Liquidity benefits and bank capital requirements*

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We analyze the lending and financing decision of banks in a setting in which banks' short-term debt can generate liquidity benefits. If there are no further frictions, the socially optimal allocation is implemented in the decentralized equilibrium: Even though equity financing is more expensive, banks issue outside equity to make short-term debt safe such that runs and subsequent fire sales are excluded. If banks, however, are subject to a limited commitment friction concerning their financing decision, a maturity rat race arises. As a consequence, banks issue risky short-term debt and become subject to runs in downturns, which in turn trigger fire sales of bank assets. In this case, there is too much short-term debt, but too few liquidity benefits generated. Moreover, a wedge between the privately optimal and the socially optimal level of investment arises due to a pecuniary externality. The implementation of a capital requirement can restore the constrained-efficient allocation. Contrary to many other papers, we find that a capital requirement increases the amount of money-like claims in the economy and increases the aggregate level of lending.

Key works: banking; capital regulation; capital structure; short-term debt; bank runs; fire sales;

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1 Introduction

After the 2007-2009 financial crisis, policymakers as well as academics have held fierce debates on the optimal degree of capital regulation for banks. Those authors that tend to be opposed to strict capital requirements (e.g., DeAngelo and Stulz, 2015; Gorton and Winton, 2016) stress the virtues of short-term debt such as its value as a transaction medium (Gorton and Pennacchi, 1990; Dang et al., 2013), and its ability to impose discipline on intermediaries (Calomiris and Kahn, 1991, Diamond and Rajan, 2001, as well as the Squam Lake Report, 2010), and emphasize that increased capital requirements come at the cost of lower investment and hence lower economic activity.[1] In contrast, proponents of strict capital requirements (e.g., Admati and Hellwig, 2013b) emphasize that financial institutions have a natural tendency to choose excessively short maturities (Brunnermeier and Oehmke, 2013; He and Milbradt, 2016), and excessively high leverage (Admati et al., 2013; Hellwig, 2015) with potentially large social costs.

The banking literature has provided several arguments for why banks are different than other corporations. In classic arguments, banks insure agents against idiosyncratic liquidity shocks by offering demand deposit contracts (Diamond and Dybvig, 1983) and credit lines (Holmström and Tirole, 1998), they provide claims that are immune to adverse selection and thereby facilitate trade among agents with liquidity needs (Gorton and Pennacchi, 1990; Dang et al., 2013, 2014), their debt claims may be informational undemanding and hence useful to minimize monitoring costs (Diamond, 1984), or they issue claims that are particularly valuable in the presence of search frictions (Gu et al., 2013; Hollifield and Zetlin-Jones, 2016). Hence, several recent papers have condensed these insights into assuming that banks' (short-term) debt - unlike other bank liabilities - is associated with a liquidity benefit which creates utility that does not stem from its expected payoff. A number of recent models that use such an assumption are for instance Van den Heuvel (2008), Stein (2012), Hanson et al. (2015), DeAngelo and Stulz (2015), Moreira and Savov (2016), and Hellwig (2015).

Such an assumption clearly violates the requirements of the Modigliani and Miller (1958) theorem, and the question arises: what is a banks' optimal capital structure when short-term debt is associated with exactly such a utility-generating liquidity benefit? And how does it affect the optimal amount of lending? In this paper, we attempt to answer these questions, and investigate whether there is a wedge between the socially

[1] For instance, see the statement made by Josef Ackermann, former CEO of Deutsche Bank, in an interview on November 20, 2009 which is referred to by Admati et al. (2103): "More equity might increase the stability of banks. At the same time however, it would restrict their ability to provide loans to the rest of the economy. This reduces growth and has negative effects for all."

optimal allocation and the equilibrium outcome, and if so, whether a capital requirement can be means to improve the allocation attained in equilibrium. The novelty of our paper is that we allow for banks's short-term debt to be risky and to become subject to fundamental runs once a bank is highly leveraged. In particular, previous papers have typically ignored the possibility of runs and did not allow for short-term debt to become risky.[2]

Our paper has three main results. First, we show that if there are no other frictions, banks nonetheless implement the socially optimal allocation. The underlying logic is as follows: Banks anticipate that increasing leverage to a point where their short-term debt becomes risky, necessarily exposes them to runs if the economy enters a downturn. High leverage, in turn, has two adverse consequences. First, it increases a bank's cost of financing, as less liquidity benefits can be obtained when short-term debt becomes risky. Hence, even though, short-term debt is generally a cheaper source of financing than equity, we argue that the overall cost of financing are smallest when there is sufficient equity to make short-term debt safe such that liquidity benefits of the short-term debt in place are maximized. As in Hellwig (2015), the liquidity of bank debt itself depends on how much equity the bank is using. Second, if banks are highly leveraged, they are required to sell their assets at fire sale prices, i.e. at discounts, in downturns. This reduces the overall expected returns of the banking enterprise and hence the expected value of the bankers inside equity. As a consequence, banks optimally only issue risk-free short-term debt, capture the maximum liquidity benefit and choose the efficient level of lending (investment).[3]

However, the presence of a limited commitment friction fundamentally changes this mechanism. As a second result, we find that when banks cannot commit to their capital structure, a maturity rat race arises, similar to Brunnermeier and Oehmke (2013): banks choose to exclusively finance themselves by short-term debt and hence become necessarily subject to runs in downturns. As a consequence, the amount of liquidity benefits generated is too low: while there is plenty of short-term debt, the fact that short-term debt is risky reduces its value as a money-like claim. Moreover, a pecuniary externality and therewith a wedge between the socially and privately optimal level of investment arises. However, the pecuniary externality does not necessarily lead to excessive lending (investment), as e.g., in Lorenzoni (2008) or Stein (2012). In contrast, it is a pecuniary

[2] While the models of Van den Heuvel (2008), Hanson et al. (2015), and DeAngelo and Stulz (2015) neither allow for runs nor for short-term debt, Stein (2012) allows for the possibility runs, but short-term debt can never be risky.

