Bank Bailout Menus[1]

Sudipto Bhattacharya

London School of Economics
and CEPR

Kjell G. Nyborg

University of Zürich,
Swiss Finance Institute,
NHH and CEPR

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Bhattacharya: London School of Economics, London, UK; CEPR email: s.bhatt@lse.ac.uk. Nyborg: Swiss Banking Institute, University of Zürich, Plattenstrasse 14, 8032 Zürich, Switzerland; Norwegian School of Economics and Business Administration; CEPR. Email: nyborg@isb.uzh.ch.

Abstract

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Bailing out banks requires overcoming debt overhang as well as dealing with adverse selection with respect to the quality of banks' balance sheets, in terms of heterogeneity in both the likelihood and extent of their potential shortfalls, of future asset values vis-a-vis contractual debt obligations. We examine bailouts that eliminate debt overhang, while attempting to minimize subsidies to banks' equityholders. When banks do not differ with respect to the extent of debt overhang, it can be fully overcome with the minimal amount of subsidies, providing each bank's equity holders no more than their pre-bailout values, with a partial new equity injection, or an asset buyout. When levels of debt overhang co-vary with underlying probabilities of default, we characterize the conditions for attaining a similar minimal subsidy outcome, with a Menu of either equity injection or asset buyout plans, satisfying suitable self-selection constraints among bank types. These involve global rather than local conditions, with multiple intersections of indifference curves among types, and imply strictly greater funds injections than those needed to make existing debt default-free. We also explore the role of coupling asset purchases with providing the bailout agency Options to buy bank equity, to enhance its capture of rents arising from new investments by banks, and compare its performance with equity injections on this dimension, as well as others such as post-bailout stakes held by prior inside equity holders, and the resulting incentives for efficient performance.

1 Introduction

The recent spate of banking crises, arising in part from "unanticipated" declines in the quality of mortgage-backed loans and securities in US markets in particular, has brought to the fore issues of optimal mechanisms for restoring banks to well-functioning entities capable of further lending to the real sector of an economy. Banks are characterized not only by their specialized role as relationship-based lenders with hard-to-replace informational advantages vis-a-vis small and medium sized firms, they are also often funded with high leverage from dispersed creditors, be they depositors with demandable claims or (increasingly) wholesale market lenders with short-term claims which may not be refinanced. These features of bank liabilities make it extremely difficult to renegotiate their debt – to a combination of debt reduction and equity-like claims for example – directly with their creditors, and hence governments often play a direct role in ameliorating the problems arising from banks having a large degree of "debt overhang" – a surfeit of future fixed claims relative to potential future values of their extant assets in adverse scenarios.

As Myers (1977) recognized in a path-breaking paper, this gives rise to underinvestment by equity holders, or management acting in their interest, since much of any future cash flows generated by such investments could accrue to extant creditors, while the costs of investments would be borne by existing equity holders; either directly, or via a reduction in their future payoffs arising from additional promised repayments to new financiers whose claims would typically be junior to those of existing creditors. At the same time, a policy of injecting funds into such banks has to take into account the fact that their existing equity holders have an option value that arises from the expectation of profits after repaying their creditors in the upper tail of the distributions of their future asset values. Thus, any payoffs demanded as a quid pro quo for fund injections must leave existing equity holders with an expected payoff that is no lower than what they would have obtained in the current status quo.

In this paper, we investigate the properties of two often-used mechanisms for providing such cash injections to troubled banks, namely Equity Injections in return for the government acquiring a partial share in a bank's equity stake, and Asset Buyouts whereby the government injects cash into a bank in return for acquiring ownership of a subset, or fraction, of its (troubled) assets. The latter may also be combined with an Option or Warrant to buy equity shares at a price higher than their current values. We do so in a setting in which bank managements, who we assume to be acting in the interests of their shareholders, are privately informed about the qualities of their illiquid and risky troubled assets. As a clear consequence of such asymmetric information, fund injection mechanisms to alleviate debt overhang must be carefully designed, so as not to leave too much surplus for banks' current equity holders, relative to the status quo values of their claims in the absence of alleviating debt overhang problems.

Not surprisingly, given the enormity of the scales of recent fund injections, or Bailouts, of especially larger banks with significant holdings of "toxic" assets, these issues have received much attention in policy-related debates as well as in the emerging academic literature. In the former sphere, policy makers have vacillated, regarding massive asset buyout programs such as TARP (Troubled Assets Relief Program) in the US, much of the budgeted resources for which were later switched to equity injections as with Citibank, for example. Bebchuk (2008) provides a summary of considerations involved in evaluating asset qualities and their likely equilibrium valuations for such buyout programs; Schaefer and Zimmermann (2009) consider a related set of issues for bank recapitalization, coupled with creation of "bad banks" to manage assets acquired by the bailout agency.

In a more detailed analytical vein, Philippon and Schnabl (2009) as well as Landier and Ueda (2008) have discussed the relative merits of some alternative bailout mechanisms that have been considered. The former set of authors, in particular, conclude that equity injections would strictly dominate asset buyout programs vis-a-vis their expected subsidies– the surfeit of cash injected relative to the overall expected values of future payoffs accruing to the government – at any level of alleviation of the investment disincentive problems arising from the debt overhang faced by heterogeneous banks, with private information about their asset qualities. As they note, this dominance arises essentially due to the participation of government equity stakes in the net present values of future investments that would be made by banks, following upon the alleviation of their debt overhang, as well as non-participation by some banks with very profitable projects in such plans, as opposed to (subsidized) asset buyout programs.

In their model banks differ, in a privately known fashion, in the probabilities with which their risky assets would default fully, so that their minimum future values would correspond to a common (scale adjusted) value of non-risky assets in their portfolios. In other words, the Support of the future distributions of banks' asset values, adjusted for the scale of their liabilities, is the same across banks. They also restrict attention to equity injections and asset buyout plans with a linear pricing structure, subject to upper bounds on the amounts of assets, or fractions of equity stakes, to be acquired. As we shall see, neither of their restrictions are without loss of generality as far as plausible environments are concerned, and a richer adverse selection setting with non-linear prices for different quantities must then be considered.

In our model, we allow for the minimum future value of a bank's assets to vary with its "type"; across types both the probability of reaching these minimum values– which are strictly lower than banks' common (scale-adjusted) debt repayment obligations – and the extent of "loss given default" may differ, potentially in a correlated manner. Indeed, it is entirely plausible that banks having lower quality of debt-related assets, with higher likelihood of not repaying their par values, would also suffer higher proportionate losses on these in adverse economic states; for example, on a less senior tranch of a set of mortgage-backed securities on the same mortgage pool. Nevertheless, we are able to show that, under plausible conditions, it is feasible to design a Menu of equity injection tuples – each specifying a fractional stake to be awarded to the government in return for a given level of cash injection to the bank choosing it – such that each bank is fully relieved of the extent of its debt overhang, while its current equity holders obtain an expected payoff equal to that in the status quo ex ante. We also show that, whenever such a "first-best" menu of equity injections exists, there exists an equivalent menu of asset buyout proposals achieving at least full debt overhang relief, and providing the same future payoffs to each bank's prior equity holders in both future states.

The equivalence result above does not take into account differential payoffs arising to the government from any future bank investments with strictly positive NPV, under alternative bailout plans. However, once a suitably designed Menu of Options are attached to each component of the asset buyout mechanism, it can perform at least as well in capturing a share of these additional rents, as compared to equity injections. Hence, when taking future investment opportunities into account, a comparison between these two types of bailouts requires augmenting asset buyout programs with options.

Our paper is organized as follows. In Section 2 we set out the notation for our space of bank types, their return distributions, and two main bailout programs. In section 3, we first derive simple results characterizing the necessary cum sufficient conditions for achieving (at least) full relief of debt overhang problems at each type of bank with equity injections, without providing its equity holders with any surplus beyond their status quo expected payoff, plus their share of the NPVs from any future investments. Our solution to this problem exploits an interesting analytical feature of the underlying self-selection problem across different bank types, within a specified menu of contracts. Namely, the indifference curves for different types, in the space of share of equity surrendered in return for a given amount of cash injected, do not satisfy the commonly assumed "single-crossing" condition, of the slopes of these curves being uniformly ordered the same way across types, as a function of their probabilities of financial distress for example. Instead, under plausible conditions, the indifference curves for two different types of banks, arising from their (differing) status quo equity payoffs, intersect at multiple points, which makes our solution feasible.

Following these steps, in Section 4 we consider issues relating to net payoffs arising from post-bailout investments by banks, and the partial capture of these by the bailout agency using equity injections, or asset buyouts, potentially coupled with receiving Options to acquire bank equity in good future states. In the concluding Section 5, we elaborate on the implications of our results for Policy choices over different types of bailout mechanisms. In doing so, we allow for consideration of the implications of these for the post-bailout incentives of banks' insiders cum managers, vis-a-vis monitoring incentives for their future lending activities for example. An appendix contains proofs that are not supplied in the text.

2 The Model

There are two dates, $t = 0, 1$. A central agency (government with powers of taxation) is considering bailing out banks at date 0. At this point, the asset values of banks are uncertain. At date 1, uncertainty about the assets in place are realized according to a binomial distribution that depends on a bank's type. Only a bank's management knows its type and management's objective is to maximizes the value of existing shares.

