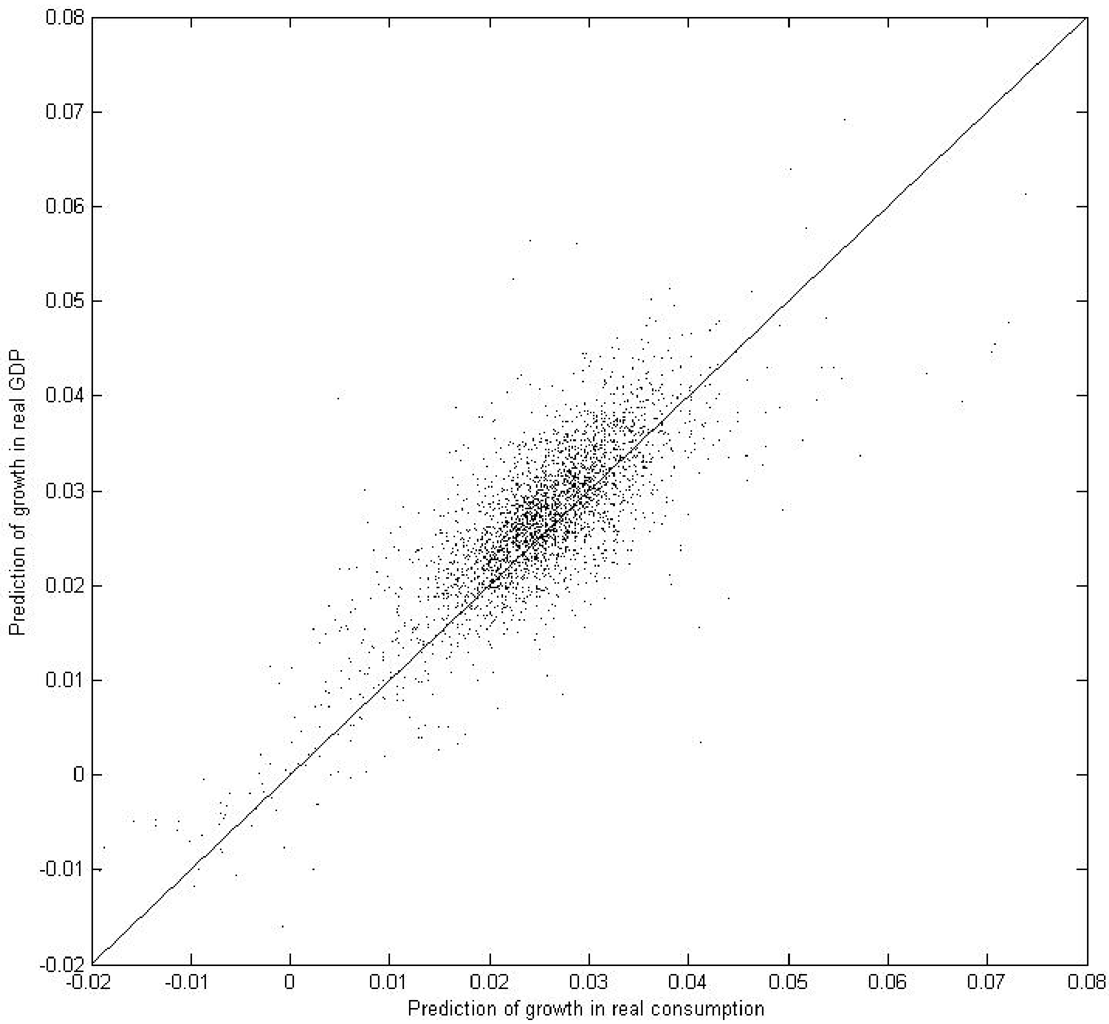
The table presents asset pricing moments from the data for the period from 1992:Q1 to 2014:Q4 and from Monte Carlo simulations of four versions of our model. The preference specifications in the first three cases are $\gamma = 2$ and $\eta = 24$, i.e., the investor is ambiguity averse. In the fourth case the investor is ambiguity-neutral with $\gamma = \eta = 10$. The model-implied values shown are the medians and the 90% confidence intervals (in brackets) from a simulation of 10,000 paths, each of the same length as the data. The data are described in Appendix C. The model labeled $q_t \equiv 0$ corresponds to the dynamics (14) with $\sigma_q = \bar{q} = 0$. The versions of the model corresponding to the restrictions $\pi_q = 0$ and $\pi_v = 0$ are obtained analogously.

