Contingent Convertible Bonds in a General Equilibrium Model

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Abstract

This contribution discusses bail-in instruments in a general equilibrium model with a continuum of banks and households. We show that if banks are already endowed with bail-in securities, they are effective in mitigating debt overhang and increase household financial wealth. However, a pure private sector debt restructuring is not sustainable and needs public sector intervention. If a bail-in program is implemented it creates incentives for high default probability banks to invest. In comparison to other interventions such as an asset purchase program, implementing a bail-in program induces larger redistributions and therefore potentially larger social costs.

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1 Introduction

Widespread agreement exists that the stability of the financial sector is crucial for a sustainable development of the real economy. The financial crisis of 2007-09 has drastically illustrated the need for mechanisms which help prevent banks from becoming overlevered and suffering from debt overhang problems. One contribution to the regulatory reform of the financial sector which has obtained widespread political consent consists of encouraging or mandating banks to make use of bail-in instruments, such as so-called contingent convertible bonds. Following the initial first issuance of contingent convertible bonds by Lloyds in 2009, the market has matured and reaches roughly 100 billion Euro in outstanding volume as of year-end 2014. Due to the regulatory treatment as additional tier 1 (AT1) capital,[1] the market is expected to have a potential of up to 400–800 billion Euro.[2]

The basic economic function of bail-in instruments is to include an additional layer of loss-absorbing securities in the capital structure which automatically recapitalize a going-concern bank, or share part of the losses in a gone-concern situation. The idea of letting private investors absorb losses instead of a taxpayer-funded public bailout is compelling and politically opportune. However, from an analytical, economic perspective the intention to relocate the solution of a debt overhang from the public into the private sector raises at least four questions: First, is a private sector solution feasible without any intervention. Second, if not, then what kind of public intervention is needed to initiate a solution. Third, how does a bail-in program compare to other available options in terms of costs and efficiency. And fourth, what kind of incentives for participating bank come along with the substantial use of bail-in instruments. This contribution attempts to provide answers to these questions from a theoretical analysis of a general equilibrium model.

The use of a general equilibrium setup follows the reasoning that bail-in instruments are by definition instruments that aim at alleviating the debt overhang problem and thereby providing a positive stimulus on credit supply.

[1] See Capital Requirement Directive IV (2013), in particular Art.52,1(n) and Art.54 of Regulation (EU) No 575/2013.

[2] See e.g. Barclays Credit Research (2013) or RBS Macro Credit Research (2014). The maturity of the market is also documented by the fact that as of 2014 an index has been constructed by BoA and CDS contracts on contingent convertible bonds are available.

This positive stimulus is intended to feed back through higher investment activity on an improved asset quality of financial institutions. The existing literature sofar has put no emphasis on this feedback mechanism. To the best of our knowledge, all theoretical contributions analyze contingent convertible bonds by assuming an exogenously given asset value or process. By construction, such a setup is unable to capture a crucial aspect of the contingent convertible bond proposal.

We model a stylized general equilibrium economy which is populated by a continuum of banks which have profitable investment opportunities available. Banks have risky senior debt outstanding and need to raise the investment outlay through external capital markets. Therefore, junior lenders will have to share investment returns with senior debt holders which makes lending conditions prohibitive for some banks. Bail-in instruments, such as contingent convertible bonds (henceforth referred to by their nickname coco bonds) can alleviate the lending conditions by letting coco bond investors take part of the losses thereby improving the expected payoffs to junior lenders. Importantly, we analyze the impact of bail-in instruments in a general equilibrium model. Thus, the model also comprises households which on the one hand owe loans to the banks and on the other hand receive financial income from the securities issued by banks. Thereby the lending conditions and investment decisions of banks feed back through household wealth on the asset quality of banks. The endogeneity of lending conditions, investment decisions and asset quality is the crucial aspect of the general equilibrium and allows to analyze the impact on social welfare, the equilibrium efficiency, and the redistribution within the economy due to different interventions to mitigate the debt overhang problem. The workhorse of our contribution is a model recently put forward by Philippon and Schnabl (2013) where they analyze the efficiency of government interventions in the banking sector such as asset purchases against cash transfer and equity investments. We build upon their analysis in extending the model towards including bail-in instruments. Our main contribution is four-fold. First, under the assumptions that banks are already endowed with bail-in securities in their capital structure, we derive the equilibrium investment set and show that bail-in instruments have a significant positive effect on the equilibrium macroeconomic state through effectively mitigating the debt overhang. Second, if banks are not yet endowed with bail-in instruments, the analysis of their participation constraints in a debt restructuring is crucial. We show that in a full-information setting where the loss-absorption feature of coco bonds are fully priced, banks will have no incentive to participate in a coco bond program, since part of the loss-absorption benefit accrues to senior debt holders. In an asymmetric information setting where coco bond investors demand an average risk compensation, the participation set is non-empty but unstable. Thus, we find that a pure private sector solution is not sustainable and demands public sector interventions. Third, by analyzing the incentives provided by bail-in securities we show that coco bonds will always induce banks with a higher default probability (lower type) to invest as compared to an equally efficient intervention in form of an asset purchase program or outright cash transfer. In a general sense, the coco bond program provides risk-shifting incentives. Finally and importantly, by assessing the efficiency of the coco bond program, we provide evidence that the use of bail-in instruments creates larger redistributions and thereby potentially larger social costs as compared to equally efficient alternative interventions.

Our contribution adds to the growing literature on bail-in instruments and coco bonds in particular. Early contributions which point out the possibility to use coco bonds to improve the stability of the financial sector have appeared in the aftermath of the financial crisis and are due to Flannery (2005, 2009), Hart and Zingales (2009), Acharya et al. (2009), or Culp (2009). The potential benefits are also mentioned in policy recommendations such as Squam Lake Working Group (2009), Duffie (2009), and Landier and Ueda (2009). More general reviews about alternative mechanisms to public sector bail-outs are discussed in Maes and Schoutens (2010), Calomiris and Herring (2011), and more recently in Dewatripont (2014) and Dutordoir et al. (2014). Besides coco bonds, suggestions have been made how to structure bail-in instruments in an alternative way. Pennacchi et al. (2010) propose so-called Coercs securities, which is the abbreviation for call option enhanced reverse convertibles and suggests that a coco bond is combined with a call feature to buy back the newly create shares upon conversion. Bolton and Samama (2012) put forward so-called capital access bonds which they conceive as a capital line commitment.

Early contributions which provide a more detailed analysis of coco bonds have focused primarily on valuation and design issues. McDonald (2013)[3] stresses that the conversion should be made contingent on a firm-specific ratio such as the capital ratio as well as on a systemic ratio such as a banking sector index in order to avoid opportunistic behavior of individual banks. A similar concern is raised in the contribution by Sundaresan and Wang (2014)[4] who discuss the scope of manipulation if triggers are defined in terms of market variables. One of the first contributions that discuss capital structure decisions and risk-taking incentives in a continuous-time model framework is due to Albul et al. (2010) and Koziol and Lawrenz (2012).[5] In particular, Koziol and Lawrenz (2012) provided the first analysis to point out that coco bonds provide risk-shifting incentives. They show that conditions exist under which the asset substitution is anticipated and priced by bond investors and still the optimal choice for banks' equity holders. Thus, although being the rational choice by banks' owners it increases the probability of default and expected default costs. The incentive effect of coco bonds have later been further analyzed by Himmelberg and Tsyplakov (2012), Martynova and Perotti (2012), and Hilscher and Raviv (2014). The question if risk-shifting incentives can be empirically supported is difficult, but Hillion and Vermaelen (2004), Dam and Koetter (2012), Berg and Kaserer (2012), and Vallee (2013) provide evidence which is consistent with theoretical predictions and underlines the importance of incentives underlying specific financing arrangements. The question of optimal participation in the use of coco bond programs has only rarely been addressed, one notable exemption being Crummenerl et al. (2014) who point out that banks may be reluctant to the use of coco bonds despite their positive impact on lending conditions. Finally, a significant part of the contributions focus on the pricing of coco bonds. Pennacchi (2010), Madan and Schoutens (2011), Glasserman and Nouri (2010), and Chen et al. (2013) put forward sophisticated valuation frameworks with jump-diffusion models for the underlying asset value and focus on partial continuous conversion and tail-risks.

Although the literature sofar has provided important insights with respect to the use of bail-in instruments, it suffers from mainly two shortcomings. First, models analyze the decisions of a representative bank and neglect the

[3] The working paper was available as of 2009.
[4] The working paper was available as of 2010.
[5] The working paper underlying Koziol and Lawrenz (2012) was available as of 2009.

heterogeneity across financial institutions. Second, and more importantly, models are set up in partial equilibrium, which means that the random asset quality is taken as exogenously given. Models differ in the sophistication of the underlying randomness which ranges from a discrete static distribution to complex continuous-time jump-diffusion processes. However, they share the property that they are not affected by the financing decisions. Thus, results have the status of comparative-static comparisons, in particular when considering risk shifting incentives. Our contribution wants to improve along these two dimensions.

The remainder of the article is structured as follows. Section 2 introduces the model framework, while section 3 describes the equilibrium investment decisions. Section 4 discusses alternative programs to mitigate the debt overhang problem. Importantly, section 5 provides the assessment of programs along incentive provision and efficiency. Section 6 concludes. Proofs are in the Appendix.

2 Model setup

We are interested in setting up a model which is able to capture feedback effects of introducing alternative programs to alleviate the debt overhang problem. In particular, our focus in on analyzing the endogenous effect on expected aggregate asset values in the economy. Therefore, we introduce the following general equilibrium model, which follows the model put forward in Philippon and Schnabl (2013). They consider a model setup in discrete time that has two periods, three dates t_0, t_1 and t_2. The economy is populated by financial institutions, which are generically called banks and households. Since the financing decisions of the corporate sector is not within the scope of our focus, we integrate the corporate sector into the household sector. So households owe loans to banks, while at the same time being also debt holder, equity holder and lender to the financial sector. The loan repayments to banks depend upon households' ability to make contractual payments, which in turn depend among others on the financial income of households, thereby endogenizing banks' investment behavior.
In order to be able to capture heterogeneity, it is assumed that the economy consists of a continuum of households as well as banks, which will differ in

the quality of their existing assets as well as in their investment opportunity. All agents are assumed to be risk-neutral. We turn to the description of each sector in more detail in the next subsections.

