Competition and regulation in a differentiated good market*

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Abstract

This paper addresses the issue of how to design the institutional structure of an industry which provides two differentiated products. One good is supplied by a regulated monopoly and the other is produced in a competitive (unregulated) segment. Two possible institutional patterns are compared. Under "concentration" the regulated firm can enter the competitive segment by owning one firm which operates there (even though the two firms must be legally unbundled). The regime of "separation" implies that regulated activities are totally unbundled from the unregulated ones, that is, common ownership is not allowed. When the regulator does not know the regulated monopoly's cost of production, we find that the pattern of separation improves (expected) social welfare as long as goods are substitutes. Conversely, concentration performs better in case of complementarity.

Keywords: competition, complementarity, concentration, regulation, separation, substitutability.

JEL Classification: D82, L11, L51.

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1. Introduction

Nowadays competition and regulation often take place simultaneously in horizontally related markets, where goods may exhibit some degree of substitutability or complementarity. This is mainly a consequence of both important technological advancements and intense deregulation, which occur at a different pace in different sectors. Regulated firms are sometimes allowed to operate in competitive segments. Local telephone untertakings such as regional Bell companies in the US provide unregulated broadband internet services. The utility Pepco also offers energy management services. In Europe GDF Suez, RWE and Enel all operate in regulated as well as unregulated markets in energy, water and other utility sectors. A further prominent example is Deutsche Post, which acts as a regulated monopoly in the distribution of all letters below a certain weight and competes as a logistic group integrated with DHL and Postbank in the liberalized market for the management and transportation of goods, information and payments. In most these cases legal unbundling is required, which implies that the regulated and competitive activities are run by separate firms with their own accounts but common ownership is allowed.

Regulatory institutions have raised some concerns about this horizontal expansion and have sometimes prevented regulated firms from exerting activities in competitive markets. This was for instance the case of the regional Bell companies, which had been prohibited from expanding into the unregulated long-distance services. The Telecommunications Act of 1996 removed this interdiction but it required these activities to be legally separated.[1] However, these companies are still not permitted to manufacture equipment.

Whether to allow a regulated firm to enter a liberalized market and how to design regulation in this case are issues of practical and theoretical relevance. There are well-known objections to horizontal diversification such as the fear that firm's expansion may distort competition in the unregulated sector (the "level playing field" argument). Conversely, it is well established in the economic literature that firm's diversification may yield efficiency gains (economies of scope, also known as "synergies") which can benefit consumers.

In this paper we introduce a different element which can play a role in the policy debate on horizontal diversification: the *differentiation* between regulated and unregulated goods. The aforementioned examples show that regulated firms tend to expand into related sectors, whose goods may exhibit a relation of substitutability or complementarity with the commodity sold in

[1]This requirement, which has been recently abandoned, was included in the Section 272 of the Telecommunications Act of 1996.

the regulated market. To this end we consider a two differentiated product industry where one good is regulated and the other is offered in a competitive way. In the European utility sector, the former may be the electricity provided in the regulated market and the latter may represent the energy sold in the free market (substitute) or energy management services (complement). In a setting of regulatory optimal control, two possible institutional structures are compared: the regime of *concentration*, where the regulated monopoly is allowed to engage in the competitive segment by owning one firm which operates there, and that of *separation*, which prescribes that regulated and competitive activities must be totally unbundled. In line with some practical evidence, we assume that even in the first case the regulator imposes a kind of unbundling and prescribes legal separation between the two entities, which must have their own accounts even though common ownership is permitted.

As Dana [1993] emphasizes, the problem of choosing the suitable organizational structure cannot be disentangled from the design of the optimal regulatory policy when the regulator has limited knowledge of the industry. This is because the organizational structure crucially affects the informational costs of policy implementation. Hence, the investigation of this interaction allows the regulator to acquire a valuable instrument to improve the performance of regulation.

When the regulator cannot observe the regulated firm's cost of production, our model predicts that the pattern of separation improves (expected) social welfare in case of substitutability between goods. The idea is that the regime of consolidation exacerbates the monopoly's incentives to raise costs since this strategy is also profitable in the competitive segment. Cost exaggeration has a *positive informational externality* on the unregulated affiliate which produces (and earns) more when goods are substitutes. This aggravates the regulator's critical control problem since the social costs of policy implementation increase. For sufficiently high levels of substitutability, the regulator must even abandon its desire to discriminate between the firm's types and can only offer a bunching mechanism. Under separation the regulator finds it easier to incentivize the regulated firm, which does *not* internalize the effect of its strategic behaviour on the competitive market.

Concentration performs better if goods are complements. The firm's strategy to inflate costs, which is as usual beneficial in the regulated segment, now penalizes the competitive subsidiary, whose quantity (and profit) decreases (in expected value). We find that *countervailing incentives* emerge, which relax the regulator's problem since the monopoly's interest in cost manipulation is weaker. For sufficiently high levels of complementarity, the regulator is even able to extract all informational rents despite the asymmetric information problem.

Our analysis suggests that demand interdependence between regulated and unregulated markets deserves careful consideration from policy makers when deciding whether to allow a regulated firm to expand into competitive activities. Even though it is only a particular aspect of a number of important considerations which warrant a close study in any complete analysis of the topic, we believe that this element yields an additional relevant trade-off which is worth *per se* investigating.

2. Related literature

The main part of the early literature on horizontal diversification of regulated firms (see, among others, Braeutigam and Panzar [1989]) emphasizes the risk of excessive expansion into unregulated markets if firms are able to engage in cost shifting, which occurs when the regulator counts as costs for regulated services those arising solely from unregulated activities. However, Anton and Gertler [1988] demonstrate that regulators can use the profit opportunities created by "external" (unregulated) markets to insulate the "internal" (regulated) markets from quantity distortions which arise in environments with asymmetric information. A necessary condition for insulation is that marginal costs vary with output levels, thus providing a cost linkage between the two markets.

Lewis and Sappington [1989a] study a model where the costs of regulated activities are positively correlated with profitability in the unregulated sector. They show that the firm's incentive to exaggerate costs in the regulated market will be mitigated because such an overstatement implies a claim that participation in the unregulated market is more profitable than it really is. Put differently, allowing the regulator to expand into the competitive segment generates "countervailing incentives" (Lewis and Sappington [1989b]) which alleviate the regulatory task of controlling cost exaggeration. They depend on the black-box description of profitability in the competitive market. We also find that countervailing incentives may operate when the regulated firm diversifies but in our paper these forces are driven by demand interdependence between markets.

More recently, Sappington [2003] has shown that diversification is undesirable if the regulator cannot control effort diversion to unregulated services and the firm exaggerates costs. In Calzolari and Scarpa [2009] a regulated domestic firm also operates in an unregulated foreign market. Although firm's expansion amplifies the distortions due to asymmetric information, home consumers are better off when costs exhibit economies of scale.[2]

The role of cost interdependence between regulated and unregulated markets has then been carefully studied in the literature. In this paper we focus our attention on a different aspect which proves to be of some relevance to the regulatory policy in case of horizontal expansion: the interdependence between markets on the *demand side*.

A handful of relevant papers analyzes the effects of asymmetric information on the optimal organizational structure in a multiproduct industry with interdependent demands.[3] Baron and Besanko [1992] investigate how a regulator should organize a production activity in which different units produce components (which are perfect complements) and each of them has private information about its costs. They show that contracting with a single firm which controls both components of production makes the regulator better off than having one independent producer for each component. The idea is that the suppliers are disciplined by threatening to terminate their operation if total reported cost is too high. Termination reduces the profit that can be generated on both units. Integrated production allows the regulator to save rents because a single firm fully internalizes the impact that an overstatement of each unit's costs has on the output and the profit of the other. Gilbert and Riordan [1995] find essentially the same results, by focusing on the case of inelastic demand.

Our paper asks the similar question of how to organize the institutional pattern in a multiproduct industry in a more general setting where goods are either substitutes or complements.[4] This is in line with the analysis in Iossa [1999], who shows that a regulator which has imperfect information about the state of demand may achieve a better performance through integration of production if goods are substitutes, while decentralization tends to be preferred in case of complementarity. Notice that we find completely reverse results in presence of regulatory limited knowledge about production costs. The kind of asymmetric information in hand clearly affects the design of industry structure. Severinov [2003] shows that under complementarity or

[2] In a companion paper [2011] Calzolari and Scarpa consider a regulated firm which is active in other unregulated sectors of the same country, and joint conduct of activities generates economies of scope that are firm's private information. They show that even though this creates distortions in the optimal regulation, allowing the firm to integrate is (socially) desirable.

[3] For a review on this topic see Armstrong and Sappington [2007].

[4] Gilbert and Riordan recognize that perfect complementarity is a quite special case when they point out that ‹‹our analysis also depends on the assumption of a fixed-proportions production technology for the final good. This is perhaps questionable even in the electricity example, because optimizing the transmission grid may reduce the need for new generation capacity›› [1995, p. 252].

small degree of substitutability the value for the producers of using two pieces of information together is lower than the sum of values of each piece of information used independently. Since the interests of producers are opposite to those of the regulator, the latter prefers centralization. The reverse occurs for sufficiently high levels of substitutability.

All papers mentioned so far presume that markets are totally regulated. As Iossa points out [1999, p. 213]

> ‹‹However, regulated firms often interact with unregulated rivals and this interaction plays a crucial role in the contractual relationship between the regulator and the regulated firm. Consequently a next step could be to consider situations where regulated and unregulated firms compete›› .

To our knowledge, no model examining the design of institutional pattern in a differentiated good market takes this aspect into account. Our aim is to seek to fill this gap and consider a setup where only a part of the industry is regulated, while competition in the market occurs in the other part. We believe that this better fits the features of modern integrated markets, where regulated activities often interact with the competitive supply of differentiated products. Within this setting we compare two institutional patterns: the regime of consolidation, where the regulated firm can enter the competitive segment by owning one enterprise which operates there, and that of separation, where there is full unbundling between regulated and competitive activities.