Banks are indexed by $s \in \boldsymbol{S}$. The assets in place of a bank of type s can be described by a three-tuple, (H_s, L_s, p_s), where $H_s > L_s$ and these represent the values of the assets in place in the up and down states, respectively, and p_s is the probability of the up state.[1] There is universal risk neutrality and the riskfree rate is 0, implying that the full information date 0 value of the assets in place of a bank of type s is

$$A_s = p_s H_s + (1 - p_s) L_s \tag{1}$$

H_s and L_s are (weakly) increasing and decreasing, respectively, in s, and p_s is strictly increasing in s. Thus, the quality of a bank is increasing in s, in the sense that the distribution of asset values for a bank of type s first order stochastically dominates that of a bank of type $s' < s$. Moreover, A_s is increasing in s.

Banks have only two types of claims outstanding, senior debt (including demand deposits) and common stock. All debt matures at date 1. The promised payment on debt

[1] Note that we are not making any assumption about the correlations between the asset value distributions of banks. Each bank can be viewed as having its own solvency and default states.

is independent of bank type and satisfies $F \in (L_s, H_s)$ for all s; this is meant as a scale normalization. Thus, without a bailout, each bank would default in its down state, but not in the up state. Furthermore, in the case of default, writedowns, $F - L_s$, are (weakly) larger for lower quality banks. Another way to put this is that debt overhang is more severe for lower quality banks.

While debt overhang is central to our paper, we do not explicitly model new investments in our baseline model. So much, though not all, of our analysis focuses on the interim problem as to how to restructure a "weak" bank's balance sheet, in preparation for it to return to normal banking activities, including raising non-bailout capital. We solve for the bailout plans that fully remove debt overhang at each type of bank. These are plans that make their debt free of default risk. However, our notation and analysis below recognizes that some bailout plans may leave bank debt risky.

A bailout plan is a pair of two-tuples

$$B = \{(\phi, C), (\lambda, E)\} \in ([0,1] \times [0,\infty)) \times ([0,1] \times [0,\infty))$$

that describe an asset buyout and an equity injection, respectively. $\phi \in [0,1]$ is the fraction of assets that are bought and $C \geq 0$ is the price that is paid for these assets. $\lambda \in [0,1]$ is the fraction of all shares the bailout agency obtains if it injects $E \geq 0$ in fresh equity capital. After a bailout, B, asset values in the up and down states for a bank of type s are, respectively

$$H_s^B = (1-\phi)H_s + C + E \quad \textit{and} \quad L_s^B = (1-\phi)L_s + C + E. \tag{2}$$

This assumes that the stochastic returns on a banks' remaining assets after a partial sale remain the same as that on its assets as a whole. It also assumes that all cash injections are (temporarily) placed in the riskfree asset.

The payoff to existing equityholders in the up and down states are, respectively,

$$Z_s^{B,H} = (1-\lambda)\max\left\{H_s^B - F, 0\right\} \quad \textit{and} \quad Z_s^{B,L} = (1-\lambda)\max\left\{L_s^B - F, 0\right\}. \tag{3}$$

The payoffs to creditors in the up and down states are, respectively:

$$D_s^{B,H} = \min\left\{H_s^B, F\right\} \quad and \quad D_s^{B,L} = \min\left\{L_s^B, F\right\}. \tag{4}$$

If there is no bailout, payoffs to equity and creditors are equivalent to those under the "null plan", $B_0 \equiv \{(\phi = 0, C = 0), (\lambda = 0, E = 0)\}$. All these payoffs are standard; they recognize that equity is a call option on the bank's assets with a strike price equal to the promised payment to debt.

Let

$$E[Z_s^B] = p_s Z_s^{B,H} + (1 - p_s) Z_s^{B,L} \tag{5}$$

denote the conditional expected payoff to existing shareholders for a bank of type s given bailout plan B. A necessary condition for the bank to accept the plan is that existing shareholders' wealth (weakly) improves, that is,

$$E[Z_s^B] \geq E[Z_s^{B_0}]. \tag{6}$$

Let $\boldsymbol{B}$ be a collection, or menu, of bailout plans and let $B_s \in \boldsymbol{B}$ satisfy[2]

$$B_s = \arg\max{}_{B \in \boldsymbol{B}} E[Z_s^B] \tag{7}$$

In other words, for any menu of bailout plans, the best plan for the existing shareholders of a bank of type s is denoted B_s. The dependence of B_s on a given menu, $\boldsymbol{B}$, is suppressed in the notation, for ease of reading. We write

$$B_s = \{(\phi_s, C_s), (\lambda_s, E_s)\}. \tag{8}$$

Let q_s be the prior (unconditional) probability that a bank is of type s. The problem faced by the bailout agency is to choose a menu that induces banks to pick bailout plans so that the total subsidy to existing claimholders is minimized, subject to the constraint that debt overhang is to be eliminated. Since the aggregate subsidy to existing claimholders

[2]We require that $\boldsymbol{B}$ be compact in the space of menus of bailout plans. If for a given s there are multiple plans that satisfy (7), B_s can be picked arbitrarily among them.

equals the cash injected by the plan less the expected value of the bailout agency's holdings under the plan, this amounts to choosing a menu $\boldsymbol{B}$ to minimize

$$\int_{\boldsymbol{S}} q_s \left\{ \underbrace{C_s - \phi_s(p_s H_s + (1-p_s)L_s)}_{\text{subsidy from asset sale}} \quad + \quad \underbrace{E_s - \frac{\lambda}{1-\lambda} E[Z_s^{B_s}]}_{\text{subsidy from equity injection}} \right\} ds \tag{9}$$

subject to (7), (6), and

$$D^{B_s,H} = D^{B_s,L} = F \quad \text{for all } s. \tag{10}$$

Constraint (7) means that the bank chooses the plan in the menu that maximizes the wealth of existing shareholders, and constraint (6) means that this leaves all the existing shareholders as least as well off as without a bailout. Constraint (10) says that under the best plan, B_s, for a bank of type s, its debt is riskfree, or equivalently, debt overhang is eliminated. Thus, we can also write the minimization problem as

$$\min_{\boldsymbol{B}} \int_{\boldsymbol{S}} q_s E[Z_s^{B_s}] ds \tag{11}$$

subject to the same constraints, $(6), (7), (10)$.

3 Analysis

We provide conditions under which it is possible to design a menu of bailout plans so that first best can be achieved, that is, for every s debt overhang is eliminated and expected subsidies equal their minimal level, $(1-p_s)(F-L_s)$, or equivalently, existing equityholders receive their status quo payoff, $E[Z_s^{B_0}]$. Our analysis also shows how to construct such optimal menus and provides the explicit form they take. In this section, we do not explicitly take new investments by banks into account. This is studied in Section 4. We start with a simple observation, from which we can deduce an irrelevance result with respect to equity injections versus asset buyouts.

Lemma 1 *There is a one-to-one mapping with respect to payoffs to (old) equityholders between equity injection and asset buyout bailout plans. In particular, the pure equity injection plan* $\{\lambda, E\}$ *and the pure asset buyout plan* $\{\phi, C\}$ *give the same state-by-state payoff to (old) equityholders provided that* $\phi = \lambda$ *and* $C = (1 - \lambda)E + \lambda F$.

This has two immediate and important implications. First, the set of payoffs to old equityholders induced by a menu of equity injection plans can be replicated with a menu of asset buyout plans (and vice versa). Second, for an equity injection plan that solves the debt overhang problem for a bank of type s, there is a corresponding asset buyout plan that gives the same state-by-state payoffs to all claimholders and the bailout agency (and vice versa). This follows immediately from the lemma by observing that if the debt overhang problem is solved, then creditors get F regardless of the state, w. As a corollary of these two observations, we have

Proposition 1 *If there is a menu of pure equity injection plans that yields first best, then there is also a menu of pure asset buyout plans that does so (and vice versa). One can go from one to the other by applying the mapping in Lemma 1.*

The one-to-one mapping in Lemma 1 (in terms of payoffs to old equityholders) holds regardless of the distribution of the returns. It follows that the equivalence between equity injections and asset buyouts described by Proposition 1 also does so.

If first best can be achieved by a menu of pure equity injection or asset buyout plans, then it can also be achieved by a menu of mixed plans (being convex combinations of the pure equity and asset buyout plans that achieve first best).

Furthermore and importantly, Lemma 1 also implies that for any mixed plan there is an equity injection plan that gives the same payoff to old equityholders. Thus, if there is a menu of mixed plans that achieve first best, there is also a set of pure equity plans that does so. In short, in terms of findings conditions under which first best can be achieved, it suffices to study pure equity injection plans. We have therefore chosen to cast our analysis in terms of these. However, we also compare the characteristics of optimal equity injection plans to their corresponding optimal asset buyout plans.

3.1 Homogeneous Debt Overhang

We begin by studying the simple setting of *homogeneous debt overhang*, that is, $L_s = L$ for all s, which is also studied by Philippon and Schnable (2009). When banks differ only in their probabilities of default, but have the same loss given default relative to their debt obligations, there is an equity injection plan, as well as an asset buyout one, that achieves first best.

Proposition 2 (Homogeneous debt overhang) *Suppose banks differ only in p_s, that is, for all s we have $L_s = L$ and $H_s = H$. Then for each type, s, full relief of debt overhang with a minimal subsidy to existing claimholders is achieved by either of the following two plans, of equity injection or asset buyout:*

$$B_E : \lambda = \frac{F - L}{H - L} \;\; \textit{and} \;\; E = F - L$$

or

$$B_A : \phi = \frac{F - L}{H - L} \;\; \textit{and} \;\; C = \phi H = F - L(1 - \phi)$$

These two plans are unique (in terms of achieving first best) in the class of pure equity injection or pure asset buyout plans, respectively.

Proposition 2 establishes that if banks differ only in the probability of default, one plan works for all banks. The choice between equity injection or asset buyout is immaterial insofar as payoffs to claimholder is concerned. But as the expressions for the optimal E and C reveals, asset buyouts involve larger cash contributions by the bailout agency than equity injections.

