Online Appendix
"Licensing Process Innovations when Losers' Messages Determine Royalty Rates"

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Introduction

This online appendix has two parts:

Part A A collection of technical proofs that are not spelled out in the paper

Part B An extension of our analysis to three firms, to show how our analysis can be generalized to more than two firms.

References without prefix, such as a reference to equation (1) or lemma 1, refer to the paper; references with a prefix, such as a reference to equation (A1) or lemma A1, refer to this appendix.

A Proofs of Lemmas and Propositions

A.1 Proof of Lemma 1

Proof. First, we show that $\pi_W'(x) > 0$ and $\pi_L^{*'}(y) < 0$. Using the envelope theorem and the fact that $q_{L_2}'(x) < 0$ and $q_{W_2}^{*'}(y) > 0$ one has:

$$\pi_W'(x) = P'(\cdot)q_{L_2}'(x)q_{W_1}(x) + q_{W_1}(x) > 0$$
$$\pi_L^{*'}(y) = P'(\cdot)q_{W_2}^{*'}(y)q_L^*(y) < 0.$$

Next, we show that $\partial_z \pi_L(x,z,y)|_{z=x} = -q_L^*(y)\gamma(y)$, where $\gamma(y) > 1$ for all y.

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By the envelope theorem we have

$$
\begin{aligned}
\partial_z \pi_L(x,z,y)|_{z=x} &= -\, q_{L_1}(x,z,y)|_{z=x} \Big(1 - P'(q_{W_2}(\cdot) + q_{L_1}(\cdot))\partial_z q_{W_2}(x,z,y)\Big)\Big|_{z=x} \\
&= -q_L^*(y)\gamma(y) \\
\text{where} \quad \gamma(y) :&= 1 - \Big(P'(q_{W_2}(\cdot) + q_{L_1}(\cdot))\partial_z q_{W_2}(x,z,y)\Big)\Big|_{z=x} \\
&> 1 \quad (\text{because } P' < 0 \text{ and } \partial_z q_{W_2}(x,z,y)|_{z=x} > 0).
\end{aligned}
$$

Note, both $P'(\cdot)$ and $\partial_z q_{W_2}(x,z,y)|_{z=x}$ are only functions of y.

If demand is linear, $P'(\cdot) = -1,\ \partial_z q_{W_2}(x,z,y)|_{z=x} = 1/3$; hence, $\gamma(y) = 4/3$ (see Section A.9). $\square$

A.2 Part 3 of the proof of Proposition 2

Compute $\beta'(x)$ from (10) for $x \geq r$. By Lemma 1 and the assumed log-concavity of the reliability function, which implies that $f'(x) > -f(x)^2/(1-F(x))$, one has

$$
\begin{aligned}
\beta'(x) &= (\pi_W'(x) - \pi_L^{*\prime}(x)) + \frac{d}{dx}\left(\frac{1}{f(x)}\int_x^c \partial_z \pi_L(x,z,y)|_{z=x}\, dF(y)\right) \\
&> (\pi_W'(x) - \pi_L^{*\prime}(x)) + q_L^*(x)\gamma(x) - \frac{1}{1-F(x)}\int_x^c q_L^*(y)\gamma(y) dF(y).
\end{aligned}
$$

There, $\pi_W(x)$ and $\pi_L^*(x)$ are the equilibrium profits in the Cournot subgames when the winner's cost reduction is x:

$$
\begin{aligned}
\pi_W(x) &= \max_q \,(P(q + q_L^*(x)) - c + x)\, q, \\
\pi_L^*(x) &= \max_q \,(P(q_W^*(x) + q) - c)\, q,
\end{aligned}
$$

and $q_W^*(x)$ and $q_L^*(x)$ are the corresponding equilibrium outputs.[1]

The first–order conditions of the above maximization problem are:

$$
\begin{aligned}
P'(q_W^*(x) + q_L^*(x))q_W^*(x) + P(q_W^*(x) + q_L^*(x)) - c + x = 0 \\
P'(q_W^*(x) + q_L^*(x))q_L^*(x) + P(q_W^*(x) + q_L^*(x)) - c = 0.
\end{aligned} \tag{A.1}
$$

Differentiating (A.1) w.r.t. x, one obtains

$$
\begin{aligned}
\big(P''(\cdot)q_W^*(x) + P'(\cdot)\big)\,(q_W^{*'}(x) + q_L^{*'}(x)) + P'(\cdot)q_W^{*'}(x) = -1 \\
\big(P''(\cdot)q_L^*(x) + P'(\cdot)\big)\,(q_W^{*'}(x) + q_L^{*'}(x)) + P'(\cdot)q_L^{*'}(x) = 0,
\end{aligned}
$$

from which one can derive

$$
\begin{aligned}
q_W^{*'}(x) &= -\frac{2P'(\cdot) + P''(\cdot)q_L^*(x)}{P'(\cdot)(3P'(\cdot) + P''(\cdot)\,(q_W^*(x) + q_L^*(x)))} \\
q_L^{*'}(x) &= \frac{P'(\cdot) + P''(\cdot)q_L^*(x)}{P'(\cdot)(3P'(\cdot) + P''(\cdot)\,(q_W^*(x) + q_L^*(x)))}.
\end{aligned} \tag{A.2}
$$

[1]Note that $q_W^*(x) = q_{W_1}(x) = q_{W_2}(x,z,y)|_{z=x=y}, q_L^*(x) = q_{L_2}(x) = q_{L_1}(x,z,y)|_{z=x=y}$.

