INTERMEDIATION ACROSS IMPERFECTLY COMPETITIVE MARKETS

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Abstract: We present a standard Cournot model of several markets for a commodity where trade across markets is conducted via intermediaries. We model the behavior of intermediaries and study the effect of intermediation across such markets.

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1. Introduction

Intermediation forms a part of the cycle of exchanges in markets, so interest in its study is motivated in a natural way. It is only a matter of casual observation that in modern economies a large number of markets are serviced (at least in part) by intermediaries, i.e., entities which are involved in the cycle of exchanges of a commodity, but they neither participate in its production process nor do they consume it. The extend and role of intermediation has been the subject of numerous studies, both empirical and theoretical. Issues which are directly or indirectly related to intermediation have cropped up and addressed within a number of fields ranging from international trade to industrial organization. The interest in the subject is reinforced by the observation that intermediation activities are becoming a substantial part of the cycle of exchanges in markets and the realization that this trend will likely continue. Indeed, it is evident that in a world of globalized markets, intermediation across markets will likely be more widespread and play an increasingly crucial role. This is equally true for markets at the level of region or economic area.

There are two main issues that attract (but do not exhaust) interest on this topic. One of them is the role and effects of intermediation in the determination of market outcomes, in various different contexts. The other, is the establishment of the basis of intermediation, i.e., an explanation of how intermediation arises. The literature has succeeded in providing a number of different theories which explain the emergence of such activities in market exchanges. It turns out that there are several factors that may constitute a basis for intermediation. Alternative theories in various contexts emphasize different aspects of markets (such as geography, transaction costs, asymmetric information etc) as the basis for the emergence of intermediation.

In this paper we do not attempt to explain how the need of intermediation arises. Instead of dwelling too much into the reasons for the emergence of intermediaries, we consider a model which provides a scope for intermediation from the outset. Our purpose is rather focused on the study of the role of intermediation. Our preoccupation is that intermediation serves as a link between markets and therefore it plays a crucial role in the interaction between markets and in particular in the way that competitive conditions spread across markets. It is this aspect of intermediation that has stimulated our interest in this area and we will try to address. More explicitly the purpose of this paper is three fold. First, to formalize the behavior of intermediaries and develop a model that incorporates intermediation. Second, to study the effects of intermediation on the determination of market outcomes. Third, demonstrate the important role of intermediation via some examples and so further motivate this line of study. To this end we provide a novel way to model the behavior of intermediaries and study the effects of their activities across markets. Our approach is developed within the partial equilibrium framework and it draws on the most basic of industrial organization models of markets. The analysis of the model that we develop in this paper sheds light on the effects of intermediation and on the way that competitive market forces spread across markets through intermediation activities.

Intermediaries may enter the cycle of exchanges either by mediating between the producers and the consumers or by purchasing a commodity in some markets and selling it in others. We use the term 'vertical' for the former and 'horizontal' for the latter type of intermediation. In the industrial organization context there has been some vivid literature on 'vertical' intermediation. There are a number of recent papers which address mediation between the producers and the consumers of a commodity by entities who make wholesale purchases from the production sector and supply the consumption sector. We will not go in that direction, although it is not entirely unrelated to what we do here. Our focus in this paper is intermediation across a set of segregated markets for a given commodity, i.e., 'horizontal' intermediation. The aim is to develop a model that allows a direct study of intermediation across markets and articulate its role in the determination of market outcomes. In particular, we are interested in modeling the behavior of intermediaries across different markets for a given commodity and the study of the effects of their behavior on the determination of the configuration of prices across markets. We are not aware of any study to this effect within the industrial organization framework.

2. The Cournot Style Intermediation Model

In order to motivate our way of modeling intermediation, let us imagine a set of islands each one with its proper market for a commodity, i.e., a consumption sector represented by a demand function and a supply sector comprising of some firms which produce the commodity in question. Intermediaries (merchants) in such a world can be thought of as 'boatmen' who link together the markets across islands. These entities purchase the commodity from some of the markets and sell it to some others. At this point we leave the stylized reasons (institutional, geographical, informational etc.) that give rise to an island configuration of markets to the reader's taste and imagination, and save the discussion of this matter for later. Instead we proceed to lay down the questions that arise in such a world.

The first step in the study of this context is to motivate and formalize the behavior of intermediaries. Motivating the behavior of intermediaries seems rather simple. We believe that few readers would resist the argument that the motive for an intermediary is the anticipation of a profit from the mediation activity: buy the commodity cheap and sell it expensive. In standard terminology in economics/finance this amounts to advocating that the intermediaries' motive is to arbitrage prices across markets.

The substance of the matter is that the intermediaries' effort to arbitrage prices would certainly lead them to transfer across markets non-negligible quantities of the commodity for, as long as there is a price difference, they would be able to profit from any additional units they transfer across markets, until those transfers are substantial enough to bear on this price difference. Thus, the intermediation activity will alter the initial price difference against the intermediary: the effort to arbitrage prices would lead the intermediary to simultaneously place a buy order in the cheap market and a sell order in the expensive market, thereby increasing (reducing) the price in the cheap (expensive) market. On the other hand a clever intermediary would never completely arbitrage prices, because by doing so (s)he would drive the profit from mediation to zero. In conclusion, the intermediary who is motivated by price arbitrage is faced with a tradeoff, i.e., arbitrage price differences but not so excessively that the profit from arbitrage is extinguished. The position of the intermediary on the extent of arbitrage of market clearing prices determines the equilibrium prices in markets.

