Flippers in Housing Market Search*

Charles Ka Yui Leung[†] and Chung-Yi Tse[‡] [§]

September 30, 2013

Abstract

We add arbitraging middlemen—investors who attempt to profit from buying low and selling high—to a canonical housing market search model. Not surprisingly, in a slack market in which it is difficult to sell, the opportunities offered to mismatched homeowners to quickly dispose of their old houses by these middlemen are particularly welcome. Less obvious is that the same opportunities are similarly welcome in a tight market in which houses can be sold quickly even without the aid of these intermediaries. To follow is the possibility of multiple equilibrium In one equilibrium, most, if not all, transactions are intermediated, resulting in rapid turnover, a high vacancy rate, and high housing prices. In another equilibrium, few houses are bought and sold by middlemen. Turnover is sluggish, few houses are vacant, and prices are moderate. The housing market can then be intrinsically unstable even when all flippers are of the liquidity-providing variety in classical finance theory.

Key words: Search and matching, housing market, liquidity, flippers

JEL classifications: D83, R30, G12

*We thank Enrique Schroth as discussant of the paper in the 2011 NYU Economics PhD Alumni Conference, Angela Mak for excellant research assistance and Kuang Liang Chang, Vikas Kakkar, Fred Kwan, Edward Tang, Min Hwang, and Isabel Yan for discussions. Tse grateful acknowledges financial support from HK GRF grant HKU 751909H.

†City University of Hong Kong

‡University of Hong Kong

§Corresponding author. Address: School of Economics and Finance, University of Hong Kong, Pokfulam Road, Hong Kong. E-mail: tsechung@econ.hku.hk.

1 Introduction

In many housing markets, the purchases of owner-occupied houses by individuals who attempt to profit from buying low and selling high rather than for occupation are commonplace. For a long time, anecdotal evidence abounds as to how the presence of these investors, who are popularly known as flippers in the U.S., in the housing market can be widespread.[1] More recently, empirical studies have began to systematically document the extent to which transactions in the housing market are motivated by buying and selling for short-term gains and how these activities are correlated with the housing price cycle. Notable contributions include Haughwout et al. (2011), Depken et al. (2009), and Bayer et al. (2011).[2] A common theme in the discussion is that housing market flippers can be of two types—the trend-chasing speculators versus the arbitraging middlemen. Whereas the speculators, as noise traders, inevitably destabilize the market, the middlemen, as liquidity providers in classical finance theory, help improve market efficiency. But is such a simple and clear-cut dichotomy warranted? To the extent that the sales and purchases of houses by flippers, arbitraging middlemen or otherwise, add to market demand and supply, it is not inconceivable that the entry and exit of these investors into and out of the housing market can be a source of volatility. Moreover, any efficiency improvement brought by arbitraging middlemen must be offset by the losses would-be buyers suffer when facing the higher market price in a more fluid market.

In this paper, we study a housing market search model along the lines of Arnold (1989) and Wheaton (1990) in which houses are demanded by flippers in addition to end-user households. The flippers are of the liquidity-providing variety in classical finance theory. A role for these agents exists because ordinary households are assumed liquidity constrained to the extent that each cannot hold more than one house at a time. In this case, a household which desires to move because the old house is no longer a good match must first sell it before the household can buy up a new house. In a buyer's market—a market in which sellers outnumber buyers by a significant margin—the wait can be lengthy. This opens up profitable opportunities for the flippers to just buy up the mismatched house at a discount in return for the time spent waiting for the eventual end-user buyer to arrive on behalf of the original owner.

[1] Out of the five transactions in a large developement in Hong Kong in August 2010, three were reported to involve investors who buy in anticipation of short-term gains (September 10, 2010, *Hong Kong Economic Times*). According to one industry insider, among all buyers of a new development in Hong Kong recently, only about 60% are buying for own occupation (November 20, 2010, *Wenweipo*).

[2] Haughwout et al. (2011) report that at the peak of housing price cycle in the U.S. in 2006, up to 45% of new purchase mortgages in the "bubble states" are taken out by individuals having two or more first-lien mortgages. Around the same period of time, Depken et al. (2009) find that 25% of housing market transactions are for houses sold again within the first two years of purchase in the Las Vegas metropolitan area. Bayer et al. (2011) report a much smaller percentage of such purchases, however, at about 4%, for their sample comprising transactions in five counties in the LA area.

This is the usual reason for why flippers can improve market liquidity. The novelty in our analysis is that we find that mismatched homeowners could similarly prefer to sell quickly to flippers in a seller's market to capitalize on the high prices in such a market sooner. In either case, transaction volumes, vacancies, and housing prices all increase with the extent of flippers' presence in the market, whereas the average Time-On-the-Market (TOM) declines in the interim.

Because flippers can survive in both slack and tight markets, multiple equilibrium is a distinct possibility in our model. With a multiplicity of equilibrium, wide swings in prices and transactions can happen without any underlying changes in technology, preference, and interest rate. Moreover, in our model, the market penetration of flippers can be rather sensitive to small interest rate shocks. In this case, a given interest rate shock will have an indirect impact on housing prices through its influence on the entry and exit of flippers, in addition to the usual direct effect of interest rates on asset prices. Then, a housing market populated by liquidity-providing middlemen can be prone to a substantial amount of volatility. In all, the presence of these agents in the housing market can be a double-edged sword. On the one hand, the flippers may help improve liquidity. On the other hand, when the extent of their presence can be fickle, the housing market can become more volatile as a result.

It would be foolhardy to suggest that the volatility arising from the activities of liquidity-providing middlemen in our analysis is an important source of the housing market bubble in the U.S. in the early- to mid-2000s. Perhaps the trend-chasing speculators have played a significantly more decisive role. In any case, we should emphasize that our model is not meant to be a candidate explanation for any episodes of housing market bubble in the U.S. and beyond. Nevertheless, our quantitative analysis indicates that housing prices can differ by up to 23 percent across steady-state equilibria and vary by 26 percent in response to a seemingly unimportant interest rate shock when the model is calibrated to the several observable characteristics of the U.S. housing market. Amid this substantial price difference, welfare differs much less across the steady-state equilibria. Aggregate welfare in a "fully-intermediated" equilibrium is at most 7 percent higher than in a "no-intermediation" equilibrium. That any welfare increase from intermediation may be modest is because the increase is bounded by the losses would-be buyers suffer in a more active and higher-priced market.

Our model has a number of readily testable implications. First, it trivially predicts a positive cross-section relation between housing prices and TOM—mismatched homeowners can either sell to flippers at a discount or to wait for a better offer from an end-user buyer to arrive—which agrees with the evidence reported in Merlo and Ortalo-Magne (2004), Leung et al. (2002) and Genesove and Mayer (1997), among others.[3]

[3] Albrecht et al. (2007) emphasize another aspect of the results reported in Merlo and Ortalo-Magne (2004), which is that downward price revisions are increasingly likely when a house spends more and more time on the market.

An important goal of the recent housing market search and matching literature is to understand the positive time-series correlation between housing prices and transaction volumes and the negative correlation between the two and average TOM.[4] In Kranier (2001), for instance, a positive but temporary preference shock can give rise to higher prices and a greater volume of transaction, whereas Diaz and Jerez's (2009) analysis implies that an adverse shock to construction will shorten TOM, and may possibly lead to higher prices and a greater volume of transaction. The paper by Ngai and Tenreyro (2010) focuses on the co-movement in prices and sales over the seasonal cycle and they argue that increasing returns in the matching technology play a key role in generating such cycles.

In our model, across steady-state equilibria, a positive relation between prices and transaction volumes and a negative relation between the two and average TOM also hold–in an equilibrium in which more houses are sold to flippers, prices and sales both increase, whereas houses on average stay on the market for a shorter period of time. In the meantime, in our model, vacancies tend to increase together with prices and transaction volumes if the increase in transaction volume is due to more houses sold to flippers, who may then just leave them vacant for the time it takes for the end-user buyers to arrive. In contrast, in both the Kranier and the Diaz and Jerez's model, the increase in sales should be accompanied by a decline in vacancy—given that when a house is sold, it is sold to an end-user, who will immediately occupy it, vacancies must decline, or at least remain unchanged. In Ngai and Tenreyro's model, households are assumed to move out of their old houses and into rental housing immediately when they become mismatched. Then, any and all houses on the market are vacant houses and given the assumed increasing-returns-to-scale matching technology, vacancies rise and fall with prices and transactions in the seasonal cycle. A mismatched household in their model, however, could well have stayed in the old house and avoided rental housing and the payment thereof until it has successfully sold the old house. In this alternative setup, the stock of vacant houses only includes houses held by people who have bought new houses before they manage to sell their old ones. Then, it is no longer clear that vacancies must rise and fall with prices and sales in the seasonal cycle of Ngai and Tenreyro.

Figure 1 depicts the familiar positive housing price-transaction volume correlation for the U.S. for the 1981Q1 to 2011Q3 time period.[5] The usual housing market

[4] Stein (1995), who explains how the down-payment requirement plays a crucial role in amplifying shocks, is an early non-search-theoretic explanation for the positive relation between prices and sales. Hort (2000) and Leung et al. (2003), among others, provide recent evidence. Kwok and Tse (2006) show that the same relation holds in the cross section.

[5] Housing Price is defined as the nominal house price, which is the transaction-based house price index from OFHEO (http://www.fhfa.gov), divided by the CPI, from the Federal Reserve Bank at St. Louis. We set Housing Price at 1981Q1 equal to 100. Transaction is measured by the *quarterly* sales in single-family homes, apartment condos, and co-ops, normed by the stock of such units. The sales data are from the *Real Estate Outlook* by the National Assoication of Realtors, complied by Moody's Analytics. The housing stock is defined as the sum of owner-occupied units and vacant

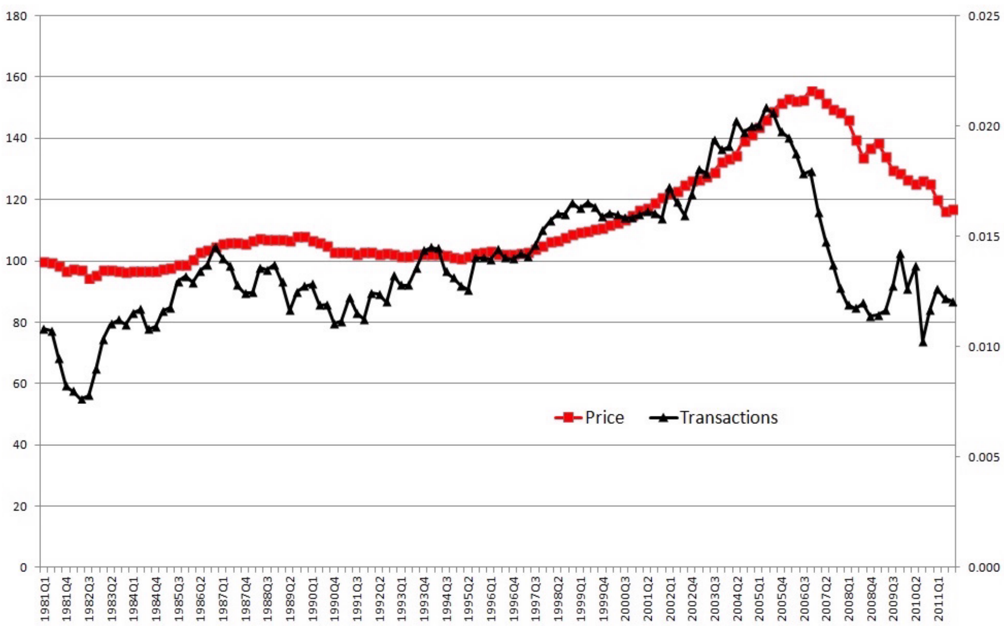

Figure 1: Price and Transactions

search model predicts that vacancies should decline in the housing market boom in the late-1990s to the mid-2000s and rise thereafter when the market collapses around 2007. Figures 2 and 3, however, show that any decline in vacancy is not apparent in the boom.[6] In fact, if there is any co-movement between vacancies on the one hand and prices and transactions on the other hand in the run-up to the peak of the housing market boom in 2006, vacancies appear to have risen along with prices and transactions. In a literal interpretation of our model, vacancies should fall very significantly to follow the market collapse since 2007. The decline in vacancy in Figures 2 and 3 since the market collapse is modest, however, compared to the increase in the boom years. Two forces absent in our analysis–the massive amount of bank foreclosures and unsold new constructions in the market bust–may have accounted for the slow decline in vacancy since 2007.

Insofar as the flippers in our model act as middlemen between the original home-owners and the eventual end-user buyers, this paper contributes to the literature on middlemen in search and matching pioneered by Rubinstein and Wolinsky (1987). Previously, it was argued that middlemen could survive by developing reputations as sellers of high quality goods (Li, 1998), by holding a large inventory of differentiated

and for-sale-only units. The data are from the Bureau of Census's CPS/HVS Series H-111 available at http://www.census.gov/housing/hvs/data/histtabs.html.

[6] Vacancy rate is obtained by dividing the number of vacant and for-sale-only housing units by the housing stock as defined in the prevous note.

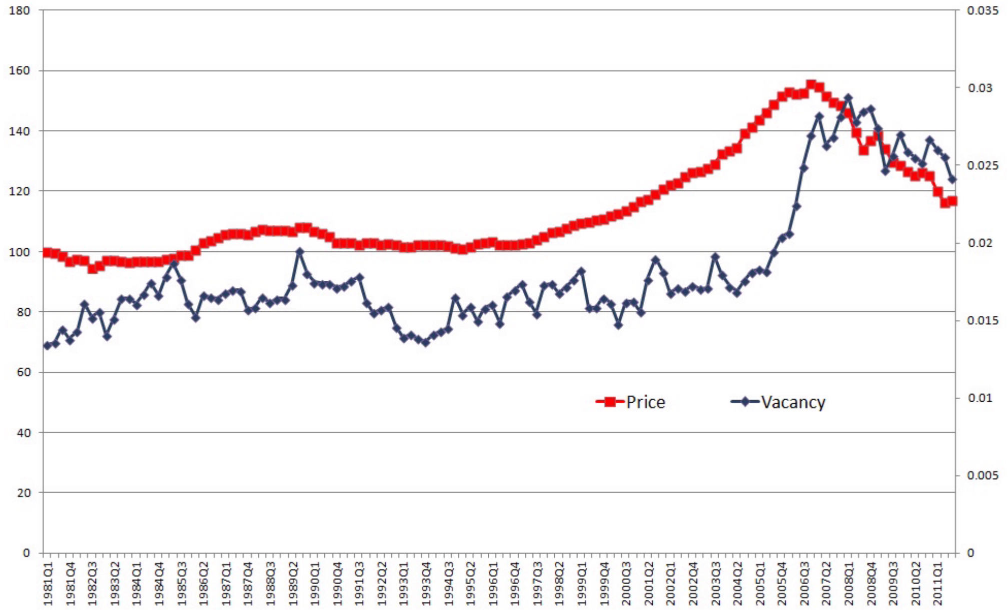

Figure 2: Price and Vacancy

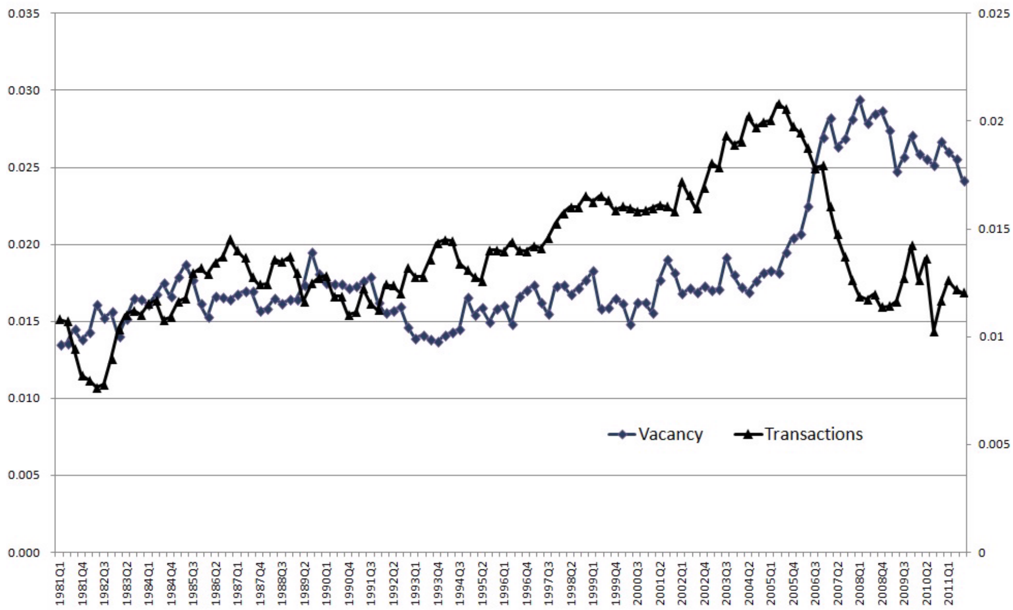

Figure 3: Transactions and Vacancy

products to make shopping less costly for others (Johri and Leach, 2002; Shevchenko, 2004; Smith, 2004), by raising the matching rate in case matching is subject to increasing returns (Masters, 2007), and by lowering distance-related trade costs for others (Tse, 2011). This paper studies the role of middlemen in the provision of market liquidity.

