The Tragedy of Complexity*

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Abstract

This paper presents an equilibrium theory of product complexity. Complex products generate higher potential value, but require more attention from consumers. Because consumer attention is a limited common resource, an *attention externality* arises: Producers distort the complexity of their own products to grab attention from other products. This externality leads to an equilibrium distortion towards intermediate complexity—products that are well understood end up being too complex, whereas products that are not well understood are too simple. The model provides a categorization of goods according to both their absolute complexity as well as their complexity relative to first best.

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1 Introduction

Products differ vastly in their complexity. Some products are exceedingly complicated: never-ending options in retail financial products, overly complex financial regulation, and endless features and settings for smartphones and software. Others appear overly simplified: the media and politicians tend to simplify complex issues, while material taught in MBA courses can sometimes seem overly simplistic. But what is the right level of complexity? And does the market deliver it? This paper proposes an equilibrium theory of complexity to shed light on these issues.

The key premise of our analysis is that complex products generate higher potential value, but require more of the consumer's limited attention. By allowing complexity to create value, our framework departs from much of the existing literature that has mostly focused on complexity as a means of obfuscation. By explicitly recognizing the consumer's limited attention, our analysis highlights a novel *attention externality*: When choosing the complexity of their goods, producers do not take into account that attention is a common resource. In equilibrium, producers therefore distort the complexity of their products, but in doing so they divert attention away from other goods. For example, an insurance company may decide to provide a more complicated, customized health insurance policy. While this can increase the stand-alone value of the health policy, the insurance company does not take into account that the more complicated health insurance policy leaves the consumer with less time to understand products from other producers, such as her pension plan or home insurance.

Our analysis yields three main results. First, equilibrium complexity is generally inefficient. Specifically, the attention externality can lead to *too much or too little complexity*, depending on the direction of the consumer's attention reallocation in response to changes in complexity. We refer to this generic inefficiency of equilibrium complexity choice as the *tragedy of complexity*. Second, we characterize which products are too complex and which are too simple. Perhaps counterintuitively, products that are relatively well understood tend to be the ones that are too complex relative the complexity a planner would choose, whereas products that are not well understood tend to be too simple. Third, we provide a set of comparative statics for equilibrium complexity. Among other things, this analysis reveals that equilibrium complexity is more likely to be excessive when available attention (per good) is abundant. This leads to a *complexity paradox*: Rather than helping consumers deal with complexity, increases in information processing capacity can lead to excessive complexity. The converse of this insight leads to the *curse of variety*: As the number of differentiated goods competing for a given amount of consumer attention increases, this can lead to an inefficient dumbing down of products.

In our model, a consumer with limited attention purchases goods from a number of differentiated producers. We model limited attention by assuming that the consumer has a fixed time budget that she allocates across all goods. producers have market power, so that they extract a share of the surplus generated by their good, and non-cooperatively choose the complexity of the good they are selling.

The consumer's valuation of a good consists of two components. First, it directly depends on the good's complexity. This captures that, all else equal, a more complex good can be worth more to the consumer, for example, because of additional features, functionality or customization. Second, the consumer's valuation is higher the more time she spends on understanding the good. Therefore, as in Becker (1965), the consumer's time acts as an input to the value of consumption goods. In particular, a deeper understanding allows the consumer to make better use of the good (e.g., its features, functionality, or customization), and more complex goods require more attention to achieve the same depth of understanding. Specifically, we assume that when the complexity of a good doubles, it takes the consumer twice as much time to reach the same depth of understanding. The consumer's understanding of a good then depends on the *effective attention* (time spent divided by complexity) paid to the good. The assumption that more complex goods are harder to understand leads to a trade-off: A more complex good is potentially more valuable to the consumer, but the consumer also has to pay more attention to reach the same depth of understanding.

When producers choose the complexity of their good, they internalize that consumers respond by adjusting the amount of attention they allocate to the good. Because producers extract a fraction of the surplus generated by the good, they have an incentive to distort the good's complexity in the direction that increase the amount of attention paid to it by the consumer. In doing so, producers do not internalize that attention is a common resource—an increase in attention paid to their good necessarily corresponds to a decrease in attention paid to other goods. These other goods decrease in value, resulting in an *attention externality*.

While in principle the direction of the externality depends on the characteristics of all goods in the economy, we show that there is a simple test to determine whether a producer has an incentive to increase or decrease the complexity of his own product relative to the social optimum: A producer has an incentive to increase the complexity of his good beyond the level that a planner would choose if and only if a consumer who keeps the attention paid to the good fixed is worse off after the change. This effect, which we call the *software update effect* (i.e., the feeling that additional features have made a product worse) is therefore a red flag for an inefficient increase in complexity.

Equilibrium complexity features too much or too little complexity depending on whether producers attract attention away from other goods by raising or lowering the complexity of their own good. Our analysis reveals that, in general, this leads to a distortion towards intermediate complexity. The reason is that goods of intermediate complexity attract the most attention from consumers: Simple goods are well understood even when little attention is paid to them, whereas very complex goods cannot be understood even if the consumer devotes all of her attention to them, giving consumers an incentive to devote their attention predominantly to intermediate complexity goods. Producers therefore distort complexity towards intermediate levels—increasing the complexity of goods that should be relatively simple and decreasing the complexity of goods that should be relatively complex. This leads to a pattern where goods that are well understood in equilibrium (i.e., high effective attention) end up being too complex. For example, consumers understand smartphones and software well, whereas banks have to understand regulation issued by various regulators, indicating that these are likely overly complex in equilibrium. On the other hand, goods that are not well understood (low effective attention) are too simple. For example, voters do not understand complicated policy issues, implying that these are oversimplified by media and politicians in equilibrium.

Our model generates a number interesting comparative statics. For example, paradoxically goods tend to be overly complex precisely when the consumer has a relatively large attention budget. Therefore, rather than helping consumers deal with the complexities of everyday life, improvements in information processing capacity may act as a driver of excessive complexity—the *complexity paradox.* For example, this may explain why instructors of MBA courses, competing for relatively small amount of time students can devote to course, end up oversimplifying courses, whereas instructors of PhD courses react to the larger amount of time that PhD students devote to coursework by making their courses overly complex and difficult. In contrast, when more goods compete for a fixed amount of consumer attention, goods can end up being inefficiently simple, an effect we call the *curse of variety*. Our model therefore provides a potential explanation for why the recent increase of online and social media outlets has gone hand in hand with a dumbing down of content.

Related literature. By viewing time as an input to the value of consumption goods, our approach to modeling complexity builds on the classic work of Becker (1965). We extend this framework by introducing complexity choice. The choice of complexity affects the value of the good directly, but also changes how the consumer transforms her time into understanding the good. By assuming a limited time budget for the consumer, our framework captures that complexity is inherently tied to bounded rationality (Brunnermeier and Oehmke, 2009) and inattention (Gabaix, 2019). The constraint on the consumer's time serves a role similar to information processing constraints models of inattention (see Sims 1998, 2003). The interaction of complexity with the consumer's attention budget also differentiates our work from models of quality choice, as analyzed by a literature going back to Spence (1975).

Our approach to complexity differs from most of the existing literature, which has focused on complexity as a means to obfuscate in order increase market power or influence a consumer's purchasing decision (Carlin 2009, Carlin and Manso 2010, Piccione and Spiegler 2012, Spiegler 2016, Hefti 2018, and Asriyan et al. 2019). In contrast to this literature, in our model complexity is value enhancing, at least potentially. Moreover, in our framework the cost of complexity is not an increase in market power or a distortion in the consumer's purchasing decision. Rather, it manifests itself as an externality that the complexity of one good imposes on the equilibrium value of other goods. A complementary interpretation of complexity is that of Basak and Buffa (2017), who analyze how introducing new more complex operations leads to operational risk.

A key aspect of our paper, competition for attention, is studied also by Bordalo et al. (2016). In contrast to our paper, their focus is on the salience of certain product attributes: Consumer attention can be drawn to either price or quality, resulting in equilibria that are price- or quality-salient. Despite the difference in focus, their analysis shares with ours an attention externality across goods. Our analysis is also related to competition for attention by producers (De Clippel et al. 2014) media outlets (Chen and Suen 2019). Liang et al. (2019) analyzes the incentives to provide precise information, while Anderson and de Palma (2012) shows that competition for attention can lead to information overload and excessive advertising. The key difference to these papers is that we link this competition for attention to the complexity of the content provided. Finally, our work is related to the literature on providing default options, see Choi et al. (2003). Specifically, privately optimal excess complexity may explain why producers are often unwilling to provide default options that would make the product less time consuming to use.

2 Model Setup

We consider an economy with N producers (he) and a single consumer (she). Goods are differentiated and there is one producer per good $i \in \{1, ..., N\}$, each endowed with an indivisible unit of good i. Because goods are differentiated, producers have some market power, which we capture in reduced form by assuming that producer i can extract a share $\theta_i \in (0, 1)$ of the value v_i of good i, while the consumer receives the remaining share $1 - \theta_i$.[1]

[1] For now we simply assume this sharing rule. We provide a more detailed discussion of this and other assumptions in Section 3.5.1. In Appendix B, we provide an alternative model in which the surplus sharing between consumer and producer is determined by an equilibrium price for the good.

The key decision for each producer is to choose the complexity c_i of the good he sells. While complexity has no direct cost (or benefit) for the producer, complexity matters because it affects the value of the good.[2] On the one hand, complexity can add value, for example, when it arises as a byproduct of customization that caters the consumer's specific needs. On the other hand, realizing the full value of a more complex good requires attention from the consumer, who needs to devote time to understand a more complex good.[3] The total value of a complex good therefore arises from the combination of its characteristics and the time the consumer allocates to the good. In this respect, our paper builds on classic work on time as an input into the utility derived from market goods pioneered by Becker (1965).

To capture these features of complexity more formally, we assume that the value to the consumer of consuming a unit of good i with complexity c_i, having allocated t_i units of time, is given by

$$v_i \left(c_i, \frac{t_i}{c_i} \right), \tag{1}$$

which we assume is twice continuously differentiable in both arguments. The first argument of $v_i(\cdot, \cdot)$ captures that the value of the good depends directly on the complexity of the good. We assume that for sufficiently low levels of complexity $\frac{\partial v_i}{\partial c_i} > 0$, such that *ceteris paribus* some complexity raises the value of the good. However, as a good becomes more complex, the direct benefit of complexity exhibits diminishing marginal returns, $\frac{\partial^2 v_i}{\partial c_i^2} < 0$. At some point, the marginal direct effect of complexity on value could even turn negative.

The second argument of $v_i(\cdot, \cdot)$ reflects that the value of the good increases with the consumer's understanding of the good. How well the consumer understands the good depends on how much attention she devotes to the good as well as on the good's complexity. In particular, as complexity increases, a unit of attention becomes less valuable. To capture this effect, we assume that the

[2] Our results would be similar if complexity had a direct benefit for the producer (e.g., by reducing litigation costs) instead of increasing the value of the good for the consumer.

[3] Attention may be devoted to the good before purchase (e.g., figuring out the specific features of a more complex product) or during the use of the good (e.g., when the use of a more complex good is more time consuming). Our model can accommodate both interpretations.

consumer's understanding is determined by effective attention, which we define as time spent on the good divided by the good's complexity, t_i/c_i (we sometimes simply denote effective attention by $e_i \equiv t_i/c_i$). Therefore, a good that is twice as complex takes twice as much time to understand (e.g., a contract that is twice as long takes twice as much time to read). We make standard assumptions on the effect of understanding on the value of the good: All else equal, a deeper understanding increases the value of the good to the consumer, $\frac{\partial v_i}{\partial(t_i/c_i)} > 0$, but with diminishing marginal returns, $\frac{\partial^2 v_i}{\partial(t_i/c_i)^2} < 0$. In addition, we make the following simplifying assumption:

Assumption 1. *The value of good i is bounded from below in the consumer's effective attention: $v_i(c_i, 0) > 0$.*

Assumption 1 implies that good i is valuable even when consumers pay no attention to it, and guarantees that all goods are consumed in equilibrium, which simplifies the analysis. Given that the consumer purchases all goods, the key decision faced by the consumer is how much attention $t_i \geq 0$ to allocate to each of these goods. In making her decision, the consumer takes the complexity of each good as given, but takes into account that she receives a share $1 - \theta_i$ of the value v_i generated by good i.[4] The key constraint faced by the consumer is that her attention is limited. Specifically, the consumer has a fixed amount of time T that she can allocate across the N goods. One interpretation of this limited attention constraint is that it introduces an element of bounded rationality that is required to make complexity meaningful (Brunnermeier and Oehmke, 2009). Finally, we assume that the consumer's utility is quasi-linear in the benefits derived from the N goods and wealth, and that the consumer is deep pocketed. This assumption implies that our results are driven by the consumer's attention constraint (introduced in more detail below) rather than a standard budget constraint.

