Organizational Structure and Investment Strategy^{*}

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Abstract

We show that a firm can use its organizational structure to commit to an investment strategy. The firm delegates sequential search and project management tasks to a manager. Ex post, the firm turns away projects that generate high project management rent. However, because the expectation of such rent serves to defray the manager's search cost, investment might be optimal ex ante. A leveraged subsidiary mitigates this time-inconsistency problem by creating ex post risk-shifting incentives that counteract underinvestment. Subsidiaries are more valuable for projects with costly search, intermediate management costs, and returns that are uncorrelated with the existing business.

Keywords: Organizational structure; multinational business; branch; subsidiary.

JEL Codes: G32, G34, L22.

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1. Introduction

When a firm expands into a new market, how should it structure its investment? One possibility is for the firm to open a new branch; alternatively, it might place the investment into a wholly owned subsidiary. This decision has a knock-on effect on the range of structures that the firm can use to finance its investment. In line with this observation, subsidiary debt issues have accounted for approximately 13% of total US non-financial corporate public debt proceeds since 1995 (Kolasinski, 2009). Our theoretical grasp of the relationship between the firm's organizational structure and its investment strategy nevertheless remains incomplete. This paper attempts to address this gap. We present a theory in which the firm selects the organizational structure that it uses for expansion in order to ensure that it opts to accept investments uncovered by a costly search whenever, from an ex ante perspective, it is desirable that it should do so. Subsidiary firms thus arise in our model as an endogenous response to a time-inconsistency problem.

We study a firm with a valuable asset-in-place, a deep-pocketed shareholder, and no cash. The firm has a profitable expansion opportunity. In order to realise its opportunity, the firm must invest in the necessary organizational infrastructure and must hire a specialist manager. If the manager exerts a costly search effort, then the firm acquires a concrete investment project. If the project is pursued, then it has a positive net present value precisely when the manager exerts privately costly effort on project management. The cost of project management is random ex ante and realises when the project is located.

There are two key frictions in this set-up. The first is that the manager's project management efforts are unobservable. The second is that the firm is unable to commit to pursue the investment opportunity that the manager finds; if it does not, then it can liquidate its infrastructure investment for a guaranteed payment that is less than the infrastructure cost.

Because project management effort is unobservable, the firm has to leave some rent to the manager. In case the project management cost is high, the manager's rent exceeds the project's net present value (NPV) and, when that happens, the firm elects not to invest, and to liquidate its organizational infrastructure. From an ex ante perspective, however, the firm may prefer to commit to invest. The reason for this is that project search is costly and, if it were possible to commit to ex post investment, the anticipation of ex post project management rents would serve to defray those costs. After the project has been located, of course, the search costs are sunk and, hence, cannot affect the firm's investment decision.

We show that the firm can resolve its commitment problem through an appropriate choice of organizational form. We consider two possibilities. First, the firm could use a branch structure to invest. In this case, the firm's asset-in-place, its infrastructure, and its new project are all owned by the same legal entity and, hence, sit on the same balance sheet. Second, the firm could expand using a subsidiary firm that has a separate legal personality from its parent company. In this case, the new infrastructure and the new investment project sit on the subsidiary firm's balance sheet, while the asset-in-place resides on the parent firm's balance sheet.

Expanding via a branch is cheaper than using a subsidiary, because, in order to create a subsidiary, it is necessary to duplicate some of the infrastructure that the parent company already has. But a firm that creates a subsidiary has access to a wider range of financing strategies. Specifically, expansion via a branch involves only one balance sheet, so that this mode of expansion can only be funded by borrowing against all of the firm's assets. In contrast, if the firm expands using a subsidiary, then it can issue debt at the subsidiary level that is backed by the subsidiary firm's assets, as well as at the parent level against the asset-in-place and the subsidiary firm's equity. The subsidiary's limited liability creates a risk-shifting incentive that renders ex post investment more attractive. Hence, it goes some way to mitigate the commitment problem that lies at the heart of our analysis.

In short, our analysis predicts that the organizational structure derives from a simple commitment problem: the firm wishes to commit to make ex post investments because those investments generate managerial rent than could defray ex ante managerial search costs. The scale of this problem is increasing in the magnitude of both managerial search costs, which render investment more attractive ex ante, and project management costs, which render investment less attractive ex post. We consider these effects in turn. First, because higher search costs increase the value of commitment, they render subsidiary firms more attractive. Second, the firm uses the cheaper branch structure for expansion when the cost of project management for the marginal project is sufficiently small to preclude a commitment problem. For a higher project management cost, the firm is willing to incur the costs necessary to use a subsidiary structure in order to resolve the ex ante commitment problem. When the project management cost is very high, the subsidiary may be unable ex post to overcome the commitment problem; moreover, the costs of subsidiary commitment may to too high to be worth incurring from an ex ante perspective. In this case, the firm expands with a branch structure and underinvests ex post.

Underinvestment arises in our model as a result of frictions that are *dynamic*. The firms we consider delegate a series of costly tasks to an agent and, because the agent's costs are sunk when the firm decides whether to invest, the firm is unable to commit to invest. The resultant underinvestment has a qualitatively different cause than the static trade-offs that arise in many models of underinvestment. For example, it is well understood that the underinvestment caused by a Myers (1977)-style debt overhang can be resolved by housing new investments in a separate legal entity. But debt overhang is a consequence of renegotiation frictions and not of commitment problems; as a result, our analysis applies in different situations.

Debt overhang also features in John's (1993) analysis of optimal organizational form. In her model, the appropriate organizational form is driven by the trade off between the tax benefits of risky debt and the costs of a debt-overhang. Her results are driven by the correlation between the new investment's returns and those of the firm's existing assets. When the two have a high correlation, the new investment exacerbates bankruptcy risk and, hence, the firm prefers to spin the new investment off into a separate firm so as to mitigate debt overhang.

In order to study the effects of correlation, we consider an extension to our basic set-up in which the firm's assets-in-place are risky. The optimal organizational structure depends upon the respective effects of creating a limited liability option in the existing firm by combining the new investment with the firm's existing assets, and of creating a new limited liability option by placing the investment into a separate subsidiary. The former approach is relatively less valuable when correlation is low and, hence, we predict that subsidiarization is relatively more likely in this case.

The most important parameters in our model are the initial search cost, the subsequent project management cost, and the correlation between the returns of the firm's existing assets and those of the new investment. Our model's empirical predictions rely upon an interpretation of these parameters. We argue that search costs are low when firms expand existing business lines when they operate in markets in which they are expert, and when they face low levels of competition. The cost of project management is low for firms operating in industries where they have pre-existing expertise, where they can easily access management accounting information, and for firms operating in countries with strong legal systems and institutions. And, finally, the correlation between a firm's asset-in-place and its new investment is high when the firm invests in businesses that are closely related to its existing activities.

In practice, a subsidiary firm is indistinguishable in our set-up from a spinoff or a carveout. Our work therefore predicts that subsidiary investment, carve-outs, and spin-offs are all associated with more competition, and that they are more likely in new markets; in contrast, branch expansion occurs in existing markets. Using a real option model, Hackbarth, Mathews, and Robinson (2014) also study the effect of competition upon the spin-off decision. In their model, spin-offs mitigate an excessive tendency to delay investment, but they generate a risk of cannibalisation. In line with our analysis, Hackbarth et al find that competition results in spin-offs when it is associated with higher cashflow risk. Fulghieri and Sevilir (2011) study a model in which spinoffs cause undesirable competition but, because they resolve a Hart and Moore (1990)-style hold-up problem, they also heighten employee incentives. The optimal organizational structure then trades these effects off against one another.

Our work also generates predictions for the relative profitability and investment scope of

branch and subsidiary firms. In order to generate testable hypotheses, we fix every parameter except for the search cost so that subsidiaries are associated with higher search costs than branches. We predict that subsidiary firms invest in a greater scope of projects. We also predict that managers are paid more in subsidiary firms than in branches; the reason for this is that, because subsidiary firms accept marginal projects with higher project management costs, their managers receive more agency rent.

Starting from Lewellen (1971) a large literature argues that firms pursuing multiple activities should select their organisational structure so as to achieve the optimal tradeoff between the tax advantage of debt and the risk of incurring an exogenous cost of bankruptcy.¹ In a more recent contribution, Leland (2007) asks how the correlation and the riskiness of the cashflows from multiple activities affect this trade-off. Luciano and Nicodano (2014) use Lealand's framework to examine loan guarantees between a parent and a subsidiary firm. They study conditional guarantees that are limited by the solvency of the guarantor, and also unconditional and mutual guarantees: the former correspond in our set-up to branch firms and the latter to subsidiaries. Then, using our terminology, Luciano and Nicodano show that branch firms dominate subsidiaries if the correlation between two firms' returns is sufficiently high.

A second strand of the literature analyses the effect that the agency problems of debt have upon optimal organisational structure choices. John (1993) and John and John (1991) consider models in which debt causes agency problems that result in underinvestment; they derive optimal organisational structure by trading this effect off against the tax advantages of debt. Chemmanur and John (1996) model firm organization as a way of achieving the optimal allocation of corporate control rights. In their framework, the entrepreneur chooses between separate and joint incorporation of two projects so as to protect himself against a loss of control rights caused by takeover or financial distress. Flannery, Houston, and Venkataraman (1993) analyses optimal organisational structure in a model with endogenous project choice: in addition to the tax benefits of debt, their model incorporates debt overhang and asset substitution.

Credit institutions often structure themselves so as to place loans with differing risk characteristics into different subsidiaries. Kahn and Winton (2004) present an explanation for this type of structure that relies upon risk-shifting incentives. Kahn and Winton model the effect that an ex ante choice of organizational structure has upon a static tradeoff. In our model, in constrast, we suggest that organizational structure might affect the dynamic incentive problem faced by an agent who performs a series of costly tasks in the face of incomplete commitment by the principal; in this set-up, the possibility of ex post risk shifting may serve to mitigate commitment problems.²

¹Early papers include Higgins and Schall (1975), Kim and McConnell (1977), and Stapleton (1982).

