Calibrating CAT Bonds for Mexican Earthquakes

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Mexico is exposed to earthquake risk (EQ):

- EQ disasters are huge and volatile
- An 8.1 Mw EQ hit Mexico in 1985: estimated payouts of 4 billion dollars

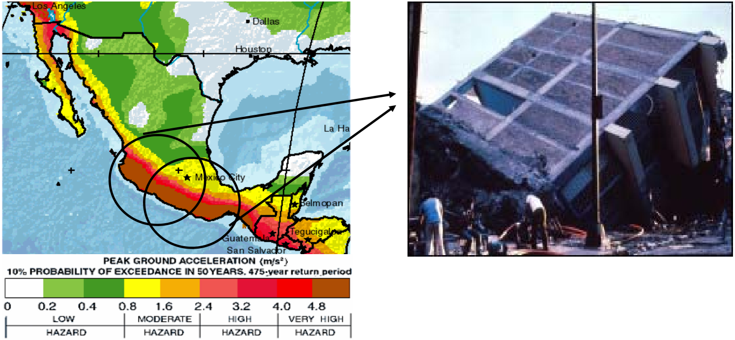


Figure 1: Location of epicenters

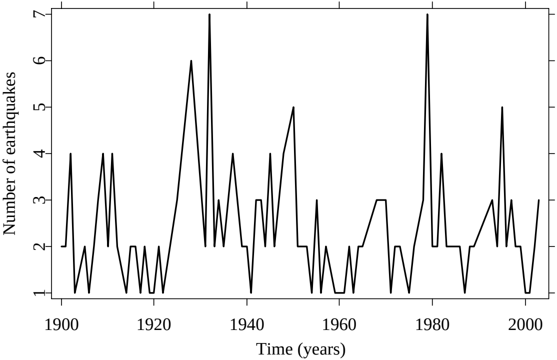


Figure 2: Number of EQs higher than 6.5 Mw in Mexico during 1900-2003.

CAT bonds

- Reconstruction can be financed by transfering the risk with CAT bonds
  - From insurers, reinsurance and corporations (sponsors) to capital market investors
- Alternative or complement to traditional reinsurance
- Supply protection against natural catastrophes without credit risk present in reinsurance
- Offer attractive returns and reduce the portfolio risk
- Attractive surplus alternatives

Calibrating CAT bonds

Pure parametric index trigger: payouts are triggered by the occurrence of a catastrophic event with certain defined physical parameters.

- The intensity rate (λ) describes the flow process of EQ:
  - Reinsurance market (λ_1): Ceding & Reinsurance company
  - Capital market (λ_2): SPV & investors
  - Historical data (λ_3): real intensity of EQ
- Comparative analysis: is $\lambda_1 = \lambda_2 = \lambda_3$? Fair?

Pricing CAT bonds

Modeled – index trigger: the physical parameters of the catastrophe are used to estimate the expected losses to the ceding company's portfolio.

- Different variables affect the value of the loss: physical parameters, building material, construction design, impact on main cities, etc.
- Minimization of basis risk borne by the sponsor, while remaining non-indemnity based.

Calibrating CAT bonds

- Does the trigger mechanism matter in pricing?
- What are the differences in pricing?
- What are the stochastic properties that influence the valuation?

Outline

1. Motivation ✓
2. What are CAT bonds?
3. Calibrating the parametric Mexican CAT Bond
4. Pricing a Modeled-index CAT bond

CAT bonds

- Ease the transfer of catastrophic insurance risk
- Coupons and principal depend on the performance of a pool or index of natural catastrophe risks
- Parties: Sponsor, SPV, collateral & investors
- If there is no event: SPV gives the principal back to the investors with the final coupon
- If there is an event: investors sacrifices fully or partially their principal plus interest and the SPV pays the insured loss

