Persuasion and Stubbornness in a Dynamic Trading Game

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Agenda

Develop models of bargaining (dynamic trading games) in which the parties generate hard evidence to gain advantage.

Instead of impatience (break-down risk) and risk aversion, the parties are motivated by their desire to convince their opponent.

The two sides may agree to disagree about the value of the good (same or different priors, commonly known).

Interesting questions:

1. Equilibrium price dynamics.
2. Delay in, and overall efficiency of, bargaining.
3. Relation to no-trade theorems & Coase conjecture.

Motivation

Why need another model of bargaining?

1) Descriptive of situations where evidence production is key:

- Financial litigation or regulatory bargaining supported by economic consultants.
- Haggling over the price of real estate, works of art, etc.

2) Insights may be generalized beyond this game.

- Persuasion and stubbornnes in debate: Search for arguments to convince the opponent and get an advantageous resolution.

Ingredients of our model

Trading game:

- One seller (he), one buyer (she), one good (it).
- Buyer's valuation depends on the state of nature.
- Bargaining power captured by right to make offers.

Persuasion:

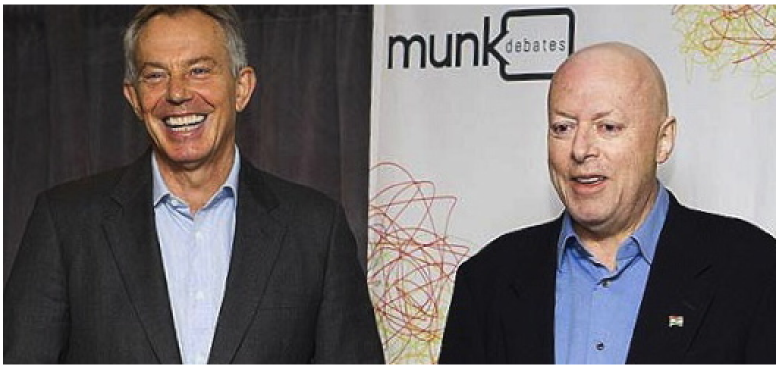
- Each period the parties may get hard signals about the state.
- Signals can be concealed, but are verifiable if disclosed.

A type of non-common (uncommon?) priors:

- The parties may disagree on the distribution of the state.
- Their disagreement is commonly known.

Why allow different (but CK) priors?

1) Realistic. Intelligent people often differ on the subjective probability of an event and "agree to disagree".



2) Technically sound and no exploitation, no paradoxes; we get interesting comparative statics in the degree of disagreement.

Model

One good; the seller's value is zero, the buyer's $v \in \{0, 1\}$.
Priors are $\sigma_0 = \Pr_S(v = 1)$, $\beta_0 = \Pr_B(v = 1)$, commonly known.

At $t = 1, 2, \ldots T$, hard signals s_t and b_t (independent across t and of each other conditional on v) may be generated:

- $\Pr(s_t = v) = 1 - \Pr(s_t = \emptyset) = r_S$, observed by S;
- $\Pr(b_t = v) = 1 - \Pr(b_t = \emptyset) = r_B$, observed by B.

If the signal's realization is v then it can be verifiably disclosed.

At the end of period t the seller proposes a price p_t.
If p_t is accepted then the good is consumed and the game ends.

Risk neutral, transferable utilities; discount factors $\delta_S, \delta_B \leq 1$.
The world ends at $T + 1$.

Equilibrium

Perfect Bayesian equilibrium:

- Strategies are best responses given beliefs (on/off eqm path), keeping track of higher order beliefs over the good's value.
- The players' strategies may depend on their private information (concealed signal's value).
- The players' beliefs are consistent with the priors and the equilibrium strategies in the usual (Bayesian) sense.

Method of finding PBE:

- Iterated dominance arguments starting at the end of the game.

Characterization of the equilibrium

We look for and find an equilibrium with these properties:

- At t, a self-serving signal ($s_t = 1$ or $b_t = 0$) is disclosed, and trade takes place immediately at $p_t = v$.
- If S observes $s_t = 0$ then he behaves as if $s_t = \emptyset$.
- Uninformed seller either sets $p_t = \bar{p}_t$ accepted for sure (*settle*), or $p_t = p'_t$ that is accepted iff the buyer has observed a 1 (*skim*).
- The seller believes on and off the equilibrium path that the buyer is uninformed if she has rejected his offer.

We will show that in this equilibrium the seller either settles at $t = 1$, or skims until a specific $t = m$ and settles then, or skims for all $t \leq T$.