 $^{^{2}}$ Segura and Zeng (2020) show that the possibility that a firm could voluntarily support debt issued

A third strand of the literature shows how agency problems within the firm shape organisational choices. Laux (2001) presents a model in which an agent makes an effort choice at a single time. The principal is able to terminate the investment but, absent information, will never do so. It follows that, if the threat of termination is to incentivise the manager, the principal must structure its investment so as to ensure that, ex post, it prefers to monitor. Subsidiary investment accomplishes this. Hence, in contrast to our model, in which subsidiarization counters an underinvestment problem, it serves in Laux's model to prevent over-investment. Once again, the most important difference between the two models is that Laux considers a single agential effort choice, while we consider a sequence of related choices.

Finally, Ayotte (2017) analyzes the costs and benefits of spinning-off innovation in an Aghion and Tirole (1997)-style princial-agent model. Managers in his model have better incentives to innovate in subsidiary firms, both because subsidiaries have a higher risk of bankruptcy and because they experience less interference from the headquarters. Information revealed at an intermediate date makes it hard for the headquarters in this set-up to commit to implement certain types of project. The principal in Ayotte's model can achieve commitment by setting up a subsidiary in which decision-making power is delegated to the manager. In contrast, the headquarters in our model retains decision-making powers even when it expands via a subsidiary, and commitment is achieved by placing debt in the subsidiary so as to render ex post investment more attractive.

2. Model description

We analyse a model with three dates, t = 0, 1, 2, in which every agent is risk neutral and the interest rate is normalised to zero. There is an unlevered firm with an existing asset-inplace that will pay $Y \ge 1$ at time 2. The firm has limited liability and it is run by its sole shareholder. The shareholder has deep pockets but the firm has no funds at time 0.

2.1 Business expansion and contracts

At time 0, the firm has an opportunity to invest 1 to acquire the infrastructure needed to expand its business into a new market; we discuss the organizational implications of this decision in Section 2.3. Note that the firm's asset-in-place ensures that it can raise external finance to cover the cost of expansion. Business expansion requires specialised skills and, if the time 0 investment occurs, the firm's owner hires a specialist manager to manage the expansion. The manager makes an unobservable time 0 decision to search for a project in

by a separate structure can help that firm to alleviate future adverse selection when financing follow-up investments. However, the separate structure has limited liability and, hence, introduces a fresh source of moral hazard. In their model, the optimal organizational structure trades off the cost of that moral hazard against the signalling value of voluntary support.

the new market. The manager incurs a private disutility $\zeta > 0$ from search and, if it occurs, then she finds a project with probability 1; if the manager does not search for a project, then she does not find one. Project discovery is verifiable. Notice that the unobservability of the manager's search decision is inconsequential because it is perfectly revealed by the presence (or absence) of a project.

At time 1, the state of the world $\sigma \in \{h, e\}$ realises. We refer to the project as *hard* and easy in case the state σ is h or e, respectively; the probability that $\sigma = h$ is λ . The firm then decides whether to pursue the project that the manager finds. If it does not, or if the manager does not find a project, then it can liquidate its time 0 infrastructure investment at time 2 to return $L \leq 1$. If the firm pursues the project then the manager makes an unobservable decision to incur a personal effort cost m_{σ} : $m_h = M > 0$ and $m_e = 0$. The project's return \tilde{R} is a random variable whose value is R (success) or 0 (failure). The project succeeds with probability p if the manager exerts effort $(m_{\sigma} = M)$, and with probability $p - \delta$ otherwise $(m_{\sigma} = 0)$.

We impose two parametric assumptions.

Assumption 1.

$$M \le \overline{M}_1 \equiv pR - 1 < \delta R; \tag{1}$$

$$M \le \overline{M}_2(\zeta) \equiv \frac{1}{\lambda} \left(pR - 1 - \zeta \right) \tag{2}$$

Equation (1) implies that $pR - M - 1 \ge 0$, so that hard projects have a positive NPV if the manager exerts effort M, and that $(p - \delta)R - 1 < 0$, so that projects always have negative NPV if the manager exerts no effort. Conditional upon finding a project, managerial effort is therefore optimal. Moreover, because Equation (2) implies that $pR - 1 - \lambda M - \zeta \ge 0$, it is ex ante optimal to search for a project. The expected NPV of the firm's asset-in-place and its business expansion under efficient decisions is therefore given by Equation (3):

$$V^{FB} = Y + (pR - 1 - \lambda M - \zeta). \tag{3}$$

2.2 Compensation contracts

The relationship between the manager and the firm is governed by a compensation contract. It is impossible to contract upon the manager's unobservable time 1 effort choice, but the compensation contract can be contingent upon project discovery; if there is a project, the contract can also reference the state of the world $\sigma \in \{e, h\}$, the firm's decision to invest, and the success or failure of the project in case of investment. Our baseline analysis rules out payments to the manager in case a project is not found, and in case a project is found but not pursued. We therefore consider contracts of the form $(w_{\sigma}^{R}, w_{\sigma}^{0})_{\sigma \in \{e, h\}}$, where w_{σ}^{pay} is the payment that the manager receives in case the project goes ahead in state $\sigma \in \{e, h\}$ and generates a time 2 payment $pay \in \{R, 0\}$; managers are protected by limited liability and, hence, $w_{\sigma}^{pay} \geq 0$. The restrictions that we place upon the contracting space do not materially affect our analysis. First, ruling out payments in case there is no project is without loss of generality, because this restriction is the cheapest way to incentivise project search. Second, we demonstrate in the Appendix that our main results are qualitatively unchanged in case we allow the firm to make a non-zero wage payment to the manager when it does not pursue a project.

2.3 Organizational structure and financing

At time 0, the firm selects the organizational structure that it uses for business expansion, and it borrows to raise the unit investment that expansion requires from competitive, deep-pocketed outside investors.³ The firm can adopt a *branch* or a *subsidiary* structure.

If the firm opts to expand using a branch structure, then the infrastructure that supports expansion and any business expansion projects that it pursues share the firm's legal personality. They therefore sit on the same balance sheet, and any debt that the firm issues at time 0 is backed by returns from both the new business and from the firm's asset-in-place. In this case, the firm issues debt with face value D_B to raise 1.

If the firm uses a subsidiary structure, then it incurs running costs, such as duplicated IT and accounting system overheads, that amount to C > 0 to maintain a subsidiary firm that has a separate legal personality from the original firm, which we refer to in this case as the "parent." The subsidiary firm owns the infrastructure that supports expansion as well as the expansion project, if it is pursued. In turn, the subsidiary's equity is entirely owned by the parent firm, which is protected by limited liability from subsidiary losses. Because the subsidiary is a separate firm, it can issue debt that is backed by its assets. It is also possible in this case for the parent to issue debt that is backed by its asset-in-place as well as by its equity holding in the subsidiary firm. The parent firm issues debt with face value D_P to raise d_P and the subsidiary issues debt with face value D_S to raise d_S , where $d_P + d_S = 1$.

In our model, the critical difference between branch and subsidiary expansion is that subsidiarization allows the parent to partition its assets and to borrow against only some of them, while branch expansion allows for debt backed by all of the firm's assets. In practice, subsidiary asset partitioning is achieved because the subsidiary has a separate legal personality, but it could be partially unwound by contract: for example, the parent firm could commit to cover the subsidiary's losses. Similarly, the firm may be able to limit its exposure to its branch's activities by buying protection from a third party. But the fact

 $^{^{3}}$ In Section 6, we show that our results remain qualitatively identical when we allow the firm to raise the funds through a combination of debt and equity.

$time \ 0$	time 1	time 4
Firm chooses branch or subsidiary structure. The unit investment required for expansion is raised using debt	Manager makes search decision. State of the world σ realizes.	Returns realize, incentive contract is settled, and in-
Firm offers a contract to manager.	If a project is found, firm decides whether to pursue it.	vestors are repaid.
Manager decides whether to accept the contract.	If project is pursued, manager selects effort level M or 0.	

Figure 1. Model timeline.

that this type of contracting is possible simply serves to highlight the fact that there are several ways of achieving the separation accomplished through a straightforward subsidiary structure. The fact that firms use subsidiaries for this purpose is strong prima facie evidence that alternative approaches would incur higher costs.

2.4 Timeline

Figure 1 illustrates the timeline for our model. At time 0, the firm decides whether to use branch or subsidiary structure for business expansion and uses debt to raise 1. If it uses a branch structure then the expansion project is combined with the asset-in-place and debt with face value D_B is issued against the combination. If it uses a subsidiary then the subsidiary issues debt with face value D_S against the expansion project to raise d_S and the parent firm issues debt with face value D_P against its asset-in-place and its equity in the subsidiary to raise $d_S = 1 - d_P$. The firm then offers a compensation contract $(w_{\sigma}^R, w_{\sigma}^0)_{\sigma \in \{e,h\}}$ to the manager.

At time 1, the manager makes an unobservable decision whether or not to search for a project, and the state of the world $\sigma \in \{e, h\}$ realises. If a project is found, the firm decides whether or not to pursue it. The manager makes an unobservable effort choice if the project is pursued.

At time 2, returns realise and all wage and financing contracts are settled.

3. Strategies and payoffs

Consider a firm in which the manager has a contract $(w_{\sigma}^{R}, w_{\sigma}^{0})_{\sigma \in \{e,h\}}$. If the firm pursues a project at time 1, then managerial effort is incentive compatible in state σ if and only if $\delta(w_{\sigma}^{R}-w_{\sigma}^{0}) \geq m_{\sigma}$. This condition is always satisfied for any $w_{e}^{R} \geq w_{e}^{0}$ when $\sigma = e$. Managerial effort is incentive compatible for hard projects precisely when the following condition is satisfied:

$$w_h^R - w_h^0 \ge \overline{w} \triangleq \frac{M}{\delta}.$$
(4)

Without loss of generality, we assume that the firm requires its managers to exert effort

for every project. It follows that every contract $(w_{\sigma}^{R}, w_{\sigma}^{0})_{\sigma \in \{e,h\}}$ that the firm offers must satisfy the managerial effort constraint (4). At time 1, the manager therefore expects to earn the following rent if the firm pursues a project in state σ :

$$\rho_1(\sigma) \triangleq w_{\sigma}^0 + p(w_{\sigma}^R - w_{\sigma}^0) - m_{\sigma}.$$
(5)

At time 1, the firm knows the state σ and decides whether to pursue the project. Its investment strategy can therefore be expressed as an indicator function

$$\mathbf{1}_{\sigma} = \begin{cases} 1, & \text{if the firm pursues projects in state } \sigma; \\ 0, & \text{otherwise.} \end{cases}$$
(6)

The firm can always induce managerial effort at zero cost when $\sigma = e$ and hence, by Equation 1, $\mathbf{1}_e = 1$ in equilibrium. It follows that two possible strategies are possible:

Definition 1. The firm pursues a selective strategy if $\mathbf{1}_h = 0$, so that it pursues easy projects but not hard projects; it pursues an unselective strategy if $\mathbf{1}_h = 1$, so that it pursues every project.