Moving into formalizing now the behavior of an intermediary is no simple task, as there are a number of different ways that an intermediary can act across markets. We are now at a point where we have to make a decision, as to what is the set of activities that an intermediary is allowed to undertake in markets. In game theoretic terms we have to decide as to what is the strategy set of an intermediary. As a first step, in this paper we will consider what can be most accurately described as the 'pure Cournot' model, namely a model where all producers and intermediaries use quantity signals as their strategy whereas prices in markets adjust to clear markets. In the following section we formally develop this context.

Let n denote the number of markets (trading posts) for a given commodity. In each market the consumption sector is summarized by an (inverse) demand function $p_i = F_i(Q_i)$, which is assumed to be differentiable, and the supply sector comprises of a number of firms

k_i, $i = 1, 2, \ldots, n$, each characterized by a cost function $c_{i,f}(q_{i,f})$, $f = 1, 2, \ldots, k_i$. In short we think of n standard oligopolistic markets.

Those n markets are distinguished by the premise that demand by consumers as well as supply by the corresponding firms in each market is immobile across those markets, i.e., each consumer and firm is associated with one and only market. This is the idea of 'islands' that we suggested in the introduction.

In addition to consumers and producers of the commodity in question, our model features a number m of intermediaries who link markets together by buying and selling the commodity at will. Since this is a benchmark model for simplicity we assume that intermediation is costless. The intermediaries can be thought of as the 'boatmen' in the discussion of this world in the introduction.

As a first shot at the subject let us consider the case where all strategic participants in markets are Cournot competitors, that is their strategic signals are quantities. In this case the strategy set of a firm $f = 1, 2, \ldots, k_i$, operating in market $i = 1, 2, \ldots, n$ can be simply described as $S_{i,f} = \Re_+$. As usual the standard behavior of each firm (i, f), in Cournot style competition is to solve the following problem:

$$\max_{q \in S_{i,f}} p_i q_{i,f} - c_{i,f}(q_{i,f}) \tag{1}$$

where $c_{i,f}(q_{i,f})$ represents the costs of production.

The behavior of a 'Cournot' intermediary, i.e., one whose strategic signals are in terms of quantities to be purchased/sold in markets, can be formalized as follows: the strategy set of each intermediary $j = 1, 2, \ldots, m$ is given by $S_j = \left\{q \in \Re^n : \sum_{i=1}^n q^i \leq 0\right\}$, with the convention that $q^i > 0$ ($q^i < 0$) represents a supply to (demand from) the corresponding market. Following our discussion about the objectives of intermediation the behavior of intermediary $j = 1, 2, \ldots, m$ formally is:

$$\max_{q \in S_j} \sum_{i=1}^{n} p_i q^i \tag{2}$$

Given a profile of strategies for the firms and intermediaries in each market the total quantity of the commodity that arrives in the ith market is $Q_i = \sum_{f=1}^{k_i} q_{i,f} + \sum_{j=1}^{m} q_j^i$. If $Q_i \geq 0$ the price in the market is determined as usual via the inverse demand function. If $Q_i < 0$, indicating that the demands of intermediaries cannot be covered by the total supply in the market, then we postulate that $p_i = \infty$. Thus, the problem for each firm and intermediary can be written as follows:

$$\max_{q \in S_{i,f}} F_i \left(\sum_{f=1}^{k_i} q_{i,f} + \sum_{j=1}^{m} q_j^i \right) q_{i,f} - c_i(q_{i,f}) \tag{3}$$

$$\max_{q \in S_j} \sum_{i=1}^{n} F_i \left(\sum_{f=1}^{k_i} q_{i,f} + \sum_{j=1}^{m} q_j^i \right) q_j^i \tag{4}$$

An example of the model we just presented might be helpful at this point.

Example 1. Consider two monopolies linked by a single intermediary. This example is the simplest one that captures our ideas. Let $n = 2$, $p_i = a_i - b_i Q_i$, $k_1 = k_2 = 1$ and $m = 1$. Suppose that the two firms have constant returns to scale technologies with marginal costs c_1 and c_2 respectively. In this case for each $i = 1, 2$, $Q_i = q_i + q^i$, where q_i is the output produced by the firm associated with market $i = 1, 2$ and q^i is the quantity supplied (if nonnegative) or demanded (if negative) by the intermediary. Notice that in this case since it must be $q^1 + q^2 = 0$, the problem of the intermediary can be simplified to a single dimensional one:

$$\max_{q \in \Re} \left[a_1 - b_1 \left(q_1 + q \right) \right] q - \left[a_2 - b_2 \left(q_2 - q \right) \right] q \tag{5}$$

The profit maximization problems of the two firms in this example are given by:

$$\max_{q_i \in \Re_+} \left[a_i - b_i \left(q_i + q \right) \right] q_i - c_i q_i \tag{6}$$

Before we even start to discuss equilibrium in such a setup, we can already discuss an implication of the behavior of intermediaries on a very important issue.