That the same equity injection plan works for all banks may seem to be inconsistent with the idea expressed by Myers and Majluf (1984) that overvalued firms that wish to raise equity financing will necessarily have to dilute their equity excessively. However, what is happening in Proposition 2 is that the while the bailout plan leaves old equityholders equally well off as under the status quo, regardless of the type of their bank, the expected subsidy is decreasing in the type, s. Thus, the values of the new shares are not equal to the equity injection; they are less by an amount equal to these subsidies.

Proposition 2 also asserts that the optimal pure equity injection or asset sale bailout plans involve the minimal injection of capital that resolves the debt overhang problem, since $E = F - L$ and $C = F - L(1-\phi)$. Focusing without loss of generality on equity injections, one way to understand the logic behind this result is in terms of share-cash indifference curves that leave equityholders indifferent between a bailout and the status quo. Specifically, this indifference curve is given by the locus of λ's and E's that satisfy

$$p_s(H_s - F) = \begin{cases} (1-\lambda)p_s(H_s + E - F) & \text{if } E \leq F - L_s \\ (1-\lambda)\left\{p_s(H_s + E - F) + (1-p_s)(L_s + E - F)\right\} & \text{if } E > F - L_s. \end{cases} \tag{12}$$

Under the assumptions of Proposition 2, $H_s = H$ and $L_s = L$. Thus, it is straightforward that for all $E \leq F - L$, the indifference curves of different bank types coincide. However, if the bailout overcapitalizes the bank, in the sense that $E > F - L$, λ is decreasing in p_s. Intuitively, as the probability of the up state increases, the relative contribution of the up state to overall payoff must increase, which can only be accomplished by decreasing λ. This can be seen by rewriting the bottom part (the case that $E > F - L_s$) of equation (12) as

$$(1-\lambda)\left\{\frac{H + E - F}{H - F} + \frac{1-p_s}{p_s}\,\frac{L_s + E - F}{H - F}\right\} = 1 \tag{13}$$

and keeping in mind that, for now, $L_s = L$ for all s.

Thus, under homogeneous debt overhang, the indifference curves $\lambda_s(E)$ coincide for $E \leq F - L$. But for $E > F - L$, the indifference curve of higher bank types lie below those of lower bank types. This is illustrated in Figure 1. Put differently, the indifference curves have a single point in common where debt overhang is solved, namely the (λ, E)

combination referred to as plan B_E in Proposition 2, and this plan also provides the minimal level of equity injection that resolves debt overhang.

3.1.1 Upside Heterogeneity

If H_s varies across types, the above argument breaks down because the indifference curves of different types never intersect. The following result holds in the general case where we consider equity injections, asset buyouts, and combinations thereof.

Proposition 3 *Suppose banks have the same debt overhang but have different asset values in the up state, that is, for all s, $L_s = L$, while H_s is strictly increasing in s. Then a first best bailout outcome cannot be achieved.*

If two bank types are equivalent in all respects expect that one has a higher asset value in the up state, the bailout agency needs to hold a larger fraction of the shares for the lower quality bank, ceteris paribus, in order to achieve first best. Thus, any plan that leaves the higher quality type with its status quo payoff will give an excess subsidy to the lower quality type. Since there is nothing to stop the lower quality type from choosing the plan tailored for the higher quality type, a first best outcome cannot be achieved. This is essentially a standard Akerlof (or Myers and Majluf) type result. The contrast to Proposition 2 arises because it is now not possible to give the same payoff in the up and down states to different quality banks.

3.2 Heterogeneous Debt Overhang

In the *heterogeneous debt overhang* setting, L_s varies across types. This is most relevant to settings in which assets held by banks are of the debt(-based security) type, with contractual upper bounds (Par values) on their payoffs. However, lower quality assets may suffer from greater loss given default. In contrast to the homogeneous debt overhang setting, it is therefore not possible to find a single bailout plan that for all bank types injects the minimal amount of capital necessary to eliminate debt overhang. As we shall show, overcapitalization of lower-quality bank types will now have to be part of any optimal bailout menu which minimizes the subsidies to bank equity holders. Note that, in what follows, the homogeneity of banks in their (scale-adjusted) high state net payoffs, $(H-F)$, may be thought of as arising from similar ex ante capital requirements.

Proposition 4 *Suppose a bank is one of two types, $s = 1, 2$, where $L_1 < L_2$, $H_s = H$, $p_1 < p_2$, so that the future asset values of the two banks can be strictly ordered by first order stochastic dominance (FOSD). There is a unique pure equity injection bailout plan, $B_E(1,2) = \{\lambda^{(1,2)}, E^{(1,2)}\}$, that for both banks resolves debt overhang and achieves the minimal subsidy. Under this plan, both banks obtain an equity injection which is strictly greater than the minimum needed to overcome debt overhang fully, that is $E > F - L_1$. The bailout plan is given by:*

$$\lambda^{(1,2)} = \frac{(H-F)(p_2-p_1)}{p_2 H + (1-p_2)L_2 - (p_1 H + (1-p_1)L_1)} = 1 - \frac{(H-F)(p_2-p_1)}{A_2 - A_1} \tag{14}$$

and

$$E^{(1,2)} = F - \frac{p_2\left(Hp_1 + L_1\left(1-p_1\right)\right) - p_1\left(Hp_2 + L_2\left(1-p_2\right)\right)}{p_2-p_1} = F - \frac{p_2 A_1 - p_1 A_2}{p_2 - p_1}. \tag{15}$$

Proof: Equation (12) implies that for any type s, its share-cash indifference curve vis-a-vis its status quo expected equity payoff is given by

$$\lambda_s(E) = \begin{cases} 1 - \frac{H-F}{H+E-F} & \text{if } E \leq F - L_s \\ 1 - \frac{p_s(H-F)}{p_s(H+E-F)+(1-p_s)(L_s+E-F)} & \text{if } E > F - L_s. \end{cases} \tag{16}$$

We see that the indifference curve changes functional form and shape as E goes from representing undercapitalization, $E < F - L_s$, to overcapitalization, $E > F - L_s$. It is straightforward to verify that (a) $\lambda_s(E)$ is strictly increasing and, on each region, strictly concave;[3] (b) $\lambda_s'(E)$ rises strictly as E enters the overcapitalization region; and (c) $\lambda_s'(E) > \frac{d}{dE}(H-F)/(H+E-F)$ for $E > F - L_s$.

[3] That is, $\lambda_s(E)$ is piecewise strictly concave; it is strictly concave on $[0, F-L_s]$ and on $[F-L_s, \infty)$.

We also see that $\lambda_1(E)$ and $\lambda_2(E)$ coincide for $E \leq F - L_2$. Thus, $\lambda_2(E) > \lambda_1(E)$ immediately to the right of $E = F - L_2$, by observation (b) above. Furthermore, by observation (c), $\lambda_2(E) > \lambda_1(E)$ for all $E \in (F - L_2, F - L_1]$. To establish the proposition, we therefore need to show that $\lambda_2(E)$ and $\lambda_1(E)$ intersect once and only once for $E > F - L_1$.

Observe now that for $E > F - L_s$, $\lambda'_s(E)$ is strictly decreasing in p_s. Therefore, $\lambda_2(E)$ and $\lambda_1(E)$ can at most intersect once. Now, inspection of (16) shows that for sufficiently large E, we have $\lambda_2(E) < \lambda_1(E)$, since p_2 is strictly larger than p_1. Thus, since the two curves are continuous, there is a (unique) point of intersection. Indeed, straightforward calculations show that the two curves intersect at E as given by (14), and at this point, λ_1 and λ_2 are as given by (15).[4] □

Remarks: The subsidy minimizing debt relief program can also be implemented with a menu of two bailout plans. Any menu $\{(\lambda_1, E_1), (\lambda_2, E_2)\}$ satisfying the following conditions will do: (i) (λ_i, E_i) is on the indifference curve of type i; (ii) $E_2 \in [F - L_2, E^{(1,2)}]$; and (iii) $E_1 \geq E^{(1,2)}$, where $E^{(1,2)}$ is given by (14). The common feature of all optimal bailout menus is that the low type must be overcapitalized so as to ensure that the high type is not oversubsidized. If, other things equal, the higher type prefers to maximize its equity "cushion" in the bad state, then it would choose the single contract identified in the Proposition above, if it were available. On the other hand, the high type may choose a separating contract with a lower level of equity injection, as that would leave its current (inside) equity holders with a larger proportional stake.

The proposition is illustrated in Figure 2. The two indifference curves coincide as long as E is so small that debt overhang is not resolved for either type. However, as E moves into the region where debt overhang is solved for the higher-quality type, the curve for

[4]The proposition (and the proof) assumes that $H_2 = H_2 = H$. If $H_2 - H_1$ is sufficiently small, the share-cash indifference curves will still cross and first best will be possible. However, if $H_2 - H_1$ is sufficiently large (and this will depend on the difference in debt overhang between the two types), first best is not possible (along the lines of Proposition 3).

the higher type moves above that for the lower type. The slope for the lower type starts to increase as E increases beyond the minimal amount that solves debt overhang for the lower type. As illustrated, at $E = F - L_1$, the higher type's indifference curve is above that of the lower type. However, the greater slope of the indifference curve of the lower curve eventually allows it to cross the curve of the higher type at the point described in the proposition. With a bailout plan defined by this point, debt overhang is solved, and by construction of the indifference curves, the minimal level of subsidy to existing claimholders is obtained.