[4]We discuss the underlying timing assumptions in more detail in Section 3.5.1

3 The Tragedy of Complexity

In this section, we present the main conceptual result of our paper: equilibrium complexity is generally inefficient. We solve the model by backward induction. We first characterize the consumer's attention allocation problem for given product complexities. We then derive and contrast the complexities chosen by profit-maximizing producers' and a benevolent social planner.

3.1 The Consumer's Problem

As discussed above, given Assumption 1, the consumer receives positive utility from consuming good i even when paying no attention to it. It is therefore optimal for the consumer to purchase all N goods. The consumer's maximization problem then reduces to choosing the amount of attention she allocates to each good, taking as given complexity c_i,

$$\max_{t_1,..t_N} \sum_{i=1}^{N} (1-\theta_i) \cdot v_i \left(c_i, \frac{t_i}{c_i} \right), \tag{2}$$

subject to the attention constraint[5]

$$\sum_{i=1}^{N} t_i \leq T. \tag{3}$$

Using λ to denote the Lagrange multiplier on the attention constraint, the consumer's first-order condition is given by

$$(1-\theta_i) \cdot \frac{\partial v_i \left(c_i, \frac{t_i(c_1,\ldots,c_N)}{c_i} \right)}{\partial \left(\frac{t_i}{c_i} \right)} \cdot \frac{1}{c_i} \leq \lambda, \tag{4}$$

which holds with equality when $t_i > 0$. The first-order condition states that, if the consumer pays attention to the good, the marginal value of an additional unit of attention paid to good i must equal the shadow price of attention λ. Because the consumer can only extract a fraction $1-\theta_i$ of the value

[5] By rewriting this constraint as $\sum_{i=1}^{N} \frac{t_i}{c_i} \cdot c_i \leq T$ (i.e., multiplying and dividing by c_i), we see that one can think of the attention constraint as a standard budget constraint, where the good purchased by the consumer is effective attention t_i/c_i, the price of effective attention for good i is the complexity of that good, c_i, and the consumer's wealth is her endowment of time, T. We show in Section 3.5.3 that this interpretation can be useful because it allows us to draw parallels to classic results from consumer demand theory.

generated by good i, all else equal, it is optimal to allocate more time to goods for which this fraction is large.

3.2 Equilibrium Complexity: The Producer's Problem

We now turn to the producer's choice of complexity. Producer i's objective is to maximize profits, given by a fraction θ_i of the value generated by good i. The producer's only choice variable is the complexity c_i of his good. However, in choosing c_i, the producer anticipates that the chosen complexity affects the amount of attention allocated to the good by the consumer. Like a Stackelberg leader, the producer internalizes that the attention the consumer pays to his good, $t_i(c_1, \ldots, c_N)$, is function of c_i. The producer's objective function is therefore

$$\max_{c_i} \theta_i \cdot v_i\left(c_i, \frac{t_i(c_1, \ldots, c_N)}{c_i}\right), \tag{5}$$

with an associated first-order condition of

$$\theta_i \cdot \frac{d}{dc_i} v_i\left(c_i, \frac{t_i(c_1, \ldots, c_N)}{c_i}\right) \leq 0, \tag{6}$$

which holds with equality whenever $c_i > 0$. Assuming that c_i is indeed an internal solution[6] and taking the total derivative, the first-order condition (6) can be rewritten as

$$\frac{\partial v_i\left(c_i, \frac{t_i}{c_i}\right)}{\partial c_i} = \frac{\partial v_i\left(c_i, \frac{t_i}{c_i}\right)}{\partial\left(\frac{t_i}{c_i}\right)} \cdot \frac{t_i}{c_i{}^2} - \frac{\partial v_i\left(c_i, \frac{t_i}{c_i}\right)}{\partial\left(\frac{t_i}{c_i}\right)} \cdot \frac{1}{c_i} \cdot \frac{\partial t_i}{\partial c_i}. \tag{7}$$

This condition states that, from the producer's perspective, the optimal level of complexity equates the marginal increase in value from additional complexity (the left-hand side of (7)) to the value reduction that arises from lower levels of effective attention holding the consumer's attention to the good constant (the first term on the right-hand side), net of the change in the good's value that arises

[6] A sufficient condition for $c_i > 0$ is that a standard Inada condition holds with respect to complexity.

from the consumer's change in attention paid to good i in response to an increase of the complexity of that good (the second term on the right-hand side). In equilibrium, this first-order condition must hold for each producer i.

The key observation is that producers take into account that changing the complexity of their good affects the amount of attention that the consumer will allocate to the good, as indicated by the $\frac{\partial t_i}{\partial c_i}$ term in Equation (7). Producers perceive additional attention that is paid to their good in response to a change in complexity as a net gain, even though in aggregate changes in attention are merely a reallocation—any additional attention paid to good i would otherwise be allocated to goods of other producers. Because the producer of good i is essentially diverting attention away from other goods, we refer to this as the *attention grabbing* effect.

Using the consumer's first-order condition (4), we can rewrite the producer's optimality condition (7) in terms of the shadow price of attention λ, which for $c_i > 0$ and $t_i > 0$ yields

$$\frac{\partial v_i\left(c_i, \frac{t_i}{c_i}\right)}{\partial c_i} = \frac{\lambda}{1-\theta_i}\left(\frac{t_i}{c_i} - \frac{\partial t_i}{\partial c_i}\right). \tag{8}$$

Expressing the first-order condition in this more concise way is useful when comparing the producer's first-order condition to the planner's optimality condition derived in the next section.

3.3 Optimal Complexity: The Planner's Problem

We now turn to the planner's choice of product complexity. The key difference compared to the producer's profit-maximization problem described above is that the planner takes into account that the consumer reallocates attention across *all* goods. Therefore, the planner internalizes the effect of a change in the complexity of good i not only on the value of good i but also, via the consumer's attention reallocation, on all other goods $j \neq i$.

More formally, the planner chooses the product complexities of all N goods to maximize total surplus,

$$\max_{c_1,\ldots c_N} \sum_{i=1}^{N} v_i \left(c_i, \frac{t_i(c_1, \ldots, c_N)}{c_i} \right). \tag{9}$$

Following the same steps as in the derivation of the producer's first-order condition (including the assumption that c_i^* is an internal solution), the optimality condition for the planner's complexity choice c_i^* for good i is given by

$$\frac{\partial v_i\left(c_i, \frac{t_i}{c_i}\right)}{\partial c_i} = \frac{\partial v_i\left(c_i, \frac{t_i}{c_i}\right)}{\partial\left(\frac{t_i}{c_i}\right)} \cdot \frac{t_i}{c_i^2} - \sum_{j=1}^{N} \frac{\partial v_j\left(c_j, \frac{t_j}{c_j}\right)}{\partial\left(\frac{t_j}{c_j}\right)} \cdot \frac{1}{c_j} \cdot \frac{\partial t_j}{\partial c_i}. \tag{10}$$

This optimality condition highlights the difference between the planner's solution and the producers' privately optimal complexity choice characterized by Equation (8). In particular, whereas the producer of good i only takes into account the change in the valuation of good i that results from the reallocation of attention to or from good i, the planner takes into account the changes in valuation that result from the reallocation of attention across *all* goods, resulting in $N-1$ additional $\frac{\partial t_j}{\partial c_i}$ terms on the right hand side. The producer's privately optimal complexity choice therefore generally differs from the planner's solution—the reallocation of attention from other goods to good i represents an externality that is not taken into account by the producer of good i.

As before, using the consumer's first-order condition (4), we can rewrite the planner's optimality condition (10) in terms of the shadow price of attention λ, which yields

$$\frac{\partial v_i\left(c_i, \frac{t_i}{c_i}\right)}{\partial c_i} = \frac{\lambda}{1-\theta_i} \cdot \left(\frac{t_i}{c_i} - \frac{\partial t_i}{\partial c_i} \right) - \sum_{j \neq i} \frac{\lambda}{1-\theta_j} \cdot \frac{\partial t_j}{\partial c_i}, \tag{11}$$

where the second term on the right hand side captures the externality that is neglected by the producer.

A particularly simple case arises when all producers have equal market power, such that $\theta_i = \theta$. In this case, the planner's optimality condition reduces to

$$\frac{\partial v_i\left(c_i, \frac{t_i}{c_i}\right)}{\partial c_i} = \frac{\lambda}{1-\theta} \cdot \left(\frac{t_i}{c_i} - \sum_{j=1}^{N} \frac{\partial t_j}{\partial c_i}\right) = \frac{\lambda}{1-\theta} \cdot \frac{t_i}{c_i}, \tag{12}$$

where the last step makes use of the fact that, when viewed across all goods, attention is merely reallocated (i.e., $\sum_{j=1}^{N} t_j = T$ implies that $\sum_{j=1}^{N} \frac{\partial t_j}{\partial c_i} = 0$).

3.4 The Complexity Externality

A comparison between the producer's and the planner's first-order condition reveals that there is an externality in complexity choice. Under equal market power of producers ($\theta_i = \theta$), a simple comparison of the first-order conditions (8) and (12) shows that the producer of good i has an incentive to deviate from the socially optimal level of complexity c_i^* whenever at c_i^* the attention grabbing effect is nonzero, $\frac{\partial t_i}{\partial c_i} \neq 0$. When $\frac{\partial t_i}{\partial c_i} > 0$ the producer of good i has an incentive to increase the complexity of his good beyond the socially optimal level, whereas when $\frac{\partial t_i}{\partial c_i} < 0$ the producer if good i wants to decrease complexity below the socially optimal level. In both cases, the direction of the externality is driven by the desire to divert the consumer's attention away from other goods. While this result is seen most easily under equal market power for producers, the result is true also when market power differs across producers, as stated formally in the following proposition.

Proposition 1. The Tragedy of Complexity. *Starting from the planner's solution $(c_1^*, ...c_N^*)$ and keeping the complexity of all other goods $j \neq i$ fixed at c_j^*, the producer of good i*

(i) has an incentive to increase complexity c_i above its optimum c_i^* if $\frac{\partial t_i^*}{\partial c_i} > 0$;

(ii) has an incentive to decrease complexity c_i below its optimum c_i^* if $\frac{\partial t_i^*}{\partial c_i} < 0$;

(iii) has no incentive to change complexity c_i from its optimum c_i^* if $\frac{\partial t_i^*}{\partial c_i}$=0.

Proposition 1 states that the complexity externality has the same sign as the attention grabbing effect: Locally, producers have an incentive to increase the complexity of their good beyond the optimal level if consumers respond by increasing the amount of attention paid to the good. In contrast, if consumers respond by decreasing the amount of attention allocated to the good when its complexity increases, producers have a local incentive to reduce the complexity of their product below the socially optimal level. Note, however, that the equilibrium distortion is not necessarily in the same direction as the local incentive to distort starting from the socially optimal complexity due to the equilibrium feedback once other producers have reacted. We provide a full analysis of equilibrium complexity choices for a specific functional form for $v(\cdot,\cdot)$ in Section 5.

Proposition 1 characterizes the externality in complexity choice in terms of the attention grabbing effect, $\frac{\partial t_i}{\partial c_i}$. However, there are a number other sufficient statistics that can be used to characterize the direction of the externality. As stated in Lemma 1 below, one can equivalently look at (1) the effect of a change in complexity on the shadow cost of attention, (2) the attention grabbing effect holding fixed the shadow cost of attention, and (3) a complementarity condition between complexity and attention.