2.1 Banks

Assume a continuum of banks with mass 1. In t_0, banks are endowed with existing assets and a given liability structure. Assets-in-place are modeled as a binomial random variable A assuming $a > 0$ in the good state and 0 otherwise, $A \in \{0, a\}$. Payoffs occur in t_2. In the intermediate time t_1, banks get a new investment opportunity. While banks are identical in t_0, they will be heterogenous as of t_1, where the type of a bank will become known to financial investors. Banks' type is two-dimensional and denoted by the pair (τ, v). The first element τ influences the success probability of assets, while the second element v refers to the quality of the new investment. More specific, we denote the success probability for existing assets as p, and assume that it depends upon the idiosyncratic type τ as well as an aggregate macroeconomic state which we denote by α. Thus, we define

$$p(\alpha, \tau) = Prob\{A = a \,|\, \alpha, \tau\},$$

and therefore obviously $1 - p(\alpha, \tau) = Prob\{A = 0 \,|\, \alpha, \tau\}$. Letting $p(\alpha, \tau)$ be increasing in both arguments, i.e. $p'_\alpha > 0$ and $p'_\tau > 0$, gives meaning to the type and the macro state in the sense that larger values for α and τ imply higher success probabilities. The macro state α will play a decisive role in the following as it provides the endogeneity channel from investment decisions on asset quality. Assuming a general feedback from a good macroeconomic environment on decreasing default rates is an empirically sensible choice. While α will be determined in equilibrium, the distribution of type τ is exogenous to the model and assumed to be given by the continuous distribution measure $\tau \sim F_\tau$ defined over the admissible range $\mathcal{T} = [\tau_{min}, \tau_{max}]$. From this it follows that the expected value of assets-in-place in the economy is equal to

$$\mathbb{E}(A|\alpha) = \int_{\mathcal{T}} a \cdot p(\alpha, \tau) \, \mathrm{d}F_\tau = a \cdot \bar{p}(\alpha),$$

where $\bar{p}(\alpha) = \int_{\mathcal{T}} p(\alpha, \tau) \, \mathrm{d}F_\tau$ denotes the average or aggregate success probability which only depends upon the macro state α. From this, α is implicitely defined through the equality

$$\alpha = a \cdot \bar{p}(\alpha). \tag{1}$$

The macro state is determined through the expected value of assets-in-place which is economically meaningful, and from implicit differentiation $\bar{p}'_\alpha = (1 - \bar{p}(\alpha))/a > 0$ consistent with the technical assumption above.
The second dimension of heterogeneity across banks is the quality of their new investment project in t_1. Every bank has the possibility to invest in a project which requires the same amount of investment, denoted I and which provides a riskfree, sure return v. However, the gross return v to the project is drawn randomly from the interval $\mathcal{V} = [0, \nu]$ with a continuous distribution function $v \sim F_v$ over $\mathcal{V}$. Thus, heterogeneity of banks occurs in the state space $\Omega = \mathcal{T} \times \mathcal{V}$. A plane, in which each bank is located by its type pair (τ, v).

Let the decision to invest be represented by an indicator function denoted by $1_\mathcal{I}$ which assumes 1 if the bank invests. The individual investment decisions will be crucially important for the determination of the equilibrium. In general, it will depend upon the type of the bank as well as the macroeconomic state α, so $1_\mathcal{I}$ will be a function of (τ, v, α).[6] Total assets of a bank can thus be denoted as: $y = A + 1_\mathcal{I}\, v$.
Note that every bank for which $v > I$ has a riskfree positive NPV project, which according to standard investment decisions should be taken absent any frictions.[7] However, we precisely consider banks facing a debt overhang problem. Therefore, we assume that initially all banks have financed their assets-in-place by having issued senior unsecured debt with a face value of d. Due to existing debt covenants, the investment I cannot be raised through asset sales, so that the bank needs to raise I through new junior lenders in the market by issuing debt claims with a notional amount of l.[8] The payoffs, which are distributed in t_2 to banks' claimholders follow the usual seniority

[6] More precisely, we will in later sections use $\mathcal{I}$ to denote the investment set which among others depends upon α, so $1_\mathcal{I}$ is short-hand notation for $1_{(\tau,v)\in\mathcal{I}(\alpha)}$.

[7] Assuming a riskfree project is for ease of exposition, as introducing a risky project payoff would not add substantially more insights for our purposes

[8] Allowing the bank to have a non-zero cash balance would again not add insights to the model, since we can then consider I to be the investment needed on top of available cash.

structure where senior debt holder are paid off before junior lenders, and equity holders being the residual claimants. Denoting payoffs by y^d, y^l and y^e respectively, they are formally defined as

$$y^d = \min(y, d); \qquad y^l = \min(y - y^d, l); \qquad y^e = y - (y^d + y^l).$$

For the debt overhang to occur, senior debt d needs to be risky even if investment occurs, which means that the restriction $a > d > \nu$ has to be imposed. Without investment, payoffs are $y^d = \{0, d\}$ and $y^e = \{0, a - d\}$, while with investment this is $y^d = \{v, d\}$, $y^l = \{0, l\}$ and $y^e = \{0, a - d + (v - l)\}$. In section 4.2 we will extend the setup by introducing claims of coco bond investors.

2.2 Households

As for the banks, we assume a continuum of households with mass 1 which are identical in t_0. Initially, we endow each household with the same portfolio of financial claims to the banks, i.e. households hold senior debt as well as an equity share. In t_1, they can make an additional investment by lending to those banks that take the project. Thus, in t_2, households obtain the financial income $y^d + y^l + y^e$. In order to be able to lend, each household is equipped with an endowment ϵ in t_1. The endowment can be used for lending to banks or carried forward for consumption in t_2 with a safe storage technology. The storage technology has an interest rate normalized to zero. If households lend, they provide an amount l^* in t_1 for the risky payoff y^l in t_2. Furthermore, households receive a random income shock in t_2, which may be interpreted as risky labor income or unexpected consumption shock. The income shock is denoted w. In sum, total household income, denoted ω is therefore:

$$\omega = (\epsilon - l^*) + w + (y^d + y^l + y^e); \qquad \text{with: } w \sim F_w, \tag{2}$$

where F_w is the continuous distribution function of the income shock over the interval $\mathcal{W} = [w_{min}, w_{max}]$. Thus, in t_2 households are heterogenous with respect to their total available income. Since we think of the private sector as including the corporate sector, household income determines the repayments to the financial sector. Thus, we consider households to owe all loans to the banks with the face value of a at t_2. In case that available income of households is insufficient to repay the notional amount, i.e. for $\omega < a$, default occurs and banks recover ω. Note that we assume the bankruptcy process to be efficient in the sense that it does not cause direct or indirect deadweight losses. So, individual payments to banks are $\min(\omega, a)$ and the aggregate flow from the household to the financial sector is therefore $\int_{\mathcal{W}} \min(\omega, a)\,\mathrm{d}F_w$. The model is closed by linking the household debt to banks' assets through endogenizing the macro state α. Therefore, as in Philippon and Schnabl (2013), we assume that the aggregate flow of the private sector determines α implicitly through the equality

$$\alpha = \int_{\mathcal{W}} \min(\omega, a)\,\mathrm{d}F_w. \tag{3}$$

Together with equation (1), this means that the aggregate private sector payments is required to be equal to the expected aggregate asset values of banks via the macro state α, i.e.

$$\int_{\mathcal{T}} a \cdot p(\alpha, \tau)\,\mathrm{d}F_\tau = \alpha = \int_{\mathcal{W}} \min(\omega, a)\,\mathrm{d}F_w. \tag{4}$$

To recognize the feedback effect, note that from equation (2), ω consists of financial income y that is determined through the investment decision, which in turn will depend upon the asset success probability $p(\alpha, \tau)$ because of its impact on lending conditions. Thus, the conditions in the lending market (in particular the debt overhang problem) will determine the macro state in a general equilibrium way. We turn to the description of the equilibrium in the next section.

3 Investment decisions and equilibrium state

3.1 Aggregation

Due to the continuum of banks and households, we need to distinguish between individual and aggregate variables. In general, we will use a bar to indicate the aggregate values, as e.g. already introduced for $\bar{p}(\alpha)$. With this convention, aggregate bank and household income is

$$\bar{y} = \bar{y}^d + \bar{y}^l + \bar{y}^e = \bar{A} + \iint\limits_{\Omega} 1_{\mathcal{I}} \cdot v \, \mathrm{d}F_{\tau,v} = \bar{A} + \bar{v}_{\mathcal{I}} \tag{5a}$$

$$\bar{\omega} = (\bar{\epsilon} - \bar{l}^*) + \bar{y} + \int_{\mathcal{W}} w \, \mathrm{d}F_w, \tag{5b}$$

where for ease of notation, we introduced the notation $\bar{v}_{\mathcal{I}}$ for aggregate gross investment return given by $\bar{v}_{\mathcal{I}} = \iint_{\mathcal{I}} v \, \mathrm{d}F_{\tau,v} = \iint_{\Omega} 1_{\mathcal{I}} \cdot v \, \mathrm{d}F_{\tau,v}$ for the second term in (5a).[9] Note that for variables which are assumed to be identical over banks and households (i.e. which are independent of λ, v, w), the aggregate variables equal individual variables due to the fact that the continuum has a mass normalized to 1.

Aggregate investment at t_1 is the sum over I for all investing banks, so with similar notation, this is $\bar{I}_{\mathcal{I}} = \iint_{\mathcal{I}} I \, \mathrm{d}F_{\tau,v}$. The entire aggregate investment sum is financed by households who lend the amount $\bar{l}^*$ for a junior debt claim with notional amount $\bar{l}$. In order to make lending by households feasible, their t_1 endowment is assumed to be larger than aggregate investment, i.e. that households have excess savings at time t_1: $\epsilon > \bar{I}_{\mathcal{I}}$.

Junior lenders will want to break even, which means that the amount lent has to equal the expected payoff of the claim, i.e. it has to satisfy $l^* = \mathbb{E}(y^l)$. The budget constraint then implies that $\bar{I}_{\mathcal{I}} = \bar{l}^*$.

3.2 First-best investment set

The first-best investment decision in this setup is almost trivial. The first-best follows from firm value maximization of banks at t_1. Expected firm value is $\mathbb{E}(y) - \mathbb{E}(y^l)$. The first term is equal to $\mathbb{E}(A) + 1_{\mathcal{I}} \, v$, while the second term is l^*, which is according to the budget constraint $l^* = I$. Therefore, the bank maximizes $\mathbb{E}(A) + 1_{\mathcal{I}} \, (v - I)$, which has the obvious solution $1_{\mathcal{I}} = 1$ for all $v > I$. This determines the investment set $\mathcal{I}^{fb} = \{(\tau, v) | v > I\}$, which is independent of type τ. We label it with the superscript fb to denote the first-best set. All banks having a positive NPV project invest irrespective of the quality of their assets-in-place. We next turn to the investment set under debt overhang.

[9] Note also that we write $\mathrm{d}F_{\tau,v}$ for the joint distribution given by the marginal distributions F_τ and F_v.

3.3 Investment set under debt overhang

Consider banks to maximize equity holders value at t_1. As banks learn their type in t_1, the objective is to choose investment such as to maximize $\mathbb{E}(y^e|(\tau, v))$.[10] From the definition of y^e, the optimization is

$$\max_{1_{\mathcal{I}}} \{p(\alpha, \tau) \cdot (a - d + 1_{\mathcal{I}}(v - l))\},$$

because the payoff to equity holders is zero in the downstate. As the maximization is conditional on (τ, v), i.e. pointwise, it is straightforward to see that the solution is $1_{\mathcal{I}} = 1$ if and only if $v > l$. However, note that l is the notional amount pledged to junior lenders. From the above discussion, junior lenders' lending decision follows the break-even constraint, i.e. they will provide an amount satisfying $l^* = \mathbb{E}(y^l)$. So the budget constraint $I = l^*$ and the definition of y^l determines the required notional amount l. It is easy to see that

$$I = \mathbb{E}(y^l) = p(\alpha, \tau) \cdot l, \qquad \text{leads to:} \quad l = \frac{I}{p(\alpha, \tau)}.$$

The inverse of the success probability is the premium demanded by junior lenders to compensate for the losses on the downstate. Given that τ is known in t_1 to potential investors, banks with lower quality assets face tighter credit conditions. Note that due to $1/p > 1$, the first-best solution will never be obtained. Plugging in the equilibrium principal l in the maximization problem, we have $\max_{1_{\mathcal{I}}} \left\{p(\alpha, \tau) \cdot \left(a - d + 1_{\mathcal{I}}(v - \frac{I}{p(\alpha,\tau)})\right)\right\}$ and again, it is obvious that investment is optimal if and only if $v > \frac{I}{p(\alpha,\tau)}$. Since this depends upon v as well as τ, the optimal investment set is therefore determined by

$$\mathcal{I}^0 = \left\{(\tau, v) \,|\, p(\alpha, \tau) > \frac{I}{v}\right\}. \tag{6}$$