At time 0, a manager with contract $(w_{\sigma}^{R}, w_{\sigma}^{0})_{\sigma \in \{e,h\}}$ earns the following expected rent from searching for a project:

$$\rho_0 \triangleq \mathbb{E}[\mathbf{1}_{\sigma}\rho_1(\sigma)] - \zeta. \tag{7}$$

The contract must therefore satisfy the following search compatibility constraint:

$$\rho_0 \ge 0. \tag{8}$$

The firm's debt is fairly priced. At time 0, its expected payoff Π is equal to the sum of the income generated by its asset-in-place and the value of the projects that it pursues, minus information costs and any rent payments it makes to its manager. If the incentive constraints (4) and (8) for managerial effort and search are satisfied, then Π is given by Equation (9):

$$\Pi = Y + \mathbb{E}[\mathbf{1}_{\sigma}(pR - 1 - m_{\sigma}) + (1 - \mathbf{1}_{\sigma})(L - 1)] - \zeta - \rho_0.$$
(9)

4. Commitment benchmark

We now consider the case where the firm is able to commit at time 0 to an investment strategy. Conditional upon a strategy, the firm selects the non-negative contract $(w_{\sigma}^{R}, w_{\sigma}^{0})_{\sigma \in \{e,h\}}$ that maximizes its time 0 expected payoff subject to the manager's effort constraint (4) and search constraint (8). We write Π_{Un} and Π_{Sel} for the firm's time 0 expected payoff in case the firm follows an unselective and selective strategy, respectively, under the optimal compensation contract.

Consider the selective case. If $\mathbf{1}_h = 0$ then the firm can set w_e^0 and w_e^R to make the search constraint (8) bind and, in this case, the effort constraint (4) does not apply. With the manager's ex ante rent $\rho_0 = 0$, Equation (9) can be written as follows:

$$\Pi_{\text{Sel}} = Y + (1 - \lambda)(pR - 1) + \lambda(L - 1) - \zeta = V^{FB} - \lambda(pR - M - L).$$
(10)

Because the manager earns no net rent, the firm extracts all of the value generated by its investment strategy. That value is equal to the first best payment V^{FB} , less a deadweight cost equal to the NPV of the foregone hard investment opportunity, net of its salvage value L.

Now consider the unselective case. The cheapest way for the firm to satisfy the effort constraint (4) is to use contract \underline{w}_{U} , defined as follows:

$$\underline{\boldsymbol{w}}_{\boldsymbol{U}} \triangleq (w_h^0 = 0, w_e^R = 0, w_e^0 = 0, w_h^R = \overline{\boldsymbol{w}}).$$
(11)

Substituting contract \underline{w}_U and $\mathbf{1}_{\sigma} \equiv 1$ into Equation (7) yields the following expression for the manager's time 0 expected rent

$$\rho_0 = \mathbb{E}[\rho_1(\sigma)] - \zeta = \lambda \frac{p - \delta}{\delta} M - \zeta.$$
(12)

If this expression is positive, then the search compatibility constraint (8) is slack: the firm uses contract \underline{w}_{U} . If it is negative, then the firm increases wage payments until it binds. Writing x^{+} for max(x, 0), we therefore have

$$\Pi_{\rm Un} = V^{FB} - \rho_0$$

= $V^{FB} - \left(\lambda \frac{p - \delta}{\delta} M - \zeta\right)^+$ (13)

$$=\begin{cases} V^{FB}, & \text{if } M \leq \frac{\delta}{\lambda(p-\delta)}\zeta;\\ Y+pR-1-\frac{\lambda p}{\delta}M, & \text{otherwize.} \end{cases}$$
(14)

By Equation (12), the second term in Equation (13) is the expected rent ρ_0 that accrues to the manager.

The firm earns more by committing to an unselective strategy than it earns from a selective strategy precisely when $\Pi_{\text{Un}} \geq \Pi_{\text{Sel}}$. This requirement is equivalent to Condition

(15):

$$M \le \max\left(\frac{\delta\zeta}{\lambda(p-\delta)}, \frac{\delta}{p}\left(pR - L + \frac{\zeta}{\lambda}\right)\right).$$
(15)

It is easy to check that $\frac{\delta\zeta}{\lambda(p-\delta)} > \frac{\delta}{p}(pR - L + \zeta/\lambda)$ if and only if $\zeta \ge \lambda(p-\delta)(pR - L)/\delta$, which is true if and only if $M \ge pR - L$, which exceeds the maximum value for M. Hence, for every feasible M, Condition (15) reduces to Condition (16):

$$M \le M^*(\zeta) \triangleq \frac{\delta}{p} \left(pR - L + \frac{\zeta}{\lambda} \right).$$
 (16)

When Condition (16) is satisfied, the firm expands unselectively if and only if $\Pi_{\text{Un}} \geq Y$, which is equivalent to the following condition:

$$M \le \overline{M}_U \triangleq \frac{\delta}{p\lambda} (pR - 1). \tag{17}$$

Conversely, when Condition (16) is violated, so that selective investment dominates unselective, the firm expands if and only if $\Pi_{\text{Sel}} \geq Y$, which reduces to the following condition:

$$\zeta \le \zeta_S \triangleq (1 - \lambda)(pR - 1) + \lambda(L - 1).$$
(18)

Proposition 1 characterizes the firm's equilibrium organizational form and investment strategy in case it can commit to an investment strategy.

Proposition 1. Suppose that the firm can commit to an investment strategy. Then it uses a branch structure if it expands. Its expansion decision depends upon the cost M of managerial effort in the following way:

- 1. If $M \leq \min(M^*(\zeta), \overline{M}_U)$, then the firm expands with an unselective investment strategy.
- 2. If $M^*(\zeta) < \overline{M}_U$, then the firm expands with a selective investment strategy for $M > M^*(\zeta)$.

Otherwize, the firm does not expand.

Proof. With commitment, the only difference between branch and subsidiary expansion is that the latter is costly. The firm therefore expands via a branch. Part 1 follows immediately from Equations (16) and (17). Part 2 follows from the facts first, that selective investment is preferred whenever $M > M^*(\zeta)$ and, second, that Condition (18) is satisfied whenever $M^*(\zeta) < \overline{M}_U$.

Unselective expansion is worthwhile when the firm does not give up too much rent to the manager; that is the case when the cost M of managerial effort is low enough, as in Equation (17). And selective investment is valuable if the profit generated by accepting easy projects



(a) $\frac{\delta}{p\lambda} < 1$. In this case, the selective region extends vertically to the maximum effort cost pR - 1 given by Equation (1).

(b) $\frac{\delta}{p\lambda} > 1$. In this case, the maximum value pR-1 for the managerial effort cost is lower than the maximum cost \overline{M}_U for which unselective investment is individually rational. The consequence is that selective investment occurs for a lower range of search costs ζ than in panel (a) of this figure.

Figure 2. Optimal investment strategies under commitment. Unselective investment is preferred to selection investment for any M less than or equal $M^*(\zeta)$. Unselective investment is individually rational for $M \leq \overline{M}_U$. It follows that, in each panel, the light-shaded region contains the parameter values for which the firm elects to invest unselectively. Selective investment is individually rational for $\zeta \leq \zeta_S$ and will occur when, in addition, $M > M^*(\zeta)$, as illustrated in both panels. It is easy to check that, as illustrated in both panels, \overline{M}^* is equal to pR - 1 when $z = \zeta_S$.

and liquidating hard ones is sufficient to defray the cost ζ of search, as in Equation (18). Hence, the firm prefers an unselective investment strategy to a selective strategy for low Mand high ζ ; the boundary $M^*(\zeta)$ between the Selective and Unselective regions in Figure 2 is therefore increasing in ζ .

To understand why it is possible to express part 2 of Proposition 1 entirely in terms of the effort cost M, note that, at the point where $M^*(\zeta) = \overline{M}_U$, the firm is indifferent between selective and unselective investment $(M = M^*(\zeta))$ and, in addition, the firm derives no profit from unselective investment $(M = \overline{M}_U)$. It follows that the firm generates zero profit from selective investment at this point and, hence, that it must occur where $\zeta = \zeta_S$.

5. Equilibrium without commitment

We now consider the case where the firm is unable at time 0 to commit to invest at time 1. In addition to the effort constraint (4) and the search constraint (8), the firm's wage contract

 $(w_{\sigma}^{R}, w_{\sigma}^{0})_{\sigma \in \{e,h\}}$ and its investment strategy $\mathbf{1}_{\sigma}$ must therefore satisfy a time consistency constraint, that investment be the dominant time 1 strategy whenever $\mathbf{1}_{\sigma} = 1$. We consider the implications of this requirement for branch and subsidiary structures in turn.

5.1 Time-consistent branch firm strategies

If the firm expands via a branch, then its debt is backed by the firm's asset-in-place, which has value $Y \ge 1$. It follows that the branch firm's debt is riskless and, hence, that its debt has face value $D_B = 1.^4$ The time-consistency requirement for investment to be incentive compatible at time 1 in state σ is that $(Y+pR-D_B)-(w_{\sigma}^0+p(w_{\sigma}^R-w_{\sigma}^0)-m_{\sigma}) \ge Y+L-D_B$, which can be written as follows:

$$(pR - m_{\sigma} - L) - \rho_1(\sigma) \ge 0. \tag{19}$$

That is, the firm will invest at time 1 if and only if the incremental NPV of the investment exceeds the agency rent incurred at time 1.