To state these results concisely, it is useful to introduce some additional notation. First, at times it will be useful to write attention as a function of the good's own complexity and the shadow cost of attention (based on Equation (4)). We will denote this function by $\tilde{t}_i(c_i, \lambda)$. Second, for the last equivalence result in Lemma 1, we rewrite the value of good i in terms of attention instead of effective attention (i.e., we define $\tilde{v}(c,t) = v(c, t/c)$).

Lemma 1. Attention Grabbing: Equivalence Results. *For any given vector of product complexities $(c_1, \ldots, c_N)$, the following have the same sign:*

(i) the attention grabbing effect for good i, $\frac{\partial t_i(c_1,..c_N)}{\partial c_i}$*;*

(ii) the effect of good i's complexity on the shadow cost of attention, $\frac{\partial \lambda(c_1,..c_N)}{\partial c_i}$*;*

(iii) the attention grabbing effect for good i keeping the shadow cost of complexity fixed, $\frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial c_i}\big|_{\lambda}$*;*

(iv) the complementarity (or substitutability) of attention and complexity, $\frac{\partial^2 \tilde{v}(c_i,t_i)}{\partial c_i \partial t_i}$*.*

Lemma 1 contains some useful intuition for the attention externality. For example, statements (i) and (ii) imply that the condition under a producer increases complexity above the efficient level given in Proposition 1 is equivalent to the statement that, at the optimal level of complexity, the shadow price of attention increases when complexity is increased. Through their complexity choice, producers drive up the shadow price of attention, which reduces attention allocated to other goods. The observation that the externality works through the shadow price also highlights the importance of the assumption of limited attention. If attention could be bought or sold at a fixed cost (i.e., if λ were independent of the producers' complexity choices), there would be no externality, because increasing the amount of attention allocated to one good would not mean that the attention paid to other goods has to diminish. Statement (iv) in Lemma 1 provides a useful microeconomic interpretation of the complexity externality: There is an incentive for producers to increase complexity beyond the optimal level when attention and complexity are complements. In contrast, when attention and complexity are substitutes, producers have an incentive to decrease complexity below the optimal level.

Even though the complexity externality can lead to too much or too little complexity, there is a simple diagnostic that allows us to determine whether an increase in complexity is inefficient. To do so, it is useful to divide the producer's first-order condition into two parts, the effect of an increase in complexity *holding fixed the consumer's attention* and the additional effect that results from attention reallocation. Using the notation $\tilde{v}(c, t)$ introduced above, the first-order condition (7) then becomes

$$\frac{\partial \tilde{v}_i(c_i, t_i)}{\partial c_i} + \frac{\partial \tilde{v}_i(c_i, t_i)}{\partial t_i} \cdot \frac{\partial t_i}{\partial c_i} = 0. \tag{13}$$

Because $\frac{\partial \tilde{v}_i(c_i, t_i)}{\partial t_i}$ is strictly positive it follows that the effect of increased complexity, holding fixed the consumer's attention, has the opposite sign to the attention reallocation effect $\frac{\partial t_i}{\partial c_i}$. This leads to the following proposition.

Proposition 2. The Software Update Effect. *Producer i has a local incentive to increase complexity beyond its optimal level ($\frac{\partial t_i}{\partial c_i} > 0$) if and only if the value of good i to the consumer decreases when time allocated to the good is held constant, $\frac{\partial \tilde{v}_i(c_i, t_i)}{\partial c_i} < 0$.*

Proposition 2 explains a familiar phenomenon: Often an updated product initially appears worse than before. For example, following the release of a new version of Excel, a user complained: *"I hate the new product I bought. It has far too many features that I will never use and cannot get rid of. [...] Why do u have to make things so difficult?"* Another user replied: *"That's normal. Many people found that the new interface in Excel 2007 was a nightmare [... However,] there are so much cool functions and features added. Just take some time to adapt to the new interface."*[7]

Our model reconciles these seemingly contrasting views. Without investing more time it often seems that an updated product has become worse than it was before. Proposition 2 states that these are exactly the circumstances under which a producer has an incentive to choose excessive complexity. Moreover, our model rationalizes why, despite the apparent reduction in value that arises when attention is held constant, the producer engages in this type of behavior. Once we account for the extra attention allocated to the good by the consumer in response to the change in complexity, the value of this particular good increases, and some of this additional value is extracted by the producer. The flip side, not internalized by the producer, is that the extra time allocated to the good is taken away from other goods, so that the valuation of those goods decreases. In fact, the value of these other goods decreases so much that the consumer is worse off overall. In line with the example above, we refer to the result in Proposition 2 as the software update effect.

3.5 Discussion

In this section we provide some further discussion of key assumptions and results. Section 3.5.1 discusses our key modeling assumptions. Section 3.5.2 contrast the tragedy of complexity with the traditional tragedy of commons. Section 3.5.3 uses tools from consumer demand theory to intepret the complexity externality.

[7] https://answers.microsoft.com/en-us/msoffice/forum/all/why-is-excel-so-complicated/a2fc9495-1fb6-4bf0-965a-07c2b037606b (August 14, 2015), last accessed July 14, 2019.

3.5.1 Discussion of Key Modeling Assumptions

In the analysis presented above, we made a number of assumptions to keep the model simple. One key simplifying assumption is that the producer receives a fixed share θ_i of the value of the good. This assumption captures, in reduced form, that producers of differentiated goods have market power that allows them to extract surplus. However, our results do not depend on this share of surplus to be fixed. Rather, the crucial assumption is that the producer can extract *some* of the increase in the value of the good that results when the consumer allocates more attention to it.

In the following, we discuss four settings which this assumption captures well. First, in non-market transactions, our assumption is equivalent to assuming a benevolent producer (or provider) who is interested in the surplus generated by the good. An example for this setting is financial regulators that design regulations to maximize the value generated by the market segments they oversee. Our model then implies that, in a setting with competing regulators (e.g., multiple regulators that oversee a large financial institution, as is common in the U.S.), the complexity of financial regulation generally does not coincide with the social optimum. Second, if producers and consumers bargain over the price of the good, our model is equivalent to a setting in which consumers allocate attention to understand the good before bargaining over the price. In this case, θ_i simply corresponds to producer i's Nash bargaining weight vis-à-vis the consumer. This interpretation applies to many retail financial products, where consumers usually have to allocate attention and make choices before finalizing and signing the contract (e.g., choosing a custom-tailored insurance contract or pension product). Third, in Appendix B we show that our results are unchanged if we replace the exogenous surplus-sharing rule with a setting in which the good's price is determined by market clearing. Specifically, if consumers decide how much of each good to consume, market clearing leads to the same equilibrium conditions for complexity choice as in our model. Fourth, our results would be qualitatively unchanged if instead of receiving a fraction of the good's value, the producer benefits directly from the time devoted to the good t_i. This is a natural assumption when modeling media outlets that provide free content but sell the consumer's attention to advertisers.

Another important assumption we made is that limited attention takes the form of a hard constraint on the consumer's time budget. This is meant to capture the limited amount of time (after work and rest) that a consumer can spend on analyzing and consuming goods. This assumption seems particularly fitting for settings that involve retail consumers, such as retail financial products. In the case of companies, one may argue that by employing more people or purchasing additional IT the company can relax its attention constraint. However, as long as expanding the attention budget is associated with an increasing cost (a reasonable assumption given the technological and organizational costs involved), the implications of such a setting would be equivalent to those under a hard attention constraint. As shown in Lemma 1, the key force that leads to the complexity externality is that complexity choice affects the (shadow) price of attention.

Finally, we make an important assumption about timing: complexity is chosen before the consumer makes her choices. This results in market power for the consumer, similar to that of a Stackelberg leader. Here, the crucial assumption is not the specific timing, but that the consumer cannot choose (or shop around for) the complexity she prefers. In many markets this timing is realistic, given that goods and services are often designed before they reach the market and their complexity cannot be easily altered afterwards.

3.5.2 The Tragedy of Complexity and the Tragedy of the Commons

The difference between equilibrium complexity and the planner's solution has parallels with the classic tragedy of the commons. Like grass on a common grazing meadow, attention is a shared resource. However, in contrast to the classic tragedy of the commons, attention grabbing can manifest itself in too much or too little complexity, depending on whether "overcomplicating" or "dumbing down" leads to an increase in consumer attention paid to a particular product. Yet, whereas the complexity externality can go either way, the scarce resource of attention is always overused irrespective of the direction of the externality. Competition for the consumer's attention implies that the shadow price of attention is higher in equilibrium than it would be under the planner's solution, $\lambda^e \geq \lambda^*$, with strict inequality whenever $c_i^e \neq c_i^*$ for at least one good. In words, the consumer constantly feels short of time when producers compete for her attention.

Proposition 3. The Consumer is Short of Time. *Suppose that equilibrium and optimal complexity differ for at least one good. Then the equilibrium shadow price of attention strictly exceeds the shadow price of attention under the planner's solution, $\lambda^e > \lambda^*$.*

Thus the conventional tragedy-of-commons intuition holds for the fixed-supply common resource used by all goods, attention. The contribution of our paper is to show that the classic tragedy of commons with respect to consumer attention leads to equilibrium complexity that is generically inefficient and can be above or below the efficient the efficient complexity level—the *tragedy of complexity*.[8]

3.5.3 Complexity through the lens of demand theory

The conditions that lead to excess complexity can be cast in the language of consumer demand theory. For simplicity, we demonstrate this in a two-good setting. Rewriting the attention constraint in terms of effective attention, $e_i = \frac{t_i}{c_i}$, we obtain

$$c_1 e_1 + c_2 e_2 = T. \tag{14}$$

This version of the attention constraint shows that we can think of product complexity as the price of a unit of effective attention. Under this interpretation, we can then express the consumer's choice of effective attention for good $i = 1, 2$ as

$$e_i(c_1, c_2, T), \tag{15}$$

[8]What type of policy intervention would solve the tragedy of complexity? According to the above analysis, a regulation that simply aims to reduce complexity is not the right policy. After all, complexity can be too high or too low. Rather, the optimal regulation would have to induce producers to internalize the shadow cost of attention. In principle, this could be achieved via tradable permits for attention, although such a policy seems difficult to implement.

the Marshallian demand for effective attention as a function of the complexity of the two goods, c_1 and c_2, and the attention budget, T.

We can now use standard concepts from consumer demand theory to characterize when excess complexity emerges in equilibrium. Suppose producer 1 increases the complexity of his good. Now consider a Slutsky decomposition that divides the change in effective attention the consumer allocates to good 2, $\frac{\partial e_2(c_1,c_2,T)}{\partial c_1}$, into a substitution effect and an income effect. The substitution effect results in a reallocation of effective attention from good 1 to good 2: When the price of effective attention for good 1 is increased, the consumer optimally increases the effective attention paid to good 2. The income effect, on the other hand, results in a decrease in effective attention paid to both goods. When the income effect outweighs the substitution effect, then the increase in the complexity of good 1 leads to reduction in the effective attention paid to good 2. Because c_2 is unchanged, this implies that t_2 decreases and t_1 increases (because $t_1 + t_2 = T$). Therefore, excess complexity arises ($\frac{\partial t_1}{\partial c_1} > 0$) if and only if the income effect for effective attention outweighs the substitution effect.[9]

Writing the consumer's budget constraint in terms of effective attention also provides useful intuition for the source of the externality in our model. In contrast to the budget constraint in standard consumer theory, the price in (14) is not a market price that is determined in equilibrium, but is chosen directly by the producer of good i.

4 An Explicit Characterization

In this section, we use an explicit functional form for the value of the good to characterize the direction of the complexity externality and the resulting equilibrium. Specifically, we assume that v_i is given

[9]Alternatively, one can show that complexity is excessive when the demand for effective inattention is inelastic. Under the interpretation of complexity as the price of a unit of effective attention, the time spent on good i is equal to the consumer's expenditure on that good (i.e., $t_i = c_i e_i$). A standard result from consumer demand theory is that an increase in the price of good i leads to an increase in the total expenditure on that good if and only if the own-price demand elasticity of that good is smaller than one, $\eta_i = \left|\frac{\partial e_i}{\partial c_i}\frac{c_i}{e_i}\right| < 1$.

by:[10]

$$v_i\left(c_i, \frac{t_i}{c_i}\right) = w_i \cdot \left(f_i(c_i) + \delta_i \cdot \frac{\frac{t_i}{c_i}}{1 + \frac{t_i}{c_i}}\right). \tag{16}$$

Under this functional form, the direct benefit from complexity is given by $f_i(c_i)$. In addition to this direct benefit, the value of good is increasing in the consumer's understanding of the good, captured by the effective attention, t_i/c_i. The remaining parameters are introduced to perform comparative statics: w_i captures the utility weight of good i in the consumer's consumption basket, whereas δ_i captures the importance of understanding the good. This functional form satisfies all assumptions we made in Section 2.