2.1. **Intermediation and the 'Law of One Price'.** One fundamental principle in equilibrium analysis of markets is the 'law of one price', which dictates that at equilibrium there is a unique price which clears all markets for a commodity. The validity of this principle presumes the possibility of arbitrage activity across markets. In the present model, there is a possibility to arbitrage across markets. However, this arbitrage possibility is attributed to a limited number of entities who use this privilege strategically. The following proposition highlights the importance of strategic considerations in arbitrage activity.

Proposition 1. *Suppose that all inverse demand functions are downward slopping, i.e.,* $\frac{dF_i}{dQ_i} < 0$*, for* $i = 1, 2, \ldots, n$*. Then the law of one price across this system of markets obtains at equilibrium if and only if* $q_j = 0$ *for each* $j = 1, 2, \ldots, m$*.*

Proof: Let us fix one intermediary j. From the first order conditions of (4) we obtain:

$$q_j^i \frac{dF_i}{dQ_i}\frac{dQ_i}{dq_j^i} + F_i(Q_i) - \lambda_j = 0 \tag{7}$$

or

$$q_j^i \frac{dF_i}{dQ_i} + p_i = \lambda_j, \;\; i = 1, 2, \ldots, n \tag{8}$$

The implication of the last equation for the behavior of intermediaries is that:

$$q_j^i \frac{dF_i}{dQ_i} + p_i = q_j^t \frac{dF_t}{dQ_t} + p_t, \;\; i \neq t \tag{9}$$

or

$$p_i - p_t = q_j^t \frac{dF_t}{dQ_t} - q_j^i \frac{dF_i}{dQ_i}, \;\; i \neq t \tag{10}$$

Hence $p_i = p_t \Leftrightarrow q_j^t \frac{dF_t}{dQ_t} = q_j^i \frac{dF_i}{dQ_i}, i \neq t$. Since by hypothesis $\frac{dF_i}{dQ_i} < 0$ for $i = 1, 2, \ldots, n$, it must be $q_j^i q_j^t > 0$, $i, t = 1, 2, \ldots, n$. But then $\sum_{i=1}^n q_j^i \neq 0$. Hence, it must be $q_j^i = 0$ for $i = 1, 2, \ldots, n$. □

The above proposition makes clear that the law of one price will obtain at equilibrium if and only if there is no intermediation activity across markets. In other words when (strategic) intermediation activity takes place at equilibrium, it will never exhaust price differences. The message from this analysis is that the mere possibility to arbitrage prices is not enough in itself to bring about the 'law of one price'. The possibility to arbitrage prices across markets has to be available to 'sufficiently many' intermediaries in order to eliminate strategic considerations which would inhibit the level of arbitrage activity. Intermediaries serve as the channel through which competition spreads across the segregated markets. A small number of intermediaries can then control the flow of competitive forces across markets to their advantage, i.e., there is not enough 'bandwidth' in the channel for the full force of competition to spread across markets.

2.2. **Equilibrium.** A natural candidate for equilibrium in this model is a pure strategy Cournot-Nash equilibrium. It turns out that due to the way prices are defined when there is excess demand, the simultaneous move game does not behave very well. It is easy to see that the example above does not have an equilibrium except in rather special cases. Indeed, in the example above suppose that the strategy profile (q_1, q_2, q) is a pure strategy Cournot-Nash equilibrium. If $q < 0$ (the intermediary buys from market 1 and sells to market 2) then in order for q to be best response to (q_1, q_2) it must be $q \geq -q_1$. However, in order for q_1 to be best response to q it must be $q_1 < -q$, which is a contradiction. A similar reasoning establishes that $q > 0$ cannot hold either. Finally, if $q = 0$ then $q_i = \frac{1}{2b_i}(a_i - c_i)$ is the best response of firm $i = 1, 2$. In this case $q = 0$ is a best response if and only if $\frac{a_1 - c_1}{b_1} = \frac{a_2 - c_2}{b_2}$. Thus, the pure strategy Cournot-Nash equilibrium concept does not seem very interesting in the present framework.

Given that the motive of intermediaries is to arbitrage prices, it stands to reason that they will choose to act once they observe an arbitrage opportunity. Hence, it serves our intuition to consider a two stage game where in the first stage firms take production decisions. Once firms' decisions are observed, in the second stage intermediaries position themselves in the markets by naming quantities they wish to buy and sell, in anticipation of an arbitrage opportunity. In view of this description of the game, the natural candidate for an equilibrium in this model, is a subgame perfect Nash equilibrium of the two stage game that we just described. This is the equilibrium concept that we will focus on for the rest of this paper.

3. Cournot Intermediation As A Two Stage Game

3.1. **Monopolistic markets with intermediation.** In order to proceed further with a more concrete analysis of this model let us consider the simpler case where inverse demand functions and cost functions are linear, which allows for explicit solutions for the equilibria of the model. To this effect let $F_i(Q_i) = a_i - b_i Q_i$. Furthermore, let each of those demand functions be associated with one firm which produces under constant marginal cost c_i for $i = 1, 2, \ldots, n$. Finally, let there be m intermediaries who can operate across this set of markets. The sequence of moves in those markets are as follows: in the first stage each of the n firms takes a decision to produce a quantity of output. Production decisions by firms are observed by intermediaries who in the second stage place demand and supply orders in the different markets. Given the moves of all participants, prices clear markets and each participant collects payoffs. An equilibrium of such a game is a subgame perfect Nash equilibrium of this two stage game.