Table 10: **Return prediction via the variance premium**

Ambiguity attitude		averse	averse	neutral
	Data	$\pi_q = 0$, $\pi_v > 0$	$\pi_q > 0$, $\pi_v = 0$	$\pi_q > 0$, $\pi_v = 0$
$\beta(1)$	12.68	0.14 [-14.36,14.07]	34.03 [1.73,13.24]	12.13 [-5.74,30.02]
t-stat	5.22	0.02 [-2.03,2.02]	4.07 [1.84,6.76]	1.40 [-0.64,3.69]
$R^2(1)$	5.72	0.16 [0.00,1.78]	6.27 [1.73,13.24]	0.86 [0.00,5.16]
$\beta(3)$	10.23	0.04 [-12.16,12.03]	23.47 [9.94,36.72]	8.32 [-6.51,22.85]
t-stat	6.81	0.01 [-2.25,2.18]	3.86 [1.41,7.01	1.25 [-0.93,3.80]
$R^2(3)$	9.94	0.35 [0.00,3.88]	10.13 [1.89,22.80]	1.34 [0.00,9.86]
$\beta(6)$	6.21	-0.06 [-10.07,10.22]	15.05 [4.45,25.84]	5.33 [-6.60,17.51]
t-stat	4.07	-0.01 [-2.33,2.37]	3.31 [0.86,6.44]	1.05 [-1.23,3.59]
$R^2(6)$	6.60	0.51 [0.00,5.59]	9.30 [0.84,24.35]	1.24 [0.00,10.37]
$\beta(12)$	2.57	0.02 [-7.90,8.00]	8.05 [0.15,16.18]	2.85 [-6.22,11.95]
t-stat	1.87	0.72 [-2.45,2.48]	2.55 [0.04,5.43]	0.78 [-1.65,3.40]
$R^2(12)$	2.08	0.68 [0.00,6.89]	6.12 [0.08,21.08]	1.04 [0.00,10.35]
$\beta(24)$	1.62	0.01 [-6.04,6.02]	4.03 [-1.69,9.99]	1.45 [-5.27,8.10]
t-stat	1.46	0.00 [-2.57,2.52]	1.85 [-0.70,4.66]	0.57 [-1.93,3.19]
$R^2(24)$	1.46	0.79 [0.00,8.29]	3.59 [0.01,17.69]	0.99 [0.00,9.88]
$\beta(36)$	0.61	-0.04 [-5.12,5.09]	2.66 [-2.20,7.78]	0.95 [-4.66,6.67]
t-stat	0.67	-0.02 [-2.66,2.62]	1.54 [-1.06,4.44]	0.48 [-2.09,3.16]
$R^2(36)$	0.31	0.84 [0.00,9.21]	2.75 [0.01,17.28]	0.99 [0.00,10.42]