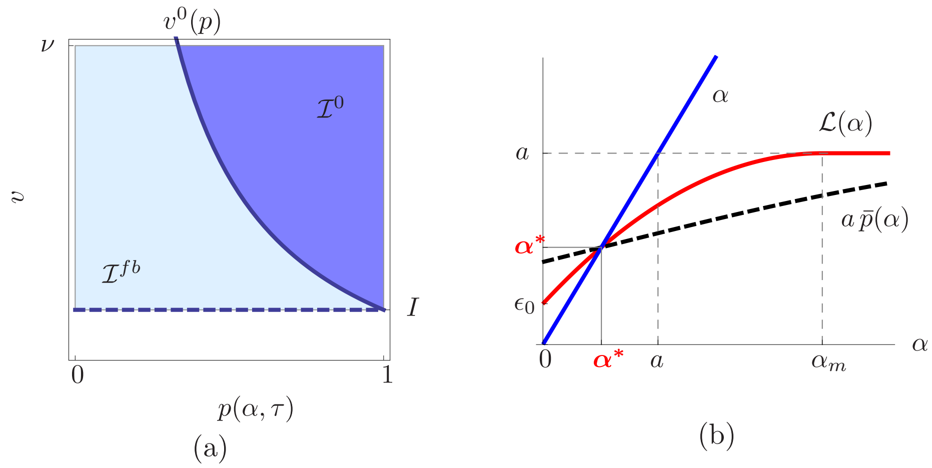
We label set $\mathcal{I}$ with the superscript 0 to indicate the benchmark debt overhang situation against we compare the introduction of programs for allevi-

[10] As discussed by Philippon and Schnabl (2013) the assumption that bank managers maximize equity value is not entirely consistent in the model setup as equity holders (i.e. households) are also debt holders and would therefore internalize valuation effects on debt claims. However, since maximization of equity value is an realistic assumption, this simplification facilitates the model without having a substantial impact on results.

ating the financing constraints. For further reference, when introducing a program $\mathcal{P}$, we will denote the corresponding investment set as $\mathcal{I}(\mathcal{P})$. Investment sets $\mathcal{I}^{fb}$ and $\mathcal{I}^0$ are illustrated in Figure 1 as subsets of Ω, i.e. regions in the two-dimensional plane. While $\mathcal{I}^{fb}$ is the region above the dashed

Figure 1: Investment sets and equilibrium macroeconomic state.

Panel (a) illustrates the investment set $\mathcal{I}^0$ (dark shaded area) in the two-dimensional state-space $\mathcal{T} \times \mathcal{V}$ under the benchmark debt overhang problem. The solid line represents the boundary of the investment set $v^0(p) = I/p$. The dashed line represents the boundary of the first-best investment set at $v = I$. In panel (b), the solid red line is the graph of $\mathcal{L}(\alpha) = \int_{\mathcal{W}} \min(\epsilon + w + \alpha + \overline{\triangle v_{\mathcal{I}}}, a)\, \mathrm{d}F_w$. ϵ_0 is the point defined by $\mathcal{L}(0) = \int_{\mathcal{W}} \min(\epsilon + w, a)\, \mathrm{d}F_w$. The blue straight line is just α. The dashed, thick line is the graph of $\bar{p}(\alpha) \cdot a$. The equilibrium α^* is determined by the requirement $\alpha = \mathcal{L}(\alpha) = a\,\bar{p}(\alpha)$.



line at $v = I$, the investment region under debt overhang is the dark-shaded region to the right and above the solid line, indicated by the boundary $v^0(p) = I/p$. Banks with lower success probability will only take sufficiently profitable projects. The difference in the investment region, i.e. $\mathcal{I}^{fb} \setminus \mathcal{I}^0$ is the loss which occurs due to debt overhang.

3.4 Equilibrium macroeconomic state

The investment sets derived above depend upon the macroeconomic state α. Thus, to determine the general equilibrium investment, we turn to the derivation of the equilibrium macro state. Using the aggregate values, we first state the balance sheet identity for banks, which is at time t_2

$$\bar{p}(\alpha)\bar{A} + \iint\limits_{\mathcal{I}} v \, \mathrm{d}F_{\tau,v} = \bar{y}^d + \bar{y}^l + \bar{y}^e$$

The LHS represents aggregate bank income, while the RHS is the liabilities side, or the payments to the private sector. Using the aggregate household income from (5b), aggregate financial income $\bar{y}$ is replaced by the LHS above, to yield

$$\bar{\omega} = (\bar{\epsilon} - \bar{l}^*) + \int_{\mathcal{W}} w \, \mathrm{d}F_w + \bar{p}(\alpha)\bar{A} + \iint\limits_{\mathcal{I}} v \, \mathrm{d}F_{\tau,v}$$

Recall that the budget equation required $\bar{l}^* = \bar{I}_{\mathcal{I}} = \iint_{\mathcal{I}} I \, \mathrm{d}F_{\tau,v}$. Combining $\bar{l}^*$ with the last term is the aggregate net investment which we denote with similar notation as $\overline{\triangle v_{\mathcal{I}}} = \iint_{\mathcal{I}} (v - I) \, \mathrm{d}F_{\tau,v}$. Further from (1), $\bar{p}(\alpha)\bar{A}$ equals α. Due to the assumption that the continuum has mass 1, aggregate variables can be broken down to individual variables, so that each household income is equal to

$$\omega = \epsilon + w + \alpha + \overline{\triangle v_{\mathcal{I}}} \tag{7}$$

Combining this with condition (3) by plugging in ω yields the time t_2 equilibrium value of the macroeconomic state α through the equality

$$\alpha = \int_{\mathcal{W}} \min(\epsilon + w + \alpha + \overline{\triangle v_{\mathcal{I}}},\, a) \, \mathrm{d}F_w. \tag{8}$$

The RHS represents the equilibrium aggregate loan repayments as function of α, so we abbreviate it as $\mathcal{L}(\alpha)$. An explicit solution for the equation $\alpha = \mathcal{L}(\alpha)$ is not directly available, but inspection of the equation shows that at t_2 for a given investment set $\mathcal{I}$ the $\mathcal{L}(\alpha)$ is increasing in α with a slope smaller or equal to 1, since the derivative is the distribution function $F_w(.) \in [0, 1]$. Given that the LHS obviously has slope one, a meaningful equilibrium requires the RHS to have a positive value at $\alpha = 0$. Therefore, the assumption that $\mathcal{L}(0) = \int_{\mathcal{W}} \min(\epsilon + w,\, a) \, \mathrm{d}F_w > 0$ is imposed. In economic terms, the assumption requires that on average the income shock at t_2 does not wipe out completely the initial endowment, ruling out that house-

holds have on average a negative net worth in default.[11] The equilibrium value α^* satisfying 8 is illustrated in panel (b) of Figure 1. The solid (red) line shows the graph of $\mathcal{L}(\alpha)$, which increases from $\epsilon_0 = \mathcal{L}(\alpha) > 0$ monotonically to approach a for $\alpha > \alpha_m = a - (\overline{\triangle v_{\mathcal{I}}} + \epsilon + w_{min})$.[12] The thick, dashed (black) line represents the graph of equation (1) showing the expected aggregate asset value $\mathbb{E}(A|\alpha)$ being an increasing function of α. Due to the endogeneity requirement from equation (4), the equilibrium value α^* is determined by the requirement $\alpha = \mathcal{L}(\alpha) = a\,\bar{p}(\alpha)$.
With $\overline{\triangle v_{\mathcal{I}}}$ being given at t_2, the comparative statics of α^* with respect to initial endowment ϵ and net investment surplus $\overline{\triangle v_{\mathcal{I}}}$ are found from implicit differentiation. Both partial derivatives are positive, i.e.

$$\frac{\partial \alpha}{\partial \epsilon} = \frac{\partial \alpha}{\partial \overline{\triangle v_{\mathcal{I}}}} = \frac{F_w(\cdot)}{1 - F_w(\cdot)} > 0$$

Thus, in particular a higher net investment surplus leads to an increased equilibrium macro state. The investment surplus in t_2 is determined through the investment decision taken one time period earlier in t_1 and which depends upon the investment region $\mathcal{I}$. So, the time t_1 equilibrium takes into account the t_2 feedback effect from investment on the equilibrium macro state α. Therefore, in line with our notation in the previous section, we can label the investment set in (6), by a star to indicate the equilibrium set $\mathcal{I}^{0,*} = \big\{(\tau, v) \,|\, p(\alpha^*, \tau) > \frac{I}{v}\big\}$.[13] This concludes the description of the equilibrium.

4 Programs to mitigate the debt overhang

This section discusses three different ways how to mitigate the debt overhang problem in the basic model setup. First, for reference we briefly show how pure cash transfers affect the equilibrium. Second, we consider an asset purchase program. And third, we analyze the effects of introducing coco bonds.

[11] Furthermore, the assumption rules out multiple equilibria. Refer to Philippon and Schnabl (2013), p. 12 for a more detailed discussion.

[12] The upper boundary α_m is the largest value for α such that even the household with the lowest income shock (i.e. w_{min}) does not default.

[13] Note that the first-best investment set $\mathcal{I}^{fb}$ is independent of α.

4.1 Cash transfer program

Within the pure cash transfer program, the government is able to raise cash by imposing taxes on households in t_0 and distribute the aggregate amount to banks at time t_1. To begin with, assume that this redistribution can be organized in an efficient way, so that no deadweight losses occur. Let $k \geq 0$ denote the cash injection granted to an individual bank, and thus $\bar{k}$ being the aggregate cash redistributed to the financial sector. The tax is levied from each household so that the initial endowment is reduced to $\omega - k$. Note that due to the assumption of excess savings and the fact that the government has no incentive to redistribute more than I, households always retain a positive endowment, i.e. $\omega - k > 0$ at t_0. For banks, the cash transfer relaxes their budget constraint. Instead of lending I, they only need to raise $I - k$ from junior lenders. Since lenders want to break even, it is straightforward to see that the notional amount is determined by the requirement $\mathbb{E}(y^l) = I - k$, leading to $l = \frac{I-k}{p(\alpha,\tau)}$. Taking the relaxed budget constraint into account, the banks' investment decision solves the maximization of equity value which is given by

$$\max_{1_{\mathcal{I}}} \left\{ p(\alpha,\tau) \cdot \left(a - d + 1_{\mathcal{I}} \left(v - \frac{I-k}{p(\alpha,\tau)} \right) + (1 - 1_{\mathcal{I}})k \right) \right\},$$

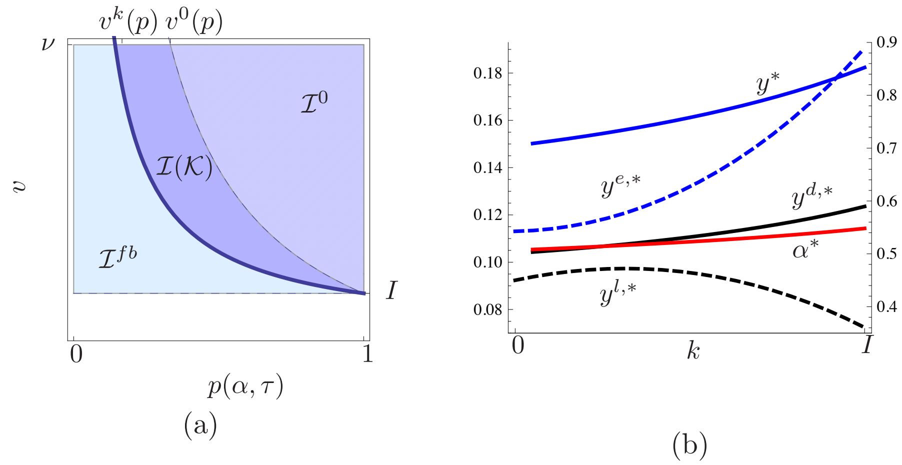
leading to $1_{\mathcal{I}} = 1$ if and only if $v - k - \frac{I-k}{p(\alpha,\tau)} > 0$. So, the investment set under the cash transfer program (denoted by $\mathcal{K}$), is therefore

$$\mathcal{I}(\mathcal{K}) = \left\{ (\tau, v) \,|\, p(\alpha,\tau) > \frac{I-k}{v-k} \right\}. \tag{9}$$

Writing the boundary of $\mathcal{I}(\mathcal{K})$ as a function of p, denoting it as $v^k(p) = k + \frac{I-k}{p}$ and comparing it to the previous boundary $v^0(p)$ (following equation (6)), it can be seen that both boundaries are equal in $p = 1$, but that $v^k(p)$ has a smaller slope for each $p \in (0,1)$ than $v^0(p)$, implying that the boundary lies always below of $v^0(p)$ in the state space. Figure 2 (a) shows the extended region $\mathcal{I}(\mathcal{K})$ which is to the upper right of the bold line indicating the boundary $v^k(p)$. The additional investment which occurs due to the cash transfer is shown as the dark blue region between the former boundary $v^0(p)$ (dashed line) and the bold line $v^k(p)$. Overall, more banks invest. Note that for $k = I$, i.e. if the government transfers the entire investment

Figure 2: Investment sets under cash transfer program.

