Prisoner's Dilemma with Talk*

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Abstract

When players in a game can communicate they may learn each other's strategy. It is then natural to define a player's strategy as a mapping from what he has learned about the other players' strategies into actions. In this paper we investigate the consequences of this possibility in two-player games and show that it expands the set of equilibrium outcomes in one-shot games. When strategies are observable with certainty, any feasible and individually rational outcome can be sustained in equilibrium. Our framework can account for both cooperation and correlation in the players' strategies in the prisoner's dilemma.

KEYWORDS: Cooperation in Prisoner's Dilemma, Games with Communication, Talk, Program Equilibrium, Delegation.

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1 Introduction

A few salient facts emerge from a large body of experimental evidence on the Prisoner's Dilemma (henceforth PD). Communication enlarges the range of possible payoffs, even in the PD, where cheap talk should make no difference theoretically. Frank (1998) reports experimental results that show that when subjects are allowed to interact for 30 minutes before playing the PD, they are able to predict quite accurately their opponent's behavior. Moreover, roughly 84% of the subjects who predict that their opponent will cooperate (defect) respond with the same action. A longer period of communication also leads to a higher probability of cooperation. Both the level of cooperation and the accuracy of the predictions drop when players are allowed to interact only for 10 minutes.

Sally (1995) did a meta-analysis of experiments from 1958 to 1992. Combining data from 37 different experiments, he showed that communication increases the rate of cooperation by roughly 40%. Interestingly, communication was one of the few variables that had a significant effect on cooperation (see Sally (1995) and references therein).

Kalay et al. (2003) consider data obtained from a TV game similar to the PD, in which two players accumulate a substantial amount of money, and then divide it as follows: Players communicate for several minutes, and each player then chooses one of the two actions cooperate or defect. If they both cooperate, each obtains half of the money they accumulated. If one cooperates and the other defects, the one that defected receives everything and the other nothing. In case both defect, both receive nothing. As in the PD, the dominant strategy is to defect. However, 42% of the time the players cooperated. Moreover, the data reveals a correlation between the actions chosen by the two players (21% of the time both players cooperated, compared to 17.64% if there had been no correlation); this implies a correlation coefficient of 0.14.

More recently, Belot et al. (2010) and den Assem et al. (2010) have studied similar game-shows. Belot et al. find that making a promise to cooperate prior to the decision is positively related to the actual cooperation rate if the promise was voluntary, but not if it was elicited by the host. Using data from the TV game show 'Golden Balls' den Assem et al. find that while co-operation decreases with the stakes, players still cooperate about 50 percent of the time even at the higher end of the stakes. The authors also find evidence that people have reciprocal preferences. That is they prefer to cooperate with players who cooperate with them and defect against players who defect against them.

Standard game theory tends to ignore the fact that in strategic situations people often have the opportunity to communicate before choosing their actions. When communication is included it is taken to be "cheap talk" that does not oblige the players in any way. However there are situations where communication leads players to either betray their true intentions or learn the intentions of the other players. A player may then want to condition his action on the information he learns from his opponents during the interaction.

Consider the PD, where each player's action set is $\{C, D\}$ and payoffs are as follows.

	C	D
C	$3,3$	$0,4$
D	$4,0$	$1,0$

In game theoretic models strategies are the same as actions in a one-shot game. We enrich this notion of a strategy to include not just actions but also richer notions about the person's *intention*. Suppose each player has 3 strategies — $C, D, nice$. We shall show that *nice*, to be defined shortly, can reconcile theory and evidence, even within the scope of equilibrium notions.

Players simultaneously pick a strategy from $S_1 = S_2 = \{C, D, nice\}$ and then engage in talk. When players in a game can talk to each other they may learn the other player's strategy with a probability p. The *nice* strategy is a mapping from what a player has learned into an action as follows; it amounts to saying "If I learn that the other player picked *nice* then I will pick C, and pick D if he has picked C or D or if I get no signal about his pick." We assume that during real talk, each player either receives a correct signal about the other's player strategy (with probability p), or learns nothing. An incorrect signal is impossible in this setting, which means that either each player knows the other's strategy, none of them does, or one does and the other does not. For simplicity, we further assume that players receiving signals are independent events. This assumption is relaxed below.

For intuition consider the simplest case, where $p = 1$, i.e. players learn their opponent's strategies perfectly during the real talk phase. Each pair of *strategies* translates into a pair of *actions*. The interesting entries are contained in row 3 and column 3, corresponding to the choice of *nice* by at least one player.

	C	D	nice
C	C,C	C,D	C,D
D	D,C	D,D	D,D
nice	D,C	D,D	C,C

This leads to the following payoff matrix:

	C	D	nice
C	(3,3)	(0,4)	(0,4)
D	(4,0)	(1,1)	(1,1)
nice	(4,0)	(1,1)	(3,3)

There are two desirable strategy profiles that give the best payoff— (C, C) and $(nice, nice)$. The first is not an equilibrium but the second is. In fact the conclusion is stronger— *nice* weakly dominates both C and D! In particular, D is not a weakly dominant strategy in the augmented game. The paper generalises this observation to other games, other values of p (not necessarily 1), and considers the possibility of correlation between players' signals.

The plan of the paper is as follows. Section 2 consists of a review of related literature. In Section

3 we present the model and basic definitions. In section 4 we find what payoffs may be sustained in equilibrium of general two player games. Section 5 analyzes the prisoner's dilemma. Section 6 concludes.

2 Related Literature

This paper concerns true information that is transferred between two people during communication. Although we label this as "talk" it can be interpreted as any form of information transmission, either as leakage (as in Matsui, 1989) or espionage (as in Solan and Yariv, 2004). This is very different from cheap talk, which Farrell and Rabin (1996) describe as "costless, non-binding, non-verifiable messages that may affect the listener's beliefs." Even though real talk is costless, the messages (or signals) that pass between the players are true, and thus binding. Adding real talk to a game may expand the set of equilibria in games when cheap talk fails to. The best example is the PD: cheap talk does not add any equilibrium to the game, whereas with real talk players can achieve full cooperation. Real talk is also different from Aumann's (1974) correlated equilibrium, as cooperation cannot emerge even in a correlated equilibrium of the PD.

2.1 Computer Programs

Howard (1988) analyzes a game in which two computer programs play the PD against each other. He shows that it is possible to write a program that receives the code of the program running on the other computer as input, and tells if it is identical to itself or not. This program can be slightly modified to also choose an action as output. In such a way, Howard constructs a program that plays C when receiving itself as an input, and D otherwise. Clearly, if both computers run this program, it will lead to an equilibrium in which both computers cooperate. Moreover, it is possible to write two different programs, P and Q, such that P recognizes Q, and vice versa. By doing so other equilibria may be sustained.

Tennenholtz (2004) shows that in this setting any payoff profile that is both individually rational and feasible can be achieved in equilibrium. Fortnow (2009) extends Tennenholtz's program equilibrium to an environment in which the player's payoffs are discounted based on the computation time used. See Binmore (1987), Anderlini (1990), and Rubinstein (1998) for some paradoxes that emerge from thinking of players as computer programs or Turing machines.

2.2 Delegation Models

In Fershtman et al. (1991) players use agents to play on their behalf. If this delegation is done by an observable contract, cooperation can emerge. Kalai et al. (2009) offer a similar model to the program equilibrium, in which each player chooses a commitment device rather than a computer program. A commitment device is a function that takes the other player's commitment device as its input, and returns a strategy (a probability distribution over the player's actions). Peters and Szentes (2009) explore games where each player writes a contract that obligates a player to respond with a specified action depending on the opponent's contract. They prove a similar folk theorem for any number of players and further show that this result does not hold in an environment with incomplete information.

