Wage Floors and Optimal Job Design*

Jenny Kragl† and Anja Schöttner‡

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Abstract

We analyze the effects of lower bounds on wages, e.g., minimum wages or liability limits, on job design within firms. In our model, two tasks contribute to non-verifiable firm value and affect an imperfect performance measure. The tasks can be assigned to either one or two agents. In the absence of a wage floor, it is optimal to assign the tasks to different agents whenever the agents' reservation utility is not too large. Under such a job design, the principal can tailor incentives according to each task's marginal productivity. By contrast, with a relatively large wage floor, the principal gradually lowers effort incentives to avoid rent payments to the agents, even before the wage floor exceeds the agents' reservation utility. If the wage floor is sufficiently large, the principal hires only one agent even though this leads to a distortion of effort across tasks or the non-execution of one task altogether.

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†Department of Governance & Economics, EBS Universität für Wirtschaft und Recht, Gustav-Stresemann-Ring 3, D-65189 Wiesbaden, Germany, email: jenny.kragl@ebs.edu.

‡Department of Economics, BWL II, University of Bonn, Adenauerallee 24-42, D-53113 Bonn, Germany, email: anja.schoettner@uni-bonn.de.

"When the [minimum] wage went up on Sept. 1 he halved her hours. Meantime, full-timers have taken up that slack. Nowadays, one person sets up the registers, then starts the biscuits, then does assorted odd tasks before business picks up at lunch time. Mr. Isah freely concedes that people are working twice as hard for their modest raise."

Wysocki Jr. (1997), The Wall Street Journal[1]

1 Introduction

In the present paper, we theoretically analyze how optimal job design in firms is affected by the existence of a wage floor given that performance signals are imperfect. As one major result, we find that lower bounds on wages may induce firms to redistribute tasks within the organization amongst fewer employees at the expense of efficient effort incentives. Accordingly, firms offer fewer jobs which are, however, characterized by broader task assignments. Depending on the particular characteristics of the tasks, firms may even find it optimal to entirely exclude less important tasks from the production process.

As Holmström and Milgrom (1991) show in their seminal paper, job design is an important instrument for the control of incentives in multitask principal-agent problems. In the real world, 'ideal' performance measures that perfectly reflect an agent's contribution to firm value are rare. Frequently if not typically, an employee's overall contribution to firm value is too complex and subtle to be objectively measured. In particular, an agent's job generally involves several potentially conflicting dimensions such as the production of a desired quantity, the provision of good quality, the care for the production equipment, or the cooperation in a team.[2] Thus, available measures of employee performance are usually imperfect; and rewards that depend on such measures cannot perfectly align an employee's incentives with the firm's objectives.[3] If, however, the tasks of a job can be separated into several jobs carried out by different agents, the problem

[1] Wysocki Jr. (1997), p. A1, on the effects of the 1996 minimum wage increase in the fast-food sector for the case of a Popeyes Chicken & Biscuits restaurant in West Philadelphia. Mr. Isah is the store manager.

[2] Moreover, an employee's behavior normally only partially determines outcomes because firm value and performance measures are also affected by measurement errors or other random factors. When agents are risk averse, this leads to the well-known trade-off between risk sharing and incentives. See, e.g., Prendergast (1999) and the numerous references therein.

[3] See Kerr (1975) for an extensive number of examples.

of distortive incentives can be mitigated or even eliminated by an appropriate choice of the wage scheme (see, e.g., Holmström and Milgrom (1991) or Ratto and Schnedler (2008)).

In practice, however, the firm's discretion in wage setting is often restricted by law, personal wealth constraints, contractual agreements on liability limits, collective bargaining agreements, or (other) labor market institutions, which has not found attention in the job design literature so far. For example, in many countries workers are guaranteed a legal minimum wage, i.e., a wage floor that prescribes a (positive) lower bound on the wage paid to individual workers.[4] Moreover, in many occupational sectors, obligatory standard wages exist. Beyond that, for upper-level managers limits on their personal liability may be established.[5] We show that such wage floors may render the separation of tasks suboptimal from the firm's point of view.

These findings bear relevance for a multitude of occupational sectors. For example, in the case of a legal minimum wage, the jobs that are most likely to be directly affected are the ones that pay a wage close to the minimum, i.e., low-wage sectors. Amongst them, jobs in food preparation and serving-related occupations have been subject to much empirical investigation.[6] As suggested by the introductory quotation on Popeyes Chicken & Biscuits, anecdotal evidence from several low-wage employers in the fast-food sector indicates that employers are not only more carefully scrutinizing who they hire in response to minimum wage increases but also cut hours, increase workloads, and assign more tasks to a single worker (see Wysocki Jr. (1997), Duff (1996)). These empirical observations are perfectly consistent with our model's predictions.[7] Underlining that, along with "the harsh business environment", such developments may have a clear negative impact on overall firm value, Wysocki Jr. (1997) notes that "crew hours were cut back, and cleanliness suffered" while the Popeyes store manager is worried that the "[q]uality of work will fall". This evidence is in accordance with our result that wage floors may lead to the negligence of 'less important tasks' such as cleaning compared to cooking. Moreover, in line with our findings, several empirical studies

[4] See, e.g., Boeri and van Ours (2008), p. 29.

[5] Formally, this can be seen as a negative wage floor.

[6] See, e.g., the studies by Neumark and Wascher (2000) or Card and Krueger (2000).

[7] We consider a moral-hazard model with incentive pay for worker motivation. Note that many fast-food companies implement incentive-based performance programs, amongst them McDonald's and Popeyes Chicken & Biscuits which are subject of the articles by Wysocki Jr. (1997) and Duff (1996), see, e.g., QSRweb.com (2010a), QSRweb.com (2010b), or McDonald's (2010).

show that a minimum wage can have significant impact on job-finding and job-loss probabilities. Positive effects on job-loss probabilities of affected workers in the US have been reported by, e.g., Currie and Fallick (1996) and Zavodny (2000) and by Abowd, Kramarz, and Margolis (1999) for both French and US workers. Investigating the 1987 minimum wage increase for Portuguese teenagers, Portugal and Cardoso (2001) report that minimum wages reduce the probability that firms hire workers from the affected group.[8]

To illustrate the main concern of our paper, consider the example of a fast-food chain. Typically, in fast-food restaurants various tasks have to be performed, such as preparing the food, selling the meals, serving the customers, maintaining the equipment, and keeping the restaurant clean. For the sake of exemplification, we focus on two tasks; 'selling' and 'cleaning'. Effort in both tasks contributes to the fast-food chain's firm value. In particular, cleanliness of the restaurant affects customer satisfaction and, thereby, future sales of the chain. Moreover, both tasks influence the individual store's divisional profit, which, we suppose, is the only available performance measure. However, while cleanliness has a positive impact on the divisional profit, the effect on firm value is much more significant because the fast-food chain also cares about its reputation in the long run (e.g., there are externalities of the cleanliness of one restaurant on all other chain stores). If only one employee is responsible for both tasks, incentives based on divisional profits are likely to distort the employees's effort towards the selling task. Holmström and Milgrom (1991) argue that, to counteract a misallocation of effort, the tasks 'selling' and 'cleaning' should be assigned to different agents. These agents can then be provided with individual incentives based on divisional profits, resulting in efficient effort in both tasks. In the absence of a wage floor, this job design is in general also optimal in our setting.[9] However, if a lower bound on wages such as a minimum wage is introduced, the separation of tasks becomes relatively more expensive: Not only needs the firm to pay the minimum wage twice. To provide efficient incentives, the firm may also be forced to leave a share of the surplus to its

[8] While our study analyzes the effects of wage floors at the firm level, a large body of empirical research investigates the overall employment effects of minimum wages on an aggregate level. Yet there is a lack of consensus about its overall impact. Neumark and Wascher (2007) review the minimum wage research for the US and some other countries; they find that the majority of the studies indicates a negative employment effect. However, in a recent comprehensive long-term study on the county level, Dube, Lester, and Reich (2010) find no adverse employment effects for the US labor market.

[9] More precisely, without a wage floor the separation of tasks is optimal if the employees' reservation utility is not too large.

employees, who thus earn rents. This makes incentivizing two employees relatively more costly. This trade-off between the feasibility of efficient effort incentives with two employees but possibly smaller wage costs with just one employee is central to the results in our paper. For the given example, our analysis predicts that the tasks 'selling' and 'cleaning' are less likely split into two jobs when a wage floor exist.

Formally, in line with the above example, we consider a situation in which two tasks contribute to firm value. The tasks can either be assigned to just one job carried out by one agent ('broad task assignment' or 'multitasking'), or tasks can be split into two jobs such that each task is delegated to a different agent ('specialization').[10] The principal cannot observe the effort exerted by an agent in any task. In the basic model, the tasks are technologically independent in the agent's cost function. Then, in the first-best solution, the principal employs one agent and assigns both tasks to him. This solution cannot be achieved as the generated firm value is non-verifiable. There is, however, an imperfect performance measure whose realization randomly depends on both tasks and which the principal can thus use to provide the agents with effort incentives. When designing the incentive contract, the principal's discretion is restricted by the wage floor and the agents' reservation utility. Both impose lower bounds on the agents' payoffs, but in quite different ways. While the former dictates an agent's minimum ex-post wage payment, the latter determines the minimum ex-ante expected income net of effort costs that ensures an agent's participation. As a result of these different types of wage restrictions, we will show that the effect of wage floors on the strength of incentives and, consequently, optimal job design is in sharp contrast to that of the workers' reservation utility such as their alternative wage, unemployment benefits, or social benefits.[11]

In a first step, we separately analyze the two possible job regimes. Broad task assignment never induces first-best effort in both tasks unless firm value and performance measure are perfectly aligned. The principal's profit increases in the

[10] We focus on a setting where it is not feasible or reasonable to split a task between two agents. For instance, in the above example, only one person can operate a particular cash register or clean a particular table.

[11] In particular, the introduction of a wage floor that forces the firm to adjust ex-post wage payments inevitably entails inefficient effort incentives. By contrast, an increase of the workers' reservation utility does not alter the efficient incentive scheme but only leads to a redistribution of the surplus from the firm to its employees, as long as the reservation utility does not get too large. Focussing particularly on minimum wages vs. unemployment benefits, we discuss this point in greater detail in Section 7.

alignment of the performance measure with firm value as a better aligned performance signal allows for a more effective provision of incentives.[12] By contrast, firm profit decreases in the size of the wage floor. In particular, with an increasing wage floor, the principal gradually lowers incentives to avoid paying a rent to the agent, even before the wage floor exceeds the agent's reservation utility. However, if the wage floor is sufficiently large, the principal cannot avoid leaving a share of the generated surplus to the agent. Moreover, if the marginal productivities of the tasks differ sufficiently or the performance measure is sufficiently distortive, the principal excludes the less important task from the job. This allows for providing efficient incentives in the remaining task because the agent can no longer misallocate effort across tasks.[13]

By contrast, under specialization, the principal can tailor incentives according to each task's true marginal productivity by offering separate incentive contracts to the agents. This leads to first-best effort in both tasks whenever the wage floor is sufficiently small so that the principal does not need to pay the agents a rent. Naturally, this is true in the absence of any wage floor.[14] With an increasing wage floor, the principal again lowers incentives to reduce rent payments to the agents. Moreover, once the wage floor exceeds the agents' reservation utility, the principal implements an incentive payment only for the agent who is responsible for the more important task. Incentivizing both agent is then too costly: Raising one agent's effort incentives also always increases the other agent's rent because both agents are rewarded according to the same performance measure.

In a second step, we compare the two job regimes. We find that specialization is optimal when it entails first-best effort and the agents' reservation utility is not too large. Then, the additional profit from efficient incentives in both tasks under specialization exceeds the additional costs of hiring a second agent. This is the more likely, the worse the alignment of the performance measure with firm value. The specialized job regime becomes, however, less attractive if there is a relatively large wage floor. Once the wage floor is so large that the principal would pay at

[12]This is a well-known result in the literature on incentive provision with multitasking and distortive performance measures, see, e.g., Baker (2002) or Gibbons (2005).

[13]Such a task exclusion requires the principal to be able to prevent an agent from engaging in a task that is not assigned to his job and not performed by another agent either. This could be achieved, for instance, by not granting the agent access to indispensable task-specific tools.

[14]Importantly, though a specialized job design may induce first-best effort in both tasks despite the problem of imperfect performance measurement, it does nevertheless not resemble the first-best solution. The reason is that two agents are employed, leading to additional costs in the amount of an agent's reservation utility.

least one of the agents a rent under specialization, she will hire only one agent. In particular, this is the case if (but not only if) the wage floor exceeds the agents' reservation utility. Employing only one agent is then superior even though it leads to a distortion of effort across tasks or the exclusion of one task altogether. Our model thus implies that the existence of lower bounds on wages leads to broader task assignments and thus employment of fewer workers within the firm at the expense of efficient effort incentives.

To complement our basic analysis, we show that our main results continue to hold in two extensions of our basic model. In the first one, we consider a situation where tasks may be complements or substitutes in an agent's cost function. In the second extension, we allow wage floors to vary across the different job designs. This accounts for situations in which the wage floor is due to an hourly minimum wage and, compared to broad task assignment, the agent's working hours can be reduced when he carries out only a single task.

The present paper brings together important aspects of the literature on multitasking and job design and that on wage floors.[15] For more than two decades, economists have been concerned with incentive distortions and inefficiencies that result from limited liability in principal-agent models.[16] However, to the best of our knowledge, we are the first to introduce liability limits (or, more generally speaking, wage floors) in a multitasking setting with imperfect performance measures. Given that an agent has to carry out more than one task and performance measures are imperfect, Holmström and Milgrom (1991) and Baker (1992) present the basic rationale for distortive effort incentives.[17] As discussed above, if tasks are separable, assigning each task to a different agent may mitigate or even overcome the problem of effort misallocation (see, e.g., Holmström and Milgrom (1991); Ratto and Schnedler (2008)). We complement this literature by highlighting that, in the presence of wage floors, the advantage of separating tasks is diminished,

[15] Our paper is also related to standard neoclassical labor market models (see, e.g., Boeri and van Ours (2008)). They predict negative effects of minimum wages on aggregate employment in competitive markets. By contrast, conclusions are ambiguous for non-competitive labor markets, depending on the size of the minimum wage. For a comprehensive discussion of imperfect competition in labor markets see Manning (2003) and Manning (2010). In contrast to aggregate models of the labor market, we offer an explanation for unemployment at the firm level based on incentive considerations and worker motivation when workers perform different tasks and performance measures are imperfect.

