Spectrum license auctions with exit (and call) options: Alternative remedies for the exposure problem☆

Luke Hu[a], Elmar G. Wolfstetter[b]

[a] *Institute of Economic Theory I, Humboldt University at Berlin Spandauer Str. 1, 10178 Berlin, Germany*
[b] *Institute of Economic Theory I, Humboldt University at Berlin, Spandauer Str. 1, 10178 Berlin, Germany*

Abstract

Inspired by some spectrum auctions we consider a stylized license auction with incumbents and a potential entrant. Whereas the entrant values only the bundle of several spectrum blocks (synergy), incumbents' marginal valuations are non-increasing. The seller proactively encourages entry and subjects incumbents to a spectrum cap. In this framework, a simultaneous multi-round auction (SMRA) gives rise to an exposure problem that distorts efficiency and yields low revenue. We consider three remedies and rank their performance: a combinatorial Vickrey auction, a SMRA with exit option that allows the entrant to annul his bid, and a SMRA with exit and call option that lifts the spectrum cap if entry failed to occur and then allows the successful incumbent to acquire stranded spectrum at a fixed price.

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1. Introduction

Radio spectrum is an essential input to deliver mobile voice and data service and is required for capacity and coverage purposes. If an operator does not gain sufficient access to spectrum, he may not be able to deliver service and recover his cost.

During the early years of the mobile phone industry operators were awarded radio spectrum either by a managed assignment process or a beauty contest which often lead to a quagmire of litigation and delay. However, during the past 15 years, regulators all over the world successfully employed auctions to award spectrum, which however pose their own challenges.

In spectrum auctions the regulator typically has several concerns that are difficult to reconcile:

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Email addresses: huluke3@yahoo.com (Luke Hu), wolfstetter@gmail.com (Elmar G. Wolfstetter)

- the regulator favors entry and wants to prevent foreclosure by incumbents, but not at all cost
- entrants require a minimum endowment of spectrum, a spectrum floor, which implies a strong synergy
- synergies may give rise to an exposure problem which in turn leads to cautious bidding, a low probability of entry, and low revenue
- revenue matters, not only because debt-ridden governments are short of revenue but also because license fees are a distortion-free method of taxation
- the regulator wants to avoid that some spectrum remains stranded.

Typically, the regulator responds to these concerns by restricting the bidding rights of incumbent bidders[1] or by adding provisions that deal with the exposure problem, for example by adopting combinatorial auctions.[2] Some regulators proactively encourage entry without regard of cost by reserving some designated lots for entrants.[3]

An interesting alternative response to these concerns was adopted in the 3G and 4G spectrum auctions in Germany in the years 2000 and 2010. There bidders were entitled to state a minimum requirement for spectrum, with the provision that if a bidder ends up winning less than his stipulated minimum, his bids are annulled. In addition, provisions were made to re-auction any left-over or "stranded" spectrum to minimize the risk that spectrum is wasted.[4]

The present paper is inspired by these rules and contributes to evaluate their merit. In particular, we ask the questions: does an "exit option" that allows an entrant to annul his bids if his minimum requirement is not satisfied resolve the entrants' exposure problem? Does a "call option" that allows an incumbent to acquire stranded spectrum if entry has failed ensure that no spectrum is wasted without compromising the preference for entry? And how does an auction that includes exit and call options perform in terms of entry and seller's revenue compared to a combinatorial Vickrey auction that is usually recommended as a remedy for the exposure problem?

[1] An alternative to such spectrum caps has been used recently in Greece. There, entrants were granted the right to buy an essential endowment of spectrum at fixed prices prior to the main auction. Similarly, in the 2013 spectrum auction in Austria, entrants could bid for a reserved part of the spectrum prior to the main auction. However, entry did not occur and all blocks were auctioned to incumbent bidders.

[2] Combinatorial auctions have been used recently, for example in Austria, Denmark, France, India, the Netherlands, Nigeria, the UK, and Switzerland. Some of these were sealed-bid first-price auctions (France, Nigeria) and others (like Switzerland and Austria) employed a combinatorial clock auction format, supplemented by a final round of combinatorial sealed bids. For a detailed survey of auction formats employed in some recent 4G auctions see Nett and Stumpf (2011); for the rules of the most recent CCA auctions in Switzerland and in Austria see TKK (2013) and BAKOM (2010).

[3] This policy was adopted in the 3G auction in the U.K. in the year 2000, and more recently in post-3G auctions in the Czech Republic and in the Netherlands.

[4] In the 3G auction that minimum requirement was 2 blocks of 2×5 MHz spectrum in the 1900-2025 MHz band. This requirement was requested by the industry on the ground that building a 3G network involves a fixed cost in the order of 8 billion DM (roughly 4 billion Euros), which can only be recovered with sufficient capacity. In the 4G auction the minimum requirement was not mandatory, it was only optional.

We address these issues in the framework of a stylized model with one entrant and two incumbents. The entrant is subject to a strong synergy effect, due to his need for a minimum capacity, whereas incumbents already own complementary spectrum and their marginal valuations are non-increasing. The seller favors entry and restricts incumbent bidders. The seller auctions two blocks of spectrum of which each incumbent prefers one (because they may be neighbors to already owned spectrum).

We first consider a simultaneous multi-round English clock auction (*SMRA*) and show that it leads to a double-exposure problem that causes a low probability of successful entry: in equilibrium the entrant may win only one block of spectrum that has no value to him, or win both blocks yet pay more than the bundle value, and in both cases suffer a loss.

Both exposure risks can be remedied by allowing the entrant to annul his bid if he failed to win a stipulated number of units ("exit option").[5] However, the possibility of stranded spectrum remains an issue. Yet, this problem can also be remedied by further adding a call option that allows the successful incumbent to buy stranded spectrum at a fixed price if the entrant exercised his exit option. Of course, a combinatorial Vickery auction (*CVA*) can also remedy the exposure problem and avoid that some spectrum is stranded. However, compared to the other proposed remedies, it performs poorly in terms of generating revenue to the auctioneer.

There is a small literature on synergies in auctions (see Krishna and Rosenthal, 1996; Rosenthal and Wang, 1996). The classical reference on combinatorial auctions is Ausubel and Milgrom (2002); a comprehensive collection of later contributions is Cramton, Shoham, and Steinberg (2006).

For an account of the 3G auction in Germany see Grimm, Riedel, and Wolfstetter (2004), van Damme (2002), and Klemperer (2002, 2004), and for the rules of some of the more recent 4G auctions see Nett and Stumpf (2011) and Salant (2013).

The plan of the paper is as follows: After stating the model in section 2 we analyze the standard *SMRA* in Section 3 and the *CVA* in Section 4. The *SMRA* is supplemented by an exit option for the entrant in Subsection 5, and by further adding a call-option in Subsection 5.2. The performance of all three remedies of the exposure problem is ranked in Section 6 and the quantitative significance of this ranking is assessed by means of a simulation in Section 7. The robustness of our analysis is discussed in Section 8. The paper concludes with a brief discussion in Section 9.

