Reward (only) the superprofessor[1]

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Achim I. Czerny[2] and Peter-J. Jost[3]

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[2]WHU – Otto Beisheim School of Management, achim.czerny@whu.edu.
[3]WHU – Otto Beisheim School of Management, pjjost@whu.edu.

Abstract

We address the question whether a best teacher award and a best researcher award should be rewarded in parallel or whether one big prize for the "superprofessor" who performs best in both categories yields the better incentives to invest in efforts. We show that from the principal's perspective the ranking of tournament designs crucially depends on a prize and a noise effect. The prize effect captures the observation that because multiple tournaments require several winner prizes, the agents' incentives to exert effort in each tournament diminish. This effect makes single tournaments more attractive. However, the noise effect which captures the ratio of the marginal winning probabilities under different tournament designs, makes a single tournament less attractive because the aggregation of random terms reduce the agents' incentives to invest in effect. We analyze this trade-off in different setups. In particular, we find a clear advantage of a single tournament relative to multiple tournaments when random errors are identically normally distributed.

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1 Introduction

Multidimensional tasks are ubiquitious in the business world (Holmstrom and Milgrom, 1991): managers and employees may sell and develop products and services, treat their customers, motivate their colleagues and subordinates, develop future strategies and so forth.[1] In the academic world professors who are supposed to hold insightful and entertaining lectures, publish in top academic journals and develop and manage study programs amongst others, are another prominent example.

Starting with the seminal paper by Holmstrom and Milgrom (1991), multi-tasking environments have been studied intensely in the economic literature. While these authors concentrate on piece-rate schemes where the agent's payment consists of a fixed element and an element that is linear in stochastic outputs, the present paper focusses on labor market tournaments where the winner and loser prizes are allocated to agents according to their ranking in stochastic outputs.[2] Franckx et al. (2004) concentrate on multiple tasks and a single tournament and analyze how to optimally aggregate stochastic outcomes to a single index, which is then used to determine the winning agent. Clark and Konrad (2007) also concentrate on a single tournament. In their scenario, the principal specifies a rule that determines in how many dimensions the winner must beat

[1] Clark and Konrad (2007) provide an excellent discussion of the practical relevance of multitasking environments.

[2] Since the seminal contributions of Lazear and Rosen (1981) and Holmstrom (1984), an extensive body of literature on labor market tournaments has been developed, see Lazear (1991), Prendergast (1999) and Kräkel (2004) for excellent overviews. Recent analyzes investigated single-task environments with an arbitrary number of heterogenous agents, where each agent can win one prize that depends on his ranking position, e.g., Clark and Riis (1998), Moldovanu and Sela (2001), Budde (2009), Akerlof and Holden (2012). Kräkel and Schöttner (2010) consider heterogenous workers in an environment with endogenous technologies. For dynamic tournament and contest structures see Jost and Kräkel (2005, 2008), Fu and Lu (2009) and Ederer (2010). Dixit (1987) analyzes strategic behavior in contests, while Glazer and Hassin (1988) derive optimal contest design. A comparison between tournaments and contests is provided by Green and Stokey (1983). A comparison between tournaments and piece-rates under limited liability is provided by Kräkel (2004).

the opponent in order to become the winning agent. Our contribution is to study the optimal number of tournaments in a multi-tasking environment and analyze whether a single tournament that aggregates individual task-specific achievements in a single index (called the single-tournament) or multiple parallel and task-specific tournaments (called the multiple-tournaments) may be better for the principal. Note that the number of winner prizes doesn't need to be equal to the number of winning agents in the case of the multiple-tournaments. Taking reference to the problem of professors: we try to answer the question whether a best teacher award and a best researcher award should be provided in parallel or whether one big prize for the professor who performs best in all categories yields the better incentives to invest in efforts. Since the same professor may win the best teacher and the best researcher award, the number of winner prizes can be different from the number of winning agents.

We consider a three-stage model, where the principal chooses between the single-tournament or the multiple-tournaments as well as the corresponding winner and loser prizes in the first stage, while agents choose efforts in the second stage, and winner and loser prizes are allocated in the third stage. The basic model involves two identical agents, two identical tasks and effort costs that depend on total efforts and are independent of the distribution of efforts between tasks.[3] This basic model leads to a unique symmetric equilibrium in the case of a single tournament; thus, the basic analysis also concentrates on a symmetric equilibrium in the case of multiple tournaments.

At the core of our analysis is the factor we call *prize multiplier*. The basic idea is that the single-tournament strictly dominates the multiple-tournaments if it can increase equilibrium efforts in all tasks for any given aggregate budget of the principal. Taking equilibrium efforts of the single-tournament as given, the prize multiplier then determines the exact change in budget that is needed for replicating these efforts with the multiple-tournaments. Although a change between tournament designs may increase efforts for some tasks and decrease efforts for other tasks, the prize multiplier

[3] Our model thus captures that a reduction of winner prizes in one task may increase the incentives to invest efforts in the other task, which is a useful property (e.g., Holmstrom and Milgrom, 1991).

leads to clear-cut rankings of tournament designs in surprisingly many environments.

Our main finding in the basic model is that the prize multiplier completely determines the ranking between a single-tournament and the two-tournaments. In particular, the single-tournament dominates the use of the two-tournaments if the prize multiplier exceeds one. This is because replication of the single-tournament efforts with two-tournaments is only possible with an increase in the aggregate expenditure.

The prize multiplier is determined by two effects. The first effect, which we call the *prize effect*, captures that multiple tournaments require several winner prizes, while only one winner prize is needed with the single-tournament. The second effect, which we call the *noise effect*, captures that random effects may be more likely to dominate differences in efforts with the single-tournament relative to the multiple-tournaments. This is because random errors may be more likely to cumulate to extreme values when the single-tournament is used. With the multiple-tournaments, the prize effect reduces the agents' incentives to exert effort relative to the single-tournament, while the noise effect may reduce the relative incentives to exert effort with the single-tournament. Surprisingly, the prize multiplier is the same whether production functions are task specific or interdependent and this is true for an arbitrary number of tasks. The intuition is simple and based on the fact that the prize multiplier is fully independent of production functions under these conditions.

The prize multiplier becomes inconclusive if the distribution of random errors are distinct between tasks. It is well known that the agents' incentives to invest in effort increase when randomness is reduced, and a clear advantage of the multiple-tournaments is that they can be designed to fully exploit incentive structures arising from task-specific distributions of random errors. This, may lead to a situation, where optimal efforts may be increased or reduced by the single-tournament relative to the multiple-tournaments for a given aggregate budget. In these situations, the prize multiplier becomes inconclusive, if its value is greater than one and task-specific random errors' distributions are distinct. Moreover, we show that both the multiple- and the single-tournament structures may be used as a handicap for the more capable agent.

We illustrate our findings with the help of normal distributions of random errors. We demonstrated that the prize multiplier is greater than one in the case of identically independently normally distributed random errors, which provides a relative advantage to the single-tournament from the principal's viewpoint. Furthermore, this relative advantage of the single-tournament is increasing in the number of tasks.

The remainder of the paper is structured as follows. The next section, Section 2, introduces the basic model. Section 3 derives the prize multiplier and discusses the prize and noise effects. Section 4 considers several extensions including different shapes of production functions, variations in the number of tasks and the distributions of random errors as well as differences in the agents' capabilities. Conclusions are provided in the final Section 5.