[16] Important contributions include Sappington (1983), Park (1995), Kim (1997), Demougin and Fluet (2001), and Lewis and Sappington (2000, 2001).

[17] Building upon these seminal papers, multitasking problems are also analyzed by, e.g., Feltham and Xie (1994), Datar, Kulp, and Lambert (2001), Baker (2002), and Schnedler (2008).

if not eliminated, as the principal might be forced to share the surplus with several agents. Moreover, also with broad task assignments, wage floors may have detrimental effects on the efficiency of the resulting outcome.

The literature on job design provides additional reasons as to why broad task assignments may be optimal. Itoh (1994, 2001) also considers a joint performance measure for different tasks, which are interdependent in the agent's cost function. However, in contrast to our model, agents are risk averse and wage floors are absent. Assigning all tasks to one agent is optimal when the degree of substitutability between tasks is sufficiently low because then the effect of paying only one risk premium dominates. Focussing on performance measurement, Zhang (2003) and Hughes, Zhang, and Xie (2005) demonstrate that complementarities between tasks may lead to task bundling, which is in line with the results of our model extension to interdependent tasks. Schöttner (2008) shows that broad task assignments may enhance relational employment contracts.

Finally, potential pros and cons of task bundling are also analyzed in the literature on optimal contracting under limited liability. In contrast to our analysis, that literature focusses on environments where individual (task-dependent) performance measures are available. For example, Laux (2001) provides a rationale as to why incentive problems lead to the assignment of multiple projects to a single manager. Similar to our findings, such a project assignment relaxes the limited-liability problem and reduces managerial rents and, thus, expected wage costs. Furthermore, Schmitz (2005) considers the organization of a project that consists of two stages, at each of which one action has to be undertaken, under limited liability. Incentive considerations due to moral hazard can explain the optimality of either separation of the tasks to different agents or integration, i.e., assigning both tasks to only one agent.

The remainder of the paper proceeds as follows. The next section introduces the model and the first-best benchmark. Section 3 derives the optimal contract when the principal hires only one agent to perform both tasks (multitasking). Section 4 analyzes the case of two agents (specialization). In Section 5, we compare the different job designs and show under which circumstances one regime dominates the other. Subsequently, we present two extensions of our model in Section 6. In particular, in Subsection 6.1 we extend the model to interdependent tasks while we analyze the case of job-design dependent wage floors in Subsection 6.2. Finally, Section 7 discusses important implications of our results and concludes.

2 The Model

We consider a production process that requires the completion of two tasks. Non-observable effort in task i ($i = 1, 2$) is denoted by $e_i \geq 0$. The effort level reflects the diligence exercised by the worker who carries out task i. A task cannot be split between different workers. The effort levels stochastically determine firm value Y, which is either high or low, $Y \in \{0, 1\}$. The probability for $Y = 1$ is given by

$$\Pr[Y = 1 \,|e_1, e_2] = \min\{f_1 e_1 + f_2 e_2, 1\}. \tag{1}$$

Here, $f_i > 0$ is task i's marginal productivity with respect to expected firm value. We assume that task 1 is weakly more important for the firm, i.e., $f_1 \geq f_2$. Firm value Y is non-verifiable and thus non-contractible. However, there is a verifiable performance measure $P \in \{0, 1\}$ with

$$\Pr[P = 1 \,|e_1, e_2] = \min\{g_1 e_1 + g_2 e_2, 1\}. \tag{2}$$

The parameter $g_i > 0$ reflects task i's marginal impact on the expected value of the performance measure. Given f_i, g_i, and e_i, the realizations of Y and P are independent. Since both f_i and g_i are positive, increasing effort in either task raises the expected realization of both firm value and performance measure. However, because in general $f_i \neq g_i$, a task's true productivity differs from its impact on the performance measure. Thus, the performance measure is usually imperfect.[18]

The firm owner (principal) cannot perform any of the tasks herself. For execution of the tasks, she can choose between two different job designs, *multitasking* and *specialization*. Under multitasking, she hires a single agent to complete both tasks. As a special case of this work arrangement, the principal can exclude one task from the agent's job. In this case, the agent is forbidden to exert effort in the excluded task and, consequently, this task is not performed at all.[19] Under spe-

[18] We could also assume that task 1 is indispensable for realizing a high firm value and/or a high performance measure, i.e., $\Pr[Y = 1 \,|e_1 = 0, e_2] = 0$ and/or $\Pr[P = 1 \,|e_1 = 0, e_2] = 0$ for all $e_2 \geq 0$, whereas (1) and (2) apply if $e_1 > 0$ and $e_2 \geq 0$. For example, task 1 is indispensable to obtain $P = 1$ if this task is a production task and $P = 1$ means that the good has been produced (while task 2 could be the maintenance of the asset required for production). Assuming that task 1 is indispensable would lead to exactly the same results as the above specification because our optimal contract will always induce strictly positive effort in task 1.

[19] We thus assume that the principal can enforce that a task is not carried out (compare Footnote 13 in the Introduction). If this is not possible, however, our analysis still applies when we simply neglect the case of task exclusion.

cialization, the principal employs two agents, and each agent carries out a different task.

Timing is as follows. First, the principal determines the job design. If she chooses multitasking, she offers one agent an employment contract. The contract specifies the task assignment (either both tasks, or only task 1, or only task 2), a fixed wage s, and a bonus b to be paid if the performance measure is favorable, i.e., if $P = 1$. Thus, the agent receives s if $P = 0$ and $s + b$ if $P = 1$. If the agent accepts the contract, he exerts effort. Then, P and Y are realized and the payments are made.

By contrast, under specialization, the principal proposes each of two agents a separate contract. For simplicity, each agent is identified with the task i he is supposed to perform. Thus, the contract for agent i specifies that he will carry out task i, receive a fixed wage s_i, and a bonus b_i if $P = 1$. Given that both agents accept the contract, they simultaneously exert effort in their tasks. Afterwards, P and Y are realized and the agents are paid.

Agents are homogeneous and risk neutral. An agent's cost of exerting effort is $c(e_1, e_2) = \frac{1}{2}(e_1^2 + e_2^2)$.[20] He accepts the principal's job offer if it guarantees him an expected wage payment net of effort costs of at least $u \geq 0$, i.e., u denotes an agent's reservation utility. Moreover, due to exogenous restrictions, the wage of the agent must meet or exceed the wage floor w in each state of the world. We allow w to take values from the interval $[-\infty, \infty)$. In case $w \geq 0$, we can interpret w as a minimum wage. By contrast, if $-\infty < w < 0$, the firm can extract payments from the agent, but the latter is protected by limited liability (or has limited wealth). Finally, the case $w = -\infty$ corresponds to a situation without any restrictions on wage payments. In our basic model, we assume that the wage floor w is identical for all job designs. This assumption is appropriate when w is a liability limit or when w is due to an hourly minimum wage and an agent's working hours are independent of his task assignment.[21] If, however, an agent's working hours can be reduced when he performs only one task, a wage floor dictated by an hourly minimum wage would decrease. In Section 6.2 we extend our basic model to capture such a situation.

[20] Thus, the tasks are independent in the agent's cost function $\left(\text{i.e., } \frac{\partial^2 c}{\partial e_1 \partial e_2} = 0\right)$ and equally costly. However, as we show in Section 6.1, our analysis can be extended to a more general quadratic cost function incorporating substitutabilities or complementarities between the tasks.

[21] The latter case can incorporate a situation where an agent's nominal working hours (i.e., the stipulated working time during which he must be present at the workplace) do not vary but the time he is indeed performing his tasks or his working pace may change.

For ease of exposition, we introduce vector notation and define $f^T = (f_1, f_2)$, $g^T = (g_1, g_2)$, and $e^T = (e_1, e_2)$.[22] The expression $|| \cdot ||$ denotes the length of a vector, i.e., $||f|| = \sqrt{f_1^2 + f_2^2}$. We assume that $||f||, ||g|| < 1$, which avoids corner solutions because it guarantees that the probabilities in (1) and (2) remain strictly below one at the optimal (first- and second-best) solution.[23] Furthermore, assuming that the principal realizes a profit of zero when she does not employ any agents, our analysis is relevant only if the principal's expected profit is positive under at least one job regime. The following assumption ensures that multitasking leads to a positive expected profit, which is the case if the agent's reservation utility u and the wage floor w do not exceed certain thresholds.[24]

Assumption 1 *Define* $D := \frac{(f^T g)^2}{g^T g}$*. We assume that* $w < \bar{w} := \frac{1}{4} \min \{D, f_1^2\}$ *and* $u < \bar{w} + \frac{1}{8} \min \{D, f_1^2\}$*.*

As a benchmark, we consider the first-best solution, which would be implemented if effort was contractible. In this case, because tasks are independent in the agent's cost function, the principal would employ only one agent. Thus, the first-best effort levels $e^{FB} = (e_1^{FB}, e_2^{FB})^T$ solve the optimization problem

$$\max_{e_1, e_2} \; f_1 e_1 + f_2 e_2 - \frac{1}{2}(e_1^2 + e_2^2) - u \quad = \quad \max_{e} \; f^T e - \frac{1}{2} e^T e - u, \tag{3}$$

yielding $e^{FB} = f$ and a first-best profit of $\pi^{FB} = \frac{||f||^2}{2} - u$.

3 Multitasking

We start the analysis of the model with the case of multitasking, where the principal hires only one agent. First, we assume that the principal assigns both tasks to the agent. Afterwards, we investigate whether the principal may be better off by excluding one task from the agent's job.

[22] All vectors are column vectors. Superscript T denotes transpose.

[23] Hence, from now on we will simply write $\Pr[Y = 1|e_1, e_2] = f_1 e_1 + f_2 e_2 = f^T e$ and $\Pr[P = 1|e_1, e_2] = g_1 e_1 + g_2 e_2 = g^T e$.

[24] Assumption 1 refers to the multitasking case since, as explained in the Introduction, we will characterize situations where the adoption of wage floors may induce the principal to switch from a specialized job regime to multitasking. This is only reasonable if multitasking generates a non-negative profit. How the specific upper bounds on u and w arise from the model becomes clear in the proofs of Proposition 1 and 2, which determine the optimal profit under multitasking.

Given that the agent will perform both tasks, the principal's problem of choosing the optimal fixed payment s and the optimal bonus b reads as follows:

$$\max_{e,s,b} \quad f^T e - (s + g^T e \cdot b) \tag{I$_M$}$$

$$\text{s.t.} \quad e = \arg\max_{\widehat{e}} s + g^T \widehat{e} \cdot b - \frac{1}{2} \hat{e}^T \hat{e} \tag{IC$_M$}$$

$$s + g^T e \cdot b - \frac{1}{2} e^T e \geq u \tag{PC$_M$}$$

$$s \geq w \tag{WC$_M$}$$

$$s + b \geq w \tag{WC$'_M$}$$

The principal maximizes the expectation of firm value minus wage payments, subject to the agent's incentive-compatibility constraint (IC$_M$), his participation constraint (PC$_M$), and the wage-floor constraints (WC$_M$) and (WC$'_M$).

From the incentive-compatibility constraint it follows that, for a given bonus b, the implemented effort levels satisfy

$$e = b \cdot g \quad \Leftrightarrow \quad e_i = b \cdot g_i, \quad i = 1, 2. \tag{4}$$

Consequently, the principal is extremely restricted in the set of effort levels she is able to induce: feasible are only those effort levels e that are multiples of the vector g. Intuitively, because the agent cares solely about the realization of the performance measure P, his effort allocation across tasks reflects the tasks' marginal impact on P rather than their importance for firm value Y. This also implies that, in general, first-best effort $e^{FB} = f$ is not feasible because there is no bonus that makes the agent internalize the tasks' true productivities. Such a bonus exists if and only if the vectors f and g are perfectly aligned, i.e., if there is a real number λ such that $f = \lambda \cdot g$. However, as we will see, even if the principal is able to induce first-best effort levels in both tasks, it might not be optimal for her to do so.

Equation (4) implies that we can focus on non-negative bonuses.[25] Therefore, we henceforth neglect the second wage-floor constraint (WC$'_M$). Furthermore, we can use equation (4) to replace e in the principal's problem. Then, the participation constraint (PC$_M$) and the wage-floor constraint (WC$_M$) yield that, for a given b,

[25] Any negative bonus would imply zero effort in both tasks and thus lead to the same results as a bonus of zero.

the optimal fixed wage s must satisfy

$$s = \max\left\{u - g^T g \cdot \frac{b^2}{2}, w\right\}. \tag{5}$$

Hence, by substituting s, the principal's problem can be written as a function of the bonus b only:

$$\begin{aligned} &\max_b \left[f^T g \cdot b - g^T g \cdot b^2 - \max\left\{u - g^T g \cdot \frac{b^2}{2}, w\right\}\right] \\ &= \max_b \left[f^T g \cdot b - \max\left\{u + g^T g \cdot \frac{b^2}{2}, w + g^T g \cdot b^2\right\}\right]. \end{aligned} \tag{II$_M$}$$

In the last line, the first term in square brackets is the expected firm value as a function of b, and the second term corresponds to the principal's expected wage costs for a given b. These wage costs can be explained as follows. If the bonus is such that

$$u + g^T g \cdot \frac{b^2}{2} \geq w + g^T g \cdot b^2, \tag{6}$$

then the principal can choose the fixed payment s such that the agent's participation constraint is satisfied with equality. Then, the agent's expected payment net of effort costs just equals his reservation utility. By contrast, if b is such that (6) is violated, then only the wage-floor constraint (WC$_M$) holds with equality. In this case, the agent earns a rent, i.e., his expected payment net of effort costs is strictly larger than his reservation utility u.