[3] In our model, the banks lending operations is represented by a production technology. We use the expression investment and lending interchangeably.

externality of the redistributive type à la Dávila and Korinek (2016) and – depending on the parameters – there is under- or over-investment compared to the socially optimal allocation under limited commitment.

It then follows that a capital requirement is a natural candidate to act as a commitment device for banks, similar as in Admati et al. (2013), and address and alleviate the limited commitment friction. In our setup, a capital requirement allows to implement the allocation that is socially optimal in absence of the commitment friction. I.e., it allows the economy with limited commitment to mimic the economy without commitment friction.

Interestingly, the capital requirement has two effects that contradict findings that capital requirements dampen the possibility of banks to produce money-like claims (e.g., as in Van den Heuvel (2008), DeAngelo and Stulz (2015), and Gorton and Winton (2016)) or dampen aggregate lending. First, the requirement allows to increase the available money-like claims and hence the available liquidity benefits in the economy. The underlying logic is that only equity can make short-term debt safe and hence exclude the possibility of runs and the subsequent fire sales. This is in turn reduces the available funds that can be paid out to short-term debt holders in downturns and hence the available money-like claims. Second, it always leads to an increase in the aggregate investment. Here, the underlying logic is that in the absence of fire sales, investment incentives are not distorted as the bank is never required to sell its assets at discounts to outsiders.

Our model is a variant of the model discussed by Stein (2012), which in turn is similar to Lorenzoni (2008). There are three periods, and intermediaries can borrow from households and invest on their behalf. When intermediaries issue short-term debt claims that are safe, households require a lower return as they obtain a liquidity benefit that generates utility. If households desire to demand their claims back in the interim period, the intermediaries are required to sell some of their technology to other investors at a fire sale price. Such investors are themselves endowed with a productive technology, but only have finite funds. Hence they reduce their otherwise efficient investment in a fire sale, making it potentially socially costly (Diamond and Rajan, 2011; Shleifer and Vishny, 2011).

The key difference to the model by Stein (2012) is the that we assume that the intermediaries' technology always yields a positive cash flow. In Stein's model, agents holding short-term debt claims withdraw from banks because they fear that the ultimate return of the banks assets may become zero. Hence, a fire sale in Stein's model is necessary to make short-term debt claims safe. In contrast, in our model, a fire sale is a consequence of a run which can only happen when short-term debt is risky. Not allowing for zero cash flows generates scope for a capital requirement to be an effective and a natural instrument that makes short-term debt risk free and hence prevents fire sales.[4]

The slight change in the setup changes has somewhat drastic implications for the outcomes. If we don't add any further frictions, there is no wedge between the social planner choice and the competitive equilibrium. If, however, there is a maturity rat race that sets the stage for runs and fire sales in downturns, a pecuniary externality arises. However, the pecuniary externality here does not operate along collateral constraint as in Lorenzoni (2008) or Stein (2012), and hence does not necessarily lead to over-investment. In contrast, it is a pecuniary externality of the redistributive type in the sense of Dávila and Korinek (2016), that can potentially lead to over **or** over-investment. Finally, the wedge can be entirely eliminated by requiring all banks to issue sufficient equity to exclude runs, i.e., by using the capital requirement to alleviate the limited commitment friction.

We proceed as follows: in Section 2, we describe the basic setup. The constrained efficient allocation under commitment is derived in Section 3 before Section 4 derives the competitive equilibrium under commitment. Finally, we derive the socially optimal allocation and the equilibrium under limited commitment in Section 5 as well as the optimality of the capital requirement in Section 6, before concluding in Section 7.

2 Setup

The setup is similar to the model by Stein (2012). The economy goes through a sequence of three dates, $t = 0, 1, 2$. There is one good which can be used for investment and consumption, and there are two different states of the world, high and low, $s \in \{H, L\}$. s is revealed to all agents in $t = 1$ and the ex-ante probability of $s = H$ is given by π, and $s = L$ with $1 - \pi$.

The economy is populated by three types of risk neutral agents: households, intermediaries, and late investors. We begin by describing the agents' preferences, endowments and technologies. Altogether, there are two types of technologies: an early production technology, available at $t = 0$, as well as a late production technology, available at $t = 1$.

[4] In fact, we argue that our model is more robust than the version provided by Stein. In Stein's model, it is the fire sale that makes money safe. Hence, if there is a zero probability of zero cash flow in up state, a fire sale is triggered.

Households

Households are endowed with one unit of the good at $t = 0$. They maximize their expected lifetime consumption, given by $U = c_0 + E[c_1 + c_2] + \gamma m$, where m is the fraction of the households' short-term claims that generate a liquidity benefit $\gamma > 0$.