Structure of Cash flows

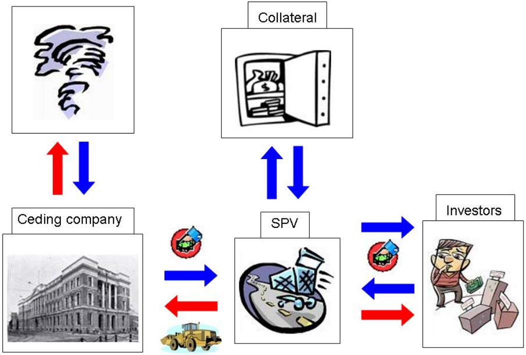


Figure 3: Cash Flows Diagram. Event (red), no event (blue)

Trigger mechanisms

1. **Indemnity:** Actual loss of the ceding company
2. **Modeled loss:** A third party projects the expected losses to the ceding company´s portfolio
3. **Industry index:** The ceding recovers a % of total industry losses in excess of a predetermined point
4. **Parametric index:** weighting boxes exposure
   - Hurricane index value $= K \sum_{i=1}^{I} w_i (v_i - L)^n$
5. **Pure parametric index:** Richter Scale

CAT-MEX bond

Issue Date	May-06
Sponsor	Mexican government
SPV	CAT-Mex Ltd
Reinsurer	Swiss Re
Total size (P)	$160 mio.
Risk Period	3 year
Risk	Earthquake
Structure	Pure Parametric
Spread (z)	LIBOR plus 235 basis points
Total coverage	$450 mio.
Premiums	$26 mio.

Table 1: Mexican parametric CAT bond

Cash flows CAT-MEX

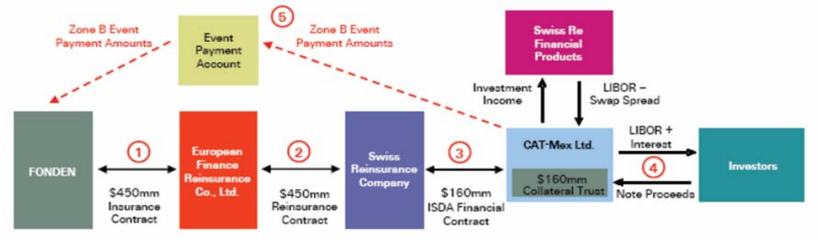


Figure 4: The cash flows diagram for the mexican CAT bond.

- Air Worldwide Corporation modeled the seismic risk
- Given the federal governmental budget plan: 3 zones

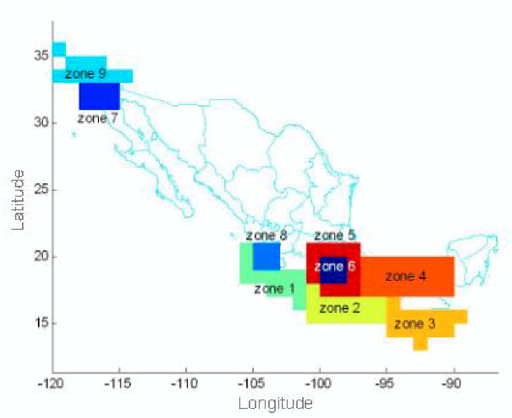


Figure 5: Map of regions

- The CAT bond payment would be triggered if:

Zone	Threshold u in $Mw \geq$ to
Zone 1	8
Zone 2	8
Zone 5	7.5

Table 2: Thresholds u's of the Mexican parametric CAT bond

- In case of a trigger event:
  - Swiss Re pays the covered insured amount to the government
  - Investors sacrifice full principal and coupons
- Premium & proceeds are used to pay coupons to bondholders

Assumptions

The arrival process of EQs $N_t, t \geq 0$ uses the times between EQ $\tau_i = T_i - T_{i-1}$:

$$N_t = \sum_{n=1}^{\infty} \mathbf{1}(T_n < t)$$

EQ suffer loss of memory: $\mathrm{P}(X > x + y | X > y) = \mathrm{P}(X > x)$

N_t is a Homogeneous Poisson Process (HPP) with intensity rate $\lambda > 0$:

- N_t is governed by the Poisson law
- The waiting times τ_i are i.i.d. $\exp(\lambda)$

Assumptions

The probability of occurrence of an EQ in the interval $(0, t]$ is:
$\mathrm{P}(\tau_i < t) = 1 - \mathrm{P}(\tau_i \geq t) = 1 - e^{-\lambda t}$