According to (6), whether or not the agent earns a rent under a given bonus crucially depends on the relative size of the wage floor w and the agent's reservation utility u. Consequently, the optimal employment contract will also crucially depend on the relationship between w and u. To determine the optimal contract, it is instructive to first consider two special cases (i) and (ii). In case (i), there is no wage floor ($w = -\infty$), meaning that inequality (6) is satisfied for all bonuses b and, therefore, the agent will not obtain a rent. By contrast, in case (ii), we assume that $w > u$, implying that the agent will earn a rent because (6) is violated for all b.

In case (i), the bonus that maximizes the principal's objective function (II$_M$) is

$$b^{PC} = \frac{f^T g}{g^T g} = \frac{||f||}{||g||} \cos\theta. \tag{7}$$

Here, we follow Baker (2002) by introducing θ as the angle between the vectors f

and g.[26] This notation has the advantage that $\cos\theta$ can serve as a measure of the alignment between f and g or, equivalently, of the usefulness of the performance measure for effectively directing effort to the different tasks. The lower $\cos\theta$, the larger the angle θ and hence the worse aligned are f and g. According to (7), poor alignment results in a low optimal bonus and, consequently, in low effort. Intuitively, high-powered incentives have to be avoided because they would entail a severe misallocation of effort across tasks. Now consider the case of perfect alignment where $f = \lambda \cdot g$ for some real number λ and thus $\cos\theta = 1$. Then, the optimal bonus is $b^{PC} = \lambda$ and first-best effort is induced (compare (4)). These results are well-known in the literature on multitasking problems without wage floors (see, e.g., Baker (2002) or Gibbons (2005)).

By contrast, in case (ii), the principal does not only encounter a multitasking problem due to misaligned incentives. In addition, the relatively high wage floor forces her to leave a rent to the agent. From (II_M), we obtain the optimal bonus

$$b^{WC} = \frac{1}{2} b^{PC}. \tag{8}$$

Thus, compared to case (i), the principal reduces incentives ($b^{WC} < b^{PC}$). This is optimal from the principal's point of view because it lowers the agent's rent while the principal's share of the generated surplus increases relative to a situation where the bonus is b^{PC}. Furthermore, for the same reason as in case (i), the optimal bonus decreases if the alignment between f and g becomes worse. However, contrary to case (i), even under perfect alignment incentives will be inefficiently low due to the relatively high wage floor.

Having analyzed the above two cases, it remains to determine the optimal bonus if $w \in (-\infty, u]$. In this case, inequality (6) shows that the participation constraint (PC_M) is binding for small bonuses while the wage-floor constraint (WC_M) is binding for high ones. Let $\widehat{b}$ denote the bonus for which both (PC_M) and (WC_M) are binding, implying that (6) is satisfied with equality. We thus have

$$\widehat{b} = \frac{\sqrt{2(u - w)}}{||g||}. \tag{9}$$

Hence, $\widehat{b}$ depends on the relative size of u and w. Using the above derivations, the following proposition characterizes the complete solution to the principal's

[26]The second equation then follows from $f^T g = ||f|| \cdot ||g|| \cdot \cos\theta$ and $g^T g = ||g||^2$.

problem, showing that for $w \in (-\infty, u]$ the principal chooses one of the three bonuses b^{PC}, b^{WC}, and $\widehat{b}$.

Proposition 1 *If the principal hires one agent and assigns both tasks to him, she implements the bonus*[27]

$$b^M(u,w) = \begin{cases} b^{PC} & \text{if } w \leq u - \frac{D}{2} \\ \widehat{b} & \text{if } u - \frac{D}{2} < w \leq u - \frac{D}{8} \\ b^{WC} & \text{if } u - \frac{D}{8} < w \end{cases} \tag{b^M}$$

and earns the positive expected profit

$$\pi^M(u,w) = \begin{cases} \frac{D}{2} - u & \text{if } w \leq u - \frac{D}{2} \\ \sqrt{2(u-w)D} + w - 2u & \text{if } u - \frac{D}{2} < w \leq u - \frac{D}{8} \\ \frac{D}{4} - w & \text{if } u - \frac{D}{8} < w \end{cases} . \tag{π^M}$$

The agent obtains a rent of $w + \frac{D}{8} - u$ if and only if $u - \frac{D}{8} < w$.

All proofs are relegated to the Appendix.

The optimal bonus and the principal's profit are illustrated in Figure 1 for $u = 0$.[28] According to Proposition 1, the bonus b^{PC} remains optimal if a wage floor exists but is rather small ($w \leq u - D/2$). Then, only the participation constraint (PC_M) is binding at the optimal solution. Similarly, the bonus b^{WC} is implemented for a range of wage floors below the agent's reservation utility ($u - D/8 \leq w$). In this case, only the wage-floor constraint (WC_M) is binding and, thus, the agent receives a rent. However, there also is an interval of intermediate wage floors ($u - D/2 < w \leq u - D/8$), where both the participation constraint and the wage-floor constraint are binding. For such wage floors, the principal already diminishes incentives, and the optimal incentive distortion completely avoids rent payments to the agent. Thus, the wage floor strictly reduces the overall surplus from the relationship without allocating part of the remaining surplus to the agent. By contrast, if $u - D/8 < w$, an increase in the wage floor raises the agent's rent by exactly this amount, while the principal's profit is reduced by the same amount.

[27] Recall that D has been defined in Assumption 1 as $D = \frac{(f^T g)^2}{g^T g} = ||f||^2 \cos^2 \theta$.

[28] The expected profit $\pi^M(u,w)$ is always smaller than the bonus $b^M(u,w)$. To see this, consider e.g. the case $w \leq u - \frac{D}{2}$. Then, $b^M(u,w) = \frac{f^T g}{g^T g} = \frac{D}{f^T g}$ and $\pi^M(u,w) = \frac{D}{2}$. Because $f^T g = ||f|| \cdot ||g|| \cdot \cos\theta < 1$ by our assumption $||f||, ||g|| < 1$, the claim follows. Figure 1 is sketched for $f^T g = 0.8$. If u increases, both curves shift to the right. In addition, $\pi^M(u,w)$ shifts downwards such that profit is again zero at $w = \frac{D}{4}$.

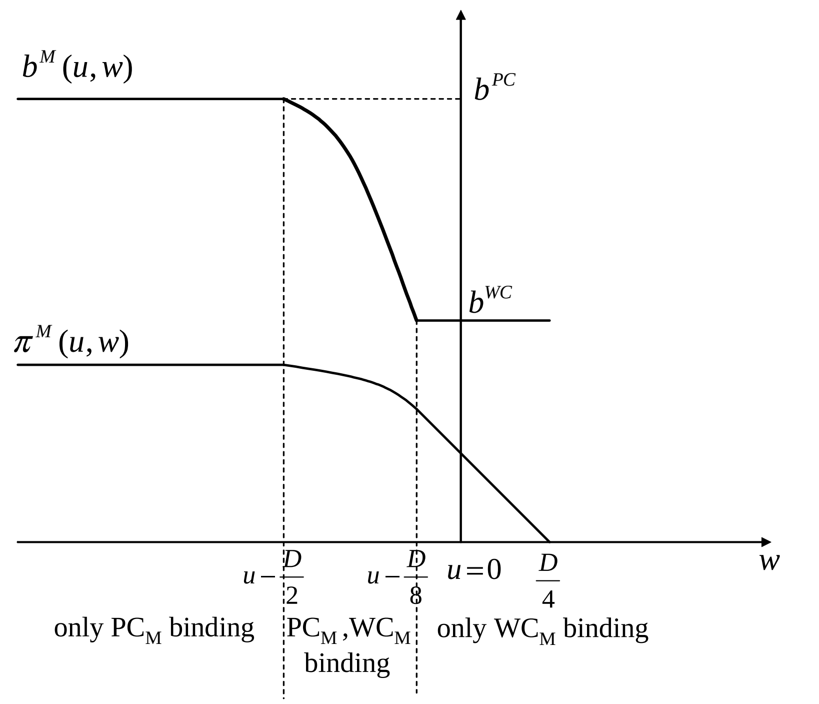

Figure 1: The optimal bonus and profit with one agent and multitasking for $u = 0$.

Furthermore, Proposition 1 shows that the principal's profit is increasing in D and, hence, in the alignment between firm value and performance measure, $\cos\theta$. As already explained above, the more useful the performance measure for providing incentives, the higher powered will be the agent's bonus contract. As a result, the principal's profit increases. However, a high bonus also implies that the agent is more likely to earn a rent. Thus, the higher $\cos\theta$, the lower the threshold on the wage floor above which a rent is paid to the agent.

Finally, Proposition 1 tells us that first-best profit $\pi^{FB} = \frac{||f||^2}{2} - u$ is attained only if $\cos\theta = 1$ and, additionally, w is sufficiently small, $w \leq u - \frac{||f||^2}{2}$. In particular, because $\pi^{FB} > 0$ by Assumption 1, this means that the wage floor must be negative. Thus, a major drawback of hiring one agent and assigning both tasks to him is that the first-best effort allocation e^{FB} is in general not implemented. This raises the question whether the principal could benefit from excluding one task from the agent's job. Excluding a task would allow the principal to provide efficient incentives in the remaining task because the agent can no longer misallocate effort across tasks. Formally, the bonus can always be set such that the agent has efficient incentives for fulfilling the remaining task i and thus exerts first-best effort e_i^{FB} in this task. However, if the wage floor is so high that the principal has to leave a rent to the agent for implementing e_i^{FB}, she may prefer to induce less than first-best effort. The next proposition shows that exclusion of a task may indeed be optimal.

Proposition 2 *If the principal hires one agent, she should exclude the less important task 2 from the job if and only if $f_1 \geq ||f|| \cdot \cos\theta$. The principal then pays the bonus*

$$b^{M1}(u,w) = \begin{cases} \frac{f_1}{g_1} & \text{if } w \leq u - \frac{f_1^2}{2} \\ \frac{\sqrt{2(u-w)}}{g_1} & \text{if } u - \frac{f_1^2}{2} < w \leq u - \frac{f_1^2}{8} \\ \frac{f_1}{2g_1} & \text{if } u - \frac{f_1^2}{8} < w \end{cases} \tag{b^E}$$

and earns the positive expected profit

$$\pi^{M1}(u,w) = \begin{cases} \frac{f_1^2}{2} - u & \text{if } w \leq u - \frac{f_1^2}{2} \\ \sqrt{2(u-w)}f_1 + w - 2u & \text{if } u - \frac{f_1^2}{2} < w \leq u - \frac{f_1^2}{8} \\ \frac{f_1^2}{4} - w & \text{if } u - \frac{f_1^2}{8} < w \end{cases}. \tag{π^E}$$

The agent obtains a rent of $w + \frac{f_1^2}{8} - u$ if and only if $u - \frac{f_1^2}{8} < w$.

Proposition 2 shows that, if the principal excludes a task, this should always be the task with the lower marginal productivity, i.e., task 2. Holding alignment (i.e., $\cos\theta$) fixed, the exclusion of this task is optimal if the relative marginal productivities of the tasks differ sufficiently, i.e., if f_1 is sufficiently larger than $||f||$. Then, the principal benefits strongly from more effective incentives for task 1 and does not lose much by giving up task 2. On the other hand, for given marginal productivities f_1 and f_2, task 2 should not be performed if the performance measure is sufficiently distortive (i.e., $\cos\theta$ is low). In this case, excluding one task prevents a relatively severe misallocation of effort. Moreover, whether task exclusion is optimal or not does not depend on u and w. This is because the impact of these parameters on the optimal provision of incentives is analogous to the case of broad task assignment.

To summarize, when the principal hires one agent, excluding the less productive task from his job is sometimes optimal because it allows to improve the incentives for the task with the higher productivity. This result suggests that the principal might be even better off by hiring a second agent and assigning task 2 to him. By paying different bonuses to the agents, the principal can then set first-best incentives for *both* tasks. However, the drawback of employing two agents is that the principal has to pay bonuses and fixed wages to both of them. We now proceed by analyzing the principal's contracting problem if she allocates the tasks to two different agents.

4 Specialization

Under specialization, the principal offers each agent i a contract specifying that the agent is supposed to carry out task i, receives the fixed wage s_i and the bonus b_i if the performance measure is favorable.

Given that agent i has accepted the contract, he chooses his effort e_i to maximize his expected wage net of effort costs, i.e.,

$$e_i = \arg\max_{\widehat{e}_i} s_i + (g_i \widehat{e}_i + g_j e_j) b_i - \frac{1}{2}\widehat{e}_i^2, \;\; i = 1, 2. \tag{10}$$

It follows that agent i chooses the effort level $e_i = b_i g_i$.[29] Thus, in sharp contrast to the case of multitasking, the principal is able to induce every arbitrary effort pair (e_1, e_2) under specialization. In particular, the first-best effort allocation $e^{FB} = f$ is always feasible under specialization; the bonus $b_i = f_i / g_i$ induces first-best effort $e_i^{FB} = f_i$ in task i.

Anticipating the agents' effort choices under a given contract, the principal solves the following optimization problem:

$$\max_{\substack{e, s_i, b_i \\ i=1,2}} \;\; f^T e - s_1 - s_2 - g^T e \cdot (b_1 + b_2) \tag{I$_S$}$$

$$\text{s.t.} \quad e_i = b_i g_i, \;\; i = 1, 2 \tag{IC$_S$}$$

$$s_i + g^T e \cdot b_i - \frac{1}{2} e_i^2 \geq u, \;\; i = 1, 2 \tag{PC$_S$}$$

$$s_i \geq w, \;\; i = 1, 2 \tag{WC$_S$}$$

When maximizing expected firm value minus wage costs, the principal has to take into account the agents' incentive compatibility and participation constraints,

[29] Agent i's effort choice does not depend on the effort of his co-worker because the marginal effect of effort in task i on P is independent of effort in task j, i.e., $\frac{\partial^2 \Pr[P=1|e_i,e_j]}{\partial e_i \partial e_j} = 0$.