Panel (a) illustrates the investment set $\mathcal{I}(\mathcal{K})$ under a cash transfer program in the two-dimensional state-space $\mathcal{T} \times \mathcal{V}$. The solid line represents the boundary of the investment set $v^k(p) = k + (I - k)/p$. The downward-sloping dashed line $(v^0(p))$ represents the boundary of the investment set without any intervention. The dark shaded area illustrates the set of additionally investing banks due to the cash transfer. Panel (b) provides comparative-statics for an increasing amount of cash transfer k. The solid red line (α^*) shows the equilibrium macro state, The solid blue line y^* represents household wealth, while $y^{e,*}, y^{d,*}$, and $y^{l,*}$ are equilibrium values for equity holders, debt holders, and junior lenders respectively.
Dashed lines are measured on left-hand axis, solid lines are on right-hand axis.



amount to banks than, the first-best solution would be obtained, because the investment condition $v - k - \frac{I-k}{p(\alpha,\tau)} > 0$ reduces to $v - I > 0$.
Panel (b) in Figure 2 shows the aggregate equilibrium macro state α^* for $k \in (0, I)$ (solid red line) which is monotonically upward sloping, verifying the interpretation that the cash transfer mitigates debt overhang and thereby has a positive general equilibrium effect on the economy. The graph denoted by $y^* = \mathbb{E}(y)$ shows the expected aggregate firm value which increases due to the improved investment set. In the same sense, graphs denoted by $y^{e,*}$, $y^{d,*}$, and $y^{l,*}$ show the impact of k on the expected aggregate equity, senior debt, and junior debt values respectively. While junior debt declines (since less lending is needed), aggregate equity and senior debt values increase.

Overall, the cash transfer shows that if renegotiation were possible then the debt overhang could be solved. However, it is unrealistic to assume that the government can easily redistribute cash from taxpayers to banks ex ante for at least two reasons. First, even if it were politically feasible, deadweight losses due to distorting taxes are likely to occur. Second, in order to minimize the burden on taxpayers, governments will inject cash only in return for a stake in the financial sector. One possibility is to buy assets from banks against cash in the sense of the Troubled Asset Relief Program (TARP) in the US 2008.[14] Philippon and Schnabl (2013) focus on the question which way to recapitalize banks is most efficient. In this contribution, our focus is on the impact of using coco bonds for the mitigation of the debt overhang problem. Therefore, the next section discusses the way how coco bonds are introduced in the model.

4.2 Coco bond program

As discussed in the introduction, coco bonds, and more generally contingent capital has been issued in a variety of different specific forms. The prevalent form are currently (principal) write-down bonds which means that the notional amount is (entirely or partly) lost once the trigger has been hit. Thus, the main economic function of coco bonds is their loss absorption feature in adverse states. The very intention of coco bonds is to combine the debt-like fixed payoff characteristic in good states with equity-like loss-absorbing characteristic in bad states. So it seems economically plausible, to model coco bonds in the following way.

We start by assuming that a bank has a capital structure at t_0 which already includes coco bonds. In line with previous notation, we label the nominal value of coco bonds as c and the corresponding payoff for coco bond holders by y^c. Coco bonds replace part of the senior debt, so we label the principal of senior debt in this context as d^c. While the payoff function to senior debt and junior lenders is still $y^d = \min(y, d^c)$ and $y^l = \min(y - y^d, l)$, we now have also $y^c = \min(y - (y^d + y^l), c)$. Equity as residual claimant is therefore $y^e = y - (y^d + y^c + y^l)$. The definition of y^c reflects the assumption that

[14]TARP has been made possible through the Emergency Economic Stabilization Act (EESA) of 2008. See `www.treasury.gov/initiatives/financial-stability/about-tarp/` for the official information, or Bayazitova and Shivdasani (2012) and Duchin and Sosyura (2014) for two academic accounts.

coco bond holders cover losses before junior lenders are affected. More precisely, we need to put restrictions on admissible values to make the setup interesting. We assume the inequality

$$a > (d^c + c) > \nu > d^c > \nu - I,$$

which ensures that (i) the bank is not overlevered up to the point where default occurs in both states ($a > (d^c + c)$), (ii) the bank has sufficient coco bonds to have an impact on lending conditions ($(d^c + c) > \nu > d^c$), and (iii) the bank has not issued coco bonds up to the point where they make junior lenders risk-free ($d^c > \nu - I$). With this restriction the payoffs in the two states of our model to claimholders when investment took place are:

$$y^d = \{\min(v, d^c),\, d^c\}; \quad y^c = \{0, c\}; \quad y^l = \{\max(v - d^c, 0),\, l\}; \quad y^e = \{0,\, a - (d^c + c) + (v - l)\}. \tag{10}$$

Without investment they reduce to: $y^d = \{0, d^c\}$, $y^c = \{0, c\}$, and $y^e = \{0, a - (d^c + c)\}$ being similar as in the scenario without coco bonds. From (10) we see that in case of investment, coco bonds can relax lending conditions as their loss-absorption feature will leave junior lenders with a positive payoff even in the bad state if the investment project is sufficiently profitable or if banks have substituted enough senior bonds for coco bonds. The condition $\nu > d^c$ ensures that there exists a positive mass of banks for which this will happen. The relaxation of the lending condition will have an impact on the investment decision, so we now turn to the determination of the investment set.

As in the preceding sections, the derivation of the investment set under coco bond financing, which we denote by $\mathcal{I}(\mathcal{C})$ follows the principle of equity value maximization. Maximizing the expected equity holders' payoff $\mathbb{E}(y^e)$ is the optimization program

$$\max_{1_{\mathcal{I}}} \left\{ p(\alpha, \tau) \cdot (a - (d^c + c) + 1_{\mathcal{I}}(v - l) \right\},$$

for which we have $1_{\mathcal{I}} = 1$ if and only if $v > l$. The impact comes directly through the nominal amount l of junior lenders, so the determination of l is crucial. Junior lenders break-even condition $I = \mathbb{E}(y^l)$ leads to

$$\begin{aligned} I &= \mathbb{E}(y^l) = p(\alpha,\tau)\cdot l + (1-p(\alpha,\tau))\cdot(v-d^c)^+ \\ \Leftrightarrow \quad l &= \frac{I-(1-p(\alpha,\tau))(v-d^c)^+}{p(\alpha,\tau)}, \end{aligned} \tag{11}$$

where $(v-d^c)^+$ is shorthand for $\max(v-d^c,0)$. Plugging in (11), leads to the equilibrium investment set

$$\mathcal{I}(\mathcal{C}) = \left\{(\tau,v)\,|\,p(\alpha,\tau) > \frac{I-(v-d^c)^+}{v-(v-d^c)^+}\right\}. \tag{12}$$

On first glance, the condition which determines $\mathcal{I}(\mathcal{C})$ looks very similar to the one determining the set under cash injection $\mathcal{I}(\mathcal{K})$. In comparison to (9) k seems to be only replaced by $(v-d^c)^+$. However, the small difference is crucial as in contrast to the fixed cash amount k it depends upon the state v. Actually, expressing the boundary of the investment set by solving the equality for v as function of p yields

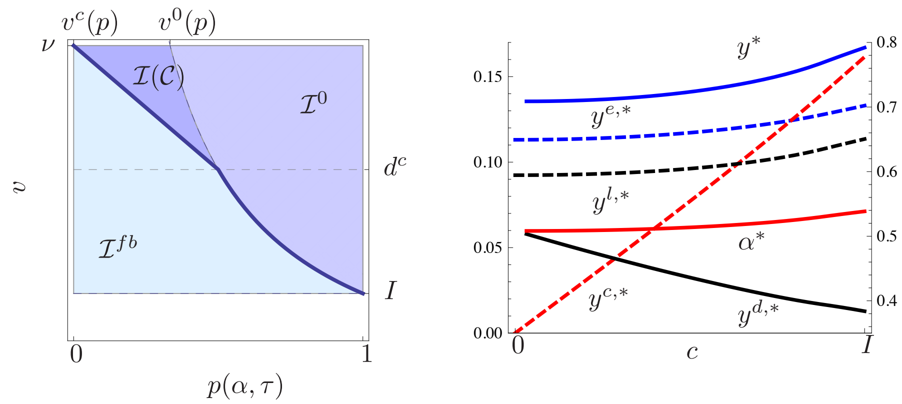
$$v^c(p) = \begin{cases} I/p(\alpha,\tau) & \text{for: } v \le d^c \\ I + (1-p(\alpha,\tau))\,d^c & \text{for: } v > d^c \end{cases}$$

For $v \le d^c$, the boundary is identical with $\mathcal{I}^0$, which is clear from recognizing that for $v \le d^c$, lending conditions are unaffected by coco bonds. However, for $v > d^c$ they are, and thus the boundary is $v^c(p) = I+(1-p(\alpha,\tau))\,d^c$ which is actually linear in p with slope $-d^c$. Figure 3 illustrates the investment set $\mathcal{I}(\mathcal{C})$ in panel (a). We observe that additional investments in comparison to the set $\mathcal{I}^0$ are made in the upper left region, starting with a type determined by $v = d^c$, and $p(\alpha,\tau) = I/d^c$. For d^c large, i.e. if banks have a high fraction of senior debt outstanding, the additional investment set is smaller than if the banks had substituted a significant part of their senior debt by coco bonds. Note further that the additional investment region $\mathcal{I}(\mathcal{C})\setminus\mathcal{I}^0$ includes also banks with a very low success probability $p(\alpha,\tau)$. A property to which we return more extensively in section 5.1.

Panel (b) of Figure 3 illustrates the impact of varying the degree of coco bond financing on the macro state and the aggregated expected payoffs $y^{c,*}$, $y^{l,*}$, $y^{e,*}$ (dashed lines, left scale) and y^*, $y^{d,*}$, α^* (solid lines, right scale). We observe that the equilibrium macroeconomic state variable (solid red

Figure 3: Investment sets under coco bond financing.

Panel (a) illustrates the investment set $\mathcal{I}(\mathcal{C})$ under a coco bond program in the two-dimensional state-space $\mathcal{T} \times \mathcal{V}$. The solid line represents the boundary of the investment set $v^c(p)$ given in equation ((12)). The downward-sloping dashed line ($v^0(p)$) represents the boundary of the investment set without any intervention. The dark shaded area illustrates the set of additionally investing banks due to the cash transfer. Panel (b) provides comparative-statics for an increasing extent of coco bond financing c. The solid red line (α^*) shows the equilibrium macro state, The solid blue line y^* represents household wealth, while $y^{e,*}, y^{d,*}, y^{c,*}$, and $y^{l,*}$ are equilibrium values for equity holders, senior debt holders, coco bond holders, and junior lenders respectively.
Dashed lines are measured on left-hand axis, solid lines are on right-hand axis.