Define *stopping time* equal to:

$$\tau = \min \{t : N_t > 0\}$$

where $f_\tau(t) = \lambda e^{-\lambda t}$ is the density of occurrence of event

Calibrating Parametric CAT bond

The intensity rate (λ) describes the flow process of an EQ:

- Reinsurance market (λ_1): Ceding & Reinsurance company
- Capital market (λ_2): SPV & investors
- Historical data (λ_3): real intensity of EQ

Assumptions:

- Flat term structure of continuously compounded discount interest rates
- N_t is a HPP

Reinsurance market intensity: λ_1

Let H be the total premium paid by the government (26 mio.) and let $J = 450 \cdot \mathbf{1}(\tau < 3)$ be the payoff.

A compounded discounted *actuarially fair insurance price* at $t = 0$ is:

$$
\begin{aligned}
H &= \mathsf{E}\left[Je^{-\tau r_\tau}\right] \\
&= \mathsf{E}\left[450 \cdot \mathbf{1}(\tau < 3) e^{-\tau r_\tau}\right] \\
&= 450 \int_0^3 e^{-r_t t} f_\tau(t)\,dt
\end{aligned}
$$

where $f_\tau(t) = \lambda_1 e^{-\lambda_1 t}$ is the density of occurrence of event.

LIBOR in May 2006 $r_t = \log(1.0541)$,

$$26 = 450 \int_0^3 e^{-\log(1.0541)t} \lambda_1 e^{-\lambda_1 t} dt \tag{1}$$

Hence λ_1=0.0215, i.e. Swiss Re expects:

- 2.15 events in 100 years
- Probability of occurrence of an event in 3 years equal to 0.0624

Capital market intensity: λ_2

- CAT bond with coupons every 3 months and payment of the principal P at T
- Coupon bonds pay a fixed spread z=230 bp. over LIBOR
- Coupons equal to $C = \left(\frac{r+z}{4}\right) \cdot P = \$3.1055$ mio
- In case of an event: investor sacrifices principal P & coupons
- Let G be the investors' gain

A discounted *fair bond price* at time $t = 0$ is given by:

$$
\begin{aligned}
P &= \mathsf{E}\left[G \left(\frac{1}{1+r_\tau} \right)^{\tau} \right] \\
&= \mathsf{E}\left[\sum_{t=1}^{12} C \cdot \mathbf{1}(\tau > \frac{t}{4}) \left(\frac{1}{1+r_t} \right)^{\frac{t}{4}} + P \cdot \mathbf{1}(\tau > 3) \left(\frac{1}{1+r_t} \right)^{3} \right]
\end{aligned}
$$

Then,

$$
160 = \sum_{t=1}^{12} 3.1055 \left(\frac{e^{-\lambda_2}}{1.0541} \right)^{\frac{t}{4}} + \frac{160 e^{-3\lambda_2}}{(1.0541)^3} \qquad (2)
$$

Hence, $\lambda_2 = 0.0241$. The capital market estimates a probability of occurrence of an event in 3 years equal to 0.0644, equivalently to 2.4 events in 100 years

Historical Intensity: λ_3

Descriptive	time(t)	magnitude(Mw)
Minimum	1900	6.50
Maximum	2003	8.20
Mean	-	6.90
Median	-	6.90
Sdt. Error	-	0.37
25% Quantile	-	6.60
75% Quantile	-	7.10
Excess	-	0.25
Nr. obs.	192	192.00
Distinct obs.	82	18.00

Table 3: Descriptive statistics of EQ data from 1900 to 2003 (SSN)

Intensity model

- Let Y_i be i.i.d rvs. indicating Mw of the i^{th} EQ at time t
- Let $\varepsilon_i = \mathbf{1}(Y_i \geq \bar{u})$ characterizing EQ with Mw higher than a defined threshold for a specific location
- N_t is a HPP with intensity $\lambda > 0$

A new process B_t defines the trigger event process:

$$B_t = \sum_{i=1}^{N_t} \mathbf{1}\left(\varepsilon_i > 0\right) \tag{3}$$

- Data contains only 3 events: the calibration of the intensity of B_t is based on 2 waiting times τ_i