A simple model of housing market flippers as middlemen is also in Bayer et al. (2011). The model though is partial equilibrium in nature and cannot be used to answer many of the questions we ask in this paper. Intermediaries who serve to improve liquidity in the housing market are also present in the model of the interaction of the frictional housing and labor markets of Head and Lloyd-Ellis (2012), which is fully general equilibrium in nature. Analyzes of how middlemen may serve to improve liquidity in a search market also include Gavazza (2012) and Lagos et al. (2011). However, none of these studies allows end-user households a choice of whether to deal with the middlemen and for the multiplicity of equilibrium and how the market share of these middlemen may vary across the equilibria. Multiple equilibrium in a search and matching model with middlemen can also exist in Watanabe (2010). The multiplicity in that model, however, is due to the assumption that the intermediation technology is subject to increasing returns to scale. Moreover, only one of the two steady-state equilibria in that model is stable, whereas there can be two stable steady-state equilibria in our model.

The next section presents the model. Section 3 contains the detailed analysis. Section 4 takes a more systematic look at the patterns shown in Figures 1-3 with reference to the model's implications. In Section 5, we calibrate the model to several observable characteristics of the U.S. housing market to assess the amount of volatility that the model can generate. Our model, interpreted literally, is a model of buy-and-sell flips. In Section 6, we conclude by arguing that the model, conceptually, can also encompass buy-renovate-sell flips. All proofs are relegated to the Appendix. For brevity, we restrict attention to analyzing steady-state equilibria in this paper. A companion technical note (Leung and Tse, 2013) covers the analysis of the dynamics for the special case in which all agents possess the same bargaining power.[7]

2 Model

2.1 Basics

The model housing market is populated by a continuum of measure one risk-neutral households, each of whom discounts the future at the same rate r_H. There are two types of housing in the market: owner-occupied, the supply of which is perfectly inelastic at $H < 1$ and rental, which is supplied perfectly elastically for a rental payment of q per time unit. A household staying in a matched owner-occupied house

[7] Not for publication, available for download in http://www.sef.hku.hk/~tsechung/index.htm

enjoys a flow utility of $\upsilon > 0$, whereas a household either in a mismatched house or in rental housing none. A household-house match breaks up exogenously at a Poisson arrival rate δ, after which the household may continue to stay in the house but it no longer enjoys the flow utility υ. In the meantime, the household may choose to sell the old house and search out a new match. An important assumption is that households are liquidity constrained to the extent that each can hold no more than one house at a time. Then a mismatched homeowner must first sell the old house before she can buy a new one. The qualitative nature of our results should hold as long as there is a limit, not necessarily one, on the number of houses a household can own at a time. The one-house-limit assumption simplifies considerably.

The search market The flow of matches in the search market is governed by a concave and CRS matching function $M(B, S)$, where B and S denote, respectively, the measures of buyers and sellers in the market. Let $\theta = B/S$ denote market tightness. Then, the rate at which a seller finds a buyer is

$$\eta = \frac{M(B,S)}{S} = M(\theta, 1),$$

whereas the buyer's matching rate is $\mu = \eta/\theta$. Given that M is increasing and concave in B and S,

$$\frac{\partial \eta}{\partial \theta} > 0, \qquad \frac{\partial \mu}{\partial \theta} < 0.$$

We impose the usual regularity conditions on M to ensure that

$$\lim_{\theta \to 0} \eta = \lim_{\theta \to \infty} \mu = 0, \qquad \lim_{\theta \to \infty} \eta = \lim_{\theta \to 0} \mu = \infty.$$

Prices in the search market fall out of the Nash bargaining between pairs of matched buyers and sellers.

The Walrasian investment market Instead of waiting out a buyer to arrive in the search market, a mismatched homeowner may sell her old house right away in a Walrasian market populated by specialist investors—agents who do not live in the houses they have bought but rather attempt to profit from buying low and selling high. Because homogeneous flippers do not gain by selling and buying houses to and from one another, the risk-neutral flippers may only sell in the end-user search market and will succeed in doing so at the same rate η that any household-seller does in the market. We allow for flippers to discount the future at a possibly different rate r_F than the households in the city. In the competitive investment market, prices adjust to eliminate any excess returns on real estate investment.

We recognize that the assumption of a Walrasian investment market seemingly completely contradicts the motivations for applying the search and matching framework to the study of the housing market. What is needed in the analysis, however, is not an investment market altogether free of search frictions of any kind, but only one in which the frictions are less severe than in the end-user market.[8] If flippers are entirely motivated by arbitrage considerations and do not care if the houses to be purchased are good matches for their own occupation, search should not a particularly serious problem. In reality, we imagine that households who intend to sell quickly and are willing to accept a lower price will convey their intentions to real estate agents, who in turn will alert any specialist investors the availability of such deals. The competition among flippers should then bid prices up to just eliminate any excess returns on investment. A Walrasian market assumption captures the favor of such arrangements in the simplest possible manner.

2.2 Accounting identities and housing market flows

Accounting identities At any one time, a household can either be staying in a matched house, in a mismatched house, or in rental housing. Let n_M, n_U, and n_R denote the measures of households in the respective states. Given a unit mass of households in the market,

$$n_M + n_U + n_R = 1. \tag{1}$$

Each owner-occupied house must be held either by an ordinary household or by a flipper. Hence,

$$n_M + n_U + n_F = H, \tag{2}$$

where n_F denotes both the measures of active flippers and houses held by these individuals.

If each household can hold no more than one house at any moment, the only buyers in the search market are households in rental housing; i.e.,

$$B = n_R. \tag{3}$$

On the other hand, sellers in the search market include mismatched homeowners and flippers, so that

$$S = n_U + n_F. \tag{4}$$

Housing market flows In each unit of time, the inflows into matched owner-occupied housing are comprised of the successful buyers among all households in rental housing (μn_R), whereas the outflows are comprised of those who become mismatched in the interim (δn_M). In the steady state,

$$\mu n_R = \delta n_M. \tag{5}$$

Households' whose matches just break up may choose to sell their old houses right away to flippers in the investment market or to wait out a buyer to arrive in the

[8] Let's say, for example, the meetings in the investment market are given by another CRS matching function $M_F(B, S)$, whereby $M_F(B, S) > M(B, S)$ for any $\{B, S\}$ pair.

search market. Let α denote the fraction of mismatched households who choose to sell in the investment market and $1-\alpha$ the fraction who choose to sell in the search market. In each time unit then, the measure of mismatched homeowners selling in the search market increases by $(1-\alpha)\,\delta n_M$, whereas the exits are comprised of the successful sellers (ηn_U) in the meantime. In the steady state,

$$(1-\alpha)\,\delta n_M = \eta n_U. \tag{6}$$

Households moving into rental housing are mismatched households who just sell their properties to flippers $(\alpha\delta n_M)$ and to end users (ηn_U), respectively. The exits are comprised of the successful buyers among all households in rental housing (μn_R). In the steady state,

$$\alpha\delta n_M + \eta n_U = \mu n_R, \tag{7}$$

In each time unit, the measure of houses held by flippers increases by the measure of houses recently mismatched households decide to dispose right away in the investment market $(\alpha\delta n_M)$ and declines by the measure of houses flippers manage to sell to end-users (ηn_F). In the steady state,[9]

$$\alpha\delta n_M = \eta n_F. \tag{8}$$

2.3 Flippers' market share, market tightness, and turnovers

Since houses bought by flippers will next be put up for sale in the search market, when more houses are bought by flippers, the search market should be increasingly dominated by these agents.

Lemma 1 *In the steady state,*

$$\frac{n_F}{n_U + n_F} = \alpha;$$

i.e., the fraction of houses held by flippers among all houses offered for sale in the search market is equal to the fraction of mismatched homeowners selling to flippers in the first place.

Equations (1)-(8), for the *turnovers* of houses and households, can be combined to yield a single equation,

$$\delta + \eta\,(1-H) - (1-\alpha+\theta)\,H\delta = 0, \tag{9}$$

in θ and α.[10]

[9] Where (1) and (2) are two equations in four unknowns, once any two of the four variables are given, the other two are uniquely determined. In this connection, it is straightforward to verify that only two of the four steady-state flow equations (5)-(8) constitute independent restrictions.

[10] See the proof of Lemma 2 in the Appendix for the derivation of the equation.

Lemma 2 *An implicit function $\theta = \theta_T(\alpha)$, for $\alpha \in [0,1]$, defined by (9), is guaranteed single-valued, and that $\partial\theta_T/\partial\alpha > 0$. Both the lower and upper bounds, given by, respectively, $\theta_T(0)$ and $\theta_T(1)$, are strictly positive and finite. Furthermore, $\theta_T(1) > 1/H > 1$.*

A priori, it seems clear that when no houses are sold to flippers ($\alpha = 0$), n_F must equal zero in the steady state. On the other hand, when all mismatched houses are sold to flippers in the first instance ($\alpha = 1$), n_U, in the steady state, should just be equal to zero. And then in general, as α increases from 0 toward 1, n_F should increase along, whereas n_U should decline in the meantime. Lemma 3 confirms these intuitions.

Lemma 3

a. *At $\alpha = 0$,*

$$n_F = 0, \quad n_R = 1 - H, \quad n_M = H - n_U,$$

whereas n_U is given by the solution to (61) *in the Appendix.*

b. *As α increases from 0 toward 1,*

$$\frac{\partial n_F}{\partial\alpha} > 0, \ \frac{\partial n_R}{\partial\alpha} > 0, \ \frac{\partial n_M}{\partial\alpha} > 0, \text{ whereas } \frac{\partial n_U}{\partial\alpha} < 0.$$

c. *At $\alpha = 1$,*

$$n_U = 0, \quad n_R = 1 - H + n_F, \quad n_M = H - n_F,$$

whereas n_F is given by the solution to (62) *in the Appendix.*

What is less obvious in the Lemma is that, besides n_F, both n_R and n_M also increase along with α. The first relation follows from the fact that if both the population of households and the housing stock are given, a unit increase in the measure of houses held by flippers must be matched by a unit decline in the measure of houses occupied by ordinary households. To follow then is the same unit increase in the population of these households in rental housing. For the second relation, at a larger α, fewer households spend any time at all selling their old houses in the search market before initiating search for a new match. In the meantime, the increase in θ (Lemma 2), through lowering μ, lengthens the time a household spends on average in rental housing before a new match can be found. By Lemma 3(b), the first effect dominates, so that more households are in matched owner-occupied housing in the steady state.

Now, if $\partial n_R/\partial\alpha > 0$ and given that $B = n_R$, there will be more buyers in the search market to follow an increase in α. Second, if $\partial n_M/\partial\alpha > 0$ and given that

$S = n_U + n_F = H - n_M$ by (2), there will be fewer sellers in the market in the meantime. With more buyers and fewer sellers, market tightness,

$$\theta = \frac{B}{S} = \frac{n_R}{n_U + n_F},$$

can only increase when more transactions are intermediated by flippers. These tendencies, of course, are the forces behind the comparative statics in Lemma 2.

In the model housing market, the entire stock of vacant house is comprised of houses held by flippers. With a given housing stock, the vacancy rate is simply equal to n_F/H. A direct corollary of Lemma 3(b) is that:

Lemma 4 *In the steady state, the vacancy rate for owner-occupied houses is increasing in α.*

Housing market transactions per time unit in the model are comprised of (i) $\alpha\delta n_M$ houses sold from households to flippers, (ii) ηn_F houses flippers sell to households, and (iii) ηn_U houses sold by one household to another, adding up to an aggregate transaction volume,

$$TV = \alpha\delta n_M + \eta n_F + \eta n_U. \tag{10}$$

Lemma 5 *In the steady state, TV is increasing in α.*

The usual measure of turnover in the housing market is the time it takes for a house to be sold, what is known as Time-On-the-Market (TOM). Given that houses sold in the investment market are on the market for a vanishingly small time interval and houses sold in the search market for a length of time equal to $1/\eta$ on average, we may define the model's average TOM as

$$\frac{\alpha\delta n_M}{TV} \times 0 + \frac{\eta n_F + \eta n_U}{TV} \times \frac{1}{\eta}. \tag{11}$$

Lemma 6 *In the steady state, on average, TOM is decreasing in α.*

TOM is a measure of the turnover of houses for sale, and as such Lemma 6 in itself does not carry any direct welfare implications. A more household-centric measure of turnover is the length of time a household (rather than a house) has to stay unmatched. We define what we call Time-Between-Matches (TBM) as the sum of two spells: (1) the time it takes for a household to sell the old house, and (2) the time it takes to find a new match thereafter. While the first spell (TOM) on average is shorter with an increase in α, the second is longer as the increase in θ to accompany the increase in α causes μ to fall. A priori then it is not clear what happens to the average length of the whole spell. The old house is sold more quickly. But it also takes longer on average to find a new match in a market with more buyers and fewer sellers. To examine which effect dominates, write the model's average TBM as

$$\alpha\frac{1}{\mu}+(1-\alpha)\left(\frac{1}{\eta}+\frac{1}{\mu}\right). \tag{12}$$

where $1/\mu$ is the average TBM for households who sell in the investment market[11] and $1/\eta+1/\mu$ for households who sell in the search market.[12]

Lemma 7 *In the steady state, on average, TBM is decreasing in α.*

Lemma 7 may be taken as the dual of Lemma 3(a) ($\partial n_M/\partial\alpha>0$). When matched households are more numerous in the steady state, on average, people must be spending less time between matches

Up to this point, the model is purely mechanical. Given α, market tightness θ is completely isomorphic to the determination of housing prices in equilibrium. The same conclusion carries over to the determination of vacancies, turnover, and transaction volumes. If not for the inclusion of flippers in the model housing market, α is identically equal to 0 and Lemma 2 would have completed the analysis of everything that seems to be of any interest. With the inclusion of flippers and their market share measured by α, Lemmas 6 and 7 show how changes in the latter affect the turnovers of houses and households, which can have important consequences on welfare, a question we shall address in the following. But first α obviously should be made endogenous to which we next turn.

2.4 Asset values and housing prices

Asset values for flippers Let V_F be the value of a vacant house to a flipper and p_{FS} the price she expects to receive for selling it in the search market. In the steady state,

$$r_F V_F=\eta\left(p_{FS}-V_F\right). \tag{13}$$

Let p_{FB} be the price the flipper has paid for the house in the competitive investment market in the first place. In equilibrium, where any excess returns on real estate investment are eliminated,

$$p_{FB}=V_F. \tag{14}$$

[11] The household sells the old house instantaneously. Given a house-finding rate μ, the average TBM is then $1/\mu$.

[12] Let t_1 denote the time it takes the household to sell the old house in the search market and t_2-t_1 the time it takes the household to find a new match after the old house is sold. Then the household's TBM is just t_2. On average, $E\left[t_2\right]=\int_0^\infty \eta e^{-\eta t_1}\left(\int_{t_1}^\infty t_2\mu e^{-\mu(t_2-t_1)}dt_2\right)dt_1=1/\eta+1/\mu$.

Asset values for households There are three (mutually exclusive) states to which a household can belong at any one time,

1. in a matched house; value V_M,
2. in a mismatched house; value V_U,
3. in rental housing; value V_R.