Given production decisions $q \in \Re_+^n$ by firms, in the second stage intermediaries $j = 1, 2, \ldots, m$ are faced with the following problem:

$$\max \sum_{i=1}^{n} \left[a_i - b_i \left(q_i + \sum_{j=1}^{m} q_j^i \right) \right] q_j^i, \ s.t. \ \sum_{i=1}^{n} q_j^i = 0 \tag{11}$$

For $i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, m$ the solution to this problem turns out to be:

$$q_j^i(q) = \frac{1}{b_i(m+1)} \sum_{t \neq i} r_t \left[(a_i - b_i q_i) - (a_t - b_t q_t) \right] \tag{12}$$

where $r_i = \frac{\prod_{i \neq j} b_j}{\sum_{i=1}^{n} \prod_{i \neq j} b_j}$.

Observe that the strategies of all intermediaries in the second stage game are identical, so at equilibrium each of the best responses depends only on the number of intermediaries along with the production decisions of firms in the first stage. Moving now to the first stage, each firm $i = 1, 2, \ldots, n$ is faced with the problem:

$$\max \left[a_i - b_i \left(q_i + \sum_{j=1}^{m} q_j^i(q) \right) \right] q_i - c_i q_i, \ s.t. \ q_i \geq 0 \tag{13}$$

The solution to this problem is:

$$q_i(q) = \frac{1}{b_i(mr_i + 2)} \left[a_i - c_i(m+1) + \sum_{i=1}^{n} r_i(a_i - b_i q_i) \right] \tag{14}$$

Thus, at equilibrium the quantities produced are for $i = 1, 2, \ldots, n$:

(15)

$$q_i = \frac{1}{mr_i + 2} \left[\hat{a}_i - \hat{c}_i(m+1) + \frac{mr_i}{1 + \sum_{i=1}^{n} \frac{mr_i}{mr_i + 2}} \sum_{i=1}^{n} \frac{1}{mr_i + 2} \left[(mr_i + 1)\hat{a}_i + (m+1)\hat{c}_i \right] \right]$$

where r_i is as before, $\hat{a}_i = \frac{a_i}{b_i}$ and $\hat{c}_i = \frac{c_i}{b_i}$. Equations (15) along with (12) determine the equilibrium.

Some special cases of our model might help understand the role of intermediation in this model.

Example 2. In order to make more transparent the importance of the degree of competition in the intermediation sector let us consider the special case of m intermediaries operating across identical demand functions $p_i = a - bQ_i$, where $i = 1, 2, \ldots n$ and corresponding firms have identical marginal costs c.

In this case we have that formula (15) becomes:

(16) $$q_i = \frac{n\,(m+1)}{2n + m\,(n+1)}\left(\frac{a-c}{b}\right)$$

Hence $q_j^i = 0$, for all $i = 1, 2, \ldots n$, $j = 1, 2, \ldots m$.
More especifically for $n = 2$, $m = 1$ the equilibrium quantities are $q_1 = q_2 = \frac{4}{7}\left(\frac{a-c}{b}\right)$, $q\,(q_1, q_2) = 0$.
By way of comparison the monopoly solution of each of the two markets in isolation (without the presence of an intermediary) is given by $q_i = \frac{1}{2}\left(\frac{a-c}{b}\right)$, $i, j = 1, 2$. Also, the duopoly output in each of the two markets served by two identical firms is $q_i = \frac{2}{3}\left(\frac{a-c}{b}\right)$, $i, j = 1, 2$. Finally the duopoly output in the two markets combined together is $q_i = \frac{2}{3}\left(\frac{a-c}{b}\right)$, $i = 1, 2$. Notice that the intermediary is inactive (and thus transparent in the cycle of exchanges) at equilibrium. Each firm remains the sole supplier of the respective market but *does not behave as a monopolist*: it supplies a quantity which is more than the monopoly and less than the duopoly quantities.

This outcome might at first seem somewhat counter-intuitive: why is it that since the intermediary is inactive, the two firms do not behave as monopolists? Indeed, in the above example it can be verified that a strategy profile, where each firm produces the monopoly output and the intermediary is idle, results in a Pareto dominating set of payoffs. Yet this is not an equilibrium. If one firm produces their monopoly level of output it is in the strategic interest of the other to expand its output beyond the monopoly level. The intuitive reason is that each of the firms tries to expand output in order to attract the intermediary on their demand side, by way of fending off competition and at the same time serving (through the intermediary) a part of the other market. The strategic effort to this effect by both firms finally keeps the intermediary out of business but the two firms expand output to fend off potential competition.

The only way for the monopoly outputs to prevail in the two markets is if the two firms could came to an agreement to ignore the presence of intermediaries in their output decision process. Clearly such an agreement is beneficial for both firms and its effect is self fulfilling: at equilibrium the intermediary is idle. Nevertheless, such an agreement has a similar problem to the cooperative solution in 'prisoner's dilemma'. Indeed, it is not sustainable from the noncooperative point of view, as each of the firms would have an incentive to deviate from it.