In all these models, the players don't actually play the game themselves— the game is played by a computer program, a commitment device, or an agent. Formally commitment devices or contracts play the role of interaction in our model; however we extend the framework to allow noisy signals. Players don't know the strategy of their opponent with certainty. This allows us to answer additional questions. How does the equilibrium payoff set depend on p, the probability of learning? Under what conditions can we get (C, C) as an equilibrium outcome of the PD? We also allow for correlation in learning each other's strategies.

2.3 Informal Models

An informal commitment model without any external mechanisms is Frank's (1998) commitment model. Emotions are the commitment devices. It is argued that feelings such as love, anger or revenge can sometimes make people act in way that are not in their best interests. Hence, a person's feelings commit him to act in a certain way. Since psychological research shows that emotions are both observable and hard to fake (see Frank (1988) and references within), an agent can use them as signals in a game. This enables each player to discern his opponents emotional predispositions through physical and behavioral clues, and play accordingly.

Gauthier (1986) proposes an environment in which there are two types of agents: straightforward maximizers (SM) and constrained maximizers (CM). SM simply maximize their utility; CM are more sophisticated. They take into account the utilities of the other players and base their actions on a joint strategy: "A CM is conditionally disposed to cooperate in ways that, followed by all, would yield nearly optimal and fair outcomes, and does cooperate in such ways when she may actually expect to benefit." Gauthier assumes that an agent's type is known to everybody else (or at least with some positive probability). Thus, in the PD, when a CM meets another CM, they will both cooperate. In any other interaction between two players, they will both defect.

These last two works resemble ours but are not posed in a formal game theoretic framework. Binmore (1994), for example, criticised Gauthier for lacking microeconomic foundations. This paper provides a formal game-theoretic model that captures the intuition above.

2.4 Non Simultaneous Models

A different line of research considers players who do not play simultaneously; the second player chooses a strategy conditional on the first player's choice.

One example of such a model is Howard's (1971) metagame model. A 2-metagame is a game in which player 1 chooses a "regular" strategy (an action), while player 2 chooses a function from player 1's actions to his own action space. For instance, in the PD player 1 can either play C or D, and player 2 can play CC, DD, CD, DC where the first letter describes the action he plays if player 1 plays C and the second is the action to be played if player 1 plays D. The strategy CD can be interpreted as “I will cooperate if, and only if, you will”. However, (C, CD) is not an equilibrium since given the fact that player 1 plays C, player 2 will deviate to DD.

Similarly, a 1-2-metagame is a game in which player 2’s strategies are functions from player 1’s actions to his own, and player 1’s strategies are functions from player 2’s strategies, as just defined, into actions. In the PD example, since player 2 has 4 strategies, player 1 now has 16. Interestingly, now $(DDCD, CD)$ is an equilibrium yielding cooperation by both players. In Howard’s words: ”Player 2 says, “I’ll cooperate if you will” (implying “not if you won’t", i.e., the policy CD), and 1 replies “in that case (meaning if CD is your policy) I’ll cooperate too (implying ”not otherwise,”, i.e., the policy $DDCD$).”

In Solan and Yariv’s (2004) model of espionage, player 2 chooses an action. If player 1 then purchases information about it, with some probability he receives a signal about the action 2 chose. Finally player 1 chooses an action. In their model strategies are not chosen simultaneously; also, only one player can obtain information about the other player’s strategy, while in this model both can. Finally, espionage is costly but in our model information is free.

Matsui (1989) models espionage in an infinitely repeated game. Each player chooses a strategy for the entire game. Then one player might be informed of the other player’s strategy and can then revise his strategy in this light. Then, the repeated game is played out. Matsui shows that any subgame perfect equilibrium pair of payoffs is Pareto-efficient as long as the probability of information leakage is small enough. The models are very different— one-shot versus repeated game, one-sided espionage versus two-sided, simultaneous choice of strategies versus the possibility of revising one’s strategy after obtaining information.

"Secret handshakes” in evolutionary game theory are related to our strategy "nice”. Robson (1990) considers any evolutionary game possessing several evolutionarily stable strategies (ESS) with differing payoffs. A mutant is introduced which will "destroy” any ESS which yields a lower payoff than another. This mutant possesses a costless signal and also conditions on the presence of this signal in each opponent. The mutant then can protect itself against a population playing an inefficient ESS by matching this against those who do not signal. At the same time, the mutants can achieve the more efficient ESS against the mutant population itself. The main difference in results is that in a one-shot prisoner’s dilemma a superior outcome (which is not induced by an ESS) may be temporarily but not permanently attained; in our framework (C, C) is an equilibrium outcome even in a one-shot PD. In the case of the repeated prisoner’s dilemma, the “evolution of co-operation” becomes ultimately inevitable.

3 The Real Talk Model

Let $G = \langle A_1, A_2, \Pi_1, \Pi_2 \rangle$ be a two-person game in normal form, where A_i is a finite set of actions for player i $(i = 1, 2)$, and $\Pi_i : A_1 \times A_2 \to R$ is the payoff function for player i. A mixed action Nash equilibrium in G is a pair of mixed actions, (α_1^*, α_2^*) such that neither player can increase his expected payoff by deviating to another (mixed) action. Formally:

Definition 1 *A mixed action Nash equilibrium in G is a pair of mixed actions* (α_1^*, α_2^*) *such that for* $i = 1, 2$*:* $\Pi_i\left(\alpha_i^*, \alpha_{-i}^*\right) \geq \Pi_i\left(\alpha_i, \alpha_{-i}^*\right)$ *for any* $\alpha_i \in \Delta(A_i)$*.*

Definition 2 *A strategy* $s_i \in S_i$ *for player i in the game with real talk* $\widehat{G}$ *that is induced by G is a function from* $S_{-i} \cup \{\emptyset\}$ *to* $\Delta(A_i)$*, where* S_{-i} *is the opponent's strategy set,* $\emptyset$ *represents learning nothing:*

$$S_i \subseteq \{f : S_{-i} \cup \{\emptyset\} \rightarrow \Delta(A_i)\}.$$

A game with real talk $\widehat{G}$ induced by the game G consists of three stages and is played as follows:

1. Both players choose a strategy simultaneously;
2. Each player observes his opponent's chosen strategy with probability p and with probability $1 - p$ he sees nothing;
3. Each player then uses his own strategy and the signal in stage 2 to choose an action in $\triangle A_i$.

Definition 3 *The game with real talk,* $\widehat{G}$*, that is induced by G, is a tuple* (G, S, p, ρ) *where:*

- *G is a two-person game in normal form.*
- $S = S_1 \times S_2$*, where* S_i *is the set of feasible strategies of player i.*
- $p \in [0, 1]$ *is the probability that each player observes the other player's strategy in stage 2.*
- $\rho \in [0, 1]$ *is the correlation coefficient*[1] *between the two events* $\{X_1 = 1\}$ *and* $\{X_2 = 1\}$*, where* $X_i = 1$ *if i observes j's strategy and* $X_i = 0$ *if i gets no signal.*

Proposition 1 *For at least one player* $S_i \neq \{f : S_j \cup \{\emptyset\} \rightarrow \Delta(A_i)\}$*.*

Proof Otherwise $S_1 = |\Delta(A_1)|^{|S_2|+1}$ and $S_2 = |\Delta(A_2)|^{|S_1|+1}$ which is impossible by Cantor's Theorem. ■

The strategies that the players choose in the first stage will determine what action they play in the last stage given the signal at stage 2. Strategies are fixed, and cannot be changed once chosen.

Remark 1 *It is possible to construct finite strategy spaces, which contain as few as just one strategy for each player. For example,* $S_i = \{s_i\}$ *where for each player,* s_i *is a strategy that always plays some pure action* $a_i \in A_1$*. It is also possible to construct infinite strategy spaces such that* S_i *includes all function from* $S_{-i} \cup \{\emptyset\}$ *to* $\Delta(A_i)$ *that can be described in finite sentences (using Godel encoding); see Peters and Szentes (2009).*

[1] Note that even though technically the correlation coefficient could be also negative, under our interpretation it makes less sense: if one player detects his opponent's strategy in a conversation, then it should increase the probability that the opponent detects his, rather than decreases it.