2. Model

Consider a stylized spectrum auction with two incumbent bidders (A and B), one potential entrant bidder (E), and two blocks of spectrum for sale (a and b). The entrant E is free to bid on both blocks and values only the bundle of a and b (because he does not value capacity below two blocks). In other words, E

[5] Another remedy is to auction the bundled spectrum in the SMRA format and grant entrants a bonus. However, this tends to yield a less competitive market structure. For a general discussion of the relationship between the auction rules and promoting competition in the mobile phone market see Cramton, Kwerel, Rosston, and Skrzypacz (2011).

is subject to a strong synergy. Incumbents are subject to a spectrum cap which restricts them to bid on only one block (A wants a and B wants b), and if that restriction is lifted they value the second block not more than the first.

Incumbents' valuations for their favored block are denoted by V_A, V_B and the entrant's valuation for the bundle of two blocks is denoted by $2V_E$. V_A, V_B, V_E, are *i.i.d.* random variables,[6] drawn from the c.d.f. F with support $[0, 1]$, and p.d.f. $f(v) > 0$, and their realizations are bidders' private information. The entrant's valuation for one block is equal to zero, and incumbents' valuation for the bundle of two blocks is either equal to or less than $2V_A, 2V_B$.

For convenience we denote the order statistics of the sample of incumbents' valuations by $V_{(1)} > V_{(2)}$, their joint p.d.f. by $f_{(12)}(y, z) = 2f(y)f(z)$ (for $y > z$ and equal to zero otherwise), and the associated marginal p.d.f.s by $f_{(1)}(y), f_{(2)}(z)$.

The seller's own valuation is normalized to zero and the seller neither uses a strategic reserve price nor an entry fee.

The two blocks of spectrum are auctioned simultaneously, using either a SMRA or a CVA.

Two variations of the base model SMRA are considered:

1) SMRA with exit option (SMRA):* the entrant has the option to return one block if he failed to win both blocks. If that option is exercised, one block is not sold and the seller collects payment only from the incumbent who won one block.

*2) SMRA with exit plus call option (SMRA**):* the auctioneer lifts the restriction on bidding if the entrant exercised his exit option. In that case, the incumbent who won the first block has the option to buy the second at a fixed price equal to the price (or a fraction of that price) he paid for the first.

In the following, we will say that entry occurred only if the entrant have acquired two blocks of spectrum, because the entrant cannot offer services if he fails to satisfy his minimum capacity requirement.

3. Simultaneous multi-round (English clock) auction (SMRA)

In the SMRA bidders indicate their demand for block a or b at given clock prices. Price clocks start at zero, and go up continuously in response to excess demand. If there is excess demand for both blocks, both price clocks go up in tandem; but if there is excess demand for only one block, that price clock goes up while the other remains unchanged. The auction terminates when there is no excess demand. Then, the last round bidders are awarded the blocks they demanded at terminal clock prices.

3.1. Equilibrium bid strategies

There are several possible outcomes: The entrant may be the first to quit. In that case, each incumbent wins his favored block and pays the price at which the entrant quit. Alternatively, one incumbent, say A, may be the first to quit. Then, price clock a stops and E is the preliminary winner of a, while bidding continues on b. If thereafter the entrant is the first to quit auction on b, the

[6] As a rule, random variables are denoted by capital letters and their realizations by lower case letters.

auction ends: B wins b at the price at which E quit and E wins a at the price at which A quit and E suffers a loss. Whereas if B quits the auction on b before E quits, E wins both blocks and pays the prices at which the two incumbents quit.

The entrant suffers a winner's curse if he wins only one block, which is useless for him, and if he wins both blocks but pays more than the bundle value. As we will show below, in equilibrium both kinds of winner's curse occur with positive probability, which in turn induces the entrant to play a cautious initial stop rule that reduces the probability of entry as well as the seller's revenue.

The details of the solution of the game are as follows:

Bidders' strategies are stop rules. Incumbents bid only on their favored block; therefore, their bid strategy is a stop rule that prescribes incumbent $i \in \{A, B\}$ to quit bidding for his favored block at price $\beta_i(v_i)$. The entrant's strategy is a collection of stop rules $(s(v_E), \sigma(v_E, p))$: 1) the initial stop rule, s, prescribes to quit bidding at price $s(v_E)$ as long as no incumbent has quit before, and 2) the continuation stop rule, σ, prescribes to quit the remaining active auction at price $\sigma(v_E, p)$, after one incumbent has quit first at price p.

Evidently, incumbents have (weakly) dominant strategies and the entrant has a (weakly) dominant continuation strategy (but not a dominant initial stop rule):[7]

Proposition 1. *Incumbents have (weakly) dominant strategies: bid on your favored block up to its value, $\beta_i(v_i) = v_i, i \in \{A, B\}$, and the entrant has a (weakly) dominant continuation strategy, σ: having won one block at price p, bid on the other block up to the bundle value, $\sigma(v_E, p) = 2v_E$ for all p.*

Once the entrant has won one block, he is driven to "overbid" on the second block. The entrant is thus exposed to a double winner's curse problem: he risks winning only one block and he risk winning the bundle at a price that exceeds its value,[8] and in either event suffers a loss.

The entrant responds to this double exposure problem by playing a cautious initial stop rue, $s(v_E)$, which exhibits bid shading to counteract the equilibrium overbidding for the second block, as follows:

Proposition 2. *The entrant plays a cautious initial stop rule that exhibits bid shading, $s(v_E) < v_E$. This stop rule is strictly increasing, approaches v_E as v_E approaches 1, and is implicitly defined as the solution of the condition:*

$$s + \int_s^{\min\{2v_E, 1\}} y \frac{f(y)}{1 - F(s)} dy = 2v_E \int_s^{\min\{2v_E, 1\}} \frac{f(y)}{1 - F(s)} dy. \tag{1}$$

An interpretation of this condition is provided in Corollary 1.

Proof. Given the continuation strategy $\sigma(v_E) = 2v_E$ and incumbents' equilibrium strategy, the entrant's expected payoff is a function of the initial bid s, as

[7] Like in a standard single-object Vickrey auction, there are other equilibria. These are, however, not meaningful because they violate "trembling hand perfection".