Outline of the exposition

Case 1: In the continuation equilibrium after period t the seller skims in every period; the buyer accepts the offer as soon as a 1-signal is observed either by her, or by the seller (in which case it is disclosed).

Case 2: After period t, the seller is expected to skim until $t + k$, when a settlement at $p_{t+k} = \bar{p}$ is reached.

In each case we derive conditions under which the seller skims or settles at t. Inspection of these conditions yields the equilibrium.

We state the conditions for $\delta_S = \delta_B = 1$. Since time is discrete, the results do not change for δ_S, δ_B close to 1. We will say more on steep discounting in the discussion of the results.

Notation

The uninformed seller's belief at $t = 1$ (and, in our equilibrium, at any t) that $v = 1$ conditional on the buyer being either uninformed or concealing a just-received 1-signal is

$$\sigma_1 = \Pr_S(v = 1 | b_1 \in \{\emptyset, 1\}) = \frac{\sigma_0}{\sigma_0 + (1 - r_B)(1 - \sigma_0)}.$$

The uninformed buyer's belief at t that $v = 1$ conditional on the seller not having disclosed a 1-signal at or before t is

$$\beta_t = \Pr_B(v = 1 | s_\tau \in \{\emptyset, 0\}, \forall \tau \leq t) = \frac{(1 - r_S)^t \beta_0}{(1 - \beta_0) + (1 - r_S)^t \beta_0}.$$

Note that β_t is strictly decreasing in t.

Derivation of the equilibrium: $t = T$

Suppose we are at $t = T$. By the construction of the equilibrium strategies and off-path beliefs, the seller believes the buyer is uninformed right before the beginning of the period.

If the buyer remains uninformed at T then she believes the good's value is $v = 1$ with probability β_t.

If no signal is revealed at T then an uninformed seller believes $v = 1$ with probability σ_1, and that the buyer has just generated a 1-signal at T, that is $b_T = 1$, with probability $r_B \sigma_1$.

Therefore the seller offers $p_T = 1$ (accepted only if $b_T = 1$) or $p_T = \beta_T$ (accepted for sure); he sets $p_T = 1$ iff $r_B \sigma_1 \geq \beta_T$.

Case 1: Skimming expected to the end

Let $V_T = r_B \sigma_1$, the seller's expected profit from "skimming" at T. If $V_T \geq \beta_T$, then the buyer's continuation value at $T - 1$ is zero.

Hence at $T - 1$, the seller can sell for sure at price $p_{T-1} = \beta_{T-1}$, or alternatively skim with $p_{T-1} = 1$.

The seller could also *delay* (not sell for sure) by setting $p_{T-1} > 1$, but that can be shown to be inferior to skimming for any $\delta_S < 1$.

At any $t < T$: If for all $t + 1, \ldots, T$ the seller is expected to skim with a price of 1, then his payoff at t from skimming (with $p_t = 1$) is

$$V_t = \sigma_1 r_B + \sigma_1 \delta_S (1 - r_B)[1 - (1 - r_B)(1 - r_S)] + \ldots$$
$$+ \sigma_1 \delta_S^{T-t} (1 - r_B)^{T-t} (1 - r_S)^{T-t-1} [1 - (1 - r_B)(1 - r_S)]\}.$$

Case 1: Skimming expected to the end

Let $\delta_S = 1$ (simpler formula for V_t, no loss compared to δ_S close to 1).

Then the finite geometric series V_t can be summed as

$$V_t = \left[1 - (1 - r_B)^{(T-t)+1}(1 - r_S)^{T-t}\right] \sigma_1.$$

V_t is decreasing in t, increasing in T, r_S, r_B, and σ_1; $\lim_{T\to\infty} V_t = \sigma_1$.

Lemma 1: If $V_t \geq \beta_t$ for all t, then there is an equilibrium in which

(i) $s_t = 1$ or $b_t = 0$ is disclosed, followed by trade at $p_t = v$;

(ii) for all t without such disclosure the seller sets $p_t = 1$, and the buyer buys the good as soon as she observes a 1-signal.

Case 2: Settling expected at $p_{t+k} = \bar{p}$

If $V_T \equiv r_B\sigma_1 < \beta_T$ then the seller settles at T, price $p_T = \beta_T$.

We characterize the seller's optimal decision at t if he is expected to settle at price $p_{t+k} = \bar{p}$ in period $t + k$ and skim from $t + 1$ until then.

If $k > 1$ then the seller is skimming at $t + k - 1$. A buyer who has seen a 1-signal is indifferent between accepting p'_{t+k-1} and going to period $t + k$ by rejecting it iff $1 - p'_{t+k-1} = \delta_B(1 - r_S)(1 - \bar{p})$.