In conjunction with a quadratic benefit function, $f_i(c_i) = \alpha_i \cdot c_i - c_i^2$, the above functional form allows us to solve for equilibrium attention allocation and complexity in closed form. The parameter α_i captures the direct benefit of complexity. Note that the quadratic nature of the benefit function implies increasing complexity beyond some level reduces the value of the good to the consumer. Therefore, even without a constraint on consumer attention, the optimal level of complexity of good i is finite.[11]

The key to signing the complexity externality is to determine how attention allocated to good i changes when the producer changes the complexity of the good, keeping the complexity of all other goods unchanged. Mathematically, we are therefore interested in the shape of the function $t_i(c_i)$. Recall from Lemma 1 that holding λ fixed does not change the sign of the slope of $t_i(c_i)$, so that in order to sign the externality it is sufficient to characterize the slope of $\tilde{t}_i(c_i, \lambda)$ with respect to c_i. Focusing on $\tilde{t}_i(c_i, \lambda)$ is convenient because it allows us to derive an explicit expression for the amount of attention paid to good i. Substituting the functional form (16) into the consumer's first-order condition (4), we find that

$$\tilde{t}_i(c_i, \lambda) = \max\left(0, \sqrt{\frac{c_i \cdot \delta_i \cdot (1 - \theta_i) \cdot w_i}{\lambda}} - c_i\right). \tag{17}$$

[10] We make this specific functional assumption because it greatly facilitates the exposition. We can show that the qualitative results in this section are true more generally, see Footnote 13.

[11] Without a constraint on attention, the value of the good would be maximized by choosing $c_i = \frac{\alpha_i}{2}$.

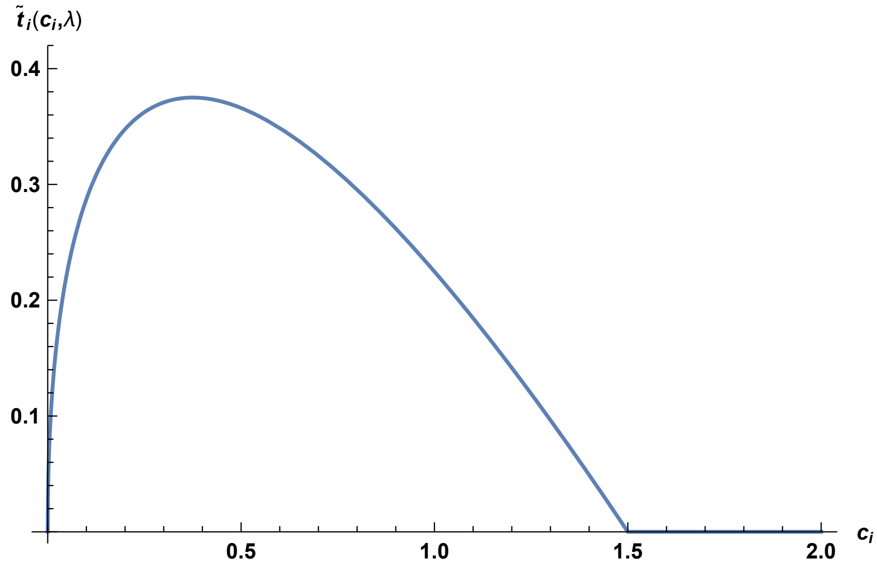
Figure 1 plots $\tilde{t}_i(c_i, \lambda)$ as a function of c_i, holding the shadow cost of attention λ fixed. As we can see, consumer attention follows a hump shape. For low levels of complexity, an increase in the complexity of good i leads to an increase in the attention paid to the good ($\frac{\partial t_i}{\partial c_i} > 0$). In this region, the producer of good i has an incentive to increase the complexity of his good. For higher levels of complexity, the direction of the externality reverses, and an increase in the complexity of good i leads to a reduction in attention paid to good i ($\frac{\partial t_i}{\partial c_i} < 0$), so that the producer of good i has an incentive to decrease the complexity of his good. Finally, above some critical level of complexity, the consumer pays no attention to good i (even though she still buys the good).[12] Even if the consumer were to allocate all of her attention to the good, she would not understand it well, so that it becomes preferable for the consumer to focus her attention on other goods. In this "giving up" region there is no externality, so that equilibrium complexity coincides with the social optimum.[13]

The hump shape illustrated in Figure 1 implies that the producer of good i has an incentive to make goods that are relatively simple too complex and goods that are relatively complex too simple. Of course, whether a good is relatively simple or complex (i.e., on the upward-sloping or downward-sloping segment of $t_i(c_i)$) is an equilibrium outcome that depends on all the parameters of the model. Nonetheless, there is a simple way to determine whether a good is relatively simple or complex. Since $t_i(c_i)$ is hump-shaped in c_i, a given level of consumer attention t_i can be achieved in two ways: by choosing a low level of complexity or a high level of complexity. While the consumer allocates the same amount of attention to these two goods, the simpler one is well understood (high effective attention $\frac{t_i}{c_i}$), whereas the more complex one is less well understood (low effective attention $\frac{t_i}{c_i}$). Therefore,

[12] Real world examples of this phenomenon include terms and conditions associated with online purchases or software updates, both classic instances of information overload. See Brunnermeier and Oehmke (2009).

[13] The result that attention choice follows a hump shape holds even outside of the specific functional form assumed in this section. One can show that $t_i(c_i)$ is increasing for small c_i, decreasing for higher c_i and equal to zero above some level as long as the value of the good remains bounded even when the depth of understanding is infinite (i.e., $v(c_i, \infty) < \infty$) and the value of the good is additively separable in the benefit of complexity and the depth of understanding (i.e., $\frac{\partial^2 v_i}{\partial c_i \partial (t_i/c_i)} = 0$). The assumption $v_i(c_i, \infty) < \infty$ ensures that very simple goods ($c_i \to 0$) have finite value, even when these goods are extremely well understood ($t_i/c_i \to \infty$). The result that the consumer gives up for high levels of c_i follows from Assumption 1: Given that v_i is bounded from below, the marginal value of attention is bounded from above even when $t_i = 0$.

Figure 1: **Attention as a function of complexity (with fixed shadow cost of attention)**



This figure illustrates $\tilde{t}_i(c_i, \lambda)$, the consumer's attention choice as a function of the complexity of good i, c_i holding fixed the shadow price of attention λ. Attention allocation is hump shaped: Initially, $\tilde{t}_i(c_i, \lambda)$ is increasing, then decreasing, and at some point the consumer chooses to pay no attention to good i. Parameters: $\delta_i = 0.9$, $w_i = 1$, $\alpha_i = 2$, $\theta_i = 0.5$, $\lambda = 0.3$.

goods that receive relatively high effective attention lie on the upward-sloping part of the $t_i(c_i)$ curve, whereas goods that receive relatively low effective attention are located on the downward-sloping part.

To see this formally, recall from Lemma 1 that $\frac{\partial t_i(c_1,..c_N)}{\partial c_i}$ has the same sign as $\frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial c_i}|_\lambda$. Assume that we are in the interesting case $t_i > 0$. Using Equation (17), the condition $\frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial c_i}|_\lambda > 0$ can be rewritten as

$$\frac{(1-\theta_i)\cdot\delta_i\cdot w_i}{4\cdot c_i\cdot\lambda} > 1 \tag{18}$$

Then, noting that we can rewrite Equation (17) as

$$\frac{t_i}{c_i} = 2\cdot\sqrt{\frac{(1-\theta_i)\cdot\delta_i\cdot w_i}{4\cdot c_i\cdot\lambda}} - 1, \tag{19}$$

it becomes apparent that $\frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial c_i}\Big|_\lambda > 0$ if and only if $\frac{t_i}{c_i} > 1$. Therefore, producers have an incentive to overcomplicate those goods that are relatively well understood (effective attention larger than one) by the consumer.[14]

Proposition 4. Complexity and the Depth of Understanding. *When goods are ex ante heterogeneous, the producer has an incentive to*

(i) overcomplicate goods that are relatively well understood in the planner's solution, $\frac{t_i^*}{c_i^*} > 1$*;*

(ii) oversimplify goods that are not well understood in the planner's solution, $\frac{t_i^*}{c_i^*} < 1$*.*

Proposition 4 provides a simple characterization of the distortion of product complexity. Goods that in the planner's solution are relatively simple end up being too complex in equilibrium, whereas goods that in the planner's solution are complex end up being too simple (or dumbed down). This result stems from the fact that it is goods of intermediate complexity that attract the most attention from consumers: Simple goods are well understood even when little attention is paid to them, whereas very complex goods cannot be understood even if the consumer devotes all of her attention to them. To attract the consumer's attention, the producer therefore distorts complexity towards intermediate levels—increasing the complexity of goods that should be relatively simple and decreasing the complexity of goods that should be relatively complex.

The distortion towards intermediate complexity generates a number of interesting predictions. For example, based on Proposition 4, goods that are plausibly too complex include smartphones and checking accounts. These goods are relatively simple and arguably well understood under the planner's solution, but producers have an incentive to complexify them. In the case of smartphones, this manifests itself in the development of additional apps that turn the phone into a time sink. In the case of checking accounts, banks add contingent fees, promotional interest rates, and other features that makes checking accounts more complex than they should be. In contrast, our model implies that

[14] Under the functional form (16), effective attention larger than one implies that the consumer realizes more than half of the potential benefit of understanding the good. This can be seen by noting that, for $\frac{t_i}{c_i} = 1$, $\delta_i \cdot \frac{\frac{t_i}{c_i}}{1+\frac{t_i}{c_i}}$ becomes $\frac{\delta_i}{2}$.

intricate policy debates that are hard to understand may end up being oversimplified by politicians and the media. For example, despite the apparent complications, the question of whether the UK should leave the EU was often dumbed down to how much the UK contributes to the EU budget.

5 Equilibrium Complexity and Comparative Statics

In the analysis so far, we used first-order conditions to characterize the producer's incentives to deviate from the socially optimal level of complexity. In this section, we characterize the full equilibrium, starting with ex-ante homogeneous goods in Section 5.1, and then extending the analysis to ex-ante heterogeneous goods in Section 5.2.[15] In order to be able to explicitly solve for the equilibrium, we continue to assume that the value of the good takes the functional form given in Equation (16).

5.1 Ex-Ante Homogeneous Goods

When goods are ex-ante homogeneous, the producer's first-order condition (8) and the planner's first-order condition (11) can be written as follows.

Lemma 2. First Order Conditions with Ex-Ante Homogeneous Goods. *Assume the value of good i takes the functional form (16). In a symmetric equilibrium with ex-ante homogenous goods, the equilibrium first-order condition is*

$$f'(c) = \left[\frac{T}{N \cdot c} - \frac{N-1}{2 \cdot N} \cdot \left(\frac{T}{N \cdot c} - 1\right)\right] \cdot \frac{\delta}{c \cdot \left(1 + \frac{T}{N \cdot c}\right)^2}, \tag{20}$$

whereas the planner's first order condition is

$$f'(c) = \frac{T}{N \cdot c} \cdot \frac{\delta}{c \cdot \left(1 + \frac{T}{N \cdot c}\right)^2}. \tag{21}$$

[15] By ex-ante homogeneous we mean that the parameters that determine the functional form for v (i.e., w, δ, α, and θ) are equal across goods.

Under the quadratic benefit function, the marginal benefit of complexity is $f'(c) = \alpha - 2c$, so that solving for the equilibrium level of complexity requires solving a third-order polynomial. The solution can be expressed in closed form but it is relatively complicated. We therefore present the key features of the equilibrium by comparing the relevant first-order conditions and by plotting the equilibrium solution.