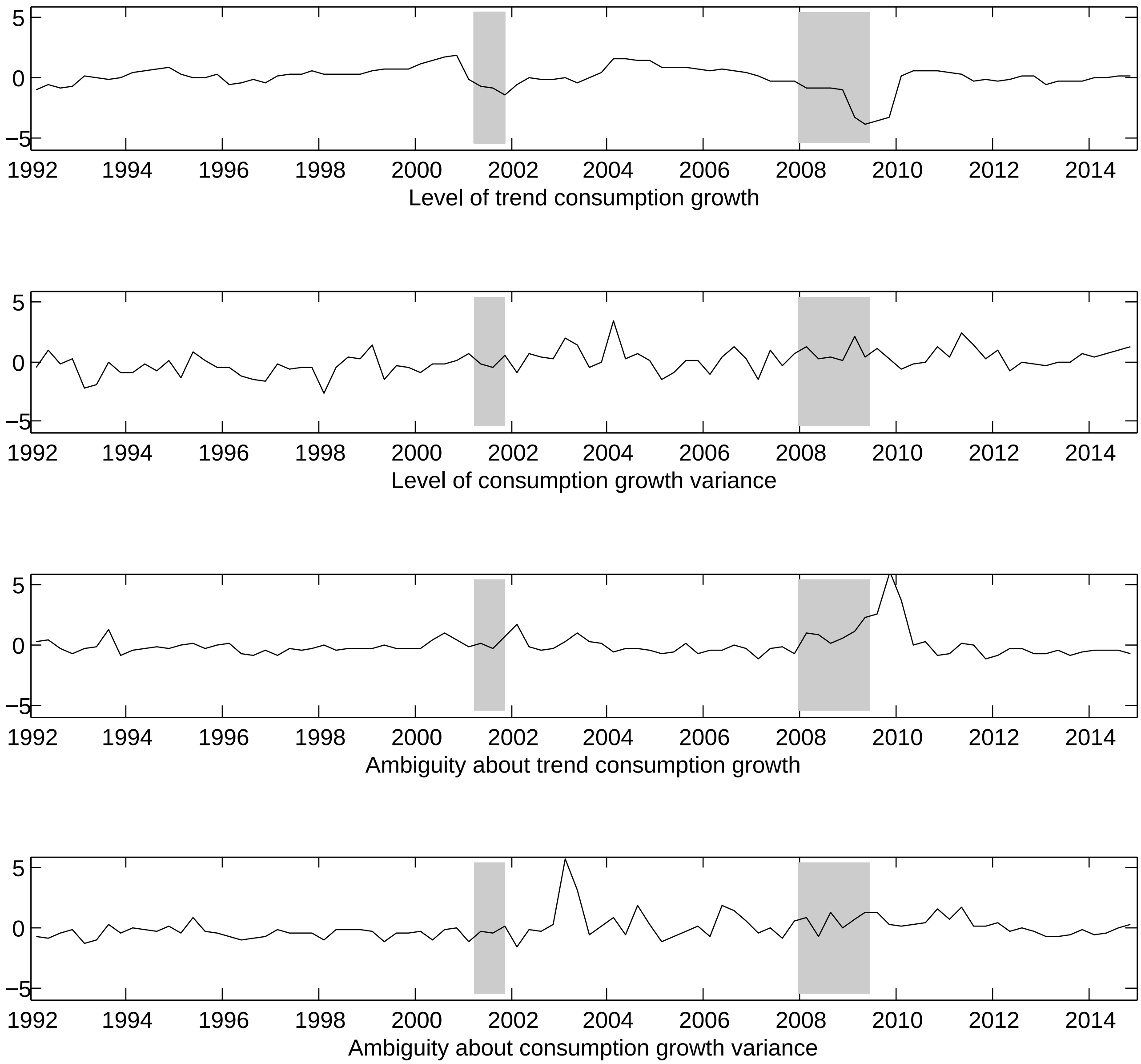
This table presents results of regressions as specified in Equation (15) in the data between 1992:Q1 and 2014:Q4 and on simulated data from our model. It reports regression coefficients $\beta(h)$, Newey and West (1987) t-statistics, and R^2s for return horizons of 1, 3, 6, 12, 24, and 36 months. The median values and 90% confidence intervals (in brackets) reported in columns 2-5 are from 10,000 simulation runs of equivalent length to the data.