Our contribution is also related to the literature on the institutional design in vertically related markets (see Vickers [1995]). In line with some empirical evidence, this literature has recently considered the presence of legal unbundling. Sibley and Weisman [1998] and Cremer *et al.* [2006] have introduced the idea that the subsidiary independently maximizes its own profits, while the parent company cares about joint profits. Very recently, Höffler and Kranz [2011] have suggested that legal unbundling can be a "golden mean" between ownership separation and full vertical integration. We follow the common approach in these papers, which presume that legal unbundling works perfectly in separating the interests of the affiliate from the rest of the integrated group.[5]

[5] As Höffler and Kranz [2011, p. 579] recognize, this does not always reflect the practice of legal unbundling. However, legislation in Europe and in the US often explicitly excludes direct instructions of the parent company or prescribes arm's length relations. Other requirements may help to implement more severely legal unbundling, like strict personnel separation which ensures that professional interests of the subsidiary's employees are separated from those of the parent company.

The rest of paper is organized as follows. Section 3 describes the basic structures of the model. In Section 4 we consider the case of regulation under complete information, where the two institutional structures turn out to be equivalent. Section 5 derives the regulatory outcome with asymmetric cost information. In Section 6 we make welfare comparisons between the two regimes. Finally, Section 7 is devoted to some concluding remarks. All relevant proofs are relegated in the Appendix.

3. The model

The setting under consideration is a differentiated good market with two commodities, which can be either substitutes or complements. The production of one good is regulated. The other good is provided in a competitive way. Two institutional patterns are compared: the regime of consolidation, where the regulated monopoly owns one competitive firm, and that of separation, under which regulated and competitive activities are completely unbundled.

Following Singh and Vives [1984], the consumers' gross utility from the marketplace is represented by a quadratic utility function of the form

$$U\left(q_r, q_u\right) = \alpha q_r + \alpha q_u - \frac{1}{2}\left(\beta q_r^2 + 2\gamma q_r q_u + \beta q_u^2\right), \tag{1}$$

where q_r denotes the quantity for the regulated good and q_u represents the quantity for the good provided in the unregulated (competitive) segment. Moreover, α, β are positive parameters, and $|\gamma| < \beta$ captures the degree of product differentiation with $\gamma \in (0, \beta)$ if goods are substitutes and $\gamma \in (-\beta, 0)$ if they are complements. For $\gamma = 0$, the two markets are perfectly independent.

The consumer surplus net of expenditures on goods is given by

$$CS\left(q_r, q_u\right) = U\left(q_r, q_u\right) - p_r q_r - p_u q_u. \tag{2}$$

The inverse demand function $p_i\left(q_i, q_j\right)$ for good i, with $i, j = r, u$, $i \neq j$, is thus

$$p_i\left(q_i, q_j\right) = \alpha - \beta q_i - \gamma q_j. \tag{3}$$

A benevolent regulator maximizes social welfare, which is defined as

$$W(q_r, q_u, S_r) = CS(q_r, q_u) - S_r, \tag{4}$$

i.e. consumer surplus in (2) net of subsidies S_r received by the monopoly via the regulatory process (see below).[6]

Notice that (4) is a social welfare function à la Baron and Myerson [1982] with zero weights on profits. The choice of this function is not crucial for the results we obtain. As Armstrong and Sappington [2007] emphasize, we can neglect without any loss of generality the shadow cost of public funds à la Laffont and Tirole [1986] arising from distortionary taxation, which increases even more the weight of taxpayer welfare in the social welfare function and makes the analysis less transparent without affecting qualitatively the results. Interestingly enough, our formulation also allows to interpret S_r as the fixed payment of a two-part tariff. Finally, a positive weight on profit in (4) would not change the relevant trade-offs, the only difference being that the higher regulatory concern about profits clearly creates lower quantity distortions and higher profits.

The regulated firm's profit is

$$\pi_r(q_r, q_u, S_r; c_r) = p_r(q_r, q_u)\, q_r - c_r q_r + S_r, \tag{5}$$

which is the revenue from the marketplace plus subsidies.[7]

Two firms operate in the competitive segment with respective profits

$$\pi_u^1\left(q_u^1, q_u, q_r; c_u^1\right) = p_u(q_r, q_u)\, q_u^1 - c_u^1 q_u^1 \tag{6}$$

$$\pi_u^2\left(q_u^2, q_u, q_r; c_u^2\right) = p_u(q_r, q_u)\, q_u^2 - c_u^2 q_u^2, \tag{7}$$

where $q_u \equiv q_u^1 + q_u^2$. The two firms compete on prices (à la Bertrand) and exhibit different costs. Without loss of generality firm 1 is more efficient than firm 2, i.e. $\Delta c_u \equiv c_u^2 - c_u^1 > 0$. As stressed in Section 1, competition in

[6] Notice that the regulator cares about welfare generated in the entire market, even though it is not allowed to control the competitive part. The idea is that the deregulation process may imply the loss of some regulatory instruments but does not affect the regulatory concern about the aggregate welfare. This is particularly true if one indivisible market for differentiated goods is involved.

[7] Possible fixed costs which make the activity naturally monopolistic are irrelevant for our analysis and then ignored without any loss of generality.

markets connected with the regulated ones is often the result of a deregulation process. Hence, the competitive market can be thought of as a segment where a single firm still operates (firm 1, see Section 4) but the removal of entry barriers and consequential deregulation allow a rival (or competitive fringe, firm 2) to exert (potential) competitive pressure.[8] Interestingly enough, our results are not crucially driven by the assumption of strategic complementarity, that is, price competition. In the Appendix we show that the main conclusions remain valid if we consider strategic substitutability in the unregulated market, that is, quantity (Cournot) competition.

Under concentration the monopoly owns firm 1 and is entitled to receive its profits. As discussed in Section 2, this implies that it cares about joint profits, while the competitive subsidiary maximizes its own profits.

Notice from (5), (6) and (7) that we do not consider economies of scope or any other cost linkage between markets. Put differently, the two institutional patterns we investigate are equivalent in terms of productive efficiency. In line with the main literature which is relevant for our purposes (see Iossa [1999]), this allows to focus on the role played by the interdependence of demands.

4. Complete information

To better appreciate how our main results are driven by asymmetric cost information we start with the case of a fully informed regulator. Under complete information the two institutional patterns yield exactly the same outcome: the regulated monopoly has no degree of freedom and then the (possibly) different strategic behaviours induced by the two organizational structures are inconsequential.

Our model is a two-stage game. After choosing the institutional pattern, at the first stage the regulator makes a take-it-or-leave-it offer of a policy $\{q_r, S_r\}$ to the regulated firm, which accepts or refuses the offer. In case of rejection, the firm obtains its reservation utility (normalized to zero). At the

[8] The more efficient firm may be the former monopolist. There are good reasons for thinking that this is the case, like the longer experience of the incumbent in running the activity. For our purposes it is however sufficient that any one competing firm turns out to be more efficient than the others, otherwise price competition implies zero profits for any firm and the monopoly's expansion into the unregulated sector does not change its profits (and incentives). Imperfect competition which allows room for profits seems to be a natural assumption in markets which have been recently liberalized. The decision to deregulate is in line with empirical evidence (see Section 1) and it can stem from a number of sensible reasons (like the policy makers' desire to promote competition in segments which are not natural monopolies), whose analysis is outside the scope of this paper.

second stage, two firms compete in the unregulated market.

We solve this game by backward induction. At the second stage, using (6) and (7) Bertrand competition implies that the equilibrium price is (almost) equal to the marginal cost of (inefficient) firm 2, i.e. $p_u = c_u^2$,[9] and (efficient) firm 1 will serve all the market, i.e. $q_u^1 = q_u$, with

$$q_u(q_r) = \frac{1}{\beta}\left(\alpha - c_u^2 - \gamma q_r\right) \tag{8}$$

by (3). Notice that when goods are substitutes ($\gamma > 0$) a trade-off emerges in their consumption ($\frac{dq_u}{dq_r} < 0$), while complementarity ($\gamma < 0$) implies a benefit in their joint consumption ($\frac{dq_u}{dq_r} > 0$). Hereafter we denote by $\pi_u = \pi_u^1$ the profit of firm 1, which is the unique firm indeed operating in the unregulated market.[10]

At the first stage the regulator designs a policy $\{q_r, S_r\}$, which specifies the quantity q_r and the subsidy S_r for the regulated firm in order to maximize social welfare defined in (4). We replace the choice variable S_r in (5) with π_r given their bijective correspondence in the monopoly's profit function for a given q_r. Using (5) and the second-stage outcome in (8), the regulator's problem in (4) becomes

$$\max_{\{q_r, \pi_r\}} \alpha q_r - c_r q_r - \frac{1}{2}q_r^2 + \frac{1}{2\beta}\left(\alpha - c_u^2 - \gamma q_r\right)^2 - \pi_r \quad s.t. \quad \pi_r \geq 0, \tag{9}$$

where the constraint ensures the participation of the regulated firm, which receives a non-negative utility level from the regulatory relationship.[11]

Standard calculations lead to the following conclusion.

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[9] There is a very minor technical detail here, since at the Bertrand-Nash equilibrium firm 2 charges a price equal to its marginal cost and firm 1 a price which is a shade ω below it. Of course firm 1 wants to choose ω as close as possible to zero, but for any given small number ω it is always possible to find another number smaller than it. To solve this technical problem we use an *escamotage* which is common in the literature, by assuming that for identical prices all the demand goes to the firm with lower costs.

[10] We assume that, despite its cost advantage, firm 1 cannot set a monopoly price (see Appendix B). Otherwise, firm 2 would be completely irrelevant, and the effect of (potential) competition in the market would disappear.

[11] Under consolidation the original participation constraint is $\pi_r + \pi_u \geq 0$. In line with some relevant literature (see Iossa [1999]), we restrict our attention to the *ex ante* case where the institutional pattern cannot be made dependent on the particular characteristics of the firm. This still allows the regulator to prohibit the firm from consolidating if it rejects the contract, so that its outside option is zero. Moreover, because of legal separation, each branch of the consolidated group must be viable *per se* when operating in the market (for instance because cross subsidization is not allowed), which implies $\pi_r \geq 0$.