Comparing (20) with (21) shows that equilibrium complexity c^e is higher than planner's choice of complexity if and only if

$$c^e < \frac{T}{N}. \tag{22}$$

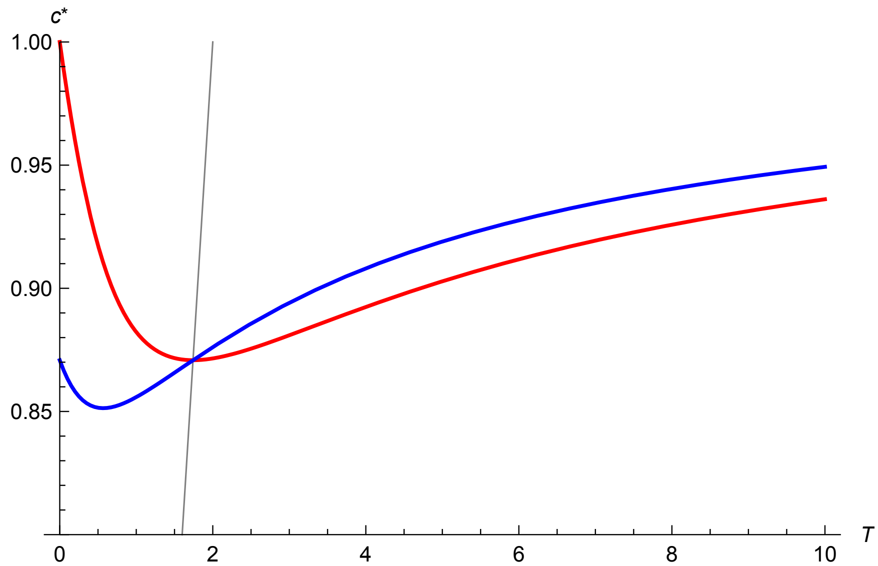
This condition defines a separating hyperplane in the parameter space. On one side of this hyperplane ($c^e < \frac{T}{N}$) complexity is inefficiently high, whereas on the other side ($c^e > \frac{T}{N}$) complexity is inefficiently low. The grey line in Figure 2 illustrates this separating hyperplane for the case of two goods. To the left of the grey line, equilibrium complexity c^e (the blue line) lies below the optimal level of complexity c^* (red line). To the right of the grey line, equilibrium complexity is higher than the optimal level of complexity.

A particularly interesting observation in Figure 2 is that, for relatively high consumer attention budgets T, the equilibrium level of complexity (blue line) lies above that chosen by the planner (red line). Therefore, complexity rises to inefficiently high levels precisely when information processing capacity grows. The figure therefore illustrates a *complexity paradox*: Rather than helping the consumer deal with complexity, increased information processing capacity can be a source of excessive complexity in the economy.[16] The following proposition formalizes this insight and establishes a number of additional comparative statics.

Proposition 5. The Complexity Paradox and Other Comparative Statics. *When goods are ex-ante homogenous and the benefit of understanding a good δ is not too large, all goods receive*

[16] This result confirms the intuition gained from Equation (22): The first order effect of raising T is that it is more likely that equilibrium complexity c^e lies below the separating hyperplane. The adjustment of c^e in response to raising T does not overturn this first order effect.

Figure 2: **Equilibrium and optimal complexity as a function of attention capacity T**



The figure shows equilibrium complexity (blue line) and optimal complexity (red line) as a function of the attention budget T. The grey line illustrates the separating hyperplane defined by Equation (22). Equilibrium and optimal complexity converge to the unconstrained optimal level of complexity of 1 as $T \to \infty$. Homogenous goods with parameters: $N = 2$, $\delta = 0.9$, $w = 1$, $\alpha = 2$, $\theta = 0.5$.

the same amount of attention. Equilibrium complexity is inefficiently high compared to the planner's solution if

(i) the benefit from paying attention δ is large;

(ii) attention T is abundant;

(iii) the direct benefit of complexity α is small;

(iv) the number of goods N is small.

The ranking of equilibrium and optimal complexity does not depend the fraction θ of the surplus that the producer can extract.

In addition to the complexity paradox, another interesting prediction of Proposition 5 is that an increase in the number of goods that the consumer consumes (which one may argue is a natural byproduct of economic development) leads to goods that are overly simple. We call this phenomenon the *curse of variety*. To understand this result, note that in a symmetric equilibrium all goods receive the same amount of attention, $\frac{T}{N}$. Therefore, more goods necessarily lead to less attention paid to each good.[17] Hence, based on the same logic by which lower T leads to overly simplistic goods, so does increasing the number of goods N. By a similar argument, a lower direct benefit of complexity α and a higher cost of inattention δ lead to too much complexity by lowering the equilibrium complexity c_i^e, making condition (22) more likely to hold.

Together with Figure 2, Proposition 5 also highlights the importance of distinguishing between absolute and relative complexity. For example, while a decrease in the attention budget T (or an increase in the number of goods N) leads to overly simple goods relative to the planner's solution, it is not necessarily the case that goods become simpler in an absolute sense. Figure 2 illustrates that decreasing T (equivalent to increasing N) can lead to an increase in absolute complexity. The reason is that when attention is severely limited, goods no goods are well understood, so that it becomes optimal for producers to focus on the direct benefit of complexity.

Finally, note that Proposition 5 assumes that the benefit of understanding the good δ is not too large. This assumption ensures that the equilibrium is symmetric. When δ is large, there is potentially also an asymmetric equilibrium: For high δ and small attention capacity T it can optimal (both in equilibrium and in the planner's solution) to choose complexities asymmetrically across otherwise identical goods: One good is very simple ($c_i = 0$) and receives no attention, whereas the other good is complex ($c_j > 0$) and receives all of the consumer's attention. Therefore, fundamentally similar products can have very different levels of complexity. As usual, the equilibrium does not pin down which good ends up being the complex one.

5.2 Ex-ante Heterogenous Goods

We now consider the case in which goods are allowed to differ in characteristics other than complexity (i.e., w, δ, α, and θ differ across goods). The condition for excess complexity (18) provides intuition for which goods tend to be well understood and therefore too complex: those that have a large utility

[17]Note that this is similar to decreasing the overall attention capacity T for a fixed number of goods

weight (high w_i), those for which the consumer gets to keep more of the value (low θ_i), and those goods for which paying attention is very important (high δ_i). According to condition (18), for these types of goods, the equilibrium level of complexity is likely to be on the upward-sloping part of the $\tilde{t}_i(c_i, \lambda)$ function illustrated in Figure 1. However, while indicative, simply reading these results from Equation (18) is not correct, because equilibrium complexity c_i^e changes when the above parameters change. Proposition 6 therefore formalizes the above intuition and characterizes absolute and relative complexity in an equilibrium setting.

Table 1: **The Complexity Matrix**

	too simple	too complex
simple	low w_i ∼ less important good high θ_i ∼ producer's share high	high δ_i ∼ attention important low α_i ∼ complexity not beneficial
complex	low δ_i ∼ attention not important high α_i ∼ complexity beneficial	high w_i ∼ more important good low θ_i ∼ consumer's share high

Proposition 6. Absolute and relative complexity with heterogenous goods. *Assume that there are two goods ($N = 2$) that are initially identical in all parameters $\{w, \delta, \alpha, \theta\}$, and that at these parameters equilibrium complexity and planner's choice of complexity coincide. Introducing heterogeneity in one of the good-specific parameters $\pi \in \{w, \delta, \alpha, \theta\}$, such that for the first good $\pi_1 = \pi - \epsilon$ and for the second good $\pi_2 = \pi + \epsilon$, for small enough $\epsilon > 0$, the following holds:*

(i) Importance of attention ($\pi = \delta$). The good for which attention is less important (low δ_i) is more complex but too simple relative to the planner's solution. The good for which attention is more important (higher δ_i) is simpler but too complex relative to the planner's solution.

(ii) *Utility weight* ($\pi = w$). *The good with the smaller utility weight* w_i *is simpler in an absolute sense and too simple relative to the planner's solution. The more important good (larger* w_i*) is more complex and too complex relative to the planner's solution.*

(iii) *Producer's share of value* ($\pi = \theta$). *The good for which the producer can extract a larger share of value (higher* θ_i*) is simpler in equilibrium and too simple relative to the planner's solution. The good for which producers can extract less (lower* θ_i*) is more complex and too complex relative to the planner's solution.*

(iv) *Direct benefit of complexity* ($\pi = \alpha$). *The good for which complexity has a larger direct benefit (higher* α_i*) is more complex but too simple relative to the planner's solution. The good for which complexity is less valuable (lower* α_i*) is simpler but too complex relative to the planner's solution.*

The results presented in Proposition 6 lead to a categorization of goods according to the complexity matrix illustrated in Table 1. The complexity matrix categorizes goods based on (i) whether they are simple or complex in an absolute sense (absolute complexity) and (ii) whether they are too simple or too complex relative to the planner's solution (relative complexity). Depending on their characteristics, the complexity matrix assigns goods to four categories according to their absolute and relative complexity. Therefore, while Proposition 4 provides a diagnostic as to whether goods are too complex or too simple based on the observed depth of understanding, Proposition 6 links both absolute and relative complexity to the deep parameters of the model.

Relative to Proposition 4, the complexity matrix implied by Proposition 6 provides additional clues regarding the link between complexity and the deep parameters of the model. For example, based on Proposition 4 we argued that the observation that smartphones are typically well understood by their users implies that, in the context of our model, they are likely too complex. But what drives this result? Using Table 1, we can identify a number of potential reasons. One the one hand, it could be the case that smartphones are too complex because attention is an important component of the value they generate (high δ). However, according to Table 1, in this case it must also be the case that smartphones are simple in an absolute sense. On the other hand, it could be the case that smartphones are too complex because either their weight w_i in the consumer's utility is high or the consumer gets to keep most of the value (high $1-\theta_i$). In this case, Table 1 indicates that smartphones would be too complex as well as complex in an absolute sense.

6 Conclusion

Complexity is an important choice variable for the producers of goods and services, including financial products. Nonetheless, this complexity does not feature in most economic models, and when it does its only role is usually to obfuscate. This paper develops an equilibrium model of product complexity that explicitly allows for both the positive and negative side of complexity—complexity can generate value but it uses the consumer's limited attention. The main result of our analysis is that, in a world in which consumers have limited attention, equilibrium complexity choices generally involve an *attention externality*: When choosing the complexity of their goods, producers do not take into account that attention is a common resource that is shared across products. In equilibrium, producers therefore distort the complexity of their products in order to divert attention from the goods of other producers. Depending on the consumer's reaction to an increase in complexity—does the consumer devote more or less time when a product becomes more complex?— this can lead to *too much or too little complexity* in equilibrium.

Our model yields a number of interesting insight. Perhaps surprisingly, the goods that are likely to be excessively complex are those that, in equilibrium, are relatively well understood by the consumer. We also show that, paradoxically, equilibrium complexity is more likely to be excessive when attention is abundant. Therefore, rather than helping consumers deal with complexity, increases in information processing capacity make it more likely that complexity is excessive—the *complexity paradox*. Finally, with heterogeneous goods, our model allows us to characterize the factors that determine which goods are simple and which complex, both in an absolute sense and relative to the efficient level of complexity.

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A Proofs

Proof of Proposition 1.