We first consider the hard state. When $\sigma = h$, the time consistency requirement (19) reduces to the requirement that $w_h^0 + p(w_h^R - w_h^0) - M \le pR - M - L$, or

$$w_h^0 + p(w_h^R - w_h^0) \le pR - L.$$
(20)

By the effort constraint (4), the lowest value that the left hand side of this expression can attain is achieved by setting $w_h^0 = 0$ and $w_h^R = M/\delta$. It follows that the hard-state time consistency constraint (20) can be satisfied for any M satisfying Condition (21):

$$M \le \underline{M}^B \triangleq \frac{\delta}{p} (pR - L).$$
(21)

Note that

$$\underline{M}^B = M^*(0) < M^*(\zeta).$$

It follows that there is a range $(\underline{M}^B, M^*(\zeta)]$ of effort costs within which a firm that expands through a branch prefers an unselective strategy at time 0 but cannot commit to follow it because, at time 1, it will not invest in hard projects. The reason is that, at time 0, the firm views the ex post rent $\rho_1(h)$ that it pays to its manager in the hard state as a way of defraying the search cost ζ incurred by the manager. In contrast, at time 1 the search cost has been sunk and the firm views the manager's rent simply as a reduction in the ex post income it makes from proceeding with the project. Note that, when the search cost ζ is equal to zero, this time inconsistency does not arise and, as a result, the ex post threshold

⁴Because debt issued when $Y \ge 1$ is repaid with certainty, it is equivalent to equity financing.

 \underline{M}^{B} is equal to the ex ante threshold $M^{*}(0)$.

When $M > \underline{M}^B$, the firm cannot commit to an unselective strategy and it therefore adopts a selective strategy whenever it is both time consistent and profitable. As Lemma 1 shows, this is the case whenever unselective investment is profitable with commitment.

Lemma 1. Suppose that the firm expands using a branch structure and that it cannot commit at time 0 to its time 1 investment strategy.

- 1. If $M \leq \underline{M}^B$, then the firm expands using an unselective investment strategy.
- 2. If $M > \underline{M}^B$, then the firm expands using a selective strategy if $\zeta \leq \zeta_S$.

Otherwize, the firm does not expand.

Proof. For the first part, the firm expands unselectively when unselective expansion is time consistent, when it is preferred to selective expansion, and when it dominates non-expansion. Equation (21) guarantees that the time consistency condition is satisfied when $\sigma = h$. Furthermore, note that $M_U = \underline{M}^B + \delta((pR - 1)(1 - \lambda) + pL)/(p\lambda) > \underline{M}^B$, so that unselective banking is preferred to selective whenever $M \leq \underline{M}^B$. Now, without loss of generality, assume that the easy state contract promises a flat payment w_e . Setting the hard-state rent $\rho_1(h)$ equal to pR - M - L, which is the maximal value consistent with managerial effort that it has when $M = \underline{M}^B$, and setting $w_e = pR - L$ so that the state e time consistency requirement binds, the search constraint (Equation (8)) can be written as $pR - L - \lambda M - \zeta \geq 0$, which is true because $L \leq 1$ and $pR - 1 - \lambda M - \zeta \geq 0$ by Equation (2).

For the second part, the easy state time consistency requirement $pR - L \ge w_e$ can be satisfied simultaneously with the search constraint (8) for selective firms if and only if $\zeta \le (1 - \lambda)(pR - L) = \zeta_S + 1 - L$; this is guaranteed to be true whenever $\zeta \le \zeta_S$, which is the condition that obtains for selective investment with commitment.

The optimal branch strategies without commitment are illustrated in Figure 2. The selective region in each panel of the Figure is smaller than the corresponding region in Figure 1, because, for $M > \underline{M}^B$, an unselective investment strategy violates the time consistency constraint (19). In the region bounded by a dashed line, selective investment is performed instead of unselective investment; in the region bounded by a dotted line, the firm would have performed unselective investment had it been able to commit but, without this ability, it does not invest at all.

5.2 Time-consistent subsidiary firm strategies

If the firm expands via a subsidiary, then it issues subsidiary debt $D_S \leq R$ and parent debt $D_P \leq Y$ and signs a contract $(w_{\sigma}^R, w_{\sigma}^0)_{\sigma \in \{e,h\}}$ with the manager. If the firm invests in state σ at time 1, then it repays its parent debt in full; it pays its subsidiary debt D_S in full if the



Figure 3. Optimal branch investment strategies without commitment. Panels (a) and (b) of this Figure illustrate the investment strategies of branch firm that cannot commit in the respective cases where δ is less than, and greater than $p\lambda$. In the region that is bounded by a dashed line, the branch without commitment adopts a selective strategy because the unselective strategy that is ex ante optimal is not time consistent. In the very light gray region that is bounded by a dotted line no investment occurs, again because the optimal unselective investment strategy is not time consistent.

project succeeds and otherwise defaults and pays nothing; and it pays its manager's wages. The firm's net income from investment is therefore

$$Y - D_P + p(R - D_S) - (w_{\sigma}^0 + p(w_{\sigma}^R - w_{\sigma}^0)).$$
(22)

If the firm does not invest, then it repays its parent debt in full; if the liquidation value L exceeds the subsidiary debt then it repays the debt in full and retains the residual sum $L - D_S$ and if $L < D_S$ it defaults, pays L and receives nothing. The firm's net income from not investing is therefore

$$Y - D_P + (L - D_S)^+.$$
 (23)

The time-consistency constraint for time 1 investment to be incentive compatible in state σ is that Expression (22) exceed Expression (23). This requirement can be expressed as follows:

$$(pR - m_{\sigma} - L) - \rho_1(\sigma) + \Delta(D_S) \ge 0, \qquad (24)$$

where

$$\Delta(D_S) \triangleq \min(D_S, L) - pD_S \tag{25}$$

		State	
		s_1 (success)	s_2 (failure)
Debt repayment	Invest	D_S	0
	Liquidate	$\min(L, D_S)$	$\min(L, D_S)$
Marginal effect		$\min(L, D_S) - D_S$	$\min(\overline{L,D_S})$

Figure 4. **Debt repayment with and without subsidiary investment.** The Figure illustrates the subsidiary's debt repayments in case it does, and does not invest. The marginal effect of investment is to increase the firm's income by the reduction in debt service.

is the marginal reduction in the costs of debt servicing caused by investment. To understand $\Delta(D_S)$, we write s_1 and s_2 for the respective states in which subsidiary investment succeeds and fails, and in Figure 4 we show the state-dependent debt repayment in case investment does, and does not, occur. Investment increases the firm's income by the resultant drop in debt servicing costs, which is given by the marginal effect row in Figure 4; the probabilityweighted value of this row is the incremental effect in Equation (24).

Investment reduces debt servicing costs by $\min(L, D_S)$ in state s_2 because the subsidiary has limited liability; it increases them by $D_S - \min(L, D_S)$ in state s_1 , when investment enables full repayment of the debt. The following result is immediate:

Lemma 2. Suppose that the firm expands with a subsidiary structure and that it cannot commit to an investment strategy. Then the marginal reduction $\Delta(D_S)$ in debt servicing costs is maximized by borrowing $D_S = L$ at the subsidiary level.

Because the firm's debt is fairly priced, it is indifferent between any debt levels that satisfy the time-consistency condition (24). By Lemma 2, this condition is slackest when $D_S = L$ and we therefore assume without loss of generality that

$$D_S = L \tag{26}$$

so that

$$\Delta(D_S) = (1-p)L. \tag{27}$$

Now consider state $\sigma = h$. In this case, the time-consistency requirement (24) can be written as follows:

$$M \le \delta(R - L). \tag{28}$$

Hence, for $M \in [\underline{M}^B, \delta(R-L) = \underline{M}^B + (1-p)L]$, the firm is able to commit to unselective investment with a subsidiary but not with a branch. As noted above, the reason is that investing allows the firm to profit from the limited liability option on its subsidiary: if the subsidiary borrows L, then that option has value (1-p)L and the firm can sustain its commitment to invest in hard states for a correspondingly higher managerial effort cost.

This discussion provides the intuition for the following result.

Lemma 3. Suppose that the firm expands using a subsidiary structure and that it cannot at time 0 commit to its time 1 investment strategy. Let

$$\underline{M}^{S} = \min\left(\delta(R-L), M^{*}(\zeta), \overline{M}_{U}\right).$$
⁽²⁹⁾

Then

1. If $M \leq \underline{M}^S$, then the firm expands using an unselective investment strategy;

2. If $M > \underline{M}^S$, then the firm expands using a selective strategy if $\zeta \leq \zeta_S$.

Otherwize, the firm does not expand.

Proof. For $M \leq \underline{M}^S$, unselective investment dominates selective investment, returns more than non-investment, and is time consistent. This establishes part 1.

For part 2, we need only demonstrate that selective investment is time consistent when $M > \underline{M}^S$. Without loss of generality, we assume that the state e contract is a flat wage w_e . The time consistency requirement in state e is then $p(R - L) \ge w_e$, and the search constraint is that $(1 - \lambda)w_e \ge \zeta$. Allowing this to bind and substituting back into the time consistency constraint yields the requirement that $p(R - L)(1 - \lambda) \ge \zeta$. Observe that $p(R - L)(1 - \lambda) = (1 - \lambda)(pR - 1) + \lambda(L - 1)(1 - pL) + \lambda(1 - L)$, which exceeds ζ whenever the participation constraint (18) is satisfied.

The optimal subsidiary investment strategy without commitment is illustrated in Figure 5. As in Figure 3, the inefficiencies caused by time inconsistencies are indicated in two regions. The first is bordered by a dashed line, and indicates parameter values for which a firm with a subsidiary opts for a selective strategy when it would have opted for an unselective strategy with commitment. The second is bordered by a dotted line. Within this region, the firm would opt for an unselective strategy if it were able to commit to invest but, because it cannot make this commitment, it is unable to cover the high search cost ζ and, hence, does not invest. It is clear from a comparison between Figures 3 and 5 that the scale of the inefficiency caused by an inability to commit is smaller with a subsidiary structure than it is with a branch. As discussed above, the reason for this is that the subsidiary can profit from its limited liability option if it invests, and this renders it easier to honour a time 0 promise to invest at time 1.