The flow payoff for a matched owner-occupier begins with the utility she derives from staying in a matched house υ. The match will be broken, however, with probability δ, after which the household may sell the house right away in the investment market at price p_{FB} and switch to rental housing immediately thereafter. Alternatively, the household can continue to stay in the house while trying to sell it in the search market. In all,

$$r_H V_M = \upsilon + \delta \left(\max \left\{ V_R + p_{FB}, V_U \right\} - V_M \right). \tag{15}$$

Let p_H denote the price a household-seller expects to receive in the search market. Then, the flow payoff of a mismatched owner-occupier is equal to

$$r_H V_U = \eta \left(V_R + p_H - V_U \right). \tag{16}$$

Two comments are in order. First, in (16), the mismatched owner-occupier is entirely preoccupied with disposing the old house while she makes no attempt to search for a new match. This is due to the assumption that a household cannot hold more than one house at a time and the search process is memoryless. Second, under (15) and (16), the household has only one chance to sell the house in the investment market, at the moment the match is broken. Those who forfeit this one-time opportunity must wait out a buyer in the search market to arrive. This restriction is without loss of generality in a steady-state equilibrium, in which the asset values and housing prices stay unchanging over time. No matter, after the old house is eventually sold, the household moves to rental housing to start searching for a new match. Hence, with α equal to the fraction of houses offered for sale in the search market held by flippers and $1 - \alpha$ the fraction held by ordinary households,

$$r_H V_R = -q + \mu \left(V_M - \left(\alpha p_{FS} + (1 - \alpha)\, p_H \right) - V_R \right), \tag{17}$$

where q is the exogenously given flow rental payment.

Bargaining Prices in the search market fall out of Nash bargaining between matched buyer-seller pairs. There is only one buyer type in the search market—households in rental housing. The sellers can be either flippers, assumed to possess bargaining power β_F, or mismatched homeowners, assumed to possess bargaining power β_H.

Hence, when a household-buyer is matched with a flipper, the division of surplus in Nash Bargaining satisfies

$$\beta_F (V_M - p_{FS} - V_R) = (1 - \beta_F) (p_{FS} - V_F), \tag{18}$$

whereas when the household-buyer is matched with a household-seller, the division of surplus in Nash Bargaining satisfies[13]

$$\beta_H (V_M - p_H - V_R) = (1 - \beta_H) (V_R + p_H - V_U). \tag{19}$$

If flippers are agents specializing in buying and selling, it is most reasonable to assume that $\beta_F \geq \beta_H$.

2.5 Which market to sell?

Define

$$\Delta \equiv V_R + p_{FB} - V_U \tag{20}$$

as the difference in payoff for a mismatched homeowner between selling in the investment market ($V_R + p_{FB}$) and in the search market (V_U). By (13), (14), (18), and (19),

$$\Delta = (1 - \beta_H)^{-1} \left(\frac{\beta_H r_F + \beta_F \eta}{\beta_F \eta} p_{FB} - p_H \right). \tag{21}$$

That is, mismatched homeowners prefer to sell right away in the investment market if the given instantaneous reward (p_{FB}) dominates an appropriately-discounted reward of selling in the search market $\left(\frac{\beta_F \eta}{\beta_H r_F + \beta_F \eta} p_H \right)$ to be received at some future date.

Lemma 9 in the Appendix presents the solutions of p_{FB} and p_H, together with those of the various asset values, from (13)-(19). Substituting in the solutions to (21), Δ is seen to have the same sign as

$$D(\theta, \alpha) \equiv \left(\beta_F \frac{r_H}{r_F} - \beta_H - z\beta_H \right) \eta + (1 - \beta_H - \alpha (\beta_F - \beta_H)) \mu - (\delta + r_H) z, \tag{22}$$

where $z = q/v$.[14] Clearly, if $D(\theta, \alpha) > 0$ (< 0), for all $\alpha \in [0, 1]$, an individual mismatched homeowner prefers to sell in the investment (search) market at the given θ no matter what others choose to do. For certain θ, however, there may exist some $\alpha_D(\theta) \in [0, 1]$ such that $D(\theta, \alpha_D(\theta)) = 0$. In this case, equilibrium requires a certain

[13] With multiple types, the assumption of perfect information in bargaining is perhaps stretching a bit. We could have specified a bargaining game with imperfect information as in Harsanyi and Selten (1972), Chatterjee and Samuelson (1983), or Riddell (1981), for instance. It is not clear what may be the payoffs for the added complications.

[14] Lemma 9 in the Appendix presents two sets of prices and asset values, one derived under the assumption that $\Delta \leq 0$ and the other $\Delta \geq 0$. In either case, Δ is seen to have the same sign as D in (22).

fraction $\alpha_D(\theta)$ of mismatched homeowners selling in the investment market and the rest selling in the search market. In all, we can define a correspondence

$$\alpha_A(\theta) = \begin{cases} 1 & D(\theta, 1) > 0 \\ \alpha_D(\theta) & D(\theta, \alpha_D(\theta)) = 0 \\ 0 & D(\theta, 0) < 0 \end{cases},$$

that gives the fraction of mismatched homeowners selling in the investment market from the households' optimal decisions formed by *arbitraging* between selling in the two markets.

To further characterize $\alpha_A(\theta)$, we need to sign $\partial D/\partial\theta$. Given that η is increasing and μ is decreasing in θ, $D(\theta, \alpha)$ is guaranteed decreasing in θ throughout if

$$r_F \geq \frac{\beta_F}{\beta_H}\frac{r_H}{1+z} \equiv \widehat{r}_H;$$

otherwise, it can be shown that $D(\theta, \alpha)$ as a function of θ is U-shaped, first decreasing but eventually increasing under a fairly weak condition on η,[15] which we assume holds in the following.

Lemma 8 Suppose $\beta_F > \beta_H$.

a. *If* $r_F \geq \widehat{r}_H$, $\alpha_A(\theta)$ *is non-increasing throughout, given by*

$$\alpha_A(\theta) = \begin{cases} 1 & \theta \leq \widehat{\theta}_a \\ \alpha_D(\theta) & \theta \in \left(\widehat{\theta}_a, \widehat{\theta}_b\right) \\ 0 & \theta \geq \widehat{\theta}_b \end{cases},$$

where $\widehat{\theta}_a < \widehat{\theta}_b$ *are defined by, respectively,* $D\left(\widehat{\theta}_a, 1\right) = 0$ *and* $D\left(\widehat{\theta}_b, 0\right) = 0$, *and that* $\partial\alpha_D(\theta)/\partial\theta < 0$.

b. *If* $r_F < \widehat{r}_H$, *there are three possibilities:*

$$(i) \qquad \alpha_A(\theta) = 1 \text{ for } \theta \geq 0.$$

$$(ii) \qquad \alpha_A(\theta) = \begin{cases} 1 & \theta \leq \widehat{\theta}_1 \\ \alpha_D(\theta) & \theta \in \left(\widehat{\theta}_1, \widehat{\theta}_2\right) \\ 1 & \theta \geq \widehat{\theta}_2 \end{cases}.$$

where $\widehat{\theta}_1 < \widehat{\theta}_2$ *are defined by* $D\left(\widehat{\theta}_1, 1\right) = 0$ *and* $D\left(\widehat{\theta}_2, 1\right) = 0$ *along the decreasing and increasing portions of* $D(\theta, \alpha)$, *respectively. Here,* $\alpha_D(\theta)$ *is*

[15] The condition is $2\frac{\partial\eta}{\partial\theta}\left(\eta - \theta\frac{\partial\eta}{\partial\theta}\right) + \theta\frac{\partial^2\eta}{\partial\theta^2}\eta \leq 0$, which is guaranteed to hold if η is isoelastic.

first decreasing, reaching a minimum above zero, and then increasing toward 1.

$$(iii) \qquad \alpha_A(\theta) = \begin{cases} 1 & \theta \leq \widehat{\theta}_{1a} \\ \alpha_D(\theta) & \theta \in \left(\widehat{\theta}_{1a}, \widehat{\theta}_{1b}\right) \\ 0 & \theta \in \left[\widehat{\theta}_{1b}, \widehat{\theta}_{2a}\right] \\ \alpha_D(\theta) & \theta \in \left(\widehat{\theta}_{2a}, \widehat{\theta}_{2b}\right) \\ 1 & \theta \geq \widehat{\theta}_{2b} \end{cases},$$

where $\widehat{\theta}_{1a} < \widehat{\theta}_{1b} < \widehat{\theta}_{2a} < \widehat{\theta}_{2b}$ *are defined by, respectively,* $D\left(\widehat{\theta}_{1a}, 1\right) = 0$ *and* $D\left(\widehat{\theta}_{1b}, 0\right) = 0$ *along the decreasing portion of* $D(\theta, \alpha)$ *and* $D\left(\widehat{\theta}_{2a}, 0\right) = 0$ *and* $D\left(\widehat{\theta}_{2b}, 1\right) = 0$ *along the increasing portion of* $D(\theta, \alpha)$*. For* $\theta \in \left(\widehat{\theta}_{1a}, \widehat{\theta}_{1b}\right)$*,* $\partial\alpha_D(\theta)/\partial\theta < 0$*, whereas for* $\theta \in \left(\widehat{\theta}_{2a}, \widehat{\theta}_{2b}\right)$*,* $\partial\alpha_D(\theta)/\partial\theta > 0$*.*

Part (a) is concerned with where $D(\theta, \alpha)$ is decreasing in θ throughout. In this case, for the small θ, $D(\theta, \alpha) > 0$ and for the large θ, $D(\theta, \alpha) < 0$ for any $\alpha \in [0, 1]$. In between, there exists an interval $\left(\widehat{\theta}_a, \widehat{\theta}_b\right)$ along which $D(\theta, \alpha_D(\theta)) = 0$ for some unique $\alpha_D(\theta) \in (0, 1)$. Panels E and F of Figure 4 illustrate two such $\alpha_A(\theta)$ functions that follow. Part (b) is concerned with where $D(\theta, \alpha)$ is U-shaped, first decreasing but eventually increasing in θ. Now if the U-shaped $D(\theta, \alpha)$ stays above zero for all θ and $\alpha \in [0, 1]$, $\alpha_A(\theta) = 1$ always as in part (b.i). Panel A of Figure 4 depicts such an $\alpha_A(\theta)$ function. On the other hand, the U-shaped $D(\theta, \alpha)$ may only stay above zero for the smallest and largest θ for all $\alpha \in [0, 1]$. Part (b.ii) is concerned with the case of $D(\theta, 0)$ never falling to zero. Then, $\alpha_A(\theta)$ never reaches zero too. Panel B of Figure 4 illustrates an example of such an $\alpha_A(\theta)$. On the other hand, if $D(\theta, 0)$ does fall below zero for a range of θ, over the given range of θ, $\alpha_A(\theta) = 0$. Part (b.iii) defines such an $\alpha_A(\theta)$ function and Panels C and D of Figure 4 illustrate two examples.

In Lemma 8, we restrict attention to where $\beta_F > \beta_H$. In case $\beta_F = \beta_H$, $D(\theta, \alpha)$ in (22) is independent of α, which means that at a given θ, it is either greater than, equal to, or less than zero for all α. Then, if $D = 0$ holds at some θ', $\alpha_D(\theta') = [0, 1]$ and $\alpha_A(\theta)$ becomes a multi-valued correspondence. In the counterpart to Lemma 8(a), $\widehat{\theta}_a$ and $\widehat{\theta}_b$ would collapse into a single $\widehat{\theta}$, while in the counterpart to Lemma 8(b.iii), $\widehat{\theta}_{1a}$ and $\widehat{\theta}_{1b}$ would collapse into one $\widehat{\theta}_1$ and $\widehat{\theta}_{2a}$ and $\widehat{\theta}_{2b}$ into another $\widehat{\theta}_2$. At such θs, $\alpha_A(\theta) = [0, 1]$. In Panels C-F of Figure 4, the downward and upward-sloping segments would become vertical segments. Analogously, the two $\widehat{\theta}$ in Lemma 8(b.ii) would become one at which $\alpha_A(\theta) = [0, 1]$, while the U-shaped segment in Panel B of Figure 4 would vanish. Since $\alpha_A(\theta)$ for $\beta_F = \beta_H$ is merely the limit of $\alpha_A(\theta)$ for $\beta_F > \beta_H$ as $\beta_F \to \beta_H$ from above, for brevity and for ease of exposition, we shall only

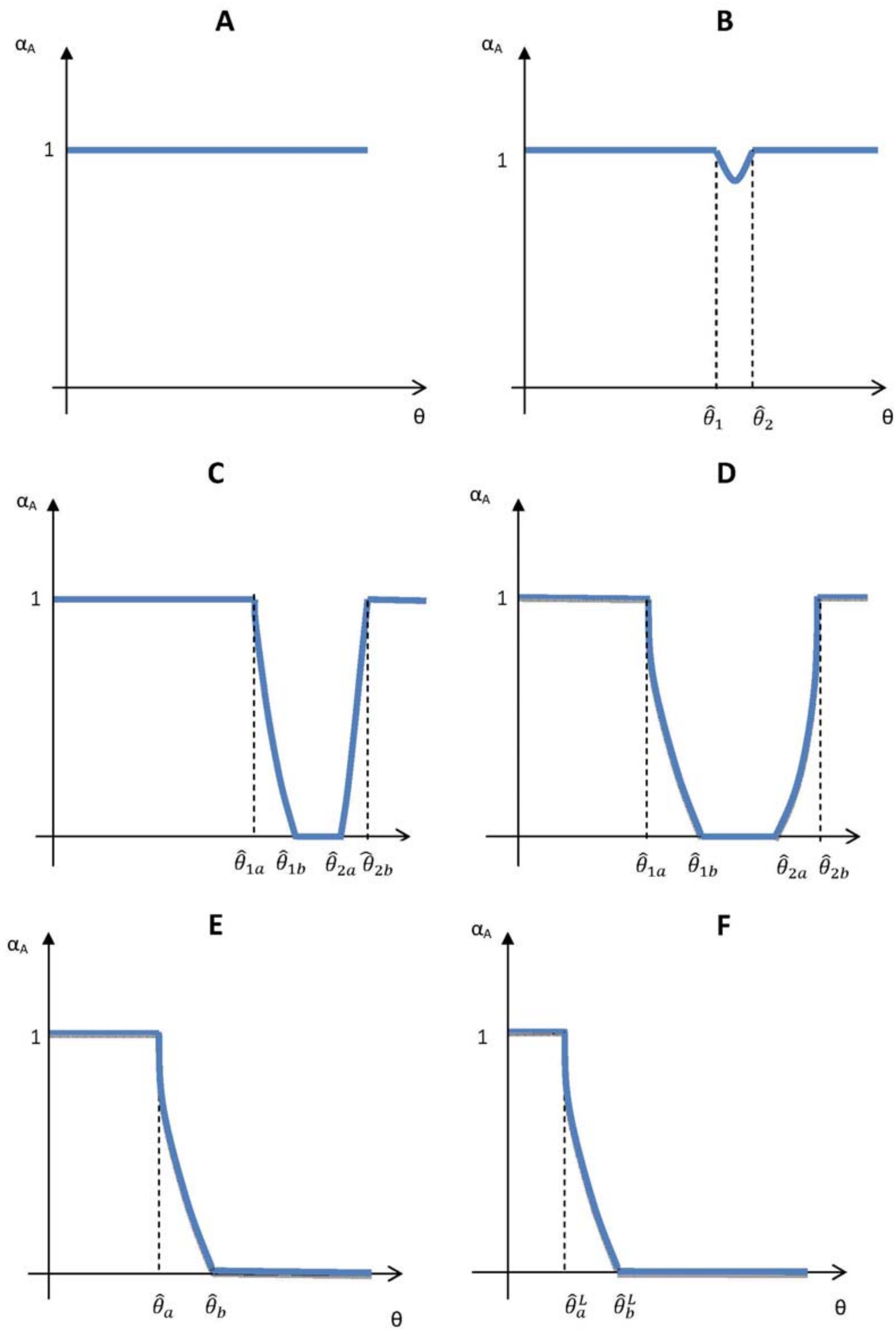

Figure 4: The $\alpha_A(\theta)$ function

present results for the case of $\beta_F > \beta_H$ hereinafter. The ommision is without loss of generality. In the quantitative analysis in Section 5, we will relax the restriction though and also consider the case of $\beta_F = \beta_H$.