(IC$_S$) and (PC$_S$), respectively. Moreover, the wage-floor constraints (WC$_S$) must be satisfied.[30]

We can use (IC$_S$) to replace e_i in the principal's problem. Then, we obtain from agent i's participation and wage-floor constraint that, for given bonuses b_i and b_j, the optimal fixed wages must satisfy

$$s_i = \max\left\{u - \frac{1}{2}g_i^2 b_i^2 - g_j^2 b_j b_i, w\right\}, \quad i = 1, 2. \tag{11}$$

Hence, after substituting s_i, the principal's optimization problem becomes:

$$\max_{b_1, b_2} \quad \left[f_1 g_1 b_1 + f_2 g_2 b_2 - \max\left\{u + \frac{1}{2}g_1^2 b_1^2,\ w + g_1^2 b_1^2 + g_2^2 b_1 b_2\right\} - \max\left\{u + \frac{1}{2}g_2^2 b_2^2,\ w + g_2^2 b_2^2 + g_1^2 b_1 b_2\right\}\right] \tag{12}$$

The term $f_1 g_1 b_1 + f_2 g_2 b_2$ is the expected firm value for given bonuses b_1 and b_2. The next expression is the principal's expected wage payment to agent 1. If the bonuses are such that $u + \frac{1}{2}g_1^2 b_1^2 \geq w + g_1^2 b_1^2 + g_2^2 b_1 b_2$, then the fixed payment s_1 can be chosen such that agent 1's participation constraint is binding. Otherwise, the agent earns a rent. Importantly, in the latter case, agent 1's expected payment also depends on the bonus paid to agent 2. The reason is that agent 2's incentives affect agent 1's probability of earning his own bonus: The higher b_2, the harder agent 2 works. Consequently, the probability that the agents' joint performance measure P is favorable rises and, thus, agent 1's expected bonus payment also increases. The part of agent 1's expected payment that results from agent 2's effort is exactly $g_2^2 b_1 b_2$ because

$$\Pr[P = 1 | e_1 = 0, e_2] \cdot b_1 = g_2 e_2 \cdot b_1 = g_2^2 b_2 \cdot b_1, \tag{13}$$

where the last equation follows from the incentive-compatibility constraints (IC$_S$). An analogous explanation holds for agent 2's expected wage, which is given by the term in the second line of (12).

Let $\pi^S(u, w)$ denote the principal's profit under the solution to problem (12). Using (IC$_S$), we can rewrite (12) as a function of effort, which will be useful for

[30] We drop the constraints $s_i + b_i \geq w$ since from (IC$_S$) it is clear that we can focus on non-negative bonuses.

the further analysis. We thus obtain:

$$\pi^S(u,w) = \max_{e_1,e_2} \left[f_1e_1 + f_2e_2 - \max\left\{u + \frac{1}{2}e_1^2,\ w + e_1^2 + \frac{g_2}{g_1}e_1e_2\right\} - \max\left\{u + \frac{1}{2}e_2^2,\ w + e_2^2 + \frac{g_1}{g_2}e_1e_2\right\}\right] \tag{II$_S$}$$

Figure 2 depicts whether the principal has to pay rents to agent 1 and 2, respectively, for inducing a given effort pair (e_1, e_2), assuming that $g_1 = g_2$ and $u - w > 0$. If the effort pair belongs to area A_1, then no agent earns a rent.[31] In area A_2, agent 1 obtains a rent but not agent 2, whereas area A_3 corresponds to the opposite case. Finally, in area A_4, both agents earn rents. As $u - w$ decreases, A_4 becomes larger relative to the other areas. Moreover, if $u - w \leq 0$, then A_1, A_2, and A_3 disappear. Thus, analogous to the case of multitasking, if the wage floor is strictly larger than the reservation utility, both agents earn rents for every pair of effort levels.

The next proposition characterizes the circumstances under which the principal indeed makes use of the possibility to induce first-best effort under specialization.

Proposition 3 *Under specialization, the principal induces the agents to choose first-best effort levels e^{FB} if and only if*

$$R := \max\left\{\frac{1}{2}f_1^2 + \frac{g_2}{g_1}f_1f_2, \frac{1}{2}f_2^2 + \frac{g_1}{g_2}f_1f_2\right\} \leq u - w. \tag{FB}$$

The principal's expected profit then is $\pi^S(u,w) = \frac{||f||^2}{2} - 2u$.

According to Proposition 3, first-best effort will be implemented if the reservation utility u is sufficiently large or the wage floor w is sufficiently low. In particular, in the absence of a wage floor ($w = -\infty$), specialization always leads to first-best effort. More precisely, inequality (FB) ensures that u and w are such that the principal does not need to pay rents for making the agents exert first-best effort, i.e., e^{FB} belongs to area A_1 in Figure 2.

We now turn to the case $u - w \leq 0$ such that only area A_4 remains and, therefore, agents earn rents for each pair of strictly positive effort levels.

[31] Define $q := \frac{g_2}{g_1}$. From (II$_S$), agent 1 does not earn a rent if $u + \frac{1}{2}e_1^2 \geq w + e_1^2 + qe_1e_2 \Leftrightarrow \sqrt{q^2e_2^2 + 2(u-w)} - qe_2 \geq e_1$. Analogously, agent 2 does not earn a rent if $\sqrt{q^{-2}e_1^2 + 2(u-w)} - q^{-1}e_1 \geq e_2$.

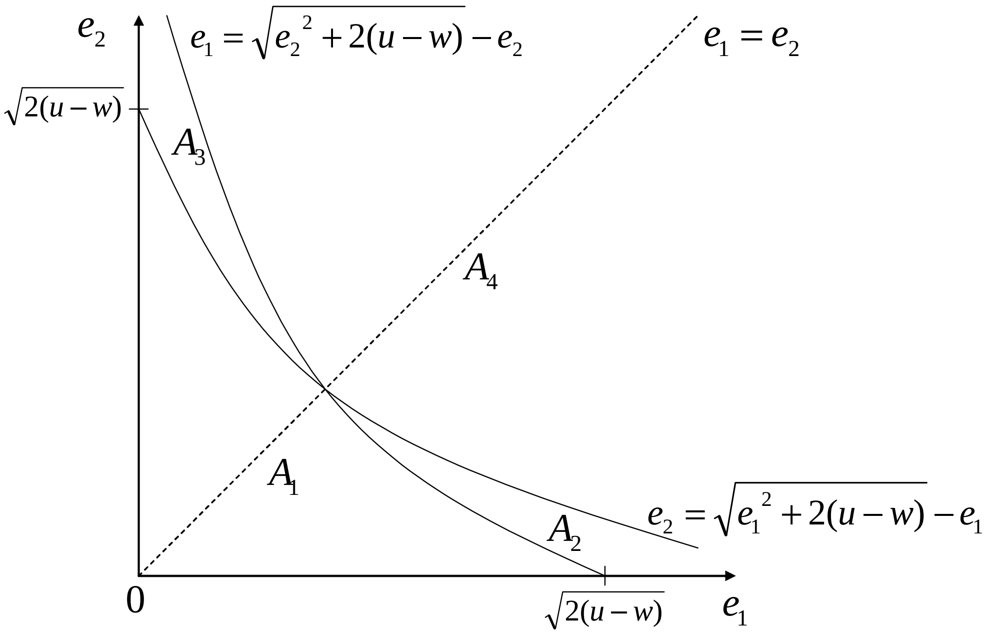

Figure 2: Workers' rents under specialization for given effort levels (e_1, e_2).

Proposition 4 *If $u - w \leq 0$, then the effort levels $(e_1, e_2) = \left(\frac{f_1}{2}, 0\right)$ solve the principal's problem (II_S) under specialization. The principal's expected profit is $\pi^S(u, w) = \frac{f_1^2}{4} - 2w$.*

Surprisingly, Proposition 4 shows that the principal does not provide incentives for the less important task 2 whenever w is so large or u is so small that both agents earn rents for each pair of positive effort levels or, equivalently, bonuses. Then, providing incentives for agent 2 is too costly because a positive bonus b_2 increases the rent of both agents (compare (12)) but makes only agent 2 to work harder. Therefore, the principal prefers to exclusively focus on the more important task 1.

In Proposition 2, we have shown that inducing zero effort in task 2 may also be optimal if only one agent is employed. However, note that task 2 is neglected for entirely different reasons under the two job regimes. With one agent, the principal excludes task 2 from the job when the performance measure is a poor proxy for the firm value. Then, the problem of misallocating effort across tasks would be too severe. The relative size of w and u does not play a role for the decision whether to neglect a task. By contrast, under specialization, the principal can tailor incentives to the different tasks. Therefore, the quality of the performance measure is not relevant for the decision whether to neglect a task. Instead, the size of the rent payments to the agents is crucial.

It remains to analyze the case where u and w are such that the Propositions 3 and 4 do not apply, i.e., $0 < u - w < R$. Then, in Figure 2, all the areas $A_1 - A_4$ exist, however, the first-best effort levels e^{FB} are not part of A_1. In this case, it is not possible to explicitly solve problem (II_S). We can, however, identify important characteristics of the optimal solution, which will be useful for the comparison of the different job regimes.

Proposition 5 *Assume that $0 < u - w < R$. (i) If $f_1 > f_2$ and/or $g_1 \neq g_2$, then at most one agent earns a rent under the optimal solution to (II_S). (ii) If $f_1 = f_2$ and $g_1 = g_2$, then the optimal solution to (II_S) may not be unique. In particular, there may be an optimal solution where both agents earn a rent. However, there still is an optimal pair of effort levels for which at most one agent receives a rent.*

Proposition 5 shows that the optimal employment contracts usually do not incorporate rents for both agents. Rent payments to both agents can occur only in the special case when $f_1 = f_2$ and $g_1 = g_2$ when the solution to problem (II_S) may not be unique. However, it still exists an optimal solution where only one agent earns a rent. Thus, in the search for an optimal solution, one can neglect the interior of A_4 in Figure 2.

Depending on the parameter constellation, it is also possible that none of the agents receives a rent when $0 < u - w < R$. For example, consider the case $g_1 = g_2$. From (II_S) we can see that, for $g_1 = g_2$, the principal's wage costs are symmetric in e_1 and e_2. Therefore, neither task has a cost advantage over the other. However, task 1 is more important for the firm value ($f_1 \geq f_2$). Consequently, the principal will induce (weakly) higher effort in task 1 than in task 2. Together with Propositions 3 and 5 it follows that, if $0 < u - w < R$, the optimal solution will be in area A_2 of Figure 2. Intuitively, because the principal prefers higher effort in task 1 than in task 2, only agent 1 may earn a rent. Furthermore, for $f_2 \leq \frac{f_1}{2}$ it can be shown that the optimal effort levels always lie on the boundary between A_1 and A_2. Thus, no agent receives a rent; due to task 2's relatively low productivity, it does not pay to implement such a high effort in task 2 that agent 1 will earn a rent. By contrast, if $\frac{f_1}{2} < f_2 < \frac{2}{3}f_1$, there is a range of values of $u - w$ for which the optimal solution lies in the interior of A_2, i.e., agent 1 obtains a rent.

5 Optimal Job Design

In this section, we compare the two previously analyzed job designs and show under which circumstances one dominates the other. The above results imply that multitasking and specialization crucially differ with respect to their respective sets of feasible effort levels. When the principal hires only one agent and assigns both tasks to him, she cannot tailor incentives to the different tasks. Therefore, she can induce only those effort levels e that are multiples of the vector g (see equation (4)). Excluding one task from the agent's job makes every effort level feasible in the remaining task, but the excluded task is not carried out. By contrast, under specialization, the possibility to pay different bonuses for different tasks allows the principal to induce every arbitrary pair of effort levels, including first-best effort e^{FB} (see equation (IC_S)). By Proposition 3, the principal indeed provides first-best incentives if the reservation utility u and the wage floor w are such that agents do not earn rents for exerting first-best effort. However, being able to induce e^{FB} then comes at the cost of having to compensate another agent for his forgone outside option u. This suggests that the principal will prefer specialization to multitasking if the benefit of having first-best effort in both tasks exceeds u. The next proposition verifies this intuition.

Proposition 6 *Assume that first-best effort is optimal under specialization, i.e., inequality (FB) holds. Then, the principal prefers specialization to multitasking if and only if*

$$u \leq \frac{1}{2}\left(||f||^2 - \max\{||f||^2 \cos^2\theta,\ f_1^2\}\right). \tag{14}$$

The proof of Proposition 6 shows that, whenever w and u are such that first-best effort is induced under specialization, the participation constraint of the single agent under the competing job regime would also be binding. Hence, under either job design, the principal's expected wage payment equals the sum of the agents' effort and opportunity costs. Therefore, the left-hand side of condition (14) is the increase in opportunity costs when two agents are employed, while the right-hand side gives the associated increase in expected firm value net of effort cost. If the latter exceeds the additional opportunity costs, then specialization is optimal. In the absence of a wage floor ($w = -\infty$), specialization always entails first-best effort because inequality (FB) holds for all f, g, and u. In this case, condition (14) shows that, the better aligned performance measure and firm value (i.e., the larger $\cos\theta$), the less likely it is that specialization is superior. Then, a single agent can be more effectively provided with incentives, making specialization less attractive to the principal. In the special case of perfect alignment ($\cos\theta = 1$), multitasking is always optimal because it then leads to the first-best solution.

Even though first-best effort is feasible under specialization, Proposition 4 has shown that this job design becomes highly ineffective if the wage floor exceeds the reservation utility. In such a situation, given that the principal has hired two agents, it is optimal not to provide incentives to the agent to whom the less productive task is assigned. Clearly, under these circumstances, the principal would be better off by hiring only one agent and excluding task 2 from his job. Then, with one agent, the principal can induce the same effort in task 1 but does not need to pay w to a second, idle agent. Moreover, by Proposition 2, if the performance measure is not too distortive, the principal can even do better by assigning both tasks to the agent. The next proposition summarizes these findings.