[8] Note, this can only happen if $v_E < 1/2$; for, if $v_E \geq 1/2$, $2v_E$ exceeds incumbents' valuation with probability 1.

follows:

$$\begin{aligned}\pi_E(s) = 2v_E \int_0^s \int_z^{\min\{2v_E,1\}} f_{(12)}(y,z)dydz - \int_0^s \int_z^1 z f_{(12)}(y,z)dydz \\ - \int_0^s \int_z^{\min\{2v_E,1\}} y f_{(12)}(y,z)dydz.\end{aligned} \tag{2}$$

There, the first term is the expected benefit of winning both blocks (winning only one block has no value), and the second and third terms are the expected payments for the first and second block.[9]

The equilibrium strategy $s(v_E)$ must solve the equilibrium requirement: $s(v_E) = \arg\max_s \pi_E(s)$, which yields the first-order condition:

$$0 = \frac{\partial \pi_E(s)}{\partial s} = \int_s^{\min\{2v_E,1\}} (2v_E - y)2f(s)f(y)dy - \int_s^1 2sf(s)f(y)dy. \tag{3}$$

If $v_E \geq 1/2$ Equation (3) has the trivial solution $s = 1$, which is however not a maximizer, because $\left.\frac{\partial^2 \pi_E(s)}{\partial s \partial s}\right|_{s=1} = 4f(1)^2(1 - v_E) > 0$. Having ruled out $s = 1$, we show that Equation (3) has a solution $s \in (0,1)$. This follows from

$$\begin{aligned}\left.\frac{\partial \pi_E(s)}{\partial s}\right|_{s=0} &= 2f(0)\int_0^{\min\{2v_E,1\}} (2v_E - y)f(y)dy > 0 \\ \left.\frac{\partial \pi_E(s}{\partial s}\right|_{s=v_E} &= 2f(v_E)\left(\int_{v_E}^{\min\{2v_E,1\}} (2v_E - y)f(y)dy - \int_{v_E}^1 v_E f(y)dy\right) \\ &< 2f(v_E)\left(\int_{v_E}^{\min\{2v_E,1\}} (2v_E - v_E - v_E)f(y)dy\right) \\ &= 0,\end{aligned}$$

and the fact that $\pi_E(s)$ is continuously differentiable.

Dividing Equation (3) by $1 - F(s) > 0$, it follows that the $s(v_E)$ solves Equation (1).

To show that $s(v_E)$ exhibits bid shading, consider the following evaluation that begins with the equilibrium condition Equation (3):

$$\begin{aligned}0 &= 2v_E \int_s^{\min\{2v_E,1\}} f(y)dy - \int_s^{\min\{2v_E,1\}} yf(y)dy - s(1 - F(s)) \\ &= 2v_E (F(\min\{2v_E,1\}) - F(s)) - \int_s^{\min\{2v_E,1\}} yf(y)dy - s(1 - F(s)) \\ &< 2v_E (F(\min\{2v_E,1\}) - F(s)) - s(F(\min\{2v_E,1\}) - F(s)) - s(1 - F(s)) \\ &< 2v_E (F(\min\{2v_E,1\}) - F(s)) - 2s(F(\min\{2v_E,1\}) - F(s)) \\ &= 2(v_E - s)(F(\min\{2v_E,1\}) - F(s)).\end{aligned}$$

[9] Note, the first block is won if and only if $V_{(2)} < s(v_E)$, which occurs with probability $\int_0^s \int_z^1 f_{(12)}(y,z)dydz$ and the second block is won if and only if the first block is won and $V_{(1)} < 2v_E$, which occurs with probability $\int_0^s \int_z^{\min\{2v_E,1\}} f_{(12)}(y,z)dydz$.

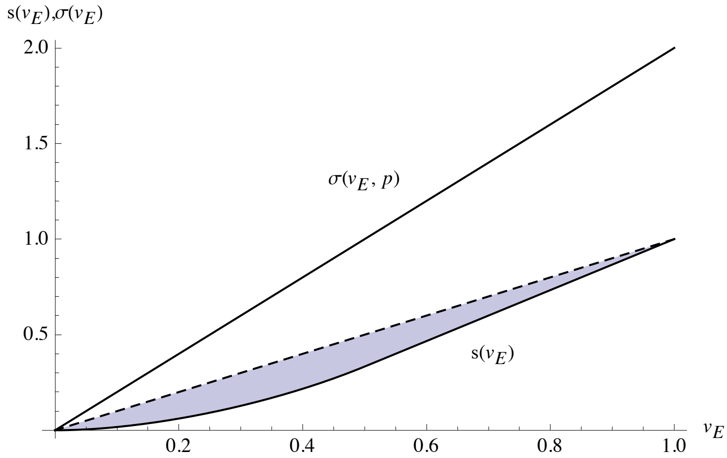

Figure 1: Entrant's equilibrium strategies, $s(v_E), \sigma(v_E, p)$, for the uniform distribution

Hence, $s(v_E) < v_E$ for all $v_E \in (0, 1)$, as asserted.

Differentiating (1) with respect to v_E gives, for all $v_E \in (0, 1)$:

$$s'(v_E) = \frac{2\left(F(\min\{2v_E, 1\}) - F(s(v_E))\right)}{1 - F(s(v_E)) + 2\left(v_E - s(v_E)\right) f\left(s(v_E)\right)} > 0.$$

Hence, s is a differentiable and strictly increasing function for all $v \in (0, 1)$.

To show that $\lim_{v_E \to 1} s(v_E) = 1$, consider the inverse relationship between v_E and s (which involves no loss of generality because $s(v_E)$ is strictly increasing). For large v_E the equilibrium condition (1) can be written as: $2v_E = s + \frac{1}{1-F(s)} \int_s^1 y dF(y)$. By L'Hôpital's rule $\lim_{s\to 1} \left(\frac{1}{1-F(s)} \int_s^1 y dF(y)\right) = 1$. Therefore, as s approaches 1, v_E approaches 1. □

As an illustration, suppose $F(v) \equiv v$ (uniform distribution). Then $s(v_E)$ takes the form:

$$s(v_E) = \begin{cases} \frac{1}{3}\left(1 + 2v_E - \sqrt{4v_E(1 - 2v_E) + 1}\right) & \text{if} \quad v_E \in [0, 1/2] \\ \frac{1}{3}\left(4v_E - 1\right) & \text{if} \quad v_E \in [1/2, 1]. \end{cases} \tag{4}$$

This function is plotted in Figure 1. There, the shaded area indicates the extent of bid shading during the first phase of the auction.

Corollary 1. *The equilibrium stop rule s has a simple interpretation: Bid for the first block up to the level $s(v_E)$ at which the conditional expected payment, conditional on the "worst case" event of winning the first block at the highest price, $s(v_E)$, is equal to the bundle value times the conditional probability of winning also the second block. Stated formally, the equilibrium stop rule $s(v_E)$ that solves (1) can be interpreted as the solution of the "break-even" condition:*

$$s + \int_s^{\min\{2v_E,1\}} y f_{(1)}(y \mid V_{(2)} = s) dy = 2v_E \int_s^{\min\{2v_E,1\}} f_{(1)}(y \mid V_{(2)} = s) dy \tag{5}$$

where $f_{(1)}(y \mid V_{(2)} = s)$ denotes the conditional p.d.f. of the order statistic $V_{(1)}$, conditional on the event $V_{(2)} = s(v_E)$.

Proof. If the entrant wins the first block and pays the maximum price $s(v_E)$, he infers that $V_{(2)} = s(v_E)$. Therefore, given the equilibrium continuation strategies (see Proposition 1), the entrant wins also the second block only if, conditional on the event $V_{(2)} = s(v_E)$, one has $V_{(1)} \leq \min\{2v_E, 1\}$.