By the envelope theorem, one obtains

$$\begin{aligned}\pi_W'(x) &= \Big(P'(\cdot)q_L^{*'}(x)+1\Big)\, q_W^*(x)\\ \pi_L^{*\prime}(x) &= P'(\cdot)q_W^{*'}(x)q_L^*(x).\end{aligned} \tag{A.3}$$

To compute $\gamma(x)$, consider the Cournot subgame in the event when both firms' reports met the threshold level and firm 1 has lost. The first–order conditions of that subgame are:

$$\begin{aligned}P'(\cdot)q_{W_2}(\cdot)+P(\cdot)-c+y &= 0\\ P'(\cdot)q_{L_1}(\cdot)+P(\cdot)-c+x-z &= 0.\end{aligned} \tag{A.4}$$

Differentiating (A.4) w.r.t. z, one obtains

$$\partial_z q_{W_2}(x,z,y) = -\frac{P'(\cdot)+P''(\cdot)q_{W_2}(x,z,y)}{P'(\cdot)(3P'(\cdot)+P''(\cdot)\left(q_{W_2}(x,z,y)+q_{L_1}(x,z,y)\right))}. \tag{A.5}$$

Setting $z=y=x$ and using definition (8), we get

$$\gamma(x) = 1+\frac{P'(\cdot)+P''(\cdot)q_W^*(x)}{3P'(\cdot)+P''(\cdot)\left(q_W^*(x)+q_L^*(x)\right)} \tag{A.6}$$

Combining (A.2), (A.3) and (A.6), we have

$$\begin{aligned}(\pi_W'(x)-\pi_L^{*\prime}(x))+q_L^*(x)\gamma(x) &= 2q_L^*(x)+q_W^*(x)\\ &\quad+\frac{\left(P'(\cdot)+P''(\cdot)q_L^*(x)\right)q_W^*(x)}{3P'(\cdot)+P''(\cdot)\left(q_W^*(x)+q_L^*(x)\right)}\\ &> 2q_L^*(x)+q_W^*(x) \;\; (\text{because } \; P'\le 0, P'' \le 0).\end{aligned} \tag{A.7}$$

Evaluation (A.5) at $z=x$ and using (8), one obtains $\gamma(y)$, which has the same form as (A.6) with x replaced by y. Hence, we have

$$-q_L^*(y)\gamma(y) = -2q_L^*(y)+\frac{\left(2P'(\cdot)+P''(\cdot)q_L^*(y)\right)q_L^*(y)}{3P'(\cdot)+P''(\cdot)\left(q_W^*(y)+q_L^*(y)\right)} > -2q_L^*(y). \tag{A.8}$$

From (A.7) and (A.8), it follows that

$$\begin{aligned}\beta'(x) &> 2q_L^*(x)+q_W^*(x)-\frac{1}{1-F(x)}\int_x^c 2q_L^*(y)dF(y)\\ &> 2q_L^*(x)+q_W^*(x)-\frac{1}{1-F(x)}\int_x^c 2q_L^*(x)dF(y)\\ &= q_W^*(x) > 0,\end{aligned}$$

the second inequality holds because $q_L^*(\cdot)$ is a decreasing function.

A.3 Part 4 of the proof of Proposition 2

Proof. Consider a firm with cost reduction x. If that firm participates, and the other firm tells the truth, that firm's payoff is $\Pi_p(x)$, whereas if it does not participate, its payoff is $\Pi_{np}(x)$:

$$\begin{aligned}\Pi_p(x) &= F(r)\left(\pi_W(x) - R\right) + \int_r^x \left(\pi_W(x) - \beta(y)\right) dF(y) + \int_x^c \pi_L^*(y) dF(y) \\ &= -F(r)R + F(x)\pi_W(x) - \int_r^x \beta(y) dF(y) + \int_x^c \pi_L^*(y) dF(y) \\ \Pi_{np}(x) &= F(r)\pi_A + \int_r^c \pi_L^*(y) dF(y).\end{aligned}$$

Let

$$\begin{aligned}\psi(x) &:= \Pi_p(x) - \Pi_{np}(x) \\ &= -F(r)(\pi_A + R) + F(x)\pi_W(x) - \int_r^x \left(\pi_L^*(y) + \beta(y)\right) dF(y).\end{aligned}$$

Differentiate ψ with respect to x, and one obtains, using (10), for $x \geq r$:

$$\begin{aligned}\psi'(x) &= f(x)\pi_W(x) + F(x)\pi_W'(x) - \left(\pi_L^*(x) + \beta(x)\right) f(x) \\ &= F(x)\pi_W'(x) - \int_x^c \partial_z \pi_L(x,x,y) dF(y) > 0.\end{aligned}$$

By definition of r, one has $\psi(r) = 0$; hence, r is implicitly defined as the solution of (1), and we conclude that firms participate if and only if $x \geq r$. □

A.4 Proof of Proposition 3

Proof. $\Pi(x,z)$ is pseudoconcave if the cross derivative is positive:

$$\partial_{zx}\Pi(x,z) = \left(\partial_x \pi_W(x) - \partial_x \pi_L(x,z,z)\right) f(z) + \int_z^c \partial_{xz} \pi_L(x,z,y) dF(y) > 0. \tag{A.9}$$

We show that this is true if x and z are sufficiently large. Because $z \geq x \geq r$, this holds if and only if r is sufficiently large.

Let $r \to c$. Then the integral on the RHS of (A.9) vanishes; however, the first term does not vanish and is positive. Indeed, as we show below, $\lim_{x,z \to c}(\partial_x \pi_W(x) - \partial_x \pi_L(x,z,z)) > 0$. Hence, $\partial_{zx}\Pi(x,z) > 0$ for $x = z = c$. By continuity, pseudoconcavity holds true for $x, z \geq r$ if r is sufficiently large.

Finally, we prove the inequality:

$$\lim_{x,z \to c} \left(\partial_x \pi_W(x) - \partial_x \pi_L(x,z,z)\right) = \frac{(4P'(Q) + P''(Q)Q)(q_{W_1}(c) - q_{L_1}(c,c,c))}{3P'(Q) + P''(Q)Q} > 0. \tag{A.10}$$

By the envelope theorem one has

$$\partial_x \pi_W(x) = q_{W_1}(x)\left(1 + P'(Q)\partial_x q_{L_2}(x)\right) \tag{A.11}$$

$$\partial_x \pi_L(x,z,z) = q_{L_1}(x,z,z)\left(1 + P'(Q)\partial_x q_{W_2}(x,z,z)\right). \tag{A.12}$$

The profile of unit costs that underlies $\pi_W(x)$ is $(c_1, c_2) = (c - x, c)$, and the profile that underlies $\pi_L(x,z,z)$ is $(c_1, c_2) = (c - x + z, c - z)$. In each case, the sum of unit costs is equal to $2c - x$. By a well-known fact, the aggregate equilibrium output, Q, is only a function of the sum of unit costs (see Bergstrom and Varian, 1985). Therefore, Q is the same in both equations.