Lemma 1 *Under complete information, the regimes of consolidation and separation yield the same outcome, which exhibits the following properties*

$$q_r = \frac{\alpha - c_r - z\left(\alpha - c_u^2\right)}{\beta\left(1 - z^2\right)} \tag{10}$$

$$p_r = c_r \tag{11}$$

$$\pi_r = 0 \tag{12}$$

$$q_u = \frac{\alpha - c_u^2 - z\left(\alpha - c_r\right)}{\beta\left(1 - z^2\right)} \tag{13}$$

$$p_u = c_u^2, \tag{14}$$

where $z \equiv \frac{\gamma}{\beta} \in (-1, 1)$.

The quantities for the two goods in (10) and (13) are interdependent through the parameter z, which captures the relative degree of substitutability ($z > 0$) or complementarity ($z < 0$). Notice from (14) that the unregulated firm produces at a price above its marginal costs c_u^1, which arises from competition and cannot be directly affected by regulation. The marginal cost pricing outcome in the regulated market (see (11)) reveals that the regulator finds it too costly to reduce allocative inefficiency in the competitive segment by distorting the regulated price away from marginal cost. It prefers to tolerate this social cost and maximize allocative efficiency in the regulated part of the industry while the monopoly is given zero profits (see 12)).

5. The case of asymmetric information

The result of equivalence of the institutional structure emphasized in Lemma 1 breaks down as long as the regulator is no longer omniscient. We now assume that it does not know the cost c_r of producing the regulated good.[12]

[12] Asymmetric information concerns only the regulated part of the industry. The competitive supply of a homogeneous good should allow the regulator to acquire quite easily all widespread information. More relevantly, our results do not change significantly if costs in the two markets are unknown and independently distributed, when the consolidated firm does not know its affiliate's costs, as we expect with legal separation which usually prescribes a system of "Chinese walls" against the exchange of information within the group (the main difference being that the firm takes the expected competitive profits into account). As the choice of the institutional pattern does not affect competition in the unregulated market, when costs are (sufficiently highly) correlated the regulator can achieve the complete information result under the two regimes by implementing a mechanism of "yardstick competition" which conditions the subsidy to the regulated firm on the outcome at the second stage.

For the sake of convenience, this cost can only take two possible values, that is, $c_r \in \{c_r^-, c_r^+\}$, with (common knowledge) probabilities ν and $(1-\nu) \in (0,1)$ respectively, and $\Delta c_r \equiv c_r^+ - c_r^- > 0$.

The sequence of events is the following.

(I) The regulator chooses the institutional pattern.

(II) Nature draws a type $c_r \in \{c_r^-, c_r^+\}$ for the firm and privately informs the firm.

(III) Invoking revelation principle (Myerson [1979]), the regulator makes a take-it-or-leave-it offer $\{q_r(\widehat{c}_r), S_r(\widehat{c}_r)\}$, which specifies the quantity $q_r(.)$ and the subsidy $S_r(.)$ as functions of the regulated firm's report $\widehat{c}_r \in \{c_r^-, c_r^+\}$, such that $\widehat{c}_r = c_r$ in equilibrium. The firm accepts or refuses the offer. In case of rejection, the firm obtains its reservation utility (normalized to zero).

(IV) Two firms compete in the unregulated segment.

Let us analyze now the regulatory game under the two alternative institutional patterns.

5.1. The regime of separation

The presence of asymmetric cost information allows the monopoly to exploit strategically its informative advantage. The economic literature has long ago emphasized that the firm has a natural incentive to overstate its costs in order to obtain a higher price and then a larger profit. This conclusion definitely applies to the pattern of separation.[13]

As the second-stage outcome is unchanged, the regulator's problem at the first stage is the maximization of (9) in expected terms. Formally, we have

$$\max_{\{(q_r(c_r^-),\pi_r(c_r^-));(q_r(c_r^+),\pi_r(c_r^+))\}} \nu\left[\alpha q_r(c_r^-) - c_r^- q_r(c_r^-) - \frac{1}{2}\beta q_r^2(c_r^-)\right.$$

$$\left. + \frac{1}{2\beta}\left(\alpha - c_u^2 - \gamma q_r(c_r^-)\right)^2 - \pi_r(c_r^-)\right] + (1-\nu)\left[\alpha q_r(c_r^+) - c_r^+ q_r(c_r^+)\right.$$

$$\left. - \frac{1}{2}\beta q_r^2(c_r^+) + \frac{1}{2\beta}\left(\alpha - c_u^2 - \gamma q_r(c_r^+)\right)^2 - \pi_r(c_r^+)\right] \quad s.t. \tag{15}$$

[13] An efficient firm which claims to be inefficient, by declaring $\widehat{c}_r = c_r^+$ instead of c_r^-, gets a positive profit, i.e. from (5) $\pi_r(c_r^+; c_r^-) = \pi_r(c_r^+) + \Delta c_r q_r(c_r^+) > 0$, as $\pi_r(c_r^+) = 0$ if the regulator attempts to implement the complete information policy. Conversely, understating costs is self-defeating, since $\pi_r(c_r^-; c_r^+) = \pi_r(c_r^-) - \Delta c_r q_r(c_r^-) < 0$, as $\pi_r(c_r^-) = 0$.

$$\pi_r\left(c_r^-\right) \geq 0 \tag{16}$$

$$\pi_r\left(c_r^+\right) \geq 0 \tag{17}$$

$$\pi_r\left(c_r^-\right) \geq \pi_r\left(c_r^+; c_r^-\right) = \pi_r\left(c_r^+\right) + \Delta c_r q_r\left(c_r^+\right) \tag{18}$$

$$\pi_r\left(c_r^+\right) \geq \pi_r\left(c_r^-; c_r^+\right) = \pi_r\left(c_r^-\right) - \Delta c_r q_r\left(c_r^-\right), \tag{19}$$

where (16) and (17) are the participation constraints for the two types c_r^- and c_r^+ of the firm, while (18) and (19) represent their respective incentive compatibility constraints. Clearly, each type of the firm must receive at least what it would get by mimicking the other.[14]

The following Proposition summarizes the main results.

Proposition 1 *Under asymmmetric information, the regime of separation yields*

$$q_r^S\left(c_r\right) = \begin{cases} \frac{\alpha - c_r^- - z\left(\alpha - c_u^2\right)}{\beta(1-z^2)} & \text{if } c_r = c_r^- \\ \frac{\alpha - c_r^+ - z\left(\alpha - c_u^2\right)}{\beta(1-z^2)} - \frac{\phi \Delta c_r}{\beta(1-z^2)} & \text{if } c_r = c_r^+ \end{cases} \tag{20}$$

$$p_r^S\left(c_r\right) = \begin{cases} c_r^- & \text{if } c_r = c_r^- \\ c_r^+ + \phi \Delta c_r & \text{if } c_r = c_r^+ \end{cases} \tag{21}$$

$$\pi_r^S\left(c_r\right) = \begin{cases} \frac{\alpha - c_r^+ - z\left(\alpha - c_u^2\right) - \phi \Delta c_r}{\beta(1-z^2)} \Delta c_r & \text{if } c_r = c_r^- \\ 0 & \text{if } c_r = c_r^+ \end{cases} \tag{22}$$

$$q_u^S\left(c_r\right) = \begin{cases} \frac{\alpha - c_2^u - z\left(\alpha - c_r^-\right)}{\beta(1-z^2)} & \text{if } c_r = c_r^- \\ \frac{\alpha - c_2^u - z\left(\alpha - c_r^+\right)}{\beta(1-z^2)} + \frac{z \phi \Delta c_r}{\beta(1-z^2)} & \text{if } c_r = c_r^+ \end{cases} \tag{23}$$

where $\phi \equiv \frac{\nu}{1-\nu}$.

[14] The right-hand side in (18) and (19) is derived using (5), see also the previous footnote.

Standard forces operate here.[15] The c_r^--firm's production in (20) is first-best optimal ("no distortion at the top" result), and the corresponding price in (21) equals marginal costs. Clearly, the competitive output in (23) is affected by asymmetric information, neither. The regulated quantity in (20) of the inefficient firm is downward distorted - and the corresponding price in (21) is set above marginal costs - in order to curb the informational rent in (22) extracted by the efficient firm. The competitive output in (23) reacts according to the sign of product differentiation, and thus it increases (decreases) when goods are substitutes (complements) with respect to the complete information outcome.

5.2. The regime of consolidation

Under consolidation the regulated firm internalizes the profits of its affiliate in the unregulated market. This might affect the natural sign of incentives to manipulate information. The following Lemma guarantees that this is not the case.

Lemma 2 *If the regulator implements the complete information policy despite the problem of asymmetric information, the regulated firm has only an incentive to exaggerate costs.*

The regime of consolidation does *not* distort the usual direction of misreporting costs. The efficient monopoly still pretends to be inefficient. Conversely, the high-cost firm does not find it convenient to understate its information.

However, the internalization of its subsidiary's profits crucially affects the strategic behaviour of the regulated firm. Incentive compatibility constraints are now written as

$$\pi_r\left(c_r^-\right)+\pi_u\left(c_r^-\right)\geq\pi_r\left(c_r^+;c_r^-\right)+\pi_u\left(c_r^+;c_r^-\right) \tag{24}$$

$$\pi_r\left(c_r^+\right)+\pi_u\left(c_r^+\right)\geq\pi_r\left(c_r^-;c_r^+\right)+\pi_u\left(c_r^-;c_r^+\right). \tag{25}$$

The regulator designs a policy which guarantees to the firm higher *joint* profits from truth-telling. This means that the overall profit from revealing its own type (left-hand side of (24) and (25)) outweighs the gain from manipulating information (right-hand side of (24) and (25)).

[15] For an analysis of the impact of asymmetric information on the optimal regulatory policy in a single market, see the seminal paper of Baron and Myerson [1982].