This pins down the skimming price at $t + k - 1$. By the same argument, for all $i = 0, 1, \ldots, k - 1$, the skimming price is

$$p'_{t+i} = 1 - \delta_B^{k-i}(1 - r_S)^{k-i}(1 - \bar{p}).$$

Case 2: Settling expected at $p_{t+k} = \bar{p}$

If an uninformed buyer rejects the seller's offer at t then her continuation value is

$$U_t = \delta_B \beta_t (1 - r_S) r_B (1 - p'_{t+1}) + \delta_B^2 \beta_t (1 - r_S)^2 (1 - r_S) r_B (1 - p'_{t+2})$$
$$\ldots + \delta_B^{k-1} \beta_t (1 - r_S)^{k-1} (1 - r_B)^{k-2} r_B (1 - p'_{t+k-1})$$
$$+ \delta_B^k \beta_t (1 - r_S)^k (1 - r_B)^{k-1} (1 - \bar{p}) - \delta_B^k (1 - \beta_t)(1 - r_B)^k \bar{p}.$$

This simplifies to

$$U_t = \delta_B^k \left[(1 - r_S)^k \beta_t (1 - \bar{p}) - (1 - r_B)^k (1 - \beta_t) \bar{p} \right].$$

An uninformed buyer accepts any $p_t \le \beta_t - U_t$.

Case 2: Settling expected at $p_{t+k} = \bar{p}$

It is easy to show that the maximum price accepted by a buyer concealing a 1-signal, $p'_t = 1 - \delta_B^k (1 - r_S)^k (1 - \bar{p})$, exceeds that accepted by an uninformed buyer, $\bar{p}_t = \beta_t - U_t$.

Therefore, at t the seller has three choices:

1. Offer p'_t to *skim* the buyer concealing a 1-signal;
2. Offer $\bar{p}_t$, accepted for sure, to *settle*;
3. Offer $p_t > p'_t$ rejected for sure, to *delay*.

It can be shown that *skim* dominates *delay* for all $\delta_S < 1$.
[Delay 'loses money' on the buyer currently concealing a 1.]

Case 2: Settling expected at $p_{t+k} = \bar{p}$

The seller's payoff from skimming at t is

$$V_t^k = \sigma_1 r_B p_t' + \sigma_1(1-r_B)\delta_S[r_S + (1-r_S)r_B p_{t+1}'] + \dots$$
$$+ \sigma_1(1-r_B)^{k-1}(1-r_S)^{k-2}\delta_S^{k-1}[r_S + (1-r_S)r_B p_{t+k-1}']$$
$$+ \sigma_1(1-r_B)^k(1-r_S)^{k-1}\delta_S^k[r_S + (1-r_S)\bar{p}]$$
$$+ (1-\sigma_1)(1-r_B)^k\delta_S^k\bar{p}.$$

For $\delta_S = \delta_B = 1$ the formula greatly simplifies to

$$V_t^k = \left[1-(1-r_S)^k\right]\sigma_1 + \left[(1-r_S)^k\sigma_1 + (1-r_B)^k(1-\sigma_1)\right]\bar{p}.$$

Case 2: Settling expected at $p_{t+k} = \bar{p}$

For $\delta_B = 1$ the settling price $\bar{p}_t = \beta_t - U_t$ becomes

$$\bar{p}_t = \beta_t - \left[(1-r_S)^k\beta_t(1-\bar{p}) - (1-r_B)^k(1-\beta_t)\bar{p}\right]$$
$$= \left[1-(1-r_S)^k\right]\beta_t + \left[(1-r_S)^k\beta_t + (1-r_B)^k(1-\beta_t)\right]\bar{p}.$$

Skimming is better than settling for the seller at t iff $V_t^k \geq \bar{p}_t$, which is equivalent to $\sigma_1 \geq \beta_t$.

Lemma 2: $\delta_S = \delta_B = 1$. Let m be the greatest $t \leq T$ with $\beta_t > V_t$. Absent signal disclosure at or before m, trade occurs at $p_m = \beta_m$.

Absent signal disclosure at or before $t < m$: If $\sigma_1 \geq \beta_t$ then skim at $p_t = 1-(1-r_S)^{m-t}(1-\beta_m)^{m-t}$, accepted iff $b_t = 1$; if $\sigma_1 < \beta_t$ then settle, $p_t = \beta_t - [(1-r_S)^{m-t}\beta_t(1-\beta_m) - (1-r_B)^{m-t}(1-\beta_t)\beta_m]$.