Proposition 7 *Assume that $u - w \leq 0$. Then, the principal prefers multitasking to specialization, i.e., she hires only one agent. If $f_1 < ||f|| \cdot \cos\theta$, the agent carries out both tasks. If $f_1 \geq ||f|| \cdot \cos\theta$, the principal excludes task 2 from the agent's job.*

Propositions 6 and 7 have established the optimal job design for $R \leq u - w$ and $u - w \leq 0$, respectively. It remains to discuss the case $0 < u - w < R$. For this situation, due to the fact that it is not possible to provide an explicit solution to the principal's problem under specialization, a comparison of the two job regimes is more complex. Therefore, we cannot present a set of conditions under which one job regime dominates the other. We can, however, show that, whenever specialization is preferred to multitasking, the former job design does not involve any rent payments to agents. In other words, as soon as only one of the agents earns a rent under specialization, the principal (weakly) prefers to employ only one agent.

Proposition 8 *Assume that $0 < u - w < R$. If specialization leads to a strictly higher profit than multitasking, neither of the two agents earns a rent under specialization.*

The previous results have revealed that the introduction of wage floors has substantial effects on the work organization within firms: They lead to increased multitasking and thus employment of fewer agents. In particular, in our model, a wage floor exceeding the workers' reservation utility always renders specialization suboptimal (Proposition 7). However, wage floors below the reservation utility may also result in multitasking (Propositions 6 and 8). In any case, wage floors never give rise to the employment of two agents, both earning a rent.

6 Extensions

6.1 Interdependent Tasks

So far, we have focussed on a situation where tasks are independent in the agent's cost function (i.e., $\partial^2 c/\partial e_i \partial e_j = 0$). However, in practice, tasks may compete for the agent's attention in the sense that an agent who is already responsible for one task finds it harder to engage in another one. On the other hand, tasks may also interact in an advantageous way such that the costs of one task decrease if the agent already performs a complementary one. In the former case, tasks are substitutes ($\partial^2 c/\partial e_i \partial e_j > 0$) whereas they are complements in the latter case ($\partial^2 c/\partial e_i \partial e_j < 0$). In the introductory fast-food example, serving customers and selling food are presumably complementary tasks whereas cooking and cleaning are likely to be substitutes. In this section we show that our model can be extended to interdependent and differently costly tasks. To do this, we consider the cost function

$$c(e_1, e_2) = \frac{1}{2}(c_1 e_1^2 + c_2 e_2^2) + c_{12} e_1 e_2, \tag{15}$$

where $c_1, c_2 > 0$ and $c_{12} \in (-\sqrt{c_1 c_2}, \sqrt{c_1 c_2})$.[32] Interdependencies between the tasks exist if the parameter c_{12} is different from zero. If $c_{12} > 0$, the two tasks are substitutes. In this case, assigning both tasks to a single agent leads to a new disadvantage because, holding effort levels fixed, effort costs will increase. By contrast, if $c_{12} < 0$, total effort costs decrease if the tasks are performed by a single agent.

We first characterize the *first-best job design*, which now crucially depends on the parameters in the cost function. To do so, we derive the surplus-maximizing effort levels under each job regime and compare the resulting profits. We start

[32] The restriction $c_{12} \in (-\sqrt{c_1 c_2}, \sqrt{c_1 c_2})$ ensures that the cost function is strictly convex.

with the case where the principal hires only one agent. If the agent performs both tasks, the surplus-maximizing effort levels (e_1^M, e_2^M) solve

$$\max_{e_1,e_2} \; f_1e_1 + f_2e_2 - \frac{1}{2}(c_1e_1^2 + c_2e_2^2) - c_{12}e_1e_2 - u, \tag{16}$$

which yields

$$e_1^M = \frac{c_2f_1 - c_{12}f_2}{c_1c_2 - c_{12}^2}, \quad e_2^M = \frac{c_1f_2 - c_{12}f_1}{c_1c_2 - c_{12}^2}. \tag{17}$$

We assume that these effort levels are positive, i.e., c_{12} is not too large. The corresponding profit is

$$\pi_{\max}^M = \frac{1}{2}\frac{c_2f_1^2 + c_1f_2^2 - 2c_{12}f_1f_2}{c_1c_2 - c_{12}^2} - u. \tag{18}$$

If the principal assigns only task i to the agent, while task j is not carried out, the efficient effort level e_i^{M1} maximizes $f_ie_i - \frac{c_i}{2}e_i^2 - u$. This yields $e_i^{M1} = \frac{f_i}{c_i}$ and the profit

$$\pi_{\max}^{M1}(i) = \frac{1}{2}\frac{f_i^2}{c_i} - u. \tag{19}$$

Finally, if the tasks are assigned to different agents, the surplus-maximizing effort levels $e_i^S = \frac{f_i}{c_i}$ maximize

$$f_1e_1 + f_2e_2 - \frac{1}{2}\left(c_1e_1^2 + c_2e_2^2\right) - 2u, \tag{20}$$

yielding the profit

$$\pi_{\max}^S = \frac{1}{2}\left(\frac{f_1^2}{c_1} + \frac{f_2^2}{c_2}\right) - 2u. \tag{21}$$

We assume that $\frac{f_1^2}{c_1} \geq \frac{f_2^2}{c_2}$, implying that, under specialization, task 1 makes a larger contribution to firm profit than task 2. Furthermore, to ensure that specialization may indeed be optimal, we assume that $\frac{1}{2}\frac{f_2^2}{c_2} - u > 0$, i.e., the highest possible net contribution of the second agent is positive. It follows that $\pi_{\max}^{M1}(i) < \pi_{\max}^S$ for $i = 1, 2$. Thus, task exclusion cannot be part of the first-best job design. Consequently, the first-best job design incorporates multitasking if and only if $\pi_{\max}^M \geq \pi_{\max}^S$. This condition is always satisfied if the tasks are independent, i.e., $c_{12} = 0$. Furthermore, because $\pi_{\max}^M$ is decreasing in c_{12}, hiring only one agent is first-best if the tasks are complementary, i.e., $c_{12} < 0$. By contrast, if the tasks are substitutes and u is sufficiently close to zero, specialization is first-best.

We now turn to the principal's problem under *specialization*. Quite similarly to problem (I_S) – (WC_S) in Section 4, it is now given by:

$$\max_{\substack{e,s_i,b_i\\ i=1,2}} \quad f^T e - s_1 - s_2 - g^T e \cdot (b_1 + b_2) \tag{22}$$

$$\text{s.t.} \quad e_i = \frac{g_i}{c_i} b_i, \;\; i = 1, 2 \tag{23}$$

$$s_i + g^T e \cdot b_i - \frac{c_i}{2} e_i^2 \geq u, \;\; i = 1, 2 \tag{24}$$

$$s_i \geq w, \;\; i = 1, 2 \tag{25}$$

Analogously to the procedure presented in Section 4, we can simplify the principal's problem to:

$$\pi_I^S(u,w) = \max_{e_1,e_2} \quad \Bigg[f_1 e_1 + f_2 e_2 - \max\left\{ u + \frac{c_1}{2} e_1^2, \; w + c_1 e_1^2 + c_1 \frac{g_2}{g_1} e_1 e_2 \right\} - \max\left\{ u + \frac{c_2}{2} e_2^2, \; w + c_2 e_2^2 + c_2 \frac{g_1}{g_2} e_1 e_2 \right\} \Bigg] \tag{II_S^I}$$

We must have that $\pi_I^S(u,w) \leq \pi_{\max}^S$. The principal can indeed realize the profit $\pi_{\max}^S$ if she does not have to pay rents to the agents for implementing the corresponding effort levels $e_i^S = \frac{f_i}{c_i}$. Similarly to Proposition 3, this is the case if

$$R_I := \max\left\{ \frac{1}{2}\frac{f_1^2}{c_1} + \frac{1}{c_2}\frac{g_2}{g_1} f_1 f_2, \frac{1}{2}\frac{f_2^2}{c_2} + \frac{1}{c_1}\frac{g_1}{g_2} f_1 f_2 \right\} \leq u - w. \tag{MAX}$$

Moreover, in accordance with Proposition 5, agents earn rents for each pair of positive effort levels if $u - w \leq 0$. Then, the principal does again not provide incentives for task 2 because it is less important to firm value than task 1 $\left(\frac{f_1^2}{c_1} \geq \frac{f_2^2}{c_2}\right)$. The optimal effort levels thus are $(e_1, e_2) = \left(\frac{f_1}{2c_1}, 0\right)$ and the corresponding profit is $\pi_I^S(u,w) = \frac{f_1^2}{4c_1} - 2w$.

The analysis of the *multitasking* case also proceeds analogously to the procedure presented in Section 3. The only difference is that, to determine the usefulness of the performance measure for effectively directing effort to the different tasks, we now have to appropriately weight the vectors f and g with the parameters of the cost function. Because these parameters also affect how the agent will allocate effort across tasks, the angle between f and g does no longer reflect the quality of the performance measure, as it was the case in the basic model. Here, we skip the derivation of the associated weighting process, which is explained in detail in Schöttner (2008).[33] The resulting weighted vectors are $f_c = Sf$ and $g_c = Sg$, were S is a 2x2-matrix with $S^T S = C^{-1}$ and C is the matrix of the parameters in the agent's cost function,

$$C = \begin{pmatrix} c_1 & c_{12} \\ c_{12} & c_2 \end{pmatrix}. \tag{26}$$

To understand the intuition, consider the example $f^T = \frac{1}{2}(1,1)$, $g^T = \frac{1}{2}(1,2)$, and $c_{12} = 0$. Then, the relative overemphasis of task 2 in the performance measure is mitigated as the cost parameter for task 2, c_2, increases. The reason is that cost considerations make the agent direct relatively more effort towards task 1. Thus, even though f and g do not change, alignment between Y and P and, consequently, the optimal bonus for the agent increases. The alignment between Y and P can now be measured by the angle between f_c and g_c, which we denote by θ_c.[34]

To obtain the results for the case where the principal assigns both tasks to the agent, we just have to replace f, g, and θ with f_c, g_c, and θ_c, respectively, in Proposition 1. Moreover, as the counterpart to Proposition 2 we obtain that the principal will exclude task 2 from the agent's job if and only if

$$\frac{f_1}{\sqrt{c_1}} \geq ||f_c||^2 \cos^2 \theta_c. \tag{27}$$

Thus, the essential intuition that task 2 will be excluded if the performance measure is sufficiently distortive (i.e., $\cos \theta_c$ is low) is still valid. Furthermore, in Schöttner (2008, pp. 148-9) it is shown that $||f_c|| \cdot \cos \theta_c$ decreases in c_{12}. Consequently, the degree of task interdependencies c_{12} has an unambiguous and intuitively clear effect on the principal's profit: As task complementarities decrease or substitutabilities increase, assigning both tasks to one agent becomes less attractive.

Based on the above results, we can derive the *optimal job design*, which is in accordance with Propositions 6 and 8 in the foregoing section.[35]

Proposition 9 *Consider the effort cost function*

$$c(e_1, e_2) = \frac{1}{2}(c_1 e_1^2 + c_2 e_2^2) + c_{12} e_1 e_2, \tag{28}$$

[33] See Schöttner (2008), pp. 143-4.

[34] If $c_1 = 1$, we obtain for the weighted vectors $f_c^T = \frac{1}{2}\left(1, \frac{1}{\sqrt{c_2}}\right)$ and $g_c^T = \frac{1}{2}\left(1, \frac{2}{\sqrt{c_2}}\right)$. It can be shown that the angle θ_c is decreasing in c_2.

[35] The proof of this Proposition is available from the authors upon request. It proceeds analogously to the proofs of Propositions 6 and 8, which are given in the Appendix.

where $c_1, c_2 > 0$, $c_{12} \in (-\sqrt{c_1 c_2}, \sqrt{c_1 c_2})$, *and* $\frac{f_1^2}{c_1} \geq \frac{f_2^2}{c_2}$.
(i) Assume that $R_I \leq u - w$, *i.e.,* $\pi_{\max}^S$ *is realized under specialization. Then, the principal prefers specialization to multitasking if and only if*

$$u \leq \frac{1}{2}\left(\frac{f_1^2}{c_1} + \frac{f_2^2}{c_2} - \max\left\{||f_c||^2 \cos^2\theta_c,\ \frac{f_1^2}{c_1}\right\}\right). \tag{29}$$

This condition is more likely to hold as c_{12} *increases.*
(ii) Assume that $u - w \leq 0$. *Then, the principal prefers multitasking to specialization, i.e., she hires only one agent. The agent carries out both tasks if and only if* $\frac{f_1}{\sqrt{c_1}} \geq ||f_c||^2 \cos^2\theta_c$. *Otherwise, he performs only task one. Task exclusion becomes more likely as* c_{12} *increases.*

Thus, our main results on optimal job design in the presence of wage floors carry over to a cost function with differently costly tasks that can be substitutes or complements. The additional and quite intuitive insight from this extension is that weaker task complementarities or stronger task substitutabilities make the assignment of both tasks to one agent less often optimal. According to Proposition 9, the principal becomes more likely to switch either to specialization or task exclusion as c_{12} increases.

6.2 Variable Wage Floors

In this subsection, we return to the assumption of technologically independent and equally costly tasks but allow for wage floors that depend on the agent's task assignment. Such variable wage floors are relevant if the wage floor is due to a per-hour minimum wage and, compared to broad task assignment, the agent's working hours can be reduced when he carries out only a single task. To reflect such a situation, we now assume that the wage of a worker who performs only task i must be at least $k_i w$, where $k_i \in (0, 1]$ is an exogenously given parameter. Accordingly, the lower k_i, the less time-consuming is a job consisting only of task i relative to broad task assignment. To shorten notation, we define $w_i := k_i w$ as the wage floors under single task assignment. Furthermore, we assume that $\min\{k_1, k_2\} < 1$, implying that at least one of the wage floors w_1 and w_2 is strictly lower than w.