(a) (b)

line) increases with the use of coco bonds, which follows from the fact that the investment region $\mathcal{I}(\mathcal{C})$ increases with c due to the improved lending conditions. Therefore, more investment takes place in the aggregate which has a positive feedback effect on banks' asset quality and aggregate firm value (y^*). The decrease in $y^{d,*}$ reflects the substitution of senior debt by coco bonds and mirrors the increase in the aggregate expected coco bond value $y^{c,*}$. Finally, $y^{l,*}$ as well as $y^{e,*}$, i.e. junior lenders' and equity holders' value increases in the economy. We summarize these findings as:

Result 1. *(Investment) Under the model assumptions, in particular for $a > (d^c + c) > \nu > d^c > \nu - I$, the initial substitution of senior debt for coco bonds with the same notional amount enlarges the investment set,* *i.e.* $\mathcal{I}(\mathcal{C}) \setminus \mathcal{I}^0 > 0$. *The additional investment increases the equilibrium macroeconomic state* α^*, *which in turn increases aggregate asset quality* $\bar{p}(\alpha)$ *and firm value* y^*.

The result basically follows from inspection of the boundary function $v^c(p)$ which determines that $\mathcal{I}(\mathcal{C}) > \mathcal{I}^0$ for $v > d^c$. The inequality ensures that there always exists a non-zero mass of banks for which this is true. Since the aggregate expected firm value is given by equation (5a) as $\bar{y} = \bar{A} + \iint_{\mathcal{I}} v \, \mathrm{d}F_{\tau,v}$ it follows that a larger investment set always increases y^* and α^*.

This section has shown that coco bonds in the initial capital structure do have a positive impact on equilibrium in comparison with only senior straight bonds. Economically, the explanation is given by the mitigation of the lending conditions through the loss-absorption feature of coco bonds in the bad state. Put shortly, coco bond investors provide insurance. In this section, we considered the situation that banks are already endowed with coco bonds, without discussing the conditions under which it is possible to substitute senior debt for coco bonds. Since coco bond investors will not provide insurance for free, we next turn to the description when banks will participate in issuing coco bonds.

4.3 Participation constraints

We now assume that in t_1 banks will evaluate the possibility to restructure their debt structure before making their investment decision. Restructuring and investment decisions are done on the basis of learning their type (τ, v) and follow the maximization of equity holders' time t_1 expectations. Denoting the decision to participate in a restructuring program with similar notation as the indicator $1_{\mathcal{P}}$, then the optimization is

$$\max_{(1_{\mathcal{P}}, 1_{\mathcal{I}})} \; y^{e,*}(\tau, v; 1_{\mathcal{P}}, 1_{\mathcal{I}}),$$

where $y^{e,*} = \mathbb{E}(y^e | \tau, v; 1_{\mathcal{P}}, 1_{\mathcal{I}})$ it the t_1 expectation of equity holders conditional on their type and participation and investment decision. Since the indicators can assume two values, the maximization is over four combinations. Define the pairwise comparisons by[15]

$$\begin{aligned}
\delta_i &= y^{e,*}(1,0) - y^{e,*}(0,0) && \text{[inefficient participation]}\\
\delta_e &= y^{e,*}(1,1) - y^{e,*}(0,0) && \text{[efficient participation]}\\
\delta_o &= y^{e,*}(1,1) - y^{e,*}(0,1) && \text{[opportunistic participation]}\\
\delta_c &= y^{e,*}(1,0) - y^{e,*}(0,1) && \text{[counterproductive participation].}
\end{aligned}$$

The description in brackets follows from the economic interpretation if δ would turn out positive. If e.g. $\delta_i > 0$, then a bank that would not invest without the program, would participate in the program but without investing. Therefore, $\delta_i > 0$ characterizes inefficient participation, while $\delta_e > 0$ characterizes the goal of introducing the program which is inducing banks who would not invest without the program, to invest if they participate in the program. The value of δ depends upon the bank's type, so we can determine the set of banks for which $\delta > 0$. Denote the sets by $\Delta_z = \{(\tau, v)|\delta_z > 0\}$. Then, conditional on not investing without the program, the union of the sets Δ_i and Δ_e determines the set of participation, and likewise, conditional on investing without the program, the union of Δ_o and Δ_c determines participation. Formally, let

$$\Pi_0 = \neg\mathcal{I} \cap (\Delta_i \cup \Delta_e); \qquad \Pi_1 = \mathcal{I} \cap (\Delta_o \cup \Delta_c)$$
$$\Pi = \Pi_0 \cup \Pi_1 \tag{13}$$

The set Π is the set of banks which participate in the program. As noted above, the participation decision will crucially depend upon the way how coco bond investors are compensated for their loss-absorption, i.e. insurance provision. In the following, we will discuss three relevant scenarios. First, for reference, we assume that for some reason, the bank can substitute senior debt for coco bonds with the same notional amount without making any additional compensation.[16] Second, we consider the case where coco bond investors demand a higher principal conditional on knowing the individual bank's type. Depending on their asset quality and investment opportunity,

[15] Dropping the reference to the type for notational convenience, but keeping in mind that the differences depend on the type.

[16] An explanation justifying this assumption would be the case of a compulsory restructuring imposed by the regulator, which deliberately forces losses on debt holders.

banks will have to compensate coco bond investors with a varying risk premium. Third, if coco bond investors do not know banks' type they cannot condition on individual asset quality, so all banks are faced with the same average additional risk premium.

Starting with the first scenario, where the same notional amount of senior debt is substituted by coco bonds. In this case, the participation set is determined by the pairwise differences

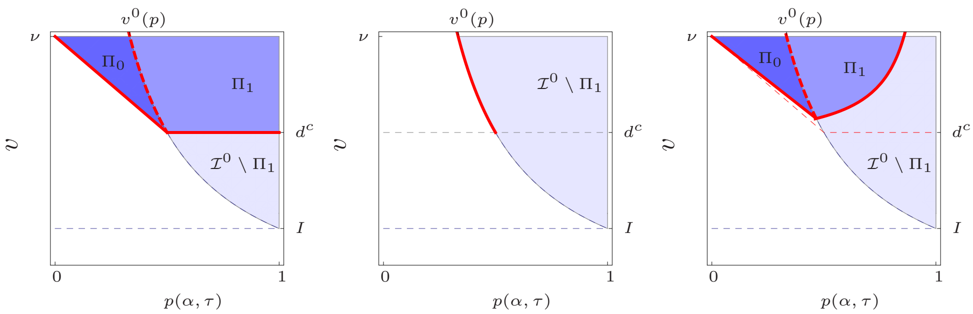
$$
\begin{aligned}
\delta_i &= p(a-(d^c+c)) - p(a-d) \\
\delta_e &= p(a-(d^c+c)+v-\frac{I+(1-p)(v-d^c)^+}{p}) - p(a-d) \\
\delta_o &= p(a-(d^c+c)+v-\frac{I+(1-p)(v-d^c)^+}{p}) - \left(p(a-d+v-\frac{I}{p})\right) \\
\delta_c &= p(a-(d^c+c)) - \left(p(a-d+v-\frac{I}{p})\right)
\end{aligned}
$$

Due to the assumption that the senior debt is replaced with coco bonds with the same principal, we have $d = d^c + c$, and thus $\delta_i = 0$ for all types, so inefficient participation never occurs. Likewise, for counterproductive participation $\delta_c > 0$ occurs for $I/v > p$ contradicting the investment decision. Efficient participation occurs for $\delta_e > 0$ which happens when $p > \frac{I-(v-d^c)^+}{v-(v-d^c)^+}$ conditional on not having yet invested. Finally, opportunistic investment takes place for δ_o which occurs for all $p < 1$ and $v > d^c$, conditional on investing. These regions are illustrated in panel (a) of Figure 4. The dark shaded region Π_0 is the efficient participation of banks that are induced to participate in the program and choose to invest, while region Π_1 is the set of banks which would have invested anyway, but which find it beneficial to participate in the program. The entire participation region is within the boundary of the bold solid red line. The region indicated by $\mathcal{I}^0 \setminus \Pi_1$ are the set of banks which will invest without the program and have no incentive to participate, which we can call efficient non-participation region.

The first scenario assumes that equity holders can decide to participate in the coco bond program and swap senior debt for coco bonds with the same principal. However, since the claim of coco bond investors provides the loss absorption, their market value will be less than a senior debt claim

Figure 4: Participation sets under coco bond financing.

The three panels illustrate the participation sets under coco bond financing under the scenarios (i) of a costless debt swap (left panel), (ii) full information compensation (middle panel), and (iii) asymmetric information compensation (right panel). The area indicated by Π_0 represents efficient participation while the area indicated by Π_1 represents the set of banks which opportunistically participate in the coco bond program.



(a) (b) (c)

with same principal. Thus, the the bank would either have to compensate the debt holders by the difference or issue coco bonds with a larger notional amount c. Since we assume that the bank has no liquid assets or funds available and we allow government intervention only in a later section, the only way to initiate the debt restructuring is to issue coco bonds with a larger principal. This is the second scenario we consider. The necessary increase in the notional amount of the coco bond (denoted as Δc) follows from the reasoning that a potential investor will only invest in a coco bond if it offers the same expected value as a an investment in senior debt with the same notional amount. The expected value of senior debt $\mathbb{E}(y^d|\tau, v)$ with the initial principal d is $p \cdot d + (1-p) \cdot v$.[17] Now, split this value into $\phi \cdot \mathbb{E}(y^d|\tau, v)$, and $(1-\phi) \cdot \mathbb{E}(y^d|\tau, v)$. Let $\phi = \frac{d^c}{d}$ be the fraction of senior debt after the restructuring, and likewise $1 - \phi = \frac{c}{d}$ be the fraction of coco bonds. Then Δc can be determined from the equation

$$p(\alpha, \tau) \cdot c + (1 - p(\alpha, \tau))(1 - \phi)v = p(\alpha, \tau) \cdot (c + \Delta c).$$

[17] The discussion above has shown that we can restrict our attention to the cases where investment occurs, as non-investing banks will not issue coco bonds.

The LHS is the pro rata value of a senior debt claim, while the RHS is the expected value of a coco bond with notional amount $c + \Delta c$. Solving for Δc yields

$$\Delta c = \frac{(1 - p(\alpha, \tau)) \cdot (v - \phi v)}{p(\alpha, \tau)}.$$

Note that this is derived under the assumption that coco bond investors know the bank's type (τ, v), so they can condition on this knowledge and demand an individual notional amount. The solution shows that Δc compensates for the excess of v over ϕv which is not captured by senior debt holders in the bad state. Note further that from our assumption that $d^c < \nu < d^c + c$, we have

$$v \leq \nu < d^c + c \quad \Leftrightarrow \quad \frac{d^c}{d^c + c} v < d^c \quad \Leftrightarrow \quad \phi v < d^c.$$

If banks have to issue coco bonds with a principal of $c + \Delta c$ instead of only c in order to initiate the debt restructuring, this will have an impact on their participation constraints, i.e. on the participation set Π. Substituting in the above equations for δ, the principal of coco bonds by $c + \frac{(1-p)\cdot(v-\phi v)}{p}$, we find that $\delta_i > 0$ occurs for $p > 1$, and $\delta_c > 0$ occurs for $p < \frac{I-(v-\phi v)}{v-\phi v}$ which contradicts with initial investment. Thus, inefficient or counterproductive investment will not occur.
Importantly, $\delta_o > 0$ is equivalent to $(1 - p)(\phi v - d^c) > 0$ which (due to the above inequality) will only occur for $p > 1$. Finally, $\delta_e > 0$ occurs for $p > \frac{I-(\phi v - d^c)}{v-(\phi v - d^c)}$, which is strictly inside $\mathcal{I}^0$ and therefore, Π_0 is empty. This means that neither opportunistic nor efficient participation will occur. The result is illustrated in panel (b) of Figure 4, showing that the region Π_0 as well as Π_1 are empty.