Figure 1: **Point forecasts of consumption vs output growth**



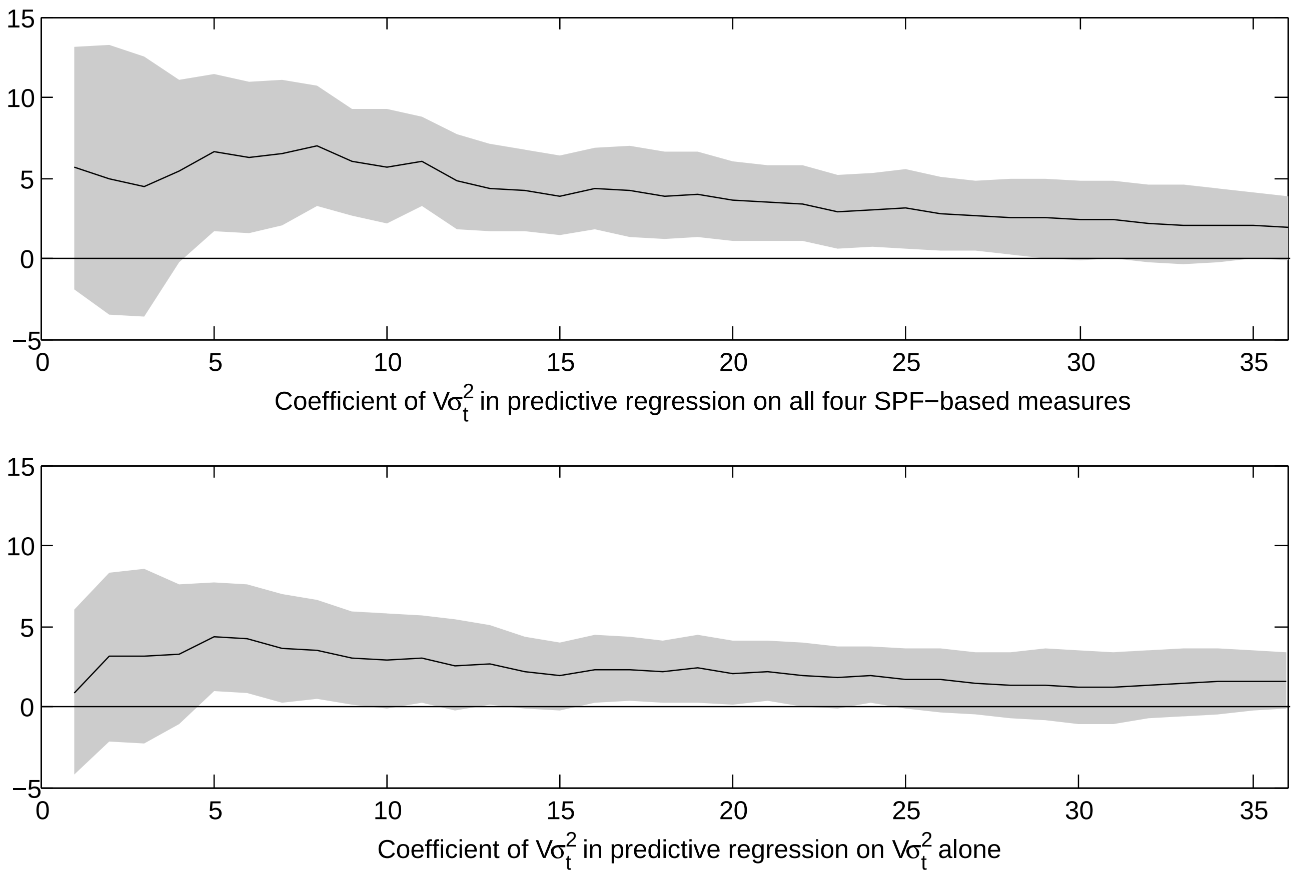
The figure shows the individual point forecasts of annual average over annual average growth in real GDP plotted against the individual point forecasts of annual average over annual average growth in real consumption. The sample comprises all analysts' forecasts between 1992:Q1 and 2014:Q4.