Characterization Theorem

For δ_S, δ_B close to 1, the equilibrium is as follows:

1) If $\beta_t \leq V_t$ for all $t = 1, \ldots, T$ then the seller offers $p_t = 1$ for all t. Trade occurs at t iff a 1-signal is generated at t.

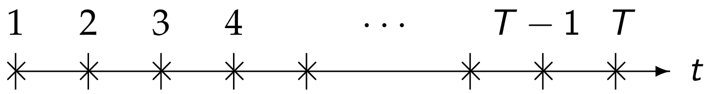
2) Otherwise let m be the greatest $t \leq T$ such that $\beta_t > V_t$.

Suppose $\sigma_1 \geq \beta_1$. There is no trade before m unless either the seller observes $s_t = 1$ (disclosed, trade at $p_t = 1$), or the buyer sees $b_t = 0$ (disclosed, trade at $p_t = 0$), or the buyer observes $b_t = 1$ (concealed) in which case she accepts $p_t = 1 - \delta^{m-t}(1 - r_S)^{m-t}(1 - \beta_m)$.
In all other cases trade takes place at m, price $p_m = \beta_m$.

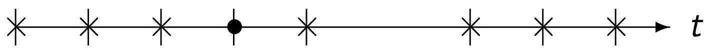
If $\sigma_1 < \beta_1$ then, in the absence of signal disclosure at $t = 1$, trade at $p_1 = \beta_1 - \left[(1 - r_S)^{m-1}\beta_1(1 - \beta_m) - (1 - r_B)^{m-1}(1 - \beta_1)\beta_m\right]$.

Graphical illustration of the timing of trade

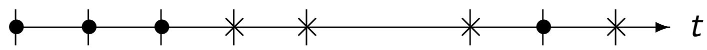
1) If $\beta_t \leq V_t, \forall t$, then skim with $p_t = 1$ to the end.



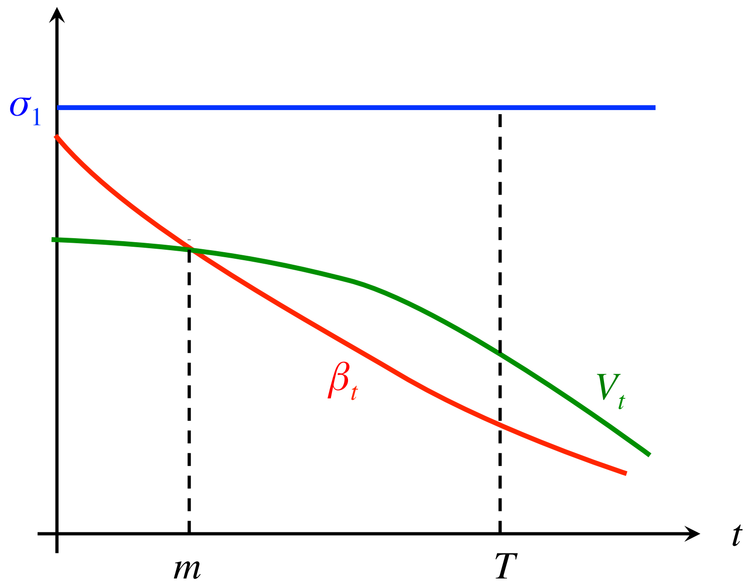
2) Otherwise, if $\beta_1 \leq \sigma_1$ then skim for all $t < m = \max\{t | \beta_t > V_t\}$ and settle at $p_m = \beta_m$.



If $\beta_1 > \sigma_1$ then settle at $t = 1$ and at all $t < m$ such that $\beta_t > \sigma_1$, settle again at m, skim otherwise:



An example with $\sigma_1 > \beta_1 > V_1 > V_T > \beta_T$



$$\sigma_1 > \beta_1 > V_1 \equiv [1-(1-r_B)^T(1-r_S)^{T-1}]\sigma_1 > V_T \equiv r_B\sigma_1 > \beta_T$$

Comparative statics of m

Suppose $\sigma_1 > \beta_1$: The seller is more enthusiastic about the good's value than the buyer at the time they meet.

Then, provided $\beta_1 > V_1 \equiv [1-(1-r_B)^T(1-r_S)^{T-1}]\sigma_1$, trade without signal realization takes place at $m = \max\{t \leq T : \beta_t > V_t\}$.

Proposition: If the seller is a priori more optimistic, or if the buyer is less optimistic, then m is lower (wait less until a compromise).

A larger T (longer horizon) shifts V_t right inducing a lower m.