The third scenario refers to the determination of the coco bond risk premium under asymmetric information. As before, coco bond investors require a higher notional amount due to the loss-absorption. However, unlike in the previous paragraph, assume that the type of the bank is unknown.

The unconditional value of a senior debt claim with notional d is

$$\begin{aligned}\mathbb{E}(y^d) &= \iint\limits_{\Omega} p(\alpha,\tau)\, d + (1-p(\alpha,\tau))\, 1_{\mathcal{I}}\, v \,\mathrm{d}F_{\tau,v} \\ &= \bar{p}(\alpha)\, d + \iint\limits_{\mathcal{I}} (1-p(\alpha,\tau))\, v \,\mathrm{d}F_{\tau,v}\end{aligned}$$

Therefore, equating the pro rata share of $(1-\phi)\mathbb{E}(y^d)$ to the unconditional expectation of a coco bond is

$$\bar{p}(\alpha)\, c + (1-\phi) \iint\limits_{\mathcal{I}} (1-p(\alpha,\tau))\, v \,\mathrm{d}F_{\tau,v} = \iint\limits_{\Omega} p(\alpha,\tau)\,(c+\Delta c)\,\mathrm{d}F_{\tau,v}$$

Since Δc does not depend on the type, we have $\iint_\Omega p(\alpha,\tau)\,(c+\Delta c) = \bar{p}(\alpha)(c+\Delta c)$. Solving for Δc and denoting the solution in the unconditional case as Δc_0 yields

$$\Delta c_0 = \frac{(1-\phi)}{\bar{p}(\alpha)} \iint\limits_{\mathcal{I}} (1-p(\alpha,\tau))\, v \,\mathrm{d}F_{\tau,v} \tag{14}$$

Note that $\Delta c_0 > 0$. Using the principal of $c + \Delta c_0$ in the equations δ for deriving the participation sets, we find that (i) $\delta_i = -p\,\Delta c_0$ which is always negative. $\delta_c > 0$ occurs for $p < \frac{I}{\Delta c_0 + v}$ which contradicts investment, so again, inefficient and counterproductive participation does not happen.
For opportunistic participation, we find δ_o to be positive for $p < \frac{(v-d^c)}{\Delta c_0 + (v-d^c)}$. The boundary describes an increasing graph in the investment region $\mathcal{I}^0$. The efficient participation set is determined by $\delta_e > 0$ which is equivalent to $p > \frac{I-(v-d^c)}{d^c - \Delta c_0}$ whose boundary in decreasing with a slope of $-(d^c - \Delta c_0)$ which is less negative as in the first scenario. Therefore, the set of efficient participation is smaller. Panel (c) of Figure 4 illustrates the participation set Π. Note that both the efficient participation set Π_0 as well as the opportunistic participation set Π_1 is smaller, but non-zero. Economically, since the additional risk premium to coco bonds is determined from the unconditional expectation across all bank types, banks which turn out to have a high success probability are deterred from issuing coco bonds because they are very likely to pay coco bond investors the full notional amount. In other terms, they consider coco bonds to be too expensive. Similarly, banks with

intermediate project returns will also find coco bonds too expensive. Thus, the participation region boundaries in the (v, p)-space shift upwards and to the left. We summarize our findings in the following

Result 2. *(Participation) Under the model's assumptions, participation in the three scenarios are: (i) If the bank can substitute senior debt for coco bonds with the same principal such that* $d = d^c + c$*, then opportunistic participation as well as efficient participation takes place, i.e. the regions* Π_0 *and* Π_1 *are non-empty. (ii) If the bank has to offer coco bond investors an increase in the notional amount to compensate for the loss-absorption, i.e.* $d < d^c + c + \Delta c(\tau, v)$*, and coco bond investors know the type of banks, then the opportunistic and efficient participation region collapses to a set with mass zero. (iii) If the bank has to offer an increased notional amount to coco bond investors, who do not know banks' type, i.e.* $d < d^c + c + \Delta c_0$*, then the participation set is non-zero but smaller as under (i) since banks with high success probability and low project values will find coco bonds too expensive.*

The result shows that if banks' are already endowed with coco bonds, the debt overhang can partly be mitigated. However, in light of an already existing debt overhang problem, the restructuring of the debt structure under full information about banks' type is not feasible as banks will not find it optimal to participate. This finding illustrates that the private sector is not able to solve the debt overhang problem on its own.
The inability of the private sector calls for an intervention by the government to provide incentives for banks to participate in a debt restructuring. One way of doing this would be to provide the necessary capital injection Δc which compensates coco bond holders and therefore provides the incentives for banks to use coco bond financing. The interesting question in this context is the efficiency of such an intervention which can only be assessed by comparing it to some alternative intervention, such as the already mentioned cash injection. Another more realistic intervention is the purchase of (part of) banks' assets by the government, in the sense of the earlier mentioned TARP (Troubled Asset Relief Program). An asset purchase program has been a key focus of Philippon and Schnabl (2013) in assessing recapitalization schemes, and provides a meaningful benchmark against which the coco bond program can be compared. The next section describes the im-

plications of an asset purchase program before we turn to the comparison between both programs.

4.4 Asset purchase

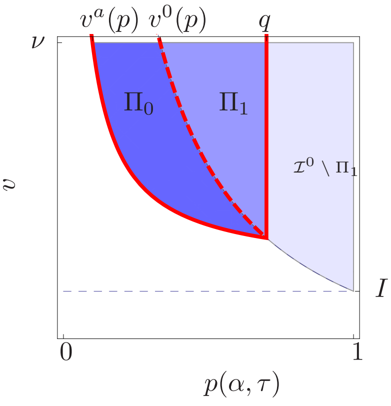
Instead of an intervention on the liabilities side, an asset purchase program focuses on the asset side. The idea of an asset purchase is that the government overcomes the market friction by committing to liquidate part of the existing banks' assets, which in turn relax the funding constraints. In line with this general economic mechanism, we assume in the model that the government offers banks to buy assets with face value z of their existing assets for price q. Thus, the asset purchase program is denoted as $\mathcal{A}$ and characterized by the pair (z, q). The bank receives an amount $q \cdot z$ as proceeds from the transaction which can be used to partly finance the new investment opportunity. Two restrictions make economic sense: First, $I \geq q \cdot z$, i.e. the government will not buy more assets than to completely relax the budget constraint. Second, $a - z \geq d$, i.e. the government will not buy assets to the extent that the bank is overindebted in both states.
Given that a bank participates in an asset purchase, its investment decision follows from the discussion in section 4.1 since the liquidation provides available funds of qz which reduces the amount that needs to be raised from junior lenders to $I - qz$. Thus, the investment decision is equivalent to the cash transfer with $k = qz$. Therefore, the investment set is analogous to equation (9).
While conditional on participating in the program, the investment decision follows the cash transfer scenario, the participating decision itself is crucial to the efficiency of the program. Applying the steps to determine the participation set from the previous section, we first find

$$
\begin{aligned}
\delta_i &= p(a-d-z) - p(a-d) = -p \cdot z > 0 \Leftrightarrow p < 0 \\
\delta_e &= p(a-d-z) + pv - I + qz - (p(a-d)) > 0 \Leftrightarrow p(v-z) > I - qz \\
\delta_o &= p(a-d-z) + pv - I + qz - (p(a-d) + pv - I) > 0 \\
&\quad \Leftrightarrow (q-p)z > 0 \\
\delta_c &= p(a-d-z) - (p(a-d) + pv - I) > 0 \Leftrightarrow p < \frac{I}{z+v}.
\end{aligned}
$$

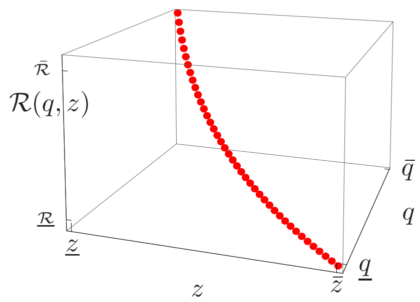
Inefficient and counterproductive investment will not occur, as it is only possible for $p < 0$ or contradicts initial investment. Opportunistic participation will occur for any bank whose success probability is smaller than q. Efficient participation is described by a set with a boundary of $v^a(p) = \frac{I}{p} - \frac{(q-p)z}{p}$ for $q > p$, which is decreasing in the (v, p)-space. Therefore, both sets Π_0 and Π_1 are non-empty, and illustrated in panel (a) of Figure 5. Note that

Figure 5: Participation set and cost function for the asset purchase program.

Panel (a) shows the participation set under an asset purchase program. The area indicated by Π_0 represents efficient participation while the area indicated by Π_1 represents the set of banks which opportunistically participate in the program. Panel (b) shows the total costs (redistributions) for different forms of asset purchase programs, i.e. for combinations of (q, z), which achieve the same equilibrium state α^*. The graph confirms that the least-cost asset purchase program chooses the lowest possible q and highest z.



(a)



(b)

set Π_1 is determined (on the right) by the horizontal boundary at $p = q$. Any bank whose success probability is below the price q will participate in the asset purchase program, since it can sell assets with an expected payoff of $p \cdot z$ for a sure amount $q \cdot z$, which is clearly beneficial. The extent of opportunistic participation can be influenced by choosing a lower q. However this will also reduce the set of efficient participation Π_0. If the program wants to achieve a target increase in the macroeconomic condition via an increase in investment activity, it can either choose to buy a small amount of assets (low z) at a high price (high q), or buy a large amount of assets (high z) at low prices (q). From the graphical inspection, the latter choice will reduce the set of opportunistic participation and might more efficient. This intuition is confirmed by the plot in panel (b) of Figure 5, where we show possible programs (q, z) from $q \in [\underline{q}, \bar{q}]$, and $z \in [\underline{z}, \bar{z}]$ which induce the same equilibrium macro state α_0^*. Formally, we (numerically) determine the set $\mathcal{A}(\alpha_0^*) = \{(q, z) \,|\, \alpha^*(q, z) = \alpha_0^*\}$. The efficiency of a program can be determined from recognizing that the government will buy assets at price qz which turn out to have an expected value of $p(\alpha, \tau) \cdot z$. Thus, by denoting the cost function as $\mathcal{R}$, the cost of the asset purchase program is given by

$$\mathcal{R}(q, z) = \iint\limits_{\Pi} (q - p(\alpha, \tau))\, z \,\mathrm{d}F_{\tau,v}, \tag{15}$$

which is plotted for any (q, z) combination of $\mathcal{A}(\alpha_0^*)$ on the vertical axis. The graph shows that costs $\mathcal{R}$ monotonically decrease as z is larger. Thus, the cost-minimizing program consists of buying a large fraction of assets for a low price. We turn our focus now towards a comparison between the cash transfer, the asset purchase and the coco bond program.

5 Assessment of interventions

We assess the efficiency of the coco bond program along two aspects. First, we compare the incentives of the investment and participation set between programs $\mathcal{A}$ and $\mathcal{C}$. Second, we compare the efficiency in the sense of the associated costs involved in implementing the programs.