INSERT FIGURES 2 and 3 HERE

If the FOSD condition of Proposition 4 is NOT satisfied, for example if $p_2 > p_1$ but $L_2 < L_1$, then there is no equity injection menu which fully relieves debt overhang for both bank types and also achieves the minimal level of subsidy. This is illustrated in Figure 3, where the indifference curve of type 1 is above that of type 2 at any level of equity injection that solves the debt overhang problem for type 2. Thus, any plan that works for type 2 will oversubsidize the equity of type 1. The assumption that the future asset values of different types is ordered by strict FOSD, i.e., p_2 is strictly larger than p_1, is explicitly used in the proof of Proposition 4. Without *strict* FOSD, the result would not hold.

Proposition 5 *Suppose L_s is strictly increasing in s. Suppose also that $H_s = H$ and $p_s = p$ for all s. First best cannot be achieved.*

In this case, FOSD holds, but only weakly. First best is now not possible because higher quality types could improve their payoffs relative to the status quo by adopting plans tailored to lower quality types. The reason is that high types have the same payoffs as low types in the up state, but strictly higher payoffs in the down states, and the probabilities of these states do not vary across types under the conditions of the proposition.

When there are more than two types of banks differing in their p_s as well as L_s values, then their Binomial future value distributions being ordered by strict first order stochastic dominance is no longer sufficient for the existence of a first best bailout menu. We also need the intersection points of the share-cash indifference curves (in $E \times \lambda$ space) to be monotonic in the following sense:

Definition 1 (Monotonicity of indifference curve intersection points) *Suppose there are $S \geq 3$ bank types and for each type $s > 1$ let $\{\lambda^{(s-1,s)}, E^{(s-1,s)}\}$ be the intersection point of the share-cash indifference curves of bank types $s-1$ and s, as defined by (14) and (15).*[5] *These $S-1$ intersection points are monotonic if $E^{(s-1,s)} \leq E^{(t-1,t)}$ whenever $s > t$.*

The intersection points referred to here represent the unique first best common equity injection bailout plans for two adjacent bank types, as given in Proposition 4. Intersection point monotonicity is necessary to ensure that no type is oversubsidized. A graphical illustration of intersection point monotonicity and its importance for first best is provided in Figure 4 below.

INSERT FIGURE 4 HERE

Figure 4 depicts the indifference curves of three types, $s = 1, 2, 3$, with $p_1 < p_2 < p_3$, and $L_1 < L_2 < L_3$. The red, blue, and green lines are the curves for types $1, 2$ and 3, respectively. Focusing on the segments of these curves after they no longer coincide, we see that the green (type 3) and blue (type 2) curves intersect at a smaller E than does the blue and red (type 1) curves. This is intersection point monotonicity.

As a result of intersection point monotonicity, there is also indifference curve monotonicity with respect to which type has the maximum λ for a given equity injection. Specifically, we see in Figure 4 that for low E, type 3 is willing to give away a larger share of the equity than types 2 and 1. For intermediate E's, type 2 is willing to give away a larger λ than the other two types. And for high E's, type 1 is willing to give away a larger share. Thus, it is possible to design a menu of three bailout plans that achieves first best

[5] Substitute s for 2 and $s-1$ for 1 in these equations.

by offering: (i) a plan with a low E and a λ that falls on the indifference curve of type 3; (ii) a plan with an intermediate E and a λ that falls on the indifference curve of type 2; (iii) a plan with a high E and a λ that falls on the indifference curve of type 1.[6]

If intersection point monotonicity does not hold, this construction is not possible. Indeed, obtaining a first best outcome is not feasible. This can be seen most easily by focusing on Figure 5. Here, types are ordered by FOSD, yet the indifference curves of types 1 and 2 intersect before those of types 2 and 3. Hence, the indifference curve of type 2 (blue) is never above the indifference curves of type 1 (red) and type 3 (green) simultaneously. As a result, any plan that seeks to separate out the intermediate type will leave either type 3 or type 1 better off than under the status quo. In other words, first best cannot be achieved.

INSERT FIGURE 5 HERE

Theorem 1 (Heterogeneous debt overhang) *Suppose L_s is strictly increasing in s, and $H_s = H$ for all s. A menu of equity injection plans which (a) achieves full relief of debt overhang for each bank type, $s = 1, \ldots, S$ and (b) provides the equity holders of each type with an expected payoff equal to the pre-bailout level, exists if and only if: (i) the Binomial future value distributions are ordered by strict FOSD; and (ii) the indifference curve intersection points are monotonic, which holds if and only if*

$$\frac{p_{s+1}(1-p_s)L_s - p_s(1-p_{s+1})L_{s+1}}{p_s(1-p_{s-1})L_{s-1} - p_{s-1}(1-p_s)L_s} > \frac{p_{s+1}-p_s}{p_s-p_{s-1}} \qquad (17)$$

for $s = 2, \ldots, S-1$.

Proof: See the Appendix.

Remark: Using the mapping from equity injections to equivalent asset buyouts described in Lemma 1, any optimal menu of equity injections can be replicated with a corresponding menu of asset buyouts.

[6] "Low" E means $E < E^{(2,3)}$; "intermediate" E means $E \in [E^{(2,3)}, E^{(1,2)}]$; and "high" E means $E > E^{(1,2)}$

3.2.1 Example

Here, we present an example that illustrates the role of the indifference curve intersection points in constructing first best menus of bailout plans. The example also illustrates the equivalence result between equity injections and asset buy back plans (Proposition **??**.

The example considers three types, all with $F = 100$, $H_s = 120$, and L_s and p_s as follows:

Type:	$s = 1$	$s = 2$	$s = 3$
L_s	80	85	90
p_s	.5	.55	2/3
$E[Z^{B_0}]$	10	11	40/3

The final row provides the status quo payoffs to inside (old) equity. These parameter values satisfy (17) so that indifference curve intersection point monotonicity is satisfied. FOSD is also satisfied. First best menus of bailout plans therefore exist. The intersection points of the indifference curves are as follows:

	λ	E
Types 2 and 3	.59	22.89
Types 1 and 2	.76	42.50

By the analysis above, a menu consisting of these intersection-points contracts will achieve first best. A feature of such a menu is that type 2 is, by construction, indifferent between the two plans. As discussed in Remarks above, the intersection points also can be used to construct other menus consisting of three plans that strictly separate out the three types.

Table 1 presents the payoffs to equity holders and the bailout agency in the high and low states as well as the expected subsidy under the two "intersection point plans". Both plans completely solve the debt overhang problem for all three types, implying that the ordering of the plans according to payoff to existing equityholders is equivalent to their ordering by the total subsidy to all claimholders. Thus, the plan that gives the higher total subsidy to some type s also gives the higher subsidy to its old equityholders (as pointed out in the discussion leading up to equation (11)). This ordering equivalence can be seen in the final two columns of Table 1.

The table shows that Plan 1, which corresponds to the intersection point of types 2 and 3, leaves equityholders of these types with their status quo payoff (zero subsidy to inside equity); whereas it leaves equityholders of type 1 worse off.

In contrast, Plan 2, which corresponds to the intersection point of types 1 and 2, leaves equityholders of these types indifferent; whereas it makes type 3 equityholders worse off.

Thus, Plan 1 will be chosen by type 3 banks, while Plan 2 will be chosen by type 1 banks. Type 2 banks may choose either plan.

Table 2 illustrates the one-to-one mapping between equity injections and asset buyouts. This table presents asset buyout plans that leaves all existing debt and equityholders with identical payoffs as under the equity injection plans in Table 1.

Table 1: Example of First Best Menu of Equity Injections

Type	State	Prob	Assets		Equity		Bailout	$\frac{\text{Equity}}{\text{Assets}}$	$\frac{\text{Inside E}}{\text{Assets}}$	**Expected Subsidy**	
		low	w/o bailout	w/ bailout	Total	Inside	Agency			**Total**	Inside Equity
Panel a: Plan 1. $\lambda = .59$ *and* $E = 22.86$											
All types	H		120	142.86	42.86	17.39	25.47	30.00%	12.17%		
Type 1	L_1	.5	80	102.86	2.86	1.16	1.70	2.78%	1.13%	**9.28**	-.72
Type 2	L_2	.45	85	107.86	7.86	3.19	4.67	7.29%	2.96%	**6.75**	0
Type 3	L_3	.33	90	112.86	12.86	5.22	7.64	11.39%	4.62%	**3.33**	0
Panel b: Plan 2. $\lambda = .76$ *and* $E = 42.50$											
All types	H		120	162.50	62.50	14.71	47.79	38.46%	9.05%		
Type 1	L_1	.5	80	122.50	22.50	5.29	17.21	18.37%	4.32%	**10.00**	0
Type 2	L_2	.45	85	127.50	27.50	6.47	21.03	21.57%	5.08%	**6.75**	0
Type 3	L_3	.33	90	132.50	32.50	7.65	24.85	24.53%	5.77%	**2.35**	-.98

Table 2: Example of First Best Menu of Asset Buyouts

Type	State	Prob	Assets		Equity		Bailout	$\frac{\text{Equity}}{\text{Assets}}$	$\frac{\text{Inside E}}{\text{Assets}}$	**Expected Subsidy**	
		low	w/o bailout	w/ bailout	Total	Inside	Agency			**Total**	Inside Equity
Panel a: Plan 1. $\phi = .59$ *and* $C = 68.70$											
All types	H		120	117.39	17.39	17.39	71.30	14.82%	14.82%		
Type 1	L_1	.5	80	101.16	1.16	1.16	47.54	1.15%	1.15%	**9.28**	-.72
Type 2	L_2	.45	85	103.19	3.19	3.19	50.51	3.09%	3.09%	**6.75**	0
Type 3	L_3	.33	90	105.22	5.22	5.22	53.48	4.96%	4.96%	**3.33**	0
Panel b: Plan 2. $\phi = .76$ *and* $C = 86.47$											
All types	H		120	114.71	14.71	14.71	91.76	12.82%	12.82%		
Type 1	L_1	.5	80	105.29	5.29	5.29	61.17	5.03%	5.03%	**10.00**	0
Type 2	L_2	.45	85	106.47	6.47	6.47	65	6.08%	6.08%	**6.75**	0
Type 3	L_3	.33	90	107.65	7.65	7.65	68.82	7.10%	7.10%	**2.35**	-.98

3.2.2 Minimal Capital Outlay

The menus focused on in the example above induce first best outcomes, but they do not minimize the bailout agency's capital outlay. If this were to be considered as a secondary objective, we would need S (the number of bank types) plans in the menu, in contrast to $S-1$ as in the example above.