After a strategy profile is chosen, there are four possibilities for the information the players have: Both players receives signals, player 1 receives a signal and 2 does not, player 2 receives a signal and 1 does not, and none of the players receives a signal. The following table shows the probabilities for the four cases:

	signal	no signal
signal	$p^2+\rho p\,(1-p)$	$p(1-p)-\rho p\,(1-p)$
no signal	$p(1-p)-\rho p\,(1-p)$	$(1-p)^2+\rho p\,(1-p)$

As expected, correlation increases the probabilities along the main diagonal, and decreases those on the secondary diagonal by $\rho p\,(1-p)$. If a strategy profile (s_1, s_2) is chosen by the two players, each one plays one of two possible actions, according to the signal he receives. The action profiles for the four different possibilities are shown in the following table:

	signal	no signal
signal	$(s_1(s_2), s_2(s_1))$	$(s_1(s_2), s_2\,(\emptyset))$
no signal	$(s_1\,(\emptyset)\,, s_2(s_1))$	$(s_1\,(\emptyset)\,, s_2\,(\emptyset))$

Let $\widehat{\Pi}_i(s_1, s_2)$ be the expected payoff for player i if the strategies chosen by player 1 and player 2 are s_1 and s_2, respectively. Using the above two tables and the action payoff function, Π, we obtain:

$$\begin{aligned}\widehat{\Pi}_i(s_1,s_2) = {} & \left[p^2+\rho p\,(1-p)\right]\cdot\Pi_i\,(s_1(s_2),s_2(s_1)) + \left[p(1-p)-\rho p\,(1-p)\right]\cdot\Pi_i\,(s_1(s_2),s_2\,(\emptyset)) + \\ & + \left[p(1-p)-\rho p\,(1-p)\right]\cdot\Pi_i\,(s_1\,(\emptyset)\,,s_2(s_1)) + \left[(1-p)^2+\rho p\,(1-p)\right]\cdot\Pi_i\,(s_1\,(\emptyset)\,,s_2\,(\emptyset))\,.\end{aligned}$$

As mentioned before, the simplest possible strategies are constant mixed actions, i.e. ones that always play an action $\alpha_i \in \Delta(A_i)$ regardless of what player i learns about player j's strategy. If both players choose such strategies, then their payoff would be: $\widehat{\Pi}_i\,(s_1, s_2) = \Pi_i\,(\alpha_1, \alpha_2)$.

Definition 4 *A strategy space S is natural if it contains all constant mixed actions. That is, for $i = 1, 2$ and $\alpha_i \in \Delta(A_i)$ the strategy set S_i contains a strategy that always plays the (mixed) action α_i regardless of the opponent's strategy.*

Definition 5 *A Nash equilibrium in the game with real talk $\widehat{G}$ is a pair of strategies (s_1^*, s_2^*) such that $\widehat{\Pi}_1\,(s_1^*, s_2^*) \geq \widehat{\Pi}_1\,(s_1, s_2^*)$ for any $s_1 \in S_1$ and $\widehat{\Pi}_2\,(s_1^*, s_2^*) \geq \widehat{\Pi}_2\,(s_1^*, s_2)$ for any $s_2 \in S_2$.*

The following proposition follows immediately from these definitions.

Proposition 2 *If $\widehat{G}$ is the real talk game induced by G and S is "natural" then*

1. *Every strategy in G has a corresponding (constant) strategy[2] in $\widehat{G}$.*

2. *Given any mixed action Nash equilibrium (α_1^*, α_2^*) in the original game G, there is a corresponding Nash equilibrium (s_1^*, s_2^*) in the game with real talk $\widehat{G}$ such that $s_i^* \equiv \alpha_i^*$.*

[2]Note that this does not imply that all Nash equilibria in $\widehat{G}$ are in constant strategies.

3. *Furthermore if $p = 0$ then $\hat{G}$ is strategically equivalent to the original game G.*

Proof If $p = 0$ and a player chooses a strategy s, then the only possible input that the strategy would receive is $\emptyset$, and the action that will be played in $\widehat{G}$ is $s(\emptyset)$ with probability 1 (players play $s(\emptyset)$ regardless of the opponent's strategy). Therefore, by choosing a strategy all the players do is simply choose a probability distribution over their own action space A_i. Since S is natural, for every probability distribution in A_i player i has a constant strategy that always plays it (of course, there may be many other strategies that play the same mixed action when receiving no signal, but they are all equivalent in this case). Hence, strategically the players face exactly the same choices in $\widehat{G}$ as they do in G. Clearly, the feasible payoff profiles and Nash equilibria are the same in the two games. ■

If S is natural, then the players *can* play the original game G, whereas when $p = 0$ they have no other choice.

4 A Real Talk Folk Theorem

Let G be a two person game and let w_i be the minmax value for player i G, i.e.

$$w_i = \min_{\alpha_j \in \Delta A_j} \max_{a_i \in A_i} \Pi_i(a_i, \alpha_j).$$

Definition 6 *A payoff π_i for player i is individually rational if $\pi_i \geq w_i$.*

Let ψ_i be player i's minmax strategy. That is, when ψ_i is played, player j can achieve a payoff of at most w_j. Formally, $\psi_i \in \arg\min_{i \in A_i} \max_{a_j \in A_{-i}} \Pi_{-i}(a_i, a_j)$. For any $\alpha_i \in \triangle A_i$, let $\alpha_i(a_i)$ be the probability of the action a_i.

Definition 7 *A payoff profile (π_1, π_2) is feasible* [3] *if $\exists \alpha_i \in \triangle A_i; i = 1, 2$ such that*

$$(\pi_1, \pi_2) = \sum_{a_1 \in A_1} \sum_{a_2 \in A_2} \alpha_1(a_1)\alpha_2(a_2)\Pi(a_1, a_2).$$

Proposition 3 *[Folk Theorem] For any game G there exists a game with real talk, $\widehat{G}$, such that any individually rational and feasible payoff profile (π_1, π_2) of G is the payoff profile of some Nash equilibrium of $\widehat{G}$.*

Proof Let (π_1, π_2) be an individually rational and feasible payoff profile. Let α_1 and α_2 be *any* pair of probability distributions over A_1 and A_2 respectively such that

$$(\pi_1, \pi_2) = \sum_{a_1 \in A_1} \sum_{a_2 \in A_2} \alpha_1(a_1)\alpha_2(a_2)\Pi(a_1, a_2)$$

[3] Note that this definition is not standard because of independent mixing of actions, and it does not always coincide with the standard definition of a feasible payoff profile. The standard definition requires (π_1, π_2) to be a convex combination of all outcomes in G, that is $\sum_{\alpha \in A} \alpha(a_1, a_2)\Pi(a_1, a_2)$ where α is a probability distribution over the joint action space A.

Define $s^{\pi} := \left(s_1^{(\pi_1,\pi_2)}, s_2^{(\pi_1,\pi_2)}\right)$ as follows:

$$s_i^{(\pi_1,\pi_2)}\left(s_j\right) = \begin{cases} \alpha_i \text{ if } s_j = s_j^{(\pi_1,\pi_2)} \\ \psi_i \text{ otherwise} \end{cases}.$$

Let S_1 and S_2 be arbitrary mutually consistent strategy sets such that $s_1^{(\pi_1,\pi_2)} \in S_1$ and $s_2^{(\pi_1,\pi_2)} \in S_2$ for all individually rational and feasible payoffs (π_1, π_2). Let $\widehat{G} = (G, S, 1, 1)$. That is $\widehat{G}$ is the real talk game that is induced from G when $S_1 \times S_2$ is the strategy set, $p = 1$, and $\rho = 1$.