The first order conditions concerning the subgame that underlies the first equation are:

$$P(Q)+P'(Q)q_{W_1}(x)-(c-x)=0 \tag{A.13}$$
$$P(Q)+P'(Q)q_{L_2}(x)-c=0. \tag{A.14}$$

Differentiating (A.13) and (A.14) with respect to x and solving for $\partial_x q_{L_2}(x)$ one has

$$\partial_x q_{L_2}(x)=\frac{P'(Q)+P''(Q)q_{L_2}(x)}{P'(Q)\left(3P'(Q)+P''(Q)Q\right)}. \tag{A.15}$$

Similarly, one finds

$$\partial_x q_{W_2}(x,z,z)=\frac{P'(Q)+P''(Q)q_{W_2}(x,z,z)}{P'(Q)\left(3P'(Q)+P''(Q)Q\right)}. \tag{A.16}$$

Combining (A.11)-(A.16) and using the facts that $q_{L_1}(c,c,c)=q_{L_2}(c), q_{W_1}(c)=q_{W_2}(c,c,c)$ proves inequality (A.10). □

A.5 Proof of Proposition 4

Proof. Substituting $\pi_W(x)$, and $\pi_L(x,z,y)$ from Section A.9 into (A.9), one can easily confirm that $\forall x,z\geq r$:

$$\partial_{zx}\Pi(x,z)=\frac{4}{3}zf(z)-\frac{8}{9}(1-F(z))\geq 0 \quad\iff\quad x,z\geq r\geq r_{\min}. \tag{A.17}$$

Existence and uniqueness of $r_{\min}$ follow from the fact that the LHS of (12) in the paper is negative at $r=0$, positive at $r=c$, and strictly increasing in r by the assumed hazard rate monotonicity. □

A.6 Proof of Lemma 3

Proof. The proof of part 1) is the same as the proof of Lemma 1 (spelled out in Section A.1) except that in model II one has $\partial_z q_{W_2}(\cdot)=0$; therefore, $\gamma(y)=1$, see the definition of γ in (8) in the paper. Part 2) follows from the envelope theorem:

$$\begin{aligned}\frac{d}{dx}\pi_W(x,x) &= \left(P'(\cdot)\partial_x q_{L_2}(z)q_{W_1}(x,z)+q_{W_1}(x,z)+P'(\cdot)\partial_z q_{L_2}(z)q_{W_1}(x,z)\right)\Big|_{z=x}\\ &= \left(q_{W_1}(x,z)+P'(\cdot)\partial_z q_{L_2}(z)q_{W_1}(x,z)\right)\Big|_{z=x} \quad \text{(because } \partial_x q_{L_2}(z)=0\text{)}\\ &> 0 \quad \text{(because } P'(\cdot)<0 \text{ and } \partial_z q_{L_2}(z)<0\text{)}\\ \pi_L^{*'}(y) &= P'(\cdot)\partial_y q_{W_2}(y)q_{L_1}(x,z,y)<0 \quad \text{(because } \partial_y q_{W_2}(y)>0\text{)}\,.\end{aligned}$$

□

A.7 Proof of Proposition 8

Proof. The cross derivative of the payoff function is

$$\begin{aligned}\partial_{zx}\Pi(x,z) &= \left(\partial_x\pi_W(x,z)-\partial_x\pi_L(x,z,z)\right)f(z)+\partial_{xz}\pi_W(x,z)F(z)\\ &\quad+\int_z^c \partial_{xz}\pi_L(x,z,y)dF(y).\end{aligned} \tag{A.18}$$

1) Sufficiency: We show that this cross-derivative is positive if x, z are sufficiently large. Therefore, Π is pseudoconcave if r is sufficiently large.

Let $r \to c$. Then the integral on the RHS of (A.18) vanishes; however, the sum of the first and second terms on the RHS of (A.18) is positive, as we show next.

By the envelope theorem,

$$\partial_x \pi_W(x,z) = q_{W_1}(x,z), \quad \partial_x \pi_L(x,z,z) = q_{L_1}(x,z,z). \tag{A.19}$$

Hence, $\lim_{x,z\to c}(\partial_x \pi_W(x,z) - \partial_x \pi_L(x,z,z)) = q_{W_1}(c,c) - q_{L_1}(c,c,c) > 0$. By $\partial_{xz}\pi_W(x,z)|_{x=z=c} > 0$, it follows that for $x = z = c$, $\partial_{zx}\Pi(x,z) > 0$. By continuity, pseudoconcavity holds true for all $x, z \geq r$ if r is sufficiently large.

2) Necessity: We show that pseudoconcavity is violated if r is not sufficiently large. For this purpose, suppose $r = 0$. Consider a firm with cost reduction $x > 0$ that reports $z = 0$. We show that $\partial_z \Pi(x,z)|_{z=0} < 0$, which contradicts pseudoconcavity, because pseudoconcavity requires that $\Pi(x,z)$ is monotone increasing for all $z < x$.