The p.d.f. of the second highest order statistic, $V_{(2)}$, is $f_{(2)}(s) = 2f(s)(1 - F(s))$. Therefore, for all $y \geq s$:

$$f_{(1)}(y \mid V_{(2)} = s) = \frac{f_{(12)}(y, s)}{f_{(2)}(s)} = \frac{2f(y)f(s)}{2f(s)(1 - F(s))} = \frac{f(y)}{1 - F(s)}. \tag{6}$$

Therefore, conditions (5) and (1) are equivalent. □

Obviously, if $v_E \geq 1/2$, winning the first block while playing the equilibrium continuation strategy $\sigma(v_E) = 2v_E$ implies winning also the second. Therefore, in this case, the conditional expected benefit, conditional on winning the first block at price $s(v_E)$, which is the RHS of (5), is equal to the bundle value, $2v_E$.

3.2. The seller's revenue

To compute the seller's equilibrium expected revenue, one needs to distinguish between three kinds of events and associated equilibrium prices:
1) $s(V_E) < V_{(2)} \Rightarrow$ the entrant quits first at prices equal to $s(v_E)$ and each incumbent wins one block; seller's revenue $\Pi_0 = 2s(V_E)$.
2) $s(V_E) > V_{(2)}$, $2V_E < V_{(1)} \Rightarrow$ one incumbent quits first and the entrant quits second; therefore, one block is won by the entrant and one by an incumbent; seller's revenue $\Pi_0 = V_{(2)} + 2V_E$.
3) $s(V_E) > V_{(2)}$, $2V_E > V_{(1)} \Rightarrow$ the entrant wins both blocks; seller's revenue $\Pi_0 = V_{(2)} + V_{(1)}$.

Based on this distinction of events, we compute the seller's equilibrium expected revenue and probability of entry in Appendix A.1. These computations are required for the example summarized in Table 1 and the simulation reviewed in Section 7.

Altogether, the SMRA is not favorable for the entrant because it exposes him to a double winner's curse problem. The entrant responds to this by playing a cautious initial stop rule. Therefore, it is also not favorable for the seller's revenue. Altogether, it is in the interest of the entrant as well as of the seller to remedy the exposure problem.

4. Combinatorial Vickrey-Clarke-Groves auction (CVA)

In the combinatorial Vickrey-Clarke-Groves auction (in brief: Vickrey auction) the auctioneer selects the allocation that maximizes the sum of valuations, and the winner(s) have to pay a price equal to the sum of valuations crowded out by their participation.[10] In order to be comparable we consider this auction subject to the regulatory constraint that incumbents cannot be awarded more

[10] The sum of valuations crowded out by a bidder's participation is the difference between the sum of others' valuations of the blocks they are awarded if that bidder does not participate (based on the above stated allocation rule) and the sum of others' valuations of the blocks they are awarded if that bidder participates.

than one block.[11] Like in a single-object Vickrey auction, truthful bidding is an equilibrium (see Ausubel and Milgrom, 2002).[12]

In order to characterize the equilibrium outcome and compute the seller's equilibrium revenue one needs to distinguish between the following events and associated equilibrium prices:

1. $2V_E > V_A + V_B \Rightarrow$ the entrant wins the bundle and pays the incumbents' valuations which he crowds out; seller's revenue $\Pi_0 = V_A + V_B$.

2. $2V_E < V_A + V_B \Rightarrow$ the incumbents win their favored blocks.

a) $V_{(2)} > 2V_E \Rightarrow$ no single incumbent crowds out the entrant; seller's revenue $\Pi_0 = 0$.[13]

b) $V_{(1)} > 2V_E > V_{(2)} \Rightarrow$ only the high value incumbent crowds out the entrant; seller's revenue $\Pi_0 = 2V_E - V_{(2)}$.

c) $V_{(1)} < 2V_E \Rightarrow$ each incumbent crowds out the entrant; seller's revenue $\Pi_0 = 4V_E - (V_A + V_B)$.

Based on this distinction of events, we compute the seller's equilibrium expected revenue and probability of entry in Appendix A.2. Again, these computations are required for the example summarized in Table 1 and the simulation reviewed in Section 7.

Altogether, the CVA eliminates the exposure problem because it ensures that the entrant never wins only one block and never pays more than the bundle value, and it ensures that no block is stranded. However, as we will confirm later, it performs poorly in terms of the seller's expected revenue. Moreover, (combinatorial) Vickrey auctions have other well-known deficiencies such as their vulnerability to collusion and shill-bidding (see Ausubel and Milgrom, 2002; Bulow, Levin, and Milgrom, 2009; Rothkopf, 2007; Salant, 2013). This is why some authors have recommended to use a first-price package auction which are however plagued by multiplicity of equilibria and whose properties are not fully understood as yet (see Milgrom, 2004).

5. *SMRA* with exit (and call) option(s)

5.1. SMRA *with exit option (SMRA*)*

We now modify the SMRA by allowing the entrant to exit and annul his bid if he has won only one block. If that option is exercised, the entrant returns the

[11] We do not consider the particular two-stage combinatorial auction that has been designed to minimize the risk of collusion. In the first "clock-stage" of that combinatorial auction bidders bid on *one* bundle until there is no excess demand; in the second "sealed-bid stage", bidders may submit sealed bids on *all* possible bundles, subject to restrictions that reflect the history of bidding during the first stage (see Marsden, Sexton, and Siong, 2010). This format has been used in the LTE auctions in Switzerland and Austria (see BAKOM, 2010; TKK, 2013).

[12] Adding the constraint does not change the fact that truthful bidding is an equilibrium. This follows from the fact that the CVA with the spectrum caps is equivalent to an unconstrained CVA in which incumbents demand only one unit.

[13] We explain this one case on more detail as follows: In this case both incumbents win one block. In order to determine how much each incumbent has to pay, consider the counterfactual case that one incumbent, say incumbent A does not participate. In that case the CVA would still allocate one block to the other incumbent, B, and nothing to the entrant (because $V_{(2)} > 2V_E$). Therefore, the allocation of blocks to bidders other than bidder A is not affected by A's participation and hence bidder A pays nothing. The same is applies to the other incumbent B. Therefore, the seller's revenue is equal to zero.

block he has won and pays nothing; in that event one block remains unsold.

The exit option induces the entrant to radically change his behavior in three ways: the entrant always exercises the exit option if he fails to win both blocks, gives up his cautious initial stop rule, and changes his continuation strategy in such a way that he never suffers a winner's curse. Of course, the incumbents' equilibrium strategy is unchanged.

By elimination of (weakly) dominated strategies we find:

Proposition 3. *In the SMRA with exit option (SMRA*), the entrant plays the bid strategy:*

$$s(v_E) = v_E \tag{7}$$
$$\sigma(v_E, p) = 2v_E - p, \tag{8}$$

and exercises his exit option if he won only one block.

In order to characterize the equilibrium outcome and compute the seller's equilibrium revenue one needs to distinguish between the following events and associated equilibrium prices:

1) $V_{(2)} > V_E \Rightarrow$ the entrant quits first at price equal to V_E and each incumbent wins one block; seller's revenue $\Pi_0 = 2V_E$.