The key feature that defines our model as a "banking model", is the assumption that short-term debt creates utility beyond its expected payoff, i.e., it is associated with a liquidity benefit.[5] In particular, we assume that short-term debt, including risky short-term debt, can generate liquidity benefits. That is, in our setup, a short-term claim does not have to be absolutely safe to generate a liquidity benefit. However, a liquidity benefit only accrues for the "safe part" m of a claim, i.e, for the minimum cash flow that is generated across all states and dates. This implies that a short-term claim with an expected payoff of $\bar{X}$ and a minimum payoff of X_{min} across dates and states gives the household a utility of $\bar{X} + \gamma X_{min}$. Hence, even if a bank is subject to a fundamental run and hence if its short-term debt is risky, any cash flow that accrues to the claims holders at any contingency will nonetheless generate a liquidity benefit.[6] Similar to Stein (2012), the gross real return on a risky bond or risky equity claim is thus $R^B = 1$ and on any safe part of a claim that is risk-less, a "money" claim, is thus $R^M = \frac{1}{1+\gamma}$. Note that our setup is robust to assuming that only entirely safe claims generate liquidity benefits (e.g., as in Stein 2012).

Intermediaries

Households are assumed to be unable to invest their endowments in physical projects directly, and need intermediaries to do so.[7] In $t = 0$, a continuum of mass one of such intermediaries stand ready to provide this service. Intermediaries are risk neutral and also derive utility from consuming in the final period, i.e. $U^I = E[c_2^I]$. They have no goods endowment, but have access to the early production technology. Access to the technology is meant to represent the banks ability to issue loans/lend to firms and households, which we do not model explicitly.

The technology is available at $t = 0$ and specified as follows: Investing I units yields

[5] As indicated in the introduction, there are several papers in the banking literature that argue why bank liabilities, in particular short-term debt, create value by themselves. E.g., compare Diamond and Dybvig (1983), Diamond (1984), Gorton and Pennacchi (1990), and Dang et al. (2014).

[6] As will become clear later, this is only the case if a bank pays out an equal amount to all short-term debt holders, i.e., pro rate. If there is a sequential service constraint, then no liquidity benefits can be created if runs are possible.

[7] We do not explicitly model the friction that gives rise to such a restriction. One may argue that households may not have the monitoring and/or screening technology that intermediaries have, e.g. as in Diamond (1984) and Holmström and Tirole (1997).

an output at $t = 2$ given by $f(I) > I$ if the state is high, $s = H$, and where $f(I)$ is a concave function. In contrast, in the low state, $s = L$, the return is only λI, where $\lambda < 1$. The realization of the state becomes public knowledge at date 1.

The key difference to the model by Stein (2012), is to assume that the payoff from the intermediaries' assets is always strictly positive. In Stein's model, the ultimate cash flow can always become zero in the low state and hence makes a capital requirement genuinely unsuited to create safe claims. Hence, changing this assumption creates scope for a capital requirement to play a crucial role in the economy.

In order to finance their investment, intermediaries raise funds from households. It is assumed that intermediaries have all the bargaining power when contracting with households.[8] It is further assumed that the intermediary can only use two types of financial contracts: she can raise funds by issuing **short-term debt**, which generates a liquidity benefit as described above, and **outside equity**, which does not generate any utility beyond it expected payoff.[9] Households do not observe the intermediary's capital structure when deciding what type of claim to buy initially. The un-observability of the capital structure gives rise to a maturity rat race à la (Brunnermeier and Oehmke, 2013), whenever intermediaries cannot commit to their capital structure, as argued below.

Per unit of funding which is raised in the form of short-term debt in $t = 0$, the intermediary promises to pay an interest rate r_0 at $t = 1$. At $t = 1$, after the realization of the state, the capital structure of the intermediaries becomes public information, the intermediary offers a rollover interest rate $r_{1,s}$. The short-term debt holders can then decide to rollover the debt and are hence promised $r_0 r_{1,s}$ at $t = 2$. Denote by d the fraction of initial investment financed by short-term debt, i.e., the total amount of short-term debt is dI. The roll over decision is denoted by ω_s, where $\omega_s = 1$ indicates roll over, and $\omega_s = 0$ refusal to roll over. If all short-term debt holder refuse to roll over at the same time, we will refer to this as a bank run.

The intermediary raises the remaining part $(1 - d)I$ of the investment by issuing outside equity. In exchange for providing this funding, households in aggregate receive a claim on a fraction $\kappa(1 - d)$ of the residual that remains after repaying debt holders at $t = 2$. The intermediary receives the other fraction $1 - \kappa(1 - d)$ of the residual, which is referred to as the inside equity and hence the object that the intermediary desires to maximize.

[8] An alternative way of describing the structure of the economy is as follows: each intermediary shares an island populated by some households that each have an endowment of one unit. She is hence a monopolist in providing intermediation services to households. However, intermediaries are assumed to price takers when trading in $t = 1$.

[9] The setup can easily be extended by allowing for other risky long-term claims to be issued.

Late investors

Finally, in $t = 1$, there is a continuum of mass one of late investors.[10] They are risk neutral and maximize their final period consumption $U^{LI} = c_2^{LI}$. Late investors are each endowed with A units of the good and they have access to the **late production technology**. Besides investing, they can also use their endowment to purchase assets from intermediaries.

The late production technology is specified as follows: an investment of k in $t = 1$ yields a gross output of $g(k)$ at $t = 2$ with certainty, where $g(\cdot)$ is a strictly concave function with $g'(\cdot) > 1$ and $g''(\cdot) < 0$.