In the example above, the menu consists of the two "intersection point plans," that is, (i) (λ_1, E_1) being the intersection point of the indifference curves of types 2 and 3, $(.59, 22.89)$; and (i) (λ_2, E_2) being the intersection point of the indifference curves of types 1 and 2, $(.76, 42.5)$. To this menu, we add (λ_3, E_3) equal to $(\lambda_3(F-L_3), F-L_3)) = (1/3, 10)$. That is, we add the plan that just solves debt overhang for the highest type while leaving equityholders of that type with their status quo payoff.

A menu of these three plans achieves minimal capital outlay subject to first best being obtained – provided that, on the margin, banks prefer lower capital injections. This is important, since under the structure we have just described, all banks have exactly two plans between which they are indifferent. If we cannot be certain that banks will choose the plan with the lowest capital injection, then we can get arbitrarily close to minimal capital outlay by replacing the "intersection point plans" with plans immediately to the right of them. This construction works regardless of how many types there are, provided, of course, that the conditions described in Theorem 1 are met. By Proposition 1, a similar construction can be employed for asset buyouts.

4 New Investments

Our setup when there are new investments assumes that their NPV accrue to a bank's shareholders. Thus we allow for the possibility that an advantage to equity injections is that the bailout agency gets to share in the NPV of new investments, as argued by Paul Krugman, among others.[7] An alternative assumption is that new investments follow the

[7] See, e.g.,"Gordon Does Good" (op ed) by Paul Krugman, New York Times, October 13, 2008.

assets rather than the bank itself. In this case, the bailout agency would get a share of the NPV under an asset buyout as well, and the analysis above would go through with minor modifications. In particular, there would be no difference between asset buyouts or equity injections with respect to their ability to achieve first best.

We consider two possibilities with respect to the timing of new investments; they arrive and must be taken either ex post or at an interim stage. That is, (i) when the up or down states are realized, or (ii) after the bailout has been implemented but before the up or down state is revealed. If projects must be taken at the interim stage, then the NPV must be calculated with the information at hand at that stage. New investments are financed by the extant shareholders. An issue is whether the bailout removes the debt overhang problem sufficiently for this to be possible.

4.1 Model 1: New Investments Arrive Ex Post

A bank of type s has new investments with positive NPV of $X_{H,s}$ and $X_{L,s}$ in the up and down states, respectively. The extra NPV at date 1 implies that full resolution of debt overhang requires a smaller cash injection than what was considered in Section 3. An equity injection, for example, now needs to provide $E \geq F - (L_s + X_{L,s})$ in cash for a bank of type s.

More generally, denoting the realized state by w and the corresponding NPV of the new investment by X_w, the payoff to old equityholders under a pure equity injection plan is

$$(1 - \lambda) \max[(w + X_w + E - F), 0], \tag{18}$$

and under a pure asset buyout plan it is

$$\max[(1 - \phi)w + X_w + C - F, 0]. \tag{19}$$

Thus, with respect to payoffs to old equityholders, a one-to-one mapping between equity injections and asset buyouts no longer exists. For now, a one-to-one mapping would require $\lambda = \phi$ and

$$C = (1 - \lambda)E + \lambda F - \lambda X_w \tag{20}$$

Thus, we would need X_w to be independent of the state, i.e., $X_{H,s} = X_{L,s}$.

If $X_{H,s} = X_{L,s} = X_s$, then there is a one-to-one mapping, but unless the NPV, X_s, is independent of s, the mapping would also depend on s. Hence, we could not generally conclude that for a given menu of pure equity injection plans, there is a menu of pure asset buyout plans that induce the same set of payoffs to all bank types. In short, there is no equivalent result to Proposition 1 unless $X_w = X$ for all types in all states.

However, it bears emphasis that if X_w is a constant, *there is* a one-to-one mapping from equity injections to asset buyout plans with respect to payoffs to old equityholders. So if we can achieve first best with equity injections, we can do so with asset buyouts too. That equity injections work is straightforward, since the same analysis as in Section 3 applies – simply replace H by $H + X$ and L_s by $L_s + X$.

This illustrates the fallacy of arguing that equity injections are necessarily better because they allow the bailout agency to participate in the up state. The "absence of upside" in an asset buyout can be dealt with, in some cases, by transferring the assets at a comparatively low price. The price of the assets is essentially reduced by the "lost" share in the NPV.

Of course, if the NPV depends upon the state and the bank type, equivalence between equity injections and asset buyouts breaks down exactly because the transfer price in an asset buyout would have to be a function of the state and bank type, as seen in (20). However, assuming NPVs are sufficiently small, if first best is achievable under equity injections, it will also be so under asset buyouts, since λX_w would then have a small effect on incentive compatibility constraints.

An additional implication of the equations above is that if one combines an asset buyout with the right to share in new investments, then equivalence to equity injections (with respect to the payoff to old equityholders) can be retrieved even in cases when X_w varies across states and types. The right to share in new investments under the modified asset buyout plan could, for example, be represented by warrants or convertible bonds with appropriate conversion terms. In turn, this would imply that if first best is achievable under equity injections, it is also achievable under modified asset buyout plans (paralleling Proposition 1).

Finally, we know from the analysis in the previous section that first best is not always achievable through pure plans. When equity injections and asset buyouts are not equivalent, this raises the possibility that efficiency can be improved in some circumstances by mixed bailout plans.

4.2 Model 2: New Investments Arrive at an Interim Date

Denote the NPV of the new investment of a bank of type s at the interim date by $X_s = p_s X_{H,s} + (1-p_s)X_{L,s}$. Shareholders will make the investment if and only if the NPV at the interim date exceeds the debt overhang. For equity injections, this is $X_s > (1-p_s)(F - E - L_s)$, or $E \geq F - L_s - \frac{X_s}{1-p_s}$. Thus, under a pure equity injection plan, the expected payoff to old equityholders is given by

$$E[Z_s^{B=(\lambda,E)}] = \begin{cases} (1-\lambda)(A_s + X_s + E - F) & \text{if} \quad E \geq F - L_s - \frac{X_s}{1-p_s} \\ (1-\lambda)p_s(H + E - F) & \text{if} \quad E < F - L_s - \frac{X_s}{1-p_s}. \end{cases} \tag{21}$$

The corresponding expression for a pure asset buyout plan is

$$E[Z_s^{B=(\phi,C)}] = \begin{cases} (1-\phi)A_s + X_s + C - F & \text{if} \quad C \geq F - (1-\phi)L_s - \frac{X_s}{1-p_s} \\ p_s[(1-\phi)H + C - F] & \text{if} \quad C < F - (1-\phi)L_s - \frac{X_s}{1-p_s}. \end{cases} \tag{22}$$

Thus equity injections and asset buyouts are not equivalent. To see this more clearly, note first that for the break-even points for investing at the interim dates to be the same, we need to have $C = E + \phi L_s$.[8] A necessary condition for having a one-to-one mapping with respect to payoffs to old equityholders is that $\phi = \lambda$. But with these values for ϕ and C, asset buyouts always yield a smaller (larger) expected payoff to old equityholders than the corresponding equity injection in those cases when the cash injection is large enough (too small) for the investment to be made at the interim date.

[8]This follows by comparing the expressions on the right hand side of the "if" in (21) and (22).

4.3 Asset Buy Backs with Call Options

Here we come back to the argument that equity injections are superior to asset buyouts because the former gives the bailout agency a part of the upside from new investments. However, it is straightforward that if an asset buyout plan can be augmented, via giving the bailout agency an additional option to buy shares, at the projected post-bailout share price of a rescued bank in the good state in the absence of such additional NPV, its sharing in any additional value generated by such bank investments would be feasible. It is even possible that the option augmented asset buyout plan can provide the bailout agency with a larger share of the new NPV than an equity injection plan.