5.3 Optimal organizational structure

Section 5.2 demonstrated that, because a firm with a subsidiary can take advantage of a limited liability option on its debt, the time consistency constraint is slacker for firms with



Figure 5. Optimal subsidiary investment strategies without commitment. Panels (a) and (b) of this Figure illustrate the investment strategies of branch firm that cannot commit in the respective cases where δ is less than, and greater than $p\lambda$. Analogously with Figure 3, the dashed lines indicate regions in which, due to commitment problems, a selective firm operates when an unselective firm's ex ante value is higher; and the regions bounded by dotted lines, the subsidiary performs no investment because commitment problems prevent it from running an unselective firm. Note that, because the subsidiary can use the limited liability option on its debt to overcome commitment problems, the scale of the distortion caused by time inconsistency is smaller with a subsidiary than it is with a branch.

subsidiaries that it is for firms with branches. Hence, the firm may opt to spend C to set up a subsidiary structure in order to commit to invest in the hard state. It is worth doing so when $\Pi_{\text{Un}} - C \ge \Pi_{\text{Sel}}$. This condition reduces to

$$M \le \frac{\delta}{p} \left(pR - L + \frac{\zeta - C}{\lambda} \right). \tag{30}$$

The line $M = (\delta/p) (pR - L + (\zeta - C)/\lambda)$ is illustrated in Figure 6. It sits at a distance $\delta C/(p\lambda)$ below the line $M = M^*(\zeta)$ that separates selective from unselective expansion in Figure 5, which illustrates the subsidiary case when the cost C has already been sunk.

Setting up a subsidiary is profitable precisely when $\Pi_{\text{Un}} - C \ge Y$: equivalently, when

$$M \leq \begin{cases} \hat{M}_U \triangleq \overline{M}_U - \frac{\delta}{p\lambda}C = \frac{\delta}{p\lambda}(pR - 1 - C), & \text{if } M \leq \frac{\delta}{\lambda(p - \delta)}\zeta; \\ \hat{M}_2(\zeta) \triangleq \overline{M}_2(\zeta) - \frac{1}{\lambda}C = \frac{1}{\lambda}(pR - 1 - \zeta - C), & \text{otherwize.} \end{cases}$$
(31)

Once again, \hat{M}_U and \hat{M}_2 are illustrated in Figure 6.



Figure 6. Optimal organizational form and investment strategy. The Figure illustrates the case where $\delta < p\lambda$. Branch expansion is cheaper than subsidiary expansion and the firm therefore uses subsidiary expansion only when it is necessary to commit to an unselective investment strategy, and when that is more profitable than selective branch investment. Hence, when $M \leq \underline{M}^B$ and the firm can commit to unselective investment with a branch structure, it operates as an unselective branch. The line $\hat{M}^*_U(\zeta)$ is indicated as a bold, inverted-U shape in the Figure and below this line subsidiary expansion achieves commitment and is cost-effective. Above both $\hat{M}^*_U(\zeta)$ and \underline{M}^B , selective branch investment is the most profitable way to expand; it is worth performing when $\zeta \leq \zeta_S$, as indicated in the Figure.

Define

$$\hat{M}_{U}^{*}(\zeta) \triangleq \min\left(\frac{\delta}{p}\left(pR - L + \frac{\zeta - C}{\lambda}\right), \hat{M}_{U}, \hat{M}_{2}(\zeta)\right).$$
(32)

We can now state our main result:

Proposition 2. If the firm cannot commit to an investment strategy, then its optimal organizational form for business expansion depends upon the effort cost M as follows:

1. If $M \leq \underline{M}^B$, then the firm expands as an unselective branch.

2. If $\underline{M}^B < M \leq \hat{M}_U^*(\zeta)$, then the firm expands as an unselective subsidiary.

3. If $M > \max(\underline{M}^B, \hat{M}^*_U(\zeta))$ and $\zeta \leq \zeta_S$, then the firm expands as a selective branch.

Otherwize, the firm does not expand.

Proposition 2 is illustrated in Figure 6 in the case where $\delta < p\lambda$; the case with $\delta > p\lambda$ is similar and can be drawn by analogy to panel (b) in Figures 3 and 5. For $M < \underline{M}^B$, the firm is able to commit to invest in both types of project if it expands as a branch; as branch expansion is cheaper than subsidiary expansion, the firm uses the branch structure. The function $\hat{M}_U^*(\zeta)$ is illustrated in the Figure as a bold, inverted-U shape. For M < $\hat{M}_U^*(\zeta)$, the firm is able to commit to invest in both types of project when it expands as a subsidiary, and the additional profit that it makes from this commitment exceeds the cost C of setting up a subsidiary. Hence, in the dark-shaded region of Figure 6, the firm elects to expand as a subsidiary and it adopts an unselective investment strategy. Finally, for $M > \max(\underline{M}^B, \hat{M}_U^*(\zeta))$, the firm is unable to commit to invest in both types of project as a branch, and either it cannot commit to invest in hard projects even with a subsidiary structure, or it does not find it worthwhile to form a subsidiary in order to achieve commitment. It therefore expands as a selective branch when it is profitable to do so; that is the case throughout the gray region labelled "Selective branch" in Figure 6, where $\zeta < \zeta_S$.

Proposition 2 states that the firm's optimal organizational structure is a non-monotonic function of the cost M of managerial effort in a hard project. If M is low enough, then the firm can commit to invest in hard projects without the additional risk-shifting incentive that comes from the subsidiary limited liability structure, and so expands as a branch. For high M, the firm cannot commit to invest in hard projects (either because it cannot comit to invest in hard projects even with a subsidiary structure, or because it does not find it profitable to do so), and so uses the branch structure to expand, because it is cheapest. But, for intermediate values of M, if the firm uses a subsidiary structure, then it can create a time 1 risk-shifting incentive by borrowing at the subsidiary level; it then has a heightened time 1 incentive to invest, because investment allows it to default if its project fails. The firm therefore uses a subsidiary structure in this region.

Note that, although the organizational form depends non-monotonically upon M, the firm's equilibrium investment strategy does not: it invests unselectively for $M \leq \max(\hat{M}_U^*(\zeta), \underline{M}^B)$, and it invests selectively for $M > \max(\hat{M}_U^*(\zeta), \underline{M}^B)$,.

We conclude this section by considering the effect that variations in the search cost ζ and the success probability p have upon organizational form.

First, note that, as ζ increases, the upper and lower bounds of the subsidiary region in Figure 6 do not change; the left-hand boundary shifts leftward, and the right-hand boundary shifts to the left, also. Hence, if a higher search cost does not force a firm to cease expansion, it renders the firm more likely to expand as a subsidiary. The reason is that, when the search cost is higher, the firm has to give up more to its managers if it is to operate at all. It is therefore more willing give up the rent that it loses when it operates unselectively and, hence, to operate as a subsidiary.

Second, as p increases, the value of \underline{M}^B increases, so that the range of parameters in which the firm is willing to operate as an unselective branch also expands. At the same time, the inverted-U of $\hat{M}_U^*(\zeta)$ shifts up. In other words, the range of parameters within which unselective investment occurs expands. This happens because a higher probability of project success increases the value of hard projects; consequently, the firm is more willing to

incur the costs of setting up a subsidiary in order to commit to invest in hard projects.

6. Extension: Risky asset-in-place

We now consider a version of our model in which the firm's asset-in-place is risky with time 2 value $\tilde{Y} \in \{0, \overline{Y}\}$, with $\overline{Y} > 1$ and $\mathbb{P}[\tilde{Y} = \overline{Y}] = q$. We write

$$Y \triangleq q\overline{Y}.$$
(33)

In order to analyse the correlation between the respective time 2 values \tilde{Y} and \tilde{R} of the firm's asset-in-place and its investment, we partition the time 2 state space Ω when the manager exerts effort into four as illustrated in Figure 7.

Ω	$ \begin{array}{c} \boldsymbol{\omega}_1 \\ \tilde{Y} = \overline{Y} \end{array} $	$egin{array}{c} \omega_2 \ ilde{Y} = 0 \end{array}$
	$\tilde{R} = R$	$\tilde{R} = R$
	ω_3	$oldsymbol{\omega}_4$
	$\tilde{Y} = \overline{Y}$	$\tilde{Y} = 0$
	$\tilde{R} = 0$	$\tilde{R} = 0$

Figure 7. State space partition with risky asset-in-place. The asset-in-place succeeds in states 1 and 3; the investment succeeds in states 1 and 2. Hence, more correlated returns on the asset-in-place and the investment are indicated by higher probabilities p_1 and p_4 of states 1 and 4, respectively.

Hence, for example, conditional upon managerial effort, the asset-in-place and the investment both succeed in state 1, and they both fail in state 4. It is convenient to write p_i for the probability measure $\mathbb{P}[\omega = \omega_i]$ of state *i*. It is clear from inspection of Figure 7 that

$$p_1 + p_2 = p;$$
 (34)

$$p_1 + p_3 = q, (35)$$

where p and q are the respective success probabilities of the investment and the asset-inplace. We use the partition of Figure 7 to study the effect of higher and lower correlations between the investment and the asset-in-place. To accomplish this, define

$$q^R \triangleq \mathbb{P}[\tilde{Y} = \overline{Y} | \tilde{R} = R] = \frac{p_1}{p_1 + p_2};$$
(36)

$$q^0 \triangleq \mathbb{P}[\tilde{Y} = \overline{Y} | \tilde{R} = 0] = \frac{p_3}{p_3 + p_4}.$$
(37)

		State			
		ω_1	ω_2	ω_3	ω_4
Debt repayment Marg	Invest	D_B	D_B	D_B	0
	Liquidate	D_B	$\min(L, D_B)$	D_B	$\min(L, D_B)$
	rginal effect	0	$\min(L, D_B) - D_B$	0	$\min(L, D_B)$

Figure 8. **Debt repayment with and without investment.** The Figure illustrates the branch debt repayments by a branch with risky asset-in-place in case it does, and does not invest. The marginal effect of investment is to increase the firm's income by the reduction in debt service.