The incentives of producer i to change the level of complexity c_i of good i starting from the planner's optimum $(c_1^*, ...c_N^*)$ depend on the difference between the producer's first-order condition (8) and that of the planner (11), both evaluated at the planner's choice $(c_1^*, ...c_N^*)$. We rewrite the difference between the right-hand side of the two first-order conditions as

$$\sum_{j \neq i} \frac{\lambda^*}{1-\theta_j} \cdot \frac{\partial t_j(c_1^*, ..., c_N^*)}{\partial c_i} = \sum_{j \neq i} \frac{\lambda^*}{1-\theta_j} \cdot \frac{d\tilde{t}_j(c_j^*, \lambda(c_1^*, ..., c_N^*))}{dc_i} = \frac{\partial \lambda(c_1^*, ..., c_N^*)}{\partial c_i} \cdot \sum_{j \neq i} \frac{\lambda^*}{1-\theta_j} \cdot \frac{\partial \tilde{t}_j(c_j^*, \lambda^*)}{\partial \lambda}. \quad \text{(A1)}$$

The first step in (A1) uses the fact that we can rewrite the consumer's attention choice $t_j(c_1^*, ..c_N^*)$ as a function of the complexity of good j and the shadow cost of attention λ, $\tilde{t}_j(c_j^*, \lambda(c_1^*, ..c_N^*))$. The second step applies the chain rule, $\frac{d\tilde{t}_j(c_j^*, \lambda(c_1^*, ..c_N^*))}{dc_i} = \frac{\partial \tilde{t}_j(c_j^*, \lambda^*)}{\partial \lambda} \cdot \frac{\partial \lambda(c_1^*, ..c_N^*)}{\partial c_i}$ and moves the first (constant) term outside of the summation sign. λ^* denotes the shadow cost of attention at the planner's solution, $\lambda(c_1^*, ..c_N^*)$.

Note that raising the shadow cost of attention leads to less attention being paid to all goods, because the inverse function theorem implies

$$\frac{\partial \tilde{t}_i(c_i, \lambda)}{\partial \lambda} = \frac{1}{\frac{\partial \tilde{v}_i^2(c_i, \tilde{t}_i(c_i, \lambda))}{\partial \tilde{t}_i^2}} < 0, \quad \text{(A2)}$$

where we used $\tilde{v}_j(c_j, \tilde{t}_j) = v_j(c_j, \frac{t_j}{c_j})$ and $\frac{\partial v_j^2(c_j, t_j)}{\partial {t_j}^2} = \frac{\partial v_j^2\left(c_j, \frac{t_j}{c_j}\right)}{\partial \left(\frac{t_j}{c_j}\right)^2} \cdot \frac{1}{c_j^2} < 0$, which holds by assumption.

Therefore, from (A1) we see that the externality is negative if $\frac{\partial \lambda(c_1^*, ..., c_N^*)}{\partial c_i} > 0$, meaning that the planner's optimum must entail a lower level of c_i, which in turn increases the left hand side of (11) (due to the decreasing benefits of complexity we have assumed). We show that $\frac{\partial \lambda(c_1^*, ..., c_N^*)}{\partial c_i}$ has the same sign as $\frac{\partial t_i(c_1^*, ..c_N^*)}{\partial c_i}$ in the proof of Lemma 1 below. □

Proof of Lemma 1.

Equation (4) implicitly defines the attention allocated to good i, t_i, as $\tilde{t}_i(c_i, \lambda)$. Attention grabbing $\frac{\partial t_i(c_1, ..c_N)}{\partial c_i}$ can then be written as:

$$\frac{\partial t_i(c_1, ..c_N)}{\partial c_i} = \frac{d\tilde{t}_i(c_i, \lambda(c_1, ..c_N))}{dc_i} = \frac{\partial \tilde{t}_i(c_i, \lambda)}{\partial c_i} + \frac{\partial \tilde{t}_i(c_i, \lambda)}{\partial \lambda} \cdot \frac{\partial \lambda(c_1, ..c_N)}{\partial c_i}, \quad \text{(A3)}$$

where the first term is the effect of c_i on t_i keeping λ fixed, while the second term captures the indirect effect through the shadow price of attention λ.

The equilibrium shadow price $\lambda(c_1, ..c_N)$ is implicitly defined by the binding attention constraint

$$T = \sum_j t_j = \sum_j \tilde{t}_j (c_j, \lambda). \tag{A4}$$

Without a specific functional form for v we cannot express $\lambda(c_1, ..c_N)$ explicitly. However, we can take a total derivative of (A4) with respect to c_i to get:

$$0 = \frac{\partial \tilde{t}_i (c_i, \lambda)}{\partial c_i} + \sum_{j=1}^{N} \frac{\partial \tilde{t}_j (c_j, \lambda)}{\partial \lambda} \cdot \frac{\partial \lambda}{\partial c_i}, \tag{A5}$$

from which it follows that

$$\frac{\partial \lambda}{\partial c_i} = \frac{\frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial c_i}}{\sum_{j=1}^{N} -\frac{\partial \tilde{t}_j(c_j,\lambda)}{\partial \lambda}}. \tag{A6}$$

Plugging this into (A3) yields

$$\frac{\partial t_i(c_1, ..c_N)}{\partial c_i} = \frac{\partial \tilde{t}_i(c_i, \lambda)}{\partial c_i} + \frac{\partial \tilde{t}_i(c_i, \lambda)}{\partial \lambda} \cdot \frac{\frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial c_i}}{\sum_{j=1}^{N} -\frac{\partial \tilde{t}_j(c_j,\lambda)}{\partial \lambda}} = \frac{\partial \tilde{t}_i(c_i, \lambda)}{\partial c_i} \cdot \left[1 - \frac{-\frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial \lambda}}{\sum_{j=1}^{N} -\frac{\partial \tilde{t}_j(c_j,\lambda)}{\partial \lambda}} \right] \tag{A7}$$

The second term is positive as $\frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial \lambda} < 0$ for all $i \in \{1, N\}$ (see (A2)). Thus $\frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial c_i}$ has the same sign as $\frac{\partial t_i(c_1,..c_N)}{\partial c_i}$, proving the equivalence of (i) and (iii). By the same argument, it is obvious from (A6) that $\frac{\partial \lambda}{\partial c_i}$ also has the same sign as $\frac{\partial t_i(c_1,..c_N)}{\partial c_i}$, proving the equivalence of (i) and (ii).

We now turn to the equivalence of (i) and (iv). We first rewrite the consumer's problem (2) in terms of $\tilde{v}$:

$$\max_{t_1,..t_N} \sum_{i=1}^{N} (1 - \theta_i) \cdot \tilde{v}_i (c_i, t_i), \tag{A8}$$

which is maximized subject to the attention constraint (3). This yields the counterpart of Equation (4), which for interior solutions for t_i can be written as

$$\frac{1}{1 - \theta_i} \cdot \lambda = \frac{\partial \tilde{v}_i (c_i, t_i)}{\partial t_i}. \tag{A9}$$

Differentiating this expression with respect to c_i (keeping all c_j, $j \neq i$, fixed), we obtain:

$$\frac{1}{1-\theta_i} \cdot \frac{\partial \lambda}{\partial c_i} = \frac{\partial^2 \tilde{v}_i\left(c_i, t_i\right)}{\partial c_i \partial t_i} + \frac{\partial^2 \tilde{v}_i\left(c_i, t_i\right)}{\partial t_i^2} \cdot \frac{\partial \tilde{t}_i(c_i, \lambda)}{\partial c_i} + \frac{\partial^2 \tilde{v}_i\left(c_i, t_i\right)}{\partial t_i^2} \cdot \frac{\partial \tilde{t}_i(c_i, \lambda)}{\partial \lambda} \cdot \frac{\partial \lambda}{\partial c_i} \tag{A10}$$

where we take into account that t_i is a function of c_i and λ, $\tilde{t}_i(c_i, \lambda)$ and that λ is a function of all c_i. From Equation (A2) we know that $\frac{\partial^2 \tilde{v}_i(c_i,t_i)}{\partial t_i^2} \cdot \frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial \lambda} = 1$, so that:

$$\frac{\theta_i}{1-\theta_i} \cdot \frac{\partial \lambda}{\partial c_i} = \frac{\partial^2 \tilde{v}_i\left(c_i, t_i\right)}{\partial c_i \partial t_i} + \frac{1}{\frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial \lambda}} \cdot \frac{\partial \tilde{t}_i(c_i, \lambda)}{\partial c_i}. \tag{A11}$$

Using (A6) to substitute $\frac{\partial \lambda}{\partial c_i}$, we then obtain

$$\frac{\theta_i}{1-\theta_i} \cdot \frac{\frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial c_i}}{\sum_{j=1}^N -\frac{\partial \tilde{t}_j(c_j,\lambda)}{\partial \lambda}} = \frac{\partial^2 \tilde{v}_i\left(c_i, t_i\right)}{\partial c_i \partial t_i} + \frac{1}{\frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial \lambda}} \cdot \frac{\partial \tilde{t}_i(c_i, \lambda)}{\partial c_i}, \tag{A12}$$

which can be rearranged to yield

$$\frac{\frac{\theta_i}{1-\theta_i} \cdot \left(-\frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial \lambda}\right) + \sum_{j=1}^N -\frac{\partial \tilde{t}_j(c_j,\lambda)}{\partial \lambda}}{\left(-\frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial \lambda}\right) \cdot \sum_{j=1}^N -\frac{\partial \tilde{t}_j(c_j,\lambda)}{\partial \lambda}} \cdot \frac{\partial \tilde{t}_i(c_i, \lambda)}{\partial c_i} = \frac{\partial^2 \tilde{v}_i\left(c_i, t_i\right)}{\partial c_i \partial t_i}. \tag{A13}$$

From Equation (A2) we know that $\frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial \lambda} < 0$ for all $i \in \{1, ..N\}$, which implies that $\frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial c_i}$ and $\frac{\partial^2 \tilde{v}_i(c_i,t_i)}{\partial c_i \partial t_i}$ have the same sign. □

Proof of Proposition 2.

See text. □

Proof of Proposition 3.

As shown in Proposition 1, the direction of the complexity distortion is determined by the sign of $\frac{\partial t_i(c_1,..c_N)}{\partial c_i}$. Lemma 1 shows that this distortion has the same sign as $\frac{\partial \lambda}{\partial c_i}$. Therefore, producers have an incentive to distort complexity in the direction that raises the shadow price of attention λ. Thus starting from the social planner's solution and allowing producers to follow a best-response strategy, in every step of the iteration the shadow cost of attention weakly increases (strictly if $\frac{\partial t_i(c_1,..c_N)}{\partial c_i} > 0$). Thus $\lambda^e > \lambda^*$ whenever equilibrium complexity and the planner's solution do not coincide (which is the case only if $\frac{\partial t_i(c_1,..c_N)}{\partial c_i} = 0$ for all producers at the planner's solution). □

Proof of Proposition 4.

See text. □

Proof of Lemma 2.

We use the heterogenous setup until noted otherwise. The consumer's first-order condition for a given shadow price of attention λ is:

$$\tilde{t}_i(c_i,\lambda) = \sqrt{\frac{c_i \cdot \delta_i \cdot (1-\theta_i) \cdot w_i}{\lambda}} - c_i. \tag{A14}$$

Plugging this expression into the (binding) attention constraint (3), we can express λ as:

$$\lambda = \frac{\left(\sum_{k=1}^{N} \sqrt{c_k \delta_k \left(1-\theta_k\right) w_k}\right)^2}{\left(\sum_{j=1}^{N} c_j + T\right)^2}. \tag{A15}$$

Substituting this expression back into Equation (A14) we obtain

$$t_i(c_1, ..c_N) = \frac{\sqrt{c_i \delta_i \left(1-\theta_i\right) w_i}}{\sum_{k=1}^{N} \sqrt{c_k \delta_k \left(1-\theta_k\right) w_k}} \cdot \left(\sum_{j=1}^{N} c_j + T\right) - c_i. \tag{A16}$$

Partially differentiating with respect to c_i yields

$$\frac{\partial t_i(c_1,..c_N)}{\partial c_i} = \frac{\sum_{j\neq i} \sqrt{c_j \delta_j \left(1-\theta_j\right) w_j}}{\left(\sum_{k=1}^{N} \sqrt{c_k \delta_k \left(1-\theta_k\right) w_k}\right)^2} \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{c_i}} \cdot \sqrt{\delta_i \left(1-\theta_i\right) w_i} \cdot \left(\sum_{j=1}^{N} c_j + T\right) + \frac{\sqrt{c_i \delta_i \left(1-\theta_i\right) w_i}}{\sum_{k=1}^{N} \sqrt{c_k \delta_k \left(1-\theta_k\right) w_k}} - 1. \tag{A17}$$

Imposing symmetry ($c_i = c \forall i$), this implies that

$$\frac{\partial t}{\partial c_i} = \frac{N-1}{2 \cdot N} \cdot \left(\frac{T}{N \cdot c} - 1\right) \tag{A18}$$

and

$$\lambda = \frac{w \cdot \delta \cdot (1-\theta)}{c \cdot \left(1 + \frac{T}{N \cdot c}\right)^2}. \tag{A19}$$

Plugging these into Equations (8) and (12), using that fact that under the assumed functional form $\frac{\partial v_i\left(c_i, \frac{t_i}{c_i}\right)}{\partial c_i} = w \cdot f'(c)$, and observing that in symmetric equilibrium all goods get the same amount of attention $t = \frac{T}{N}$, we then arrive at the equations stated in the lemma. □

Proof of Proposition 5.