The preceding discussion raises some challenging empirical and policy issues. For instance, suppose that a competition authority observes such a pair of markets where the monopoly output and price level prevails. How can it determine whether there has been some collusion between the two firms to keep intermediaries out of both markets, or they simply produce their respective monopoly levels because there are no intermediaries willing to mediate between the two markets? We believe that this point is worthwhile investigating, but it falls beyond the scope of the present study.

Continuing our study of the example, in view of (16) an asymptotic argument shows that as $m \to \infty$ we have $q_i \to \frac{n}{n+1}\left(\frac{a-c}{b}\right)$, which is the oligopolistic output for the *ith* market (for instance, if $n = 2$ then $q_i \to \frac{2}{3}\left(\frac{a-c}{b}\right)$ which is the duopoly output level for market $i = 1, 2$). Observe that the same picture arises in this situation as well, where each firm is the sole supplier in its respective market but acts *as if* competitors were present. The more important message arising from this analysis however, is that intermediation cannot induce more competition than there is inherent in the markets: at best, if intermediation is competitive enough, it will lead to the oligopoly outcome. In particular, notice that for $n \geq 2$:

$$(17) \qquad \frac{n\,(m+1)}{2n+m\,(n+1)}\left(\frac{a-c}{b}\right) \leq \frac{n}{n+1}\left(\frac{a-c}{b}\right)$$

Another asymptotic conclusion drawn from this example is that as $n \to \infty$ we have $q_i \to \frac{m+1}{m+2}\left(\frac{a-c}{b}\right)$, which makes precisely the point that strategic intermediation inhibits the flow of competition across markets[1]. Sensible as this conclusion might appear, there is something intriguing about it. It is worthwhile pointing out that, in this asymptotic result, each firm acts as if it were competing in its own market with m other identical firms[2]. In other words each firm treats the intermediaries in the same way as competing producers (firms with marginal costs identical to its own). However, the marginal cost that the intermediaries incur is the unit price they have to pay their suppliers, which in equilibrium is higher than the (common) marginal cost of producers. Thus, firms appear to act under the false conjecture that prices in other markets (i.e., intermediaries' marginal cost) are competitive (i.e., equal to the supplier's marginal cost)! However, their decisions under this conjecture leads to an equilibrium outcome which refutes this conjecture.

The preceding example and discussion demonstrates that the role of intermediaries is non trivial, even though these entities are idle at equilibrium. Since intermediation activity

[1] Notice that for all m: $\frac{n(m+1)}{2n+m(n+1)}\left(\frac{a-c}{b}\right) \leq \frac{n}{n+1}\left(\frac{a-c}{b}\right)$

[2] Recall that $\frac{m+1}{m+2}\left(\frac{a-c}{b}\right)$ is the oligopolistic outcome of $m+1$ identical firms with marginal cost c operating in the *ith* market.

is triggered by price dispersion across markets, it appears that when the characteristics of markets (production costs and inverse demand functions) are identical, the symmetry of the model would never leave room for intermediation activity, i.e., intermediaries would be active only if there are different demand/cost conditions in two markets for a commodity. This is not so as the following example shows.

Example 3. Let us consider two markets for a commodity with identical demand functions $p_i = 36 - 24Q_i$, each one served by a single producer with cost function $c_i = 35 \log(1 + q_i)$ where $i = 1, 2$. Trade across markets is conducted via one intermediary.
Given first stage output decisions (q_1, q_2) the intermediary solves the problem:

$$\max_{q \in \Re} [36 - 24(q_1 + q)]\, q - [36 - 24(q_2 - q)]\, q \tag{18}$$

Solving this problem results in the following second stage best response for the intermediary:

$$q(q_1, q_2) = \frac{1}{4}(q_2 - q_1) \tag{19}$$

The profits of the two firms are given by $\pi_i(q_i, q(q_1, q_2))$. Hence, in the first stage the two firms are faced with the problems:

$$\begin{aligned} \max \quad & [36 - 24(q_1 + q)]\, q_1 - 35 \log(1 + q_1) \\ s.t. \quad & q = \frac{1}{4}(q_2 - q_1) \end{aligned} \tag{20}$$

$$\begin{aligned} \max \quad & [36 - 24(q_2 - q)]\, q_2 - 35 \log(1 + q_2) \\ s.t. \quad & q = \frac{1}{4}(q_2 - q_1) \end{aligned} \tag{21}$$

Upon substitution of the constraint into the objective functions, these problems can be compactly writhen as:

$$\max \left[36 - 24\left(\frac{3}{4}q_i + \frac{1}{4}q_j\right)\right] q_i - 35 \log(1 + q_i) \tag{22}$$

where $i, j = 1, 2$. Solving this problem we obtain:

$$\frac{\partial \pi_i}{\partial q_i} = 36 - 36q_i - 6q_j - \frac{35}{1 + q_i} \tag{23}$$

$$\frac{\partial^2 \pi_i}{\partial q_i^2} = -36 + \frac{35}{(1+q_i)^2} < 0 \tag{24}$$

Hence, the best responses of the two firms are solutions to:

$$36q_i^2 + 6q_iq_j + q_j - 1 = 0$$

The best responses turn out to be for $i, j = 1, 2$:

$$q_i = \begin{cases} \frac{-q_j + \sqrt{q_j^2 - 24q_j + 4}}{12} & \text{if} \quad q_j \leq \frac{1}{6} \\ 0 & \text{if} \quad \frac{1}{6} \leq q_j \leq 12 - 2\sqrt{35} \\ 0 & \text{if} \quad q_j \geq 12 + 2\sqrt{35} \end{cases}$$