Let M_s denote the state-dependent amount of funds used to buy assets, the state-dependent investment in the late technology can be written as $k_s = A - M_s$. Let q_s denote the price at $t = 1$ per unit of the intermediary's asset in state s. Given some price q_s, spending one unit of liquidity to buy assets gives a return of λ/q_s. The late investor's investment profit in state s for a given price q_s is given by

$$\Pi_{LateI}(M_s) = g(A - M_s) + M_s \frac{\lambda}{q_s}.$$

In case of an interior optimum, it must hold that M_s is such that $g'(A - M_s) = \lambda/q_s$. To put it differently, if a late investor spends an amount M_s to buy assets, his willingness to pay for the asset is given by

$$q_s = \frac{\lambda}{g'(A - M_s)}. \tag{1}$$

As we will see later, it always holds that $M_H = 0$, and so q_H will not play a role. However, if $M_L > 0$, this equation will be interpreted as a binding "IC constraint" of late investors, i.e., a constraint that tells us how much funds late investors will spend to buy assets for a given equilibrium fire-sale price of assets.

In our setup, the inefficiency that may arise from fire sales does not result from late investors having a lower utility for intermediaries' assets (e.g., as in Shleifer and Vishny, 1992), but from a lower investment in their productive technology (see, e.g., Diamond and Rajan, 2011; Shleifer and Vishny, 2011, as well as Stein, 2012). Moreover, assuming $g'(\cdot) > 1$ ensures an interpretation of q_s as a fire sale price. Therefore, M_s is a crucial variable in this model as it describes the amount of liquid funds that late investors devote to purchasing assets from intermediaries in a fire sale instead of operating

[10] We treat late investors and intermediaries as different agents mostly for expositional purposes. They could be treated as a similar type of intermediary as described above. However, we would be required to assumed that they interact as price takers in $t = 1$, and are not subject to the same macroeconomic risk. In particular, late investors can be interpreted as intermediaries that have not been affected by the downturn.

their productive technology. The size of M_s determines the inefficiency that arises from intermediaries fire selling their assets in the low state.

3 Constrained-efficient allocation

We start the analysis by deriving the constrained-efficient allocation when intermediaries can commit to their capital structure. In the constrained-efficient allocation, welfare is maximized by choosing the amount of initial investment I, the fraction of financing via short-term debt d, as well as the state-dependent level of late investment M_s, subject to the restrictions that the incentive constraint of late investors as well as the incentive constraint of households are respected. While households that hold short-term debt claims need to be willing to participate in financing intermediaries ex-ante (depositing), they also need to be willing to continue financing the intermediary ex-interim in $t = 1$ (rollover). Hence, we start by analyzing the short-term debt holders' decision to withdraw, $\omega = 0$, or to roll over $\omega = 1$. Note that throughout the entire paper, we exclude the possibility of self-fulfilling, panic-based runs à la Diamond and Dybvig, 1983.

Rollover, liquidation and liquidity benefits

At $t = 1$, households observe the state s. Households holding short-term debt claims can decide whether to roll over their claims or not. Whenever some short-term debt holders refuse to roll over their debt, the intermediary will be required to liquidate an amount of assets z_s.

Observe that whenever $s = H$, households have no incentive to refuse to rollover their debt (ignoring the possibility of panic-based runs), i.e. $\omega_H = 1$. However, in state L, short-term debt holders find it optimal to refuse to roll over their debt claims if $dIr_0 > \lambda I$. Observe that by focusing on fundamental runs, the bank will either have to sell its entire portfolio of assets or nothing at all. This results from the fact that runs can only occur if short-term debt claims are risky. However, in order for short-term debt claims to be risky, a fundamental run must occur and a fundamental run can only occur if not all short-term debt holders claims can be fully served. This implies that the bank cannot fully serve all withdrawing short-term debt holder by even selling its last unit of the asset. Hence, it holds that

$$z_L = \begin{cases} 0 & \text{if } d \leq \lambda(1+\gamma) \\ 1 & \text{otherwise,} \end{cases} \tag{2}$$

Moreover, whenever short-term debt is not risky, then the interest rate paid to short-term debt holders is the risk-free rate on money $r_0 = R^M = 1/(1+\gamma)$.

Observe that because runs never occur in state $s = H$, no fire sale can take place in this state, and the funds transferred from late investor to intermediaries will be zero, i.e., $M_H = 0$. Hence, in this state, the late investors' funds are invested fully in the late technology. Without loss of generality, we can exclusively focus on the late investors investment decision M_L in state L.

The market clearing condition in the low state is given by

$$q_L z_L I = M_L. \tag{3}$$

From the liquidation function z_L if follows directly that the safe component of short-term debt m (the amount of claims that are associated with a liquidity benefit) is given by

$$m = \begin{cases} dI/(1+\gamma) & \text{if } d \leq \lambda(1+\gamma) \\ q_L I & \text{otherwise .} \end{cases} \tag{4}$$

I.e., whenever all debt claims are safe, there is are no runs, and the potential liquidity benefit that can be realized is given by the technological restrictions on the cash flow in the low state. In contrast, if debt claims are risky, the highest possible cash flow realized by an intermediary in the low state is given by the intermediaries assets valued at the fire sale price. Hence, once a run becomes possible, the liquidity benefit decreases as the cash flow of the intermediary in the low state drops.