Figure 2: **Time series of state variables extracted from SPF data**



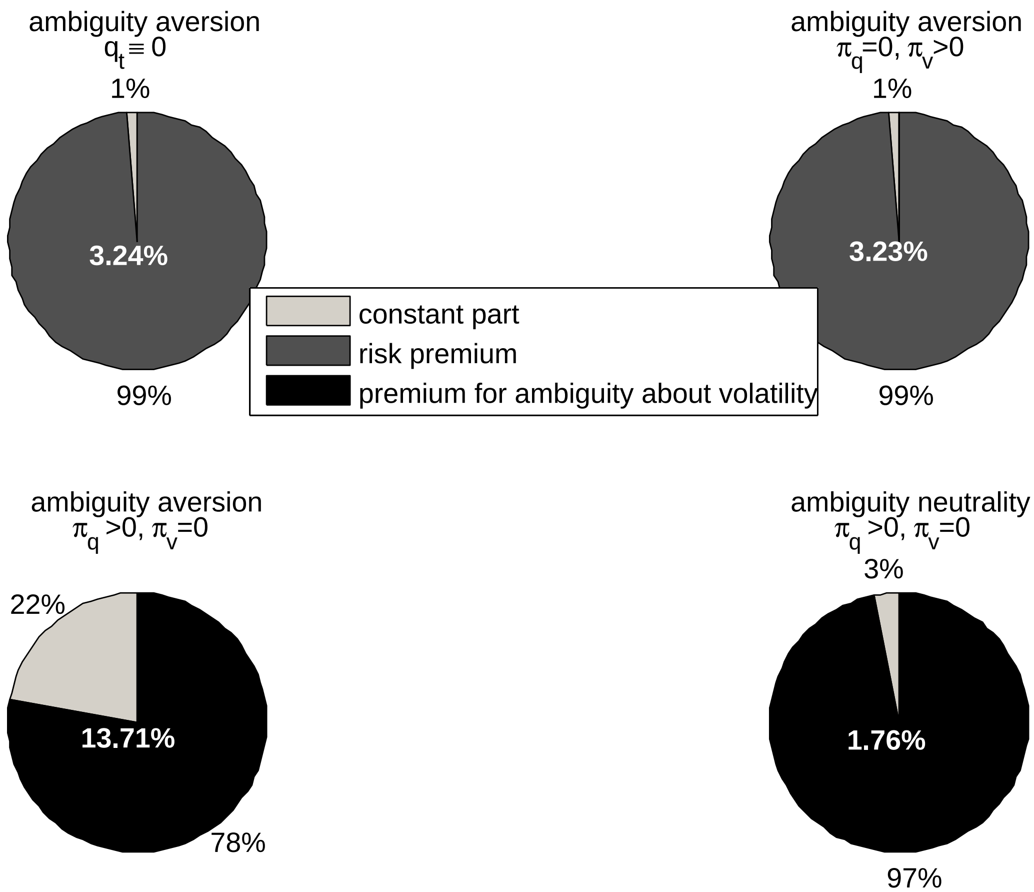
The figure shows plots of the time series for the state variables (from top to bottom) Ex_t, $E\sigma_t^2$, Vx_t, and $V\sigma_t^2$. All time series are standardized to have a mean of zero and a standard deviation of one. The sample period is from 1992:Q1 to 2014:Q4. Shaded areas represent NBER recessions.

Figure 3: **Predictability of excess returns over different horizons by $V\sigma_t^2$**



The figure shows the coefficients β_τ in the regressions $r_{d,t+\tau} - r_{f,t} = \alpha_\tau + \beta_\tau V\sigma_t^2 + \varepsilon_{t+\tau}$ plotted against τ (expressed in months). $r_{d,t+\tau}$ is the excess return on the CRSP stock market index (annualized and expressed in percent) and $V\sigma_t^2$ is normalized to have a mean of zero and a variance of one. In the upper panel, the full set of SPF-measures are used in the regression, while in the lower panel, only $V\sigma_t^2$ is used in the regression. The sample period is from 1992:Q1 to 2014:Q4. Shaded areas represent 90%-confidence bounds.

Figure 4: **Decomposition of the equity premium in the four specifications**



The figure shows the size of the different parts of the equity premium for the four specifications referred to in Table 9. The decomposition is according to Equation (11), where we introduce the unconditional average reference variance $\bar{v}$ for v_t and the unconditional average ambiguity about the volatility $\bar{q}$ for q_t. The white numbers in the center of the pies are the unconditional mean excess returns in annual terms as reported in Table 9.