The first general lesson from Lemma 8 is that facing a slack search market with a small θ, mismatched homeowners always welcome the opportunity to quickly dispose of their old houses in the investment market, whether or not $r_F \geq \widehat{r}_H$. In a slack search market where sellers outnumber buyers significantly, it can take a long time to sell. Selling to flippers quickly is obviously optimal. The novelty here is that mismatched homeowners can prefer to sell to flippers even when the search market is tight so that houses can be sold quickly there. Intuitively, when markets are tight, houses can be sold not just quickly but also at high prices. Then, mismatched homeowners may prefer to just sell right away in the investment market to capitalize on the high housing price sooner. According to Lemma 8(b), this can happen when flippers can finance investment at a low r_F or, by virtue of bargaining with a large β_F, they are able to negotiate a high price when selling in the search market later on. In either case, flippers can pay particularly high prices to attract mismatched homeowners to sell in the investment market.

2.6 Equilibrium

We now have two steady-state relations between α and θ: the $\theta_T(\alpha)$ function in Lemma 2 from the turnover equations and the $\alpha_A(\theta)$ function in Lemma 8 from mismatched homeowners arbitraging between selling in the two markets. A steady-state equilibrium is any $\{\alpha, \theta\}$ pair that simultaneously satisfies the two relations.

3 Analysis

3.1 Existence of equilibrium

To show the existence of equilibrium, it is useful to define $F(\alpha) \equiv \alpha_A(\theta_T(\alpha))$, a continuous function mapping $[0, 1]$ into itself. A steady-state equilibrium is any fixed point of F.

Proposition 1 *Equilibrium exists for all $\{r_H, r_F, \upsilon, q, \delta, \beta_F, \beta_H, H\}$ tuple.*

3.2 Multiplicity

To check for multiplicity, we begin with inverting $\theta_T(\alpha)$ in Lemma 2 to define $\alpha_T \equiv \theta_T^{-1}$, whereby $\alpha_T : [\theta_T(0), \theta_T(1)] \to [0, 1]$. Given that $\partial\theta_T/\partial\alpha > 0$, likewise, $\partial\alpha_T/\partial\theta > 0$ for $\theta \in [\theta_T(0), \theta_T(1)]$.[16] That is, $\alpha_T(\theta)$ increases continuously from 0 at $\theta = \theta_T(0)$ to 1 at $\theta = \theta_T(1)$, as illustrated in Figure 5.

[16] The closed-form solution for $\alpha_T(\theta)$ is given by (56) in the Appendix.

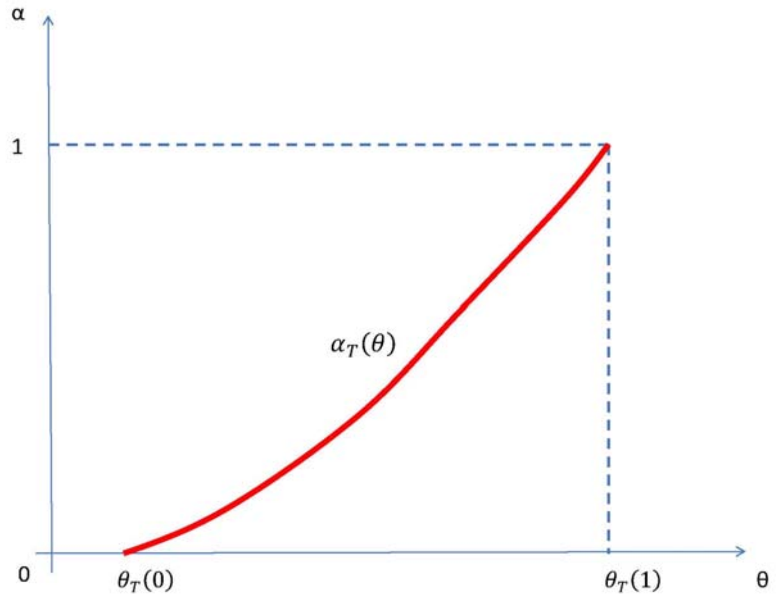

Figure 5: The $\alpha_T(\theta)$ function

With $\alpha_T(\theta)$ from Figure 5 and the applicable $\alpha_A(\theta)$ from Figure 4, a steady-state equilibrium is any θ at which $\alpha_A(\theta) = \alpha_T(\theta)$. Now, if $\alpha_A(\theta_T(0)) = 0$, then $\theta = \theta_T(0)$ and $\alpha = 0$ is a steady-state equilibrium since by construction, $\alpha_T(\theta_T(0)) = 0$. In this equilibrium, all sales and purchases are between two end-users while turnover is slowest. On the other hand, if $\alpha_A(\theta_T(1)) = 1$, then $\theta = \theta_T(1)$ and $\alpha = 1$ is a steady-state equilibrium since by construction, $\alpha_T(\theta_T(1)) = 1$. In this equilibrium, all transactions are intermediated and turnover is fastest. In between, there can be equilibria at where $\alpha = \alpha_D(\theta) \in (0, 1)$ for some $\theta \in (\theta_T(0), \theta_T(1))$. In such equilibria, with mismatched homeowners indifferent between selling in the investment and search markets, a fraction, but only a fraction, of all transactions are intermediated.

Since $\alpha_T(\theta)$ is strictly increasing over $[\theta_T(0), \theta_T(1)]$, equilibrium is unique if $\alpha_A(\theta)$ is non-increasing throughout. But $\alpha_A(\theta)$ can be strictly increasing over a given range of θ for certain parameter configurations, in which case the multiplicity of equilibrium becomes a distinct possibility.

Proposition 2 *If $r_F \geq \widehat{r}_H$, equilibrium is guaranteed unique. If $r_F < \widehat{r}_H$, $\alpha_A(\theta)$ is as given in Lemma 8(b.iii), and if $\theta_T(0) \leq \widehat{\theta}_{2a} < \widehat{\theta}_{2b} \leq \theta_T(1)$, there exist at least two steady-state equilibria.*

Figures 6 and 7 illustrate the situation covered by the second part of the Proposition. In both figures, there are indeed three equilibria. Consider first the $\theta_T(1)$ equilibrium. At $\theta = \theta_T(1)$, there is a tight search market in which houses are sold quickly and also at high prices. Mismatched homeowners then find it advantageous

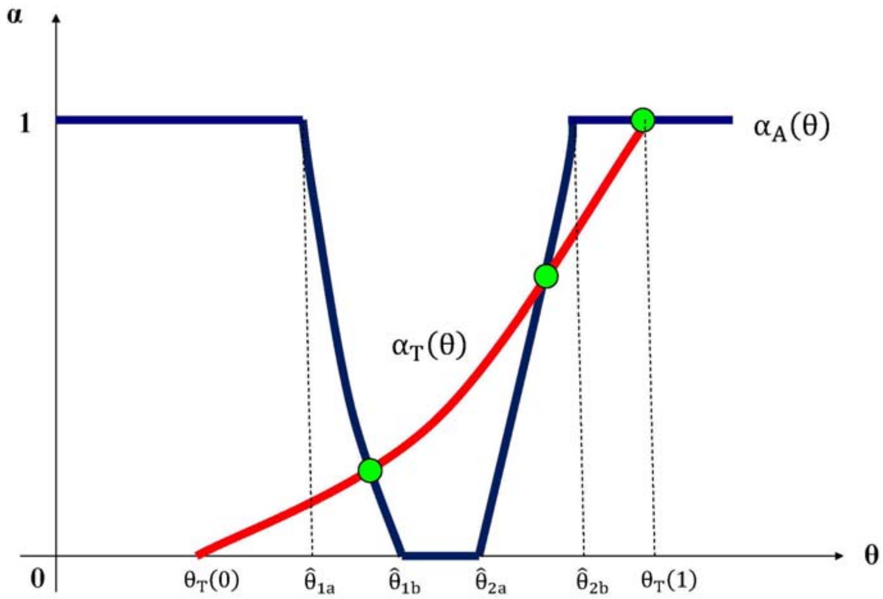

Figure 6: Multiple Equilibrium: $\widehat{\theta}_{1b} > \theta_T\left(0\right)$

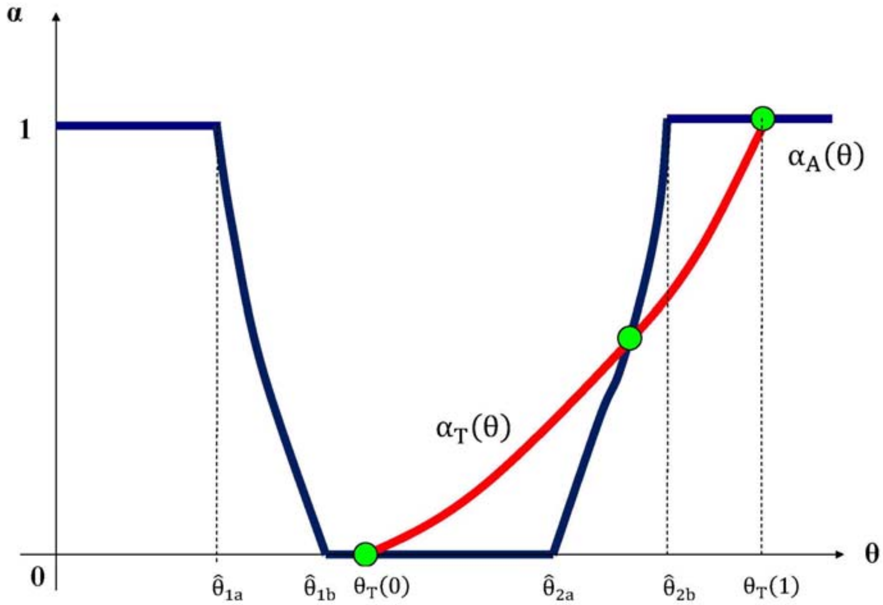

Figure 7: Mutliple Equilibrium: $\theta_T\left(0\right) > \widehat{\theta}_{1b}$

to capitalize on the high prices sooner by selling in the investment market right away. In the meantime, if all mismatched houses are sold in the investment market in the first instance, the rapid turnover will indeed give rise to a tight market. In this way, $\theta = \theta_T(1)$ and $\alpha = 1$ is equilibrium in Figures 6 and 7. For smaller θ, mismatched homeowners' incentives to sell in the investment market are weakened. But precisely because fewer or none at all mismatched houses are sold in the investment market, a relatively slack market will emerge from the slow turnover. As a result, a smaller α and a smaller θ is also equilibrium in Figures 6 and 7.

Consider a small perturbation from the middle equilibrium in Figures 6 and 7 that knocks the $\{\theta, \alpha\}$ pair off to the right of the $\alpha_A(\theta)$ schedule. Then, $D(\theta, \alpha) > 0$ since $\partial D/\partial\alpha < 0$, after which all mismatched homeowners would find it better to sell in the investment market. The increase in turnover would then raise θ. Eventually the market should settle at the $\theta_T(1)$ equilibrium. Conversely, a perturbation that knocks the $\{\theta, \alpha\}$ pair off to the left of $\alpha_A(\theta)$ schedule from the middle equilibrium in Figures 6 and 7 should send the market to a smaller θ and α equilibrium. In general, an equilibrium that occurs at where $\alpha_A(\theta)$ is increasing should be unstable. By analogous arguments, the other equilibria in the two figures should be locally stable. Hence, there are not just multiple steady-state equilibria but also multiple locally *stable* steady-state equilibria.

With multiple steady-state equilibria, the presence of flippers' in the market can be fickle, especially when the equilibrium the market happens to be in is unstable. In general, where there are multiple equilibria, any seemingly unimportant shock can dislocate the market from one equilibrium and move it to another, causing catastrophic changes in flippers' market share, turnover, and transaction volume. To follow such discrete changes in the activities of flippers can be significant fluctuations in housing price, a subject we shall address in Section 3.4. And then in Section 5, we will calibrate the model to several observable characteristics of the U.S. housing market to assess quantitatively the importance of such a channel of volatility.

3.3 Cost of financing and flippers' market share

Flippers can afford to pay the highest price when they can finance investment at the least cost. As r_F increases, other things equal, their presence should diminish. More precisely, by Lemma 10 in the Appendix:

(A) For sufficiently small but positive r_F, $D(\theta, \alpha) > 0$ for all θ and $\alpha \in [0, 1]$ so that the $\alpha_A(\theta) = 1$ throughout as shown in Panel A of Figure 4.

(B) As r_F increases to some given level below $\widehat{r}_H$, $D(\theta, 1) = 0$ begins to hold at two values for θ, while in between $D(\theta, \alpha_D(\theta)) = 0$ for some $\alpha_D(\theta) \in (0, 1)$. Panel A turns into Panel B. In the meantime, $\widehat{\theta}_1$ decreases and $\widehat{\theta}_2$ increases along with the increase in r_F.

(C) Thereafter, as r_F continues to increase, $D(\theta, 0) = 0$ also begins to hold at two values for θ. In between $D(\theta, 0) < 0$. Panel B turns into Panel C.

(D) While r_F remains below $\widehat{r}_H$, $\partial\widehat{\theta}_{1i}/\partial r_F < 0$ and $\partial\widehat{\theta}_{2i}/\partial r_F > 0$ for $i = a, b$. Panel C turns into Panel D.

(E) In the limit as $r_F \to \widehat{r}_H$, $\widehat{\theta}_{2i} \to \infty$ for $i = a, b$, whereas $\widehat{\theta}_{1i}$, $i = a, b$ tend to the respective unique roots of $D(\theta, 1) = 0$ and $D(\theta, 0) = 0$ at which $r_F = \widehat{r}_H$ just holds. Panel D turns into Panel E.

(F) For $r_F \geq \widehat{r}_H$, $\partial\widehat{\theta}_i/\partial r_F < 0$ for $i = a, b$. The limits $\widehat{\theta}_i^L \equiv \lim_{r_F\to\infty} \widehat{\theta}_i > 0$, for $i = a, b$. If $\beta_H \geq 1/2$, $\widehat{\theta}_i^L < 1$. The $\alpha_A(\theta)$ in Panel E gradually evolves toward the one in Panel F.

Granted that $\alpha_T(\theta)$ is independent of r_F, the effects of an increase in r_F on equilibrium $\{\alpha, \theta\}$ can then be read off by superimposing the same given $\alpha_T(\theta)$ successively into Panels A to F of Figure 4. We can conclude from this exercise the following.

Proposition 3

a. *For sufficiently small but positive r_F, the unique equilibrium is $\theta = \theta_T(1)$ and $\alpha = 1$.*

b. *For larger r_F, α must fall below unity and θ* below *$\theta_T(1)$ in equilibrium if $\beta_H \geq 1/2$.*

   i. *If $\theta_T(0) \geq \widehat{\theta}_b^L$, as r_F becomes large enough, $\theta = \theta_T(0)$ and $\alpha = 0$.*

   ii. *Otherwise, α stays positive for arbitrarily large r_F.*

c. *If $\theta = \theta_T(1)$ and $\alpha = 1$ is not equilibrium at a certain r_F, the pair is not equilibrium for any larger r_F. If $\theta = \theta_T(0)$ and $\alpha = 0$ is equilibrium at a certain r_F, the pair remains equilibrium for any larger r_F.*

d. *In any $\alpha = \alpha_D(\theta)$ equilibrium at which $\partial\alpha_D/\partial\theta < 0$, α is decreasing in r_F.*

e. *In any $\alpha = \alpha_D(\theta)$ equilibrium at which $\partial\alpha_D/\partial\theta > 0$, α is increasing in r_F.*

Parts (a)-(d) of the Proposition conform to the intuitive notion that an increase (decrease) in r_F should have a negative (positive) impact on flippers's market share. What seems surprising is that α can remain strictly positive in equilibrium even for an arbitrarily large r_F (part (b.ii)), under which flippers can only finance investment at a huge disadvantage vis-a-vis ordinary households. The condition for this to be the case, $\theta_T(0) < \widehat{\theta}_b^L$, holds for small $\theta_T(0)$. By (54) in the proof of Lemma 2, $\partial\theta_T(0)/\partial H < 0$, and that $\lim_{H\to1} \theta_T(0) = 0$. With a larger housing stock, there are more units for sale, other things being equal. In the meantime, if there are fewer households in rental housing, there will be fewer buyers in the search market. Then, a role for flippers can remain no matter what, if the slack in the end-user search market makes selling in the market very difficult.

Part (e) that an increase (decrease) in r_F can have a positive (negative) impact on flippers' market share in equilibrium is counterintuitive too. In Figures 6 and 7, the upward-sloping portion of $\alpha_A(\theta)$ along with the horizontal segment to its right are for values of θ for which mismatched homeowners prefer to sell in the investment market to quickly capitalize on the high housing price. Now, at a larger r_F, flippers can only pay lower prices, moving that portion of $\alpha_A(\theta)$ out to the right and making it intersect the upward-sloping $\alpha_T(\theta)$ at a larger α.