Differentiating $\Pi(x,z)$ with respect to z, and using the candidate transfer rule β stated in Proposition 7, one has for all $x \geq r$

$$\begin{aligned}
\partial_z \Pi(x,z)|_{z=0} &= (\pi_W(x,0) - \pi_L(x,0,0)) f(0) + (\pi_L^*(0) - \pi_W(0,0)) f(0) \\
&\quad + \int_0^c \left(\partial_z \pi_L(x,z,y)|_{z=0} - \partial_z \pi_L(x,z,y)|_{x=0,z=0} \right) dF(y) \\
&= \int_0^c \int_0^x \partial_{\tau z} \pi_L(\tau,z,y)|_{z=0} \, d\tau dF(y) \\
&= \int_0^c \int_0^x \partial_z q_{L_1}(\tau,z,y)|_{z=0} \, d\tau dF(y) < 0.
\end{aligned}$$

There, the second equality follows from $\pi_W(x,0) \equiv \pi_L(x,0,0)$, $\pi_L^*(0) \equiv \pi_W(0,0)$, the definition of the definite integral and a change in the order of differentiation of π_L; the third equality follows from (A.19), and the final inequality follows from the fact that $\partial_z q_{L_1}(\tau,z,y)|_{z=0} < 0$. □

A.8 Proof of Proposition 9

Proof. Denote the transfer rules in models I and II by β_n^I, β^I and β_n^{II}, β^{II}, and define $\Delta\beta^I := \beta_n^I(x) - \beta^I(x)$ and $\Delta\beta^{II} := \beta_n^{II}(x) - \beta^{II}(x)$. Using Lemmas 1 and 3, for all $x \geq r$ from the intersection of the domains of the functions β_n, β in models I and II:

$$\begin{aligned}
\Delta\beta^I &= -\frac{1}{f(x)} \int_x^c \partial_z \pi_L^I(x,z,y)\big|_{z=x} dF(y) \\
&> \frac{1}{f(x)} \int_x^c q_L^*(y) dF(y) \quad \text{(by Lemma 1)} \\
&= \Delta\beta^{II} > 0 \quad \text{(by Lemma 3).}
\end{aligned}$$

□

A.9 Model I with linear demand

Suppose $P(Q) = 1 - Q$, $Q := q_1 + q_2$. In the game with royalty scheme the equilibrium output strategies of firm 1 are: $q_{W_1}(x) = {}^{(1-c+2x)}/_3$, $q_{L_1}(x,z,y) = {}^{(1-c+2x-2z-y)}/_3$. The associated equilibrium profits are $\pi_W(x) = q_{W_1}(x)^2$ and $\pi_L(x,z,y) = q_{L_1}(x,z,y)^2$. The equilibrium profit when no firm has access to the innovation is $\pi_A = {}^{(1-c)^2}/_9$.

In the game without royalty scheme $q_{L_1}(x,z,y)$ should be replaced by $q_L(y) = {}^{(1-c-y)}/_3$ and $\pi_L(x,z,y)$ by $\pi_L(y) = q_L(y)^2$.

The transfer rules, β, β_n, are equal to $\beta(x) = \beta_n(x) = R = {}^{4(1-c+r)r}/_9$ for $x < r$, and for all $x \geq r$:

$$\beta_n(x) = \frac{x(2-2c+x)}{3}, \quad \beta(x) = \beta_n(x) - \frac{4}{9f(x)} \int_x^c (1-c-y)dF(y).$$

A.10 Model II with linear demand

In the game with royalty scheme, the equilibrium output strategies of firm 1, are $q_{W_1}(x,z) = {}^{(2-2c+3x+z)}/_6$, $q_{L_1}(x,z,y) = {}^{(2-2c+3x-3z-2y)}/_6$. The associated equilibrium profits are $\pi_W(x,z) = q_{W_1}(x,z)^2$ and $\pi_L(x,z,y) = q_{L_1}(x,z,y)^2$. The equilibrium profit when no firm has access to the innovation, π_A, is the same as in model I.

In the game without royalties, $q_{L_1}(x,z,y)$ should be replaced by $q_L(y) = {}^{(1-c-y)}/_3$ and $\pi_L(x,z,y)$ by $\pi_L(y) = (q_L(y))^2$.

The transfer rules β, β_n are equal to $\beta(x) = \beta_n(x) = R = {}^{4(1-c+r)r}/_9$ for $x < r$, and for $x \geq r$:

$$\beta_n(x) = \frac{x(2-2c+x)}{3} + \frac{(1-c+2x)F(x)}{9f(x)}$$
$$\beta(x) = \beta_n(x) - \frac{1}{3f(x)} \int_x^c (1-c-y)dF(y).$$

B Extension to more than two firms

In this section we show how our analysis can be extended to more than two firms.

Specifically, we consider the case of three firms, allowing the innovator to award either one or two unrestricted licenses, whichever is more profitable.

All other assumptions are maintained.

We proceed as follows: first, we consider the case when one unrestricted license is offered and show that adding the royalty scheme is profitable; second, we consider the case when the innovator offers two unrestricted licenses and again show that adding the royalty scheme is profitable; finally, we show that offering one unrestricted license together with the royalty scheme is optimal.

We consider only the more plausible model II.

B.1 One unrestricted license

One unrestricted license is offered to three firms. Firms simultaneously report their cost reductions. Denote the highest, the second highest, and the third highest reported cost reductions by $\hat{x}_{(1)}$, $\hat{x}_{(2)}$

and $\hat{x}_{(3)}$, respectively. The firm that has reported the highest cost reduction not smaller than r is awarded the unrestricted license and pays $\beta(\hat{x}_{(2)})$. The firm(s) that reported a cost reduction smaller than the highest cost reduction and not smaller than r obtain a royalty license with a royalty rate equal to their reported cost reduction. Firms that reported a cost reduction below r do not obtain a license and pay nothing.

For convenience we refer to the firm that obtains an unrestricted license as "winner" and to royalty licensee(s) as "loser(s)". A firm that reported a cost reduction smaller than the threshold level r qualifies neither as winner nor loser.

After licensing, all reported cost reductions are publicly revealed to all firms.

B.1.1 Cournot "subgames"

Consider one firm, say firm 1, that has drawn the cost reduction x but reports $z \geq x$, while the two other firms with cost reductions y and v tell the truth. As a convention let $y > v$.