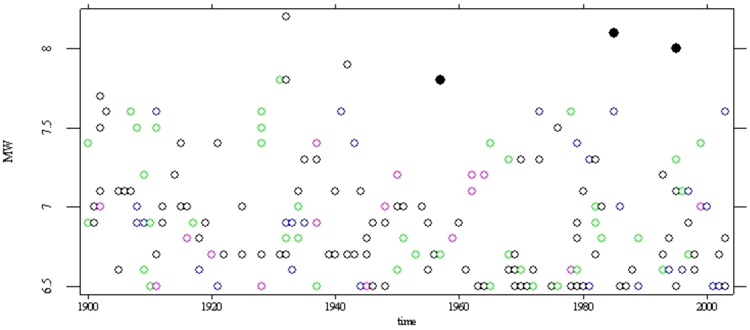


Figure 6: Mw of trigger events (filled circles), EQs in zone 1 (black circles), EQs in zone 2 (green circles), EQs in zone 5 (magenta circles), EQs out of insured zones (blue circles) eq65thMexcase.xpl

Consider B_t and define p as the probability of occurrence of a trigger event conditional on the occurrence of the earthquake. The probability of no event up to time t:

$$\begin{aligned} \mathrm{P}(B_t = 0) &= \sum_{k=0}^{\infty} \mathrm{P}(N_t = k)(1-p)^k \\ &= \sum_{k=0}^{\infty} \frac{(\lambda t)^k}{k!} e^{(-\lambda t)}(1-p)^k \\ &= e^{-\lambda p t} = e^{-\lambda_3 t} \end{aligned} \tag{4}$$

The annual historical intensity rate for a trigger event is equal to $\lambda_3 = \lambda p = 1.8504 \left(\frac{3}{192}\right) = 0.0289$

Calibration of intensity rates

	λ_1	λ_2	λ_3
Intensity (10^{-2})	2.15	2.41	2.89
Prob. of event in 1 year (10^{-2})	2.12	2.19	2.84
Prob. of event in 3 year (10^{-2})	6.24	6.44	8.30
No. expected events in 100 years	2.15	2.41	2.89

Table 4: Intensity rates

$\lambda_1 \neq \lambda_2$:

- Absence of the public & liquid market of EQ risk in the reinsurance market: limited information is available
- Contracts in the capital market are more expensive than in the reinsurance market: cost of risk capital & risk of default

$\lambda_1 \neq \lambda_2 \neq \lambda_3$:

- λ_3 is based on the time period of the historical data
- If λ_3 would be the "real" intensity rate:
  - The Mexican government paid total premiums of \$26 mio. that is 0.75 times the real actuarially fair one:

$$\int_0^3 450\lambda_3 e^{-t(r_t+\lambda_3)}dt = 34.49$$

  - Savings of \$8.492 mio.? NO
  - Probability of defaults of the reinsurer over the 3 three years $\approx$ the price discount that the Government gets in the risk transfer of EQ risk
  - The mix of the reinsurance contract and the CAT bond: 35% of the total seismic risk to the investors

Modeled-Index CAT bond for earthquakes

- Minimization of basis risk borne by the sponsor, while remaining non-indemnity based.
- Other variables can affect the value of losses: magnitude (Mw), depth (DE), location, impact on Mexico city $\texttt{IMP}(0,1)$.
  - Losses are $\propto$ Mw & time t & inversely $\propto$ DE of EQ
- We built loss data from EQs during 1900-2003
- Losses $\{X_k\}_{k=1}^{\infty}$ adjusted to population, inflation, exchange rate
- Missing loss data treatment: Expectation-Maximum (EM) algorithm