2) $V_{(2)} < V_E$ and $V_{(1)} > 2V_E - V_{(2)} \Rightarrow$ one incumbent quits first at price $p = V_{(2)}$, and the entrant quits second at price $2V_E - V_{(2)}$ and then exercises his exit option; seller's revenue $\Pi_0 = 2V_E - V_{(2)}$.

3) $V_{(2)} < V_E$ and $V_{(1)} < 2V_E - V_{(2)} \Rightarrow$ the entrant wins both blocks, the first one at price $p = V_{(2)}$ and the second at price $V_{(1)}$; seller's revenue $\Pi_0 = V_{(2)} + V_{(1)}$.

Again, we compute the seller's expected revenue and the probability of entry, based on these distinction of events, in Appendix A.3.

Evidently, the exit option completely eliminates the exposure risk. This impacts the seller's expected revenue in two ways: on the one hand, the entrant no longer plays a cautious initial stop rule, which contributes to higher revenue; on the other hand, the entrant plays a less aggressive continuation strategy, which reduces the seller's revenue.

5.2. SMRA *with exit and call options (SMRA**)*

Now we further fine-tune the *SMRA** by allowing the incumbent who wins one block to buy the second at the price he paid for the first (or a fixed fraction of that price if his marginal valuation for the second block is lower than that of the first).[14] In other words we supplement the entrant's exit option with a call option. That call option can only be exercised by the incumbent who wins the auction when the entrant has exercised his exit option.

Adding the call option expands the incumbents' strategy to include a decision rule that determines when to exercise the call option. In equilibrium, that decision rule prescribes to exercise the call option whenever the price the incumbent paid for the first block is not greater than the value of the second

[14]The case of diminishing marginal valuations is covered in Section 8 below. Of course, one could also stipulate that the incumbent who quit first can buy the returned block either at a fixed price or at a price determined in a supplementary auction. However, this would unnecessarily complicate the mechanism.

block which is, by assumption, equal to the value of the first block. Of course, an incumbent is only called upon to exercise the call option if the other incumbent quits first and the entrant wins one block but quits the second, in which case the price this incumbent pays per block is equal to the price at which the entrant quits on bidding the second block.

Next, we show that adding the call option does not affect equilibrium bidding strategies in the auction as we have described above in the SMRA*. Suppose one incumbent has quit first. Then the other incumbent bids up to his value, for the following reasons. If he deviates and bids up to more than his value, this makes a difference only if he wins (and would have lost otherwise), which makes him pay more for the first block than his value and leads him to not exercise the call option. Suppose one entrant has quit first. Then the auction ends. Similarly, if he deviates and bids up to less than his value, this makes a difference only if he thus loses (and would have won otherwise), in which case he forgoes a profit opportunity. Because adding the call option does not affect the incumbent's continuation strategy and does not directly affect the payoff of the entrant, it does not affect the entrant's continuation strategy either. Applying similar reasoning to the bidding strategies that prescribe the play before one bidder has quit first, while using the equilibrium payoffs of the continuation games, confirms that the equilibrium bidding strategies of the SMRA* also apply to the SMRA**.

Obviously, adding the call option ensures that no spectrum remains stranded, and it enhances the seller's revenue. Indeed, it increases the seller's revenue to such an extent that it always exceeds that of the CVA.

In order to characterize the equilibrium outcome and compute the seller's equilibrium revenue, one needs to distinguish between the following events and associated equilibrium prices:

1) $V_{(2)} > V_E \Rightarrow$ the entrant quits first at price equal to V_E and each incumbent wins one block; seller's revenue $\Pi_0 = 2V_E$.

2) $V_{(2)} < V_E$ and $V_{(1)} > 2V_E - V_{(2)} \Rightarrow$ one incumbent quits first at price $p = V_{(2)}$, and the entrant quits second at price $2V_E - V_{(2)}$; the entrant exercises his exit option and the incumbent who won one block exercises his call option and buys the second block at price $2V_E - V_{(2)}$; seller's revenue $\Pi_0 = 4V_E - 2V_{(2)}$.

3) $V_{(2)} < V_E$ and $V_{(1)} < 2V_E - V_{(2)} \Rightarrow$ the entrant wins both blocks, the first one at price $p = V_{(2)}$ and the second at price $V_{(1)}$; seller's revenue $\Pi_0 = V_{(2)} + V_{(1)}$.

Again, we compute the seller's expected revenue and the probability of entry, based on these distinction of events, in Appendix A.4.

6. Performance ranking

We now rank the performance of the proposed remedies of the exposure problems. Our results apply for all continuous probability distributions of marginal valuations.

Proposition 4. *The SMRA with exit and call option (SMRA**) is superior to the CVA: it gives rise to higher revenue of the seller, the same entry profile, a superior allocation if entry fails to occur, and zero probability of stranded spectrum. SMRA** is also superior to the SMRA*.*

Proof. First we show that the seller's revenue in the SMRA** is never lower and sometimes higher than that in the CVA:

1) $2V_E > V_A + V_B \Rightarrow$ the entrant wins both blocks and pays $V_A + V_B$, just like in the CVA.

2) $2V_E < V_A + V_B \Rightarrow$ the incumbents win just like in the CVA.

2a) $V_{(2)} > V_E \Rightarrow$ the entrant quits first; therefore, the seller's revenue is equal to $\Pi_0 = 2v_E$. This exceeds the seller's expected revenue in the CVA, Π_0', which is equal to

$$\Pi_0' = \begin{cases} 0 & \text{if} \quad 2V_E < V_{(2)} \\ 2V_E - V_{(2)} & \text{if} \quad 2V_E \in \big(V_{(2)}, V_{(1)}\big) \\ 2V_E - (V_A + V_B) & \text{if} \quad 2V_E > V_{(1)}. \end{cases} \tag{9}$$

2b) $V(2) < V_E \Rightarrow$ the weaker incumbent stops first, and the entrant stops second; the entrant exercises his exit option and the stronger incumbent exercises his call option; therefore, the seller's revenue is equal to $\Pi_0 = 2\big(2v_E - V_{(2)}\big) = 4V_E - 2V_{(2)}$. This exceeds the seller's revenue in the CVA, Π_0', which is equal to

$$\Pi_0' = \begin{cases} 2V_E - V_{(2)} & \text{if} \quad 2V_E < V_{(1)} \\ 4V_E - (V_A + V_B) & \text{if} \quad 2V_E > V_{(1)}. \end{cases} \tag{10}$$

Second, we show that entry occurs in the SMRA** auction if and only if it occurs in the CVA. Recall, in the SMRA** entry occurs if and only if $V_E > V_{(2)}$ and $2V_E - V_{(2)} > V_{(1)}$, which is equivalent to $2V_E - V_{(2)} > V_{(1)}$, and in the CVA entry occurs if and only if $2V_E > V_A + V_B$. It follows immediately that SMRA** and CVA have the same entry profile.

Third, notice that if entry does not occur, the CVA allocates one block to each incumbent, whereas the SMRA** allocates two blocks to the stronger incumbent if $V_{(2)} > V_E$, which creates more surplus.

Moreover, notice that spectrum is never stranded neither in CVA nor in SMRA**. This completes the performance ranking of SMRA** relative to CVA.