Case 1): All three reports met the threshold level and firm 1 won ($\mathbf{z \geq y > v \geq r}$) The innovator allocates the unrestricted license to firm 1 and the royalty licenses to the other firms and charges them a royalty rate equal to their reported cost reduction.Because firms 2 and 3 tell the truth, they believe to play a Cournot game with the profile of unit costs $(c_1, c_2, c_3) = (c - z, c, c)$. Denote the associated equilibrium strategies of the game the losers believe to play by $(q_W(z), q_{L_2}(z), q_{L_3}(z))$.

However, firm 1 privately knows that its cost reduction is x rather than z. Therefore, firm 1 plays the best reply to $(q_{L_2}(z), q_{L_3}(z))$:

$$q_{W_1}(x,z) = \arg\max_q \, (P(q + q_{L_2}(z) + q_{L_3}(z)) - c + x)\, q. \tag{B.1}$$

The reduced form profit function of firm 1 is

$$\pi_W(x,z) := (P(q_{W_1}(x,z) + q_{L_2}(z) + q_{L_3}(z)) - c + x)\, q_{W_1}(x,z). \tag{B.2}$$

Case 2): All three reports met the threshold level and firm 1 lost ($\mathbf{y > z}$ and $\mathbf{z, y, v \geq r}$) In that case firms 2 and 3 believe to play a Cournot game with the profile of unit costs $(c_1, c_2, c_3) = (c, c - y, c)$. Denote the associated equilibrium strategies of the game firms 2 and 3 believe to play by $(q_{L_1}(y), q_{W_2}(y), q_{L_3}(y))$.

If no royalty scheme is used, the equilibrium play of firm 1 depends only on the winner's cost reduction y. The associated reduced form profit function of firm 1 is

$$\pi_{L_n}(y) := (P(q_{L_1}(y) + q_{W_2}(y) + q_{L_3}(y)) - c)\, q_{L_1}(y). \tag{B.3}$$

Whereas if the royalty scheme is adopted, firm 1 privately knows that its cost reduction is x and that it pays a royalty rate equal to $z \geq x$. Therefore, firm 1 plays the best reply to $(q_{W_2}(y), q_{L_3}(y))$:

$$q_{L_1}(x,z,y) = \arg\max_q \, (P(q + q_{W_2}(y) + q_{L_3}(y)) - c + x - z)\, q. \tag{B.4}$$

The associated reduced form profit function of firm 1 is

$$\pi_L(x,z,y) := (P(q_{L_1}(x,z,y) + q_{W_2}(y) + q_{L_3}(y)) - c + x - z)\, q_{L_1}(x,z,y). \tag{B.5}$$

On the equilibrium path, i.e., for $z = x$, the payoff of firm 1 in the event when it loses is only a function of the highest cost reduction of other firms, y; therefore, we denote the on-the-equilibrium path output and profit of firm 1 when it loses by

$$q_L^*(y) = q_L(x,z,y)\big|_{z=x}, \quad \pi_L^*(y) = \pi_L(x,z,y)\big|_{z=x} = \pi_{L_n}(y). \tag{B.6}$$

Case 3): At least one report did not meet the threshold level If no one's report met the threshold level, the equilibrium profit of firm 1 is equal to π_A. If firm 1 was the only one whose report met the threshold level, its equilibrium profit is $\pi_W(x,z)$, and if the rival firm with a cost reduction y was the only one whose report met the threshold level, the equilibrium profit of firm 1 is the same as in the game without royalty scheme, i.e. $\pi_{L_n}(y)$.

Example Suppose inverse demand is linear, $P(Q) = 1-Q, Q = q_1+q_2+q_3$. Then, the equilibrium payoffs of the relevant Cournot equilibrium subgames are:

$$\pi_W(x,z) = \frac{(1-c+2x+z)^2}{16}, \quad \pi_L(x,z,y) = \frac{(1-c+2x-2z-y)^2}{16} \tag{B.7}$$

$$q_L^*(y) = \frac{1-c-y}{4}, \quad \pi_{L_n}(y) = \pi_L^*(y) = \frac{(1-c-y)^2}{16}, \quad \pi_A = \frac{(1-c)^2}{16}. \tag{B.8}$$

B.1.2 Licensing mechanism

We now construct the transfer rule β that induces truth-telling as a Bayesian Nash equilibrium, assures voluntary participation, and extracts the highest possible surplus for given r.

Similar to the case of two firms, we first construct $\beta(x)$ for $x < r$ to assure voluntary participation for these types.

Lemma B.1. *Let*

$$R = \pi_W(r,r) - \pi_A, \tag{B.9}$$

If one sets $\beta(x) = R$ for all $x < r$, all firms with cost reductions $x \leq r$ are indifferent between participation and non-participation, assuming the other firms report truthfully. Hence, voluntary participation is assured for these types.

Proof. Consider the marginal firm, with cost reduction $x = r$. If that firm participates and all other firms tell the truth, that firm's payoff is $\Pi_p(r)$, whereas if it does not participate, its payoff is $\Pi_{np}(r)$:

$$\Pi_p(r) = F(r)^2\left(\pi_W(r,r) - R\right) + \int_r^c \int_0^r \pi_L^*(y) f_{(12:2)}(y,v)\,dv\,dy + \int_r^c \int_r^y \pi_L^*(y) f_{(12:2)}(y,v)\,dv\,dy$$

$$\Pi_{np}(r) = F(r)^2\pi_A + \int_r^c \int_0^r \pi_{L_n}(y) f_{(12:2)}(y,v)\,dv\,dy + \int_r^c \int_r^y \pi_{L_n}(y) f_{(12:2)}(y,v)\,dv\,dy,$$

where $f_{(12:2)} = 2f(y)f(v)$ is the joint p.d.f. of the highest and lowest order statistics in a sample of two *i.i.d.* random variables.

The marginal firm with $x = r$ is indifferent between participation and non-participation, $\Pi_p(r) = \Pi_{np}(r)$, which implies $R = \pi_W(r,r) - \pi_A$.