First consider the case of *multitasking*. If the principal assigns both tasks to a single agent, the wage-floor constraint is $s \geq w$ as in the basic model. Thus, the principal realizes the profit $\pi^M(u, w)$ as given in equation (π^M). By contrast, if the principal excludes task i from the agent's job, the wage-floor constraint is $s \geq w_i$. If $w_1 \leq w_2$, the principal prefers to exclude task 2 rather than task 1. Task 1 then does not only make a higher contribution to firm profit ($f_1 \geq f_2$), it also leads to a lower wage floor. If, however, $w_1 > w_2$, task 2 has the advantage of a lower wage floor. For simplicity, we assume that this advantage is not strong enough to compensate for the loss in profits if only task 2 is performed.[36] Consequently, as in the basic model, if the principal excludes a task, that will always be task 2. Thus, the principal's profit under task exclusion is $\pi^{M1}(u, w_1)$, where the profit function $\pi^{M1}(u, w)$ is given in equation (π^E). As a result, task exclusion is optimal if $\pi^M(u, w) \leq \pi^{M1}(u, w_1)$. Compared to the basic model, task exclusion is more likely to occur because $\pi^{M1}(u, w_1) \geq \pi^{M1}(u, w)$. Intuitively, as task exclusion allows the principal to reduce the agent's working hours, it has an additional benefit relative to the basic model and is, therefore, more often preferred to broad task assignment.

Under *specialization*, the analysis proceeds analogously to Section 4. Similarly to (II_S), the principal's optimization problem can be simplified to:

$$\max_{e_1,e_2} \Bigg[f_1e_1 + f_2e_2 - \max\left\{u + \frac{1}{2}e_1^2,\ w_1 + e_1^2 + \frac{g_2}{g_1}e_1e_2\right\} - \max\left\{u + \frac{1}{2}e_2^2,\ w_2 + e_2^2 + \frac{g_1}{g_2}e_1e_2\right\}\Bigg] \qquad (\text{II}_S^R)$$

The principal will induce first-best effort levels e^{FB} if and only if doing so does not require rent payments to any of the agents. This is the case if the term R, which we defined in Proposition 3, does neither exceed $u - w_1$ nor $u - w_2$, or, equivalently,

$$R \leq u - \max\{k_1, k_2\} \cdot w. \qquad (\text{FB}^R)$$

As in the basic model, the principal's profit then is $\frac{||f||^2}{2} - 2u$. Compared to condition (FB), which ensures first-best effort under specialization in the basic model, condition (FB^R) is satisfied for a larger range of minimum wages if $\max\{k_1, k_2\} < 1$. That is, if working hours can be reduced for both agents under specialization, the principal more often induces first-best effort. The reason is that, with lower wage floors, the agents are less likely to earn rents for the bonuses that provide first-best

[36] This assumption is also consistent with presuming that task 1 is indispensable for a favorable outcome, as suggested in Footnote 18.

incentives. Nevertheless, as in the basic model, the principal will provide inefficient effort incentives when w is sufficiently large.

Equivalently to Proposition 4, if the agents earn rents for each pair of strictly positive effort levels, the principal does not provide incentives for the second task. By (II_S^R), this case occurs if $u-w_1$ and $u-w_2$ are both below zero, or, equivalently,

$$u - \min\{k_1, k_2\} \cdot w \leq 0. \tag{30}$$

The principal's profit then is $\frac{f_1^2}{4} - w_1 - w_2$. Since $\min\{k_1, k_2\} < 1$, condition (30) is less likely to hold than the corresponding condition in the basic model, $u - w \leq 0$. Intuitively, because one wage floor is strictly lower than w, a situation where both agents earn a rent occurs less often. Still, there are rent payments to both agents when the minimum wage w is sufficiently large. However, in contrast to the basic model, w now needs to be strictly above the reservation utility u.

The foregoing arguments lead to the following results concerning the *optimal job design.*

Proposition 10 *(i) Assume that first-best effort is optimal under specialization, i.e., inequality (FB^R) holds. Then, the principal prefers specialization to multitasking if and only if*

$$u \leq \frac{1}{2}\left(||f||^2 - \max\{\pi^M(u,w),\ \pi^{M1}(u,w_1)\}\right). \tag{31}$$

(ii) Assume that condition (30) holds. Then, the principal prefers multitasking to specialization, i.e., she hires only one agent. The agent carries out both tasks if and only if $\pi^M(u,w) \geq \pi^{M1}(u,w_1)$.

Part (i) of Proposition 10 is the counterpart to Proposition 6 in the basic model. It says that, when specialization leads to first-best effort, it is preferred to multitasking if the costs of hiring an additional agent, u, are smaller than the gain from having first-best effort in both tasks, which is given by the right-hand side of condition (31). In contrast to the basic model, this gain now depends on the wage floors w and w_1 since they affect the principal's decision on task exclusion. Moreover, this gain is (weakly) smaller than in the basic model because $w_1 \leq w$ makes the case of task exclusion more attractive for the principal. Thus, even though first-best effort is more often implemented under specialization when wage floors are variable, this does not necessarily make the principal employ two agents more often.

Part (ii) of Proposition 10 corresponds to Proposition 7 in the basic model. It establishes that the principal should hire only one agent whenever she finds it optimal not to provide incentives for the second task under specialization. Since this case is less likely to occur under variable wage floors, this argument now makes a weaker case for multitasking.

7 Discussion and Conclusion

We theoretically analyze how the existence of wage floors affects optimal job design in firms. We assume that two tasks contribute to firm value. These tasks can be assigned to only one job carried out by one agent, or each task can be delegated to a different agent with a specialized job. While firm value is non-verifiable, there is an imperfect performance measure whose realization randomly depends on both tasks such that the principal can use the latter to provide both agents with effort incentives. We show that, in the absence of any wage floor, splitting the tasks into specialized jobs is optimal whenever the agents' reservation utility is not too large. Under such a job design, the principal can tailor incentives according to each task's true marginal productivity by paying different bonuses to the agents. Consequently, the principal induces first-best effort.[37] With a sufficiently large wage floor (that is still below the reservation utility), however, the principal begins to gradually reduce effort incentives to avoid sharing the surplus with the agents. Once the wage floor becomes sufficiently large, she cannot prevent paying a rent under specialization; as a result, she hires only one agent to perform the tasks within one job. Multitasking is then superior even though it leads to a distortion of effort across tasks. Moreover, if the performance measure is sufficiently distortive, the principal may even exclude the less important task from the agent's job so that this task is no longer carried out. Altogether, our model thus implies that the existence of wage floors may lead to broader task assignments with an inefficient effort allocation across tasks and the employment of fewer workers within the firm. These insights are based solely on incentive considerations in production environments with multiple tasks and imperfect performance measures.

In our analysis, we assume that there is one exogenously given distortive per-

[37] In this section, we exclusively refer to our basic model with technologically independent tasks. As we have shown in the previous section, the case of interdependent tasks leads to similar results with respect to optimal job design. However, specialization then does not necessarily result in first-best effort levels in the absence of wage floors, e.g., if tasks are complements.

formance measure, accounting for the fact that most available performance signals are affected by more than a single task and do not perfectly reflect an agent's true contribution. In practice, however, firms can often improve the precision of performance evaluation by spending resources on additional performance measures. Returning to our introductory example, fast-food chains could for instance measure the restaurant's 'cleanliness' in addition to store profits. In this respect, our results imply that the existence of wage floors may make it more desirable for the firm to invest in performance measurement. To see this, assume that, in our model, the principal can now generate an additional, costly performance measure. If she does so, she can induce first-best effort under broad task assignment by appropriately weighting the two performance measures in an incentive contract. However, suppose that, in the absence of a wage floor, the principal finds it optimal to rely only on the distortive performance measure P and, consequently, favors specialization.[38] With large wage floors, however, specialization becomes too expensive relative to multitasking. Then, the efficiency loss under broad task assignment due to a distorting performance measure may exceed the costs of generating the additional performance signal. Fast-food chains may thus find it optimal to incur the costs for an assessment of the restaurant's 'cleanliness' because these investments pay off in the presence of minimum wages.

Moreover, our results highlight that the relative size of the wage floor compared to the agents' reservation utility is decisive for optimal job design. In particular, the detrimental effects of a wage floor may be diminished by good outside opportunities on the side of the agents. Sufficiently large reservation utilities, however, also lead to multitasking and, thus, inefficient effort allocation across tasks even if wage floors are absent or do not actively restrict the firm's contracting problem. This may explain why, in practice, we often observe excessive workloads for higher-level employees such as managers or researchers, for which wage floor restrictions such as a minimum wage are typically not relevant.

Our findings apply to a large variety of jobs. Frequently, if not typically, performance measures are affected by different tasks but do not perfectly reflect the tasks' true contributions to firm value. In addition, examples of wage floors prevail. Below we discuss the case of minimum wages in more detail. Moreover, in many occupational sectors, collective bargaining agreements guarantee associated workers standard wages. In addition, civil service pay for officers, teachers, or professors

[38] Formally, this means that the costs of creating the additional performance measure exceed an agent's reservation utility u.

prescribes mandatory wage floors for different career levels. Another important real-world application of wage floors are liability limits due to wealth constraints, law, or private contractual agreements. Such limits on personal liability are of particular importance for upper management.

A wage-floor example of high empirical (and political) relevance is the legal minimum wage.[39] Moreover, a natural interpretation of an agent's reservation utility u in our model is the worker's unemployment benefit. Since we consider a one-period model, this is saying that, if the worker rejects the firm's contract offer at the beginning of the period, he cannot find a new job within the same period. His reservation utility is therefore equal to the unemployment benefit. This assumption is reasonable if the considered period is sufficiently short and there is some unemployment on the labor market. In the following, we briefly discuss implications of our model with respect to the different effects of changes in minimum wages and unemployment benefits on the work organization in firms.[40]

Our results show that the firm can never achieve the first-best allocation if performance measures are imperfect. However, if there is no minimum wage and unemployment benefits are not too large, the firm nevertheless implements first-best effort levels by employing two specialized workers (compare Proposition 6). In that situation, the introduction of a minimum wage has a quite different effect on work organization than an increase in the unemployment benefit: The establishment of a minimum wage that forces the firm to increase workers' fixed wages necessarily implies that the firm also induces an inefficient effort allocation. By contrast, a higher unemployment benefit does not destroy effort efficiency as long as it remains below a certain threshold (i.e., inequality (14) in Proposition 6 still holds). The reason is that the two instruments affect the firm's incentive contracting problem in fundamentally different ways: The minimum wage stipulates a lower bound on *ex-post* payments to workers whereas the unemployment benefit determines the minimum *ex-ante* expected payment to a worker. Thus, when the unemployment benefit increases, the firm can avoid higher (expected) wage costs only by employing fewer workers. However, dismissing a worker inevitably entails a distortion of effort across tasks. The extent of this distortion is exogenously

[39] In 1938, the first minimum wage was introduced in the United States. Although the minimum wage has always been a controversial policy, by now the majority of countries have legally implemented some form of minimum wage.

[40] We would like to emphasize that, due to our model assumptions, these insights primarily apply to moral-hazard environments with multiple tasks and imperfect performance measures at the firm level.

given by the quality of the performance measure (i.e., the size of the angle θ). The firm therefore refrains from dismissals for sufficiently small increases of the unemployment benefit. By contrast, increased wage costs due to a binding lower bound on ex-post payments can and will always be counteracted by adjustments in the incentive system, even if both workers stay employed: If the firm is forced to raise its payment for low performance due to the introduction of a wage floor, it will optimally respond by decreasing the workers' rewards for high performance, implying inefficiently low effort incentives.

Hence, from an efficiency perspective, minimum wages exhibit a major disadvantage relative to labor market instruments stipulating ex-ante but not ex-post payments to workers as the former may alter the power of implemented incentives. From a regulatory perspective, however, not only efficiency but also employment and individual welfare effects are important. The commonly stated primary goals of a minimum wage are correcting for market inefficiencies due to, e.g., monopsonistic power or informational asymmetries and reducing earnings inequality and poverty by supporting low-income groups of the population.[41] Whether the minimum wage is indeed effective in achieving these goals is a matter of frequent discussion as well as theoretical and empirical investigation.[42] Though our analysis does not allow to draw conclusions at an aggregate level, it contributes to this debate by highlighting an important trade-off with respect to employment and welfare effects at the firm level: As a consequence of the firms' optimal job design and motivational concerns, workers may earn rents under a minimum wage. They, however, also face the threat of dismissal once the minimum wage is sufficiently large since the firm might then abandon a specialized job regime. Presuming that the government imposes a legal minimum wage in an attempt to achieve a fairer distribution of wealth within the society, i.e., to raise the wellbeing of affected workers, requires evaluating the minimum wage's effect also with respect to distributional concerns.[43] Due to the above-mentioned trade-off, such an evaluation

[41] Compare Boeri and van Ours (2008), p. 46.

[42] For comprehensive overviews of the theory, the politics, and empirical evidence on the minimum wage see, e.g., Card and Krueger (1995) or Neumark and Wascher (2008). In addition, there is a large body of empirical research on the overall employment effects of minimum wages. For a review see Neumark and Wascher (2007). Important studies include Baker, Benjamin, and Stanger (1999) for Canada, Stewart (2004) for Great Britain, or Neumark, Schweitzer, and Wascher (2004) for the US. The seminal article by Card and Krueger (1994) on minimum wages in fast-food restaurants in New Jersey and Pennsylvania triggered an instructive debate, see the comment by Neumark and Wascher (2000) and the reply by Card and Krueger (2000). For the recent debate in Germany see Knabe and Schöb (2008, 2009).

[43] Boeri and van Ours (2008) state that *"the strongest arguments in favor of an increase in the*

is nontrivial in our model. However, as we will argue in the following, our results indicate that, at the firm level, it may sometimes be preferable to increase unemployment benefits rather than implementing or increasing a minimum wage from both perspectives, efficiency and the workers' wellbeing.