2 The Basic Model

In the basic model, we consider two identical risk-neutral agents $i = 1, 2$, who perform two identical tasks $t = 1, 2$ each on behalf of a principal. Let e_{it} denote agent i's effort related to task t. Each effort e_{it} together with a random term ε_{it} determines agent i's performance on task t, $q(e_{it}) + \varepsilon_{it}$. To ensure that equilibrium efforts are positive for each task, we assume that productivity $q(\cdot)$ is strictly concave in efforts e_{it} in the sense that marginal productivity is infinite evaluated at zero (i.e., $q'(0) \to \infty$) and decreasing in efforts (i.e., $q'' < 0$). Moreover, we assume in the basic model that the random terms ε_{it} are identically and independently distributed for agents and tasks with density denoted as f with $f \equiv f(\varepsilon_{it})$, where f is symmetric around the zero mean with standard deviation σ and $f'(0) = 0$.[4]

Effort is costly for an agent and total effort costs for agent i are independent of how he distributes efforts between tasks 1 and 2 and strictly convex: Let e_i denote the aggregate individual efforts, i.e. $e_i \equiv e_{i1} + e_{i2}$, and C denote effort costs with $C \equiv C(e)$

[4] As it is usual in the tournament literature, we assume that σ is sufficiently large such that a pure strategy equilibrium in effort choices exists, see Lazear and Rosen (1981, Footnote 2).

then $C', C'' > 0$.

The principal can incentivize agents to choose effort only according to the ranking of their performances. She can choose between two rank-order tournament structures: (i) a single tournament for both tasks (the single-tournament) or (ii) two simultaneous tournaments, one for each task (the two-tournaments). In case, the principal organizes the single-tournament, she determines a winner prize W_1 and a loser prize W_2, while she decides upon winner and loser prizes W_{1t} and W_{2t} for each tournament $t = 1, 2$ in case of the two-tournaments. We assume that agents are limited liable which implies that $W_1 \geq W_2 \geq 0$ and $W_{1t} \geq W_{2t} \geq 0$ for $t = 1, 2$, respectively.

Altogether, the principal's expected payoff with the single-tournament or the two-tournaments can be expressed as

$$\Pi^s \equiv \sum_{i,t=1,2} q\left(e_{it}\right) - \left(W_1 + W_2\right) \tag{1}$$

and

$$\Pi^m \equiv \sum_{i,t=1,2} q\left(e_{it}\right) - \sum_{t=1,2} \left(W_{1t} + W_{2t}\right), \tag{2}$$

respectively (superscript s indicates the single-tournament, while superscript m (for multiple) indicates the two-tournaments). Agent i's corresponding expected utilities can be expressed as

$$Eu_i^s \equiv P_i W_1 + \left(1 - P_i\right) W_2 - C, \tag{3}$$

and

$$Eu_i^m \equiv \sum_{t=1,2} \left(P_{it} W_{1t} + \left(1 - P_{it}\right) W_{2t}\right) - C, \tag{4}$$

respectively, where $P_i \equiv P_i(e_{11}, e_{12}; e_{21}, e_{22})$ is the probability that agent i wins the single-tournament with

$$P_i \equiv \Pr\left(\varepsilon_{j1} + \varepsilon_{j2} - \varepsilon_{i1} - \varepsilon_{i2} < q\left(e_{i1}\right) + q\left(e_{i2}\right) - q\left(e_{j1}\right) - q\left(e_{j2}\right)\right) \tag{5}$$

and $P_{it} \equiv P_{it}(e_{1t}; e_{2t})$ is the probability that he wins the tournament related to task $t = 1, 2$ in case of two-tournaments with

$$P_{it} \equiv \Pr\left(\varepsilon_{jt} - \varepsilon_{it} < q\left(e_{it}\right) - q\left(e_{jt}\right)\right), \tag{6}$$

where $j \neq i$.[5]

Using our assumptions on the distributions of random errors and the productivity $q(\cdot)$, these probabilities satisfy the conditions:[6]

(i) $\partial P_i / \partial e_{it} > 0$, while $\partial P_i / \partial e_{jt} < 0$

(ii) $\partial P_{it} / \partial e_{it} > 0$, while $\partial P_{it} / \partial e_{jt} < 0$, and $\partial P_{it} / \partial e_{jt'} = 0$ for $t \neq t'$.

Properties (i) and (ii) imply that the agents' probabilities to win are increasing in their own efforts and decreasing in their rival agent's efforts for the single-tournament and the two-tournaments, respectively, which is intuitive. In addition, probabilities depend only on the efforts associated with the corresponding task in the case of the two-tournaments.

The relationship between the principal and the two agents follows a three-stage game: In stage one, the principal chooses the tournament structure by deciding between the single-tournament and the two-tournaments and determines the corresponding winner and loser prizes. In stage two, the agents decide whether to work on behalf of the principal. If they both accept competition, they simultaneously choose their efforts for each task. Together with the random errors, these efforts determine the agents' performances in stage three, and winner and loser prizes are allocated according to their ranking in the corresponding tournament.

3 Analysis

Suppose that for any given aggregate expenditure for winner and loser prizes, one tournament structure always incentivizes higher efforts in each task and for every agent. Our analysis uses the fact that one tournament structure is clearly dominant from the principal's viewpoint in this situation. In order to identify the relative benefits

[5] Here and in the remainder, if i and j appear in the same expression, it is to be understood that $j \neq i$.

[6] See Appendix A for the derivations of probabilities to win.

of tournament structures from the principal's perspective, we therefore compare the equilibrium efforts achieved under the single-tournament to the ones under the two-tournaments for given aggregate expenditures, which we denote as $w > 0$. We solve the game by backward induction as follows: In a first step, we consider the agents' effort choices for given tournament structures, while the principal's problem of determining the optimal tournament design, taking the first step and the agents' equilibrium effort choices as given is considered in a second step.

3.1 Equilibrium efforts

First, consider the single-tournament. Since only the spread between the winner and loser prize incentivizes effort and agents are risk-neutral, the principal will choose the loser prize as low as possible and the winner prize as high as possible. With the single tournament, the spread between the winner and the loser prize is maximized when the loser prize is equal to zero and the winner prize is equal to w (i.e., $W_1 = w$ and $W_2 = 0$). For the single-tournament, expected utilities can then be rewritten as

$$Eu_i^s \equiv P_i w - C. \tag{7}$$

Efforts are determined by the first-order conditions $\partial Eu_i^s / \partial e_{i1} = 0$ and $\partial Eu_i^s / \partial e_{i2} = 0$, which can be written as

$$P_i' q'(e_{i1}^s) w - C' = 0 \text{ and } P_i' q'(e_{i2}^s) w - C' = 0, \tag{8}$$

respectively (superscript s also indicates the equilibrium solution implied by the single-tournament).[7] The first terms on the right-hand sides (RHSs) show the increase in the agents' expected payoffs that is associated with a marginal increase in efforts in task t, which is a multiplication of marginal probabilities and marginal productivities. The second terms show the negative marginal effort costs. The first-order conditions

[7] To ensure the existence of a solution, we assume, here and below, that the agents' reservation utilities are sufficiently small and that the Hessian of expected utilities are negative definite.