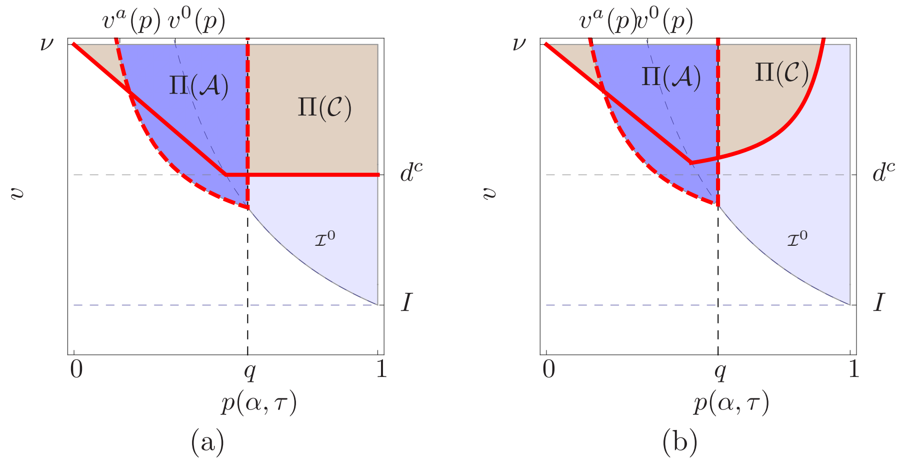
5.1 Incentives

Figure 6 combines the graphs of Figure 4 and 5 by showing regions $\Pi(\mathcal{C})$ and $\Pi(\mathcal{A})$ in one graphic for two different values of Δc_0.[18] In both panels, the two programs induce the same macroeconomic state α^*, which means both programs are equally successful. From visual inspection of the investment and participation regions, two important structural differences can be observed. First, the opportunistic participation set of the asset purchase program is bounded by the vertical line occurring at $p = q$. In contrast,

[18] In panel (a) Δc_0, while in panel (b) $\Delta c_0 > 0$.

Figure 6: Participation set and cost function for the asset purchase program.

The two panels compare the participation set between a coco bond and an asset purchase program which are equally effective, i.e. induce the same equilibrium state α^*. The area bounded by the solid red line and indicated by $\Pi(\mathcal{C})$ indicates the participation set under a coco bond program, while the blue area bounded by the dashed red line and indicated by $\Pi(\mathcal{A})$ indicates the participation set under an asset purchase program. Either under a costless debt swap (panel a), or under asymmetric information (panel b), the coco bond program will always induce a set of banks with higher default probability to participate than an equally effective asset purchase program.



the opportunistic participation under the coco bond program is bounded by either a horizontal line occurring at $v = d^c$ (left panel), or by an upward sloping boundary $v = \frac{p}{1-p}\Delta c_0 + d^c$ (increasing in p) if $\Delta c_0 > 0$ (right panel). In other words, for opportunistic participation the state space is divided vertically in the case of $\mathcal{A}$, while being divided vertically for $\mathcal{C}$. The economic explanation for this difference is the fact that program $\mathcal{A}$ intervenes on the asset side, while program $\mathcal{C}$ intervenes on the liabilities side. The difference has implications for the costs of implementing the program as we will demonstrate in the next section.

The second difference relates to the structure of the efficient participation set. The boundaries are both decreasing in p. However, while for $\Pi(\mathcal{A})$ the boundary is convex, it is linear with slope $-(d^c - \Delta c_0)$ for $\Pi(\mathcal{C})$. This difference has an important implication with respect to which type of banks will participate and invest in the two programs. As the figures suggest, the region $\Pi(\mathcal{C})$ extends over a broader range of values for p. In particular it extends farther towards low p-values, i.e. there is always a part to the left of $\Pi(\mathcal{A})$. Economically, this means that the coco bond program provides incentives that banks with a very low success probability will invest, which would not invest under the asset purchase program. The visual inspection can be made rigorous and is stated formally as

Result 3. *(Incentives of the coco bond program) Under the model's assumptions, if an asset purchase program $\mathcal{A}$ and a coco bond program $\mathcal{C}$ are structured such that they induce the same equilibrium macroeconomic state α^*, then $\Pi(\mathcal{C})$ includes banks with type τ which are strictly smaller than the minimum type in $\Pi(\mathcal{A})$. I.e. there exists $\tau \in \Pi(\mathcal{C})$ for which $\tau < \tau^{min} = \min \Pi(\mathcal{A})$.*

The proof is in the Appendix. Result 3 shows that a coco bond program provides incentives for banks with very poor asset quality to participate in the new investment opportunity. Economically, the result follows from the fact that due to the loss absorption feature of coco bonds, junior lenders share in the project return also in the bad state which in turn makes the boundary linear in p.[19] Note that the same conclusion can be drawn from comparing the coco bond program to the direct cash transfer $\mathcal{K}$, since the boundary of the investment set under program $\mathcal{K}$ is equivalent to the investment set under the asset purchase program $\mathcal{A}$ for $q = 1$.

5.2 Efficiency

Besides the type which participate in a program, we can compare them also along the cost dimension. For the asset purchase program, we already discussed the associated costs in section 4.4 and determined them in equation (15) as being the sum over $(p(\alpha, \tau) - q) \cdot z$ for types τ in the participation set. These costs have the straightforward interpretation that the government pays $q \cdot z$ in return for assets that have an expected value of $p(\alpha, \tau) \cdot z$. The costs of the cash transfer program $\mathcal{K}$ are also easily obtained by recognizing that the government provides k to every participating bank without obtaining any claim. The participation set can be obtained from (13) but

[19] See also the discussion following equation (12)

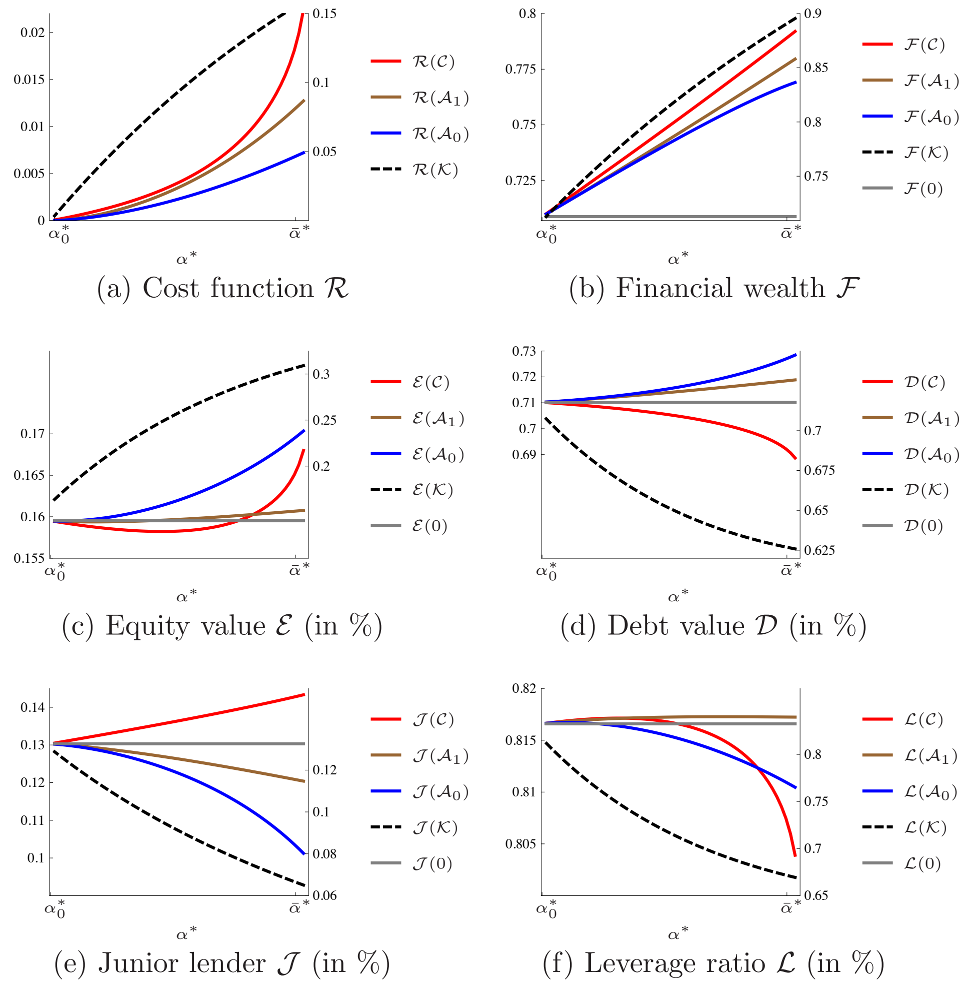
we omit the derivation, since it is intuitively obvious that any bank will participate, and so the participation set comprises the entire state space, i.e. $\Pi(\mathcal{K}) = \Omega$. Thus, the costs ($\mathcal{R}$) of the cash transfer are just k, since $\mathcal{R}(\mathcal{K}) = \iint_\Omega k \, \mathrm{d}F_{\tau,v} = k$.
For the coco bond program $\mathcal{C}$, the costs are less straightforward, since it involves the transfer needed to induce the bank to restructure its debt structure before they learn their type and investment opportunity. From section 4.3 we know that the participation set would be empty if coco bond investor knew the type of the bank and price coco bonds accordingly. If they don't, they will demand a corresponding increase in the coco bond principal, which was determined in equation 14. Analogous to the reasoning that led to 14, this determines the costs of the coco bond program, by comparing the coco bond to a senior bond with same principal. Letting as before $\phi = \frac{d^c}{d}$ then the expected value of a senior bond with principal c is $p(1-\phi)d+1_{\mathcal{I}} \cdot (1-p)(1-\phi)v$. The expected value of a coco bond with principal $c + \Delta c_0$ is just $p \cdot (c + \Delta c_0)$ as due to loss-absorption they receive nothing in the downstate. Subtracting both terms yields the difference needed to compensate coco bond holders. Aggregating these differences over the participation region, we obtain the costs of the coco bond program,

$$\mathcal{R}(\mathcal{C}) = \iint\limits_{\Pi} \left((1 - p(\alpha, \tau))(1 - \phi)v - p(\alpha, \tau)\Delta c_0\right) \, \mathrm{d}F_{\tau,v}.$$

Having determined the cost functions $\mathcal{R}(\mathcal{C})$, $\mathcal{R}(\mathcal{A})$, and $\mathcal{R}(\mathcal{K})$, we plot them in panel (a) of Figure 7. Thereby, we choose a coco bond program where we successively increase the participation region by decreasing the premium Δc_0 towards zero. In other terms, we make the coco bond issue successively cheaper. As coco bonds become more attractive, the participation region $\Pi(\mathcal{C})$ increases and more banks invest, which in turn increases the aggregate asset quality in the economy and the equilibrium macro state α^*. Then, for any such particular choice of $\mathcal{C}$, we numerically solve for program $\mathcal{A}$ and $\mathcal{K}$ which induce exactly the same equilibrium α^*, so that all three programs do have the same equilibrium effect on the economy. In the case of cash transfer, this means solving for the only free variable k, while for the asset purchase program two free variables (q, z) can be chosen. As discussed in section 4.4, the least-cost program can be obtained by choosing z as large

Figure 7: Cost function and financial wealth across different programs.

Panels (a)–(f) provide comparative-static results for a cash transfer program ($\mathcal{K}$, dashed black line), two asset purchase programs ($\mathcal{A}_0$ and $\mathcal{A}_1$, blue and brown line), and a coco bond program ($\mathcal{C}$, red line). Graphs show the magnitude of total costs (panel a), overall financial wealth (b), equity value (c), debt value (d), junior lenders (e), and leverage (f) for different levels of the equilibrium macro state α^* (horizontal axis).