To see this, consider a situation in which two tasks are important for production and the two policy measures are absent, i.e., there are zero unemployment benefits, $u_0 = 0$, and no wage restrictions. According to our model, the firm then induces first-best effort by offering two specialized jobs but the workers' net income is zero. Introducing a minimum wage $w \geq 0$ immediately destroys one of the jobs, implying inefficient effort levels. The dismissed worker receives zero unemployment benefit, $u_0 = 0$, whereas the employed worker obtains a positive rent. Now suppose that, instead of introducing a minimum wage, unemployment benefits are increased to a level $u_1 > 0$, where u_1 corresponds to the employed worker's rent under the positive minimum wage. Then two cases can arise. In case (i), u_1 is so small that both workers stay employed, obtain an expected net income of u_1, *and* effort efficiency is maintained (i.e., u_1 satisfies condition (14)). In case (ii), u_1 is so large large that the firm dismisses one of the workers (i.e., condition (14) is violated). However, the employed worker still receives a wage yielding him u_1 in expected terms while the unemployed worker gets unemployment benefits that also amount to u_1.

Thus, in case (i) increasing the unemployment benefit is preferable both from the perspective of sustaining an efficient effort allocation and to increase the workers' welfare. In case (ii), the effort allocation is inefficient, but the workers are not worse off than under the situation with a minimum wage. To sum up, in the context of our model, increasing unemployment benefits comes at no additional social costs as long as the unemployment benefit is moderate, since the workers stay employed and their income is financed by the firm (case (i)). Importantly, the sheer existence of non-zero unemployment benefits induces the firm to reward the workers accordingly and results in a more even distribution of the surplus within the firm.

Altogether, our study is useful to identify some important trade-offs that arise in response to mandatory wage floors; these insights build on the fact that profit-maximizing firms try to motivate their workers by implementing optimal incentive schemes, given all the restrictions they face. Our findings may help explain the lack of consensus among politicians and scientists with respect to the effectiveness

minimum wage rely on equity considerations." (p. 45) of a minimum wage in achieving its declared goals such as correcting for market inefficiencies and reducing earnings inequality and poverty. We contribute to the discussion by introducing asymmetric information about the workers' effort and investigating the specific characteristics of individual jobs if workers have to perform different tasks and performance measures are imperfect.

Appendix

Proof of Proposition 1. The bonus b^{PC} solves the principal's problem (II_M) if $b^{PC} \leq \widehat{b}$. By (7) and (9), the latter inequality is equivalent to $w \leq u - \frac{D}{2}$. From (II_M), the corresponding profit is

$$f^T g \cdot b^{PC} - g^T g \cdot \frac{(b^{PC})^2}{2} - u = \frac{D}{2} - u. \tag{32}$$

This profit is positive because Assumption 1 implies $u < \frac{D}{4} + \frac{D}{8} < \frac{D}{2}$. The bonus $\widehat{b}$ solves (II_M) if $b^{WC} \leq \widehat{b} < b^{PC}$. Using (7), (8), and (9), this is equivalent to $u - \frac{D}{2} < w \leq u - \frac{D}{8}$. The profit then is

$$f^T g \cdot \widehat{b} - g^T g \cdot \widehat{b}^2 - w = \sqrt{2(u-w)D} + w - 2u. \tag{33}$$

It is straightforward to check that (33) is decreasing in w. For $w = u - \frac{D}{8}$, (33) becomes $\frac{D}{4} - w$, which is positive by Assumption 1. Thus, (33) is always positive. Finally, the bonus b^{WC} is optimal if $\widehat{b} < b^{WC}$, which is equivalent to $u - \frac{D}{8} < w$. By the upper bound on u and w in Assumption 1, this case may indeed occur. Furthermore, the associated profit, $\frac{D}{4} - w$, is positive. The bonuses b^{PC} and $\widehat{b}$ both lead to a binding participation constraint (PC_M). By contrast, if the bonus b^{WC} is optimal, only the wage-floor constraint (WC_M) is binding. Thus, the agent obtains a rent if and only if b^{WC} is implemented. Using that $s = w$ and $e = b \cdot g$, the rent can be computed as the agent's expected wage payment net of effort costs and his reservation utility: $w + \frac{g^T g}{2}(b^{WC})^2 - u = w + \frac{D}{8} - u$. ■

Proof of Proposition 2. Assume that the agent performs task i, while task $j \neq i$

is excluded from the job. Then, the principal's optimization problem is

$$\max_{e_i,s,b} \quad f_i e_i - s - g_i e_i \cdot b \tag{34}$$

$$e_i = \arg\max_{\widehat{e}_i} s + g_i \widehat{e}_i \cdot b - \frac{1}{2}\widehat{e}_i^2 \tag{35}$$

$$s + g_i e_i \cdot b - \frac{1}{2} e_i^2 \geq u \tag{36}$$

$$s, s + b \geq w \tag{37}$$

This problem can be transformed analogously to the principal's problem under broad task assignment, (I_M)-(WC'_M), yielding

$$\max_b \quad \left[f_i g_i \cdot b - \max\left\{ u + g_i^2 \frac{b^2}{2}, w + g_i^2 b^2 \right\} \right]. \tag{38}$$

The principal optimally implements the bonus

$$b(f_i, u, w) = \begin{cases} \frac{f_i}{g_i} & \text{if } \ w \leq u - \frac{f_i^2}{2} \\ \frac{\sqrt{2(u-w)}}{g_i} & \text{if } \ u - \frac{f_i^2}{2} < w \leq u - \frac{f_i^2}{8} \\ \frac{f_i}{2g_i} & \text{if } \ u - \frac{f_i^2}{8} < w \end{cases} \tag{39}$$

and earns the profit

$$\pi(f_i, u, w) = \begin{cases} \frac{f_i^2}{2} - u & \text{if } \ w \leq u - \frac{f_i^2}{2} \\ \sqrt{2(u-w)} f_i + w - 2u & \text{if } \ u - \frac{f_i^2}{2} < w \leq u - \frac{f_i^2}{8} \\ \frac{f_i^2}{4} - w & \text{if } \ u - \frac{f_i^2}{8} < w \end{cases}. \tag{40}$$

The agent earns a rent if and only if $u - \frac{f_i^2}{8} < w$. Using that $s = w$, $e_i = b g_i = \frac{f_i}{2}$, the rent can be computed as the agent's expected wage payment net of effort costs and his reservation utility: $w + g_i e_i b - \frac{e_i^2}{2} - u = w + \frac{f_i^2}{8} - u$.

Next, we prove that $\pi(f_i, u, w)$ is increasing in f_i, or, equivalently,

$$\pi(f_1, u, w) \geq \pi(f_2, u, w) \ \text{ for all } w\text{, holding } u \text{ constant.} \tag{41}$$

Inequality (41) obviously holds for $w \leq u - \frac{f_1^2}{2}$, $u - \frac{f_2^2}{2} < w \leq u - \frac{f_1^2}{8}$, and $u - \frac{f_2^2}{8} < w$. Now consider the case $w \in \left(u - \frac{f_1^2}{2}, u - \frac{f_2^2}{2}\right] = A$. Since π is continuous in w, we have $\pi(f_1, u, -\frac{f_2^2}{2}) \geq \pi(f_2, u, u - \frac{f_2^2}{2})$. Furthermore, $\pi(f_1, u, w)$ is strictly decreasing in w on the interval A while $\pi(f_2, u, w)$ is constant. Thus, (41) holds on the entire interval A. Finally, consider the case $w \in \left(u - \frac{f_1^2}{8}, u - \frac{f_2^2}{8}\right] = B$. Inequality (41) holds for $w = u - \frac{f_1^2}{8}$ and $w = u - \frac{f_2^2}{8}$. Thus, because both $\pi(f_1, w)$ and $\pi(f_2, w)$ are strictly decreasing in w on B, (41) holds on the entire interval B. Therefore, (41) always holds and the principal prefers exclusion of task 2 to exclusion of task 1.

The principal prefers exclusion of task 2 to broad task assignment if $\pi(f_1, u, w) \geq \pi^M(u, w)$. Comparing ($\pi^M$) and ($\pi^E$), using that $\pi(f_1, u, w)$ is increasing in f_1, yields that exclusion of a task is optimal if and only if $f_1^2 \geq D$ or $f_1 \geq ||f|| \cdot \cos\theta$. ■

Proof of Proposition 3. For any given effort pair (e_1, e_2), the principal's wage payments to the agents must be at least as high as the agents' effort costs and reservation utilities. Thus, an upper bound for the principal's expected profit is given by

$$\max_{e_1,e_2} f_1 e_1 + f_2 e_2 - \frac{1}{2}e_1^2 - \frac{1}{2}e_2^2 - 2u = \frac{||f||^2}{2} - 2u. \tag{42}$$

This upper bound is attained for $e = e^{FB} = f$. By (II$_S$), the principal is able to realize this profit if

$$u + \frac{1}{2}f_1^2 \geq w + f_1^2 + \frac{g_2}{g_1}f_1 f_2 \text{ and } u + \frac{1}{2}f_2^2 \geq w + f_2^2 + \frac{g_1}{g_2}f_1 f_2. \tag{43}$$

This is equivalent to condition (FB). Thus, if (FB) holds, then e^{FB} is implemented. If (FB) does not hold, then e^{FB} lies in area A_2, A_3, or A_4 of Figure 2. If e^{FB} lies in the interior of one of these areas, it is straightforward to verify that e^{FB} does not satisfy the first-order conditions for a profit-maximizing effort level in the interior of the respective area. Thus, e^{FB} cannot be a solution to (II$_S$). It remains to consider the case where e^{FB} lies on the boundary of A_4 and A_j, $j \in \{2, 3\}$. First assume that e^{FB} lies on the boundary of A_2 and A_4. Then, inducing $e' = (f_1 - \varepsilon, f_2)^T$ with ε sufficiently close to zero would lead to a higher profit than inducing e^{FB} because the principal's profit is decreasing in e_1 at $e = e^{FB}$. Similarly, if e^{FB} lies on the boundary of A_3 and A_4, the principal prefers $e'' = (f_1, f_2 - \varepsilon)^T$ to e^{FB}. ■

Proof of Proposition 4. If $w \geq u$, the principal's problem (II$_S$) can be simplified to

$$\max_{e_1,e_2} \quad f_1 e_1 + f_2 e_2 - e_1^2 - e_2^2 - \frac{g_1^2 + g_2^2}{g_1 g_2} e_1 e_2 - 2w. \tag{44}$$

First, consider the case $g_1 = g_2$. Then, $\frac{g_1^2+g_2^2}{g_1 g_2} = 2$ and the principal's objective function becomes

$$\max_{e_1,e_2} \quad f_1e_1 + f_2e_2 - (e_1 + e_2)^2 - 2w. \tag{45}$$

If $f_1 = f_2$, every combination of effort levels (e_1, e_2) such that $e_1 + e_2 = f_1/2$ maximizes the principal's profit. Thus, $e_1 = f_1/2$ and $e_2 = 0$ constitute an optimal effort pair. If $f_1 > f_2$, we have a corner solution with $e_1 = f_1/2$ and $e_2 = 0$. The profit is always $\pi^S(u, w) = \frac{1}{4}f_1^2 - 2w$. Now assume that $g_1 \neq g_2$. Then, $\frac{g_1^2+g_2^2}{g_1g_2} > 2$. Hence, for any given pairs (e_1, e_2) and (f_1, f_2), the principal's profits are (weakly) lower than for the case $g_1 = g_2$. However, the principal can still realize profit $\pi^S = \frac{1}{4}f_1^2 - 2w$ by inducing $e_1 = f_1/2$ and $e_2 = 0$. Thus, these effort levels are again optimal. ■

Proof of Proposition 5. First, consider the case where under the solution to (II_S) at least one effort level is zero. Then, we can see from (II_S) that the agent exerting zero effort does not obtain a rent because $u > w$. Now consider the situation where a solution to (II_S) implies positive effort in both tasks. (i) The proof is by contradiction. Suppose the principal induces an effort pair $(\tilde{e}_1, \tilde{e}_2)$ for which both agents earn a rent. Then, $(\tilde{e}_1, \tilde{e}_2)$ must be in the interior of A_4 in Figure 2. Thus, $(\tilde{e}_1, \tilde{e}_2)$ is a local maximum of the function

$$\pi(e_1, e_2) = f_1e_1 + f_2e_2 - e_1^2 - e_2^2 - \frac{g_1^2 + g_2^2}{g_1g_2}e_1e_2 - 2w \tag{46}$$

and therefore needs to satisfy the first-order conditions

$$f_1 - 2\tilde{e}_1 - G\tilde{e}_2 = 0, \tag{47}$$

$$f_2 - G\tilde{e}_1 - 2\tilde{e}_2 = 0, \tag{48}$$

where $G := \frac{g_1^2+g_2^2}{g_1g_2}$. First assume that $G > 2$, implying that $g_1 \neq g_2$. Then, the Hessian of $\pi(e_1, e_2)$ is indefinite because it has a negative eigenvalue, $-(2+G)$, and a positive one, $-2 + G$. Thus, $(\tilde{e}_1, \tilde{e}_2)$ cannot be a local maximum of $\pi(e_1, e_2)$ and we obtain a contradiction. If $G = 2$, then $g_1 = g_2$ and, consequently, $f_1 > f_2$. Thus, the first-order conditions cannot be satisfied and we again obtain a contradiction. (ii) If $f_1 = f_2$ and $g_1 = g_2$, then all (e_1, e_2) with $e_1 + e_2 = f_1/2$ are a local maximum of $\pi(e_1, e_2)$. Thus, if $u - w$ is sufficiently close to zero, there are optimal solutions to (II_S) for which both effort levels are positive and both agents earn a rent. However, there are two pairs of effort levels on the boundary of A_4, which also satisfy $e_1 + e_2 = f_1/2$ and constitute optimal solutions. For these effort pairs, only one agent earns a strictly positive rent. ■