(8) imply that marginal probabilities are evaluated at zero in equilibrium (proofs of lemmas and propositions are delegated to Appendix B):

Lemma 1 *There is a unique symmetric equilibrium with* $e_{it}^{s} = e_{jt}^{s} = e_{i}^{s}/2$ *when the single-tournament is used.*

Second, consider the two-tournaments and impose symmetry in the sense that the agents' efforts are the same for each task in equilibrium (i.e., $e_{it}^{m} = e_{jt}^{m}$ for $t = 1, 2$). Again, to maximize the spread between the winner and loser prizes, loser prizes are set to zero (i.e., $W_{2t} = 0$). Furthermore, let $\beta \in [0, 1]$ be the share that determines the winner prize for the first tournament, which leads to $W_{11} = \beta w$. The corresponding winner prize for the second tournament W_{12} can then be written as $(1 - \beta)\, w$. For the winner prizes, this yields:

Lemma 2 *With two-tournaments and for a given aggregate winner prize* w*, the principal's expected payoff is maximized when the aggregate winner prize is evenly split between tasks* 1 *and* 2*.*

Since tasks are identical, this is an intuitive result.

According to Lemma 2, the winner prizes under the two-tournament structure are determined by an even split of the total winner prize (i.e., $\beta = 1/2$). With two-tournaments, the agents' expected utilities can then be rewritten as

$$Eu_{i}^{m} \equiv \frac{1}{2} \sum\nolimits_{t} P_{it} w - C. \tag{9}$$

Efforts are implicitly determined by the first-order conditions $\partial Eu_{i}^{m}/\partial e_{i1} = 0$ and $\partial Eu_{i}^{m}/\partial e_{i2} = 0$. With symmetry, these can be written as

$$P_{i1}' q'(e_{1}^{m}/2)\frac{w}{2} - C' = 0 \text{ and } P_{i2}' q'(e_{i}^{m}/2)\frac{w}{2} - C' = 0, \tag{10}$$

for $i = 1, 2$. Totally differentiating these first-order conditions and using Cramer's rule (for two-tournaments) yields the intuitive result that:

Lemma 3 *The equilibrium efforts for each task are increasing in the aggregate winner prize w, which is true for the single-tournament and the two-tournaments.*

Thus, equilibrium efforts are the same for agents 1 and 2 and tasks 1 and 2, which is true for both the single-tournament and the two-tournaments, while the aggregate efforts may differ when the single-tournament or two-tournaments are used. An increase in the equilibrium efforts thus requires an increase in the aggregate winner prize when the tournament structure is given. The principal's optimal choice of the tournament structure can therefore be derived by a simple comparison of the aggregate winner prizes that is required to implement a given level of equilibrium efforts.

3.2 The prize multiplier

To identify the trade-off between the tournament structures, we use a *prize multiplier* denoted as γ, which exactly determines how the total winner prize must be changed in order to ensure that the equilibrium efforts achieved by the single-tournament can be replicated by the use of the two-tournaments. If, for example, the prize multiplier is greater than one, an increase in the total winner prize is always needed to ensure that two-tournaments yield the same efforts as the single-tournament. Hence, the principal prefers the single-tournament. Specifically, the equilibrium efforts for a given aggregate expenditure w is given by e_{it}^s when the single-tournament is used. Then, to ensure that efforts are the same with the single-tournament and the two-tournaments, i.e. $e_{it}^s = e_{it}^m$, by definition of the prize multiplier, the aggregate expenditure with two-tournaments is γw. The first-order conditions (8) and (10) then imply

$$\gamma = 2\frac{P_i'(0)}{P_{it}'(0)}. \tag{11}$$

Two remarks are worth noting: First, the prize multiplier is fully independent of the aggregate expenditures w. Thus, the principal's optimal choice of the tournament structure is independent of w as well, which implies that the ranking of tournaments is independent of the optimal aggregate expenditures, which will typically be different for the single-tournament and the two-tournaments. Second, the definition in equation (11) shows that the prize multiplier is a multiplication of two terms, which represent two effects. The first effect, which we call the *prize effect*, is determined by the number of tasks when multiple tournaments are used, which is 2 in our basic scenario. The prize effect captures the fact that the winner prize is split into many winner prizes when multiple tournaments are used, and since a reduction in the winner prizes reduces equilibrium efforts by Lemma 3, this reduces the agent's incentives to exert effort. The second effect, which we call the *noise effect*, is determined by the ratio of marginal equilibrium probabilities when the single-tournament is used instead of the two-tournaments.

To illustrate the noise effect, assume that random shocks follow a standard normal distribution, see Figure 1. Note that marginal probabilities are associated with fat tails in the case of the single-tournament relative to the case of the two-tournaments. This is because the probability that the sum of random shocks are far from zero (in absolute values) increases in the number of random shocks. Thus, marginal probabilities evaluated at zero are smaller in the single-tournament case than in the two-tournaments case because the chance that an increase in effort is dominated by random effects are higher in the first than in the second case. Thus, the noise effect may work into the opposite direction relative to the prize effect and makes the use of the two-tournaments more attractive.

Specifically, for i.i.d. normally distributed random errors, the noise effect can be calculated as

$$\frac{\partial P_i(0)}{\partial e_{it}} / \frac{\partial P_{it}(0)}{\partial e_{it}} = \frac{1}{\sqrt{2}}. \tag{12}$$

Hence, the noise effect is greater than 1/2 and, therefore, the prize multiplier is greater than 1 (i.e., $\gamma = \sqrt{2}$). This implies that a single tournament clearly dominates the use of two tournaments from the principal's viewpoint: A replication of the equilibrium efforts in the single-tournament requires an increase in the total winner prize when the two-tournaments are used, independent of the aggregate expenditures w. This can be summarized as:

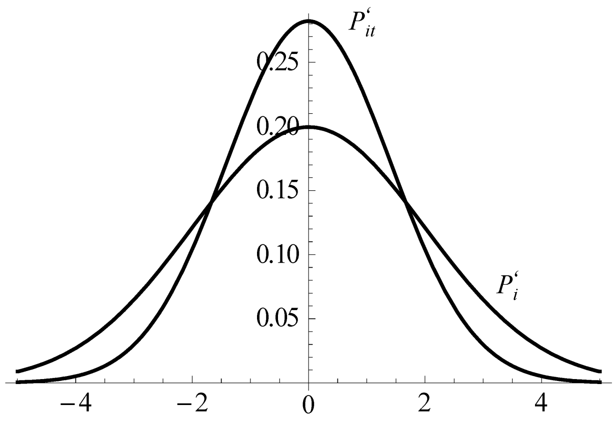

Figure 1: Marginal probabilities when random shocks follow a standard normal distribution.

Proposition 1 *(i) The principal's choice between the single-tournament and the two-tournaments is determined by the size of the prize multiplier, which is composed of a prize effect and a noise effect. If the prize multiplier is greater than 1, the principal is better off with the single-tournament, while she is better off with two-tournaments, otherwise. (ii) If random errors are i.i.d. normally distributed, the prize multiplier is strictly greater than 1 and the principal is better off with the single-tournament.*

4 Extensions

4.1 Production functions

We consider two modifications of our initial assumption, that the production function for both tasks are identical: differences in marginal productivities and interdependencies between tasks.

Let us first assume that the marginal productivity of task 1 is higher than the one of task 2, $q'_{i1}(e) > q'_{i2}(e)$, which is without loss of generality. Since agents are identical, this assumption then holds for both agents $i = 1, 2$. Our results derived in the previous section turn out to be robust with respect to those asymmetries. This is a direct consequence of the fact that the prize multiplier, γ, is independent of the shape of the principal's expected pay-offs when agents are identical.