Finally, notice that the only difference between SMRA** and SMRA* is that, in the event when the entrant exercises his exit option, the incumbent who won one block buys the returned block. This increases the seller's revenue and ensures that no block is stranded. Therefore, SMRA** is superior to SMRA* on all counts. $\square$

While SMRA* and SMRA** are primarily designed to remedy the exposure problem that plagues SMRA,[15] they are not unambiguously more profitable and do not necessarily lead to a higher probability of successful entry. This is due to fact that in SMRA the entrant bids more cautiously on the first object, yet bids more aggressively on the second object.

In order to get a quantitative idea of the performance ranking, we explicitly computed an example, assuming uniformly distributed valuations. The results, which are summarized in Table 1, indicate that SMRA** performs significantly better on all counts than all other auction formats.[16]

[15] Of course, SMRA** also solves the leftover problem that plagues SMRA.

[16] The computation uses the formula for the seller's expected revenue, entry, and leftover probabilities that are spelled out in Appendix A.

	SMRA	SMRA*	SMRA**	CVA
π_0	0.598	0.667	0.750	0.583
ρ	0.500	0.500	0.500	0.500
λ	0.030	0.167	0	0

Table 1: Seller's expected revenue (π_0), entry (ρ), and leftover (λ) probabilities

7. Simulation results

In order to gain more insight into the performance ranking in terms of the seller's revenue, we carry out simulations based on 20,000 random draws of valuations from the uniform distribution.The simulation uses Myerson (2005)'s Excel based simulation software.

The results of the simulation are summarized in Figure 2. They indicate that the probability distribution of the seller's revenue in the SMRA** first-order stochastically dominates those of all other considered auction formats. Moreover, the considered SMRAs yield a significantly higher expected revenue than the CVA.

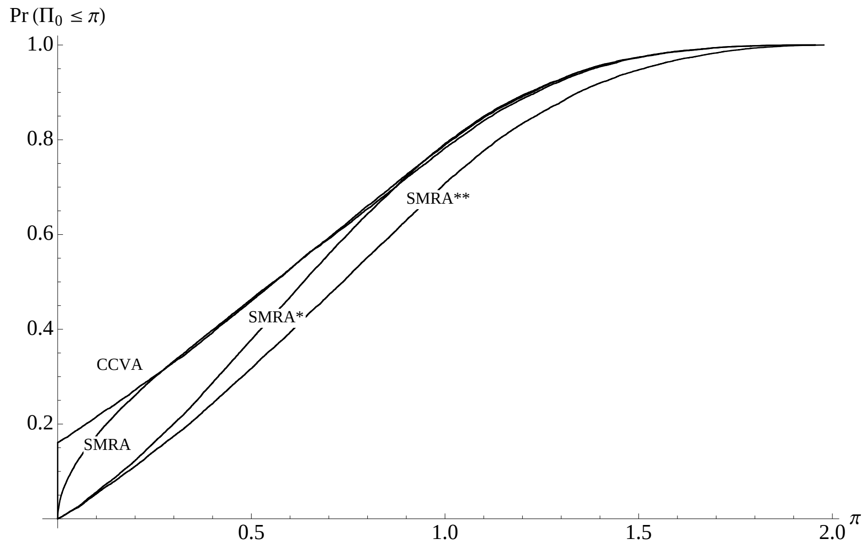

Figure 2: Simulation result: c.d.f.s of the seller's equilibrium revenue

8. Extensions and robustness

So far we used two extreme assumptions: 1) if incumbents are restricted to bid on one block, they bid only on their favored block, and 2) if incumbents are given a call option that allows them to bid on more than one block, their marginal valuations are flat, i.e., they value both blocks the same. These assumptions seem to be overly restrictive and may even appear to contradict each other. However, as we show now, these assumptions should be viewed as facilitating the exposition only. Indeed, if we replace these assumptions, our analysis extends qualitatively and our results become even stronger.

8.1. When spectrum blocks are substitutes for incumbents

As a first extension suppose incumbents view the two blocks as perfect substitutes. In that case, prices always move up *in tandem* because once an incumbent has quit the auction, the other incumbent will engage in arbitrage bidding, and alternate bidding on one and the other block. Therefore, if the entrant wins both blocks he now has to pay two times the price at which the last incumbent quit (in lieu of the sum of the two prices at which the incumbents quit), and if the entrant wins only one block, he pays the price at which he stops bidding (in lieu of the price at which the first incumbent quits). This, in turn, induces the entrant to engage in "strategic demand reduction" and apply more conservative stop rules.

Denote the entrant's equilibrium strategies in this case by $(\hat{s}(v_E), \hat{\sigma}(v_E, p))$. Comparing them with the equilibrium strategies that apply if blocks are not substitutes for the incumbents, $(s(v_E), \sigma(v_E, p))$, we find:

Proposition 5. *If spectrum blocks are perfect substitutes for incumbents, the entrant's exposure problem becomes even more severe and the entrant responds by quitting earlier, i.e., both the entrant's initial stop rule, $\hat{s}$, and the continuation game stop rule, $\hat{\sigma}$, in the basic SMRA, are lower than if blocks cannot be substituted, for all v_E, p:*

$$\hat{s}(v_E) < s(v_E), \quad \text{and} \quad \hat{\sigma}(v_E, p) \leq \sigma(v_E, p). \tag{11}$$

In particular, if F is convex and $v_E < 1/2$, the entrant quits immediately after one incumbent has quit first, $\hat{\sigma}(v_E, p) = p, \forall p$, and therefore the entrant does not bid in the first place, $\hat{s}(v_E) = 0, \forall v_E \leq 1/2$.

Proof. The proof is in the Appendix B. □

The deficiency of the SMRA is particularly extreme if F is convex and $v_E \leq 1/2$. In that case the exposure is so strong that entry never occurs and revenue is equal to zero. Therefore, in this case, the proposed remedies of the exposure problem perform considerably better on all counts.

8.2. When incumbents' marginal valuations are decreasing (rather than flat)

Our analysis also extends to the case when incumbents value the non-preferred block by a multiple γ of the preferred block's valuation. In that case the call option must take the form that the incumbent who wins the first block can buy the second at γ times the price of the first block (provided the entrant exercised his exit option). Of course, this requires that the seller knows the γ parameter. If he does not know this, the seller may re-auction the returned block among the two incumbents. However, in that case one has to take into account that bidding in the primary auction is affected. Working out the details for this case is beyond the scope of the present paper.

9. Discussion

One limitation of the present analysis is that we consider the sale of only two blocks of spectrum.

If more than two blocks of the same kind are made available, it is typically the case that neighboring spectrum is more valuable because the operator can better deal with interferences if he controls neighboring spectrum. This gives rise to yet another exposure risk, faced by incumbents and entrants alike: the risk of acquiring a fragmented portfolio of spectrum. In that regard, the designers of the spectrum auctions in Germany introduced another nice idea: the sale of generic, unnamed lots.