It can be seen that two equilibria of this example are: $\left(q_i^*, q_j^*, q^*\right) = \left(\frac{1}{6}, 0, \frac{1}{24}\right)$. The corresponding equilibrium aggregate quantities and market clearing prices in the two markets are: $\left(Q_i^*, Q_j^*\right) = \left(\frac{1}{8}, \frac{1}{24}\right)$ and $\left(p_i^*, p_j^*\right) = (33, 35)$. There is also a 'symmetric' equilibrium where $\left(q_i^s, q_j^s, q^s\right) \approx \left(\frac{1}{10}, \frac{1}{10}, 0\right)$. This equilibrium results in the price configuration $p_i^s = p_j^s = 33.63347$. By way of comparison the monopoly outputs and prices in the two markets in isolation are $Q_i^m = Q_j^m = \frac{-3+\sqrt{21}}{24} \approx \frac{1}{15}$ and $p_i^m = p_j^m = 34.41742$.

Several interesting conclusions seem to emerge from this example, which we list below:

Conclusion 1. It is clear from this example that even though the two markets are identical, there can be active intermediation at equilibrium. In this case, the 'law of one price' does not obtain across this system of markets.

Conclusion 2. It is conceivable that even though the demand and cost conditions are identical between the two markets, at equilibrium one of the two firms is driven out of business and its market is taken over by the intermediary (or, indirectly, by the other firm).

Conclusion 3. Consumers benefit from the intermediation activity in the market where the intermediary places a buy order, because they pay a lower price than the monopoly price they would have to pay in the absence of an intermediary, when the corresponding firm furnishing that market would act as a monopoly. By contrast the intermediation activity has a detrimental effect on consumers in the market where the intermediary places a sell order: these consumers end up paying a higher price than the monopoly price that would prevail in absence of the intermediary.

Conclusion 4. The effect of intermediation on consumers' welfare is ambivalent: in the 'symmetric' outcome all consumers are better off from the presence of the intermediary, as they end up paying a lower price than the monopoly price they would have to pay without the intermediary. By contrast, in the other two equilibria the presence of the intermediary favors even further the consumers in one market but at the expense of consumers in the other.

3.2. **Oligopolistic markets with intermediation.** In the previous section we discussed a version of our model presented by (3) and (4), where each market was associated with a single producer. In that model each producer was facing competition from producers in other markets (intra-market competition) which was channeled via the intermediaries, but none was facing competition from within the market. Let us now consider the model where there is inter market competition as well. Consider two markets $i = 1, 2$ each furnished by k_i firms, all with identical marginal costs c. For simplicity and clarity suppose that the inverse demand functions are also the same across the two markets $p_i = a - bQ_i$. Finally, suppose that there are m intermediaries who operate across the two marketplaces. Given production decisions of the $\sum_{i=1}^{2} k_i$ firms each intermediary solves the problem:

$$\max \sum_{i=1}^{2} \left[a - b \left(\sum_{f=1}^{k_i} q_{i,f} + \sum_{i=1}^{m} q_j^i \right) \right] q_j^i, \ \ s.t. \ \sum_{i=1}^{2} q_j^i = 0 \tag{25}$$

The solution to this problem is given by :

$$q_j^i = \frac{1}{2(m+1)} \left(\sum_{f=1}^{k_j} q_{j,f} - \sum_{f=1}^{k_i} q_{i,f} \right), \ \ i, j = 1, 2 \tag{26}$$

Hence, in the first stage each firm (i, f) solves the problem:

$$\max \left[a - b \left(\sum_{f=1}^{k_i} q_{i,f} + \frac{m}{2(m+1)} \left(\sum_{f=1}^{k_j} q_{j,f} - \sum_{f=1}^{k_i} q_{i,f} \right) \right) \right] q_{i,f} - cq_{i,f}, \ \ s.t. \ q_{i,f} \geq 0 \tag{27}$$

The solutions to those problems are:

$$q_{i,f} = \frac{2(m+1)}{(m+2)(k_i+1)} \left(\frac{a-c}{b} \right) - \frac{mk_j}{(m+2)(k_i+1)} q_{j,f}, \ \ i, j = 1, 2 \tag{28}$$

Hence the equilibrium quantities are:

$$q_{i,f} = \frac{2(m+1)(m+2) + 4(m+1)k_j}{(m+2)^2 (1 + k_i + k_j) + 4(m+1) k_i k_j} \left(\frac{a-c}{b} \right), \ \ i, j = 1, 2 \tag{29}$$

$$q_j^i = \frac{(m+2)(k_j - k_i)}{(m+2)^2 (1 + k_i + k_j) + 4(m+1) k_i k_j} \left(\frac{a-c}{b} \right), \ i, j = 1, 2 \tag{30}$$

At equilibrium the quantities offered in the two markets $i = 1, 2$ are:

$$Q_i = \frac{(m+2)^2 k_i + m(m+2) k_j + 4(m+1) k_i k_j}{(m+2)^2 (1 + k_i + k_j) + 4(m+1) k_i k_j} \left(\frac{a-c}{b} \right), \ i, j = 1, 2 \tag{31}$$

Using this solution we can draw asymptotic conclusions by taking limits as the number of firms and intermediaries increases. In particular, as $k_i \to \infty$, $i = 1, 2$ then $Q_i \to \left(\frac{a-c}{b}\right)$ irrespectively of m, which is the competitive outcome. However, if $k_j \to \infty$ while k_i remains bounded[3] then $Q_i \to \frac{m(m+2)+4(m+1)k_i}{(m+2)^2+4(m+1)k_i} \left(\frac{a-c}{b}\right)$, which is not the competitive level of output in the ith market, unless $m \to \infty$.