Like in the case of two firms, lower types with $x < r$ will also participate since they obtain no license and pay nothing and thus are not hurt by participating. Also, truth-telling is assured for all types $x < r$. □

To derive β for $x \geq r$, consider firm 1 with cost reduction x, but reports a cost reduction $z \geq x$, whereas firms 2 and 3 tell the truth. Then, using the equilibria of the duopoly subgames, the payoff function of firm 1 in the game with royalty scheme for $z \geq x \geq r$ is

$$\Pi(x,z) = F(r)^2(\pi_W(x,z) - R) + \int_r^z (\pi_W(x,z) - \beta(y)) f_{(1:2)}(y)dy + \int_z^c \pi_L(x,z,y) f_{(1:2)}(y)dy,$$

where $f_{(1:2)}(y) = 2F(y)f(y)$ is the p.d.f. of the largest order statistic in a sample of two *i.i.d.* random variables.

In the licensing game without royalty scheme, the payoff function is the same, except that in the last term $\pi_L(x,z,y)$ must be replaced by $\pi_{L_n}(y)$.

The transfer rule β must be such that $x = \arg\max_z \Pi(x,z)$, for all $x \in [r,c]$.

Proposition B.1. *In the mechanism with and without royalty scheme, set $\beta(x) = \beta_n(x) = R$ for $x < r$, and for $x \geq r$:*

$$\beta(x) = \beta_n(x) + \frac{1}{f_{(1:2)}(x)} \int_x^c \partial_z \pi_L(x,z,y)|_{z=x} f_{(1:2)}(y)dy \tag{B.10}$$

$$\beta_n(x) = \pi_W(x,x) - \pi_{L_n}(x) + \frac{F(x)}{2f(x)} \partial_z \pi_W(x,z)|_{z=x}. \tag{B.11}$$

Then β, β_n induce truthtelling provided r is sufficiently high, and β, β_n are strictly increasing for all $x \geq r$.

Proof. It is straightforward to derive the transfer rules, β and β_n, and to show that they are strictly monotone increasing. However, one also needs to assure that second order conditions for best replies are satisfied. Like in the case of two firms, this requires that the threshold level r is sufficiently large. The exact condition will be stated when we compute an example, below. The participation constraints are satisfied for all $x \geq r$, the proof is similar to that in online appendix A.3. □

B.1.3 The innovator's expected revenue

The expected revenue of the innovator in the licensing games with and without royalty scheme, $G(r)$ and $G_n(r)$, are:

$$G_n(r) = 3(1 - F(r))F(r)^2(\pi_W(r,r) - \pi_A) + \int_r^c \beta_n(y) f_{(2:3)}(y)dy$$

$$G(r) = G_n(r) + \int_r^c (\beta(y) - \beta_n(y)) f_{(2:3)}(y)dy + 6\int_r^c \int_r^x \int_r^y (v+y) q_L^*(x) f(x) f(y) f(v) dv dy dx,$$

where $f_{(2:3)}(y) = 6(1-F(y))F(y)f(y)$ is the p.d.f. of the second highest order statistic in a sample of three *i.i.d.* random variables.

In Figure B.1 we plot the functions $G(r), G_n(r)$, assuming linear inverse demand, as in (B.7)-(B.8), $c = 0.49$, and $F(x) := \frac{x}{c}$ (uniform distribution). As one can easily confirm, in the game with royalty scheme, second-order conditions (pseudoconcavity of $\pi(x,z)$ in z) are satisfied only if $r \geq r_{\min} = 0.2089$. Therefore, the dashed portion of the G function must be ignored. Comparing G and G_n, this example shows that adding the royalty scheme is profitable also in the case of three firms and one unrestricted license.

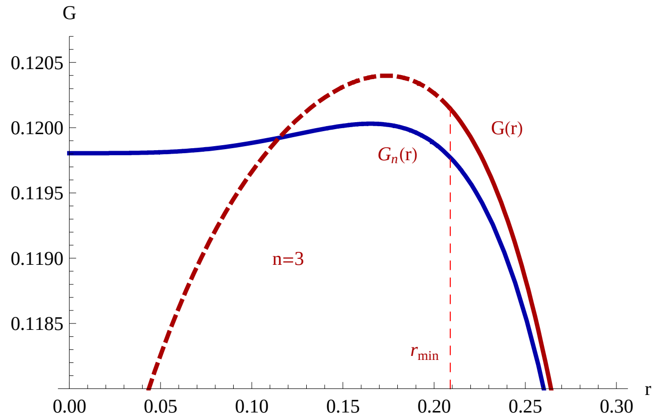

Figure B.1: Profitability of the royalty scheme with 3 firms and 1 unrestricted license

B.2 Two unrestricted licenses

When there are three firms, the innovator may also award two unrestricted licenses. Therefore, in order to determine whether the royalty scheme is profitable when there are three firms, we also need to choose the optimal number of unrestricted licenses. For this purpose we now solve the game for the case when two unrestricted licenses are offered. As we will see later, in our example it is actually optimal for the innovator to offer only one unrestricted license together with the royalty scheme.

When two unrestricted licenses are offered, the innovator employs the following rules: If $x_{(2)} \geq r$, two unrestricted licenses are awarded to the firms that reported the two highest cost reductions, and the winners pay $\beta(\hat{x}_{(3)})$. If $x_{(1)} \geq r > x_{(2)}$, one unrestricted license is awarded to the firm that reported the highest cost reduction and the winner pays $\beta(x_{(2)})$. The firms that reported a cost reduction greater or equal to r but failed to win (called losers) are awarded a royalty license and pay a royalty rate equal to their reported cost reduction. Firms that report a cost reduction below r do not obtain a license and pay nothing.

B.2.1 Cournot "subgames"

Suppose firm 1 has drawn cost reduction x but reports $z \geq x$,[2] whereas firms 2 and 3 tell the truth (again, downward deviations yield the same differential equation). Again, by convention $y > v$. The following Cournot "subgames" occur.

Case a) Only firm 1's report met the threshold level ($\mathbf{z \geq r > y \geq v}$) In this case, the equilibrium profit of firm 1 is the same as that in case 1) of the game with 3 firms and only one unrestricted license is offered, i.e., $\pi_{W_a}(x,z) = \pi_W(x,z)$.