The strategy of the proof is as follows. First, we show that there exists a $\delta^* > 0$, such that for $\delta < \delta^*$ both the planner's solution and the equilibrium outcome are symmetric. Second, we show that, starting from a set of parameters for which symmetric complexity choices coincide under the planner's solution and in equilibrium, the comparative statics stated in the proposition hold.

We first calculate a strictly positive lower bound for δ^* that is sufficient to ensure symmetric complexity choices, both under the planner's solution and in equilibrium. First note that the maximum value v that can be derived from a good is when the complexity is chosen to be the first best (unconstrained) optimum $c_i = \frac{\alpha_i}{2}$ and effective attention is chosen to be infinite $\frac{t_i}{c_i} = \infty$. Therefore, v_i can be bounded form above,

$$v_i\left(c_i, \frac{t_i}{c_i}\right) < v_i\left(\frac{\alpha_i}{2}, \infty\right) = w_i \cdot \left(\delta_i + \frac{\alpha_i^2}{4}\right). \tag{A20}$$

With N ex ante symmetric goods, if all goods have the same amount of complexity and therefore receive the same amount of attention from the consumer, v_i can be bounded from below because if $c = \frac{\alpha}{2}$ is not optimal, then v must be higher under the optimal complexity choice:

$$v_i\left(c_i, \frac{t_i}{c_i}\right) > v\left(\frac{\alpha}{2}, \frac{T/N}{\alpha/2}\right) = \frac{w\left(\alpha^3 N + 2T\left(\alpha^2 + 4\delta\right)\right)}{4\alpha N + 8T} \tag{A21}$$

To ensure that the planner's choice of complexity is symmetric, it has to be the case that the planner's first-order condition holds with equality for all goods. In particular, the planner must not have an incentive to set the complexity of one of the goods to zero. This is satisfied as long as

$$N \cdot v\left(c_s^*, \frac{t_{s,s}^*}{c_s^*}\right) > (N-1) \cdot v\left(c_a^*, \frac{t_{s,a}^*}{c_a^*}\right) + 1 \cdot v\left(0, \infty\right), \tag{A22}$$

where c_s^* is the planner's optimum in the symmetric case and c_a^* in the asymmetric one (in which one of the goods has zero complexity but other goods are symmetric in complexity). From Equations (A20) and (A21), a sufficient condition for a symmetric solution is that

$$N \cdot \frac{w\left(\alpha^3 N + 2T\left(\alpha^2 + 4\delta\right)\right)}{4\alpha N + 8T} > (N-1) \cdot w \cdot \left(\delta + \frac{\alpha^2}{4}\right) + w \cdot \delta, \tag{A23}$$

which simplifies to

$$\delta < \frac{\alpha(\alpha N + 2T)}{4N^2}. \tag{A24}$$

Note that this condition holds for small enough δ and is more likely to be violated when the attention capacity T is small. The condition holds for all $T > 0$ if $\delta < \frac{\alpha^2}{4 \cdot N}$. This is the sufficient (but clearly not necessary) condition for the planner's solution to be symmetric.

Now we show that for low enough δ there exists a symmetric equilibrium. First note that from Equation (20) it follows that if $\delta \to 0$ then $c_s^e \to \frac{\alpha}{2}$ from below. Thus for δ close enough to 0 it must hold that $\frac{\alpha}{4} < c^e < \frac{\alpha}{2}$. Because $v(.,.)$ is increasing in both arguments for $c < \frac{\alpha}{2}$, we can bound the producer's payoff in a symmetric equilibrium from below. Specifically,

$$v\left(c_s^e, \frac{t_s^e}{c_s^e}\right) > v\left(\frac{\alpha}{4}, \frac{T/N}{\alpha/2}\right) = w \cdot \left(\frac{3}{16} \cdot \alpha^2 + \delta \cdot \frac{\frac{2 \cdot T}{N \cdot \alpha}}{1 + \frac{2 \cdot T}{N \cdot \alpha}}\right). \tag{A25}$$

It remains to be shown that the producer does not want to deviate from the symmetric equilibrium to producing a good with zero complexity (and thus infinite depth of understanding by the consumer). This requires that

$$v\left(c_s^e, \frac{t_s^e}{c_s^e}\right) > v(0, \infty) = w \cdot \delta. \tag{A26}$$

Using Equation (A25) it suffices to show that

$$w \cdot \left(\frac{3}{16} \cdot \alpha^2 + \delta \cdot \frac{\frac{2 \cdot T}{N \cdot \alpha}}{1 + \frac{2 \cdot T}{N \cdot \alpha}}\right) > w \cdot \delta, \tag{A27}$$

which holds for any $T > 0$ if $\delta < \frac{3}{16} \cdot \alpha^2$. Thus we have shown that for small enough δ, there exists a symmetric equilibrium.

The separating hyperplane for which the optimal and equilibrium levels of complexity are the same happens at critical attention level T at which all goods have complexity $c = \frac{T}{N}$ (see Equation (22)). Plugging this into Equation (21) yields a quadratic equation for T, for which the unique solution that corresponds to a maximum of the social welfare function is (this can be checked by signing the second order condition):

$$T^{crit} = \frac{N}{4}\left(\alpha + \sqrt{\alpha^2 - 2\delta}\right). \tag{A28}$$

Note that this critical T only exists if $\delta \leq \frac{\alpha^2}{2}$. Equation (A28) defines a separating hyperplane in the parameter space. By continuity of the equilibrium complexity in the underlying parameters, all we have to check is whether there is too much complexity on one side of the hyperplane, arbitrarily close to the hyperplane itself.

To prove part (i) of the Proposition, we take the total derivative of the two first-order conditions (20) and (21) with respect to δ. Substituting $c = \frac{T}{N}$ (at $c^e = c^*$) and solving for $\frac{dc^e}{d\delta}$ and $\frac{dc^*}{d\delta}$ yields:

$$\frac{dc^e}{d\delta} = \frac{2NT}{\delta N(N+1) - 16T^2}, \tag{A29}$$

$$\frac{dc^*}{d\delta} = \frac{NT}{\delta N^2 - 8T^2}. \tag{A30}$$

We now need to show that if δ is slightly larger than on the hyperplane, then equilibrium complexity is higher than in planner's solution. Thus we have to show that $\frac{dc^e}{d\delta} > \frac{dc^*}{d\delta}$ at $T = T^{crit}$ which holds if

$$\frac{\sqrt{\alpha^2 - 2\delta} + \alpha}{\left(\alpha\sqrt{\alpha^2 - 2\delta} + \alpha^2 - 2\delta\right)\left(N\left(2\alpha\sqrt{\alpha^2 - 2\delta} + 2\alpha^2 - 3\delta\right) - \delta\right)} > 0, \tag{A31}$$

which in turn is satisfied if $\delta < \frac{\alpha^2}{2}$ (i.e., if T^{crit} exists).

To prove part (ii) of the Proposition, we take the total derivative of the two first order conditions (20) and (21) with respect to T. Substituting $c = \frac{T}{N}$ (at $c^e = c^*$) and solving for $\frac{dc^e}{dT}$ and $\frac{dc^*}{dT}$ yields:

$$\frac{dc^e}{dT} = \frac{\delta - \delta N}{\delta N(N+1) - 16T^2}, \tag{A32}$$

$$\frac{dc^*}{dT} = 0. \tag{A33}$$

We note that $\frac{dc^e}{dT} > \frac{dc^*}{dT}$ holds at $T = T^{crit}$ if $\delta < \frac{\alpha^2}{2}$ (i.e., if T^{crit} exists).

To prove part (iii) of the Proposition, we take the total derivative of the two first order conditions (20) and (21) with respect to α. Substituting $c = \frac{T}{N}$ (at $c^e = c^*$) and solving for $\frac{dc^e}{d\alpha}$ and $\frac{dc^*}{d\alpha}$ yields:

$$\frac{dc^e}{d\alpha} = \frac{8T^2}{16T^2 - \delta N(N+1)}, \tag{A34}$$

$$\frac{dc^*}{d\alpha} = \frac{4T^2}{8T^2 - \delta N^2}. \tag{A35}$$

We note that $\frac{dc^e}{d\alpha} < \frac{dc^*}{d\alpha}$ holds at $T = T^{crit}$ if $\delta < \frac{\alpha^2}{2}$ (i.e., if T^{crit} exists).

To prove part (iv) of the Proposition, we take the total derivative of the two first order conditions (20) and (21) with respect to N. Substituting $c = \frac{T}{N}$ (at $c^e = c^*$) solving for $\frac{dc^e}{dN}$ and $\frac{dc^*}{dN}$ yields:

$$\frac{dc^e}{dN} = \frac{\delta(N-1)T}{N\left(\delta N(N+1) - 16T^2\right)} \tag{A36}$$

$$\frac{dc^*}{dN} = 0 \tag{A37}$$

We note that $\frac{dc^e}{dN} < \frac{dc^*}{dN}$ holds at $T = T^{crit}$ if $\delta < \frac{\alpha^2}{2}$ (i.e., if T^{crit} exists). □

Proof of Proposition 6.

Heterogeneity in δ: Set $\delta_1 = \delta - \epsilon$ for good 1 and $\delta_2 = \delta + \epsilon$ for good 2, where δ is the level at which the equilibrium complexity c^e and planner's choice of complexity c^* coincide. We know from Equation (22) that the level of complexity for both goods at $\epsilon = 0$ is simply given by

$$c_1^* = c_2^* = c_1^e = c_2^e = \frac{T}{2}. \tag{A38}$$

The strategy of the proof is to compare derivatives with respect to ϵ under the equilibrium complexity choice with that under the planner's solution. For small enough ϵ this allows us to establish whether equilibrium complexity is above or below the social optimum. We can calculate these derivatives by taking total differential of the first-order conditions for the two goods (Equation (7) in the equilibrium case and Equation (10) in the planner's case) and then setting $\epsilon = 0$. In both cases (equilibrium and the planner's solution), this gives us two equations in two unknowns. For equilibrium complexity, these are $\left.\frac{\partial c_1^e}{\partial \epsilon}\right|_{\epsilon=0} = \left.\frac{\partial c_1^e}{\partial \delta_1}\right|_{\delta_1=\delta}$ and $\left.\frac{\partial c_2^e}{\partial \epsilon}\right|_{\epsilon=0} = \left.\frac{\partial c_2^e}{\partial \delta_2}\right|_{\delta_2=\delta}$, whereas for the planner's solution we have $\left.\frac{\partial c_1^*}{\partial \epsilon}\right|_{\epsilon=0} = \left.\frac{\partial c_1^*}{\partial \delta_1}\right|_{\delta_1=\delta}$ and $\left.\frac{\partial c_2^*}{\partial \epsilon}\right|_{\epsilon=0} = \left.\frac{\partial c_2^*}{\partial \delta_2}\right|_{\delta_2=\delta}$.