Second, assume that efforts are task interrelated in the sense that a change in effort e_{it} not only increases output $q(e_{it})$, but also has an effect on agent i's other task's output $q(e_{it'})$ with $t' \neq t$. This might occur in situations in which, for example, social competence or learning effects are present. We can capture those interdependencies between tasks 1 and 2 by assuming that $q \equiv q(e_{i1}, e_{i2})$ with $\partial q/\partial e_{it'} \neq 0$ for $i = 1, 2$. Again, since the prize multiplier is independent of the shape of production functions when agents are identical, these interdependencies do not affect our previous result.

Altogether, this can be summarized as follows:

Proposition 2 *The prize multiplier is identical for identical and distinct as well as independent and interdependent production functions for tasks 1 and 2.*

4.2 Number of tasks

Assume, that two identical agents have to perform n tasks, $n \geq 2$, and that the principal can choose between the single-tournament for all n tasks or n independent tournaments, that is, one for each task. In the latter case, it is optimal for the principal to split the aggregate expenditures equally for each tournament to w/n, according to Lemma 2. Given that a unique and symmetric equilibrium exists and agents' behaviors are determined by the first-order conditions $\partial Eu_i/\partial e_{it} = 0$, the corresponding prize multiplier can be derived as

$$\gamma^n \equiv n \frac{P_i'(0)}{P_{it}'(0)}, \tag{13}$$

This prize multiplier is again independent of production functions. However, the benefits of the single-tournament relative to n independent tournaments increase in n, when the prize multiplier γ^n increases in n, that is, when $\gamma^{n+1} > \gamma^n$ for $n > 1$. Since we already showed that the prize multiplier is greater than 1 when random shocks ε_{it} are i.i.d. normally distributed and $n = 2$, effort structures implemented by the single-tournament cannot be replicated by the multiple-tournaments also for multiple task $n > 2$. In fact, the prize multiplier γ^n can be written as

$$\gamma^n = \sqrt{n} \tag{14}$$

and is strictly increasing in the number of tasks n. To summarize:

Proposition 3 *(i) The prize effect is equal to the number of tasks while the structure of the noise effect is unchanged by the number of tasks in the sense that it is always represented by the ratio of marginal probabilities, $P_i'(0)/P_{it}'(0)$. (ii) If random errors are i.i.d. normally distributed, the principal's benefit of the single-tournament relative to the multiple-tournaments is increasing in the number of tasks.*

4.3 Distributions

This section allows that random errors are distributed differently between tasks 1 and 2. Specifically, let σ_1 and σ_2 denote the standard deviations of random errors associated with tasks 1 and 2, respectively. Furthermore, let $\sigma_1 \leq \sigma_2$, which is without loss of generality and means that the variance associated with the outcome of task 1 is smaller than the variance of errors associated with task 2.[8] We say that errors are homoscedastic if $\sigma_1 = \sigma_2$, while errors are heteroscedastic if $\sigma_1 < \sigma_2$.

Consider first the effect of heteroscedasticity on equilibrium behavior in a single-tournament. Since the winning probability is determined by the sum of random errors, the agents' incentive to invest in efforts are the same for all tasks. In this sense, heteroscedasticity has no direct effect on equilibrium behavior when the single-tournament is used. This conclusion, however, is different in case the principal uses the two-tournaments. For the single-task tournament, it is well known that the agents' incentives to exert effort depend upon the variances of random errors. Hence, a clear

[8] Assume that σ_1 is sufficiently large such that a pure strategy equilibrium in effort choices exists, see Lazear and Rosen (1981, Footnote 2).

advantage of the two-tournaments is that winner prizes can be chosen in order to fully exploit the incentives associated with the distinct standard deviations of random errors. Specifically, there is no reason why the total prize should be evenly split between tasks 1 and 2 when errors are heteroscedastic. Let $\beta_t w$ be the winning prize in tournament t with $\beta_1 + \beta_2 = 1$. Then the agents' expected payoffs associated with the two-tournaments are

$$Eu_i^{mh} \equiv (P_{i1}\beta_1 + P_{i2}\beta_2)\, w - C \tag{15}$$

where superscript mh indicates the use of the two-tournaments with heteroscedastic random errors. Equilibrium efforts are determined by the first-order-conditions $\partial Eu_i^{mh}/\partial e_{it} = 0$, i.e.,

$$P_{it} q'(e_{it}^{mh})\beta_t w - C' = 0. \tag{16}$$

Agent i thus increases efforts as long as the increase in the marginal winning probability weighted by the winning prize exceeds marginal effort costs. Let γ^h denote the prize multiplier when random errors are heteroscedastic.

Lemma 4 *Assume that random errors are heteroscedastic, $\sigma_1 < \sigma_2$: (i) The prize multiplier that ensures replication of single-tournament efforts by the use of the two-tournaments can be written as*

$$\gamma^h \equiv 2\overline{\left(\frac{P_i'(0)}{P_{it}'(0)}\right)}, \tag{17}$$

where $\overline{(P_i'/P_{it}')} \equiv \left(\sum_t P_i'/P_{it}'\right)/2$. (ii) The prize effect dominates the noise effect when errors are normally distributed, which is independent of whether normal random errors are identically or differently distributed.

According to condition (17), the prize multiplier can be obtained by the multiplication of the prize effect with the *average* noise effect for tasks 1 and 2 when random errors are heteroscedastic. Furthermore, the result that the prize effect dominates the noise effect when errors are normally distributed is robust and holds true for both identically or differently distributed random errors.

However, the winner-prize structure that ensures replication of the single-tournament efforts by the two-tournaments may not maximize the principal's expected pay-off for a given aggregate winner prize in case of two-tournaments. Specifically, the principal may find it beneficial to increase the winner prize attached to task 1 relative to task 2 in order to fully exploit the agents' distinct incentives to invest efforts in tasks 1 and 2. This differentiated treatment of tasks therefore is a clear advantage of the two-tournaments relative to the single-tournament. It implies, however, that the prize multiplier is only of limited use for the comparison of tournament structures because it does not take into account the potential benefits of the two-tournaments:

Proposition 4 *If the prize multiplier γ^h is less than one, the two-tournaments dominate the use of the single-tournament. Otherwise, the value of the prize multiplier is inconclusive with respect to the benefits of the two-tournaments relative to the single-tournament.*

To verify this proposition, suppose that $\gamma^h \leq 1$, i.e., the sum of winner prizes for tasks 1 and 2 are exactly equal to or lower than the winner prize in the single-tournament case. This means, that two tournaments can exactly replicate the outcome of the single-tournament for a given or lower budget w. Hence, the principal will never be worse off with two tournaments relative to the single-tournament case but might even be better off because she can better exploit the incentive structures arising from heteroscedastic random errors. Hence, the two-tournaments weakly dominate the single-tournament. On the other hand, if $\gamma^h > 1$, replication of single-tournament efforts with two tournaments requires a higher budget. This, however, does not necessarily imply that the single-tournament strictly dominates the two-tournaments. In fact, the design of two tournaments may still be better for the principal because it allows a differentiated treatment of tasks, which is useful to better exploit incentives when random errors are heteroscedastic.

Appendix C provides an example, where random errors are normally distributed, and the principal's expected pay-off can be increased by the use of the two-tournaments relative to the single-tournament.