Losses of EQ in 100 years

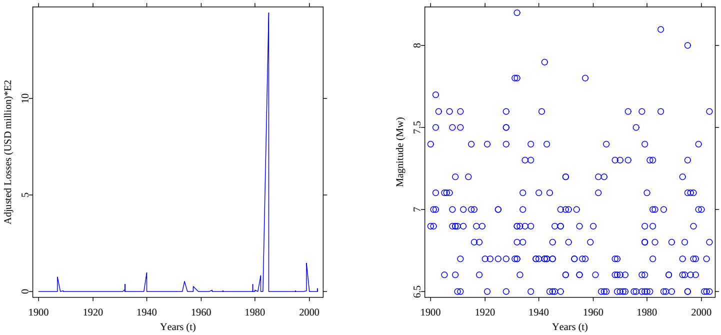


Figure 7: Adjusted Losses - Richter Scale CMX02.xpl

Compound Doubly Stochastic Poisson Pricing Model

Baryshnikov et al. (2001):

- A doubly stochastic Poisson process N_s, i.e. a Poisson process conditional on a stochastic intensity process λ_s with $s \in [0, T]$, describing the flow of a particular catastrophic natural event in a specified region.
  - HPP with an intensity $\lambda = 1.8504$
  - NHPP with intensity $\lambda_s^1 = 1.8167$

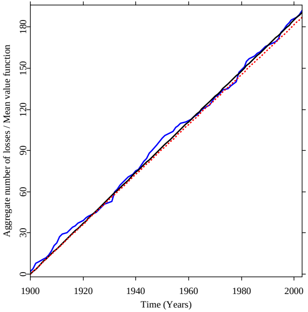


Figure 8: The accumulated number of EQs (solid blue line) and mean value functions $E(N_t)$ of the HPP with intensity $\lambda_s = 1.8504$ (solid black line) and the $\lambda_s^1 = 1.8167$ (dashed red line) CMXrisk03.xpl

- The countinuous and predictable aggregate loss process is:

$$L_t = \sum_{i=1}^{N_t} X_i \tag{5}$$

where $\{X_k\}_{k=1}^{\infty}$ at t_i are i.i.d with $F(x) = \mathrm{P}(X_i < x)$

- The threshold level D and a continuously compounded discount interest rate r: $e^{-R(s,t)} = e^{\int_s^t r(\xi)d\xi}$
- A threshold time event $\tau = \inf\{t : L_t \geq D\}$, defined it as a point of a doubly stochastic Poisson process $M_t = \mathbf{1}(L_t > D)$ with a stochastic intensity:

$$\Lambda_s = \lambda_s \{1 - F(D - L_s)\} \mathbf{1}(L_s < D) \tag{6}$$

Modeled loss:

$$\log(X) = -27.99 + 2.10Mw + 4.44DE - 0.15\mathtt{IMP}(0,1) - 1.11\log(Mw)\cdot DE$$

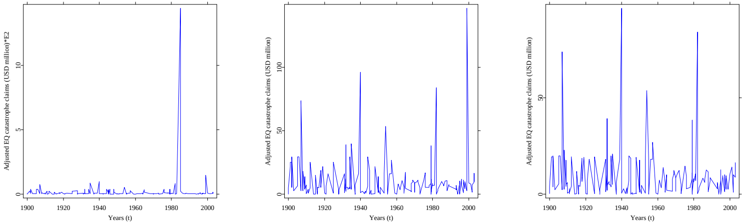


Figure 9: Historical and modeled losses of EQs from 1900-2003 (left panel), without the outlier of the EQ in 1985 (middle panel), without outliers of EQ in 1985 and 1999 (right panel) CMXmyEMalgorithm.xpl

Fitting & testing the loss distribution:

Distrib.	Log-normal	Pareto	Burr	Exponential	Gamma	Weibull
Parameter	$\mu = 1.387$ $\sigma = 1.644$	$\alpha = 2.394$ $\lambda = 12.92$	$\alpha = 3.323$ $\lambda = 16.67$ $\tau = 0.919$	$\beta = 0.143$	$\alpha = 0.143$ $\beta = -0.007$	$\beta = 0.220$ $\tau = 0.764$
Kolmogorov Sminorv	0.173	0.131	0.137	0.135	0.295	0.145
(D test)	(< 0.005)	(< 0.005)	(< 0.005)	(< 0.005)	(< 0.005)	(< 0.005)
Kuiper	0.296	0.248	0.260	0.222	0.569	0.282
(V test)	(< 0.005)	(< 0.005)	(< 0.005)	(< 0.005)	(< 0.005)	(< 0.005)
Cramér-von Mises	1.358	0.803	0.884	0.790	7.068	1.051
(W^2 test)	(< 0.005)	(< 0.005)	(< 0.005)	(< 0.005)	(< 0.005)	(< 0.005)
Anderson Darling	10.022	5.635	5.563	9.429	36.076	5.963
(A^2 test)	(< 0.005)	(0.005)	(0.01)	(< 0.005)	(< 0.005)	(< 0.005)