If generic lots are traded, at the time of bidding bidders do not know which concrete lots will be acquired. The regulator promises to allocate neighboring frequencies to achieve a contiguous holding of spectrum, after the auction. The advantage of this procedure is that bidders do not need to worry about fragmentation and at the time of the auction, all lots of the same kind are homogeneous spectrum blocks which greatly simplifies bidding.[17] However, after abstract frequencies have been auctioned, when it comes to allocate *concrete* frequencies there is typically a conflict of interest between bidders. This suggests that one adds another auction that determines who obtains which concrete frequencies.

If incumbents can bid on more than one block, the SMRA gives rise to another issue: strategic demand reduction. As we showed elsewhere, strategic demand reduction may and did give rise to very low revenue. In that case, the open-ascending bid format is not advisable, which has convinced auction designers to supplement the open ascending bid format by a final sealed-bid (phase).[18] Most current spectrum auction that employ an open ascending bid format include such a supplementary sealed-bid phase.

Another limitation of the analysis is that it does not model the strategic interaction between the spectrum auction and the aftermarket for mobile voice and data services. If one considers this linkage between markets, one deals with an auction with externalities problem, as it has been analyzed in the patent licensing literature (see, for example, Jehiel and Moldovanu (2000) and the subsequent literature). Before one can make a meaningful contribution to these issues, one must first develop a convincing and yet manageable model of the oligopolistic mobile phone market, which is beyond the scope of the present paper.

Appendix A.

Here we compute the seller's equilibrium expected revenue and the probability of entry in all considered auctions. These computations are used in the example which is summarized in Table 1 and in generating the simulation results in Section 7.

[17]Generic lots were employed in the German 2G, 3G, and in part in the recent 4G simultaneous multi-round auction. In the latter, the auctioned UMTS (2.2 GHz) frequencies were concrete (named) frequencies. These frequencies were resold in the year 2010 because two winners of the 3G auction in the year 2000 had unexpectedly returned their licenses shortly after the auction. Generic lots have also been employed in simultaneous multi-round spectrum auctions in other countries.

[18]An example of drastic demand reduction in a multi-unit SMRA is the GSM spectrum auction in Germany. For a detailed analysis of this auction case and a general result on the uniqueness of drastic demand reduction equilibrium see Grimm, Riedel, and Wolfstetter (2003) and Riedel and Wolfstetter (2006).

Appendix A.1. SMRA

Based on the distinction of events in Section 3.2, one obtains the seller's equilibrium expected revenue, π_0, and the equilibrium probability of entry, ρ (in (A.1) line 1 corresponds to event 1), line 2 corresponds to event 2), and line 3 corresponds to event 3)):

Therefore, the seller's equilibrium expected revenue, π_0, and equilibrium probability of entry, ρ, are equal to

$$\begin{aligned}\pi_0 = &\int_0^1 \int_{s(v)}^1 \int_z^1 2s(v) f_{(12)}(y,z) f(v) dydzdv \\ &+ \int_0^1 \int_0^{s(v)} \int_{\min\{2v,1\}}^1 (z+2v) f_{(12)}(y,z) f(v) dydzdv \\ &+ \int_0^1 \int_0^{s(v)} \int_z^{\min\{2v,1\}} (z+y) f_{(12)}(y,z) f(v) dydzdv\end{aligned} \tag{A.1}$$

$$\begin{aligned}\rho &= \Pr\{V_{(2)} \leq s(V_E) \,\text{and}\, V_{(1)} \leq 2V_E\} \\ &= \int_0^1 \int_0^{s(v)} \int_z^{\min\{2v,1\}} f_{(12)}(y,z) f(v) dydzdv.\end{aligned} \tag{A.2}$$

The leftover probability is $\lambda = \Pr\{V_{(2)} < s(V_E) \,\text{and}\, V_{(1)} > 2V_E\}$.

If F is the uniform distribution, the seller's expected revenue is $\pi_0 \approx 0.598$, the probability of entry is $\rho = 1/2$ and the leftover probability is $\lambda = 0.03$.

Appendix A.2. CVA

Based on the distinction of events in Section 4, one obtains the seller's equilibrium expected revenue, π_0, and the equilibrium probability of entry, ρ (in (A.3) line 1 corresponds to event 1), line 2 corresponds to event 2b), and lines 3 and 4 correspond to event 2c)):

$$\begin{aligned}\pi_0 = &\int_0^1 \int_0^{\min\{v,1\}} \int_z^{\min\{2v-z,1\}} (z+y) f_{(12)}(y,z) f(v) dydzdv \\ &+ \int_0^1 \int_0^{\min\{2v,1\}} \int_{\min\{2v,1\}}^1 (2v-z) f_{(12)}(y,z) f(v) dydzdv \\ &+ \int_0^1 \int_{\min\{v,1\}}^{\min\{2v,1\}} \int_z^{\min\{2v,1\}} (4v-z-y) f_{(12)}(y,z) f(v) dydzdv \\ &+ \int_0^1 \int_0^{\min\{v,1\}} \int_{\min\{2v-z,1\}}^{\min\{2v,1\}} (4v-z-y) f_{(12)}(y,z) f(v) dydzdv\end{aligned} \tag{A.3}$$

$$\begin{aligned}\rho &= \Pr\{V_A + V_B < 2V_E\} \\ &= \int_0^1 \int_0^1 \Pr\{V_A \leq 2v - x\} f(x) f(v) dxdv \\ &= \int_0^1 \int_0^1 F(2v-x) f(x) f(v) dxdv.\end{aligned} \tag{A.4}$$

The leftover probability is obviously $\lambda = 0$.

If $F(v) \equiv v$ (uniform distribution) one finds $\pi_0 = 7/12 \approx 0.583$, which is less than the expected revenue in the SMRA, $\rho = 1/2$, and $\lambda = 0$.

*Appendix A.3. SMRA**

Based on the distinction of events in Section 5, one obtains the seller's equilibrium expected revenue (there line 1 corresponds to event 1), line 2 corresponds to event 2), and line 3 corresponds to event 3):

$$\begin{aligned}\pi_0 = &\int_0^1 \int_{\min\{v,1\}}^1 \int_z^1 2v f_{12}(y,z) f(v) dy dz dv \\ &+ \int_0^1 \int_0^{\min\{v,1\}} \int_{\min\{2v-z,1\}}^1 (2v - z) f_{12}(y,z) f(v) dy dz dv \\ &+ \int_0^1 \int_0^{\min\{v,1\}} \int_z^{2v-z} (z + y) f_{12}(y,z) f(v) dy dz dv\end{aligned} \tag{A.5}$$

The probability of entry $\rho = \Pr\{V_A + V_B < 2V_E\}$ is evidently the same as in CVA. The leftover probability is $\lambda = \Pr\{V_E > V_{(2)} \text{ and } V_{(1)} + V_{(2)} > 2V_E\}$.

If $F(v) \equiv v$ (uniform distribution) one finds $\pi_0 = 2/3 \approx 0.667$, which is greater than the expected revenue in the SMRA, $\rho = 1/2$, and $\lambda = 0.167$.