4.4 Agents

This part considers asymmetric agents, and we model these asymmetries in two different ways: We assume first that one agent is *advantaged* in the sense that for identical efforts his output is higher than the one of his rival by a constant amount.[9] Let agent 1 be the advantaged agent and $\phi > 0$ denote the positive amount that is added to his output for each task. Agent 1's output can then be written as $q(e_{1t}) + \phi$ for tasks 1 and 2. It is well known that this may lead to a symmetric effort structure with $e_{11}^* = e_{21}^*$ and $e_{12}^* = e_{22}^*$ in equilibrium. As a consequence, the single-tournament marginal probabilities are evaluated at 2ϕ, while the two-tournament marginal probabilities are evaluated at ϕ for tasks 1 and 2. The corresponding prize multiplier, denoted as γ^A, can, thus, be written as

$$\gamma^A \equiv 2\frac{P_i'(2\phi)}{P_{it}'(\phi)}. \tag{18}$$

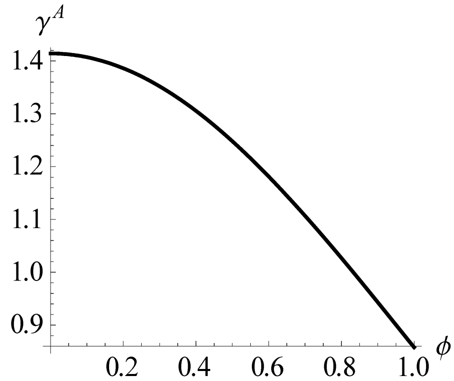

Figure 2: Prize multiplier when random errors are i.i.d. standard normally distributed and agent 1 has advantage ϕ in tasks 1 and 2.

[9] We thank Matthias Kräkel for pointing out to us that abilities may be modelled this way. There are several other ways to model differences in the agents' abilities, see, e.g., O'Keefe et al. (1984), Bull et al. (1987) and Kräkel and Gürtler (2010).

Figure 2 displays the prize multiplier γ^A for i.i.d. standard normally distributed random errors depending on ϕ. The figure shows that the prize multiplier may be decreasing when agents are heterogenous. It is well established in the literature on tournaments that differences in the agents' abilities reduce their incentives to invest in effort relative to a symmetric situation where agents' abilities are equal. To improve incentives, the principal may impose a handicap on the more able agent. In our context, the two-tournaments may serve the same purpose relative to the single-tournament. The intuition is that the doubling of agent 1's advantage under the single-tournament has a stronger negative effect on incentive structures relative to the two-tournaments. As a consequence, the prize multiplier is decreasing in agent 1's advantage measured by ϕ. The two-tournaments thus serve as a form of handicap for the advantaged agent 1.As a second alternative to model asymmetries assume that agent 1 is *more productive* in the sense that his marginal productivity is always higher than the marginal productivity of agent 2 for identical efforts, i.e. $q'_{1t}(e) > q'_{2t}(e)$ for both tasks $t = 1, 2$. In this scenario with distinct productivities, a symmetric effort structure does not exist in equilibrium, since the first-order conditions in (16) cannot be simultaneously satisfied for identical effort levels. This, however, implies that individual prize multipliers may be required to ensure that two tournaments replicate the effort structures of a single tournament.

Interestingly, the single-tournament may serve as a handicap for the more productive agent 1 in this scenario. To provide an intuition for this observation suppose for simplicity that agent 2 is absolutely incapable of performing task 2 and that the performance of agent 1's efforts can be measured with certainty for task 2, but not for task 1. If the principal arranges two tournaments, agent 1 would then exert a minimum effort to win the task-2 tournament. In the case of the single-tournament, however, his incentives to invest in task-1 effort may be positive because this effort improves his overall performance. Appendix C provides an example, which illustrates the use of the single-tournament as a handicap for the more productive agent.

To summarize:

Proposition 5 *If one agent is advantaged relative to his rival, the principal may be better off with the two-tournaments because this may be a handicap for the advantaged agent. If one agent is more productive relative to his rival, the principle may be better off with the single-tournament because this may be a handicap for the more productive agent.*

5 Conclusions

We have developed a general framework to analyze the question whether a single prize for the professor who performs best in research and teaching or a best teacher award in parallel to a best researcher award provides the better incentives. We showed that the advantage of a single prize is that the prize is relatively high compared to a situation where the winner prize is split up in two prizes (the prize effect). On the other hand, the use of a single prize reduces the incentives to invest in efforts because the probability that the aggregated random effects dominate differences in efforts increase when a single prize is rewarded (the noise effect). For the case of normally distributed errors it turned out that the prize effect clearly dominates the noise effect, which means that a single prize for the professor who performs best in all categories may provide the strongest incentives to invest in efforts. This is a robust result that does not depend on how efforts exactly translate into research and teaching output, and it remains true when the investment in research affects teaching output and vice-versa. Furthermore, we showed that the relative advantage of a single tournament increases in the number of tasks (teaching, research, program management etc.) when errors are normally distributed. Another advantage of the single tournament is that it may serve as a handicap and stimulate effort investments in all categories for a professor who may easily win in a single category when several prizes are awarded in parallel. An important advantage of several winner prizes though is that they can be designed to fully exploit incentive structures arising from task-specific distributions of random errors. An important avenue for future research would be to test these results empirically.

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Appendix

A Probabilities

A.1 General

Let $x_t \equiv \varepsilon_{1t} + \varepsilon_{2t}$ and $x \equiv \varepsilon_{11} + \varepsilon_{12} + \varepsilon_{21} + \varepsilon_{22}$, the corresponding convolutions, denoted as $g_t \equiv g_t(x_t)$ or $g \equiv g(x)$, respectively, can be calculated as

$$g_t(x_t) \equiv \int_{-\infty}^{\infty} f_t(x_t - \varepsilon_{2t}) f_t(\varepsilon_{2t}) \, d\varepsilon_{2t} \tag{19}$$

and

$$g(x) \equiv \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_1(x + \varepsilon_{12} - \varepsilon_{21} - \varepsilon_{22}) f_1(x) f_2(\varepsilon_{12}) f_2(\varepsilon_{22}) \, d\varepsilon_{22} d\varepsilon_{12} d\varepsilon_{21}. \tag{20}$$

The probabilities to win can then be written as

$$\begin{aligned} P_{it}(e_{1t}; e_{2t}) &\equiv \int_{-\infty}^{q(e_{1t})-q(e_{2t})} g_{it}(x_t) \, dx_t & (21a) \\ &\equiv F_{it}(q(e_{1t}) - q(e_{2t})) & (21b) \end{aligned}$$

and

$$\begin{aligned} P_i(e_{11}, e_{12}; e_{21}, e_{22}) &\equiv \int_{-\infty}^{q(e_{11})+q(e_{12})-q(e_{21})-q(e_{22})} g(x) \, dx & (22a) \\ &\equiv F_i(q(e_{11}) + q(e_{12}) - q(e_{21}) - q(e_{22})). & (22b) \end{aligned}$$

This yields derivatives

$$\begin{aligned} \frac{\partial P_i}{\partial e_{it}} &= \frac{\partial}{\partial e_{it}} F_i(q(e_{11}) + q(e_{12}) - q(e_{21}) - q(e_{22})) & (23a) \\ &= F_i' \frac{\partial q}{\partial e_{it}} = P_i' q'(e_t) & (23b) \end{aligned}$$

and

$$\begin{aligned}\frac{\partial P_{it}}{\partial e_{it}} &= \frac{\partial}{\partial e_{it}} F_{it}\left(q\left(e_{1t}\right)-q\left(e_{2t}\right)\right) &(24a)\\ &= F'_{it}\frac{\partial q}{\partial e_{it}} = P'_{it}q'\left(e_t\right). &(24b)\end{aligned}$$

Cross derivatives can be written as

$$\frac{\partial}{\partial e_{jt}}\frac{\partial P_{it}}{\partial e_{it}} = -q'\left(e_{1t}\right)q'\left(e_{2t}\right)F''_{it}\left(q\left(e_{1t}\right)-q\left(e_{2t}\right)\right) \tag{25}$$

and

$$\frac{\partial}{\partial e_{jt'}}\frac{\partial P_{i}}{\partial e_{it}} = -q'\left(e_{it}\right)q'\left(e_{jt'}\right)F''_{i}\left(q\left(e_{11}\right)+q\left(e_{12}\right)-q\left(e_{21}\right)-q\left(e_{22}\right)\right) \tag{26}$$

with $F''_{it}(0) = F''_{i}(0) = 0$. The latter holds true because the slope of densities at zero evaluated at zero, i.e. $f'_i(0) = 0$.