The strategy profile s^{π} is a Nash equilibrium in $\widehat{G}$ for any (π_1, π_2). To see this, assume that player 2 plays $s_2^{(\pi_1,\pi_2)}$. If player 1 plays $s_1^{(\pi_1,\pi_2)}$ then the players will play (α_1, α_2), yielding player 1 a payoff of π_1. If player 1 deviates to any other strategy, player 2 will play ψ_2 against him, giving player 1 a payoff of no more then w_1. However, since (π_1, π_2) is individually rational, $\pi_1 \geq w_1$ and therefore π_1 is at least as good as player 1's payoff if he chooses to deviate. The same argument holds for player 2. Since no player has an incentive to deviate, $\left(s_1^{(\pi_1,\pi_2)}, s_2^{(\pi_1,\pi_2)}\right)$ is a ; equilibrium of $\hat{G}$. ■

Corollary 4 *If (π_1, π_2) is an individually rational and feasible payoff profile such that $\pi_i > w_i$, then $\exists p < 1$ such that (π_1, π_2) is an equilibrium payoff of $\widehat{G} = (G, S, p, 1)$.*

Proof We now have to specify what to play when a player sees nothing. Suppose they play the same as if they saw the right strategy: $s_i^{(\pi_1,\pi_2)}\left(s_j\right) = \alpha_i$. Since π_i is strictly greater than w_i and $1 - p$ is small it will not change i's incentives. The payoff will be exactly (π_1, π_2) since on-path play involves playing (α_1, α_2). ■

For $p = 0$ or $p = 1$ the converse of the folk theorem clearly holds: Every equilibrium payoff profile of $\hat{G}$ is individually rational and feasible. When $0 < p < 1$ a partial converse holds.

Proposition 5 *[Converse of Folk Theorem] Let $\widehat{G}$ be a game with real talk comprising natural strategy spaces S_1 and S_2. Any payoff profile (π_1, π_2) of a Nash equilibrium in $\widehat{G}$ is (a) individually rational; (b) if either $p = 1$ or $\rho = 0$ it is also feasible.*

Proof Part (a): Suppose that (π_1, π_2) is the payoff profile of a Nash equilibrium in $\widehat{G}$ that is not individually rational. Then for some i, $\pi_i < w_i$. Player i can guarantee himself at least w_i by deviating to a constant strategy that plays his maxmin action.

Part (b): When $p = 1$ both players always detect their opponent's strategies and they play the action determined by the strategies with probability 1. Since players' actions are simply probability distributions over their own action spaces, they are independent. By definition, this induces a feasible payoff profile.

When $\rho = 0$ the signals the players receive are independent. Since the players' actions are simply a probability distribution over their own action spaces, they are independent. Hence, each player plays an independent lottery conditioned on an independent signal. These are compound independent lotteries, which in turn are also independent. By definition, this induces a feasible payoff profile. ■

This converse does not hold for any $\widehat{G}$ induced by G: If S_i is not *natural* it might not be rich enough for player i to be able to minmax player $j \neq i$.

Denote the set of equilibrium payoffs of a game with real talk with p and ρ by $V(p, \rho)$.

Proposition 6 *[Monotonicity] (a)* $p < p' \implies V(p, 0) \subset V(p', 0)$.
(b) If $\rho > 0$ *and players have access to a correlated randomisation device,*

$$p < p' \implies V(p, \rho) \subset V(p', \rho).$$

Proof Let $p < p'$ and let $\pi := (\pi_1, \pi_2) \in V(p, 0)$. We can find an equilibrium strategy profile of the form $s = (nice\_q_1, q_2, nice\_r_1, r_2)$ which gives the payoff vector π. Here $nice\_q_1, q_2$ is a strategy profile of player where he plays $q_1 \in \triangle(A_1)$ if he sees that strategy $nice\_r_1, r_2$, plays $q_2 \in \triangle(A_1)$ if he gets no signal, and minmaxes 2 if he sees anything else. Define a new strategy profile $s' = (nice\_q_1', q_2', nice\_r_1', r_2')$ by

$$\begin{array}{c} q_1' := \frac{p}{p'} q_1 + \left(1 - \frac{p}{p'}\right) q_2 \\ q_2' := q_2 \end{array}.$$

It can be checked that s' induces the same distribution over A as s; since $p < p'$ and s was an equilibrium so is s'. $\therefore \pi \in V(p', 0)$. ■

5 Cooperation in the Prisoner's Dilemma

The general form of a PD payoff matrix is:

	C	D
C	b, b	d, a
D	a, d	c, c

where $a > b > c > d$. However, without loss of generality, one can subtract d from all payoffs and divide by c in order to obtain a matrix of the form:

	C	D
C	b, b	$0, a$
D	$a, 0$	$1, 1$

where $a > b > 1$. We consider the last version as the general case of the PD.

Clearly, the only Nash equilibrium in this game is (D, D). If player 1 plays C with probability $x \in [0,1]$ and player 2 plays C with probability $y \in [0,1]$, the expected payoff for player 1 is

$$\Pi = x \cdot y \cdot b + (1-x) \cdot y \cdot a + (1-x) \cdot (1-y).$$

$$\implies \Pi = x \cdot y \cdot (b-a+1) - x + y \cdot (a-1) + 1.$$

Note that $\frac{\partial \Pi}{\partial x} = y \cdot (b-a+1) - 1 < 0$ and $\frac{\partial \Pi}{\partial y} = x \cdot (b-a+1) + (a-1) > 0$. Thus player 1 prefers x to be low, and would like player 2 to choose y as high as possible. Moreover, given the value of y, the incentive of player 1 to reduce x depends on the value of $b-a+1$: the higher this value is, the less player 1 has to lose by playing cooperatively. The same is true for player 2, and thus the value $c := b-a+1$ can be seen as the strength of the incentive to play cooperatively (or the inverse of the gain to defection).

5.1 Equilibria and Possible Payoffs

In this section we discuss the criteria for an equilibrium in the PD. The following two observations stem directly from the payoff matrix:

1. The mimax action for both players is D. If a player deviates to it, his opponent's payoff is at most 1.

2. Assuming that S is natural, each player can guarantee a payoff of 1 by choosing the constant strategy that always plays D. Let d be such a strategy.

For any strategy profile (s_1, s_2) we can define a new one $\left(s_1^d, s_2^d\right)$ such that $s_i^d(s_{-i}^d) = s_i(s_{-i})$, $s_i^d\,(\emptyset) = s_i\,(\emptyset)$ and otherwise $s_i^d(\cdot) = D$. The strategy s_1^d plays against s_2^d exactly as s_1 plays against s_2. However, against any other strategy it plays the minmax action D. Clearly, the payoffs from (s_1, s_2) and $\left(s_1^d, s_2^d\right)$ are exactly the same.

Proposition 7 *If the strategy set is natural and (s_1, s_2) is an equilibrium, then $\left(s_1^d, s_2^d\right)$ is also an equilibrium in any strategy set that contains it.*

Proof By (s_1, s_2) being an equilibrium, $\widehat{\Pi}_1(d, s_2) \le \widehat{\Pi}_1(s_1, s_2)$ and $\widehat{\Pi}_2(s_1, d) \le \widehat{\Pi}_2(s_1, s_2)$. By the definition of $\left(s_1^d, s_2^d\right)$, $\widehat{\Pi}_1\left(d, s_2^d\right) \le \widehat{\Pi}_1(d, s_2)$ and $\widehat{\Pi}_2(s_1^d, d) \le \widehat{\Pi}_2(s_1, d)$. Thus, $\widehat{\Pi}_1\left(d, s_2^d\right) \le \widehat{\Pi}_1(s_1, s_2)$ and $\widehat{\Pi}_2\left(s_1^d, d\right) \le \widehat{\Pi}_2(s_1, s_2)$. But $\widehat{\Pi}_1(s_1, s_2) = \widehat{\Pi}_1\left(s_1^d, s_2^d\right)$ and $\widehat{\Pi}_2(s_1, s_2) = \widehat{\Pi}_2\left(s_1^d, s_2^d\right)$, which means that when $\left(s_1^d, s_2^d\right)$ is played, neither player has an incentive to deviate to d. What remains to be shown is that deviating to d is the most profitable deviation. This completes the proof, since if the players do not have an incentive to deviate to the most profitable deviation, they do not have an incentive to deviate at all, which means (s_1^d, s_2^d) is in fact an equilibrium.