Larger r_S or r_B shifts V_t up, β_t down (constant in r_B), decreasing m.

Discontinuity of delay in σ_0

Start from a situation where $\sigma_1 > \beta_1$ (e.g., common prior), and $\beta_T < V_T$ (e.g., r_B not too small).

Equilibrium outcome: Skim for all $t < T$, settle at $p_T = \beta_T$.

Suppose the seller becomes skeptical, i.e., σ_0 is decreased.
All else equal, σ_1 as well as V_t for all t decrease.

If σ_1 falls just below β_1 then the outcome changes drastically: Settle at $p_1 = \beta_1 - \left[(1-r_S)^{T-1}\beta_1(1-\beta_T) - (1-r_B)^{T-1}(1-\beta_1)\beta_T\right]$.

As σ_0 decreases, the outcome may change from *'skim throughout'* to *'settle right away'*.

The benefits of enthusiasm

Consider an example with a common prior, $T = 1$, but assume the seller can make an *ex ante offer* p_0. Assume $\beta_1 > V_1$.

Since $\sigma_0 = \beta_0$ we have immediate agreement. Off the equilibrium path the parties settle at $p_1 = \beta_1$. Hence $p_0 = r_S\sigma_0 + (1-r_S)\beta_1$.

Suppose that the seller hires an agent with $\sigma_0' > \sigma_0$ such that $V_1' = r_B\sigma_0' > \beta_1$. At $t = 1$ the seller would skim with $p_1' = 1$.

Anticipating this, the settlement price at 0 is $\beta_0 = \sigma_0 > p_0$.

The seller can gain from 'pretending' to be overly enthusiastic because this commits him to delay off the equilibrium path.

Large, finite horizon and discounting

Suppose $\sigma_1 > \beta_1$ (e.g., common prior), $\delta \in [0, 1]$, T arbitrarily large.

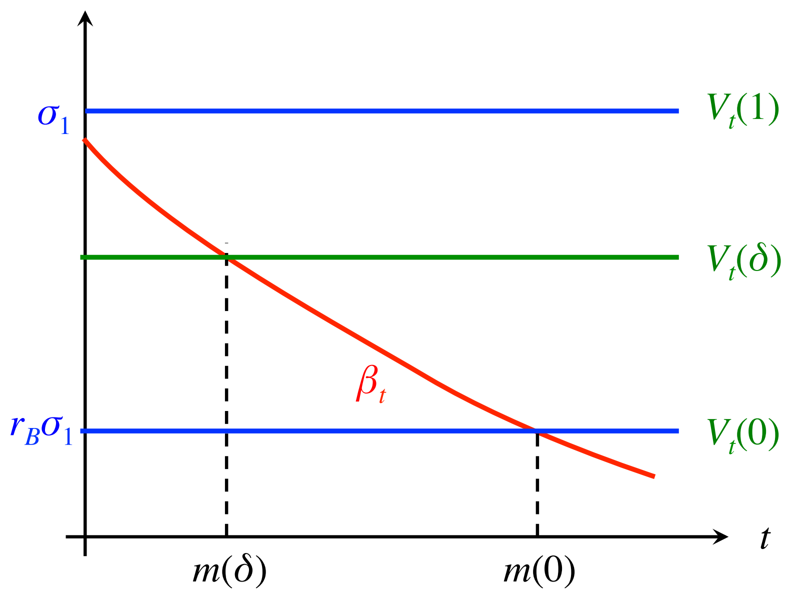
Then $V_t \approx V(\delta)$ constant for all t, where $V(0) = r_B\sigma_1$, $V(1) = \sigma_1$, strictly increasing in δ. In contrast, $\beta_T \to 0$ as $T \to \infty$.

Proposition: Assume $\sigma_1 > \beta_1$ and T large.

If $\delta \approx 0$ then in equilibrium trade occurs without signal generation at $m(0) < T$ such that $\beta_{m(0)} = r_B\sigma_1$.

As δ increases, trade without signal generation occurs at $m(\delta)$, which is decreasing in δ. For δ near 1, no trade without signal generation.

Large, finite horizon and discounting



Summary and directions

Developed a model of bilateral trading with persuasion and possible different, commonly-known priors.

- Equilibrium with stubbornness and compromise.
- Effects of discounting and length of time horizon.
- Comparative statics in priors and signal-generating probabilities.

Directions to pursue:

- More results on ex ante trade, common prior, etc.
- Extension to alternating or randomly assigned offers.
- Same information structure in other, related models of trade (e.g., possibility of negative surplus), and debate.