A.2 Normal

Assume that shocks are normally distributed. From the general result that the convolution of two normal densities with means μ_1 and μ_2 and variances σ_1^2 and σ_2^2 is again a normal density, with mean $\mu_1+\mu_2$ and variance $\sigma_1^2+\sigma_2^2$, the following properties follow in the case of single tournament or two tournaments:

For the single-tournament case

$$P_i\left(e_{11},e_{21};e_{12},e_{22}\right) = \int_{-\infty}^{q(e_{i1})+q(e_{i2})-q(e_{j1})-q(e_{j2})} \frac{1}{\sqrt{4\pi\left(\sigma_1^2+\sigma_2^2\right)}}\exp\left(-\frac{x^2}{4\left(\sigma_1^2+\sigma_2^2\right)}\right)dx, \tag{27}$$

which implies

$$\begin{aligned}\frac{\partial P_i}{\partial e_{it}} &= q'_{it}\frac{1}{\sqrt{4\pi\left(\sigma_1^2+\sigma_2^2\right)}}\exp\left(-\frac{\left(q(e_{i1})+q(e_{i2})-q(e_{j1})-q(e_{j2})\right)^2}{4\left(\sigma_1^2+\sigma_2^2\right)}\right) > 0, &(28)\\ \frac{\partial P_i}{\partial e_{jt}} &= -q'_{jt}\frac{1}{\sqrt{4\pi\left(\sigma_1^2+\sigma_2^2\right)}}\exp\left(-\frac{\left(q(e_{i1})+q(e_{i2})-q(e_{j1})-q(e_{j2})\right)^2}{4\left(\sigma_1^2+\sigma_2^2\right)}\right) < 0 &(29)\end{aligned}$$

and

$$\begin{aligned}\frac{\partial^2 P_i}{\partial e_{it}^2} &= \frac{1}{\sqrt{4\pi\left(\sigma_1^2+\sigma_2^2\right)}}\exp\left(-\frac{\left(q(e_{i1})+q(e_{i2})-q(e_{j1})-q(e_{j2})\right)^2}{4\left(\sigma_1^2+\sigma_2^2\right)}\right) \\ &\quad \left(q_{it}^{''}-(q_{it}^{'})^2\frac{\left(q(e_{i1})+q(e_{i2})-q(e_{j1})-q(e_{j2})\right)}{4\left(\sigma_1^2+\sigma_2^2\right)}\right) \\ &< 0 \end{aligned} \tag{30}$$

if $(\sigma_1^2+\sigma_2^2)$ is sufficiently large. Note that

$$\begin{aligned}\frac{\partial^2 P_i}{\partial e_{it}\partial e_{jt}} &= \frac{1}{\sqrt{4\pi\left(\sigma_1^2+\sigma_2^2\right)}} \\ &\quad \exp\left(-\frac{\left(q(e_{i1})+q(e_{i2})-q(e_{j1})-q(e_{j2})\right)^2}{4\left(\sigma_1^2+\sigma_2^2\right)}\right) \\ &\quad \frac{\left(q(e_{i1})+q(e_{i2})-q(e_{j1})-q(e_{j2})\right)}{4\left(\sigma_1^2+\sigma_2^2\right)}q'(e_{jt})q'(e_{it}) \end{aligned} \tag{31}$$

and $\partial^2 P_i/\partial e_{it}\partial e_{jt} > \partial^2 P_i/\partial e_{it}^2$ whenever

$$q^{''}(e_{it})-\frac{\left(q(e_{i1})+q(e_{i2})-q(e_{j1})-q(e_{j2})\right)}{4\left(\sigma_1^2+\sigma_2^2\right)}q'(e_{it})\left(q'(e_{jt})-q'(e_{it})\right)<0 \tag{32}$$

which is satisfied for $(\sigma_1^2+\sigma_2^2)$ sufficiently large.

For the two-tournament case

$$P_{it}\left(e_{it};e_{jt}\right)=\int_{-\infty}^{q(e_{it})-q(e_{jt})}\frac{1}{\sqrt{4\pi\sigma_t^2}}\exp\left(-\frac{x^2}{4\sigma_t^2}\right)dx, \tag{33}$$

with $\sigma_1^2 \leq \sigma_2^2$, which implies

$$\frac{\partial P_{it}}{\partial e_{it}} = q'(e_{it})\frac{1}{\sqrt{4\pi\sigma_t^2}}\exp\left(-\frac{\left(q(e_{it})-q(e_{jt})\right)^2}{4\sigma_t^2}\right)>0, \tag{34}$$

$$\frac{\partial P_{ij}}{\partial e_{jt}} = -q'(e_{jt})\frac{1}{\sqrt{4\pi\sigma_t^2}}\exp\left(-\frac{\left(q(e_{it})-q(e_{jt})\right)^2}{4\sigma_t^2}\right)<0 \tag{35}$$

and

$$\frac{\partial^2 P_i}{\partial e_{it}^2}=\frac{1}{\sqrt{4\pi\sigma_t^2}}\exp\left(-\frac{\left(q(e_{it})-q(e_{jt})\right)^2}{4\sigma_t^2}\right)\left(q^{''}(e_{it})-\left(q^{'}(e_{it})\right)^2\frac{q(e_{it})-q(e_{jt})}{4\sigma_t^2}\right)<0 \tag{36}$$

if the variance σ_t^2 is sufficiently large. Moreover, note that $\partial P_{i1}/\partial e_{i1} = q_{i1}'/\sqrt{4\pi\sigma_1^2} > \partial P_{i2}/\partial e_{i2} = q_{i2}'/\sqrt{4\pi\sigma_2^2}$.

B Proofs

Lemma 1 In the following we show that there is a unique symmetric equilibrium in the single-tournament case. The first-order conditions (8) imply

$$P_i'q'(e_{i1}^s)w = C'(e_i^s) \text{ and } P_i'q'(e_{i2}^s)w = C'(e_i^s). \quad (37)$$

Deducting the second from the first, simplifying and rearranging yields:

$$q'(e_{i1}^s) = q'(e_{i2}^s), \quad (38)$$

which directly implies $e_{i1}^s = e_{i2}^s$ for $i = 1, 2$. Thus efforts are the same for both tasks independent of the rival's efforts. To shows that effort levels are also the same, we substitute indivdual efforts by $e_i^s/2$ in order to write the first-order conditions as

$$P_1'q'(e_1^s/2)w = C'(e_1^s) \text{ and } P_2'q'(e_2^s/2)w = C'(e_2^s). \quad (39)$$

Deducting the first from the second first-order condition yields:

$$\left(P_1'q'(e_1^s/2) - P_2'q'(e_2^s/2)\right) w = C'(e_1^s) - C'(e_2^s). \quad (40)$$

Without loss of generality, suppose $e_1^s > e_2^s$. Then $C'(e_1^s) - C'(e_2^s) > 0$ by the strict convexity of effort costs and $P_1'q'(e_1^s/2) - P_2'q'(e_2^s/2) < 0$ by the strict concavity of production functions, which holds under the standard assumption of a sufficiently large standard error σ (essentially, this implies that P_i'' is sufficiently small in absolute values). A contradiction.