With out loss of generality, assume that player 2 plays the strategy s_2^d and that player 1 deviates to some strategy. If player 2 receives a signal, the deviation is detected, which results in player 2 playing D, regardless of the chosen deviation. If player 2 does not receive a signal, the deviation is not detected, and player 2's action is not at all affected by the deviation. Since in both cases, all deviations result in the same action played by player 2, playing the strictly dominant action D is optimal. ■

If we are interested only in what payoffs can be sustained in equilibrium, this proof allows us to restrict our attention only the strategies of the type $\left(s_1^d, s_2^d\right)$. Each strategy, s_1^d and s_2^d has to specify the action to be played against each other, and also what action to play when not receiving a signal. Since there are only two pure actions in this game, each strategy is a pair of probability distributions over (C, D). Each pair of strategies $\left(s_1^d, s_2^d\right)$ is equivalent to a point in $[0,1]^4$.

5.2 The Strategy Nice_q

We define the strategy *nice_q* in the following way: If the opponent's strategy is detected, *nice_q* plays C against the strategy *nice_q*, and D against any other strategy. In case it receives no signal, it plays C with probability q and D otherwise.

It should be noted that although we interpret this strategy as "nice", it is only nice if the opponent plays exactly the same strategy. If, for example, player 1 plays $nice\_\frac{1}{2}$ and player 2 plays $nice\_\frac{1}{3}$ the result would be that nobody will cooperate if they detect each other, even though they are both "nice". That *nice_q* reacts nicely only to one specific strategy is an advantage: If $(nice\_q, nice\_q)$ is an equilibrium for a pair of strategy sets S_1 and S_2, adding more strategies to either set leaves this profile as an equilibrium without re-defining the strategy *nice_q* to take into account the newly added strategies.

Going back to the symmetric case, assume that the two players choose the strategy *nice_q*. Consider the event "both players play C". This event is the union of the following three events:

1. Both players receive a signal about the opponent's strategy.

2. One player receives a signal and the other does not, but chooses to play C anyway.

3. Neither players receives a signal but both choose to play C nonetheless.

The corresponding probabilities for these events are:

1. $p^2 + \rho p\,(1-p)$

2. $2 \cdot q \cdot [p(1-p) - \rho p\,(1-p)]$ (Either player may be the one receiving the signal, hence the 2.)

3. $q^2 \cdot \left[(1-p)^2 + \rho p\,(1-p)\right]$

Since they are disjoint, the event that both players play C occurs with probability equal to the some of these three probabilities. That is,

$$[p + q(1-p)]^2 + \rho p(1-p)(1-q)^2.$$

Similarly, the probabilities for the other two possible action profiles (which can be calculated in a similar way) are:

$$p(1-p)(1-q)+(1-p)q(1-p)(1-q)-\rho p(1-p)(1-q)^2$$

for one player playing C and the other D, and

$$[(1-p)(1-q)]^2+\rho p(1-p)(1-q)^2$$

for both players playing D.

Also note that the marginal probability for each player to cooperate is $p+q(1-p)$ and to defect is $(1-p)(1-q)$. By multiplying the payoffs of the game by these probabilities we obtain the expected payoff for each of the two players:

$$\begin{aligned}\widehat{\Pi}_i(nice\_q, nice\_q) \quad = \quad & b\cdot\left[[p+q(1-p)]^2+\rho p(1-p)(1-q)^2\right] \\ & +0\cdot[p(1-p)(1-q)+(1-p)q(1-p)(1-q)-\rho p(1-p)(1-q)^2]+ \\ & +a\cdot[p(1-p)(1-q)+(1-p)q(1-p)(1-q)-\rho p(1-p)(1-q)^2]+ \\ & +1\cdot\left[[(1-p)(1-q)]^2+\rho p(1-p)(1-q)^2\right].\end{aligned}$$

By rearranging and replacing $b-a+1$ by c we obtain;

$$\begin{aligned}\widehat{\Pi}_i(nice\_q, nice\_q) \quad = \quad & c\cdot[p+q(1-p)]^2+(a-2)\,[p+q(1-p)]+1+ \\ & +c\cdot\rho p(1-p)(1-q)^2.\end{aligned}$$

Note that when holding the other parameters fixed, the higher the value of the incentive to cooperate, c, the higher the payoff for both players. If, for example, a is chosen to be 4 and b to be 3, then this expression is reduced to the simple expression:

$$\widehat{\Pi}_i(nice\_q, nice\_q)=2[p+q(1-p)]+1.$$

5.3 Conditions for $(nice\_q, nice\_q)$ to be a Nash Equilibrium when $\rho=0$

In this section we find values of q for which the strategy profile $(nice\_q, nice\_q)$ is a Nash equilibrium. This calculation will be useful in the following section. We analyze the conditions for $(nice\_q, nice\_q)$ to be a Nash Equilibrium from player 1's perspective. Since $(nice\_q, nice\_q)$ is a special case of (s_1^d, s_2^d), deviating to d is the most profitable deviation. S_1 contains d if it is natural. (Other strategies that obtain the same payoff as d may exist. For example, a strategy that plays D if it detects *nice_q*, C if it detects any other strategy, and D if it receives no signal.)

Since d is the most profitable deviation, by checking that players lose by deviating to it, we can obtain a sufficient condition for the optimality of playing *nice_q* against *nice_q*. Clearly this condition is also necessary. In the general case, player i's payoff when he deviates to playing d is

$$\widehat{\Pi}_i(d, nice\_q) = a(1-p)q + 1[p + (1-p)(1-q)].$$

Thus, $(nice\_q, nice\_q)$ is an equilibrium iff

$$c[p+q(1-p)]^2 + (a-2)\,[p+q(1-p)] + 1 + c\rho p(1-p)(1-q)^2 \geq a(1-p)q + [p+(1-p)(1-q)]$$

$$\Leftrightarrow c[p+q(1-p)]^2 + (a-2)\,p - q(1-p) + c\rho p(1-p)(1-q)^2 \geq 0.$$

Clearly, if for a certain set of parameters $(nice\_q, nice\_q)$ is an equilibrium, increasing the incentive to play cooperatively, c, does not reverse the inequality, and $(nice\_q, nice\_q)$ remains an equilibrium.

Proposition 8 *The profile $(nice\_q, nice\_q)$ is an equilibrium under the following conditions, which depend on the roots r_1 and r_2 of the previous inequality:*

1. $1 < r_1$ *and* $1 \leq r_2$*: $(nice\_q, nice\_q)$ is an equilibrium for any q.*
2. $1 < r_1$ *and* $0 \leq r_2 < 1$*: $(nice\_q, nice\_q)$ is an equilibrium only for q smaller or equal to r_2.*
3. $1 < r_1$ *and* $r_2 < 0$*: $(nice\_q, nice\_q)$ is not an equilibrium for any q.*
4. $0 < r_1 \leq 1$ *and* $0 \leq r_2 < 1$*: $(nice\_q, nice\_q)$ is an equilibrium only for q smaller or equal to r_2 or larger or equal to r_1.*
5. $0 \leq r_1 \leq 1$ *and* $r_2 < 0$*: $(nice\_q, nice\_q)$ is an equilibrium only for q larger or equal to r_1.*
6. $r_1 < 0$ *and* $r_2 < 0$*: $(nice\_q, nice\_q)$ is an equilibrium for any q.*

See the appendix for a proof.

5.4 Achieving Maximal Cooperation when $\rho = 0$

This section discusses the probability of the event that both players cooperate, i.e. the event that both players play the action C, in a real talk Nash equilibrium when $\rho = 0$ (that is, when there is no correlation between the signals). We refer to this probability as the probability for cooperation.