Proposition 6 *Suppose $X_{H,s} \geq X_{L,s}$ and let $\{\lambda_s, E_s\}$ be an equity injection plan chosen by, or designed for, a bank of type s. Attaching an Option to buy equity in the bailed out bank to an asset buyout plan for a price equal to the per share value corresponding to the equity value in the High state in the absence of any new profitable investments, that is $[(1-\phi)H + C - F]$, for a fraction of equity equaling $w_s = \frac{\lambda_s}{p_s}$, enables the bailout agency to capture at least the same share of any NPV arising from new post-bailout investments as does the equity injection plan. Such attachment is therefore feasible as long as $w_s < 1$.*

Proof: In the good state of a bank of type s, these options are worth $w_s X_{H,s}$. This has an expected value of $\lambda_s X_{H,s}$, which (weakly) exceeds the share of positive NPV from new investments accruing to the equity acquired by the bailout agency via injecting E_s, namely $\lambda_s[(p_s X_{H,s} + (1-p_s)X_{L,s}]$, from which the result follows. □

5 Policy Choices and Concluding Remarks

The main results of importance for economic policy emerging from our analysis are the following. When banks with troubled assets, having potential future below strictly below their debt cum deposit obligations, are heterogeneous on multiple dimensions – here their probabilities of default and loss given default – it is nevertheless feasible to structure a bailout program which minimizes the implied subsidies to the equity holders of different, and privately informed, bank types, holding their payoffs at the same level as their status quo equity values in the absence of such bailouts, This can be done, under the conditions spelled out in Proposition 3 and Theorem 1 above, using separating menus of either equity injection, or asset buyout menus, which induce self-selection among heterogeneous bank types. However, quite robustly, with heterogeneous losses given default that covary positively with likelihoods of default across banks, such menus necessarily induce overcapitalization – cash injections over and above the minimal amounts needed to overcome debt overhang – at weaker banks. The extent of such overcapitalization, and thus the degree to which the bailout agency must acquire equity stakes in the bailed out banks, increases as the quality of the bank declines, in terms of both its likelihood of default and loss given default. As a result, under an equity injection based bailout plan, weaker banks are left with lower inside equity stakes for their prior shareholders. That is not the case with asset buyout based bailout plans, in which prior equity holders retain full ownership stakes in the assets remaining with the bank.

These features of the two types of bailout programs may impinge on policy choices over these. As our numerical illustrations of equivalent menus above illustrate, while the equity injection menus typically involve much lower cash outlays by the bailout agency, which is of importance if there are deadweight (including political) costs associated with raising government funding. However, equity injections also leave the existing equity holders with a strictly lower proportionate stake in their bank. Such lower inside equity stakes may ill serve banks' incentives to monitor the quality of their future investments, since government capital would bear some or even much of the costs of future losses arising from such lax monitoring. A possible resolution then is to prefer equity injections for better types of banks, with higher $\{p_s, L_s\}$, in which the dilution of prior inside equity stakes are lower post-bailout, and choosing (equivalent, and separating) asset buyout plans for weaker banks in which equity injections would dilute insiders' stakes more substantially, harming their monitoring incentives. Of course, this simple criterion overlooks the problems associated with finding new private buyers for the assets acquired by the bailout agency, who would exploit these optimally.

Our analysis also shows that, when capturing a share of net present values arising from post-bailout new investments by banks would increase payoffs to the bailout agency, then augmenting (partial) asset buyout plans with options to buy the equity of bailed-out banks matters a great deal, a finding that complements the analysis of Philippon and Schnabl (2009). Especially in the current financial crisis, and unlike in prior crisis such as the US S&L Crisis of the early 1990s, potential future upside values of ”toxic” assets (cum asset-backed securities) held by banks, and the lack of transparency regarding their qualities, have given rise to vexing issues about how best to structure bank rescue programs which contribute to their healthy functioning as lenders to the economy, without overly rewarding their extant equity holders, We hope that our paper has made a useful beginning in our understanding of the (subtle) tradeoffs involved, in designing bailout menus.

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Appendix 1: Proofs

Proof of Lemma 1

In state $w \in \{H_s, L_s\}$ the pure equity plan $\{\lambda, E\}$ and the pure asset buyout plan $\{\phi, C\}$ give the following payoffs to old equityholders: $(1-\lambda)\max[(w+E-F),0]$ and $\max[(1-\phi)w+C-F,0]$, respectively. A necessary condition for these be equivalent regardless of w is that $\phi=\lambda$, which in turn implies that we must also have $C=(1-\lambda)E+\lambda F$. $\square$

Proof of Proposition 2

Pure equity injection: Since $E=F-L$ under plan B_E described in the statement of the proposition, (10) is satisfied, i.e., debt overhang is eliminated. Furthermore, in the down state, the payoff to equity of all bank types is 0. In the up state, equityholders of all bank types receive $H+E-F$. To leave old equityholders indifferent between B_E and the status quo and thus minimize expected subsidies, $\lambda \in [0,1]$ must satisfy $E[Z_s^B]=E[Z_s^{B_0}]$ for all s, or

$$p_s(H+E-F)(1-\lambda)=p_s(H-F). \tag{23}$$

This is satisfied for all s provided

$$\lambda=\frac{E}{H+E-F}=\frac{F-L}{H-L}. \tag{24}$$

Since λ as specified by (24) is in the open interval (0,1), since by assumption $F<H$, we have shown that Plan B_E works as claimed.

Next we will show that there is no other subsidy minimizing pure equity injection plan that works for all s. Since debt overhang must be eliminated, such an alternative plan must have $E>F-L$. (If $E=F-L$, we would just have plan B_E). Thus, to leave old equityholders indifferent between the plan and the status quo, we must have

$$p_s(H+E-F)(1-\lambda)+(1-p_s)(L+E-F)(1-\lambda)=p_s(H-F). \tag{25}$$

Rearranging yields

$$1-\lambda=\frac{p_s(H-F)}{p_sH+(1-p_s)L+E-F}, \tag{26}$$

implying that, given E, λ depends on bank type. This is what we wanted to show.

Pure asset buyout: This follows immediately from Proposition 1. It can also be established explicitly as follows:

Consider a bailout plan that satisfies

$$L(1-\phi)+C=F. \tag{27}$$

This eliminates debt overhang. Given (27), a necessary and sufficient condition for minimizing subsidies is that

$$H(1-\phi)+C=H\,, \tag{28}$$

since this leaves old equityholders with the same payoffs in the up and down states as under the status quo, regardless of type.

Combining (27) and (28), we see that ϕ must satisfy $(H-L)(1-\phi)=H-F$, or

$$\phi=\frac{F-L}{H-L}. \tag{29}$$

Substituting this expression for ϕ into either (27) or (28), we have

$$C=\frac{F-L}{H-L}H. \tag{30}$$

Since conditions (29) and (30) describe bailout plan B_A in the statement of the proposition (together with $\lambda=0$ and $E=0$), we have shown B_A solves the debt overhang problem while minimizing subsidies for each s, as claimed.

Next, we will show that there is no other pure asset sale bailout plan that works for all s. Consider therefore a plan with $L(1-\phi)+C>F$. Thus, to leave old equityholders indifferent between the plan and the status quo, we must have

$$p_s[H(1-\phi)+C-F]+(1-p_s)\left[L(1-\phi)+C-F\right]=p_s(H-F). \tag{31}$$

Rearranging this, we have

$$1-\phi=\frac{p_sH+(1-p_s)F-C}{p_sH+(1-p_s)L}. \tag{32}$$

Thus, given C, ϕ depends on bank type. Hence, B_A is the unique asset sale bailout plan that for all s achieves the dual objective of eliminating debt overhang and minimizing subsidies to existing claimholders. $\square$

Proof of Proposition 3

Consider any plan, B, that achieves first best for a bank of type $s > 1$. The proposition will be established if we can show that the subsidy under this plan is larger than the status quo for a bank of type $s' = 1$. Since $L_1 = L_s = L$, a plan that solves debt overhang for a bank of type s also solves it for a bank of type 1. Thus, we only need to show that, for a bank of type 1, equityholders' payoff increases (as pointed out in (11)). We will write the asset value under the plan in the down state simply as L^B.

For the plan in question, we have $L^B \geq F$ and

$$(1-\lambda)[p_s(H_s^B - F) + (1-p_s)(L^B - F)] = p_s(H_s - F). \tag{33}$$

Thus,

$$(1-\lambda) = \frac{p_s(H_s - F)}{p_s(H_s^B - L^B) + L^B - F} \tag{34}$$

The plan would yield the following expected payoff to shareholders of a bank of type 1:

$$E[Z_1^B] = \frac{p_s(H_s - F)}{p_s(H_s^B - L^B) + L^B - F} \left\{ p_1(H_1^B - L^B) + L^B - F \right\}. \tag{35}$$

This is greater than the status quo payoff for type 1, $p_1(H_1 - F)$, if and only if

$$\frac{p_s(H_s - F)}{p_s(H_s^B - L^B) + L^B - F} > \frac{p_1(H_1 - F)}{p_1(H_1^B - L^B) + L^B - F},$$

or

$$\frac{p_s(H_s - F)}{p_s(1-\phi)(H_s - L) + L^B - F} > \frac{p_1(H_1 - F)}{p_1(1-\phi)(H_1 - L) + L^B - F} \tag{36}$$

or $g(p_s, H_s) > g(p_1, H_1)$, where

$$g(p, H) = \frac{p(H - F)}{p(1-\phi)(H - L) + L^B - F}. \tag{37}$$

Now, if $L^B > F$, $g(p, H)$ is strictly increasing in both arguments, since $F > L$. Thus, (36) is satisfied. If $L^B = F$, $g(p, h)$ is unaffected by p but strictly increasing in H, since

$F > L$. Since, by assumption, $H_s > H_1$, this establishes that a bank of type 1 receives strictly more than its status quo payoff from plan B, implying that first best cannot be achieved. $\square$

Proof of Proposition 5

Consider any plan, B, that achieves first best for a bank of type $s = 1$. We will prove the proposition by showing that a bank of type $s = 2$ can achieve a payoff which is larger than its status quo payoff, $p(H - F)$, by adopting plan B. In what follows, we will write H_s^B simply as H^B, since $H_s = H$ for all s.