On the contrary, by Part (d), an increase (decrease) in r_F will have the expected negative (positive) impact on flippers' market share if equilibrium is at where $\partial\alpha_D/\partial\theta < 0$. In Figure 6, the downward-sloping portion of $\alpha_A(\theta)$ along with the horizontal segment to its left are for values of θ for which mismatched homeowners find it advantageous to sell in the investment market because it can take a long time to sell in a slack search market. An increase in r_F, by lowering the price flippers are able to offer to mismatched homeowners, shortens the interval. Then $\alpha_A(\theta)$ would only meet the downward-sloping $\alpha_T(\theta)$ at a smaller α and θ.

On the whole, one can conclude that at a larger r_F, fewer transactions would be intermediated if one is willing to dismiss any equilibrium at where $\partial\alpha_D/\partial\theta > 0$ on stability grounds and the possibility that agents may coordinate to a larger α equilibrium in case there exist multiple equilibria. However, we do not think that the analysis, strictly speaking, allows us to rule out occasions, admittedly rare, in which an increase in r_F can, rather perversely, be followed by a heightened presence of flippers in the model housing market.

3.4 Housing prices

Housing prices in no-intermediation equilibrium Absent flippers, all housing market transactions are between pairs of end-user households at[17]

$$p_H = \frac{\beta_H(\eta + r_H) - (1-\beta_H)\mu}{(r_H + \delta + \beta_H\eta)\, r_H}\upsilon + \frac{q}{r_H}, \tag{23}$$

evaluated at $\theta = \theta_T(0)$.

Housing prices in fully-intermediated equilibrium In a fully-intermediated equilibrium, all houses are first sold from mismatched homeowners to flippers at[18]

$$p_{FB} = \frac{\beta_F\eta(\upsilon + q)}{(r_F + \beta_F\eta)\, r_H + (\delta + (1-\beta_F)\mu)\, r_F}, \tag{24}$$

[17] From (37) evaluated at $\alpha = 0$. The equations for the housing prices and asset values referred to hereinafter can be found in Lemma 9 in the Appendix.

[18] From (45) evaluated at $\alpha = 1$.

in the investment market and then at[19]

$$p_{FS} = \frac{\beta_F (\eta + r_F)(\upsilon + q)}{(r_F + \beta_F \eta) r_H + (\delta + (1 - \beta_F)\mu) r_F}, \tag{25}$$

from flippers to end-user households in the search market, both evaluated at $\theta = \theta_T(1)$. Clearly, $p_{FB} < p_{FS}$. Now, houses sold from households to flippers stay on the market for a vanishingly small time interval, whereas houses sold from flippers to households in the search market stay on the market for, on average, $1/\eta > 0$ units of time. There should then be a positive cross-section relation between prices and TOM in the model housing market, as in the real-world housing market. Besides, with $p_{FB} < p_{FS}$, the model trivially predicts that houses bought by flippers are at lower prices than are houses bought by non-flippers. Both Depken et al. (2009) and Bayer et al. (2011) find the tendency to hold in their respective hedonic price regressions.

Housing prices in partially-intermediated equilibrium In a steady-state equilibrium in which mismatched homeowners sell in both the investment and search markets, in addition to the two prices[20]

$$p_{FB} = \frac{\beta_F \eta}{r_F (r_H + \delta + \beta_H \eta)} \upsilon, \tag{26}$$

$$p_{FS} = \frac{\beta_F \eta + \beta_F r_F}{r_F (r_H + \delta + \beta_H \eta)} \upsilon, \tag{27}$$

for transactions between a flipper and an end-user household, there will also be transactions between two end-user households, carried out at price[21]

$$p_H = \frac{\beta_F \eta + \beta_H r_F}{r_F (r_H + \delta + \beta_H \eta)} \upsilon. \tag{28}$$

Here, we have $p_{FB} < p_H < p_{FS}$ for $\beta_F > \beta_H$. Just as in the fully-intermediated equilibrium, a positive relation between prices and TOM holds in the cross section and houses bought by flippers are at lower prices.

Prices across equilibria Across steady-state equilibria, $\theta = B/S$ is largest in the equilibrium where flippers are most numerous. Then, prices should be highest in such an equilibrium where the competition among buyers is most intense.

Proposition 4 *Across steady-state equilibria in case there exist multiple equilibria, housing prices in both the search and investment markets are highest in the equilibrium with the tightest market and lowest in the equilibrium with the slackest market.*

[19] From (44) evaluated at $\alpha = 1$.

[20] From (39) or (45) and (38) or (44), respectively, all evaluated at $D(\theta, \alpha) = 0$.

[21] From (37) or (43), both evaluated at $D(\theta, \alpha) = 0$

Now, a direct corollary of the Proposition and Lemmas 4-7 is that:

Proposition 5 *Across steady-state equilibria in case there exist multiple equilibria, prices, vacancies, and transaction volumes increase or decrease together from one to another equilibrium, whereas average TOM and TBM move with the former set of variables in the opposite direction.*

Interest rate shocks As usual, in the present model, interest rates can play an important role in determining housing prices. First, in a no-intermediation equilibrium, a decline in r_H can lead to higher prices for sufficiently large $\theta_T(0)$ and/or q, as can be verified by differentiating (23), but market tightness, vacancies, turnover, and transaction volumes are invariant to an interest rate shock that has no effects on α. In the entire absence of flippers, not surprisingly, r_F plays no role at all.

In a fully-intermediated equilibrium, r_F should certainly be an important factor in determining housing prices. Indeed, both p_{FB} and p_{FS} are decreasing in r_F, as can be verified by differentiating (24) and (25). Just as in the no-intermedation equilibrium, such interest rate shocks will leave no impact on market tightness, vacancies, turnover, and transaction volumes, if all transactions were already intermediated in the first place.

In a partially-intermediated equilibrium, prices in the search market, p_{FS} and p_H, as well as in the investment market p_{FB}, are decreasing in r_F, just as they are in a fully-intermediated equilibrium. But where θ was not already fixed at the boundary of $\theta_T(1)$, housing prices can also vary to follow any movements in θ triggered by the given interest rate shock. As expected, the prices given in (26)-(28) are higher when the search market is tighter. Hence, if a given positive (negative) interest rate shock should cause α and therefore θ to decrease (increase), there will be lower (higher) housing prices to follow because of a direct negative (positive) effect and of an indirect effect via dampening (raising) flippers' presence in the market. In this case, when the two effects work in the same direction, a given interest rate shock can cause substantially more housing price volatility than in a model that only allows for the usual effect of interest rates on asset prices.

However, a positive interest rate shock need not cause θ and α to fall. By Proposition 3(e), along any $\alpha_D(\theta)$ equilibrium where $\partial\alpha_D(\theta)/\partial\theta > 0$, the given interest rate shock will be followed by increases in θ and α. Furthermore, in case there exist multiple equilibria, the shock can possibly dislocate the market from a given equilibrium and send it to another equilibrium. In what direction housing prices will move then cannot be unambiguously read off from (26)-(28) as the direct effect of any interest rate shock and the indirect effect via the movements in θ can affect housing prices differently. To proceed, we solve $D(\theta, \alpha) = 0$ for r_F and substitute the result into (26)-(28), respectively,

$$p_{FB} = \frac{\beta_H \eta + ((\beta_F - \beta_H)\alpha_T - (1 - \beta_H))\mu}{r_H (r_H + \delta + \beta_H \eta)} \upsilon + \frac{q}{r_H}, \tag{29}$$

$$p_{FS} = \frac{\beta_H \eta + \beta_F r_H + \left(\left(\beta_F - \beta_H\right) \alpha_T - \left(1 - \beta_H\right)\right) \mu}{r_H \left(r_H + \delta + \beta_H \eta\right)} \upsilon + \frac{q}{r_H}, \tag{30}$$

$$p_H = \frac{\beta_H \eta + \beta_H r_H + \left(\left(\beta_F - \beta_H\right) \alpha_T - \left(1 - \beta_H\right)\right) \mu}{r_H \left(r_H + \delta + \beta_H \eta\right)} \upsilon + \frac{q}{r_H}. \tag{31}$$

The three equations are independent of r_F – whatever effects a given change in r_F will have on housing prices are subsumed through the effects of changes in θ that follow the change in r_F obtained from holding $D(\theta, \alpha) = 0$. To evaluate the the effects of r_F on housing prices then is to differentiate these three expressions just with respect to θ.

Proposition 6 *Across steady-state equilibria and holding $D(\theta, \alpha) = 0$, a shock to r_F, whether positive or negative, will cause housing prices to increase (decrease), as long as to follow the interest rate shock are increases (decreases) in θ and α.*

By Proposition 6, the indirect effect of an interest rate shock on housing prices through the changes in flippers' presence and then in market tightness always dominates the direct effect shall the two be of opposite tendencies. A surprising implication then is that housing prices can actually go up in response to an increase in flippers' cost of financing, if to follow the higher interest rate is also a heightened presence of flippers' in the market. In any case, a direct corollary of Lemmas 4-7 and Proposition 6 is that:

Proposition 7 *Across steady-state equilibria and holding $D(\theta, \alpha) = 0$, a shock to r_F will cause housing prices, transaction volumes, and vacancies to move in the same direction, whereas average TOM and TBM will move in the opposite direction.*

3.5 A general interest rate shock

So far, we have restricted attention to analyzing how changes in r_F alone affect the extent of intermediation and the consequent effects on housing prices. It turns out that many of the implications continue to hold for general changes in interest rate that affect both flippers and ordinary households alike. To begin, write R for r_H/r_F in (22),

$$D(\theta, \alpha) \equiv \left(\beta_F R - \beta_H - z\beta_H\right) \eta + \left(1 - \beta_H - \alpha\left(\beta_F - \beta_H\right)\right) \mu - \left(\delta + r_H\right) z \tag{32}$$

Then equiproportionate increases in r_H and r_F, while leaving R unchanged, lower $D(\theta, \alpha)$. A general increase in interest rate thus weakens mismatched homeowners' incentives to sell in the investment market, just as an increase in r_F, holding fixed r_H, does. Analogous to Proposition 3 is that:

Proposition 8 *Holding constant R at some given level,*

a. *for sufficiently large r_H, in equilibrium, $\theta < \theta_T(1)$ and $\alpha < 1$. Eventually, as r_H rises above a certain level, $\theta = \theta_T(0)$ and $\alpha = 0$ must obtain.*

b. *if $\theta = \theta_T(1)$ and $\alpha = 1$ is not equilibrium at some r_H, the pair is not equilibrium for larger r_H. If $\theta = \theta_T(0)$ and $\alpha = 0$ is equilibrium at some r_H, the pair remains equilibrium for larger r_H.*

c. *In any $\alpha = \alpha_D(\theta)$ equilibrium at which $\partial\alpha_D/\partial\theta < 0$, α is decreasing in r_H.*

d. *In any $\alpha = \alpha_D(\theta)$ equilibrium at which $\partial\alpha_D/\partial\theta > 0$, α is increasing in r_H.*

The effects of the general increase in the cost of financing on housing prices are similar to those of an increase in r_F alone.

Proposition 9

a. *In the fully-intermediated equilibrium, equiproportionate increases in r_H and r_F lower housing prices. The same effect is felt in the no-intermediation equilibrium for sufficiently large $\theta_T(0)$* and/or *q.*

b. *In a partially-intermediated equilibrium,*

   i. *equiproportionate increases in r_H and r_F, holding θ fixed, lower housing prices;*

   ii. *across steady-state equilibria and holding $D(\theta, \alpha) = 0$, equiproportionate changes in r_H and r_F, whether positive or otherwise, cause p_{FB} to increase (decrease) as long as to follow the interest rate shocks are increases (decreases) in θ and α for $\beta_F = \beta_H = 1/2$* and *$R \in [0, 1+z]$; the same effect is felt on p_{FS}* and *p_H for R in neighborhoods of $R = 0$, 1, and $1+z$.*

Notice that by (b.i), if to follow the equiproportionate increases in r_H and r_F is a decline in flippers' presence, housing prices must unambiguously fall, just as when an increase in r_F alone causes θ and α to fall will lower housing prices for sure. More generally, (b.ii) is concerned with how prices may vary when the interest rate shocks may be followed by either an increase or a decline in θ, as in the situations covered in Proposition 6. Also as in Proposition 6, here prices will increase if θ and α happen to rise to follow the interest rate shocks, positive or otherwise, if the values of β_F, β_H, and R are chosen appropriately. These restrictions are sufficient, but not necessary, conditions and we suspect that the conclusions should hold under considerably weaker conditions.

3.6 Welfare

In a steady-state equilibrium where $D(\theta,\alpha)=0$, asset values for matched and mismatched homeowners, renters, and flippers are given by, respectively,[22]

$$V_M=\frac{(r_H+\beta_H\eta)\,\upsilon}{r_H\,(r_H+\delta+\beta_H\eta)}, \tag{33}$$

$$V_U=\frac{\beta_H\eta\upsilon}{r_H\,(r_H+\delta+\beta_H\eta)}, \tag{34}$$

$$V_R=\frac{(\beta_H r_F-\beta_F r_H)\,\eta\upsilon}{r_F r_H\,(r_H+\delta+\beta_H\eta)}, \tag{35}$$

$$V_F=\frac{\beta_F\eta\upsilon}{r_F\,(r_H+\delta+\beta_H\eta)}. \tag{36}$$

It is straightforward to verify that V_M, V_U, and V_F are all increasing in θ. Any homeowners—matched or mismatched, end-users or flippers—benefit from the higher housing prices in a tighter market. The asset value for households in rental housing V_R, however, is decreasing in θ if $\beta_H r_F<\beta_F r_H$, which is a necessary condition for the multiplicity of equilibrium (Proposition 2). In this case, would-be buyers are made worse off by the higher housing prices in the tighter market. In a comparison between two steady-state equilibria both at where $D(\theta,\alpha)=0$, homeowners are better off whereas renters are worse off in the larger θ equilibrium than in the smaller θ equilibrium. Any two such equilibria cannot then be Pareto-ranked. The same conclusion applies to comparisons between a $D(\theta,\alpha)>0$ equilibrium and a $D(\theta,\alpha)=0$ equilibrium and between a $D(\theta,\alpha)=0$ equilibrium and a $D(\theta,\alpha)<0$ equilibrium.

Even though the equilibria cannot be Pareto-ranked, perhaps they can be ranked by aggregate welfare as measured by the sum of the asset values for all agents,

$$W=n_M V_M+n_U V_U+n_R V_R+n_F V_F.$$

At where $D(\theta,\alpha)=0$, the asset values are given by (33)-(36). Again, consider a comparison between two steady-state equilibria both at where $D(\theta,\alpha)=0$. Substituting from (57)-(60) in the Appendix for the various steady-state measures of agents and simplifying,

$$W=\frac{\eta\upsilon}{r_H+\delta+\beta_H\eta}\left(\frac{H}{\eta+\delta}+\frac{\beta_F}{r_F}(H-1)+\frac{\beta_H}{r_H}\right).$$

This expression is guaranteed increasing in θ for large H. In this case, there is a larger aggregate asset value in the tighter and higher-priced equilibrium where more transactions are intermediated. For smaller H, however, W above is decreasing in θ, so that there is only a smaller W in such an equilibrium than in one in which fewer

[22] The first two equations are from (41) and (42), respectively. The last two are from (40) and (39), respectively, both evaluated at $D(\theta,\alpha)=0$.

transactions are intermediated. Thus, it seems that the equilibria cannot in general be ranked by even aggregate asset value. While all agents, except for households in rental housing, benefit from the higher housing prices in a more active equilibrium and that more households are matched in the steady state amid a shorter average Time-Between-Matches, W need not be higher. Would-be buyers in rental housing are more numerous and they suffer a lower asset value due to the higher prices. Such a negative impact on W can more than offset the positive effects of a more active market, especially when there is a small housing stock. Intuitively, when there is a small H, there are few house owners to benefit from the higher prices and faster turnover, while there are many would-be buyers to suffer from the same higher prices and the longer wait for owner-occupied housing. When owner-occupied houses are scarce to begin with, leaving more houses vacant in the hands of flippers can be disproportionately costly.[23] Conversely, in a market endowed with a large H, it is hardest to sell and flippers' role in speeding up turnover is most valued.