Substituting (8) into $\pi_u(.)$, (24) and (25) become after some manipulations

$$\pi_r\left(c_r^-\right) \geq \pi_r\left(c_r^+\right) + \Delta c_r q_r\left(c_r^+\right) + z\Delta c_u\left(q_r\left(c_r^-\right) - q_r\left(c_r^+\right)\right) \tag{26}$$

$$\pi_r\left(c_r^+\right) \geq \pi_r\left(c_r^-\right) - \Delta c_r q_r\left(c_r^-\right) - z\Delta c_u\left(q_r\left(c_r^-\right) - q_r\left(c_r^+\right)\right). \tag{27}$$

Summing (26) and (27) yields the following monotonicity constraint (also called *implementability* condition[16])

$$q_r\left(c_r^-\right) \geq q_r\left(c_r^+\right), \tag{28}$$

which clearly states that the production level requested from the c_r^--firm cannot be lower than the one requested from the c_r^+-firm.

5.2.1. Substitutes

The following Proposition emphasizes the main results under substitutability, which are investigated step by step.

Proposition 2 *Under asymmmetric information, if* $z \in [0, z_s^*]$*, with* $z_s^* \equiv \frac{\Delta c_r}{\Delta c_u} > 0$*, then the regime of consolidation yields*

$$q_r^C(c_r) = \begin{cases} \frac{\alpha - c_r^- - z\left(\alpha - c_u^2\right)}{\beta(1-z^2)} - \frac{z\Delta c_u}{\beta(1-z^2)} & \text{if } c_r = c_r^- \\ \frac{\alpha - c_r^+ - z\left(\alpha - c_u^2\right)}{\beta(1-z^2)} - \phi\frac{\Delta c_r - z\Delta c_u}{\beta(1-z^2)} & \text{if } c_r = c_r^+ \end{cases} \tag{29}$$

$$p_r^C(c_r) = \begin{cases} c_r^- + z\Delta c^u & \text{if } c_r = c_r^- \\ c_r^+ + \phi\left(\Delta c_r - z\Delta c_u\right) & \text{if } c_r = c_r^+ \end{cases} \tag{30}$$

$$\pi_r^C(c_r) = \begin{cases} \frac{\alpha - c_r^+ - z\left(\alpha - c_u^2\right) - \phi\Delta c_r + (1+2\phi)z\Delta c_u}{\beta(1-z^2)}\Delta c_r - \frac{(1+\phi)z^2(\Delta c_u)^2}{\beta(1-z^2)} & \text{if } c_r = c_r^- \\ 0 & \text{if } c_r = c_r^+ \end{cases} \tag{31}$$

[16] See Laffont and Martimort [2002, ch. 2].

$$q_u^C(c_r) = \begin{cases} \frac{\alpha - c_2^u - z\left(\alpha - c_r^-\right)}{\beta(1-z^2)} + \frac{z^2 \Delta c_u}{\beta(1-z^2)} & \text{if } c_r = c_r^- \\ \frac{\alpha - c_2^u - z\left(\alpha - c_r^+\right)}{\beta(1-z^2)} + z\phi \frac{\Delta c_r - z\Delta c_u}{\beta(1-z^2)} & \text{if } c_r = c_r^+. \end{cases} \quad (32)$$

If $z \in (z_s^*, 1)$ *then the regulatory mechanism exhibits the following properties*

$$\overline{q}_r^C = \frac{\alpha - c_r^+ - z\left(\alpha - c_u^2\right)}{\beta\left(1 - z^2\right)} \quad (33)$$

$$\overline{p}_r^C = c_r^+ \quad (34)$$

$$\overline{\pi}_r^C(c_r) = \begin{cases} \frac{\alpha - c_r^+ - z\left(\alpha - c_u^2\right)}{\beta(1-z^2)} \Delta c_r & \text{if } c_r = c_r^- \\ 0 & \text{if } c_r = c_r^+ \end{cases} \quad (35)$$

$$\overline{q}_u^C = \frac{\alpha - c_2^u - z\left(\alpha - c_r^+\right)}{\beta\left(1 - z^2\right)}. \quad (36)$$

Proposition 2 emphasizes that the regulator finds it optimal to offer a *screening* contract only if substitutability between goods is sufficiently low, i.e. $z \in [0, z_s^*]$. Interestingly enough, the production of the efficient firm in (29) is *downward* distorted, and the corresponding price in (30) is set above marginal costs. The c_r^+-firm is also allowed to produce less than under complete information (the second addend in (29) is non-negative if $z \in [0, z_s^*]$) and then the price is distorted above marginal costs (see (30)). More relevantly, this distortion is *softer* than under separation (compare (20) and (29) or equivalently (21) and (30)). The idea is that under consolidation the regulator reduces the wedge between quantities which ensures incentive compatibility (see (28)) in order to relax the relevant constraint in (26) and then curb informational rents in (31) extracted by the efficient firm. The unregulated output in (32) increases due to substitutability between goods and then the competitive branch of the consolidated group is more profitable.

If goods are sufficiently close substitutes, i.e. $z \in (z_s^*, 1)$, then the separating equilibrium characterized in the first period of Proposition 2 is *no* longer implementable.[17] The rationale is that the regulator's desire would be to have the inefficient firm producing more than the efficient one in order

[17] Notice that the interval $(z_s^*, 1)$ where the bunching solution applies is nonempty for $\Delta c_r < \Delta c_u$, that is, when the spread between costs in the regulated segment is lower than the cost asymmetry in the competitive part of the market. For higher values of Δc_r the regulator is always able to discriminate between the firm's types without violating the monotonicity condition.

to limit the high informational rents of the latter (see (26)). However, this strategy is not feasible because it violates the monotonicity condition in (28). The only option available to the regulator is then to implement the *bunching* mechanism described in the second period of Proposition 2. Both types of the firm produce the same quantity in (33) at a price equal to the cost of the inefficient firm (see (34)). This allows to the c_r^--firm to receive the informational rent given by (35), while the competitive affiliate of the consolidated group produces the quantity in (36).

5.2.2. Complements

The following Proposition shows our results in case of complementarity.

Proposition 3 *Define $z_c^* < 0$ as the (unique) value for z such that*

$$\Phi(z) \equiv -\frac{z^2 (\Delta c_u)^2}{\Delta c_r}(1+\phi)+(1+2\phi)\, z\Delta c_u+\alpha-c_r^+-z\left(\alpha-c_u^2\right)-\phi\Delta c_r = 0. \tag{37}$$

Then, under asymmetric information, if $z \in [z_c^*, 0)$ the regulatory mechanism with consolidation exhibits the same features as if $z \in [0, z_s^*]$, i.e. it is defined by (29) through (32) in the first period of Proposition 2.

If $z \in (-1, z_c^*)$, the regime of consolidation yields

$$\widetilde{q}_r^C(c_r) = \begin{cases} \frac{\alpha-c_r^- - z\left(\alpha-c_u^2\right)}{\beta(1-z^2)} - \frac{z\Delta c_u}{\nu\beta(1-z^2)}\varepsilon & \text{if } c_r = c_r^- \\ \frac{\alpha-c_r^+ - z\left(\alpha-c_u^2\right)}{\beta(1-z^2)} - \frac{\Delta c_r - z\Delta c_u}{(1-\nu)\beta(1-z^2)}\varepsilon & \text{if } c_r = c_r^+ \end{cases} \tag{38}$$

$$\widetilde{p}_r^C(c_r) = \begin{cases} c_r^- + \frac{z\Delta c_u}{\nu}\varepsilon & \text{if } c_r = c_r^- \\ c_r^+ + \frac{\Delta c_r - z\Delta c_u}{1-\nu}\varepsilon & \text{if } c_r = c_r^+ \end{cases} \tag{39}$$

$$\widetilde{\pi}_r^C(c_r) = \begin{cases} 0 & \text{if } c_r = c_r^- \\ 0 & \text{if } c_r = c_r^+ \end{cases} \tag{40}$$

$$\widetilde{q}_u^C(c_r) = \begin{cases} \frac{\alpha-c_2^u - z\left(\alpha-c_r^-\right)}{\beta(1-z^2)} + \frac{z^2\Delta c_u}{\nu\beta(1-z^2)}\varepsilon & \text{if } c_r = c_r^- \\ \frac{\alpha-c_2^u - z\left(\alpha-c_r^+\right)}{\beta(1-z^2)} + z\frac{\Delta c_r - z\Delta c_u}{(1-\nu)\beta(1-z^2)}\varepsilon & \text{if } c_r = c_r^+ \end{cases} \tag{41}$$

where $\varepsilon = \nu(1-\nu)\frac{\alpha-c_r^+-z\left(\alpha-c_2^u\right)+z\Delta c_u}{z^2(\Delta c_u)^2+\nu(\Delta c_r)^2-2\nu\Delta c_r z\Delta c_u}\Delta c_r \in (0,\nu)$.

With weak complementarity, i.e. $z \in (z_c^*, 0)$, we find from (29) an *upward* distortion of the quantity produced by the efficient firm, while the output of the inefficient one is still below the complete information level. The idea is that now the regulator prefers to *increase* the wedge between quantities, since this relaxes the incentive constraint in (26). Notice from (29) that the regulator induces an *ex ante downward* quantity distortion. Complementarity implies an analogous impact on the unregulated production (see (32)), which penalizes (in expected terms) the competitive affiliate.

The mechanism described by (29) through (32) is *no* longer feasible for sufficiently close complements, i.e. $z \in (-1, z_c^*)$.[18] This occurs because the efficient firm would incur losses (the profit in (31) would be negative) and then would not participate. The cut-off value z_c^* defined by (37) denotes the highest level of complementarity which is compatible with positive profits. When goods are close complements, the incentive constraint in (26) is so weak that the regulator can extract all informational rents (see (40)). This does not imply at all that the firm's interests perfectly align with those of the regulator and then the first-best outcome can be achieved. Regulated quantities in (38) - and corresponding prices in (39) - are qualitatively distorted as before to ensure incentive compatibility ((26) is still binding at the optimum), and the competitive subsidiary provides the output in (41).

6. Welfare comparisons

We are now in a position to state our main results.

Proposition 4 *Under asymmetric information, if goods are substitutes the regime of separation*

(i) reduces expected informational rents

(ii) increases expected consumer surplus

(iii) increases expected social welfare.

If goods are complements, the regime of concentration yields (i) through (iii).

The two regimes give the same amount of expected subsidies to the regulated firm.