(a) Cost function $\mathcal{R}$

(b) Financial wealth $\mathcal{F}$

(c) Equity value $\mathcal{E}$ (in %)

(d) Debt value $\mathcal{D}$ (in %)

(e) Junior lender $\mathcal{J}$ (in %)

(f) Leverage ratio $\mathcal{L}$ (in %)

as possible. We label the least-cost asset purchase program as $\mathcal{A}_0$. For comparison, we also report results for a choice of z strictly smaller than $\bar{z}$

and label this program as $\mathcal{A}_1$. With the short-hand notation that $\alpha^*(\mathcal{P})$ is the equilibrium macro state for a given program, we formally determine the following set

$$\mathcal{S}(\Delta c_0) = \{(q_0, q_1, k) \,|\, \alpha^*(c + \Delta c_0) = \alpha^*(q_0, \bar{z}) = \alpha^*(q_1, z_1) = \alpha^*(k)\,\}$$

The solution to $\mathcal{S}(\Delta c_0)$ is taken as input for the cost functions $\mathcal{R}(\mathcal{C})$, $\mathcal{R}(\mathcal{A}_0)$, $\mathcal{R}(\mathcal{A}_1)$, and $\mathcal{R}(\mathcal{K})$ which are then plotted against their (identical) equilibrium macro state α^*. The results are reported in panel (a). As we normalize the asset value of banks to $a = 1$, the numerical value of costs can be understood as percentage of asset value. In line with intuition, in order to obtain an improved investment activity and thus a higher macro state, the programs are more costly, i.e. the cost functions increase for all programs. By comparing the programs, it is not surprising to see that the costs of the cash transfer are an order of magnitude higher than the other programs (and therefore plotted on a separate (right) axis in all panels a–f). The excessive cost of $\mathcal{K}$ comes from the fact that it represents a pure subsidy. The more interesting and important observation is the finding that the coco bond program $\mathcal{C}$ is more expensive than both the least-cost asset purchase program $\mathcal{A}_0$ as well as $\mathcal{A}_1$. Thus, by taking the cost function as a measure of efficiency, we find that the asset purchase program is always more efficient than the coco bond program to obtain the same macroeconomic state. To highlight this finding, we summarize it formally as

Result 4. *(Efficiency of the coco bond program) Under the model's assumptions, if an asset purchase program $\mathcal{A}$ and a coco bond program $\mathcal{C}$ are structured such that they induce the same equilibrium macroeconomic state α^*, then the costs of the coco bond program $\mathcal{R}(\mathcal{C})$ are larger than those of the asset purchase program $\mathcal{R}(\mathcal{A})$*

Panels (b) to (f) in Figure 7 plot the aggregate values of the financial wealth and their components under the different programs by always using set $\mathcal{S}(\Delta c_0)$, i.e. forcing all programs to have the same equilibrium effect on α^*. In panel (b) we report the sum of the aggregate equity, debt and junior lender values which in our economy accrue to the households as financial income, thus we label it as $\mathcal{F}$. For reference, the gray constant line (labeled $\mathcal{F}(0)$) represents the household financial wealth in the base case without any program. The graphs mirror the observation from the cost functions in panel (a). Since costs represent the magnitude of the transfer to the private sector, the financial wealth under cash transfer is highest followed by the coco bond program, and the asset purchase programs. Panels (c) to (e) break down the financial wealth in the three components: Equity value $\mathcal{E}$, debt value $\mathcal{D}$, and value of junior lenders $\mathcal{J}$. In all three graphs we report their percentage value of total financial wealth $\mathcal{F}$. The interesting observation thereof is that between program $\mathcal{C}$ and $\mathcal{A}$ different redistributions between the financial claims occur. While in the asset purchase program, the transfer from the government increases both the equity as well as the senior debt claim, we observe a different impact for the coco bond program. First, the equity value is lower than for $\mathcal{A}$ and increases only for large programs. Second, the value of debt (which is the sum of senior and coco bonds) in the aggregate decrease. This is compensated by observing that $\mathcal{C}$ is the only program under which junior lenders value increases in the aggregate. Economically, the patterns are explained by the different mechanism of the programs, where $\mathcal{A}$ is an intervention on the asset side, while $\mathcal{C}$ is an intervention on the liabilities side. Thus, the subsidy in $\mathcal{A}$ reduces the reliance on junior lenders and is a transfer to both equity and debt holders. In contrast, program $\mathcal{C}$ improve the payoffs of junior lenders through the loss absorption of coco bonds. Thus, total debt values decrease. Equity holders benefit in particular of programs that target a high impact on α^* as this means that coco bonds have to be made cheaper, thereby decreasing the required principal. The lower principal value benefits equity holders as residual claimants in the good state. Panel (f) plots the aggregate leverage ($\mathcal{L}$) which summarizes the previous discussion by showing that leverage is decreasing more strongly under the coco bond program as compared to the asset purchase programs.

6 Conclusion

This paper analyzes contingent convertible bonds in a general equilibrium model. Recognizing that coco bonds are by construction intended to mitigate the debt overhang problem and to improve the credit supply, it seems evident that an analysis of coco bonds should take into account their endogenous impact on the asset quality of the financial institution, which in turn has implications on the equilibrium probability of financial distress. The literature sofar has put no emphasis on this feedback mechanism. Our stylized general equilibrium economy captures this crucial aspect of the coco bond proposal and addresses three important areas: First, are coco bonds able to mitigate the debt overhang problem in general equilibrium? In line with intuition, we find that the inclusion of coco bonds in the capital structure enlarges the investment set, which has a positive feedback on the aggregate macroeconomic state and thereby increases the asset quality of financial institutions. The economic mechanism which is responsible for this results works through alleviating the lending conditions in adverse states where the loss-absorption feature of coco bonds kicks in. However, this benefit will not come for free. Therefore, it is crucial to analyze the participation decision of financial institutions. This aspect is the second focus of our contribution. We show that in a full-information setting where the loss-absorption feature of coco bonds are fully priced, banks will have no incentive to participate in a coco bond program, since part of the loss-absorption benefit accrues to senior debt holders. In an asymmetric information setting where coco bond investors demand an average risk compensation, the participation set is non-empty but unstable. Thus, we find that a pure private sector solution is not sustainable and demands public sector interventions. This finding leads to the third focus of our contribution, which relates to an assessment of the efficiency of the intervention in comparison to alternative programs. We compare the coco bond program to an outright cash injection and an asset purchase program in the spirit of the Troubled Asset Relief Program (TARP). We assess the efficiency by focusing on two important aspects, which are (i) incentives and (ii) costs. We find that in comparison to an asset purchase program which is equally effective in the sense of achieving the same macroeconomic state, the coco bond program provides incentives such that it always induces a set of banks with higher default probability to invest.

This result generalizes findings in the previous literature who document the risk-shifting incentive of coco bond financing. The second dimension along which we assess efficiency relates to the cost of the program. Within a general equilibrium model, the notion of costs is not fully appropriate due to the endogeneity. However, we can determine the extent of wealth redistribution which takes place with different programs. By assuming that wealth redistribution creates distortions, the extent of the redistribution is a legitimate proxy for the efficiency of a program. Not surprisingly we find that an outright cash transfer necessitates the largest redistributions. More interestingly however, we also find that the coco bond program always creates larger redistributions as an asset purchase program. The economic explanation follows from the fact that while an asset purchase program makes transfers to the equity holders, the coco bond program makes transfers to equity holders *as well as* senior debt holders. In other words, coco bonds create positive external effects for senior debt holders by not only alleviating the lending conditions but by also making senior debt less risky. Thus, in order to achieve the same macroeconomic success, larger redistributions are necessary as compared to an asset purchase program.

In sum, our analysis finds rather weak support for the allegedly beneficial impact of coco bonds. Although coco bonds would have the potential to alleviate the debt overhang problem, a private sector solution is unlikely and demands public sector intervention. Such an intervention however turns out to be less efficient as compared to alternative programs. In concluding, it has to be stressed that our results rely on purely economic reasoning and do not take into account that a coco bond program may be more opportune from purely political considerations.

A Proof of Result 3.

From the intuition of the Figure 6, we want to show that if both programs induce the same additional mass of banks to invest, then there exist banks of type $\tau \in \Pi(\mathcal{C})$ for which $\tau < \tau^{min} = \min \Pi(\mathcal{A})$. I.e. banks which are induced by the coco bond program to invest, which would not invest under the asset purchase program.
The boundaries of the participation and investment region for asset purchase program $\mathcal{A}$ is determined by the condition that $(q-p)z + pv - I > 0$ for all $q > p$. Rewrite this as equality and solve for (vertical axis) v which yields: $v = \frac{I}{p} - \frac{(q-p)z}{p}$. Likewise, the investment region for the coco program $\mathcal{C}$ is determined by $p > \frac{I-(v-d^c)^+}{v-(v-d^c)^+}$. Rewrite to find: $v = \frac{I}{p}$ for $v < d^c$, and $v = I + (1-p)d^c$ for $v > d^c$.
The relevant equations are thus:

$$\begin{aligned} v^a(p) &= \frac{I}{p} - \frac{(q-p)z}{p} \\ v^c(p) &= I + (1-p)d^c \end{aligned}$$

The proof consists of showing that for all admissible parameters for $\mathcal{A}$, there is a non-zero mass of banks with $\tau < \tau^{min}$ in $\Pi(\mathcal{C})$.
The admissible range of values for $\mathcal{A}$ is determined by the following restrictions: (i) $0 \le q \le 1$, (ii) $z \le a - d$ (the program can not buy assets to the point that default occurs immediately, i.e. $d \ge a - z$), and (iii) $qz \le I$ (i.e. the proceedings of the asset purchase qz have to be smaller than the amount needed for funding the investment I).

(I) First, consider any $z \le I \le a - d$. We can set $q = 1$ without violating restriction (iii).
Now, determine the point p_0 where both frontiers intersect, i.e. the root of $v^a = v^c$. The equality determines a quadratic equation in p:

$$-d\,p^2 + (d + (I-z))\,p - (I-z) = 0; \quad \text{with solutions: } p_0 = \{\frac{I-z}{d}, 1\}$$

Next, take the first derivative of v^a and v^c at p_0, which are:

$$\frac{\partial v^a}{\partial p}\Big|_{p=p_0} = -\frac{I-z}{p_0^2} = -\frac{d^2}{I-z}; \qquad \frac{\partial v^c}{\partial p}|_{p=p_0} = -d$$

Since a solution for p_0 smaller 1 only exists for $d < (I-z)$, this shows that $\frac{\partial v^a}{\partial p}\Big|_{p=p_0} < \frac{\partial v^c}{\partial p}\Big|_{p=p_0}$, i.e. at p_0, the slope of v^a is more negative than the slope of v^c.
As v^a is decreasing and convex for all admissible p (which is verified by the first and second derivatives, being $-(I-qz)/p^2 < 0$, and $2(I-qz)/p^3 > 0$ respectively), there must be $p < p_0$ for every v, which are in $\Pi(\mathcal{C})$, but not in $\Pi(\mathcal{A})$. Since $p(\tau)$ is monotonically increasing in τ, the conclusion carries over to τ.
As $z \to 0$, the root p_0 increases to I/d. Vice versa, for any given z, decreasing q shifts v^a upwards (i.e. $\frac{\partial v^a}{\partial q} = -\frac{z}{p}$), implying again that p_0 increases.

(II) It remains to consider the case of $I \leq z \leq a-d$. Set $z = a-d$. Equating $v^a = v^c$, can be written as the quadratic equation

$$-d\,p^2 + \beta\,p + \gamma = 0; \quad \text{where: } \beta = 2d - a + I, \quad \text{and: } \gamma = I - q(a-d),$$

which has roots

$$p_{0,-} = \frac{\beta - \sqrt{4d\,\gamma + \beta^2}}{2d}; \qquad p_{0,+} = \frac{\beta + \sqrt{4d\,\gamma + \beta^2}}{2d}$$

Note that the highest possible q is $q = I/(a-d)$ which makes v^a a constant, i.e. $v^a = a-d$. In this case, the roots are $p_0 = \{0, \beta/d\}$. Now, for all $q < I/(a-d)$, γ will be strictly positive, implying that the negative root $p_{0,-}$ increases. And given that v^a is decreasing and convex for all admissible p, we derive the same conclusion as above. As decreasing z shifts v^a upwards (i.e. $\frac{\partial v^a}{\partial z} = 1 - \frac{q}{p}$ and $q > p$) the reasoning holds true for all $z \in [I, a-d]$.

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