Constrained efficient allocation

The welfare is given by

$$\mathcal{W}(I, d, M_L) = [\pi f(I) + (1-\pi)\lambda I - I] + m(d)\gamma + (1-\pi)[g(W - M_L) + M_L],$$

The constrained efficient allocation is attained the initial investment I, the late investment in the low state M_L and the fraction of short-term debt d that maximize the welfare subject to the "market IC constraint", which is a combination of the IC constraint and the market clearing constraint:

$$g'(W - M_L)M_L = z_L \lambda I. \tag{5}$$

It is straightforward to see that it is optimal to choose $d^{**} = \lambda(1+\gamma)$, implying $z_L = 0$ and $m = \lambda I$. If d was smaller, there would be scope to increase the monetary benefits without any downside. However, choosing a larger d would expose the intermediaries to a fundamental run, forcing the intermediaries to liquidate their assets, i.e., selling them to late investors. As $g'(\cdot) > 1$, late investors' resources are more efficiently used for productive investment compared to being transferred to intermediaries in exchange for claims on the intermediaries future cash flows. Thus, a larger d would lead to lower aggregate output in the low state and reduce the availability of money-like claims.

Lemma 1. *In the constrained-efficient allocation, the amount of short-term debt issues is such that no fundamental runs occur.*

Given that there are no assets sales, trivially, the optimal choice of late funds used to buy assets is given by $M_s^{**} = 0$.

Moreover, in the constrained efficient allocation, I^* is chosen such that

$$\pi f'(I^{**}) + (1 - \pi)\lambda - 1 + \lambda\gamma = 0. \tag{6}$$

We assume henceforth that f is such that an inner solution exists.[11]

4 Decentralized Equilibrium under commitment

We now turn towards deriving the decentralized equilibrium of the economy when intermediaries can commit to their capital structure.

In the decentralized equilibrium, an intermediary's investment policy I and his capital structure d (with corresponding interest rates r_0, r_1 and outside equity κ at $t = 0$) are such that they maximize the intermediary's profits (i.e., the value of inside equity), taking as given the market price. Again, the participation constraint of households (depositing) and their IC constraint (withdrawal mechanism) have to be satisfied, and the mechanics of liquidation are as described in the previous section. In equilibrium, the market price and the asset purchase of late investors M_s must be such that (1) and the market clearing constraint (5) are satisfied.

Taking into consideration the run decision by households that hold short-term debt claims, and taking as given the market price in the low state, q_L, each individual inter-

[11] Specifically, we assume that $\pi f'(1) + (1 - \pi)\lambda - 1 + \lambda\gamma < 0$.

mediary maximizes the expected value of her inside equity, given by:[12]

$$\Pi_{Bank}(I, d, z, m) = \pi f(I) + (1 - \pi)[zq_L + (1 - z)\lambda]I - I + \gamma m$$

where z and m are again given by Equations (2) and (4). In equilibrium, the IC constraint (1) and the market clearing constraint (3) have to be satisfied.

We now argue that it is privately optimal for intermediaries to choose $d^* = \lambda(1 + \gamma)$: a smaller d implies that the bank can make more profit by increasing the cheaper short-term debt finance without causing a run that would expose the intermediary to a fire sale. Moreover, a larger d would expose the intermediary to a fundamental run in $t = 1$. This has two adverse consequences: First, as $g'(\cdot) > 1$, the intermediary would be required to sell her assets at a discount and hence lose output in expectation. Second, it reduces the amount of cheap financing by capturing the liquidity benefit. Even though short-term debt is generally a cheaper source of financing than equity, equity can minimize the overall financing cost. When runs are excluded, the fund available to pay out short-term debt holders in both states increase as the bank no longer needs to liquidate the asset at the discount of $1/g'(\cdot)$.

Taken together, the intermediary's prefer a capital structure that implies that there is no liquidation at all, i.e., $z = 0$. This implies that no funds of late investors are used to buy assets in either state, $M_H^* = M_L^* = 0$, and investment is chosen such that $\pi f'(I^*) + (1 - \pi)\lambda - 1 + \lambda\gamma = 0$. Hence, we can conclude that the equilibrium and the constraint-efficient allocation coincide.

Proposition 1. *The decentralized equilibrium implements the constrained-efficient allocation.*

As described above, the result can be explained by the simple insight that choosing a higher leverage is privately costly in two ways: it makes financing more expensive because the "money-ness" of short-term claims is reduced, and it leads to a loss in the intermediary's private return as the run would expose him to a fire sale. Hence, the private and the social interests are aligned in this variant of the model.

[12] The value of the inside equity is calculate as follows. Short-term debt holder require the following interest rate r_0: If debt is risk free, $r_0 = \frac{1}{1+\gamma}$ satisfies their participation constraint. In contrast, if the intermediary's short-term debt is risky, i.e., $d \leq \lambda I(1+\gamma)$ and the intermediary becomes subject to a run in the low state, then the participation constraint is fulfilled if: $\pi r_0 + (1-\pi)q_L/d + \gamma q_L/d = 1 \Leftrightarrow r_0 = \frac{1-[(1-\pi)+\gamma]q_L/d}{\pi}$. Moreover let V^e be the expected value of the residual claims, those households that hold an outside equity claims, will require in aggregate a share κ such that $\kappa = \frac{I}{E[V^e]}$. The value of the inside equity is then given by $[1 - \kappa(1 - d)]V^e$.

5 Limited Commitment

In the following, we analyze the case when intermediaries cannot commit to their capital structure as, e.g., in Brunnermeier and Oehmke (2013), Admati et al. (2013) and Hellwig (2015). As argued in the literature We show that this gives rise to a maturity rat race, inducing intermediaries to use short-term debt as their exclusive way of raising funds.