Then we can write

$$p_1 = pq^R; (38)$$

$$p_2 = p(1 - q^R); (39)$$

$$p_3 = (1-p)q^0; (40)$$

$$p_4 = (1-p)(1-q^0). (41)$$

Note that, by Bayes' Law, The asset-in-place and investment are more correlated when p_1 and p_4 are higher: that is, when q^R is high and q^0 is low.

6.1 Branch structure with risky asset-in-place

Let D_B be the face value of debt issued by a branch firm, and let d_B be its price. If $d_B < 1$, then the firm's owners contribute $1 - d_B$ of equity. If the firm invests in state $\sigma \in \{e, h\}$, then the firm's expected income from investment is

$$p(q^{R}\overline{Y} + R - D_{B}) + (1 - p)q^{0}(\overline{Y} - D_{B}) - (w_{\sigma}^{0} - p(w_{\sigma}^{R} - w_{\sigma}^{0})).$$
(42)

The first two terms are obtained by conditioning on the success or failure of the investment, which occurs with probabilities p and 1 - p, and then, using Equations (36) and (37), conditioning upon the success or failure of the asset-in-place. The final term is the expected wage payment to the manager.

If the firm does not invest, then it repays its debt in full if and only if its asset-in-place succeeds, and otherwise pays the minimum of liquidation value L and the face value D_B of debt. Its expected income is therefore given by Equation (43):

$$q(\overline{Y} + L - D_B) + (1 - q)(L - \min(L, D_B)).$$
(43)

Branch firms whose asset-in-place is risky invest precisely when Expression (42) exceeds

Expression (43). That condition is equivalent to Condition (44):

$$(pR - m_{\sigma} - L) - \rho_1(\sigma) + \hat{\Delta}_B(D_B) \ge 0, \tag{44}$$

where

$$\hat{\Delta}_B(D_B) \triangleq (1-q)\min(L, D_B) - p(1-q^R)D_B$$
(45)

is the incremental reduction in debt servicing costs caused by investment. To understand $\hat{\Delta}_B(D_B)$, consider Figure 8, which presents the firm's debt repayment in case it does, and does not, invest at time 1. The marginal effect in the Figure is the reduction in debt repayment caused by investment; the probability-weighted value of this effect is

$$(p_2 + p_4) \min(L, D_B) - p_2 D_B = \hat{\Delta}_B(D_B).$$

In short, the marginal effect of investment in Condition (44) comprises the difference $(pR - m_{\sigma} - L)$ between the social return $pR - m_{\sigma}$ earned by continuing the project and the social return L achieved by liquidating it, less the expected rent payment $\rho_1(\sigma)$ to the manager, plus the marginal reduction $\hat{\Delta}(D_B)$ in debt service costs.

Lemma 4. Suppose that the firm has a risky asset-in-place, that it expands using a branch structure, and that it cannot commit at time 0 to a time 1 investment strategy. Then the marginal reduction $\hat{\Delta}_B(D_B)$ in debt servicing costs is maximized by borrowing $D_B = L$.

As in Section 5.2, we assume without loss of generality that

$$D_B = L \tag{46}$$

so that

$$\hat{\Delta}_B(D_B) = (1-p)(1-q^0)L.$$
(47)

With this assumption, the time-consistency Condition (44) for $\sigma = h$ reduces to Condition (48):

$$M \le \underline{M}_{\tilde{Y}}^B \triangleq \frac{\delta}{p} \left((pR - L) + (1 - p)(1 - q^0)L \right)$$
(48)

$$=\delta(R-L) - \frac{\delta}{p}(1-p)q^{0}L.$$
(49)

That is, the time consistency requirement (48) with a risky asset-in-place is slacker than the corresponding constraint (21) in case the asset-in-place is not risky by precisely $\frac{\delta}{p}(1-p)(1-q^0)L$. To understand this difference, note that, when a branch with a risky asset-in-place sets $D_B = L$, it is able to derive a profit from its limited liability option only if (1) with

probability 1 - p, its investment fails; and (2) with conditional probability $1 - q^0$ its assetin-place returns 0. It follows that the time consistency constraint is slacker with a risky asset-in-place whenever p_4 is non-zero.

As in Section 4, selective investment is preferred to unselective investment for $M \leq M^*(\zeta)$. These observations immediately yield Lemma 5:

Lemma 5. Suppose that the firm has a risky asset-in-place, that it expands using a branch structure, and that it cannot commit at time 0 to its time 1 investment strategy.

- 1. If $M \leq \min(\underline{M}_{\tilde{Y}}^B, M^*(\zeta))$, then the firm expands using an unselective investment strategy;
- 2. If $M > \min(\underline{M}_{\tilde{Y}}^B, M^*(\zeta))$, then the firm expands using a selective strategy if $\zeta \leq \zeta_S$.

Otherwize, the firm does not expand.

Unselective investment is time consistent for a larger parameter set with a risky asset-inplace because, even with a branch structure, the firm is able to take advantage of the limited liability option by investing at time 1 and, hence, finds investment correspondingly more attractive. The value of the limited liability option is maximized when the square-bracketed term in Equation (44) is maximized: that is, when $D_B = L$.

Lemma 5 states that selective investment is time consistent for a greater range of parameters when $\underline{M}_{\tilde{Y}}^B$ is higher. That is the case when $p_4 = (1-p)(1-q^0)$ is higher, so that the asset-in-place and the investment are more likely to fail together. In short, time consistency is less of a problem for branches when they have a risky asset-in-place whose returns are correlated with those of its investment opportunities.

6.2 Subsidiary structure with risky asset-in-place

The relationship between a subsidiary and its parent firm is asymmetric. Returns generated at the subsidiary level must be used to repay subsidiary debt first, and parent debt second; but, in contrast, while funds earned by the parent must be used to repay parent debt, the parent can default on subsidiary debt. To examine the consequences of this asymmetry, suppose that the firm has a subsidiary that has issued debt with face value $D_S \leq R$ and market price d_S ; and that the parent firm has issued debt with face value $D_P \leq \overline{Y}$ and market price d_P . Once again, if the total amount $d_S + d_P$ raised by debt is less than 1, the firm's owners contribute equity $1 - d_S - d_P$.

If the firm invests at time 1 in state $w \in \{l, h\}$, then its expected time 2 income is

$$p_1(\overline{Y} + R - D_P - D_S) + p_2(R - D_S - D_P)^+ + p_3(\overline{Y} - D_P) - (w_\sigma^0 + p(w_\sigma^R - w_\sigma^0)).$$
(50)

	State				
	ω_1	ω_2	ω_3	ω_4	
Invest Liquidate	$D_P + D_S$ $D_P + \min(L, D_S)$	$\frac{\min(R, D_S + D_P)}{\min(L, D_S + D_P)}$	$\frac{D_P}{D_P + \min(L, D_S)}$	$0\\\min(L, D_S + D_P)$	
Marginal effect	$\min((L, D_S) - D_S)$	$\min(L, D_S + D_P) - \min(R, D_S + D_P)$	$\min(L, D_S)$	$\min(L, D_S + D_P)$	

Figure 9. **Debt repayment with and without investment.** The Figure illustrates the subsidiary debt repayments by a firm with risky asset-in-place in case it does, and does not invest. The marginal effect of investment is to increase the firm's income by the reduction in debt service.

The first three terms in this expression are the probability-weighted firm income after debt service costs in states 1, 2, and 3 (the firm earns nothing and defaults on its debt in state 4); the final term is the firm's expected wage bill.

If the firm liquidates at time 1, then its expected income is

$$(p_1 + p_3)(\overline{Y} - D_P + (L - D_S)^+) + (p_2 + p_4)(L - D_S - D_P)^+.$$
(51)

In states 1 and 3, the asset-in-place succeeds and the firm retains its income $\overline{Y} - D_P$ after repaying the parent's debt; it also retains anything that remains after it repays its subsidiary debt D_S from the proceeds L of liquidating the project. In states 2 and 4, the firm relies upon the liquidation value L of its subsidiary to repay both subsidiary debt D_S and parent debt D_P ; only if the liquidation value covers both does it receive any income.

The subsidiary firm invests if and only if Expression (50) exceeds Expression (51). This requirement reduces to Condition (52):

$$(pR - m_{\sigma} - L) - \rho_1(\sigma) + \hat{\Delta}_S(D_S, D_P) \ge 0,$$
(52)

where

$$\hat{\Delta}_S(D_S, D_P) \triangleq q \min(L, D_S) + (1-q) \min(L, D_S + D_P) - p_1 D_S - p_2 \min(R, D_S + D_P).$$
(53)

Precisely as for Condition (44), the first two terms of Condition (52) are, respectively, the marginal social value of investment less the rent paid to the manager in case of investment. The final term $\hat{\Delta}_S(D_S, D_P)$ is the incremental effect of investment upon the costs of servicing the firm's parent and subsidiary debt. To show this, we illustrate the subsidiary's state-contingent debt-serving costs as a function of its time 1 investment decision in Figure 9; the probability-weighted marginal effect is equal to $\hat{\Delta}_S(D_S, D_P)$.

Lemma 6. Suppose that the firm has a risky asset-in-place, that it expands using a subsidiary structure, and that it cannot commit at time 0 to a time 1 investment strategy. Then



Figure 10. Derivatives of subsidiary value

the marginal reduction in debt servicing costs $\hat{\Delta}_S(D_S, D_P)$ due to investment is maximized by setting $D_P = 0$ and $D_S = L$.

Proof. Figure 10 illustrates the derivatives of $\Delta_S(D_S, D_P)$ with respect to D_S and D_P ; vertical arrows indicate in which direction D_P must move for the term to increase, and horizontal arrows indicate the direction in which D_S must move to increase the term; in the region where there are two arrows, the direction is indeterminate. Along the line $D_S + D_P$, $\hat{\Delta}_S(D_S, D_P)$ increases as D_S increases and D_P decreases, as indicated.

It is immediately obvious from this figure that, starting from any (D_S, D_P) pair, following the arrows on the figure always leads to the point (L, 0), as claimed in the Lemma.