For the plan in question, we have $L_1^B \geq F$ and

$$(1-\lambda)[p(H^B - F) + (1-p)(L_1^B - F)] = p(H - F). \tag{38}$$

Since $L_2 > L_1$, we also have $L_2^B > L_1^B$. Thus

$$E[Z_2^B] = (1-\lambda)[p(H^B - F) + (1-p)(L_2^B - F)] > p(H - F), \tag{39}$$

which is what we wanted to show. $\square$

Proof of Theorem 1

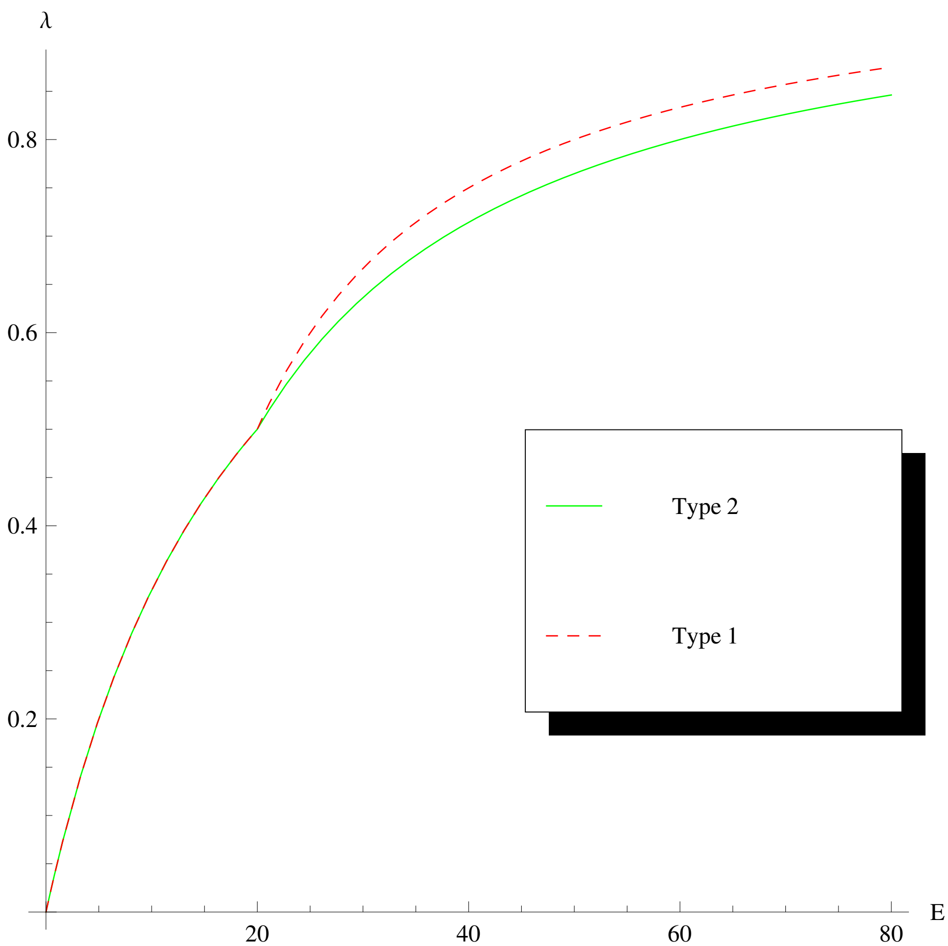
The statement that first best can be achieved if and only if (i) types can be ordered by strict FOSD and (ii) indifference curve intersection monotonicity holds, follows from preceding results and the discussion in the text (it is straightforward to formalize and extend the discussion regarding intersection monotonicity in the text to an arbitrary number S of types). However, we need to show that indifference curve intersection monotonicity holds if and only if (17) holds for all $s = 2, \ldots, S-1$. To this end, pick any $s \in \{2, \ldots S-1\}$. Indifference curve intersection monotonicity implies that $E^{(s,s+1)} < E^{(s-1,s)}$, or by (15),

$$F - \frac{p_s A_{s-1} - p_{s-1} A_s}{p_s - p_{s-1}} > F - \frac{p_{s+1} A_s - p_s A_{s+1}}{p_{s+1} - p_s}. \tag{40}$$

Using $A_s = p_s H + (1-p_s) L_s$ (and the analogous expressions for $s-1$ and $s+1$), (40) simplifies to (17). $\square$

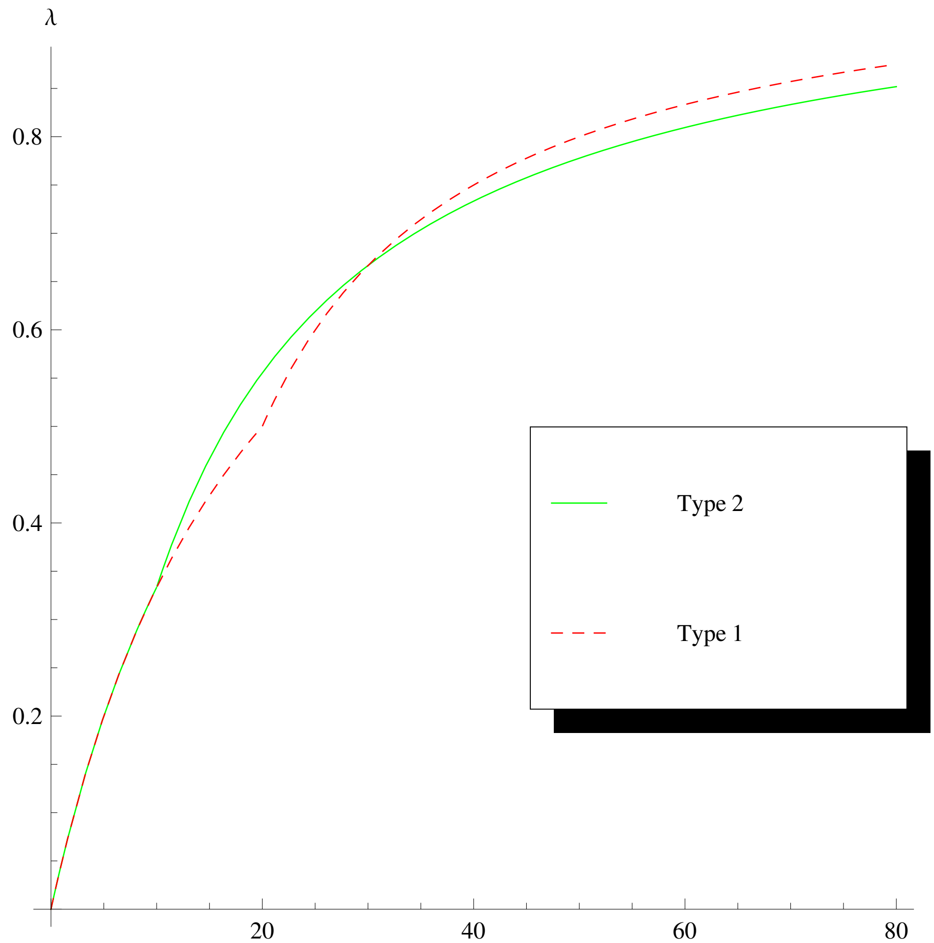
Appendix 2: Figures

Figure 1: Share-Cash Indifference Curves under Homogeneous Debt Overhang



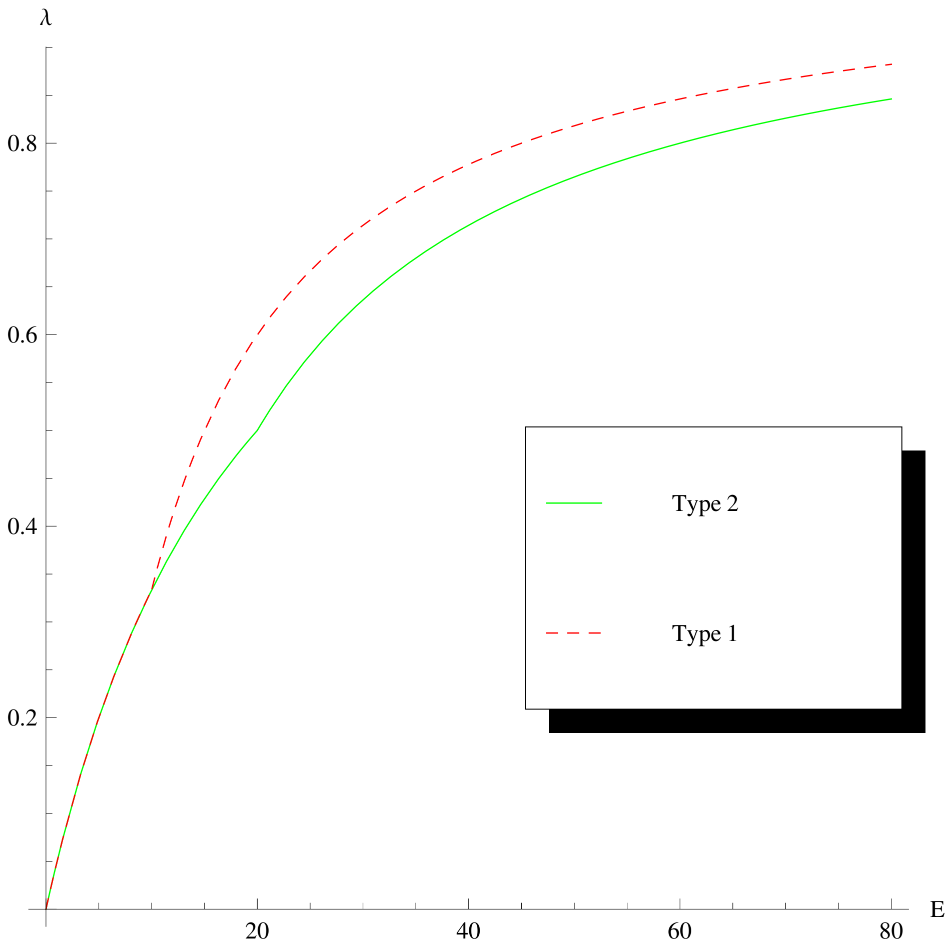
The indifference curves represent $\{\lambda, E\}$ combinations that keep existing equity to its status quo expected payoff. The parameter values used in the plots are as follows: $H = 120$, $F = 100$, $L = 80$, $p_1 = .5$, and $p_2 = 2/3$. The unique plan that implements first best is: $E = 20$ and $\lambda = .5$. For $E < 20$, debt overhang is not solved. For $E > 20$, any λ that keeps Type 1 to its status quo payoff improves the payoff to equityholders of the Type 2 bank, since Type 2's indifference curve lies below that of Type 1.