5.1 Maturity Rat Race

Assume that the intermediary's investment level I is given, and assume that the intermediary needs to raise its funds by issuing short-term debt and outside equity. We assume that the intermediary's capital structure is initially unobservable for the financiers, it only becomes public information in $t = 1$. In essence, intermediary's and households are playing a Bayesian game with hidden actions in the initial period, which has the following Percent Bayesian Equilibrium.

Lemma 2. *In the Perfect Bayesian equilibrium under limited commitment, intermediaries choose to finance themselves exclusively by short-term debt, i.e.* $d^* = 1$.

Before proving the lemma formally, let us explain the intuition underlying the result. Similar to Brunnermeier and Oehmke (2013) and Admati et al. (2013), the unobservability of actions and the inability to commit, gives the intermediary at any capital structure the incentive to increase the leverage, i.e. shorten the maturity. Why? For any given capital structure, it is optimal to give a single investor another short-term debt claim in exchange for an equity claim. While the investors that becomes a short-term debt holder is indifferent, the value of the intermediary's upside (inside equity) is increased at the expense of all other investors as they had agreed to their contracts under a lower level of leverage and hence assuming a lower risk.

We formally prove Lemma by contradiction: We can show that for any announced level of short-term debt $d < 1$, an intermediary has an incentive to deviate to $d' = 1$.

Assume a household has some belief about the overall capital structure of the intermediary. For instance, assume that she believes that the capital structure is such that there will be no run in $t = 0$, i.e., $d \leq \lambda(1 + \gamma)$. Consider the interest rate that is required order to satisfy the initial participation constraint of any household that buys a short-term debt claims, which is given by $r_0 = \frac{1}{1+\gamma}$. Alternatively, consider the case in which the household were to believe that the intermediary chooses a high leverage and hence becomes subject to a run in the low state, then the participation constraint fulfilled if:

$$\pi r_0 + (1-\pi)q_L/d + \gamma q_L/d = 1 \Leftrightarrow r_0 = \frac{1-[(1-\pi)+\gamma]q_L/d}{\pi}$$

Let $V^e(d)$ be the expected value of the residual claims (equity), i.e., the expected cash flow after repaying debt holders. Those households that hold an outside equity claims, will require a share κ such that

$$\kappa = \frac{I}{V^e}.$$

Given the unobservability of the intermediaries capital structure, we can analyze whether there is an incentive to deviate to a higher leverage, given that all households have agreed on terms of funding at a certain level of leverage d. There are two cases to distinguish. First, assume that all financiers of the intermediary were to believe that the intermediary has a low leverage and does not become subject to a run, i.e., $d \leq \lambda I(1+\gamma)$. In this case, the value of the inside equity is given by:

$$\begin{aligned} V^{ie}(d) &= (1-\kappa(1-d))[\pi f(I) + (1-\pi)\lambda I - d\frac{1}{1+\gamma}I] \\ &= \pi f(I) + (1-\pi)\lambda I - I + dI\frac{\gamma}{1+\gamma} \end{aligned}$$

If the intermediary, however deviates to a higher level of leverage, for instance, $d' = 1$, while its investors believe that $d = d$, the expected value of the inside equity becomes:

$$V^{ie}(d' = 1|d) = \pi \left[f(I) - I + I\frac{\gamma}{1+\gamma} \right]$$

which is strictly larger and hence, deviating is optimal for the intermediary and a belief of households of $d \leq \lambda I(1+\gamma)$ is not consistent.

Likewise, if households have the belief that the intermediary chooses d such that she becomes subject to a run, i.e., $1 > d > \lambda I(1+\gamma)$, the value of the inside equity is given by:

$$\begin{aligned} V^{ie}(d) &= (1-\kappa(1-d))\pi[f(I) - dr_0(d)] \\ &= \pi f(I) + (1-\pi)q_L I + \gamma q_L I - I \end{aligned}$$

Deviating to $d' = 1$ in turn yields the value of the inside equity becomes:

$$\begin{aligned} V^{ie}(d' = 1|d) &= \pi f(I) - \pi r_0(d) I \\ &= \pi f(I) + (1-\pi)q_L/dI + q_L/d\gamma I - I \end{aligned}$$

which is also strictly larger. Hence,

$$V^{ie}(d' = 1|d) > V^{ie}(d)$$

implying that it is always optimal for the intermediary to increase the leverage. Hence, in the Perfect Bayesian equilibrium of the financing game, the intermediaries choose $d^* = 1$

5.2 Constrained-efficient under Limited Commitment

We now turn towards analyzing the optimal allocation when the limited commitment problem concerning the intermediaries' financing structure is a constraint that cannot be changed, i.e., we are looking at the optimal allocation for the case that intermediaries are financed exclusively by risky short-term debt and fully leveraged. The constrained-efficient allocation is determined is determined as above, except for the fact that the leverage is no longer a choice variable, but given by $d = 1$.