Once again following Section 5.2, we assume without loss of generality that $D_P = 0$ and $D_S = L$. Condition (52) for an unselective investment strategy to be time-consistent for a subsidiary firm with a risky asset-in-place then reduces to the requirement that

$$M \le \delta(R - L). \tag{54}$$

Note that Condition (54) is identical to Condition (28). This observation gives us Lemma 7:

Lemma 7. Suppose that the firm has a risky asset-in-place, that it expands using a subsidiary structure, and that it cannot commit at time 0 to its time 1 investment strategy. Then its investment strategy is given by Lemma 3. That is, when \underline{M}^S is given by Equation (29), the firm expands using an unselective investment strategy for $M \leq \underline{M}^S$, with a selective investment strategy for $M > \underline{M}^S$ and $\zeta \leq \zeta_S$, and otherwize does not expand.

The intuition for this result follows from inspection of Figure 9. When $D_P = 0$ and $D_S = L$, the subsidiary structure experiences a limited liability profit from lower debt repayment only



Figure 11. Optimal organizational form and investment strategy with risky asset-inplace. Note that, although it is smaller than the corresponding region in Figure 6, an unselective subsidiary region remains when the asset-in-place is risky.

in states ω_3 and ω_4 : that is, exactly as in the case with a safe asset-in-place, it earns a limited liability profit of L precisely when R = 0.

6.3 Optimal organizational structure with risky asset-in-place

Precisely as in Section 5.3, the firm adopts a subsidiary structure when (1) it is impossible to commit to an unselective investment strategy with a branch: that is, when $M > \underline{M}_{\tilde{Y}}^B$; (2) it unselective investment is profitable and also dominates selective investment with a subsidiary: that is, when $M \leq M_U^*(\zeta)$. These observations immediately yield Proposition 3.

Proposition 3. If the firm has a risky asset-in-place and cannot commit to an investment strategy, then its optimal organizational form for business expansion depends upon the monitoring cost M as follows:

- 1. If $M \leq \min(\underline{M}^B_{\tilde{V}}, M^*(\zeta))$, then the firm expands as an unselective branch.
- 2. If $\min(\underline{M}^B_{\tilde{Y}}, M^*(\zeta)) < M \leq \hat{M}^*_U(\zeta)$, then the firm expands as an unselective subsidiary.
- 3. If $M > \max(\min(\underline{M}_{\tilde{Y}}^B, M^*(\zeta)), \hat{M}_U^*(\zeta))$ and $\zeta \leq \zeta_S$, then the firm expands as a selective branch.

Otherwize, the firm does not expand.

Proposition 3 is illustrated in Figure 11. When the branch firm can extract value from the limited liability option on its risky asset-in-place, it is better able to commit to invest than it is when its asset-in-place is safe. However, as illustrated in the Figure, our qualitative result survives: for some parameter values, the subsidiary structure is still useful as a device for committing to time 1 investment.

Proposition 3 also generates a new insight. Note from Equation (48) that $\underline{M}_{\tilde{Y}}^B$ is increasing in $(1-p)(1-q^0)$, which, by Equation (41), is equal to p_4 . The size of the unselective subsidiary region in Figure 11 therefore increases p_4 decreases. More correlated returns on the asset-inplace and the investment are associated with higher values of p_4 , so we have the following corollary to Proposition 3:

Corollary 1. The size of the unselective subsidiary region in Figure 11 is larger the lower is the correlation between the returns of the asset-in-place and the investment.

The intuition for Corollary 1 is as follows. The choice between subsidiary and branch expansion depends in part upon the relative value of the limited liability option in each case. The returns to a subsidiary investment do not depend upon the returns of the parent firm and, hence, the value of the subsidiary firm's limited liability option is unaffected by the correlation between the returns of the asset-in-place and of the new investment. If, on the other hand, the firm invests via a branch, then the resultant investment portfolio contains both the asset-in-place and the new investment; the volatility of that portfolio is increasing in the correlation between the two and, hence, so, too, is the value of the associated limited liability option. The value of the limited liability option associated with subsidiary expansion less the value of the limited liability option of branch expansion is therefore decreasing in the return correlation between asset-in-place and the investment. Hence, as in the statement of the Corollary, subsidiary expansion is more likely to occur for lower correlations.

7. Model implications

In this section, we discuss our model's empirical implications. We start by discussing empirical proxies for our model parameters; we then derive testable hypotheses and, when empirical work already exists, we relate it to our hypotheses.

7.1 Parameter interpretation

Our model's conclusions are expressed in terms of the search cost ζ and the effort cost M, as well as in terms of the correlation between the returns of the firm's asset-in-place and those of its investment opportunity. In this section, we discuss the interpretation of those parameters.

The search cost ζ is a measure of the difficulty of identifying new projects and of successfully investing in them. It follows that, ceteris paribus, ζ is low when the firm is expanding its existing business line, when it is operating in a country in which it has experience, and when it faces low levels of competition for other reasons. Similarly, ζ is high when competition is high, in new markets, and in new countries.

The effort cost M measures the difficult of managing a project after investment has occurred. Hence, M is lower in countries with better legal systems and stronger institutions; for industries in which labor relations are better; in industries for which management information is more easily accessed; and in industries in which the firm has pre-existing expertise. Conversely, M is high when the legal and institutional structure is weak; when information is hard to access; and when the firm lacks critical expertise.

The correlation between the firm's asset-in-place and its investment is studied in Section 6. The correlation is high when the firm invests in a business that is closely related to its existing portfolio of activities. Conversely, the correlation is low when the firm expands into activities or markets that are unrelated to its existing businesses.

7.2 Organizational form

We now develop hypotheses relating to organizational form. Our first prediction can be deduced by inspecting Figure 6. If, given M and other model parameters, both subsidiaries and branch firms exist, then the branches are selective and they lie to the left of the (unselective) subsidiaries in the Figure. That is, when a subsidiary is used for equilibrium expansion, it is deployed in markets where project search is harder. This argument yields Hypothesis 1.

Hypothesis 1. Ceteris paribus, firms that expand via subsidiaries face higher search costs than firms that expand using branches.

We argue in Section 7.1 that search costs are relatively high for firms entering new markets. Combining this observation with Hypothesis 1 yields Hypothesis 2.

Hypothesis 2. Ceteris paribus, investment in new markets is performed through subsidiaries and investment in existing markets is performed through branches.

Hypothesis 2 identifies a firm-specific effect: it predicts that, when a firm is already operating in a given market, it is likely to use a branch structure to expand within that market, and, conversely, that a firm entering the same market for the first time is likely to do so using a subsidiary structure. Hypothesis 3 idenfies a market-specific effect. Recall from Section 7.1 that every firm faces high search costs in any market in which there is a high level of competition or where it is hard to uncover information about projects. Hence, using Hypothesis 1, we have the following prediction.

Hypothesis 3. Ceteris paribus, investment performed through subsidiaries is characterized by higher competition and weaker access to market information than investment performed through branches. It is clear from inspection of Figure 6 that the optimal organizational form is nonmonotonic in the effort cost M of project management: for low and high levels of M, the firm uses a branch for expansion, while, for intermediate levels of M, the firm uses a subsidiary. Using that proxies for M that appear in Section 7.1, we therefore have Hypothesis 4.

Hypothesis 4. The organizational form that the firm uses for expansion is non-monotonic in the quality of legal institutions, labor relations, and management information systems in the target market and also in the firm's level of expertise in that market. For high and low levels of these factors, the firm uses a branch for expansion; for intermediate quality levels, it uses a subsidiary.

Recall from Corollary 1 that subsidiary expansion is more likely when the firm's new investment has low correlation with its asset-in-place, because, in this case, the value of the subsidiary firm's limited liability option is high relative to the corresponding option in case of branch expansion. This reasoning immediately yields Hypothesis 5.

Hypothesis 5. Ceteris paribus, when the returns in a new market are less correlated with their existing investments, firms are more likely to use a subsidiary firm to expand into that market.

Hypothesis 5 has implications for investment in a country within which a firm already operates. In this case, expansion using the same balance sheet occurs organically and it is impossible to create a separate branch. In a single country, then, Hypothesis 5 may be written as follows.

Hypothesis 6. Ceteris paribus, when the firm expands its existing business lines in a given country, it does so via organic investment; expansion into businesses that are uncorrelated with its existing activities may occur through a subsidiary, via a carve-out, or through a spin-off.

7.3 Profitability and scope

We now examine the implications of our model's conclusions about the portfolio of investments performed by branch and subsidiary firms. These conclusions have implications for profitability and for investment scope.

Note from Figure 6 that, if branch and subsidiary expansion are both possible for a given effort cost M, then the branch expansion is selective, and the subsidiary expansion is unselective. This observation has two implications. First, for a given M, the subsidiary firms invest in a greater range of projects than branch firms. Second, because their firms only invest in easy projects, managers in branch firms never receive expost management rent; in

contrast, managers of subsidiary firms are paid some management rent because their firms invest in both easy and hard projects. That is, for a fixed M, managerial compensation is higher in subsidiary firms than in branch firms. We summarise this discussion in Hypotheses 7 and 8.

Hypothesis 7. If corporations are matched on all parameters except for the search cost ζ , then subsidiary firms maintain a greater scope of projects.

Hypothesis 8. If corporations are matched on all parameters except for the search cost ζ , then managers in subsidiary firms are paid more than those in branch firms.

8. Conclusion

We present a model in which a firm's organizational structure and its investment choices are jointly determined. In some circumstances, a firm might discard an investment because it generates a high level of managerial rent, even though, from an ex ante perspective, it would prefer to commit to invest. In this circumstance, the firm can counter its underinvestment tendency by running its investment in a separate subsidiary firm, and raising debt within the subsidiary. In doing so, it profits from a limited liability option whose value defrays the manager's information rent.

An important difference between our approach and many prior treatments of organizational structure is that our analysis is dynamic. That is, our results are driven by a commitment friction: the firm wishes to commit up-front to a particular investment strategy but it may be unable to make that commitment because its agent earns information rent when it invests. The limited liability option created by a subsidiary investment partially offsets this effect and so may restore commitment. This effect is in contrast to earlier work that shows how subsidiary investment can resolve a debt-overhang problem.