Figure 2: Share-Cash Indifference Curves under Heterogeneous Debt Overhang: First Best Achievable



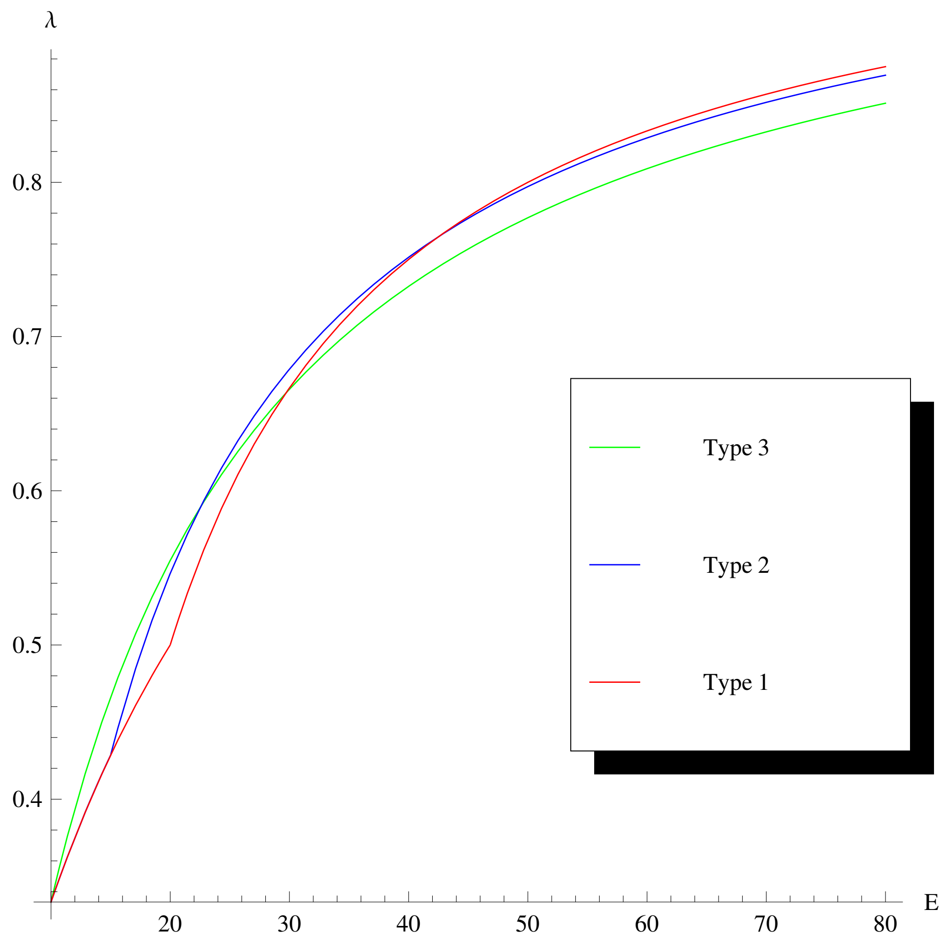
The indifference curves represent $\{\lambda, E\}$ combinations that keep existing equity to its status quo expected payoff. The parameter values used in the plots are as follows: $H = 120$, F=100, $L_1 = 80$, $L_2 = 90$, $p_1 = .5$ and $p_2 = 2/3$. Thus, first order stochastic dominance is satisfied. First best is achieved, for example, by choosing the plan represented by the intersection point $\{\lambda = 2/3, E = 30\}$ of the two curves. Any combination of two plans designed as follows will also work: Plan 1 – any point on Type 2's indifference curve with $10 < E < 30$. Plan 2 – any point on Type 1's indifference curve with $E > 30$.

Figure 3: Share-Cash Indifference Curves under Heterogeneous Debt Overhang: First Best not Achievable



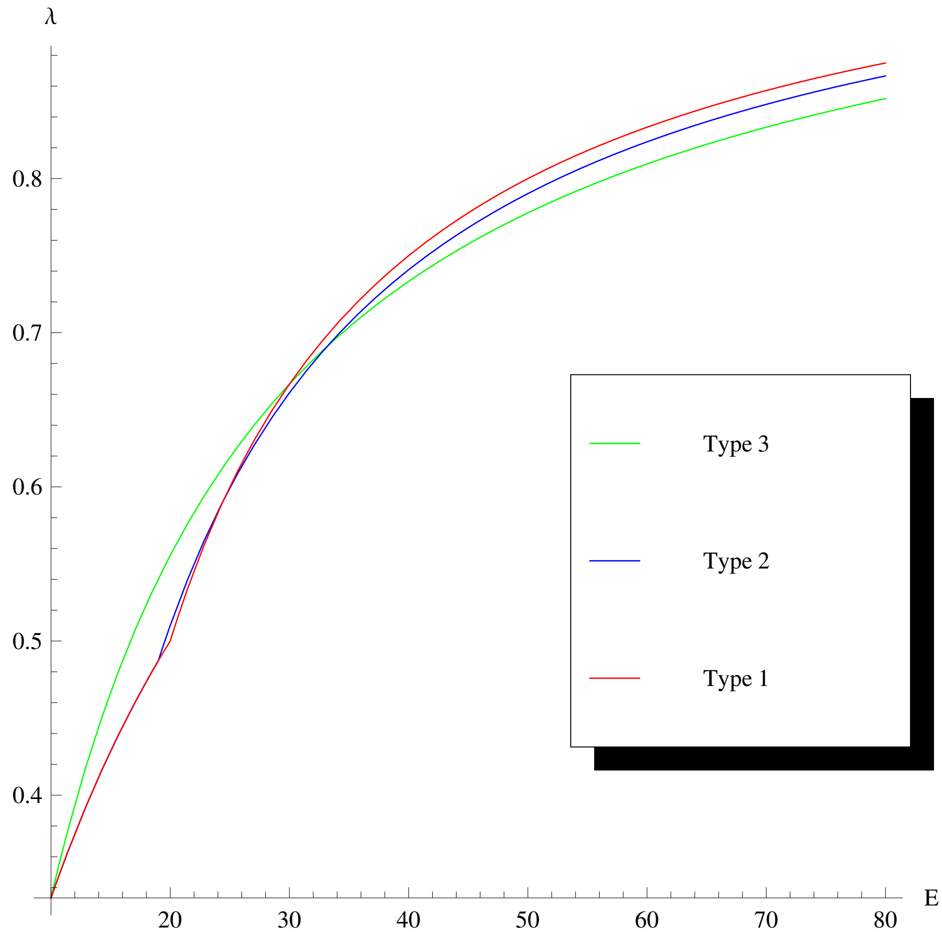
The indifference curves represent $\{\lambda, E\}$ combinations that keep existing equity to its status quo expected payoff. The parameter values used in the plots are as follows: $H = 120$, F=100, $L_1 = 90$, $L_2 = 80$, $p_1 = .5$ and $p_2 = 2/3$. Thus, first order stochastic dominance is not satisfied. First best is not achievable since any plan that solves debt overhang and keeps Type 2 to its status quo payoff will improve the payoff for Type 1. This is seen from the fact that Type 1's indifference curve lies above that of Type 2 for any equity injection, E, that fully resolves debt overhang for Type 2 (that is, any E greater than or equal to 20).

Figure 4: Share-Cash Indifference Curves under Heterogeneous Debt Overhang with Three Types: First Best Achievable



The indifference curves represent $\{\lambda, E\}$ combinations that keep existing equity to its status quo expected payoff. The parameter values used in the plots are as follows: $H = 120$, F=100, $L_1 = 80$, $L_2 = 85$, $L_2 = 90$, $p_1 = .5$, $p_2 = .55$, and $p_2 = 2/3$. Strict first order stochastic dominance and intersection point monotonicity is satisfied. The indifference curves of Types 2 and 3 intersect at $\{\lambda = .59, E = 22.86\}$, while the indifference curves of Types 1 and 2 intersect at $\{\lambda = .76, E = 42.5\}$. First best can be achieved, for example, by offering a menu of the two plans corresponding to these two intersection points.

Figure 5: Share-Cash Indifference Curves under Heterogeneous Debt Overhang with Three Types: First Best Not Achievable



The indifference curves represent $\{\lambda, E\}$ combinations that keep existing equity to its status quo expected payoff. The parameter values used in the plots are as follows: $H = 120$, F=100, $L_1 = 80$, $L_2 = 81$, $L_3 = 90$, $p_1 = .5$, $p_2 = .55$, and $p_3 = 2/3$. Strict first order stochastic dominance is satisfied but intersection point monotonicity is not. The indifference curves of Types 2 and 3 intersect at $\{\lambda = .69, E = 33.14\}$, while the indifference curves of Types 1 and 2 intersect to the left of this, specifically at $\{\lambda = .59, E = 24.50\}$. First best can not be achieved, because the Type 2 indifference curve is never above both of the other two curves at the same time. Thus, any plan that leaves Type 2 equityholders with the status quo expected payoff will improve the expected payoff of either Type 1 or Type 2.