The welfare in case of full leverage is given by

$$\mathcal{W}(I, M_L) = \pi f(I) + (1-\pi)\lambda I - I + \gamma M_L + (1-\pi)[g(W - M_L) + M_L].$$

Combining the market clearing and late investors' IC constraint, result in the combined constraint that requires that I and M_L have to satisfy the following constraint:

$$g'(W - M_L)M_L = \lambda I. \tag{7}$$

Let us to define a the implicit function $M_L(I)$ that is given by Equation (7) and observe that the following holds:

$$\frac{\partial M_L}{\partial I} = \frac{\lambda}{g'(W - M_L) - M_L g''(W - M_L)} > 0. \tag{8}$$

Using this representation, the constraint efficient has to satisfy the following FOC:

$$\frac{d\mathcal{W}}{dI} = \pi f'(I) + (1-\pi)\lambda - 1 + [\gamma + (1-\pi)(1-g')]\frac{\lambda}{g' - M_L g''} = 0, \tag{9}$$

where g' is the short form for $g'(W - M_L)$.

The constrained efficient allocation under limited commitment is fully characterized by $d^* = 1$, $M_H^* = 0$, and (I^{**}, M_L^{**}) that satisfy the FOC (9) and the market constraint (7).

5.3 Decentralized Equilibrium under Limited Commitment

Given that without commitment the intermediaries are exclusively financed by short-term debt, they are necessarily subject to a run at $t = 1$ if $s = L$. Their profit is given by

$$\Pi_{Bank}(I) = \pi f(I) + (1-\pi)q_L I - I + \gamma q_L I.$$

The decentralized equilibrium under limited commitment is characterized by (I^*, M^*) satisfying the intermediaries' FOC,

$$\frac{\partial \Pi_{Bank}}{\partial I} = \pi f'(I) + (1-\pi)q_L - 1 + q_L\gamma = 0, \tag{10}$$

and the market constraint (7).

This allows us to compare the decentralized equilibrium (DE) to the constrained efficient (CE), and to analyze whether there is under- or overinvestment in the DE compared to the CE. Observe that allocations of the CE and of the DE are satisfy the market-clearing constraint as well as the respective FOC.

In order to make the two FOCs comparable, we rewrite the derivative of the intermediaries' profit as

$$\frac{\partial \Pi}{\partial I} = \pi f'(I) + (1-\pi)\lambda - 1 + [\gamma + (1-\pi)(1 - g'(W - M_L))]\frac{\lambda}{g'(W - M_L)} \tag{11}$$

Let us evaluate the FOC of the DE, i.e., Equation (11), at the constrained efficient allocation (I^{**}, M^{**}). Using that this allocation satisfies (9) with equality, we obtain that

$$\begin{aligned}\left.\frac{\partial \Pi}{\partial I}\right|_{(I^{**},M^{**})} &= [\gamma + (1-\pi)(1 - g'(W - M^{**}))] \\ &\quad \cdot \underbrace{\left[\frac{\lambda}{g'(W-M^{**})} - \frac{\lambda}{g'(W-M^{**}) - M^{**}g''(W-M^{**})}\right]}_{>0}.\end{aligned}$$

This expression is positive if and only if the sign of

$$\gamma + (1-\pi)(1 - g'(W - M^{**})) \tag{12}$$

is positive.

Recall that the market clearing constraint is such that $\partial M/\partial I > 0$. Observe that for the set of values satisfying the market-clearing constraint, i.e., $\{(I, M(I))\}$, the intermediaries' marginal profit if monotonically decreasing in I:

$$\frac{d\frac{\partial \Pi(I,M(I))}{\partial I}}{dI} = \pi f''(I) + (\gamma + 1 - \pi)\frac{\lambda g''(W-M)}{[g'(W-M)]^2}\frac{\partial M}{\partial I} < 0.$$

Hence, the sufficient criterion that determines whether there is over or under-investment is the sign of (12). For $\gamma + (1-\pi)(1-g'(W-M^{**})) >$ it holds that $\frac{\partial \Pi(I^{**},M^{**})}{\partial I} > 0$, which is equivalent to saying that at the social optimal allocation, the marginal benefit of an additional unit of "money"-financing, γ, is larger than than the expected marginal cost of being exposed to the fire sale pricing, $(1-\pi)(1-g'(W-M^{**}))$. Hence, it follows that the decentralized equilibrium features over-investment compared to the socially optimal allocation whenever $\gamma > (1-\pi)(g'(W-M^{**})-1)$, and under-investment otherwise. We conclude with the following proposition.

Proposition 2. *Let M^{**} be the constrained efficient level of asset purchases at $t = 1$. If $\gamma < (1-\pi)(g'(W-M^{**})-1)$, there is under-investment and too little money creation in the decentralized equilibrium, and over-investment and too much money creation otherwise.*

The wedge between the constrained efficient level of investment and the one attained in the decentralized equilibrium results from a pecuniary externality which, in the sense of Dávila and Korinek (2016), is of a redistributive type. In contrast, the pecuniary externality in Stein, 2012 and Lorenzoni, 2008 operates via a collateral constraint and thus necessarily induces over-investment. In our case, the direction of the investment distortion is not clear a priori, it depends on the ratio of marginal utility or productivities across the two sectors. The decentralized equilibrium is characterized by over-investment if the parameters are such that at the constrained efficient allocation, the additional liquidity benefit of using one more unit of the late investors' budget to serve as money, γ, exceeds the expected loss resulting from forgoing the profitable investment opportunity in the low state, $(1-\pi)(g'(W-M^{**})-1)$.

As we have seen, the decentralized equilibrium does not implement the constrained efficient in the case of limited commitment. Thus, if we cannot address the limited commitment problem directly (the case where we can will be discussed in the next section), the natural question is whether how we can at least implement the CE under this friction. We can do this by target the level of I, either directly through regulation, or by imposing taxes.