Our model generates fresh empirical predictions about the relationship of organizational form to levels of competition, the quality of management accounting systems, the quality of legal institutions, and the correlation of the returns between new investments and the firm's existing business lines.

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Appendix: Fully contractible project outcomes

Our analysis of the baseline model considers contracts of the form $(w_{\sigma}^{R}, w_{\sigma}^{0})_{\sigma \in \{e,h\}}$ under which the manager is paid only if the firm pursues a project. In this appendix, we extend our analysis to contracts of the form $(w_{\sigma}^{R}, w_{\sigma}^{0}, w_{\sigma}^{N})_{\sigma \in \{e,h\}}$, where w_{σ}^{N} specifies the possibly non-zero payment that the manager receives in case the firm does not pursue a project. We demonstrate that our main results remain true with this expanded contract space.

With contract $(w_{\sigma}^{R}, w_{\sigma}^{0}, w_{\sigma}^{N})_{\sigma \in \{e,h\}}$, the manager's expected rent in case the firm pursues a project at time 1 in state σ is $\rho_{1}(\sigma)$, as in Equation (5) but the rent with the expanded contract in case the firm does not pursue a project at time 1 is w_{σ}^{N} . The manager's time 0 expected rent under contract $(w_{\sigma}^{R}, w_{\sigma}^{0}, w_{\sigma}^{N})_{\sigma \in \{e,h\}}$ is therefore given by Equation (55):

$$\tilde{\rho}_0 \triangleq w_{\sigma}^N + \mathbb{E}[\mathbf{1}_{\sigma}(\rho_1(\sigma) - w_{\sigma}^N)] - \zeta.$$
(55)

The manager's search compatibility constraint in this case is

$$\mathbb{E}[\mathbf{1}_{\sigma}(\rho_1(\sigma) - w_{\sigma}^N)] - \zeta \ge 0,$$

so the contract $(w_{\sigma}^{R}, w_{\sigma}^{0}, w_{\sigma}^{N})_{\sigma \in \{e,h\}}$ must satisfy Condition (56)

$$\tilde{\rho}_0 \ge w_{\sigma}^N. \tag{56}$$

The wage payment w_{σ}^{N} to the manager in case the firm does not pursue a project increases the manager's expected time 0 rent and thus reduces the firm's expected payoff. In particular, when $w_{0}^{N} = 0$, Equations (7)–(8) are identical to Equations (55)–(56).

Note that, if the firm is able to commit to an investment strategy, then it is optimal for it to set $w_{\sigma}^{N} = 0$ for all $\sigma \in \{e, h\}$ in order to minimize the manager's rent. Hence, the equilibrium with commitment is the same with the extended contract $(w_{\sigma}^{R}, w_{\sigma}^{0}, w_{\sigma}^{N})_{\sigma \in \{e, h\}}$ as it is in the baseline model: that equilibrium is laid out in Proposition 1.

Now suppose that the firm is unable to commit to an investment strategy. If the firm expands via a branch, then the time-consistency requirement for investment to be incentive compatible at time 1 in state σ can be written as follows:

$$(pR - m_{\sigma} - L) - (\rho_1(\sigma) - w_{\sigma}^N) \ge 0.$$
 (57)

The left hand side of Condition (57) is equal to w_{σ}^{N} plus the left-hand-side of the baseline time consistency constraint Equation (19). The additional payment w_{σ}^{N} thus renders investment more attractive to the firm at time 1, because the additional payment reduces the *incremental* agency rent that the firm pays out when it invests. Equations (56) and (57) together imply a minimum time 0 rent to the manager:

$$\tilde{\rho}_0 \ge w_h^N \ge \frac{p}{\delta}M - (pR - L).$$
(58)

It follows that the firm earns the following expected payoff if it follows a time-consistent unselective branch strategy:

$$\tilde{\Pi}_{\text{Un}}^{B} = V^{FB} - \tilde{\rho}_{0}$$

$$= V^{FB} - \max\left(\frac{p}{\delta}M - (pR - L), \lambda \frac{p - \delta}{\delta}M - \zeta, 0\right).$$
(59)

It follows from inspection of Equation (13) that, if the firm adopts a time-consistent unselective strateg with a branch structure, its expected payoff is no larger than it would be were it able to commit to an unselective strategy:

$$\tilde{\Pi}^B_{\mathrm{Un}} \leq \Pi_{\mathrm{Un}}$$

Recall that, in the baseline model, the firm would prefer to expand with an unselective strategy for every effort cost $M \geq \underline{M}^B$ but that it cannot do so for $M \in (\underline{M}^B, M^*(\zeta)]$ because it faces a time-inconsistency problem. That problem does not arise in this extension, because the firm can commit to a payment w_h^N in case of non-investment in hard projects and so reduce the marginal rent costs of investment. Resolving the time commitment problem in this way increases the firm's agency rent costs and so reduces the value of a selective investment strategy. For every effort cost M, the firm therefore faces a choice between an unselective strategy with potentially high agency costs, and a selective investment strategy. The unselective strategy is preferred if and only if $\tilde{\Pi}_{Un}^B \geq \Pi_{Sel}$. This requirement is met for every M that satisfies Condition (60):

$$M \leq \underline{\tilde{M}}^B \triangleq \min\left(M^*(\zeta), \frac{\lambda+1}{\lambda\delta/p+1}\underline{M}^B\right).$$
(60)

Note that, because $\delta < p$ and $M^*(\zeta) \geq \underline{M}^B$,

$$\underline{\tilde{M}}^B \ge \underline{M}^B,$$

with strict inequality whenever $\underline{\tilde{M}}^B < M^*(\zeta)$.

We summarise this discussion in Lemma 8:

Lemma 8. Suppose that the firm expands using a branch structure and that it cannot commit at time 0 to its time 1 investment strategy.

1. If $M \leq \underline{\tilde{M}}^B$, then the firm expands using an unselective investment strategy.

2. If $M > \underline{\tilde{M}}^B$, then the firm expands using a selective strategy if $\zeta \leq \zeta_S$. Otherwise, the firm does not expand.

Lemma 8 is analogous to Lemma 1. The key difference between the two is that, because $\underline{\tilde{M}}^B \geq \underline{M}^B$, the region in which a branch firm opts for an unselective investment strategy is larger when the firm is able to pay the manager a positive wage in case the firm does not invest.

The agency cost that the firm incurs exceeds the agency cost under commitment precisely when $\tilde{\rho}_0 > \rho_0$: that is, precisely when $M > \underline{M}^B$, where

$$\underline{M}^{B} \triangleq \frac{(pR - L) - \zeta}{\lambda + (1 - \lambda)p/\delta}.$$
(61)

Then the firm employs an unselective strategy for all $M \in (\underline{M}^B, \underline{\tilde{M}}^B]$, despite incurring higher agency cost than it would under commitment.

Now suppose that the firm expands via a subsidiary. Investment is incentive compatible at time 1 in state σ if and only if Condition (62) is satisfied:

$$(pR - m_{\sigma} - L) - (\rho_1(\sigma) - w_{\sigma}^N) + (\min(D_S, L) - pD_S) \ge 0.$$
(62)

Expression (63) for the minimum time 0 managerial rent follows immediately:

$$\tilde{\rho}_0 \ge w_h^N \ge \frac{p}{\delta}M - (pR - L) - (\min(D_S, L) - pD_S).$$
(63)

The firm minimizes $\tilde{\rho}_0$ by setting $D_S = L$. The firm's expected payoff under a time-consistent unselective subsidiary strategy is therefore given by $\tilde{\Pi}_{\text{Un}}^S$, where

$$\tilde{\Pi}_{\text{Un}}^{S} = V^{FB} - \tilde{\rho}_{0} = V^{FB} - \max\{\frac{p}{\delta}M - (pR - L) - (1 - p)L, \lambda \frac{p - \delta}{\delta}M - \zeta, 0\}.$$
(64)

Subsidiary expansion therefore alleviates the firm's time inconsistency problem by enabling it to implement an unselective investment strategy at a lower cost $\tilde{\Pi}_{\text{Un}}^S \in [\tilde{\Pi}_{\text{Un}}^B, \Pi_{\text{Un}}]$. The firm prefers an unselective investment strategy to a selective investment strategy if and only if $\tilde{\Pi}_{\text{Un}}^S \geq \Pi_{\text{Sel}}$; this is the case precisely when

$$M \leq \underline{\tilde{M}}^{S} \triangleq \min\{M^{*}(\zeta), \frac{\lambda+1}{\lambda+p/\delta}(pR-L) + \frac{1-p}{\lambda+p/\delta}L\}.$$
(65)

Lemma 9. Suppose that the firm expands using a subsidiary structure and that it cannot commit at time 0 to its time 1 investment strategy.

If M ≤ <u>M</u>^S, then the firm expands using an unselective investment strategy.
 If M > <u>M</u>^S, then the firm expands using a selective strategy if ζ ≤ ζ_S.
 Otherwise, the firm does not expand.

By analogy with \underline{M}^B , the firm incurs a higher agency cost from an unselective investment strategy than it does under commitment and nevertheless adopts an unselective investment strategy $M \in (\tilde{M}^S, \underline{\tilde{M}}^S]$, where

$$\tilde{M}^S = \frac{(pR - L) + (1 - p)L - \zeta}{\lambda + (1 - \lambda)p/\delta}.$$
(66)

In short, as in the baseline model, a firm can alleviate the time consistency problem that it would experience if it invested via a branch by setting up a subsidiary firm and taking advantage of the associated limited liability option. It is optimal for the firm to spend C to establish a subsidiary if, first, unselective investment is more profitable with a subsidiary than a branch: $\tilde{\Pi}_{\text{Un}}^S - C \ge \tilde{\Pi}_{\text{Un}}^B$; second, unselective subsidiary investment is more profitable than selective branch investment: $\tilde{\Pi}_{\text{Un}}^S - C \ge \Pi_{\text{Sel}}$; and, third, unselective subsidiary investment generates a positive profit: $\tilde{\Pi}_{\text{Un}}^S - C \ge Y$.