4 Time-series relations among housing price, transaction volume, and vacancy

By Propositions 5, 7, and 9, any movement from one to another steady-state equilibrium would involve housing prices, transaction volumes, and vacancies all moving in the same direction. The positive time-series relation between housing prices and transaction volumes is well-known and numerous models have been constructed to account for it. Unique to our analysis is that vacancies should also move in the same direction with the two variables.

The prediction is not obviously inconsistent with the pictures depicted in Figures 1-3 in the Introduction. In a more systematic analysis, we first verify that in the 1981Q1 to 2011Q3 sample period, the three variables are all $I(1)$ at conventional significance levels. Next, we test for cointegration. Assuming the absence of any time trends and intercepts in the cointegrating equations, both the Trace test and the Max-eigenvalue test indicate two such equations, whose normalized forms read

$$\text{Price} - 6045.51 \times \text{Vacancy} = 0,$$

$$\text{Transaction} - 0.74 \times \text{Vacancy} = 0,$$

which together imply that the three variables tend to move in the same direction from one to another long-run equilibrium over time. With other time trend and intercept assumptions, either one or both of the tests suggest that there exist only one or as many as three cointegrating equations. In a single cointegrating equation with nonzero coefficients for all three variables, at least two of the three coefficients must be of

[23] Masters (2007) is also a model in which intermediation in a search and matching environment can be wasteful.

the same sign. Then, the two variables concerned must co-move in opposite directions across long-run equilibria. With as many cointegrating equations as the number of variables, there exist definite long-run values for the three variables, which rules out the possibility of the system moving from one to another long-run equilibrium altogether. Restricting a priori to two cointegrating equations in the estimation, however, we always obtain two equations whose coefficients have the same signs as those in the system above whatever the trend and intercept assumptions are. Then, any long-run movements for the three variables must be in the same direction.

5 Quantitative predictions on volatility

Given the possible multiplicity of equilibrium and that an interest rate shock may have important effects on the extent of intermediation, the model can be consistent with a volatile housing market. The question remains as to how important quantitatively such channels of volitality can be. In this section, we calibrate the model to several observable characteristics of the U.S. housing market and study by how much housing prices can fluctuate across steady-state equilibria and in response to interest rate shocks.

To begin, we take a time unit in the model to be a quarter of a year and assume a Cobb-Douglas matching function whereby $\eta(\theta) = a\theta^b$. We set a priori the mismatch rate $\delta = 0.014$ to calibrate a two-year mobility rate of 11.4% for owner-occupiers reported in Ferreira et al. (2010) and $b = 0.84$, which is the elasticity of the seller's matching hazard with respect to the buyer-seller ratio reported in Genesove and Han (2012). Next, the parameters a and H and the share of mismatched households selling to flippers α are chosen to calibrate:

1. a quarterly transaction rate of owner-occupied houses of 1.78%
2. a vacancy rate of owner-occupied houses of 1.84%
3. the share of houses bought by flippers among all transactions of owner-occupied houses equal to 19%

The first two targets are, respectively, the average quarterly transaction rate and the average vacancy rate for the period 2000Q1-2006Q4, calculated from our dataset for the plots in Figures 1-3 and the estimations in Section 4. Estimates of the share of houses bought by flippers come from two sources. First, Haughwout et al. (2011) report that the share of all new purchase mortgages in the whole of the U.S. taken out by investors is around 25% on average during the period 2000Q1-2006Q4,[24] where an investor is an individual who holds two or more first-lien mortgages. Because an

[24] The investor share peaks at 35% for the whole of the U.S. and 45% for the "bubble states" in 2006.

investor in Haughwout et al. may intend to hold the house as a long-term investment, the 25% share is probably an overestimate of the true flippers' share. Second, Depken et al. (2009) report that for the same period, on average, 13.7% of housing market transactions are for houses sold again within the first two years of purchase in the metropolitan Las Vegas area.[25]. Because not all houses bought for short-term flips can actually be sold within two years, the 13.7% share is probably an underestimate of the true flippers's share. Our 19% target is obtained by taking a simple average of the two estimates.

Given the targets, denoted as x_i, $i = 1, 2, 3$, respectively, we then choose a, H, and α to

$$\min \left\{ \sum_{i=1}^{3} \left(\frac{x_i - \widehat{x}_i}{x_i} \right)^2 \right\},$$

subject to

$$a \leq 1.2,$$
$$0.6 \leq H < 1,$$
$$0 \leq \alpha \leq 1,$$

where the $\widehat{x}_i$'s are the model's calibrated values of the corresponding targets.[26] The minimization is carried out via a grid search with a grid size of 0.005 for each of a, H, and α. The first constraint is for expediency in the grid search and is not binding. Given that H in the model is the stock of owner-occupied houses relative to the population of households demanding such housing, anything near the lower bound of the second constraint is probably unreasonable, whereas the model is not well-behaved if H exceeds the upper bound of the constraint. In all, the minimization yields $a = 0.085$, $H = 0.865$, and $\alpha = 0.25$ at which the calibrated values of the three targets are reported in the second column of Table 1.

Table 1: Calibration Targets and Calibrated Model Values

	Targeted value	Calibrated value
Transaction rate (quarterly)	0.0178	0.016
Vacancy rate	0.0184	0.018
Flippers' share in transactions	0.19	0.19

Thus far in the calibration, we have effectively identified $\alpha_T = 0.25$ as equilibrium. For equilibrium to be indeed at $\alpha = 25$, we need to pick the values for $\{q, v, \beta_H, \beta_F, r_H, r_F\}$ to force $\alpha_A = 0.25$ as well. Since only the ratio $z = q/v$, but not the levels of the two parameters, matters for the value of α_A and the comparison

[25]The percentage peaks at 25% in 2005.

[26]The $\widehat{x}$s rate are equal to TV/H, r_F/H, and $\alpha\delta n_M/TV$ for the model's transaction rate, vacancy rate, and flippers' share, respectively.

of prices and welfare, we first normalize $\upsilon = 1$. We then obtain an estimate of z (or equivalently q) equal to 1.43 from the results in Anenberg and Bayer (2013). The details are in Appendix 7.2. Next, we set $r_H = 0.02$ for an annual rate of 8% to match the usual 30-year fixed-rate mortgage rate. Lastly, for the lack of any obvious empirical counterpart, we set the household-seller's bargaining power $\beta_H = 0.5$. Then for each of $\beta_F = 0.5$, 0.6, 0.65, 0.7, and 0.8, we look for the value of r_F at which $\alpha_A = 0.25$. The results are shown in Table 2.[27,28]

Table 2: Calibrated β_F and r_F for $\alpha_A = 0.25$

β_F	0.5	0.6	0.65	0.7	0.8
r_F	0.0077	0.0091	0.0099	0.0106	0.012
r_F (annual basis)	3.1%	3.7%	4%	4.23%	4.8%

For the last two pairs of β_F and r_F in Table 2, the $\alpha = 0.25$ equilibrium is the unique equilibrium. For the first three pairs, however, there are two other equilibria each besides the $\alpha = 0.25$ equilibrium. Table 3 reports the prices and aggregate asset values in these equilibria. For instance, for $\beta_F = 0.5$ and $r_F = 3.1\%$ per annum, the three equilibria are at $\alpha = 0$, 0.25, and 1, respectively.[29] The price $\overline{p}$, on the next row, is the average of p_H, p_{FB}, and p_{FS}, weighted by the shares of transactions taking place at the respective prices, with $\overline{p}$ in the smallest-α equilibrium set equal to 1. Evidently, the volatility arising from the multiplicity is non-trivial, with average prices differing by up to 23% among the equilibria. Meanwhile, welfare, as measured by the aggregate asset value W, shown on the last row, differs by at most 7%. This is not surprising in light of the analysis in Section 3.7. Whereas homeowners benefit from the higher prices and faster turnover, would-be buyers in rental housing are made worse off by the same higher prices and longer wait for owner-occupied housing. Any efficiency

[27] A r_F below r_H by a few percentage points can make sense if flippers, but not end-user households, tend to choose mortgages with zero initial or negative amortization, short interest rate reset periods, or low introductory teaser interest rates. Such mortgages obviously are ideal for flippers who plan to sell quickly for short-term gains. Amromin et al. (2012) find that borrowers who take out such "complex" mortages are usually high income individuals with good credit scores. Foote et al. (2012) find that periods of interest rate resets do not tend to trigger significant increases in defaults, consistent with the finding of Amromin et al. that the borrowers of such mortgages are sophisticated investors. Barlevy and Fisher (2010) find that interest-only mortgages are used much more heavily in cities with the most rapid increase in housing prices. And then Haughwout et al. (2011) find that states that have undergone the most rapid price increase are states where the share of transaction involving flippers is highest.

[28] Notice that the model does not require r_F to be smaller than r_H for flippers to survive (Proposition 3). For smaller values for z, we can force α_A to be equal to 0.25 at much larger r_F.

[29] At $\alpha = 1$, in the steady state, one half of all houses bought are purchases made by flippers. This is just about equal to the peak investor share in the "bubble states" reported in Haughwout et al. (2011).

gains from intermediation must be weighted against the losses buyers suffer amid a tighter market.

Table 3: Multiple Equilibrium

β_F	0.5			0.6			0.65		
r_F (annual basis)	3.1%			3.7%			4%		
α	0	0.25	1	0	0.25	1	0.25	0.71	1
$\overline{p}$	1	1.1	1.2	1	1.11	1.23	1	1.08	1.11
W	1	1.04	1.07	1	1.04	1.07	1	1.025	1.03

Table 4: Housing Prices and Interest Rates, $\beta_F = 0.7$

r_F (annual basis)	3.5%	4.21%	4.22%	4.23%	4.27%	6%
$\alpha = 0$					0.89	0.89
$\alpha = 0.25$				1		
$\alpha = 0.63$			1.07			
$\alpha = 1$	1.13	1.12				

To study the response of housing prices to interest rate shocks, we report in Table 4 average housing prices $\overline{p}$ for various small deviations of r_F from a benchmark of $r_F = 4.23\%$ and $\beta_F = 0.7$ at which equilibrium is unique at the calibrated value of $\alpha = 0.25$. Fixing $\beta_F = 0.7$, for all values of r_F under consideration, equilibrium remains unique. The entries in the table are normed by the average equilibrium price at the benchmark r_F. Here, housing prices hardly move to follow a given interest rate shock if the shock has not caused any changes in equilibrium α. But when the given interest rate shock does cause α to change significantly, it also leads to significant changes in housing prices. Specifically, a decline in r_F from 6% per annum to 4.27% per annum causes no noticeable change in $\overline{p}$ when the given movement in r_F has no effect on α. On the other hand, a further decline in r_F from 4.27% per annum to 4.21% per annum now causes $\overline{p}$ to increase by 26% as α rises from 0 to 1 in the meantime. Thereafter, $\overline{p}$ remains essentially unchanged from any additional decline in r_F as α has already reached the upper bound of 1. Hence, the response of housing prices to interest rate shocks can appear erratic and unpredictable. Before a given threshold r_F is reached, the response is at most moderate. When r_F crosses the threshold to trigger the entries of flippers, the housing market can become significantly tighter and housing prices significantly higher as a result.

6 Concluding remarks

Without any assumed or acquired heterogeneity and endogenous search efforts, our model predicts a positive relation between housing prices and TOM in the cross section, a relation found in numerous empirical studies. Our model can also generate the well-known relation between prices and transaction volumes in the time series. Previous models rely on preference and construction shocks and increasing returns in the matching technology to generate such relations. In our model, such relations are the relations among the variables across the steady-state equilibria and the relations generated from interest rate shocks. Unique to our analysis is that vacancies should move together with prices and transaction volumes. This relation appears to be borne out in the data.

If flippers can survive in both slack and tight markets, the multiplicity of equilibrium can be a natural outcome in a frictional housing market. Such a model housing market can then be consistent with a substantial amount of volatility. Undoubtedly, our analysis cannot be the complete analysis of housing market booms and busts. Credit market conditions, market psychology, and the dynamics of price movements must also feature prominently. Nevertheless, we show that even in the absence of such factors, the interaction of the strength of the incentives to sell quickly to flippers and the influence of these agents' activities on market tightness suffices to imply an intrinsically volatile housing market.

In the U.S., house flipping is thought to often involve renovating before selling rather than simply buying and then putting up the house for sale right away. In this line of thinking, the returns to flipping are more about the returns to the renovations investment than the returns to holding the house on behalf of liquidity-constrained owners. The question then is what prevents the original owners themselves from earning the returns on the investment. A not implausible explanation is that many original owners lack the access to capital to undertake the investment, just as the original owners in our model lack the access to capital to hold more than one house at a time. Thus, at a deeper level, our model is not just a model of buy-and-sell flips but should also encompass, with suitable modifications, buy-renovate-sell flips.

7 Appendix

7.1 Lemma

Lemma 9 *For* $\Delta \leq 0$, *so that* $\max\{V_R + p_{FB}, V_U\} = V_U$,

$$\begin{aligned} p_H &= \{(\beta_H(\eta + r_H)(r_F + \beta_F \eta) - (1-\beta_H)((1-\alpha)\beta_F\eta + (1-\alpha\beta_F)r_F)\mu)\upsilon \\ &\quad + (r_H + \delta + \beta_H\eta)(r_F + \beta_F\eta)q\}/G_H \end{aligned} \tag{37}$$

$$p_{FS} = \beta_F(\eta + r_F)\frac{(r_H + \beta_H\eta - \mu(1-\alpha)(1-\beta_H))\upsilon + (r_H + \delta + \beta_H\eta)q}{G_H}, \tag{38}$$

$$p_{FB} = V_F = \beta_F\eta\frac{(r_H + \eta\beta_H - \mu(1-\alpha)(1-\beta_H))\upsilon + (r_H + \delta + \eta\beta_H)q}{G_H}, \tag{39}$$

$$\begin{aligned} V_R &= \{((1-\alpha)(1-\beta_H)\beta_F\eta r_H + (1-\beta_F)\eta\beta_H\alpha r_F + (1-\beta_H - \alpha(\beta_F - \beta_H)) \\ &\quad \times r_H r_F)\mu\upsilon - (r_H + \delta + \beta_H\eta)(r_F + \beta_F\eta)r_H q\}/(r_H G_H), \end{aligned} \tag{40}$$

$$V_M = \frac{(r_H + \beta_H\eta)\upsilon}{r_H(r_H + \delta + \beta_H\eta)}, \tag{41}$$

$$V_U = \frac{\eta\beta_H\upsilon}{r_H(r_H + \delta + \beta_H\eta)}, \tag{42}$$

where

$$G_H = (r_H + \delta + \eta\beta_H)((r_F + \eta\beta_F)r_H + (1-\beta_F)r_F\mu\alpha).$$

For $\Delta \geq 0$, *so that* $\max\{V_R + p_{FB}, V_U\} = V_R + p_{FB}$,

$$\begin{aligned} p_H &= \{(\beta_H(\eta + r_H)(r_F + \beta_F\eta) - \mu(1-\beta_H)(r_F + (1-\alpha)\beta_F\eta - \alpha\beta_F r_F))\upsilon \\ &\quad + ((1-\beta_H)\delta r_F + (r_H + \beta_H\eta)(r_F + \beta_F\eta))q\}/G_F, \end{aligned} \tag{43}$$

$$p_{FS} = \frac{\beta_F(\eta + r_F)((r_H + \beta_H\eta - (1-\beta_H)(1-\alpha)\mu)\upsilon + (r_H + \beta_H\eta)q)}{G_F}, \tag{44}$$

$$p_{FB} = V_F = \frac{\beta_F\eta((\beta_H\eta + r_H - \mu(1-\alpha)(1-\beta_H))\upsilon + (\beta_H\eta + r_H)q)}{G_F}, \tag{45}$$

$$\begin{aligned} V_R &= \{(((1-\alpha)\beta_F\eta + r_F)(1-\beta_H)r_H + (1-\beta_F)\beta_H\eta\alpha r_F - (\beta_F - \beta_H)\alpha r_F r_H) \\ &\quad \times\mu\upsilon - (r_H + \eta\beta_H)(r_F r_H + \delta r_F + \beta_F\eta r_H)q\}/(r_H G_F), \end{aligned} \tag{46}$$

$$V_M = \frac{(r_H + \beta_H\eta)(((r_F + \beta_F\eta)r_H + (1-\beta_F)r_F\mu\alpha)\upsilon - r_F\delta q)}{r_H G_F}, \tag{47}$$

$$V_U = \frac{\eta\beta_H(((r_F + \beta_F\eta)r_H + (1-\beta_F)r_F\mu\alpha)\upsilon - r_F\delta q)}{r_H G_F}, \tag{48}$$

where

$$G_F = (r_H + \beta_H\eta)(r_F r_H + \delta r_F + \beta_F\eta r_H + (1-\beta_F)r_F\mu\alpha) - (1-\beta_H)(1-\alpha)\mu\delta r_F.$$