The first period of Proposition 4 reveals that with substitutability the pattern of consolidation exacerbates the monopoly's incentives to exaggerate costs. This strategy is profitable both in the regulated segment, where the

[18] The interval $(-1, z_c^*)$ is nonempty, i.e. $\Phi(z) < 0$ in (37), if Δc_u is high enough (see the Appendix for details).

firm can gain from the usual mimicking behaviour, and in the unregulated part of the market, since substitutability between goods implies an increase in production (and profits) of the competitive subsidiary. Put differently, cost exaggeration yields a *positive informational externality* for the monopoly in the unregulated market. This makes the incentive constraint in (26) more severe than in (18). Taking the expected difference between (31) and (22) yields

$$E\left[\pi_r^C\right] - E\left[\pi_r^S\right] = \nu z \Delta c_u \frac{(1+2\phi)\,\Delta c_r - (1+\phi)\,z\Delta c_u}{\beta\,(1-z^2)}, \tag{42}$$

which is positive for $z \in [0, z_s^*]$. This means that the regime of consolidation distributes higher (expected) informational rents. The more severe incentive problem under consolidation yields higher allocative inefficiency, which reduces (expected) consumer surplus and social welfare. Separation clearly performs better, since the monopoly neglects the impact of its choices on the competitive segment, and this relaxes the regulator's incentive problem. For sufficiently high levels of substitutability, i.e. if $z \in (z_s^*, 1)$, the pattern of consolidation is even less appealing, since the regulator cannot discriminate between the firm's types.

The second part of Proposition 4 shows that complementarity inverts the sign of forces at work. The regime of consolidation now softens the regulator's critical control problem, since the internalization of the loss in the competitive segment arising from cost manipulation weakens the monopoly's incentive to exaggerate costs. In other terms, consolidation creates *countervailing incentives* which make the regulatory task of controlling cost exaggeration less burdensome. The sign of (42) is now negative, which means that for $z \in [z_c^*, 0)$ consolidation allows the regulator to reduce informational rents. This improves allocative efficiency, by making consumers and society better off. With sufficiently high levels of complementarity, i.e. if $z \in (-1, z_c^*)$, the regulatory policy with consolidation is even able to capture all informational rents of the firm.

Finally notice that the regulated monopoly receives the same (expected) subsidy irrespective of the regime. The two relevant constraints in (18) and (26) differ in a term which only depends on quantities, and then there is no reason for providing different subsidies.

7. Concluding remarks

This paper has examined the design of the institutional pattern in a two-product industry with interdependent demands where one good is subject to regulation and the other is provided in a competitive way. We have found that the regime of separation improves (expected) social welfare when goods are substitutes. This is because the regulated firm does not internalize the gain from exaggerating costs in the competitive segment and then requires lower informational rents. The opposite occurs in case of complementarity, since a consolidated firm cares about the loss it imposes on the competitive affiliate when manipulating its cost information, and then the pattern of consolidation is less costly to implement.

We have also shown how the degree of product differentiation crucially affects the features of the optimal regulatory policy. If goods are sufficiently close substitutes, under consolidation the regulator can no longer discriminate between the firm's types and it is forced to offer a bunching mechanism. Conversely, for high levels of complementarity, the regulatory mechanism is able to extract all informational rents of the firm.

Our results may also shed some light on merger policies. While the welfare effects of consolidations between firms in competitive industries are well known in the literature, this analysis in environments where regulated and competitive activities interact is a stimulating field which needs further reasearch. The interplay between competition and regulation may yield new effects of some interest.

Appendix

Proof of Proposition 1 Standard arguments imply that (17) and (18) are binding at the optimum. Replacing these constraints into (15) we get the following first-order conditions

$$\alpha - c_r^- - \beta q_r\left(c_r^-\right) - z\left(\alpha - c_u^2 - \gamma q_r\left(c_r^-\right)\right) = 0$$

$$(1-\nu)\left[\alpha - c_r^+ - \beta q_r\left(c_r^+\right) - z\left(\alpha - c_u^2 - \gamma q_r\left(c_r^+\right)\right)\right] - \nu \Delta c_r = 0,$$

which yield (20). Straightforward substitutions allow to find (21) through (23).

Proof of Lemma 2 The monopoly's extra profit $\Delta\pi\left(c_r^+; c_r^-\right)$ from cost overstatement, that is, when claiming $\widehat{c}_r = c_r^+$ instead of c_r^-, amounts to

$$\Delta\pi\left(c_r^+; c_r^-\right) = \pi_r\left(c_r^+; c_r^-\right) + \pi_u\left(c_r^+; c_r^-\right) - \pi_r\left(c_r^-\right) - \pi_u\left(c_r^-\right). \tag{43}$$

We find from (5) that $\pi_r(c_r^+; c_r^-) = \pi_r(c_r^+) + \Delta c_r q_r(c_r^+) = \Delta c_r q_r(c_r^+)$, since declaring the actual costs implies zero profits when the complete information policy is implemented (see (12)). Then, we can rewrite (43) as follows

$$\Delta\pi\left(c_r^+; c_r^-\right) = \Delta c_r q_r\left(c_r^+\right) - \Delta c_u\left[q_u\left(q_r\left(c_r^-\right)\right) - q_u\left(q_r\left(c_r^+\right)\right)\right], \tag{44}$$

where the second addend arises from the second-stage Bertrand competition (see (8)). Substituting (10) and (13) into (44) yields

$$\Delta\pi\left(c_r^+; c_r^-\right) = \left[\alpha - c_r^+ - z\left(\alpha - c_u^2\right)\right] + z\Delta c_u = \left(\alpha - c_r^+\right) - z\left(\alpha - 2c_u^2 + c_u^1\right). \tag{45}$$

If $z \geq 0$ then (45) is positive, since the term in square brackets is positive ($q_r > 0$ in (10)). If $z < 0$ we find again $\Delta\pi(c_r^+, c_r^-) > 0$ for $\alpha > c_r^+$ and $\alpha > 2c_u^2 - c_u^1$.[19] Hence, the monopolist has an incentive to inflate costs.

Following the same procedure, the monopoly's extra profit $\Delta\pi(c_r^-; c_r^+)$ from cost understatement, that is, when declaring $\widehat{c}_r = c_r^-$ instead of c_r^+, is

$$\Delta\pi\left(c_r^-; c_r^+\right) = \pi_r\left(c_r^-; c_r^+\right) + \pi_u\left(c_r^-; c_r^+\right) - \pi_r\left(c_r^+\right) - \pi_u\left(c_r^+\right). \tag{46}$$

Since from (5) $\pi_r(c_r^-; c_r^+) = \pi_r(c_r^-) - \Delta c_r q_r(c_r^-) = -\Delta c_r q_r(c_r^-)$, replacing (10) and (13) into (46) yields

$$\Delta\pi\left(c_r^-; c_r^+\right) = -\left[\alpha - c_r^- - z\left(\alpha - c_u^2\right)\right] - z\Delta c_u$$

$$= -\left(\alpha - c_r^-\right) + z\left(\alpha - 2c_u^2 + c_u^1\right).$$

As the term in square brackets is positive, for $z \geq 0$ we have $\Delta\pi(c_r^-; c_r^+) < 0$. Notice from the second equality that the same conclusion applies if $z < 0$, since $\Delta\pi(c_r^-; c_r^+)$ is the sum of two negative terms. This means that deflating costs is not profitable.

[19] If firm 2's costs were large enough relative to firm 1's, then the latter could impose the monopoly price, i.e. $p_u^m = \frac{1}{2}\left(\alpha + c_u^1 - \gamma q_r\right)$. However, this clashes with the assumption of competition, since firm 2 would be completely irrelevant. Hence, we rule out the case of monopoly pricing and assume $p_u^m > c_u^2$ irrespective of the differentiation between goods, which implies $\alpha > 2c_u^2 - c_u^1$.

Proof of Proposition 2 The Lagrangian function for the problem in (15) under the constraints in (16), (17), (26) and (27) takes the following form

$$L\left(q_r\left(c_r^-\right), \pi_r\left(c_r^-\right); q_r\left(c_r^+\right), \pi_r\left(c_r^+\right)\right) = \nu\left[\alpha q_r\left(c_r^-\right) - c_r^- q_r\left(c_r^-\right) - \frac{1}{2}\beta q_r^2\left(c_r^-\right)\right.$$

$$\left. + \frac{1}{2\beta}\left(\alpha - c_u^2 - \gamma q_r\left(c_r^-\right)\right)^2 - \pi_r\left(c_r^-\right)\right] + (1-\nu)\left[\alpha q_r\left(c_r^+\right) - c_r^+ q_r\left(c_r^+\right)\right.$$

$$\left. - \frac{1}{2}\beta q_r^2\left(c_r^+\right) + \frac{1}{2\beta}\left(\alpha - c_u^2 - \gamma q_r\left(c_r^+\right)\right)^2 - \pi_r\left(c_r^+\right)\right] + \lambda\pi_r\left(c_r^-\right) + \mu\pi_r\left(c_r^+\right)$$

$$+\varepsilon\left[\pi_r\left(c_r^-\right) - \pi_r\left(c_r^+\right) - \Delta c_r q_r\left(c_r^+\right) - z\Delta c_u\left(q_r\left(c_r^-\right) - q_r\left(c_r^+\right)\right)\right]$$

$$+\iota\left[\pi_r\left(c_r^+\right) - \pi_r\left(c_r^-\right) + \Delta c_r q_r\left(c_r^-\right) + z\Delta c_u\left(q_r\left(c_r^-\right) - q_r\left(c_r^+\right)\right)\right], \quad (47)$$

where λ, μ, ε and ι are the (non-negative) Kuhn-Tucker multipliers associated respectively with (16), (17), (26) and (27). Optimizing (47) with respect to $q_r\left(c_r^-\right)$, $q_r\left(c_r^+\right)$, $\pi_r\left(c_r^-\right)$ and $\pi_r\left(c_r^+\right)$ yields after some manipulations the following conditions

$$q_r\left(c_r^-\right) = \frac{\alpha - c_r^- - z\left(\alpha - c_u^2\right)}{\beta\left(1-z^2\right)} - \frac{z\Delta c_u\left(\varepsilon - \iota\right)}{\nu\beta\left(1-z^2\right)} + \frac{\iota\Delta c_r}{\nu\beta\left(1-z^2\right)} \quad (48)$$

$$q_r\left(c_r^+\right) = \frac{\alpha - c_r^+ - z\left(\alpha - c_u^2\right)}{\beta\left(1-z^2\right)} + \frac{z\Delta c_u\left(\varepsilon - \iota\right)}{(1-\nu)\beta\left(1-z^2\right)} - \frac{\varepsilon\Delta c_r}{(1-\nu)\beta\left(1-z^2\right)} \quad (49)$$

$$\lambda + \varepsilon = \iota + \nu > 0 \quad (50)$$

$$\mu + \iota = \varepsilon + (1-\nu) > 0, \quad (51)$$

where the last two inequalities ($\nu \in (0,1)$) imply that either (16) or (26) (or both) and either (17) or (27) (or both) must be binding at the optimum (by complementary slackness conditions). Notice that under substitutability one participation constraint is binding in equilibrium. If no participation constraint were binding, the regulator could increase social welfare by reducing $\pi_r(c_r^-)$ and $\pi_r(c_r^+)$ by the same amount until one of them saturates, a contradiction.[20] If both of them were binding, then (26) would be violated.[21]