Recall that it must be the case that the parameters are such that T is given by Equation (A28) (so that equilibrium complexity and the planner's solution coincide). The derivatives of interest in the planner's solution are then

$$\left.\frac{\partial c_1^*}{\partial \theta_1}\right|_{\theta_1=\theta} = -\left.\frac{\partial c_2^*}{\partial \theta_2}\right|_{\theta_2=\theta} = \frac{T}{2\left(2T^2 - \delta\right)}, \tag{A39}$$

while for equilibrium complexity they are

$$\left.\frac{\partial c_1^e}{\partial \theta_1}\right|_{\theta_1=\theta} = -\left.\frac{\partial c_2^e}{\partial \theta_2}\right|_{\theta_2=\theta} = \frac{3T}{2\left(8T^2 - 3\delta\right)}. \tag{A40}$$

If $T > \sqrt{\frac{3}{8}\delta}$, it follows that

$$\left.\frac{\partial c_2^e}{\partial \theta_2}\right|_{\theta_2=\theta} < \left.\frac{\partial c_1^e}{\partial \theta_1}\right|_{\theta_1=\theta}. \tag{A41}$$

Thus, if ϵ is sufficiently small, then good 2 (with higher δ) is simpler than good 1 (with lower δ). The condition $T > \sqrt{\frac{3}{8}\delta}$ holds becuase T is defined by (A28) (with $N = 2$) and $\alpha^2 - 2\delta > 0$ follows from the existence of a critical T at which the planner's and equilibrium complexity coincide (as assumed in the statement of the theorem). If further $T > \frac{\sqrt{\delta}}{\sqrt{2}}$ (which again follows from (A28)), it is straightforward to show that

$$\left.\frac{\partial c_2^e}{\partial \theta_2}\right|_{\theta_2=\theta} > \left.\frac{\partial c_2^*}{\partial \theta_2}\right|_{\theta_2=\theta}. \tag{A42}$$

This proves that, for sufficiently small ϵ, good 2 (with higher δ) is too complex in equilibrium relative to the planner's solution. By the same logic, good 1 (with lower δ) is too simple relative to the planner's solution.

The rest of the proof follow a similar logic and we just report the main steps. We continue to assume that $T > \frac{\sqrt{\delta}}{\sqrt{2}}$ (which follows from (A28)).

Heterogeneity in w: The derivatives of interest in the planner's solurion are

$$\left.\frac{\partial c_1^*}{\partial w_1}\right|_{w_1=w} = -\left.\frac{\partial c_2^*}{\partial w_2}\right|_{w_2=w} = \frac{T\left(\delta + 2T^2 - 2\alpha T\right)}{2w\left(2T^2 - \delta\right)}, \tag{A43}$$

while for equilibrium complexity they are

$$\left.\frac{\partial c_1^e}{\partial w_1}\right|_{w_1=w} = -\left.\frac{\partial c_2^e}{\partial w_2}\right|_{w_2=w} = \frac{T\left(3\delta + 8T^2 - 8\alpha T\right)}{2w\left(8T^2 - 3\delta\right)}. \tag{A44}$$

It follows that

$$\left.\frac{\partial c_2^e}{\partial w_2}\right|_{w_2=w} > \left.\frac{\partial c_1^e}{\partial w_1}\right|_{w_1=w} \tag{A45}$$

and

$$\left.\frac{\partial c_2^e}{\partial w_2}\right|_{w_2=w} > \left.\frac{\partial c_2^*}{\partial w_2}\right|_{w_2=w}. \tag{A46}$$

The first inequality requires that $T \in \left(\sqrt{\frac{3}{8}}\sqrt{\delta}, \frac{1}{2}\left(\sqrt{\alpha^2 - \frac{3\delta}{2}} + \alpha\right)\right)$, the second $T > \frac{\alpha}{2}$. Both of these conditions follow from (A28).

Heterogeneity in θ: The derivatives of interest in the planner's solution are

$$\left.\frac{\partial c_1^*}{\partial \theta_1}\right|_{\theta_1=\theta} = -\left.\frac{\partial c_2^*}{\partial \theta_2}\right|_{\theta_2=\theta} = 0, \tag{A47}$$

while for equilibrium complexity they are

$$\left.\frac{\partial c_1^e}{\partial \theta_1}\right|_{\theta_1=\theta} = -\left.\frac{\partial c_2^e}{\partial \theta_2}\right|_{\theta_2=\theta} = \frac{\delta T}{2(1-\theta)\left(8T^2-3\delta\right)}. \tag{A48}$$

It follows that

$$\left.\frac{\partial c_2^e}{\partial \theta_2}\right|_{\theta_2=\theta} < \left.\frac{\partial c_1^e}{\partial \theta_1}\right|_{\theta_1=\theta} \tag{A49}$$

and

$$\left.\frac{\partial c_2^e}{\partial \theta_2}\right|_{\theta_2=\theta} < \left.\frac{\partial c_2^*}{\partial \theta_2}\right|_{\theta_2=\theta}. \tag{A50}$$

Heterogeneity in α: The derivatives of interest in the planner's solution are

$$\left.\frac{\partial c_1^*}{\partial \alpha_1}\right|_{\alpha_1=\alpha} = -\left.\frac{\partial c_2^*}{\partial \alpha_2}\right|_{\alpha_2=\alpha} = -\frac{T^2}{2T^2-\delta}, \tag{A51}$$

while for equilibrium complexity they are

$$\left.\frac{\partial c_1^e}{\partial \alpha_1}\right|_{\alpha_1=\alpha} = -\left.\frac{\partial c_2^e}{\partial \alpha_2}\right|_{\alpha_2=\alpha} = -\frac{4T^2}{8T^2-3\delta}. \tag{A52}$$

It follows that

$$\left.\frac{\partial c_2^e}{\partial \alpha_2}\right|_{\alpha_2=\alpha} > \left.\frac{\partial c_1^e}{\partial \alpha_1}\right|_{\alpha_1=\alpha} \tag{A53}$$

and

$$\left.\frac{\partial c_2^e}{\partial \alpha_2}\right|_{\alpha_2=\alpha} < \left.\frac{\partial c_2^*}{\partial \alpha_2}\right|_{\alpha_2=\alpha}. \tag{A54}$$

□

B Microfoundation with priced good

Here we provide a microfoundation of our simple model in Section 2 in which the good is priced. We show that the resulting first-order conditions with respect to complexity are the same as in the baseline model.

Suppose that the consumer buys x_i units of a divisible good i with complexity c_i for unit price p_i and devotes t_i units of time to understand the good. Assume that the consumer's valuation of good i takes the following form,

$$V_i\left(x_i, c_i, \frac{c_i}{t_i}\right) = v_1(x_i) \cdot v_2\left(c_i, \frac{t_i}{c_i}\right). \tag{B1}$$

The multiplicative structure of V ensures that we do not get a "Spence" term (see Spence (1975)) because of differences between the marginal and marginal marginal valuation of complexity. We assume that the value of consuming each good i satisfies an Inada condition $\lim_{x_i \to 0} V_i\left(x_i, c_i, \frac{c_i}{t_i}\right) = \infty$.

Consumer. The consumer chooses quantities x_i and attention t_i to solve the maximization problem

$$\max_{x_i, t_i} \sum_i v_1(x_i) \cdot v_2\left(c_i, \frac{t_i}{c_i}\right) - p_i \cdot x_i, \tag{B2}$$

subject to (3).

The FOC with respect to the attention choice t_i is given by

$$\lambda \geq v_1(x_i) \cdot \frac{\partial v_2\left(c_i, \frac{t_i}{c_i}\right)}{\partial\left(\frac{t_i}{c_i}\right)} \cdot \frac{1}{c_i} \tag{B3}$$

This FOC holds with equality for good i if $t_i > 0$.

The FOC with respect to quantity choice x_i yields:

$$p_i = \frac{\partial v_1(x_i)}{\partial x_i} \cdot v_2\left(c_i, \frac{t_i}{c_i}\right). \tag{B4}$$

Because of the Inada condition, all goods will be consumed in positive quantity ($x_i > 0$, $\forall i$), so that this FOC holds with equality.

Producer. The producer of good i chooses complexity c_i and quantity sold x_i to maximize profits:

$$\max_{c_i, x_i} x_i \cdot p_i(c_i, x_i) = \max_{c_i, x_i} x_i \cdot \frac{\partial v_1(x_i)}{\partial x_i} \cdot v_2\left(c_i, \frac{t_i}{c_i}\right) \tag{B5}$$

since the producer is a monopolist, we plugged in the equilibrium price from (B4).

For simplicity, from now on we assume an interior solution in c_i (recall that x_i will be interior because of the Inada condition). The FOC with respect to complexity c_i is given by:

$$x_i \frac{\partial v_1(x_i)}{\partial x_i}\left[\frac{\partial v_2\left(c_i, \frac{t_i}{c_i}\right)}{\partial c_i} - \frac{\partial v_2\left(c_i, \frac{t_i}{c_i}\right)}{\partial\left(\frac{t_i}{c_i}\right)} \cdot \frac{1}{c_i}\left(\frac{t_i}{c_i} - \frac{\partial t_i}{\partial c_i}\right)\right] = 0, \tag{B6}$$

which, given $x_i \frac{\partial v_1(x_i)}{\partial x_i} > 0$, can be rewritten as

$$\frac{\partial v_2\left(c_i, \frac{t_i}{c_i}\right)}{\partial c_i} = \frac{\partial v_2\left(c_i, \frac{t_i}{c_i}\right)}{\partial\left(\frac{t_i}{c_i}\right)} \cdot \frac{1}{c_i}\left(\frac{t_i}{c_i} - \frac{\partial t_i}{\partial c_i}\right). \tag{B7}$$

This condition is equivalent to (7) derived in the paper for the simple model.

The FOC with respect to quantity x_i is given by:

$$v_2\left(c_i, \frac{t_i}{c_i}\right) \cdot \frac{\partial v_1(x_i)}{\partial x_i} + v_2\left(c_i, \frac{t_i}{c_i}\right) \cdot x_i \frac{\partial^2 v_1(x_i)}{\partial x_i^2} + x_i \frac{\partial v_1(x_i)}{\partial x_i} \frac{\partial v_2\left(c_i, \frac{t_i}{c_i}\right)}{\partial c_i} \frac{1}{c_i} \cdot \frac{\partial t_i}{\partial x_i} = 0, \tag{B8}$$

where the second term is the usual monopoly term in quantity choice while the last term is an attention grabbing effect through the size of the good. That is if the producer can attract more attention by supplying more of the good, it will do so.

Planner. The planner chooses complexity c_i and quantities x_i to maximize surplus:

$$\max_{c_i, x_i} \sum_i v_1(x_i) \cdot v_2\left(c_i, \frac{t_i}{c_i}\right) \tag{B9}$$

The FOC with respect to complexity c_i is given by

$$v_1(x_i)\left[\frac{\partial v_2\left(c_i, \frac{t_i}{c_i}\right)}{\partial c_i} - \frac{\partial v_2\left(c_i, \frac{t_i}{c_i}\right)}{\partial\left(\frac{t_i}{c_i}\right)} \cdot \frac{1}{c_i}\left(\frac{t_i}{c_i} - \underbrace{\sum_j \frac{\partial t_j}{\partial c_i}}_{=0}\right)\right] = 0, \tag{B10}$$

which, given $v_1(x_i) > 0$ can be rewritten as

$$\frac{\partial v_2\left(c_i, \frac{t_i}{c_i}\right)}{\partial c_i} = \frac{\partial v_2\left(c_i, \frac{t_i}{c_i}\right)}{\partial\left(\frac{t_i}{c_i}\right)} \cdot \frac{1}{c_i} \cdot \frac{t_i}{c_i}. \tag{B11}$$

The FOC with respect to quantity x_i is given by

$$v_2\left(c_i, \frac{t_i}{c_i}\right) \frac{\partial v_1\left(x_i\right)}{\partial x_i} + \sum_j v_1(x_j) \frac{\partial v_2\left(c_i, \frac{t_i}{c_i}\right)}{\partial\left(\frac{t_i}{c_i}\right)} \frac{1}{c_i} \cdot \frac{\partial t_i}{\partial x_i} = 0. \tag{B12}$$

Under symmetry, the second term should be zero. Then, given $v_2\left(c_i, \frac{t_i}{c_i}\right) > 0$, the first order condition simplifies to

$$\frac{\partial v_1\left(x_i\right)}{\partial x_i} = 0. \tag{B13}$$

Comparing the first-order conditions for complexity choice in the competitive equilibrium (B7) with that in the social optimum (B11), the difference between the two is exactly $\frac{\partial t_i}{\partial c_i}$ as was the case in the baseline model between the difference of (8) and (12). Thus our results on the distortion of complexity derived in the paper hold in this case. Note that there are additional inefficiencies due to the usual supply effects of a monopolist (compare (B12) and (B8)) but these are orthogonal to the externalities in complexity choice.