Lemma 10

a. *For* $r_F < \widehat{r}_H$, $D(\theta, \alpha)$ *as a function of* θ *is U-shaped, with a well-defined minimum for each* $\alpha \in [0, 1]$. *Write* $D^*(\alpha) = \min_\theta D(\theta, \alpha)$. *By the Envelope Theorem,* $\partial D^*(\alpha)/\partial \alpha < 0$.

   i. *For sufficiently small* r_F, $D^*(\alpha) > 0$ *for all* $\alpha \in [0, 1]$.

   ii. $\partial D^*(\alpha)/\partial r_F < 0$. *As* r_F *increases, before* r_F *reaches* $\widehat{r}_H$, $D^*(1) = 0$ *attains.*

   iii. *As* r_F *continues to increase,* $D^*(1)$ *falls below 0, and the two roots of* $D(\theta, 1) = 0$, $\widehat{\theta}_1$ *and* $\widehat{\theta}_2$ *diverge as* $\partial\widehat{\theta}_1/\partial r_F < 0$ *but* $\partial\widehat{\theta}_2/\partial r_F > 0$.

   iv. *Then when* r_F *increases up to another threshold before reaching* $\widehat{r}_H$, $D^*(0) = 0$ *attains.*

   v. *Thereafter,* $D^*(0)$ *falls below 0, and the two roots of* $D(\theta, 0) = 0$, $\widehat{\theta}_{1b}$ *and* $\widehat{\theta}_{2a}$ *diverge as* $\partial\widehat{\theta}_{1b}/\partial r_F < 0$ *but* $\partial\widehat{\theta}_{2a}/\partial r_F > 0$

   vi. *As* $r_F \to \widehat{r}_H$, $\widehat{\theta}_{2a} \to \infty$ *and* $\widehat{\theta}_{2b} \to \infty$, *whereas* $\widehat{\theta}_{1a} \to \widehat{\theta}_a^U$ *and* $\widehat{\theta}_{1b} \to \widehat{\theta}_b^U$ *for some finite and positive limiting values* $\widehat{\theta}_a^U$ *and* $\widehat{\theta}_b^U$.

b. *For* $r_F \geq \widehat{r}_H$, $D(\theta, \alpha)$ *becomes downward-sloping throughout.*

   i. *At* $r_F = \widehat{r}_H$, *the* $D\left(\widehat{\theta}_a, 1\right) = 0$ *obtains at* $\widehat{\theta}_a = \widehat{\theta}_a^U$, *and* $D\left(\widehat{\theta}_b, 0\right) = 0$ *obtains at* $\widehat{\theta}_b = \widehat{\theta}_b^U$.

   ii. *Thereafter,* $\partial\widehat{\theta}_a/\partial r_F < 0$ *and* $\partial\widehat{\theta}_b/\partial r_F < 0$, *while as* r_F *becomes arbitrarily large,* $\widehat{\theta}_a \to \widehat{\theta}_a^L$ *and* $\widehat{\theta}_b \to \widehat{\theta}_b^L$ *for some* $\widehat{\theta}_a^L, \widehat{\theta}_b^L \in (0, 1)$ *if* $\beta_H \geq 1/2$.

7.2 Calibrating $z = q/\upsilon$

In the model housing market, the flow payoffs for matched owner-occupiers, mismatched owner-occupiers, and buyers in rental housing are equal to υ, 0, and $-q$, respectively. In this case then, the difference between the flow payoffs of matched and mismatched owner-occupiers is equal to υ, and that between mismatched owner-occupiers and buyers is q. Anenberg and Bayer (2013) report estimates on

1. mean flow payoff for matched owner-occupiers 0.0273

2. flow payoff for owner-occupiers mismatched with their old houses 0.024, comprising 30% of all mismatched households

3. flow payoff for owner-occupiers mismatched with the metro area 0.0014, comprising 70% of all mismatched households

while normalizing the flow payoff of buyers to 0. We may thus equate $v = 0.0273 - 0.024 = 0.00033$ and $q = 0.024$ so that $z = 8$ for households who are mismatched with their old houses and $v = 0.0273 - 0.0014 = 0.0259$ and $q = 0.0014$ so that $z = 0.054$ for households who are mismatched with the metro area. Taking a weighted average of the two estimates gives a value for z equal to 2.49. Alternatively, one may take the weighted average first before taking the ratio: $v = 0.0273 - 0.03 \times 0.024 - 0.7 \times 0.0014$ and $q = 0.03 \times 0.024 + 0.7 \times 0.0014$, so that $z = 0.43$. Since it is not clear in the context of our model which method is conceptually better than the other, we resort to taking a simple average of 2.49 and 0.43 to obtain a value of 1.43 for z.

7.3 Proofs

Proof of Lemma 1 Combining (5), (6), and

$$\mu = \frac{\eta}{\theta} = \eta \frac{S}{B} = \eta \frac{n_U + n_F}{n_R}$$

yields the result of the lemma.

Proof of Lemma 2 By Lemma 1,

$$n_U = \frac{1-\alpha}{\alpha} n_F. \tag{49}$$

Use (1) and (2) to write

$$n_R = 1 - H + n_F. \tag{50}$$

Then, by (49) and (50),

$$\theta = \frac{B}{S} = \frac{n_R}{n_U + n_F} = \frac{1 - H + n_F}{\frac{1-\alpha}{\alpha} n_F + n_F} = \alpha \frac{1 - H + n_F}{n_F}. \tag{51}$$

Solve the equation for n_F,

$$n_F = \frac{\alpha}{\theta - \alpha} (1 - H). \tag{52}$$

Next, by (5) and (1),

$$\mu n_R = \delta (1 - n_U - n_R).$$

Rearrange and then substitute from (49) and (50),

$$(\mu + \delta)(1 - H + n_F) = \delta \left(1 - \frac{1-\alpha}{\alpha} n_F\right).$$

Solve the equation for

$$n_F = \alpha \frac{\delta H - \mu (1 - H)}{\delta + \mu \alpha}. \tag{53}$$

Setting the LHSs of (52) and (53) equal yields (9). The LHS is equal to $(1-(1-\alpha)H)\delta > 0$ at $\theta = 0$ but is negative for arbitrarily large θ given the concavity of η. A solution is guaranteed to exist. Differentiating with respect to θ yields

$$\frac{\partial \eta}{\partial \theta}(1-H) - H\delta,$$

which is positive for small θ but negative otherwise. The solution to (9) must then be unique, and that the LHS is decreasing in θ at where it vanishes. Given that the LHS of the equation is increasing in α, $\partial\theta_T/\partial\alpha > 0$. The lower and upper bounds $\theta_T(0)$ and $\theta_T(1)$ are given by the respective solutions to

$$\delta + \eta(1-H) - (1+\theta)H\delta = 0, \tag{54}$$

$$\delta + \eta(1-H) - \theta H\delta = 0. \tag{55}$$

By straightforward manipulation of the second equation, $\theta_T(1) > 1/H > 1$.

Proof of Lemma 3 *Comparative statics–* Solve (9) for

$$\alpha = \frac{\theta\delta H - (1-H)(\eta+\delta)}{\delta H}. \tag{56}$$

Substituting (56) into (52) yields

$$n_F = \frac{\theta\delta H - (1-H)(\eta+\delta)}{\eta+\delta}. \tag{57}$$

Substituting (57) into (49) and (50), respectively, yields

$$n_U = \frac{\eta(1-H) + \delta(1-\theta H)}{\eta+\delta}, \tag{58}$$

$$n_R = \frac{\theta\delta H}{\eta+\delta}. \tag{59}$$

Finally, by (1), (58), and (59),

$$n_M = \frac{\eta H}{\eta+\delta}. \tag{60}$$

The comparative statics in the Lemma can be obtained by differentiating (57)-(60), respectively, with respect to θ, and then noting that $\partial\theta_T/\partial\alpha > 0$.

Boundary values– At $\alpha = 0$, by (8), $n_F = 0$. Then, by (50), $n_R = 1-H$ and by (2), $n_M = H - n_U$. To obtain the equation for n_U, substitute (2) for n_M into (6), set $n_F = \alpha = 0$ and solve the equation for

$$\theta_T(0) = \eta^{-1}\left[\delta\frac{H-n_U}{n_U}\right].$$

Substituting the expression into (54) yields

$$\frac{1-H}{n_U} - \eta^{-1}\left[\delta\frac{H-n_U}{n_U}\right] = 0. \tag{61}$$

At $\alpha = 1$, by (6), $n_U = 0$. And then by (50), $n_R = 1 - H + n_F$. Thus, with (1), $n_M = H - n_F$. The equation for n_F is obtained by first substituting (2) for n_M into (8), setting $n_U = 1$ and $\alpha = 0$ and solving the equation for

$$\theta_T(1) = \eta^{-1}\left[\delta\frac{H-n_F}{n_F}\right].$$

Substituting the expression into (55) yields

$$\frac{1-H}{n_F} + 1 - \eta^{-1}\left[\delta\frac{H-n_F}{n_F}\right] = 0. \tag{62}$$

Proof of Lemma 5 By (54),

$$\frac{1-H}{\delta H} = \frac{\theta_T(0)}{\delta + \eta(\theta_T(0))} \le \frac{\theta}{\delta + \eta(\theta)}, \tag{63}$$

since $\theta \ge \theta_T(0)$. Substitute (2) into (10) and then from (56) and (60),

$$TV = \alpha\delta n_M + (H - n_M)\eta = \frac{\eta\delta H}{\delta+\eta}(1+\theta) - \eta(1-H). \tag{64}$$

Differentiating and simplifying,

$$\begin{aligned}\frac{\partial TV}{\partial\theta} &= \delta H\left(\frac{\partial\eta}{\partial\theta}\frac{\delta(1+\theta)}{(\delta+\eta)^2} - \frac{\partial\eta}{\partial\theta}\frac{1-H}{\delta H} + \frac{\eta}{\eta+\delta}\right)\\ &\ge \delta H\left(\eta'\frac{\delta(1+\theta)}{(\delta+\eta)^2} - \frac{\theta\partial\eta/\partial\theta}{\delta+\eta} + \frac{\eta}{\eta+\delta}\right) > 0,\end{aligned}$$

where the first inequality is by (63) and the second inequality by the concavity of η. But then $\partial\theta_T/\partial\alpha > 0$; hence $\partial TV/\partial\alpha > 0$.

Proof of Lemma 6 Substituting from (2), (64), and (60) and simplifying, (11) becomes

$$\left((1+\theta)\eta - \frac{1-H}{\delta H}\eta(\delta+\eta)\right)^{-1}.$$

Differentiating with respect to θ yields an expression having the same sign as

$$\begin{aligned}-\left(\eta + \theta\frac{\partial\eta}{\partial\theta} - \frac{1-H}{\delta H}\frac{\partial\eta}{\partial\theta}(\delta+2\eta)\right) &< -\left(\eta + \theta\frac{\partial\eta}{\partial\theta} - \frac{\theta}{\delta+\eta}\frac{\partial\eta}{\partial\theta}(\delta+2\eta)\right) =\\ -\left(\frac{\eta}{\delta+\eta}\left(\delta+\eta-\theta\frac{\partial\eta}{\partial\theta}\right)\right) &< 0,\end{aligned}$$

where the first inequality is by (63) and the second by the concavity of η. The Lemma follows given that $\partial\theta_T/\partial\alpha > 0$.

Proof of Lemma 7 Substituting from (5), (6), and then (1), (12) becomes

$$\frac{1}{\mu}+\frac{1-\alpha}{\eta}=\frac{1-n_M}{\delta n_M},$$

a decreasing function of n_M. But where $\partial n_M/\partial\alpha>0$, there must be a smaller average TBM.

Proof of Lemma 8 With $\lim_{\theta\to 0}\eta=0$,

$$\lim_{\theta\to 0}D=\left(1-\beta_H-\alpha\left(\beta_F-\beta_H\right)\right)\mu-\left(\delta+r_H\right)z=\infty,$$

since $\lim_{\theta\to 0}\mu=\infty$. With $\lim_{\theta\to\infty}\mu=0$,

$$\lim_{\theta\to\infty}D=\left(\beta_F\frac{r_H}{r_F}-\beta_H-z\beta_H\right)\eta-\left(\delta+r_H\right)z,$$

which is negative if $r_F\geq\widehat{r}_H$ but equal to positive infinitive otherwise. Differentiating,

$$\frac{\partial D}{\partial\theta}=\left(\beta_F\frac{r_H}{r_F}-\beta_H-z\beta_H\right)\frac{\partial\eta}{\partial\theta}+\left(1-\beta_H-\alpha\left(\beta_F-\beta_H\right)\right)\frac{\partial\mu}{\partial\theta},\qquad(65)$$

which is guaranteed negative if $r_F\geq\widehat{r}_H$. In this case, D starts out equal to positive infinity and falls continuously below zero. Given that D is decreasing in α, D will first hit zero at some $\widehat{\theta}_a$ and $\alpha=1$. Further declines in θ requires corresponding declines in α to keep $D=0$. Formally, we define a decreasing function $\alpha_D(\theta)$ from $D(\theta,\alpha_D(\theta))=0$. Eventually, as θ falls to some level $\widehat{\theta}_b$, $D=0$ holds at $\alpha=0$. Thereafter $D<0$ for all α. This proves part (a) of the Lemma. On the other hand, if $r_F<\widehat{r}_H$, D starts out and ends up equal to positive infinity. It must therefore be initially decreasing but eventually increasing. If the condition in note 15 holds, (65) changes sign just once, so that D is U-shaped. This can be shown by differentiating (65) and evaluating at where it is equal to zero, which is positive if the condition in note 15 holds. Then, at where (65) vanishes, D is convex. We can next use arguments analogous to the proof of part (a) to complete the proof of part (b).

Proof of Lemma 9 Setting $\max\{V_R+p_{FB},V_U\}=V_U$ in (15) and solving (13)-(19) for the three prices and four asset values yield the solutions in the first part of the Lemma. Setting $\max\{V_R+p_{FB},V_U\}=V_R+p_{FB}$ before solving (13)-(19) yield the solutions in the second part.

Proof of Lemma 10 By (22), $\lim_{r_F\to 0}D(\theta,\alpha)=\infty$. Thus, for arbitrarily small r_F, $D^*(\alpha)>0$. This proves (a.i). Differentiating (22) and by the Envelope Theorem,

$$\frac{\partial D^*(\alpha)}{\partial r_F}=-\beta_F\frac{r_H}{r_F^2}\eta<0.$$

This proves the first part of (a.ii). As to the second part of (a.ii) and (a.iv), notice that

$$\lim_{r_F \to \widehat{r}_H} D(\theta, 1) = (1 - \beta_F)\mu - (\delta + r_H) z,$$
$$\lim_{r_F \to \widehat{r}_H} D(\theta, 0) = (1 - \beta_H)\mu - (\delta + r_H) z,$$

both of which are minimized at $\theta \to \infty$, yielding negative $D^*(1)$ and $D^*(0)$ in the limit. Given that $D^*(1)$ and $D^*(0)$ are continuous in r_F, $D^*(1) = 0$ and $D^*(0) = 0$ must hold before r_F has reached $\widehat{r}_H$. Because $D^*(1) < D^*(0)$ with $\beta_F > \beta_H$, $D^*(1) = 0$ attains first before $D^*(0) = 0$ holds. Differentiating $D(\theta, \alpha) = 0$,

$$\frac{\partial \theta}{\partial r_F} = \frac{\beta_F \frac{r_H}{r_F^2} \eta}{\partial D / \partial \theta} \gtrless 0 \Leftrightarrow \frac{\partial D}{\partial \theta} \gtrless 0. \tag{66}$$

This proves (a.iii) and (a.v). For (a.iv), notice that $\partial D / \partial \theta$, as given by (65), can only only remain positive as $r_F \to \widehat{r}_H$ if $\theta \to \infty$ in the interim. This proves $\widehat{\theta}_{2i} \to \infty$, $i = a, b$, as $r_F \to \widehat{r}_H$. The limiting values for $\widehat{\theta}_{1a}$ and $\widehat{\theta}_{1b}$ are given by the respective solutions to

$$0 = (1 - \beta_F)\mu\left(\widehat{\theta}_a^U\right) - (\delta + r_H) z,$$
$$0 = (1 - \beta_H)\mu\left(\widehat{\theta}_b^U\right) - (\delta + r_H) z,$$

which are of course also the values for $\widehat{\theta}_a$ and $\widehat{\theta}_b$ at $r_F = \widehat{r}_H$. This proves (b.i). The comparative statics in (b.ii) follow from (66). The limiting values of $\widehat{\theta}_a$ and $\widehat{\theta}_b$ as r_F becomes arbitrarily large are given by the respective solutions of $\widehat{\theta}_a^L$ and $\widehat{\theta}_b^L$ to

$$\left(\frac{1 - \beta_F}{\widehat{\theta}_a^L} - \beta_H (1 + z)\right)\eta\left(\widehat{\theta}_a^L\right) = (\delta + r_H) z,$$

$$\left(\frac{1 - \beta_H}{\widehat{\theta}_b^L} - \beta_H (1 + z)\right)\eta\left(\widehat{\theta}_b^L\right) = (\delta + r_H) z.$$

Given that the RHS of the two equations are positive and finite, the two θs are positive and finite, satisfying respectively,

$$\widehat{\theta}_a^L < \frac{1 - \beta_F}{\beta_H (1 + z)} < 1,$$

$$\widehat{\theta}_b^L < \frac{1 - \beta_H}{\beta_H (1 + z)} < 1,$$

where the two rightmost inequalities follow from $\beta_F > \beta_H \geq 1/2$. This completes the proof of (b.ii).