Given a PD game G (i.e. the values of a and b), p, and ρ, denote the maximal probability of cooperation in a symmetric real talk Nash equilibrium by $P_{\max}$. That is, in any real talk game $\widehat{G}$ that is induced by G there is no strategy profile set S and a strategy profile $(s, s) \in S$ that yields a probability for cooperation that is higher than $P_{\max}$. Let P^* denote the maximal probability that player 1 or 2 plays C in a symmetric real talk Nash equilibrium of the PD. Since$\rho = 0$, we have $P_{\max} = P^{*2}$.

In what follows we find the value of $P_{\max}$ as a function of the parameters of the game, a, b and p assuming that $\rho = 0$. Furthermore, given the parameters of the game we show that under very minor assumptions we can restrict attention to strategies of the form *nice_q*. Keeping all the parameters of the games G and $\widehat{G}$ fixed, any level of cooperation that can be achieved with some strategy s can also be achieved with strategies of the form *nice_q*. Naturally, since the original strategy profile set S might not contain $(nice\_q, nice\_q)$, we prove the following proposition for any S' containing $(nice\_q, nice\_q)$.

Proposition 9 *[Strategies] Let G be a PD and let $\widehat{G} = (G, S, p, 0)$ be a real talk game induced by G. Assume that S contains the strategy d for each player and a strategy s such that (s, s) is a Nash equilibrium. If P is the probability for cooperation when (s, s) is played, then there exist $q \in [0, 1]$ such that for any strategy set $S^{'}$ containing nice_q for both players, $(nice\_q, nice\_q)$ is a Nash equilibrium in $\widehat{G}^{'} = (G, S^{'}, p, 0)$ and the probability for cooperation is at least P.*

Proof Strategy $s \in S$ specifies what to play when it receives the signal s, and also what action to play if it receives no signal. Denote the probabilities of playing C given s and $\emptyset$ by q_1 and q_2 respectively ($q_1 = s(s)\,(C)$ and $q_2 = s\,(\emptyset)\,(C)$). The marginal probability that any given player plays C is thus $P_s := pq_1 + (1 - p)q_2$. Since the signals that the players receive are independent, the probability for cooperation is $P = P_s^2$. The payoff for each player is:

$$\pi_s = P_s^2(b - a + 1) + P_s(a - 2) + 1.$$

Let $q_3 = s(d)\,(C)$ be the probability that s plays C when it sees the signal d. The probability that s plays C against d is $P_d := pq_3 + (1 - p)q_2$. The payoff for a player who plays d against s is

$$\pi_d = P_d \cdot a + (1 - P_d) = P_d(a - 1) + 1.$$

If (s, s) is a Nash equilibrium, then π_s is greater then any other payoff that a player can receive by deviating, including π_d. Thus, $\pi_s \geq \pi_d$ and moreover, since $\pi_d \geq 1$ also $\pi_s \geq 1$. We consider two cases:

1. $p \geq P_s$.

   Consider the strategy *nice_0*. This strategy plays C against itself and D against any other strategy, including when receiving no signal. Let $S^{'}$ be a strategy space containing $(nice\_0, nice\_0)$. We need to show that a) $(nice\_0, nice\_0)$ yields a probability of cooperation of at least P and b) that it is a Nash equilibrium.

   a) If both players play *nice_0*, the probability for cooperation is p^2 and by assumption $p^2 \geq P_s^2 = P$.

   b) The payoffs for $(nice\_0, nice\_0)$ are:

   $$\pi_{nice\_0} = p^2 \cdot (b - a + 1) + p \cdot (a - 2) + 1.$$

   However, if a player deviates to any other strategy he receives a payoff of exactly 1. Hence,

$(nice\_0, nice\_0)$ can be a Nash equilibrium iff

$$p^2 \cdot (b - a + 1) + p \cdot (a - 2) + 1 \geq 1.$$

Consider the function $f = x^2(b - a + 1) + x(a - 2)$. We will show that it is positive for $x \in [0, 1]$. There are three cases to analyze:

1. $(b - a + 1) < 0$. It is easy to verify that $f = 0$ for $x_1 = 0$ and $x_2 = \frac{2-a}{b+1-a}$. Note that by the construction of the game $b > 1$ and thus $2 - a < (b - a + 1)$. Therefore also $2 - a < 0$ and $x_2 > 1$. Thus the function is not negative for any $x \in [0, 1] \subset [x_1, x_2]$, including $x = p$.

2. $(b - a + 1) > 0$. Once again, $f = 0$ for $x_1 = 0$. The other root can be either negative or positive, and the function itself is negative only for x between the two roots. If $x_2 < 0$, then clearly f is positive for any $x > 0$, including $x = p$. If $x_2 > 0$ then f is positive only for $x > x_2$. Since we know that it is positive for $x = P_s$, and since $p \geq P_s$ then it is positive also for $x = p$.

3. $(b - a + 1) = 0$. Since $b > 1$ it implies that $a > 2$. Thus $a - 2 > 0$ and f is not negative for $x \in [0, \infty)$ including $x = p$.

Hence, in all cases for any $p \in [0, 1]$, $p^2(b - a + 1) + p(a - 2) \geq 0$ and the inequality above holds. Therefore $(nice\_0, nice\_0)$ is a Nash equilibrium with a probability for cooperation $p^2 \geq P$.

2. $p < P_s$.

   Choose $q \in [0, 1]$ such that $p + (1 - p)q = P_s$. Note that by construction $q < q_2$. Let S' be a strategy profile space containing $(nice\_q, nice\_q)$. The strategy $nice\_q$ plays C when recognizing itself (an event with probability p) and plays C with probability q when not receiving a signal at all (an event with probability $1 - p$). Hence the probability that each player cooperates is $p + (1 - p)q = P_s$, and therefore the payoff for each player is exactly π_s, i.e. $\Pi_1(s, s) = \Pi_1(nice\_q, nice\_q)$.

   The profile $(nice\_q, nice\_q)$ is a Nash equilibrium, with a probability for cooperation $P_s^2 = P$, because

$$\Pi_1(s_1', nice\_q) \leq \Pi_1(d, nice\_q) \leq \Pi_1(d, s) \leq \Pi_1(s, s) = \Pi_1(nice\_q, nice\_q).$$

   Since $nice\_q$ plays the same way against any strategy other than itself, the most profitable deviation against $nice\_q$ is d; this is the first inequality. The second inequality follows from (i) $q < q_2$, which implies that $nice\_q(\emptyset)(C) < s(\emptyset)(C)$ and (ii) $nice\_q(d)(D) = 1 \geq s(d)(D)$. The final inequality is the hypothesis that (s, s) is an equilibrium and $d \in S_1$.

■

We cannot choose q as above when we have $\rho > 0$. We also have to make sure that $\Pi_1(s, s) \leq \Pi_1(nice\_q, nice\_q)$.

Maximal Probability of Cooperation when $\rho = 0$

Since the maximal probability of cooperation can be achieved by using strategies of the *nice_q* type, we will now check what is the exact value of this probability given the different parameters of the game. The probability of (C, C) under $(nice\_q, nice\_q)$ is $[p + q(1-p)]^2$.

The probability of cooperation increases in both p and q:

$$\frac{\partial [p + q(1-p)]^2}{\partial q} = 2[p + q(1-p)]\,(1-p) \geq 0, \text{ and } \frac{\partial [p + q(1-p)]^2}{\partial p} = 2[p + q(1-p)]\,(1-q) \geq 0.$$

Since p is a parameter of the game, we are interested in finding the maximal q such that $(nice\_q, nice\_q)$ is a Nash equilibrium, i.e., $P_{\max}$ is achieved by maximizing q.