*Appendix A.4. SMRA***

Based on the distinction of events in Section 5.2, one obtains the seller's equilibrium expected revenue, π_0, (there line 1 corresponds to event 1), line 2 corresponds to event 2), and line 3 corresponds to event 3):

$$\begin{aligned}\pi_0 = &\int_0^1 \int_{\min\{v,1\}}^1 \int_z^1 2v f_{12}(y,z) f(v) dy dz dv \\ &+ \int_0^1 \int_0^{\min\{v,1\}} \int_{\min\{2v-z,1\}}^1 2(2v - z) f_{12}(y,z) f(v) dy dz dv \\ &+ \int_0^1 \int_0^{\min\{v,1\}} \int_z^{2v-z} (z + y) f_{12}(y,z) f(v) dy dz dv\end{aligned} \tag{A.6}$$

Again, the probability of entry $\rho = \Pr\{V_A + V_B < 2V_E\}$ is evidently the same as in CVA, and the leftover probability is obviously equal to zero.

If $F(v) \equiv v$ (uniform distribution) one finds $\pi_0 = 0.75$, which is greater than the expected revenue in the SMRA*, $\rho = 1/2$, and $\lambda = 0$.

Appendix B. Proof Proposition 5

1) As a first step, we solve the entrant's continuation strategy $\hat{\sigma}(v_E)$. For this purpose, suppose one incumbent has quit first at price p while the remaining incumbent plays his dominant strategy and bids his value (alternating between bidding on A and B so that prices move up in tandem). For simplicity we write

$$\hat{b} := \min\{\hat{\sigma}(v_E), 1\}.$$

1a) If $v_E \geq 1/2$, the entrant's best reply is to bid up to the bundle value $2v_E$, because the remaining incumbent will quit with probability one at a price below 1, which is below the bundle value $2v_E \geq 1$.

1b) If $v_E < 1/2$, the entrant's best reply is to bid less than the bundle value $2v_E$. In particular, the entrant's expected profit is a function of his bid b (stopping point) and the price p, as follows:[19]

$$\Pi_E(b,p) = \int_p^b 2(v_E - v)\frac{f(v)}{1-F(p)}dv - b\int_b^1 \frac{f(v)}{1-F(p)}dv \tag{B.1}$$

$$\partial_b\Pi_E(b,p) = \frac{1}{1-F(p)}\varphi(b), \quad \varphi(b) := f(b)\left(2v_E - b - \frac{1-F(b)}{f(b)}\right). \tag{B.2}$$

The best reply is either one of the two corner solutions, $\hat{b} \in \{p, 1\}$ or an interior solution that solves the condition $\varphi(\hat{b}) = 0$ for some $\hat{b} \in (p, 1)$. Evidently, the particular corner solution $\hat{b} = 1$ cannot apply because $b = 1 \Rightarrow \partial_b\Pi_E(b,p) < 0$.

If there is an interior solution, one has $\varphi(b) = 0$ and hence

$$\hat{b} = 2v_E - \frac{(1-F(\hat{b}))}{f(\hat{b})} < 2v_E, \tag{B.3}$$

as asserted.

However, typically the solution is the other corner solution $\hat{b} = p$ (stop immediately). In particular, if F is convex (which includes the uniform distribution case), the best reply is $\hat{b} = p$ because there is no interior solution and $b = p \Rightarrow \partial_b\Pi_E(b,p) < 0$, because

$$\begin{aligned}
\varphi(b) &:= (2v_E - b)f(b) - \int_b^1 f(v)dv \\
&\leq (2v_E - b)f(b) - \int_b^1 f(b)dv \qquad \text{(because } f \text{ is non-decreasing)} \\
&= f(b)(2v_E - 1) \\
&< 0 \qquad \text{(because } v_E < 1/2\text{)}.
\end{aligned}$$

2) Next, we characterize the initial stop rule, $\hat{s}$, and show that $\hat{s}(v_E) < s(v_E)$, for all v_E.

Using the same qualitative distinction of events as in the proof of Proposition 2, the reduced form payoff function (incorporating the equilibrium of the continuation game) as a function of the initial stop rule, s, is

$$\begin{aligned}
\pi_E(s) =& 2v_E\int_0^s\int_z^{\hat{b}} f_{(12)}(y,z)dydz - \int_0^s\int_z^{\hat{b}} 2yf_{(12)}(y,z)dydz \\
&- \hat{b}\int_0^s\int_{\hat{b}}^1 f_{(12)}(y,z)dydz.
\end{aligned}$$

2a) Suppose $v_E \geq 1/2$. Then $\hat{b} = 1$ and $\hat{s}(v_E)$ must solve the condition $\partial_s\pi_E(s) = 0$. For all $s \in (0,1]$ the sign of $\partial_s\pi_E(s)$ is the same as the sign of $2v_E - E[2V \mid V > s]$. Therefore, $\hat{s}(v_E)$ must solve the condition: $\hat{g}(s) := E[2V \mid V > s] = 2v_E$. Similarly, the initial stop rule in the model when spectrum blocks are not substitutes, $s(v_E)$, solves the condition (see (3)): $g(s) := E[V + s \mid V >$

[19] We assume without loss of generality $b \leq 1$.

$s] = 2v_E$. Obviously, $\hat{g}(s) > g(s)$; and because $\hat{g}$ and g are strictly increasing, it follows immediately that $\hat{s}(v_E) < s(v_E)$.

2b) Suppose $v_E < 1/2$. Then $\hat{\sigma}$ has either the corner solution to stop immediately or an interior solution characterized by (B.3).

If $\hat{\sigma}$ has the corner solution, which occurs for example if F is convex, the entrant cannot ever win the bundle of spectrum blocks, and therefore does not bid at all; in that case, trivially, $\hat{s}(v_E) = 0 < s(v_E)$.

Whereas if $\hat{\sigma}$ has the interior solution, $\hat{s}$ solves the condition $\partial_s \pi_E(s) = 0$ which can be rewritten as

$$\int_{\hat{b}}^{1} s dF(y) = \int_{s}^{b^*} (2v_E - y) dF(y) - \int_{s}^{b^*} y dF(y).$$

Adding $\int_s^{\hat{b}} s dF(y)$ to both sides of the equation one has

$$\begin{aligned} \int_{s}^{1} s dF(y) &= \int_{s}^{\hat{b}} (2v_E - y) dF(y) - \int_{s}^{\hat{b}} (y - s) dF(y) \qquad \text{(B.4)} \\ &< \int_{s}^{\hat{b}} (2v_E - y) dF(y) \\ &< \int_{s}^{2v_E} (2v_E - y) dF(y). \end{aligned}$$

Now recall that the initial stop rule when spectrum blocks are not substitutable, $s(v_E)$, and $v_E < 1/2$, solves the condition

$$\int_{s}^{1} s dF(y) = \int_{s}^{2v_E} (2v_E - y) dF(y). \qquad \text{(B.5)}$$

Because the RHS of (B.4) is smaller than the RHS of (B.5) and $\int_s^{2v_E} (2v_E - y) dF(y)$ is decreasing in s, it follows that $\hat{s}(v_E) < s(v_E)$, as asserted.

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