Case b) At least two reports met the threshold level and firm 1 won (either $\mathbf{z > v}$ and $\mathbf{z, y, v \geq r}$ or $\mathbf{z, y \geq r}$ and $\mathbf{v < r}$) In that case, firms 2 and 3 believe to play a Cournot game with the profile of unit costs $(c_1, c_2, c_3) = (c - z, c - y, c)$. Denote the associated equilibrium strategies of the game firms 2 and 3 believe to play by $(q_{W_1}(z,y), q_{W_2}(z,y), q_{L_3}(z,y))$.

[2] Like in the case of one unrestricted license, downward deviations yield the same differential equation.

However, firm 1 privately knows that its cost reduction is x rather than z, and plays the best reply to $(q_{W_2}(z,y), q_{L_3}(z,y))$:

$$q_W(x,z,y) = \arg\max_q \left(P(q + q_{W_2}(z,y) + q_{L_3}(z,y)) - c + x\right) q. \tag{B.12}$$

The associated reduced form profit function of firm 1 is then

$$\pi_{W_b}(x,z,y) := \left(P(q_W(x,z,y) + q_{W_2}(z,y) + q_{L_3}(z,y)) - c + x\right) q_W(x,z,y). \tag{B.13}$$

Case c): All three reports met the threshold level and firm 1 lost ($\mathbf{v > z}$ and $\mathbf{z, y, v \geq r}$) In that case, both firm 2 and firm 3 won an unrestricted license and believe to play a Cournot game with the profile of unit costs $(c_1, c_2, c_3) = (c, c-y, c-v)$. Denote the associated equilibrium strategies of the game the winners believe to play by $(q_{L_1}(y,v), q_{W_2}(y,v), q_{W_3}(y,v))$.

If no royalty scheme is used, the equilibrium strategy of firm 1 depends only on the winners' cost reductions, y and v. The reduced form profit function of firm 1 is

$$\pi_{L_{2n}}(y,v) := \left(P(q_{L_1}(y,v) + q_{W_2}(y,v) + q_{W_3}(y,v)) - c\right) q_{L_1}(y,v). \tag{B.14}$$

Whereas if the royalty scheme is adopted, firm 1 privately knows its cost reduction is x, yet pays a royalty rate $z \geq x$. Therefore, firm 1 plays the best reply to $(q_{W_2}(y,v), q_{W_3}(y,v))$:

$$q_L(x,z,y,v) = \arg\max_q \left(P(q + q_{W_2}(y,v) + q_{W_3}(y,v)) - c + x - z\right) q. \tag{B.15}$$

The associated reduced form profit function of firm 1 is

$$\pi_L(x,z,y,v) := \left(P(q_L(x,z,y,v) + q_{W_2}(y,v) + q_{W_3}(y,v)) - c + x - z\right) q_L(x,z,y,v). \tag{B.16}$$

On the equilibrium path, i.e., for $z = x$, that payoff of firm 1 when it loses is only a function of winners' cost reductions y and v; and we write

$$q_L^*(y,v) = q_L(x,z,y,v)|_{z=x}, \quad \pi_L^*(y,v) = \pi_L(x,z,y,v)|_{z=x} = \pi_{L_{2n}}(y,v). \tag{B.17}$$

Case d) Firm 1's report did not meet the threshold level ($\mathbf{z < r}$) If firm 1's report is below r while both rival firms' reports are not smaller than r, the equilibrium profit of firm 1 depends only on the rivals' cost reductions and is equal to $\pi_{L_{2n}}(y,v)$. If firm 1's report is below r and only the rival firm with higher cost reduction (y) reported a cost reduction not smaller than r, the equilibrium profit of firm 1 depends only on y and is equal to $\pi_{L_n}(y)$. If no one's report met the threshold level, the equilibrium profit of firm 1 is equal to π_A.

Example Suppose inverse demand is linear, as in the example in section B.1. Then, the equilibrium payoffs of the relevant Cournot subgames are:

$$\pi_{W_a}(x,z) = \frac{(1-c+2x+z)^2}{16}, \quad \pi_{W_b}(x,z,y) = \frac{(1-c+2x+z-y)^2}{16},$$
$$\pi_L(x,z,y,v) = \frac{(1-c+2x-2z-y-v)^2}{16}, \quad \pi_L^*(y,v) = \pi_{L_{2n}}(y,v) = \frac{(1-c-y-v)^2}{16},$$
$$q_L^*(y,v) = \frac{1-c-y-v}{4}, \quad \pi_{L_n}(y) = \frac{(1-c-y)^2}{16}.$$

B.2.2 Licensing mechanism

We first construct β for $x < r$ to assure voluntary participation for these types.

Lemma B.2. *If one sets $\beta(x) = R^*$ for all $x < r$, where*

$$R^* = \frac{1}{2-F(r)}\Big(\left(\pi_{W_a}(r,r) - \pi_A\right)F(r) + 2\int_r^c \left(\pi_{W_b}(r,r,y) - \pi_{L_n}(y)\right)f(y)dy\Big), \tag{B.18}$$

then all firms with cost reductions $x \leq r$ are indifferent between participation and non-participation, assuming rival firms report truthfully. Hence, voluntary participation is assured for these types.

Proof. Consider the marginal firm with cost reduction $x = r$. If that firm participates, and all other firms tell the truth, that firm's payoff is $\Pi_p(r)$, whereas if it does not participate, its payoff is $\Pi_{np}(r)$:

$$\begin{aligned}\Pi_p(r) &= F(r)^2\left(\pi_{W_a}(r,r) - R^*\right) + \int_r^c \int_0^r \left(\pi_{W_b}(r,r,y) - R^*\right) f_{(12:2)}(y,v)dvdy \\ &\quad + \int_r^c \int_r^y \pi_L^*(y,v) f_{(12:2)}(y,v)dvdy \\ \Pi_{np}(r) &= F(r)^2\pi_A + \int_r^c \int_0^r \pi_{L_n}(y) f_{(12:2)}(y,v)dvdy + \int_r^c \int_r^y \pi_{L_{2n}}(y,v) f_{(12:2)}(y,v)dvdy,\end{aligned}$$

where $f_{(12:2)}(y,v) = 2f(y)f(v)$.