Proof of Proposition 6. Assume that (FB) holds. Then, by Proposition 3, the principal's profit under specialization is $\frac{1}{2}||f||^2 - 2u$. Using Propositions 1 and 2, we now determine the profit if the principal employs only one agent with one or two assigned tasks. To do so, we show that (i) $w \leq u - \frac{f_1^2}{2}$ and (ii) $w \leq u - \frac{D}{2}$, i.e., the agent's participation constraint is binding. From (FB), we obtain

$$w \leq u - \left(\frac{1}{2}f_1^2 + \frac{g_2}{g_1}f_1 f_2\right). \tag{49}$$

Because $\frac{1}{2}f_1^2 + \frac{g_2}{g_1}f_1f_2 > \frac{1}{2}f_1^2$, (i) follows immediately. Moreover, it holds that $\frac{1}{2}f_1^2 + \frac{g_2}{g_1}f_1f_2 > \frac{D}{2}$, because the latter inequality can be transformed to

$$(f_1^2 - f_2^2)g_2^2 + 2f_1f_2\frac{g_2^3}{g_1} > 0, \tag{50}$$

which is satisfied because $f_1 \geq f_2$. Hence, we obtain (ii). Consequently, by equation (π^M) in Proposition 1 and equation (π^E) in Proposition 2, the profit with one agent is $\max\left\{\frac{D}{2} - u,\ \frac{f_1^2}{2} - u\right\}$. Therefore, specialization is preferred to hiring only one agent if

$$\max\left\{\frac{D}{2} - u,\ \frac{f_1^2}{2} - u\right\} \leq \frac{1}{2}||f||^2 - 2u, \tag{51}$$

which is equivalent to (14). ■

Proof of Proposition 7. Assume that $w \geq u$. Then, by Proposition 4, profit under specialization is $\pi^S(u, w) = \frac{f_1^2}{4} - 2w$. However, the principal can do better by hiring only one agent and exclude task 2 from the job, thereby earning the profit $\frac{f_1^2}{4} - w$. Doing so is preferred to broad task assignment if $\frac{f_1^2}{4} - w \geq \pi^M$. By equation ($\pi^M$) in Proposition 1, this inequality is equivalent to

$$\frac{1}{4}f_1^2 - w \geq \frac{D}{4} - w \Leftrightarrow f_1^2 \geq D = ||f||^2 \cos^2\theta.$$

■

Proof of Proposition 8. If specialization entails a strictly higher expected profit than multitasking, the optimal effort levels under specialization, (e_1^*, e_2^*), must both be positive. Otherwise, multitasking can induce the same expected profit by excluding the task with zero effort from the agent's job. We characterize all cases where, under specialization, it is optimal to pay a rent to one of the agents. Then, we show that hiring only one agent always leads to a higher expected profit. Define $q := \frac{g_2}{g_1}$.

Assume that, under specialization, agent 1 earns a rent. Then, the optimal effort levels (e_1^*, e_2^*) must be an interior solution to

$$\max_{e_1 \geq 0, e_2 \geq 0} \; f_1 e_1 + f_2 e_2 - (e_1^2 + q e_1 e_2 + w) - \left(\frac{e_2^2}{2} + u \right) \tag{52}$$

and, furthermore, there must be an u and w such that

$$e_1^{*2} + q e_1^* e_2^* + w > \frac{e_1^{*2}}{2} + u \;\text{ and }\; e_2^{*2} + q^{-1} e_1^* e_2^* + w \leq \frac{e_2^{*2}}{2} + u \tag{53}$$

$$\Leftrightarrow \frac{e_2^{*2}}{2} + q^{-1} e_1^* e_2^* < u - w \leq \frac{e_1^{*2}}{2} + q e_1^* e_2^* \tag{54}$$

A necessary condition for the last inequality being satisfied is that

$$(q^{-1} - q) e_1^* e_2^* < \frac{1}{2}(e_1^{*2} - e_2^{*2}). \tag{55}$$

The first-order conditions of (52) are

$$f_1 - 2e_1 - q e_2 = 0 \;\text{ and }\; f_2 - q e_1 - e_2 = 0, \tag{56}$$

yielding

$$e_1 = \frac{f_1 - q f_2}{2 - q^2}, \quad e_2 = \frac{2 f_2 - q f_1}{2 - q^2}. \tag{57}$$

The Hessian of the objective function (52) is negative definite if $q < \sqrt{2}$, indefinite if $q > \sqrt{2}$, and negative semidefinite if $q = \sqrt{2}$. Thus, if $q < \sqrt{2}$, then (57) constitutes a maximum of (52). If $q > \sqrt{2}$, then (52) does not have an interior solution. Finally, if $q = \sqrt{2}$ and $f_1 \neq \sqrt{2} f_2$, then again (52) does not have an interior solution. If $q = \sqrt{2}$ and $f_1 = \sqrt{2} f_2$, then all (e_1, e_2) with $e_1 = \frac{1}{\sqrt{2}}(f_2 - e_2)$ constitute a maximum of (52) and yield a profit of $\frac{1}{4} f_1^2 - u - w$. However, by (π^E) employing only one agent will lead to at least the same profit. We therefore from now on assume that $q < \sqrt{2}$. Because $e_i^* > 0$, $i = 1, 2$, we obtain from (57)

$$e_1^* = \frac{f_1 - q f_2}{2 - q^2}, \quad e_2^* = \frac{2 f_2 - q f_1}{2 - q^2} \quad \text{for } \; q < \frac{f_1}{f_2} < \frac{2}{q}. \tag{58}$$

The associated profit can be computed as

$$\pi^S(u,w) = \frac{f_2^2 + \frac{1}{2}f_1^2 - qf_1f_2}{2-q^2} - u - w. \tag{59}$$

Now consider multitasking. We first show that the principal will always assign both tasks to the agent. By Proposition 2, this is the case if and only if:

$$f_1 < ||f|| \cdot \cos\theta = \frac{f_1g_1 + f_2g_2}{\sqrt{g_1^2+g_2^2}} \tag{60}$$

$$\Leftrightarrow f_1^2(g_1^2+g_2^2) < (f_1g_1)^2 + 2f_1f_2g_1g_2 + (f_2g_2)^2 \tag{61}$$

$$\Leftrightarrow f_1^2 g_2 < 2f_1f_2g_1 + f_2^2 g_2 \tag{62}$$

$$\Leftrightarrow \frac{f_1}{f_2} < 2\frac{g_1}{g_2} + \frac{f_2}{f_1} \tag{63}$$

$$\Leftrightarrow \frac{f_1}{f_2} < \frac{2}{q} + \frac{f_2}{f_1} \tag{64}$$

The last inequality holds because of (58). Thus, the principal's profit under multitasking is given by (π^M). We now show that $\pi^M(u,w) \geq \pi^S(u,w)$ for all u and w. First, we note that D can be transformed to $D = \frac{(f_1+qf_2)^2}{1+q^2}$. For $u - \frac{D}{8} < w$, we need to verify that

$$\frac{1}{4}\frac{(f_1+qf_2)^2}{1+q^2} - w \geq \frac{f_2^2 + \frac{1}{2}f_1^2 - qf_1f_2}{2-q^2} - u - w. \tag{65}$$

Since $u \geq 0$, it suffices to show that

$$\frac{1}{4}\frac{(f_1+qf_2)^2}{1+q^2} \geq \frac{f_2^2 + \frac{1}{2}f_1^2 - qf_1f_2}{2-q^2}. \tag{66}$$

Some transformations show that this inequality is satisfied if and only if

$$-Hf_2 \leq f_1 - \frac{1}{3}\frac{4+q^2}{q}f_2 \leq Hf_2 \tag{67}$$

where

$$H := \sqrt{\frac{1}{9}\left(\frac{4+q^2}{q}\right)^2 - \frac{1}{3}\frac{4+2q^2+q^4}{q^2}}. \tag{68}$$

The second inequality in (67) is equivalent to

$$\frac{f_1}{f_2} \leq H + \frac{1}{3}\frac{4+q^2}{q}. \tag{69}$$

Because $\frac{f_1}{f_2} < \frac{2}{q}$, the last inequality holds if $0 \leq H + \frac{1}{3}\frac{4+q^2}{q} - \frac{2}{q}$ for all $q \in (0, \sqrt{2})$. Plotting shows that this is true. The first inequality in (67) is equivalent to

$$-H + \frac{1}{3}\frac{4+q^2}{q} \leq \frac{f_1}{f_2}. \tag{70}$$

First consider the case $q > 1$. Because $q < \frac{f_1}{f_2}$, inequality (70) is satisfied if $0 \leq q + H - \frac{1}{3}\frac{4+q^2}{q}$ for all $q \in (1, \sqrt{2})$. Plotting again shows that this is true. Now assume that $q \leq 1$. Then, for (55) to hold, we need $e_1^* > e_2^* \Leftrightarrow \frac{2+q}{1+q} < \frac{f_1}{f_2}$. Since $q < \frac{2+q}{1+q} < \frac{2}{q}$, we can restrict $\frac{f_1}{f_2}$ further to $\frac{2+q}{1+q} < \frac{f_1}{f_2} < \frac{2}{q}$. Moreover, $\frac{2+q}{1+q}$ is decreasing in q and, thus, $\frac{2+q}{1+q} > \frac{2+\sqrt{2}}{1+\sqrt{2}} = \sqrt{2}$. Thus, with $q \leq 1$ it follows that $\sqrt{2} < \frac{f_1}{f_2} < 2 \Leftrightarrow \frac{1}{2} < \frac{f_2}{f_1} < \frac{1}{\sqrt{2}}$. We want to establish a lower bound on q. Inequality (55) is equivalent to

$$q^{-1} - q < \frac{1}{2}\frac{e_1^{*2} - e_2^{*2}}{e_1^* e_2^*} =: R(q, r), \quad \text{where } r = \frac{f_2}{f_1}. \tag{71}$$

It can be shown that $R(q, r)$ is strictly increasing in q for $r < \frac{1}{\sqrt{2}}$. Furthermore, $z(q) = q^{-1} - q$ is strictly decreasing in q with $z(0) = \infty$ and $z(1) = 0$. Thus, there is a unique $\bar{q} \in (0, 1)$ such that inequality (71) holds for $q \geq \bar{q}$. It can be shown that $\bar{q} > 0.6$. Plotting shows that $0 \leq \frac{2+q}{1+q} + H - \frac{1}{3}\frac{4+q^2}{q}$ for all $q \in (0.6, 1]$. Hence, (70) holds.

For $u - \frac{D}{2} < w \leq u - \frac{D}{8} \Leftrightarrow \frac{D}{8} \leq u - w < \frac{D}{2}$, we need to verify that

$$\sqrt{2(u-w)D} - (u - w) \geq \frac{f_2^2 + \frac{1}{2}f_1^2 - qf_1f_2}{2 - q^2} - w. \tag{72}$$

The left-hand side of this inequality is increasing in $u - w$. Thus, it is lowest for $u - w = \frac{D}{8}$, when it equals $\frac{3D}{8}$. It is therefore sufficient to show that

$$\frac{3D}{8} = \frac{1}{4}\frac{(f_1 + qf_2)^2}{1+q^2} + \frac{1}{8}\frac{(f_1 + qf_2)^2}{1+q^2} \geq \frac{f_2^2 + \frac{1}{2}f_1^2 - qf_1f_2}{2 - q^2} - w. \tag{73}$$

By (65), the last inequality holds if $\frac{1}{8}\frac{(f_1+qf_2)^2}{1+q^2} \geq -w$. From (53), we know that

$w > u - \frac{1}{2}e_1^{*2} - qe_1^*e_2^*$. Since $u \geq 0$, it suffices to show that

$$\frac{1}{8}\frac{(f_1+qf_2)^2}{1+q^2} - \frac{1}{2}\left(\frac{f_1-qf_2}{2-q^2}\right)^2 - q\frac{f_1-qf_2}{2-q^2}\frac{2f_2-qf_1}{2-q^2} \geq 0. \tag{74}$$

This inequality can be simplified to

$$\frac{q^2(f_2(4+q^2)-3f_1q)^2}{8(q^2-2)^2(1+q^2)} \geq 0, \tag{75}$$

which clearly holds.

For $w \leq u - \frac{D}{2}$, multitasking dominates specialization if

$$\frac{D}{2} \geq \frac{f_2^2 + \frac{1}{2}f_1^2 - qf_1f_2}{2-q^2} - w, \tag{76}$$

which holds because we verified (73).

It remains to consider the case where agent 2 but not agent 1 earns a rent. Then, the optimal effort levels (e_1^*, e_2^*) must be an interior solution to

$$\max_{e_1\geq 0, e_2\geq 0} \; f_1e_1 + f_2e_2 - \left(\frac{e_1^2}{2} + u\right) - (e_2^2 + q^{-1}e_1e_2 + w) \tag{77}$$

and, furthermore, there must be an u and w such that

$$e_2^{*2} + q^{-1}e_1^*e_2^* + w > \frac{e_2^{*2}}{2} + u \;\text{ and }\; e_1^{*2} + qe_1^*e_2^* + w \leq \frac{e_1^{*2}}{2} + u \tag{78}$$

$$\Leftrightarrow \frac{e_1^{*2}}{2} + qe_1^*e_2^* \leq u - w < \frac{e_2^{*2}}{2} + q^{-1}e_1^*e_2^* \tag{79}$$

The first-order conditions of (77) are

$$f_1 - e_1 - q^{-1}e_2 = 0 \;\text{ and }\; f_2 - q^{-1}e_1 - 2e_2 = 0, \tag{80}$$

yielding

$$e_1^* = \frac{2f_1 - q^{-1}f_2}{2-q^{-2}}, \quad e_2^* = \frac{f_2 - q^{-1}f_1}{2-q^{-2}} \;\text{ for } q^{-1} < \sqrt{2} \;\text{ and }\; \frac{1}{2q} < \frac{f^1}{f^2} < q. \tag{81}$$

Consider the case $q > 1$. For (79) to be satisfied, it is necessary that

$$e_2^* > e_1^* \Leftrightarrow f_2 - q^{-1}f_1 > 2f_1 - q^{-1}f_2 \Leftrightarrow (1+q)f_2 > (1+2q)f_1. \tag{82}$$

The last inequality cannot hold because $f_1 \geq f_2$. Now consider the case $q \leq 1$. Then, $\frac{f^1}{f^2} < q$ in (81) cannot hold. Thus, it is not possible that, at $e_i^* > 0$, agent 2 earns a rent but not agent 1. ■

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