This discussion implies that we can restrict our attention on the following four cases

(I) $\lambda > 0$; $\mu = 0$; $\varepsilon > 0$; $\iota > 0$
(II) $\lambda > 0$; $\mu = 0$; $\varepsilon = 0$; $\iota > 0$
(III) $\lambda = 0$; $\mu > 0$; $\varepsilon > 0$; $\iota = 0$
(IV) $\lambda = 0$; $\mu > 0$; $\varepsilon > 0$; $\iota > 0$.

We can immediately see that case I is impossible. In fact, $\varepsilon, \iota > 0$ yields from (26) and (27) $q_r(c_r^-) = q_r(c_r^+)$, which in turn implies $\pi_r(c_r^-) > 0$ from (26), a contradiction with $\lambda > 0$. Case II cannot be a solution either, since the binding (16) combined with (27) yields $\pi_r(c_r^+) < 0$, which violates (17). Hence, the regulator finds it optimal to implement the separating mechanism which emerges in case III. This is shown in the first period of Proposition 2 in the paper. To find this result, optimize (15) subject to the binding constraints (17) and (26). The first-order conditions are

$$\alpha - c_r^- - \beta q_r\left(c_r^-\right) - z\left(\alpha - c_u^2 - \gamma q_r\left(c_r^-\right)\right) - z\Delta c_u = 0$$

$$(1-\nu)\left[\alpha - c_r^+ - \beta q_r\left(c_r^+\right) - z\left(\alpha - c_u^2 - \gamma q_r\left(c_r^+\right)\right)\right] - \nu\left(\Delta c_r - z\Delta c_u\right) = 0,$$

which immediately yield (29). Standard substitutions allow to derive (30) through (32).

Notice that this solution is feasible as long as the monotonicity condition in (28) is satisfied, i.e. if $z \in [0, z_s^*]$, where $z_s^* \equiv \frac{\Delta c_r}{\Delta c_u}$. Otherwise, case IV applies, which yields the bunching outcome illustrated in the second period of Proposition 2.[22] To derive this solution, optimize (15) subject to the binding constraints (17) and (28). The first-order condition is

$$\nu\left[\alpha - c_r^- - \beta q_r - z\left(\alpha - c_u^2 - \gamma q_r\right) - \Delta c_r\right]$$

[20] Summing (50) and (51) yields $\lambda + \mu = 1$, which implies that at least one of the corresponding constraints must be stringent.

[21] We will see that this may be no longer the case under complementarity.

[22] The solution in case III (when feasible) clearly dominates that in case IV, since in the latter the additional constraint in (27) is relevant.

$$+(1-\nu)\left[\alpha-c_r^+-\beta q_r-z\left(\alpha-c_u^2-\gamma q_r\right)\right]=0,$$

which yields (33). Standard substitutions imply (34) through (36).

Proof of Proposition 3 Under complementarity, we may find both (16) and (17) to be binding. This implies that there are three other possible cases to investigate:

(V) $\lambda > 0$; $\mu > 0$; $\varepsilon > 0$; $\iota > 0$

(VI) $\lambda > 0$; $\mu > 0$; $\varepsilon = 0$; $\iota > 0$

(VII) $\lambda > 0$; $\mu > 0$; $\varepsilon > 0$; $\iota = 0$.

It is immediate to see that case V (which implies all constraints binding) is impossible because (26) and (27) yield $q_r\left(c_r^-\right) = q_r\left(c_r^+\right)$, which in turn implies $\pi_r\left(c_r^-\right) > 0$ using (17) and (26), and this contradicts $\lambda > 0$. Case VI can also be ruled out, since the binding (16), (17) and (27) yield $q_r\left(c_r^+\right) = \frac{\Delta c_r + z\Delta c_u}{z\Delta c_u} q_r\left(c_r^-\right)$, which implies $q_r\left(c_r^+\right) > q_r\left(c_r^-\right)$, a violation of the monotonicity condition in (28). Let us consider now case VII. Notice that this solution is implementable since the binding (16), (17) and (26) imply $q_r\left(c_r^+\right) = -\frac{z\Delta c_u}{\Delta c_r z\Delta c_u} q_r\left(c_r^-\right)$, i.e. $q_r\left(c_r^+\right) < q_r\left(c_r^-\right)$, which satisfies (28) and then ensures that (27) is fulfilled.

Notice that the aforementioned case I is impossible even under complementarity. We consider now case II with $z < 0$. Using (51), (48) and (49) can be rewritten as

$$q_r\left(c_r^-\right) = \frac{\alpha - c_r^- - z\left(\alpha - c_2^u\right)}{\beta\left(1-z^2\right)} + \phi^{-1}\frac{\Delta c_r + z\Delta c_u}{\beta\left(1-z^2\right)} \tag{52}$$

$$q_r\left(c_r^+\right) = \frac{\alpha - c_r^+ - z\left(\alpha - c_2^u\right)}{\beta\left(1-z^2\right)} - \frac{z\Delta c_u}{\beta\left(1-z^2\right)}, \tag{53}$$

where $\phi^{-1} = \frac{1}{\phi}$. Substituting (52) and (53) into the binding (27) yields after some manipulations

$$\pi_r\left(c_r^+\right) = -\frac{\alpha - c_r^- - z\left(\alpha - c_2^u\right) + \phi^{-1}\left(\Delta c_r + z\Delta c_u\right)}{\beta\left(1-z^2\right)}\Delta c_r$$

$$-z\Delta c_u \frac{\left(1+\phi^{-1}\right)\left(\Delta c_r + z\Delta c_u\right)}{\beta\left(1-z^2\right)}. \tag{54}$$

We claim that (54) is negative, which violates the c_r^+-firm's participation constraint in (17) and then case II cannot be a solution. To see this, first notice from (52) and (53) that the monotonicity condition in (28) implies $\Delta c_r \geq -z\Delta c_u$. Then, sufficient condition for (54) to be negative is that the first ratio is greater than the second ratio. This is the case since $q_r\left(c_r^+\right) > 0$ in (53).

Hence, we are left with cases III, IV and VII. The separating equilibrium in case III is implementable as long $\pi_r^C\left(c_r^-\right) \geq 0$ in (31), i.e. $\Phi\left(z\right) \geq 0$ in (37).[23] As $\Phi\left(0\right) > 0$ ($q_r^C\left(c_r^+\right) > 0$) and $\Phi''\left(z\right) = -\frac{2(1+\phi)\Delta c_u^2}{\Delta c_r} < 0$, then for $z < 0$ there exists a unique value z_c^* such that $\Phi \geq 0$ if and only if $z \in [z_c^*, 0)$. For $z \in (-1, z_c^*)$, then $\pi_r^C\left(c_r^-\right) < 0$ and the separating solution in case III is no longer feasible, as emphasized in Proposition 3 in the paper.

We claim that if case III is not feasible the regulator finds it optimal to implement case VII. Replacing $\iota = 0$ in (48) and (49) yields (38) in Proposition 3. From (26) we find after some manipulations the value for $\varepsilon \in (0, \nu)$ (see 50)). The expressions (39) through (41) are derived from straightforward substitutions.

To show that case VII outperforms case IV, we first rewrite (15) as

$$E\left[W\right] = E\left[\left(\alpha - c_r\right)q_r\right] + \frac{1}{2}\beta E\left[q_u^2 - q_r^2\right] - E\left[\pi_r\right]$$

$$= \left(\alpha - c_r^+\right)E\left[q_r\right] + \nu\Delta c_r q_r\left(c_r^-\right)$$

$$+\frac{1}{2\beta}\left[\left(\alpha - c_u^2\right)^2 - 2\gamma\left(\alpha - c_u^2\right)E\left[q_r\right] - \left(\beta^2 - \gamma^2\right)E\left[q_r^2\right]\right] - E\left[\pi_r\right],$$

using (8). Hence, the expected difference in social welfare $\Delta E\left[W\right]$ between case VII and case IV is

$$\Delta E\left[W\right] = \left(\alpha - c_r^+\right)\Delta E\left[q_r\right] + \nu\Delta c_r \Delta q_r\left(c_r^-\right)$$

$$-\frac{1}{2\beta}\left[2\gamma\left(\alpha - c_u^2\right)\Delta E\left[q_r\right] + \left(\beta^2 - \gamma^2\right)\Delta E\left[q_r^2\right]\right] - \Delta E\left[\pi_r\right]. \quad (55)$$

[23] Notice that the monotonicity condition in (28) is always satisfied for $z < 0$.