It is evident in the preceding analysis that, it is only the number of intermediaries that is relevant in the final solution. This happens because in the first stage of the game, firms' decisions depend on the aggregate intermediation activity. Since intermediaries are identical, their reactions to the observation of firms' outputs in the first stage will be identical. In this way the second stage equilibrium of the game will always be symmetric. Thus, aggregate intermediation activity can be summarized by the equilibrium strategy of a, so to say, 'representative' intermediary multiplied by the number of intermediaries. In fact, we could imagine of an intermediary who splits orders and enters markets several times. The number of intermediaries m could equally well be interpreted as the number of entries of an intermediary in markets[4].

Hence, the number of intermediaries is crucial to firms' decisions. Here lies a very important issue: the number of intermediaries should be known to each firm in advance in order to properly choose their output. This fact is incorporated in the fundamental assumption that the structure of the game (including the number of intermediaries that has been postulated at the outset of the game) is common knowledge among players in the game. We believe that dropping this hypothesis, is a very interesting direction in this line work.

[3]This situation is reminiscent to an oligopolistic market with a 'competitive fringe'.

[4]Recall that if $m = 0$, then the profit of intermediation is zero. With $m = 1$ the profit increases and drops back to zero as $m \to \infty$. This suggests that there is scope for investigating the possibility of an 'optimal' number m, be it the number of intermediaries or entries to markets. As this issue would take us astray from our purpose we leave aside for further study.

For instance, as we saw in the case of markets with identical characteristics, at equilibrium the intermediaries are 'dormant': the behavior of firms at equilibrium does not leave any scope for intermediation, so these entities remain idle at equilibrium. Knowledge of the *number* of those entities is crucial however. Moving in the direction of incomplete information, one might develop the analysis along the line of different beliefs that players in the game may have about the number of intermediaries. Indeed, it could be argued that the lack of activity might make it impossible to identify the exact number of intermediaries that would be ready to arbitrage prices across markets. In such a possibility the present model can be viewed as a special case where players in the game have identical (and correct) beliefs about the number of intermediaries. This discussion suggests that there is obvious scope for extensions in this direction, where such beliefs might be formally modeled, endogenised, evolving in a dynamic version etc.

With those ideas in mind we proceed to develop our model one step further by introducing and studying a game where intermediaries enter sequentially.

4. Cournot Intermediation With Sequential Entry

Let us consider the following model: let $F_i(Q_i) = a_i - b_i Q_i$. Each of those demand functions is associated with one firm which produces under constant marginal cost c_i for $i = 1, 2, \ldots, n$. Suppose that m intermediaries can operate across this set of markets. The sequence of moves in those markets are as follows: in the first stage each of the n firms takes a decision to produce a quantity of output. Production decisions by firms are observed by intermediaries who enter the game sequentially one by one and place demand and supply orders in the different markets. Given the moves of all participants, prices clear markets and each participant collects payoffs. An equilibrium of such a game is a subgame perfect Nash equilibrium of this $m + 1$ stage game.

We start the analysis with the study of the behavior of the *mth* intermediary. Given production decisions $q \in \Re^n_+$ by firms and $q_j \in \Re^n$ by intermediaries $j = 1, 2, \ldots, m - 1$, this player solves the problem:

$$\max \sum_{i=1}^{n} \left[a_i - b_i \left(q_i + \sum_{j=1}^{m} q_j^i \right) \right] q_m^i, \;\; s.t. \;\; \sum_{i=1}^{n} q_m^i = 0 \tag{32}$$

The solution to this problem is for $i = 1, 2, \ldots, n$:

$$q_m^i(q) = \frac{1}{2b_i}\left(\sum_{t\neq i} r_t\left[(a_i - b_i q_i) - (a_t - b_t q_t)\right] - b_i \sum_{j=1}^{m-1} q_j^i\right) \tag{33}$$

where as before $r_i = \frac{\prod_{i\neq j} b_j}{\sum_{i=1}^{n}\prod_{i\neq j} b_j}$.