The marginal bidder with $x = r$ must be indifferent between participating and not participating, $\Pi_p(r) = \Pi_{np}(r)$, which implies (B.18).

The types $x < r$ will also participate because they obtain no license and pay nothing and thus are not hurt by participation. Similar to the case of one unrestricted license, truth-telling is assured for all $x < r$. □

Now we derive β for $x \geq r$. Consider firm 1 with cost reduction x, but reports a cost reduction $z \geq x$, whereas firms 2 and 3 tell the truth. Then, using the equilibria of the duopoly subgames, the payoff function of firm 1 in the game with royalty scheme for $z \geq x \geq r$ is

$$\begin{aligned}\Pi(x,z) &= F(r)^2(\pi_{W_a}(x,z) - R^*) + \int_r^c \int_0^r (\pi_{W_b}(x,z,y) - R^*) f_{(12:2)}(y,v)dvdy \\ &\quad + \int_r^z \int_r^y (\pi_{W_b}(x,z,y) - \beta(v)) f_{(12:2)}(y,v)dvdy + \int_z^c \int_r^z (\pi_{W_b}(x,z,y) - \beta(v)) f_{(12:2)}(y,v)dvdy \\ &\quad + \int_z^c \int_z^y \pi_L(x,z,y,v) f_{(12:2)}(y,v)dvdy\end{aligned}$$

where $f_{(12:2)}(y,v) = 2f(y)f(v)$.

In the licensing game without royalty scheme, the payoff function is the same, except that in the last term $\pi_L(x,z,y,v)$ must be replaced by $\pi_{L_{2n}}(y,v)$.

The transfer rule β must be such that $x = \arg\max_z \Pi(x,z)$, for all $x \in [r,c]$.

Proposition B.2. *Set $\beta(x) = \beta_n(x) = R^*$ for all $x < r$, and for all $x \geq r$:*

$$\begin{aligned}\beta(x) &= \beta_n(x) + \int_x^c \int_x^y \partial_z \pi_L(x,z,y,v)|_{z=x} f_{(12:2)}(y,v)dvdy \\ \beta_n(x) &= \frac{1}{2f(x)(1-F(x))}\left(F(r)^2\,\partial_z\pi_{W_a}(x,z)|_{z=x} + 2F(x)\left(\int_x^c \partial_z\pi_{W_b}(x,z,y)|_{z=x} dF(y)\right)\right.\end{aligned}$$

$$+\int_r^x \partial_z \pi_{W_b}(x,z,y)|_{z=x} f_{(1:2)}(y)dy + 2f(x)\int_x^c (\pi_{W_b}(x,x,y) - \pi_{L_n}(y,x))f(y)dy\Big).$$

Then β, β_n induce truth-telling and voluntary participation, provided r is sufficiently high, and β, β_n are strictly increasing for $x \geq r$.

Proof. The derivation of β, β_n and the proof of their monotonicity are similar to that in the case of two firms. Also similar to other cases with royalty scheme, the second order conditions are satisfied only when the threshold level r is sufficiently large. □

B.2.3 The innovator's expected revenue

The expected revenue functions of the innovator without and with royalty scheme are,

$$G_n(r) = 3(1-F(r))F(r)^2R^* + 3(1-F(r))^2F(r)2R^* + 2\int_r^c \beta_n(v)f_{(3:3)}(v)dv$$

$$G(r) = G_n(r) + \int_r^c 2(\beta(v) - \beta_n(v))f_{(3:3)}(v)dv + 6\int_r^c\int_r^x\int_r^y vq_L^*(x,y)f(x)f(y)f(v)dvdydx$$

where R^* is defined in (B.18)) and $f_{(3:3)}(v) = 3\left(1-F(v)^2\right)f(v)$ is the *p.d.f.* of the lowest order statistic in a sample of three *i.i.d.* random variables.

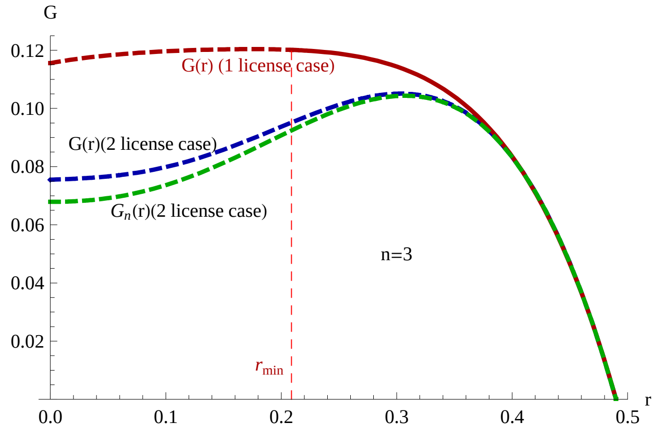

Figure B.2: Profitability of 1 *vs* 2 unrestricted licenses when there are 3 firms

In Figure B.2 we plot the innovator's expected revenue with one unrestricted license, $G(r)$ (one license case with royalty scheme), with two unrestricted licenses, $G(r)$ (two license case with royalty scheme), and $G_n(r)$ (two license case without royalty scheme). Like before, we assume linear inverse demand, $c = 0.49$, and $F(x) = x/c$ (uniform distribution). Comparing the plots in Figures B.1-B.2 we conclude that awarding one unrestricted license, combined with the royalty scheme, is strictly better than all other mechanisms.[3]

References

Bergstrom, T. C. and H. Varian (1985). "Two Remarks on Cournot Equilibria". *Economics Letters* 19, pp. 5–8.

[3]Note, the different shapes of $G(r)$ (one license case with royalty scheme) in Figures B.1 and B.2 are due to the different scales we use in these two figures.