From (33) and (38) we find after some manipulations

$$\Delta E\left[q_r\right]=\frac{v-\varepsilon}{\beta\left(1-z^2\right)}\Delta c_r \tag{56}$$

and

$$\Delta E\left[q_r^2\right]=\nu\left[\left(\frac{\alpha-c_r^- - z\left(\alpha-c_u^2\right)}{\beta\left(1-z^2\right)}-\frac{z\Delta c_u\varepsilon}{\nu\beta\left(1-z^2\right)}+\frac{\alpha-c_r^+ - z\left(\alpha-c_u^2\right)}{\beta\left(1-z^2\right)}\right)\right.$$

$$\left.\times\left(\frac{\alpha-c_r^- - z\left(\alpha-c_u^2\right)}{\beta\left(1-z^2\right)}-\frac{z\Delta c_u\varepsilon}{\nu\beta\left(1-z^2\right)}-\frac{\alpha-c_r^+ - z\left(\alpha-c_u^2\right)}{\beta\left(1-z^2\right)}\right)\right]+(1-\nu)$$

$$\times\left[\left(\frac{\alpha-c_r^+ - z\left(\alpha-c_u^2\right)}{\beta\left(1-z^2\right)}-\frac{\Delta c_r - z\Delta c_u}{(1-\nu)\,\beta\left(1-z^2\right)}\varepsilon+\frac{\alpha-c_r^+ - z\left(\alpha-c_u^2\right)}{\beta\left(1-z^2\right)}\right)\right.$$

$$\left.\times\left(\frac{\alpha-c_r^+ - z\left(\alpha-c_u^2\right)}{\beta\left(1-z^2\right)}-\frac{\Delta c_r - z\Delta c_u}{(1-\nu)\,\beta\left(1-z^2\right)}\varepsilon-\frac{\alpha-c_r^+ - z\left(\alpha-c_u^2\right)}{\beta\left(1-z^2\right)}\right)\right]. \tag{57}$$

Combining terms in (57) yields

$$\Delta E\left[q_r^2\right]=2\frac{\alpha-c_r^+ - z\left(\alpha-c_u^2\right)}{\beta^2\left(1-z^2\right)^2}(\nu-\varepsilon)\,\Delta c_r$$

$$+\frac{\left(\nu\Delta c_r - z\Delta c_u\varepsilon\right)^2}{\nu\beta^2\left(1-z^2\right)^2}+\frac{\left(z\Delta c_u-\Delta c_r\right)^2}{(1-\nu)\,\beta^2\left(1-z^2\right)^2}\varepsilon^2. \tag{58}$$

Substituting (56) and (58) into (55) we get

$$\Delta E\left[W\right]=\left(\alpha-c_r^+\right)\frac{v-\varepsilon}{\beta\left(1-z^2\right)}\Delta c_r+\nu\frac{\Delta c_r-\frac{\varepsilon}{\nu}z\Delta c_u}{\beta\left(1-z^2\right)}\Delta c_r$$

$$-\frac{1}{2\beta}\left\{2\frac{\gamma\left(\alpha-c_u^2\right)(v-\varepsilon)}{\beta\left(1-z^2\right)}\Delta c_r+\left(\beta^2-\gamma^2\right)\left[2\frac{\alpha-c_r^+ - z\left(\alpha-c_u^2\right)}{\beta^2\left(1-z^2\right)^2}(\nu-\varepsilon)\,\Delta c_r\right.\right.$$

$$+\frac{\left(\nu\Delta c_r - z\Delta c_u\varepsilon\right)^2}{\nu\beta^2\left(1-z^2\right)^2}+\frac{\left(z\Delta c_u-\Delta c_r\right)^2}{\left(1-\nu\right)\beta^2\left(1-z^2\right)^2}\varepsilon^2\Bigg]\Bigg\}+\nu\frac{\alpha-c_r^+-z\left(\alpha-c_u^2\right)}{\beta\left(1-z^2\right)}\Delta c_r. \tag{59}$$

Manipulating terms in (59) yields

$$\Delta E\left[W\right]=\nu\frac{\alpha-c_r^+-z\left(\alpha-c_u^2\right)}{\beta\left(1-z^2\right)}\Delta c_r+\frac{\nu\left(\Delta c_r\right)^2}{2\beta\left(1-z^2\right)}$$

$$-\frac{z^2\left(\Delta c_u\right)^2+\nu\left(\Delta c_r\right)^2-2\nu\Delta c_r z\Delta c_u}{2\nu\left(1-\nu\right)\beta\left(1-z^2\right)}\varepsilon^2. \tag{60}$$

If we replace $\varepsilon=\nu\left(1-\nu\right)\frac{\alpha-c_r^+-z\left(\alpha-c_2^u\right)+z\Delta c_u}{z^2\left(\Delta c_u\right)^2+\nu\left(\Delta c_r\right)^2-2\nu\Delta c_r z\Delta c_u}\Delta c_r$ into (60) we get after some computations

$$\Delta E\left[W\right]=\frac{\nu\Delta c_r}{\beta\left(1-z^2\right)}\frac{\alpha-c_r^+-z\left(\alpha-c_u^2\right)}{z^2\left(\Delta c_u\right)^2+\nu\left(\Delta c_r\right)^2-2\nu\Delta c_r z\Delta c_u}\Big[z^2\left(\Delta c_u\right)^2+\nu\left(\Delta c_r\right)^2$$

$$-2\nu\Delta c_r z\Delta c_u-\frac{1}{2}\left(1-\nu\right)\left(\alpha-c_r^+-z\left(\alpha-c_u^2\right)\right)\Delta c_r-\left(1-\nu\right)\Delta c_r z\Delta c_u\Big]$$

$$+\frac{\nu^2\Delta c_r^2}{2\beta\left(1-z^2\right)\left[z^2\left(\Delta c_u\right)^2+\nu\left(\Delta c_r\right)^2-2\nu\Delta c_r z\Delta c_u\right]}\left(z\Delta c_u-\Delta c_r\right)^2. \tag{61}$$

The expression in the first square brackets in (61) is positive if and only if

$$\frac{z^2\left(\Delta c_u\right)^2}{\Delta c_r}\left(1+\phi\right)>\frac{1}{2}\left(\alpha-c_r^+-z\left(\alpha-c_u^2\right)\right)+\left(1+2\phi\right)z\Delta c_u-\phi\Delta c_r. \tag{62}$$

Condition (62) is satisfied if $\Phi\left(z\right)<0$ in (37), that is, when case III cannot be implemented. This implies that the solution in case VII performes better than that in case IV.

Proof of Proposition 4 (substitutability) Substituting (1) and (3) into (2) the expected difference in consumer surplus $\Delta E[CS]$ between consolidation and separation is after some manipulations

$$\Delta E[CS] = \frac{1}{2}\beta\Delta E\left[q_r^2\right] + \frac{1}{2}\beta\Delta E\left[q_u^2\right] + \gamma\Delta E[q_r q_u]. \tag{63}$$

Using (8), (63) reduces to

$$\Delta E[CS] = \frac{\beta(1-z^2)}{2}\Delta E\left[q_r^2\right]. \tag{64}$$

Consider $z \in [0, z_s^*]$. Using (20) and (29), we find after some computations

$$\Delta E[CS] = \frac{\beta(1-z^2)}{2}\left[-\frac{\nu z\Delta c_u}{\beta(1-z^2)}\left(2\frac{\alpha - c_r^- - z(\alpha - c_u^2)}{\beta(1-z^2)} - \frac{z\Delta c_u}{\beta(1-z^2)}\right)\right.$$

$$\left. + (1-\nu)\frac{z\phi\Delta c_u}{\beta(1-z^2)}\left(2\frac{\alpha - c_r^+ - z(\alpha - c_u^2) - \phi\Delta c_r}{\beta(1-z^2)} + \frac{z\phi\Delta c_u}{\beta(1-z^2)}\right)\right]. \tag{65}$$

Rearranging terms in (65) yields

$$\Delta E[CS] = \frac{\nu z\Delta c_u}{\beta(1-z^2)}\left[-\Delta c_r + \frac{z\Delta c_u}{2} - \phi\Delta c_r + \frac{z\phi\Delta c_u}{2}\right]$$

$$= \frac{\nu(1+\phi)z\Delta c_u}{\beta(1-z^2)}\left[-\Delta c_r + \frac{z\Delta c_u}{2}\right]. \tag{66}$$

As $z \in [0, z_s^*]$, the term in square brackets in (66) is negative, which implies $\Delta E[CS] < 0$, that is, the regime of separation increases expected consumer surplus.

Using (5) we derive the expected subsidy difference $\Delta E[S_r] = \Delta E[\pi_r] - \Delta E[(p_r - c_r)q_r]$ between consolidation and separation. After some computations we find

$$\Delta E[S_r] = \frac{(1-\nu)z\phi\Delta c_u}{\beta(1-z^2)}\left[-\phi\Delta c_r + \alpha - c_r^+ - z\left(\alpha - c_u^2\right) - \phi\Delta c_r + z\phi\Delta c_u\right]$$

$$+\frac{\nu z\Delta c_u}{\beta(1-z^2)}\left[(1+2\phi)\Delta c_r - \phi z\Delta c_u - \left(\alpha - c_r^-\right) + z\left(\alpha - c_u^2\right)\right] = 0. \tag{67}$$

Expression (67) indicates that the two regimes give the monopoly the same expected subsidy. As from (4) $\Delta E[W] = \Delta E[CS] - \Delta E[S_r]$, then (66) and (67) immediately imply that if $z \in [0, z_s^*]$ then $\Delta E[W] < 0$, i.e. expected social welfare is higher under separation.

Consider now $z \in (z_s^*, 1)$. From (20) and (33) we find

$$\Delta E\left[q_r^2\right] = \nu \left[\left(\frac{\alpha - c_r^+ - z(\alpha - c_u^2)}{\beta(1-z^2)} \right)^2 - \left(\frac{\alpha - c_r^- - z(\alpha - c_u^2)}{\beta(1-z^2)} \right)^2 \right]$$

$$+ (1-\nu) \left[\left(\frac{\alpha - c_r^+ - z(\alpha - c_u^2)}{\beta(1-z^2)} \right)^2 - \left(\frac{\alpha - c_r^+ - z(\alpha - c_u^2)}{\beta(1-z^2)} - \frac{\phi \Delta c_r}{\beta(1-z^2)} \right)^2 \right],$$

which becomes after some manipulations

$$\Delta E\left[q_r^2\right] = -\phi \frac{(\Delta c_r)^2}{\beta^2 (1-z^2)^2}.$$

Using (64) we immediately get

$$\Delta E[CS] = -\phi \frac{(\Delta c_r)^2}{2\beta(1-z^2)} < 0, \tag{68}$$

which means that separation improves expected consumer surplus.