EXAMPLE: In the example where $a = 4$ and $b = 3$, for any $p \leq \frac{1}{3}$, we can maximize the probability for cooperation by increasing q as much as possible, which means choosing $q = \frac{2p}{1-p}$. Substituting q in the probability for cooperation equation yields $\left[p + \frac{2p}{1-p}(1-p)\right]^2$, or $P_{\max} = (3p)^2$.

For $p > 1/3$, the maximization is achieved by choosing $q = 1$ which induces cooperation with probability 1.

In sum, $P_{\max} = \min\left((3p)^2, 1\right)$. ■

In the general case, the analysis follows the same division to cases as in the previous section:

1. $c = 0$:

Similarly to the example before, for $p < \frac{1}{a-1}$ maximal cooperation occurs when $q = \frac{p(a-2)}{1-p}$. Substituting q yields $\left[p + \frac{p(a-2)}{1-p}(1-p)\right]^2$, or $P_{\max} = (p\,(a-1))^2$.

For $p \geq \frac{1}{a-1}$ maximal cooperation occurs when $q = 1$.

$P_{\max} = \min\left((p\,(a-1))^2, 1\right)$.

2. $c < 0$:

1. $r_1 < 0$ and $0 < r_2 < 1$:
   $(nice\_q, nice\_q)$ is an equilibrium only for q smaller or equal to r_2. Therefore, maximal cooperation can be achieved at $q = r_2$.
   Substituting q yields $[p + r_2(1-p)]^2$, or $\left(p + \frac{1-2cp-\sqrt{4cp-4acp+1}}{2c-2cp}(1-p)\right)^2$.
   $P_{\max} = \left(\frac{1-\sqrt{4cp-4acp+1}}{2c}\right)^2$.
2. $r_1 < 0$ and $1 \leq r_2$:

$(nice\_q, nice\_q)$ is an equilibrium for every q and maximal cooperation can be achieved by choosing $q = 1$.

$P_{\max} = 1$.

In sum,

$$P_{\max} = \min\left(\left(\frac{1-\sqrt{4cp-4acp+1}}{2c}\right)^2, 1\right).$$

3. $c > 0$**:**

If $p \geq \frac{1}{4c(a-1)}$ $(nice\_q, nice\_q)$ is an equilibrium for all $0 \leq q \leq 1$, thus maximal cooperation can be achieved by choosing $q = 1$.

$P_{\max} = 1$.

If $p < \frac{1}{4c(a-1)}$ maximal cooperation depends on r_1 and r_2, as defined earlier.

1. $1 < r_1$ and $1 \leq r_2$:

   $(nice\_q, nice\_q)$ is an equilibrium for every q and maximal cooperation can be achieved by choosing $q = 1$.

   $P_{\max} = 1$.

2. $1 < r_1$ and $0 \leq r_2 < 1$:

   $(nice\_q, nice\_q)$ is an equilibrium only for q smaller or equal to r_2. Therefore, maximal cooperation can be achieved at $q = r_2$. Substituting q yields $[p + r_2(1-p)]^2$, or

   $\left(p + \frac{1-2cp-\sqrt{4cp-4acp+1}}{2c-2cp}(1-p)\right)^2$.

   $P_{\max} = \left(\frac{1-\sqrt{4cp-4acp+1}}{2c}\right)^2$.

3. $1 < r_1$ and $r_2 < 0$:

   $(nice\_q, nice\_q)$ is not an equilibrium for any q, hence there is no cooperation.

   $P_{\max} = 0$.

4. $0 < r_1 \leq 1$ and $0 \leq r_2 < 1$:

   $(nice\_q, nice\_q)$ is an equilibrium only for q smaller or equal to r_2 or larger or equal to r_1. Maximal cooperation is reached when $q = 1$.

   $P_{\max} = 1$.

5. $0 \leq r_1 \leq 1$ and $r_2 < 0$:

   $(nice\_q, nice\_q)$ is an equilibrium only for q larger or equal to r_1. Once again, maximal cooperation is reached when $q = 1$.

   $P_{\max} = 1$.

6. $r_1 < 0$ and $r_2 < 0$:

   $(nice\_q, nice\_q)$ is an equilibrium for every q and maximal cooperation can be achieved by choosing $q = 1$.

   $P_{\max} = 1$.

Combining cases 1 through 6 we can see that cooperation can be achieved with probability 1 if $r_1 \leq 1$ or $r_2 \geq 1$. Since the condition for $r_1 \leq 1$ is $\left(c \geq 0.5 \textit{ and } p \geq \frac{1-c}{a-1}\right)$, and the condition for $r_2 \geq 1$ is $\left(c \leq 0.5 \textit{ and } p \geq \frac{1-c}{a-1}\right)$, we simply get the condition $p \geq \frac{1-c}{a-1}$.

Corollary 10 $P_{\max} = 1$ *iff* $p \geq \frac{1-c}{a-1}$.

Otherwise, that is when $r_1 > 1$ and $r_2 < 1$, the probability for cooperation is reduced:
The case $r_2 \geq 0$:

$$P_{\max} = \left(\frac{1 - \sqrt{4cp - 4acp + 1}}{2c}\right)^2 \text{ iff } \frac{2-a}{c} \leq p \leq \frac{1}{2c}.$$

The case $r_2 < 0$:

$$P_{\max} = 0 \text{ iff } \left(p > \frac{1}{2c} \text{ or } p < \frac{2-a}{c}\right).$$

A note is due regarding the last two cases, where $p < \frac{1-c}{a-1}$. On first glance it looks as if $P_{\max}$ is not monotonic in p, because for small and large ps $P_{\max}$ is zero, and in between it is positive. This is not the case, however. For given parameters a, c the following options are possible.

1. $a < 1.5$, which implies $\frac{1}{2c} < \frac{2-a}{c}$. In this case, for every p (under the assumption $p < \frac{1-c}{a-1}$) we are in the last case, where $P_{\max} = 0$.

2. $a \geq 1.5$, which implies $\frac{1}{2c} \geq \frac{2-a}{c}$. Note that $a \geq 1.5$ also implies that $\frac{1-c}{a-1} < \frac{1}{2c}$. Thus, if $p < \frac{1-c}{a-1}$, then $p < \frac{1}{2c}$. This leaves only two options: if $\frac{2-a}{c} < p$ we get some positive probability for cooperation, and for $p \leq \frac{2-a}{c}$ there is none.

$$P_{\max} = 1 \text{ iff } \left(p \geq \frac{1}{4c(a-1)} \text{ or } p \geq \frac{1-c}{a-1}\right),$$

$$P_{\max} = \left(\frac{1 - \sqrt{4cp - 4acp + 1}}{2c}\right)^2 \text{ iff} \left(\ p < \frac{1}{4c(a-1)} \text{ and } p < \frac{1-c}{a-1} \text{ and } \frac{2-a}{c} \leq p \leq \frac{1}{2c}\right) \text{ and}$$

$$P_{\max} = 0 \text{ iff } p < \frac{1}{4c(a-1)} \text{ and } \left(p < \frac{1-c}{a-1} \text{ and } \left(p > \frac{1}{2c} \text{ or } p < \frac{2-a}{c}\right)\right).$$

As can be seen, $P_{\max}$ is not necessarily a continuous function of p, but it is (weakly) monotonic increasing.

A Sufficient Class of Strategies When $\rho > 0$

When $\rho > 0$ in the PD we can prove that strategies of the form $nice\_(q_1, q_2)$ are sufficient to obtain the maximal level of cooperation.