Proceeding backwards the intermediary in the $m-1$ stage solves the problem:

$$\begin{aligned} \max \quad & \sum_{i=1}^{n}\left[a_i - b_i\left(q_i + \sum_{j=1}^{m} q_j^i\right)\right] q_{m-1}^i \\ s.t. \quad & \sum_{i=1}^{n} q_{m-1}^i = 0 \\ & q_m^i = \frac{1}{2b_i}\left(\sum_{t\neq i} r_t\left[(a_i - b_i q_i) - (a_t - b_t q_t)\right] - b_i \sum_{j=1}^{m-1} q_j^i\right) \end{aligned} \tag{34}$$

The solution to this problem is for $i = 1, 2, \ldots, n$:

$$q_{m-1}^i(q) = \frac{1}{2b_i}\left(\sum_{t\neq i} r_t\left[(a_i - b_i q_i) - (a_t - b_t q_t)\right] - b_i \sum_{j=1}^{m-2} q_j^i\right) \tag{35}$$

Proceeding inductively we obtain the solution:

$$q_1^i(q) = \frac{1}{2b_i}\sum_{t\neq i} r_t\left[(a_i - b_i q_i) - (a_t - b_t q_t)\right] \tag{36}$$

$$q_2^i(q) = \frac{1}{4b_i}\sum_{t\neq i} r_t\left[(a_i - b_i q_i) - (a_t - b_t q_t)\right] \tag{37}$$

$$q_m^i(q) = \frac{1}{2^m b_i}\sum_{t\neq i} r_t\left[(a_i - b_i q_i) - (a_t - b_t q_t)\right] \tag{38}$$

Therefore, $\sum_{j=1}^{m} q_j^i = \frac{S_m}{b_i}\sum_{t\neq i} r_t\left[(a_i - b_i q_i) - (a_t - b_t q_t)\right]$, where $S_m = \sum_{j=1}^{m}\frac{1}{2^j}$. Hence, the problem of firm $i = 1, 2, \ldots, n$ in the first stage is:

$$\begin{aligned} \max \quad & \sum_{i=1}^{n}\left[a_i - b_i\left(q_i + \sum_{j=1}^{m} q_j^i\right)\right] q_i - c_i q_i \\ s.t. \quad & \sum_{j=1}^{m} q_j^i = \frac{S_m}{b_i}\sum_{t\neq i} r_t\left[(a_i - b_i q_i) - (a_t - b_t q_t)\right] \end{aligned} \tag{39}$$

Solving this problem for $i = 1, 2, \ldots, n$ and then solving the ensuing system of best responses we obtain the solution:

$$q_i = \frac{(1-S_m)\,\hat{a}_i - \hat{c}_i}{2-2S_m+S_m r_i} + \frac{\frac{S_m r_i}{2-2S_m+S_m r_i}}{1+\sum_{i=1}^{n}\frac{S_m r_i}{2-2S_m+S_m r_i}} \sum_{i=1}^{n}\left[\frac{(1-S_m+S_m r_i)\,\hat{a}_i + \hat{c}_i}{2-2S_m+S_m r_i}\right] \tag{40}$$

The above equation along with (38) define the solution to the sequential game. In order to better grasp the above formula we provide some special cases of it that might be helpful to the reader.

Example 4. If all markets have identical characteristics ($a_i = a$, $b_i = b$, $c_i = c$) then using the fact that $S_m = 1 - \frac{1}{2^m}$, (40) becomes:

$$q_i = \frac{n2^m}{2^m\,(n+1)+n-1}\left(\frac{a-c}{b}\right) \tag{41}$$

This example makes evident once more the effect of intermediation on competition: when m remains bounded then $q_i < \frac{n}{n+1}\left(\frac{a-c}{b}\right)$, unless $m \to \infty$ in which case $q_i \to \frac{n}{n+1}\left(\hat{a} - \hat{c}\right)$, which is the Cournot oligopoly output.

5. Conclusions

This paper offers a model for the study of intermediation across markets for a homogenous commodity. We have focused on the case of Cournot style competition among producers and intermediaries because this style of competition is prominent in the literature. Other types of competition can be considered as well. Our purpose here was twofold. First, we wanted to develop a general model that addresses the effects of intermediation directly. We are not aware of any other work in this direction. Second, we attempted to motivate the study of intermediation along the lines suggested in this paper by pointing out several interesting facts associated with intermediation.

It turns out that the degree of competition in the intermediation sector is vital for the spread of competitive forces across markets. In our model intermediaries are identical in terms of characteristics, so their population is the important variable that affects market outcomes. For the purposes of the analysis in this paper this population was taken as fixed (and known to the participants in the game). More generally however, the determination of the size of the intermediaries' population turns out to be of crucial importance. This is a central message of our analysis.

The issue of the intermediaries' population, which determines the degree of competition in the intermediation sector can be addressed in our opinion in three ways. One of those would be the construction of more sophisticated dynamic games where the decision to intermediate across markets is endogenized (for example allow firms to either produce or intermediate). Another direction to this effect that should be explored is to apply results drawn from evolutionary game theory. A second way to proceed is to build more stylized models which explicitly model the fundamental sources of intermediation. For instance, it might be argued that intermediation opportunities may arise from informational asymmetries (e.g. an individual who, unlike other consumers, is aware of price differences across markets). That would require the explicit modeling of uncertainty and information in a model along the lines of the models introduced in this paper. Another usual source of opportunity to intermediate is the posession/development of a technology which allows individuals to intermediate across markets with less costs than other consumers. Again an explicit modeling of such issues would be necessary.

Finally, in closing we would like to propose two more interesting extensions that can be incorporated directly into our model. One is the introduction of 'transaction costs' in terms of licensing to intermediate across markets (phenomena like smuggling could be added to the analysis in such a model). The second is the idea of 'incomplete intermediation' whereby intermediaries are restricted to operate between alternative subsets of the markets. We believe that both of those extensions will provide very interesting insights to the effects of intermediation on markets.