Proof of Proposition 1 By Brouwer's Fixed Point Theorem, a continuous function mapping the unit interval into itself must possess a fixed point.

Proof of Proposition 2 The first part of the proposition follows from the discussion in the text. For the second part, it should be clear that superimposing an upward-sloping $\alpha_T(\theta)$ into Panel C or D of Figure 4 with the given restrictions yields at least two intersections of the two schedules.

Proof of Proposition 3 Given that for small r_F, $\alpha_A(\theta)$ is as given in Panel A of Figure 4, (a) follows immediately. For large r_F, $\alpha_A(\theta)$ tends to that in Panel F. In this case, θ can remain equal to $\theta_T(1)$ only if $\theta_T(1) \leq \widehat{\theta}_a^L$. Given that $\theta_T(1) > 1$ (Lemma 2) but $\widehat{\theta}_a^L < 1$ if $\beta_H \geq 1/2$ (Lemma 10), the condition cannot hold. Hence, for large r_F, in equilibrium, $\alpha < 1$ and $\theta < \theta_T(1)$. Next, if $\widehat{\theta}_b^L \leq \theta_T(0)$, in the limit when r_F becomes arbitrarily large, $\alpha = 0$ and $\theta = \theta_T(0)$. Otherwise, equilibrium is at some $\alpha_D(\theta) > 0$. This proves (b). For (c), note that as r_F increases, the set of θ over which $\alpha_A(\theta) = 1$ shrinks and the set of θ over which $\alpha_A(\theta) = 0$ expands. Parts (d) and (e) are direct corollaries of (a.iii), (a.v), and (b.ii) of Lemma 10, given that $\partial\alpha_T(\theta)/\partial\theta > 0$.

Proof of Proposition 4 We begin with showing search market price p_{FS} in the $\theta_T(1)$ equilibrium, given by (25), is higher than p_{FS} in an $\alpha_D(\theta)$ equilibrium, given by (27). Now, at where $\theta = \widehat{\theta}_{2b}$, $D(\theta, 1) = 0$, the two p_{FS} are by construction equal. Second, with p_{FS} in the first equation increasing in θ by the concavity of η and the p_{FS} in the second equation increasing in θ for $r_F < \widehat{r}_H$, which is a necessary condition for multiplicity, p_{FS} in the $\theta_T(1)$ equilibrium must exceed p_{FS} in an $\alpha_D(\theta)$ equilibrium, since in this case $\theta_T(1) \geq \widehat{\theta}_{2b}$ whereas p_{FS} in an $\alpha_D(\theta)$ equilibrium is at where $\theta \leq \widehat{\theta}_{2b}$. Lastly, with $p_{FS} > p_H$ in the $\alpha_D(\theta)$ equilibrium, the single search market price in the $\theta_T(1)$ equilibrium exceeds the two search market prices in the $\alpha_D(\theta)$ equilibrium. Next, in a comparison between p_{FS} in two $\alpha_D(\theta)$ equilibria, given that p_{FS} in (27) is increasing in θ, there must be higher p_{FS} in the larger θ equilibrium. The same ranking applies to the two p_H, given that p_H in (28) is similarly increasing in θ in case $r_F < \widehat{r}_H$. The final comparison is between p_H in an $\alpha_D(\theta)$ equilibrium and p_H in the $\theta_T(0)$ equilibrium, given by (23). At where $\theta = \widehat{\theta}_{2a}$, $D(\theta, 0) = 0$, the two p_H are by construction equal. With the first p_H known to be increasing in θ, the proof is completed as p_H, given by (23), is likewise is increasing in θ given the concavity of η. This completes the proof that search market housing prices across steady-state equilibria can be ranked by the value of θ. Given that $p_{FB} = \frac{\eta}{\eta + r_F} p_{FS}$, investment market housing prices are ranked in the same order as in search market housing prices.

Proof of Proposition 6 By differentiating (29)-(31) with respect to θ and noting that $\alpha_T \leq 1$ and $\partial \alpha_T / \partial \theta > 0$.

Proof of Proposition 8 Hold constant R and allow r_H to increase; by (32),

$$\frac{\partial D^* (\alpha)}{\partial r_H} = -z < 0.$$

For large r_H then, $\alpha_A (\theta)$ cannot be like the ones in Panel A of Figure 4. For $r_F \geq \widehat{r}_H$ (i.e., $R \leq \beta_H (1+z) / \beta_F$), $\partial D (\theta, \alpha) / \partial \theta < 0$ for all θ and $\alpha \in [0,1]$. Then $\partial \widehat{\theta}_i / \partial r_H < 0$, for $i = a, b$, given that $\partial D (\theta, \alpha) / \partial r_H < 0$. Moreover, in this case, $\lim_{r_H \to \infty} \widehat{\theta}_i = 0$, for $i = a, b$. For $r_F < \widehat{r}_H$, again given that $\partial D (\theta, \alpha) / \partial r_H < 0$, $\partial \widehat{\theta}_{1i} / \partial r_H < 0$ and $\partial \widehat{\theta}_{2i} / \partial r_H > 0$, for $i = a, b$. As $r_H \to \infty$, $D (\theta, \alpha) = 0$ can only hold at θ equal to zero and infinity; thus $\lim_{r_H \to \infty} \widehat{\theta}_{1i} = 0$ and $\lim_{r_H \to \infty} \widehat{\theta}_{2i} = \infty$. Parts (a)-(d) of the Proposition follows immediately.

Proof of Proposition 9 Substituting $r_F = r_H R^{-1}$ into (24) and (25) and differentiating proves the first part of (a). In a no-intermediation equilibrium, p_H is given by (23), which is independent of r_F but decreasing in r_H for sufficiently small $\theta_T (0)$ and/or q. This proves the second part of (a). For (b), substituting $r_F = r_H R^{-1}$ into (26)-(28), respectively, yields,

$$p_{FB} = \frac{\beta_F \eta}{r_H R^{-1} (r_H + \delta + \beta_H \eta)} \upsilon, \tag{67}$$

$$p_{FS} = \frac{\beta_F \eta + \beta_F r_H R^{-1}}{r_H R^{-1} (r_H + \delta + \beta_H \eta)} \upsilon, \tag{68}$$

$$p_H = \frac{\beta_F \eta + \beta_H r_H R^{-1}}{r_H R^{-1} (r_H + \delta + \beta_H \eta)} \upsilon. \tag{69}$$

all of which are decreasing in r_H. Solving $D = 0$ from (32) for

$$r_H = \frac{(\beta_F R - \beta_H - z\beta_H) \eta + (1 - \beta_H - \alpha (\beta_F - \beta_H)) \mu}{z} - \delta, \tag{70}$$

and substituting into (67)-(69), respectively, gives

$$p_{FB} = \frac{R \beta_F \eta q^2}{G}, \tag{71}$$

$$p_{FS} = \frac{((\beta_F R - \beta_H) \eta + (1 - \beta_H - \alpha (\beta_F - \beta_H)) \mu) \beta_F \upsilon - (\eta \beta_H + \delta - \eta R) \beta_F q}{G} q, \tag{72}$$

$$p_H = \frac{((\beta_F R - \beta_H)\,\eta + (1-\beta_H - \alpha\,(\beta_F - \beta_H))\,\mu)\,\beta_H \upsilon - (\beta_H\,(\eta\beta_H + \delta) - \beta_F \eta R)\,q}{G} q, \tag{73}$$

where

$$\begin{aligned} G &= (((\beta_H - \beta_F R)\,\eta - (1-\beta_H - \alpha\,(\beta_F - \beta_H))\,\mu)\,\upsilon + (\delta + \beta_H \eta)\,q) \\ &\quad \times ((\beta_H - \beta_F R)\,\eta - (1-\beta_H - \alpha\,(\beta_F - \beta_H))\,\mu)\,. \end{aligned} \tag{74}$$

Differentiating (71) with respect to θ, evaluating the resulting expression at $\beta_F = \beta_H = 1/2$ yields an expression whose sign is given by that of

$$\eta\,(R - 1 + 1/\theta - z) - 2\delta z - (R - 1 + 1/\theta)\left(\theta^2 \frac{\partial \eta}{\partial \theta}\,(R - 1 + 1/\theta - z) - \eta\right).$$

The expression is strictly positive at $R = 0$ and $R = 1 + z$ if the RHS of (70) at $\beta_F = \beta_H = 1/2$ is positive. And then differentiating twice with respect to R yields

$$-2\theta^2 \frac{\partial \eta}{\partial \theta} < 0.$$

Thus, p_{FB} in (71) must be increasing in θ for $R \in [0, 1+z]$. For p_{FS} and p_H, differentiating (72) and (73) with respect to θ and evaluating at $\beta_F = \beta_H = 1/2$ and $R = 0$, 1, and $1 + z$, respectively, all yield a strictly positive expression as long as the RHS of (70) is positive at $\beta_F = \beta_H = 1/2$. Then, p_{FS} and p_H in (72) and (73) must be increasing in θ for R in neighborhoods of 0, 1, and $1 + z$.

References

[1] Albrecht, James, Axel Anderson, Eric Smith, and Susan Vroman, 2007, "Opportunistic Matching in the Housing Market," *International Economic Review* 42, 641-664.

[2] Amormin, Gene, Jennifer Huang, Clemens Sialm, and Edward Zhong, 2012, "Complex Mortgages," memo.

[3] Anenberg, Elliot and Patrick Bayer, 2013. "Endogenous Sources of Volatility in Housing Markets: The Joint Buyer-Seller Problem,"memo.

[4] Arnott, Richard, 1989, "Housing Vacancies, Thin Markets and Idiosyncratic Tastes," *Journal of Real Estate Finance and Economics* 2, 5-30.

[5] Barlevy, Gadi and Jonas Fisher, 2010, "Mortgage Choice and Housing Speculation," memo.

[6] Bayer, Patrick, Christopher Geissler, and James W. Roberts, 2011, "Speculators and Middlemen: The Role of Flippers in the Housing Market," NBER working paper 16784.

[7] Chatterjee, Kalyan and William Samuelson, 1983, "Bargaining under Incomplete Information," *Operations Research* 31, 835-851.

[8] Diaz, Antonia and Belen Jerez, 2009, "House Prices, Sales and Time on the Market: A Search-Theortical Framework," Working Paper 09-25, Economic Series 15, Department of Economics, Universidad Carlos III de Madrid.

[9] Depken, Craig A. II, Haris Hollans, and Steve Swidler, 2009, "An Empirical Analysis of Residential Property Flipping," *Journal of Real Estate Finance and Economics* 39, 248-263.

[10] Ferreira, Fernando, Joseph Gyourko and Joseph Tracy, 2010, "Housing busts and household mobility," *Journal of Urban Economics* 68, 34-45.

[11] Foote, Christopher L., Kristopher S. Gerardi, and Paul S. Willen, 2012, "Why Did So Many People Make So Many Ex Post Bad Decisions? The Causes of the Foreclosure Crisis," Federal Reserve Bank of Altanta Working Paper 2012-7.

[12] Gavazza, Alessandro, 2012, "An Empirical Model of a Decentralized Asset Market," memo.

[13] Genesove, David and Lu Han, 2012, "Search and matching in the housing market," *Journal of Urban Economics* 72, 31-45.

[14] Genesove, David and Christopher J. Mayer, 1997, "Equity and Time to Sale in the Real Estate Market," *American Economic Review* 87, 255-269.

[15] Harsanyi, John C. and Reinhard Selten, 1972, "A Generalized Nash Solutions For Two-Person Bargaining Games with Incomplete Information," *Management Science* 18, 80-106.

[16] Haughwout, Andrew, Donghoon Lee, Joseph Tracy, and Wilbert van der Klaauw, 2011, "Real Estate Investors, the Leverage Cycle, and the Housing Market Crisis," Federal Reserve Bank of New York, Staff Reports no. 514.

[17] Head, Allen and Huw Looyd-Ellis, 2012, "Housing Liquidity, Mobility and the Labour Market," *Review of Economic Studies* 79, 1559-1589.

[18] Hort, Katinka, 2000, "Prices and turnover in the market for owner-occupied homes," *Regional Science and Urban Economics* 30, 99-119.

[19] Johri, Alok and John Leach, 2002, "Middlemen and the Allocation of Heterogeneous Goods," *International Economic Review* 43, 347-361.

[20] Krainer, John, 2001, "A Theory of Liquidity in Residential Real Estate Markets," *Journal of Urban Economics* 49, 32-53.

[21] Kwok, Hon Ho and Chung-Yi Tse, 2006, "Estimating the Liquidity Premium in the Housing Market," mimeo.

[22] Lagos, Ricardo, Guillaume Rocheteau, and Pierre-Olivier Weill, 2011, Crises and liquidity in over-the-counter markets," *Journal of Economic Theory* 146, 2169-2205.

[23] Leung, Charles K.Y., Garion C.K. Lau, and Youngman C.F. Leong, 2003, "Testing Alternative Theories of the Property Price-Trading Volume Correlation," *Journal of Real Estate Research* 23, 253-265.

[24] Leung, Charles K.Y., Youngman C. F. Leong, and Ida Y. S. Chan, 2002, "TOM: Why Isn't Price Enough?" *International Real Estate Review* 5, 91-115.

[25] Leung, Charles K. Y. and Chung-Yi Tse, 2013, "Flippers in Housing Market Search – The Dynamics," memo.

[26] Li, Yiting, 1998, "Middlemen and private information," *Journal of Monetary Economics* 42, 131-159.

[27] Masters, Adrian, 2007, "Middlemen in Search Equilibrium," *International Economic Review* 48, 343-362.

[28] Merlo, Antonio and Francois Ortalo-Magne, 2004, "Bargaining over residential real estate: evidence from England," *Journal of Urban Economics* 56, 192-216.

[29] Ngai, L. Rachel and Silvana Tenreyro, 2010, "Hot and Cold Seasons in the Housing Market", mimeo.

[30] Riddell, W. Craig, 1981, "Bargaining Under Uncertainty," *American Economic Review* 71, 579-590.

[31] Rubinstein, Ariel and Asher Wolinsky, 1987, "Middlemen," *Quarterly Journal of Economics* 102, 581-593.

[32] Shevchenko, Andrei, 2004, "Middlemen," *International Economic Review* 45, 1-24.

[33] Smith, Eric, 2004, "Intermediated Search," *Economica* 71, 619-636.

[34] Stein, Jeremy, 1995, "Prices and Trading Volume in the Housing Market: A model with down-payment effects," *Quarterly Journal of Economics* 110, 379-406.

[35] Tse, Chung-Yi, 2011, "The Spatial Origin of Commerce," *International Economic Review* 52, 349-377.

[36] Watanabe, Makoto, 2010, "A Model of Merchants," *Journal of Economic Theory* 145, 1865-1889.

[37] Wheaton, William C., 1990, "Vacancy, Search, and Prices in a Housing Market Matching Model," *Journal of Political Economy* 98, 1270-1292.