6 Capital Requirement

In the following, we will argue that a capital requirement is a natural tool to improve the economy's outcome. In particular, if there is a limited commitment friction, we argue that a capital requirement can completely offset the limited commitment friction, and allow to implement the allocation which is socially optimal under commitment.

Proposition 3. *If intermediaries cannot commit to a capital structure, a capital regulation that limits the debt ratio to $d \leq \lambda(1+\gamma)$ allows to mimic the economy under commitment. This increases aggregate investment, liquidity benefits, and welfare.*

By setting the maximum allowed fraction of short-term debt to $d = \lambda(1+\gamma)$, a regulator can enforce the implementation of the allocation that would be constrained efficient in the absence of commitment problems.

When comparing the allocation without commitment frictions and the one with commitment frictions, it can be seen that the amount if lending/investment and the liquidity benefits generated are unambiguously higher when there is no limited commitment frictions.

To formally show that investment increases, consider the following analysis. Let I_C^{**} and I_{NC}^{**} denote the constrained efficient investment levels when commitment is possible (C) and when it is not possible (NC), and let I_C^* and I_{NC}^* denote the respective investment levels attained in the decentralized equilibrium. As we have seen, it holds that $I_C^{**} = I_C^*$, where as the relationship between I_{NC}^{**} and I_{NC}^* is ambiguous. In order to evaluate the effect of capital regulation, we compare NC and C because capital regulation allows us to mimic the the economy under commitment.

We can do two different analyses: If the policy of targeting I is not feasible, than we have to compare the two allocations of the decentralized equilibrium, whereas if this policy is feasible, we have to compare the allocations of the constrained efficient. In both cases, the limited commitment case features underinvestment.

Under commitment, the level of investment is characterized by Equation (6), which can be rewritten as

$$\pi f'(I_C^{**}) = \pi f'(I_C^*) = 1 - (1-\pi+\gamma)\lambda. \tag{13}$$

Without commitment, the decentralized equilibrium investment level is given by Equation (10), which can be written as

$$\pi f'(I_{NC}^*) = 1 - (1-\pi+\gamma)q_L. \tag{14}$$

Because $q_L < \lambda$, it holds that $f'(I_{NC}^*) > f'(I_C^*)$, and $I_{NC}^* < I_C^*$.

The constrained efficient investment level is given by Equation (9), which can be written as

$$\pi f'(I_{NC}^{**}) = 1 - \left(1-\pi+\gamma - \frac{(1-\pi)(g'-1)}{g' - M_L g''}\right)\lambda. \tag{15}$$

This again implies that $I_{NC}^{**} < I_C^{**}$.

At the same time, as argued above, whenever all debt claims are safe, there is are no runs, and the potential liquidity benefit realized is given by cash flow generated by the intermediaries technology and not by the fire sale value of intermediaries assets.

Hence, in our model, a capital requirement increases investment and increases the amount of liquidity benefits in the economy. Both is contrary to the often stated view that capital requirements have a negative effect on investment as they increase the financing costs of banks. In contrast, we argue that in the presence of commitment frictions, capital requirements actually *lower* the financing costs of banks. While it is true that deposit funding of banks is cheaper than equity funding because deposits can be associated with liquidity benefits, these liquidity benefits are maximized only if the bank has issued a sufficient amount of equity such that runs are excluded. Hence, as the financing costs are lowered by a capital requirement, the actual level of investment/lending is increased.

7 Discussion

Our model makes the case for a capital requirement as a policy instrument to increase aggregate bank lending and investment, and to improve the liquidity provision by banks. The results are driven by the insights that in an economy in which banks can commit to their capital structure, private and social incentives are aligned. Only if there is a limited commitment friction, runs and fire sales take place in equilibrium and welfare decreases. In this case, the capital requirement is a natural and ideal instrument to alleviate this friction. Similar to Hellwig (2015), the capital requirement increases social welfare but also bank profits at the same time.

Do we believe that capital requirements are a panacea? The answer is no. The particular goal of this paper is to understand the role of capital requirements in an economy in which short-term debt is associated with liquidity benefits, and concludes that capital requirements are not harmful in such an economy, but in fact helpful. Our model, however, ignores other reasons for why capital requirements may have negative welfare effects. For instance, the presence of capital requirements may be detrimental for the disciplining effect of short-term debt in the sense of Calomiris and Kahn, 1991; Diamond and Rajan, 2000. If one believes that the disciplining effect of short-term debt is very important, capital requirements may yet be problematic. If one believes that the disciplining effect is an academic myth as, e.g., Admati and Hellwig, 2013a, they may yet be helpful.

An additional reason for capital requirements emerges in the presence government guarantees for banks' debt claims. In the presence of a deposit insurance or (explicit or implicit) bailout guarantees, debt finance becomes privately cheaper than other sources of finance such as equity. Hence, there are incentives to increase leverage and other types of moral hazard such as investing in negative NPV project that allow to maximize the bankers upside. This view of bank capital regulation is complementary to ours. However, as Allen et al. (2015) argue, if the non-financial sector that borrows from banks has little leverage, the actual capital requirements needed for banks may be very small.

Finally, a limitation of our analysis (and of most other work on capital requirements) is that we ignore regulatory arbitrage, i.e., the fact that capital requirements can be circumvented. E.g., Plantin, 2015 shows that capital requirements should not be too strict, because otherwise the shadow banking sector grows too large. In the presence of such regulatory arbitrage, it might be optimal to complement capital requirements with other policies to ensure financial stability.

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