Lemma 2 With two-tournaments, the agents' expected pay-offs can be written as

$$Eu_i^m \equiv P_{i1}\beta w + P_{i2}(1-\beta) w - C. \quad (41)$$

Efforts are implicitly determined by the first-order conditions $\partial Eu_i^m/\partial e_{i1} = 0$ and $\partial Eu_i^m/\partial e_{i2} = 0$, which can be written as

$$P_{i1}'q'\beta w - C' = 0 \text{ and } P_{i2}'q'(1-\beta) w - C' = 0, \quad (42)$$

respectively. Totally differentiating and using symmetry yields the system of equations

$$d\frac{\partial Eu_i^m}{\partial e_{i1}} = \left(\frac{\partial^2 Eu_i^m}{\partial e_{i1}^2} + \frac{\partial^2 Eu_i^m}{\partial e_{i1}\partial e_{j1}}\right) de_{i1}^m + \frac{\partial^2 Eu_i^m}{\partial e_{i1}\partial e_{i2}} de_{i2}^m + \frac{\partial^2 Eu_i^m}{\partial e_{i1}\partial \beta} d\beta \tag{43a}$$

$$= \left(P_{i1}' q'' \beta w - C''\right) de_{i1} - C'' de_{i2} + P_{i1}' q' w d\beta = 0 \tag{43b}$$

and

$$d\frac{\partial Eu_i^m}{\partial e_{i2}} = -C'' de_{i1} + \left(P_{i2}' q'' (1-\beta) w - C''\right) de_{i2} - P_{i2}' q' w d\beta = 0. \tag{44}$$

It is useful to denote

$$\Omega_i \equiv \det \begin{pmatrix} \left(P_{i1}' q'' \beta w - C''\right) & -C'' \\ -C'' & \left(P_{i2}' q'' (1-\beta) w - C''\right) \end{pmatrix}, \tag{45}$$

where the right-hand side is clear-cut and positive in sign. Cramer's rule can be used to derive

$$\frac{de_{i1}^m}{d\beta} = \frac{1}{\Omega_i} \det \begin{pmatrix} -P_{i1}' q' w & -C'' \\ P_{i2}' q' w & \left(P_{i2}' q'' (1-\beta) w - C''\right) \end{pmatrix} \tag{46a}$$

$$= \frac{1}{\Omega_i} \left(P_{i2}' q' w C'' - P_{i1}' q' w \left(P_{i2}' q'' (1-\beta) w - C''\right)\right) \tag{46b}$$

where the right-hand side is positive in sign and

$$\frac{de_{i2}^m}{d\beta} = \frac{1}{\Omega_i} \det \begin{pmatrix} \left(P_{i1}' q'' \beta w - C''\right) & -P_{i1}' q' w \\ -C'' & P_{i2}' q' w \end{pmatrix} \tag{47a}$$

$$= -\frac{1}{\Omega_i} \left(P_{i1}' q' w C'' - P_{i2}' q' w \left(P_{i1}' q'' \beta w - C''\right)\right) \tag{47b}$$

where the right-hand side is negative in sign. This yields the intuitive result that an increase in the winner prize associated with task 1 increases task 1's equilibrium efforts, while task 2's equilibrium efforts decrease because this reduces task 2's winner prize, since the total winner prize is given.

In the second stage, the principal chooses the winner prizes that maximize the sum of expected outputs. These winner prizes are determined by first-order condition $\frac{\partial}{\partial \beta}\left(q(e_{11}^m) + q(e_{12}^m) + q(e_{21}^m) + q(e_{22}^m)\right) = 0$, which can be written as

$$2q' \left(\frac{\partial e_{11}^m}{\partial \beta} + \frac{\partial e_{22}^m}{\partial \beta}\right) = 0. \tag{48}$$

Since $q' > 0$, this means that $\partial e_{11}^m/\partial\beta = -\partial e_{22}^m/\partial\beta$ in optimum, which holds for $\beta = 1/2$ by the relationships in (46b) and (47b).

Lemma 3 Totally differentiating the first-order conditions in (8) and (10) and using symmetry yields

$$d\frac{\partial Eu^x}{\partial e_{i1}} = \left(\frac{\partial^2 Eu_i^x}{\partial e_{i1}^2} + \frac{\partial^2 Eu_i^x}{\partial e_{i1}\partial e_{j1}} + \frac{\partial^2 Eu_i^x}{\partial e_{i1}\partial e_{i2}} + \frac{\partial^2 Eu_i^x}{\partial e_{i1}\partial e_{j2}}\right) de_{i1}N + \frac{\partial^2 Eu_i^x}{\partial e_{i1}\partial w}dw \tag{49}$$

with

$$\frac{\partial^2 Eu_i^x}{\partial e_{i1}^2} + \frac{\partial^2 Eu_i^x}{\partial e_{i1}\partial e_{j1}} + \frac{\partial^2 Eu_i^x}{\partial e_{i1}\partial e_{i2}} + \frac{\partial^2 Eu_i^x}{\partial e_{i1}\partial e_{j2}} = \begin{cases} P_i'(0)q''(e_{i1}^s)w - 2C'' & \text{for} \quad x = s \\ P_{i1}'(0)q''(e_{i1}^m)\frac{w}{2} - 2C'' & \text{for} \quad x = m \end{cases} \tag{50}$$

and

$$\frac{\partial^2 Eu_i^x}{\partial e_{i1}\partial w} = \begin{cases} P_i'(0)q'(e_{i1}^s) & \text{for} \quad x = s \\ P_{i1}'(0)q'(e_{i1}^m)/2 & \text{for} \quad x = m. \end{cases} \tag{51}$$

Altogether, this shows that $de_{i1}^N/dw > 0$ for $x = s, m$.

Proposition 2 To show that interdependencies between tasks in the sense that $q \equiv q(ei1, ei2)$ with $\partial q/\partial e_{it} \neq 0$ for $t = 1, 2$, leave the prize multiplier unchanged relative to a situation where outputs depend on the task-specific efforts only, assume that equilibrium efforts are implicitly determined by the first-order conditions

$$P_i'(0)\frac{\partial\left(q(e_{i1}, e_{i2}) + q(e_{i1}, e_{i2})\right)}{\partial e_{i1}}w - C' = 0 \tag{52}$$

and

$$P_i'(0)\frac{\partial\left(q(e_{i1}, e_{i2}) + q(e_{i1}, e_{i2})\right)}{\partial e_{i2}}w - C' = 0 \tag{53}$$

with the single-tournament, while symmetry between tasks can be used to write the first-order conditions as

$$P_{i1}'(0)\frac{\partial\left(q(e_{i1}, e_{i2}) + q(e_{i1}, e_{i2})\right)}{\partial e_{i1}}\frac{w}{2} - C' = 0 \tag{54}$$

and

$$P_{i2}'(0)\frac{\partial\left(q(e_{i1}, e_{i2}) + q(e_{i1}, e_{i2})\right)}{\partial e_{i1}}\frac{w}{2} - C' = 0 \tag{55}$$

with the two-tournaments. The derivation of the prize multiplier implies that production functions cancel out and thus that the corresponding prize multiplier is independent of the existence of interdependencies between production functions.