From (20), (21), (33) and (34) the expected subsidy difference $\Delta E[S_r] = \Delta E[\pi_r] - \Delta E[(p_r - c_r) q_r]$ is equal to

$$\Delta E[S_r] = \nu \left[\frac{\alpha - c_r^+ - z(\alpha - c_u^2)}{\beta(1-z^2)} \Delta c_r - \frac{\alpha - c_r^+ - z(\alpha - c_u^2) - \phi \Delta c_r}{\beta(1-z^2)} \Delta c_r \right]$$

$$-\nu \frac{\alpha - c_r^+ - z(\alpha - c_u^2)}{\beta(1-z^2)} \Delta c_r + \nu \frac{\alpha - c_r^+ - z(\alpha - c_u^2) - \phi \Delta c_r}{\beta(1-z^2)} \Delta c_r = 0, \tag{69}$$

which indicates that expected subsidy does not change between the two regimes. As from (4) $\Delta E[W] = \Delta E[CS] - \Delta E[S_r]$, then (68) and (69) immediately imply that $\Delta E[W] < 0$, that is, expected social welfare is higher under separation even for $z \in (z_s^*, 1)$.

Proof of Proposition 4 (complementarity) Notice from (66) and (67) that $\Delta E[W] > 0$ for $z < 0$. Then, as long as the solution characterized by (29) through (32) is feasible under complementarity, which occurs for $z \in [z_c^*, 0)$, the regime of consolidation gives higher expected social welfare.

Consider now $z \in (-1, z_c^*)$. Substituting (20) and (38) into (64) yields

$$\Delta E[CS] = \frac{\beta(1-z^2)}{2}\left[-\frac{z\Delta c_u \varepsilon}{\beta(1-z^2)}\left(2\frac{\alpha - c_r^- - z(\alpha - c_u^2)}{\beta(1-z^2)} - \frac{z\Delta c_u \varepsilon}{\nu\beta(1-z^2)}\right)\right.$$

$$+(1-\nu)\left(\frac{\phi\Delta c_r}{\beta(1-z^2)} + \frac{z\Delta c_u - \Delta c_r}{(1-\nu)\beta(1-z^2)}\varepsilon\right)$$

$$\left.\times\left(2\frac{\alpha - c_r^+ - z(\alpha - c_u^2)}{\beta(1-z^2)} - \frac{\phi\Delta c_r}{\beta(1-z^2)} + \frac{z\Delta c_u - \Delta c_r}{(1-\nu)\beta(1-z^2)}\varepsilon\right)\right]. \quad (70)$$

Rearranging terms in (70), we get

$$\Delta E[CS] = \frac{\beta(1-z^2)}{2}\left[2\frac{\alpha - c_r^+ - z(\alpha - c_u^2)}{\beta^2(1-z^2)^2}(\nu - \varepsilon)\Delta c_r - 2\frac{\Delta c_r z\Delta c_u}{\beta^2(1-z^2)^2}\varepsilon\right.$$

$$\left.+\frac{z^2(\Delta c_u)^2\varepsilon^2}{\nu\beta^2(1-z^2)^2} - (1-\nu)\left(\frac{\phi^2(\Delta c_r)^2}{\beta^2(1-z^2)^2} - \frac{(z\Delta c_u - \Delta c_r)^2\varepsilon^2}{(1-\nu)^2\beta^2(1-z^2)^2}\right)\right]. \quad (71)$$

We rewrite (71) as follows

$$\Delta E[CS] = \nu\frac{\alpha - c_r^+ - z(\alpha - c_u^2)}{\beta(1-z^2)}\Delta c_r - \frac{\alpha - c_r^+ - z(\alpha - c_u^2) + z\Delta c_u}{\beta(1-z^2)}\Delta c_r\varepsilon$$

$$\left.+\frac{z^2(\Delta c_u)^2 + \nu(\Delta c_r)^2 - 2\nu z\Delta c_u\Delta c_r}{2\nu(1-\nu)\beta(1-z^2)}\varepsilon^2 - (1-\nu)\frac{\phi^2(\Delta c_r)^2}{2\beta(1-z^2)}\right]. \quad (72)$$

As $\varepsilon = \nu(1-\nu)\frac{\alpha - c_r^+ - z\left(\alpha - c_2^u\right) + z\Delta c_u}{z^2(\Delta c_u)^2 + \nu(\Delta c_r)^2 - 2\nu\Delta c_r z\Delta c_u}\Delta c_r$, combining terms in (72) finally yields

$$\Delta E[CS] = \nu\frac{\alpha - c_r^+ - z(\alpha - c_u^2) - \phi\Delta c_r}{2\beta(1-z^2)}\Delta c_r + \nu\frac{\alpha - c_r^+ - z(\alpha - c_u^2)}{2\beta(1-z^2)}\Delta c_r$$

$$-\frac{\alpha - c_r^+ - z\left(\alpha - c_u^2\right) + z\Delta c_u}{2\beta\left(1 - z^2\right)}\Delta c_r \varepsilon > 0, \tag{73}$$

since the first ratio is positive ($q_r^S\left(c_r^+\right) > 0$ in (20)) and the difference between the other two is also positive as $\varepsilon \in (0, \nu)$. Hence, the regime of consolidation improves expected consumer surplus.

Using (20), (21), (22), (38), (39) and (40), the expected subsidy difference $\Delta E\left[S_r\right] = \Delta E\left[\pi_r\right] - \Delta E\left[\left(p_r - c_r\right) q_r\right]$ is after some manipulations

$$\Delta E\left[S_r\right] = -z\Delta c_u \left(\frac{\alpha - c_r^- - z\left(\alpha - c_u^2\right)}{\beta\left(1 - z^2\right)} - \frac{z\Delta c_u \varepsilon}{\nu\beta\left(1 - z^2\right)}\right)\varepsilon$$

$$+\left(z\Delta c_u - \Delta c_r\right)\left(\frac{\alpha - c_r^+ - z\left(\alpha - c_u^2\right)}{\beta\left(1 - z^2\right)} + \frac{z\Delta c_u - \Delta c_r}{(1 - \nu)\beta\left(1 - z^2\right)}\right)\varepsilon. \tag{74}$$

Collecting terms in (74) yields

$$\Delta E\left[S_r\right] = -\frac{\Delta c_r \varepsilon}{\beta\left(1 - z^2\right)}\left[\alpha - c_r^+ - z\left(\alpha - c_u^2\right) + z\Delta c_u\right]$$

$$+\frac{z^2\left(\Delta c_u\right)^2 + \nu\left(\Delta c_r\right)^2 - 2\nu\Delta c_r z\Delta c_u}{\nu(1 - \nu)\beta\left(1 - z^2\right)}\varepsilon^2 = 0, \tag{75}$$

using the value for ε. The two regimes give the same expected subsidy. As from (4) $\Delta E[W] = \Delta E[CS] - \Delta E\left[S_r\right]$, then (73) and (75) immediately imply that $\Delta E[W] > 0$, that is, expected social welfare is higher under consolidation.

Quantity competition The type of competition in hand clearly does not affect the result of equivalence under complete information. Hence, we focus our attention on the asymmetric information case. Using (6) and (7) the second-stage quantity competition between firms 1 and 2 implies after standard computations[24]

[24]We assume that the cost difference is such that firm 2 finds it profitable to produce. Our results do not change qualitatively if firm 2 is so inefficient that it prefers to stay out of the market.

$$q_u^1(q_r) = \frac{1}{3\beta}\left(\alpha - c_u^1 + \Delta c_u - \gamma q_r\right) \tag{76}$$

$$q_u^2(q_r) = \frac{1}{3\beta}\left(\alpha - c_u^2 - \Delta c_u - \gamma q_r\right). \tag{77}$$

Without loss of generality under consolidation the monopoly owns firm 1, whose profit is after some manipulations

$$\pi_u^1 = \frac{1}{9\beta}\left(\alpha - c_u^1 + \Delta c_u - \gamma q_r\right)^2. \tag{78}$$

Using (78) the incentive compatibility constraints in (24) and (25) become respectively

$$\pi_r\left(c_r^-\right) \geq \pi_r\left(c_r^+\right) + \Delta c_r q_r\left(c_r^+\right) + \frac{z\rho}{9}\left(q_r\left(c_r^-\right) - q_r\left(c_r^+\right)\right) \tag{79}$$

$$\pi_r\left(c_r^+\right) \geq \pi_r\left(c_r^-\right) - \Delta c_r q_r\left(c_r^-\right) - \frac{z\rho}{9}\left(q_r\left(c_r^-\right) - q_r\left(c_r^+\right)\right), \tag{80}$$

where $\rho \equiv 2\left(\alpha - c_u^1 + \Delta c_u\right) - \gamma q_r\left(c_r^+\right) - \gamma q_r\left(c_r^-\right) > 0$ (as $q_u^1(q_r) > 0$ in (75)). Summing (79) and (80) yields the following monotonicity condition

$$q_r\left(c_r^-\right) \geq q_r\left(c_r^+\right), \tag{81}$$

which is equal to (28).

As the regulatory maximization program under the two regimes only differs in the incentive compatibility constraints, using (81) it can be easily seen that (79) is more severe (relaxed) than (18) under substitutability (complementarity), which implies that separation (consolidation) performs better.[25] This corroborates our main results in Proposition 4 of the paper.

[25] Notice that (80) (and possibly (81)) cannot be the only binding constraint(s) at the optimum. If this were the case, $\pi_r\left(c_r^+\right) \geq 0$ would imply from (80) $\pi_r\left(c_r^-\right) \geq \Delta c_r q_r\left(c_r^-\right) + \frac{z\rho}{9}\left(q_r\left(c_r^-\right) - q_r\left(c_r^+\right)\right)$. This condition is more severe than that arising when (79) is the only relevant constraint (possibly with (81)) as $\pi_r\left(c_r^+\right) = 0$, by making the regulator worse off. Hence, if (80) is relevant, then (79) also binds, which means that (81) is stringent.

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