Proposition 11 *[$nice\_(q_1, q_2)$ strategies] Let G be a PD and let $\widehat{G} = (G, S, p, \rho)$ be a real talk game induced by G. Assume that S contains the strategy d for each player and a strategy s such that (s, s) is a Nash* *equilibrium. If P is the probability for cooperation when (s, s) is played, then there exist $q \in [0, 1]$ such that for any strategy set $S^{'}$ containing $nice\_(q_1^*, q_2^*)$ for both players, $(nice\_(q_1^*, q_2^*), nice\_(q_1^*, q_2^*))$ is a Nash equilibrium in $\widehat{G}^{'} = (G, S^{'}, p, 0)$ and the probability for cooperation is at least P.*

Proof See appendix B. ■

In principle, the same method we used for the case where $\rho = 0$ can be used to calculate the maximal probability of cooperation in this case as well. However, the critical inequality whose roots determined the maximal q for which a $(nice\_q, nice\_q)$ equilibrium can be sustained depends now on two parameters q_1 and q_2 rather than one as before, and so is much harder to compute. We therefore prefer to stop here.

6 Conclusion

This paper models a novel aspect of communication in games. The role of communication is to reveal a player's chosen strategy. It can help explain people's behaviour, both in a laboratory and in real-life situations. For example we show why a significant level of cooperation can emerge in a one-shot prisoner's dilemma. We characterize the maximal probability for cooperation in equilibrium as a function of the parameters when the players' signals are independent. We prove that it is sufficient to use strategies having a particular form— "nice" strategies.

Two assumptions were made about the players' signals. The first is that the players receive either a correct signal or nothing at all. Wrong signals were not allowed. The second assumption, made in order to simplify computation, is that both players have the same probability of receiving a signal. However, having a different probability for each player is not implausible: Some people are better at detecting their opponent's character, not to mention that some people are better at hiding their own. We analyzed real talk only in two-player games; generalizing this to n-player games is possible. The players' strategies would be functions from all the other player's strategies into actions; probabilities for receiving a signal and correlations would have to be redefined as well.

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Appendix A: When is (nice_q,nice_q) an equilibrium?

Since a, c, p and ρ are the parameters of the game, it is convenient to analyze this inequality as a polynomial in q:

$$\left((p-1)^2 + \rho p(1-p)\right) cq^2 + (1-p)\left(2cp\left(1-\rho\right)-1\right)q + p\left(c\left(p+\rho(1-p)\right) + (a-2)\right) \geq 0.$$

In order to solve this inequality, we consider the following three cases:
1. $c = 0$**:**
The following linear inequality is obtained:

$$p\left(a-2\right) - \left(1-p\right) qtext.$$

Hence, the condition for $(nice\_q, nice\_q)$ to be an equilibrium is $q \leq \frac{p(a-2)}{1-p}$. Since $q \leq 1$, this condition is satisfied for every q if $p \geq \frac{1}{a-1}$.

In order to simplify the computations, in what follows we assume $\rho = 0$, which implies the following condition for equilibrium:

$$(p-1)^2 cq^2 + (1-p)\left(2cp-1\right)q + p\left(cp + (a-2)\right) \geq 0.$$

When $c \neq 0$ the quadratic equation $(p-1)^2 cq^2 + (1-p)\left(2cp-1\right)q + p\left(cp + (a-2)\right) = 0$ may have two, one or no roots, depending on the discriminant:

$$\left[(1-p)\left(2cp-1\right)\right]^2 - 4\left(p-1\right)^2 c \cdot p\left(cp + (a-2)\right).$$

2. $c < 0$**:**
Since by assumption $a > 1$ and p is not negative, we obtain that $p > \frac{1}{4c(a-1)}$. Therefore, the discriminant is positive and the equation

$$(p-1)^2 cq^2 + (1-p)\left(2cp-1\right)q + p\left(cp + (a-2)\right) = 0$$

has 2 real valued roots for any feasible parameters (a, c, p). Denote the smaller root by r_1 and the larger by r_2. Explicitly:

$$r_1(a,c,p) = \frac{1-2cp+\sqrt{4cp-4acp+1}}{2c-2cp},\ r_2(a,c,p) = \frac{1-2cp-\sqrt{4cp-4acp+1}}{2c-2cp}.$$

The condition for equilibrium holds for any $r_1 \leq q \leq r_2$. Since q denotes a probability, the relevant range for q is $[0,1]$. It is possible to show that:

- $r_1 < 0$.
- $r_2 > 0$.

- $r_2 \geq 1$ iff $p \geq \frac{1-c}{a-1}$.

We can now check when $(nice\_q, nice\_q)$ is an equilibrium, depending on the possible locations of r_1 and r_2:

1. $r_1 < 0$ and $0 < r_2 < 1$:
   $(nice\_q, nice\_q)$ is an equilibrium only for q smaller or equal to r_2. For example, if $a = 5$, $c = -2$ and $p = 0.4$, then $(nice\_q, nice\_q)$ is an equilibrium only for $q \leq 0.464$.

2. $r_1 < 0$ and $1 \leq r_2$:
   $(nice\_q, nice\_q)$ is an equilibrium for any q. For example, if $a = 9$, $c = -2$ and $p = 0.4$.

3. $c > 0$**:**
If $p \geq \frac{1}{4c(a-1)}$ the discriminant is non-positive and the equation:

$$(p-1)^2 cq^2 + (1-p)(2cp-1)q + p(cp + (a-2)) = 0$$

has one or no solutions. Therefore, the condition for equilibrium always holds, which implies that $(nice\_q, nice\_q)$ is an equilibrium for any q. However, if $p < \frac{1}{4c(a-1)}$, the equation has two real valued roots. As before, denote:

$$r_1(a,c,p) = \frac{1-2cp+\sqrt{4cp-4acp+1}}{2c-2cp},\ r_2(a,c,p) = \frac{1-2cp-\sqrt{4cp-4acp+1}}{2c-2cp}.$$

It should be noted that since the denominator is now positive, r_1 becomes the larger root.

In this case, the inequality hold for $q \leq r_2$ or $q \geq r_1$. Once again, since q denotes a probability, the relevant range for q is $[0,1]$. It is possible to show that:

- $r_1 \leq 0$ iff $p \geq \max\{\frac{1}{2c}, \frac{2-a}{c}\}$.
- $r_1 \geq 1$ iff, $c \leq 0.5$ or $p \leq \frac{1-c}{a-1}$.
- $r_2 \leq 0$ iff $p \geq \frac{1}{2c}$ or $p \leq \frac{2-a}{c}$.
- $r_2 \geq 1$ iff $c \leq 0.5$ and $p \geq \frac{1-c}{a-1}$.

Appendix B: Proof of Proposition 11

Choose $q_1^*, q_2^* \in [0,1]$ such that $q_1^* = s(s)(C)$ and $q_2^* = s(\emptyset)(C)$. Let $S^{'}$ be a strategy profile space containing $(nice\_(q_1^*, q_2^*), nice\_(q_1^*, q_2^*))$. The strategy $nice\_(q_1^*, q_2^*)$ plays C w.p. q_1^* when recognizing itself and plays C w.p. q_2^* when not receiving a signal at all. Hence the probability that each player cooperates is the same as before, and therefore the payoff for each player is exactly the same, i.e.

$$\Pi_1(s,s) = \Pi_1(nice\_(q_1^*, q_2^*), nice\_(q_1^*, q_2^*)) \tag{1}$$

Note that

$$\Pi_1\left(s_1', nice\_\left(q_1^*, q_2^*\right)\right) \leq \Pi_1\left(d,, nice\_\left(q_1^*, q_2^*\right)\right) \leq \Pi_1\left(d, s\right) \leq \Pi_1\left(s, s\right). \tag{2}$$

Since $nice\_\left(q_1^*, q_2^*\right)$ plays the same way against any strategy other than itself, the most profitable deviation against it is d; this is the first inequality. The second inequality follows from
(i) $nice\_\left(q_1^*, q_2^*\right)(\emptyset)(C) = s(\emptyset)(C)$ and
(ii) $nice\_\left(q_1^*, q_2^*\right)(d)(D) = 1 \geq s(d)(D)$. The final inequality is the hypothesis that (s, s) is an equilibrium and $d \in S_1$. From the above inequalities it follows that the profile $(nice\_q, nice\_q)$ is a Nash equilibrium, with the same probability for cooperation. ■