Lemma 4 To establish part (i) In this scenario, the prize multiplier, denoted as γ^h, can be derived as follows. First, calculate the values of winner-prize shares, denoted as β_t^h, that ensure that behaviors are independent of the tournament structure, i.e. $e_{it}^{mh} = e_{it}^{sh}$ for $t = 1, 2$. To do this, use the first-order conditions in (8) and (16) and solve $\beta_t^h P_{it}' q'(e_{i1}^{mh}) \gamma^h w = P_i'(0) q'(e_{i1}^{sh}) w$ for β_t^h, which yields

$$\beta_t^h \equiv \frac{P_i'(0)}{\gamma^h P_{it}'(0)}. \tag{56}$$

Since shares, β_1^h and β_2^h, must add up to 1, the prize multiplier can finally be derived by solving $P_i' / \left(\gamma^h P_{i1}'\right) + P_i' / \left(\gamma^h P_{i2}'\right) = 1$ for γ^h, which yields

$$\gamma^h \equiv 2 \overline{\left(\frac{P_i'(0)}{P_{it}'(0)}\right)}, \tag{57}$$

where $\overline{(P_i'/P_{it}')} \equiv \left(\sum_t P_i'/P_{it}'\right)/2$.

To establish part (ii), For normally distributed random errors, it holds that

$$\gamma^h = \frac{\sum_t \sigma_t}{\sqrt{\sum_t \sigma_t^2}}, \tag{58}$$

where the RHS is greater than 1.

C Examples

C.1 Distributions

Consider production functions $q_1 = 2\sqrt{e_{i1}}$ and $q_2 = \sqrt{e_{i2}}$, i.e., the marginal productivity of task 1 is higher than the one of task 2 Furthermore, suppose that random errors are normally distributed with $\sigma_1 = 1/4$ and $\sigma_2 = 1/2$. Since random errors are normally distributed, it holds that the prize multiplier is strictly greater than one. Specifically, the

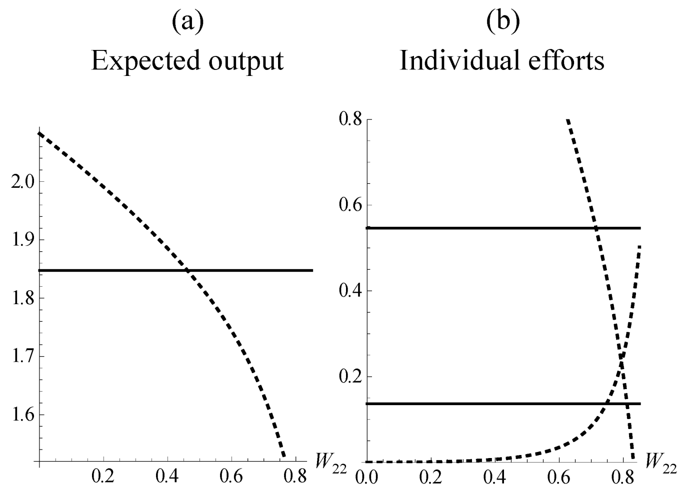

Figure 3: Diagram (a) the total expected output when a single tournament and a two-tournament structure are used (solid line and dashed line, respectively) depending on task 2's winner prize in the two tournament case (in this instance, $w = 1$, which means that $W_{11} = 1 - W_{22}$). Diagram (b) displays equilibrium efforts depending on task 2's winner prize under the two-tournament structure, W_{22}.

prize multiplier can be calculated as $\gamma^h = 1.78$. In order to replicate single-tournament efforts with two tournaments, the two-tournament budget would have to be increased by 78% relative to the single-tournament budget. Still, two tournaments are preferred by the principal because the incentives arising from the differences in the random errors' distributions can be better exploited by a two-tournament structure.

Figure 3 displays two diagrams. Diagram (a) displays the total expected output when a single tournament and a two-tournament structure are used (solid line and dashed line, respectively) depending on task 2's winner prize in the two tournament case (in this instance, $w = 1$, which means that $W_{11} = 1 - W_{22}$). Total expected output is approximately 1.85 when a single tournament is used, while the two-tournament expected output is higher for W_{22} sufficiently small (i.e., $W_{22} \lessapprox 0.42$).

Diagram (b) displays equilibrium efforts depending on task 2's winner prize under the two-tournament structure, W_{22}. Observe that the two-tournament total efforts exceed the single-tournament total efforts for $W_{22} \lessapprox 0.42$. Note this a necessary condition for the two-tournament structure to be preferred by the principal To see this, assume that the agents' individual total effort levels $e_{i1} + e_{i2}$ are given and equal. What is the optimal allocation of efforts between tasks 1 and 2 from the principal's perspective and how does this influence the choice of the tournament design? Letting $\overline{e}_i$ denote each agents' given total efforts, the answer to the first question can be derived by solving

$$\max_{e_{11},e_{12},e_{21},e_{22}} \sum_{i,t=1,2} q\left(e_{it}\right) \text{ s.t. } e_{11} + e_{12} = e_{21} + e_{22} = \overline{e}_i, \tag{59}$$

which leads to the optimum conditions $q'_{11} = q'_{12} = q'_{21} = q'_{22}$. These conditions are satisfied, if individual total efforts are evenly split between the tasks 1 and 2. The tournament structure that always induces this property, is the single tournament: This is because the agents' first-order conditions in (16) imply

$$q'\left(e^s_{it}\right) = \frac{C'_i}{wP'_i} \tag{60}$$

for the case of the single tournament and the right-hand side of this condition is constant and independent of the task-specific random shocks. Thus, a two-tournament structure will never be preferred by the principal if total equilibrium efforts are reduced in equilibrium relative to the single-tournament structure.

C.2 Agents

Suppose that $q\left(e_{it}\right) \equiv ae_{it} - e_{it}/2$ with $a > 0$ for $(i,t) \in \{(1,1),(1,2),(2,1)\}$, while $q\left(e_{22}\right) = 0$ for all $e_{22} \geq 0$, which means that agent 2 is incapable of performing task 2. For simplicity, assume that task 2's outputs are deterministic, while the difference in errors $\varepsilon_{i1} - \varepsilon_{j1}$ with $j \neq i$ is uniformly distributed with support $[-k, k]$ with $k > 0$. Effort costs are $C\left(e_{i1} + e_{i2}\right) = \left(e_{i1} + e_{i2}\right)^2/2$ for $i = 1, 2$. With two tournaments, the optimal task-1 efforts are $e^*_{i1}\left(0\right) = aw/(2k + w)$ for both agents 1 and 2, while agent 1 will only invest a minimum effort in task 2 and agent 2 will invest no effort in task 2. With a single tournament the picture changes. In this situation, agent 1 invests the same efforts in tasks 1 and 2 with $e^*_{11}(1) = e^*_{12}(1) = aw/(4k+w)$, while agent 2's efforts are unchanged relative to the case of a single tournament. Since $2aw/(4k+w) > aw/(2k+w)$, the principal is always better off with a single tournament relative to two tournaments under these circumstances.