Table 5: Parameter estimates by A^2 minimization procedure and test statistics. In parenthesis, the related p-values based on 1000 simulations

Zero Coupon CAT bonds (ZCCB)

- Pays P at T conditional on $\tau > T$
- The payment at maturity is independent from the occurrence and timing of D
- In case of a trigger event P is fully lost

The non arbitrage price of the ZCCB V_t^1, Burnecki and Kukla (2003):

$$\begin{aligned} V_t^1 &= \mathsf{E}\left[Pe^{-R(t,T)} \left(1 - M_T\right) | \mathcal{F}_t \right] \\ &= \mathsf{E}\left[Pe^{-R(t,T)} \left\{ 1 - \int_t^T \lambda_s \left\{1 - F(D - L_s)\right\} \mathbf{1}\left(L_s < D\right) d_s \right\} | \mathcal{F}_t \right] \end{aligned}$$

Coupon CAT bonds (CCB)

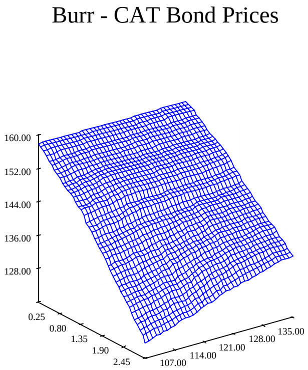
- Pays P at T & gives coupons C_s until τ
- The payment at maturity is independent from the occurrence and timing of D
- Pays a fixed spread z (bp.+LIBOR)
- In case of a trigger event P is fully lost

The non arbitrage price of the CCB V_t^2, Burnecki and Kukla (2003):

$$\begin{aligned} V_t^2 &= \mathrm{E}\left[Pe^{-R(t,T)}\left(1-M_T\right) + \int_t^T e^{-R(t,s)} C_s \left(1-M_s\right) d_s \middle| \mathcal{F}_t \right] \\ &= \mathrm{E}\left[Pe^{-R(t,T)} + \int_t^T e^{-R(t,s)} \left\{ C_s \left(1 - \int_t^s \lambda_\xi \left\{1-F(D-L_\xi)\right\} \right. \right. \right. \\ & \qquad \left.\left.\left. \mathbf{1}(L_\xi < D) d_\xi \right) - Pe^{-R(s,T)} \lambda_s \left\{1-F(D-L_s)\right\} \mathbf{1}\left(L_s < D\right) \right\} d_s \middle| \mathcal{F}_t \right] \end{aligned}$$

Calibration

- r equal to the LIBOR ($r = \log(1.0541)$)
- P=\$160 mio.
- $T \in [0.25, 3]$ years
- $D \in [\$100, \$135]$ mio. (0.7 & 0.8-quantiles of 3 yearly acc.losses)
- $z = 235$ bp. over LIBOR
- Quarterly $C_t = \left(\frac{LIBOR+235bp}{4}\right)$ \$160=\$3.1055 mio.
- N_t is an HPP with intensity $\lambda_s = 1.8504$
- 1000 Monte Carlo simulations



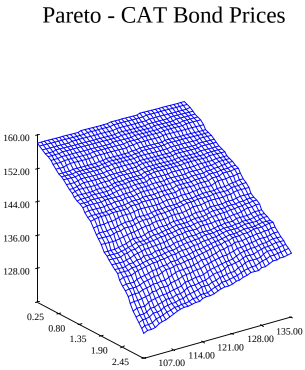


Figure 10: The CCB price (vertical axis) with respect to T (horizontal left axis) & D (horizontal right axis) in the Burr-HPP (left panel) & Pareto-HPP (right panel) for the modeled loss CMX07e.xpl

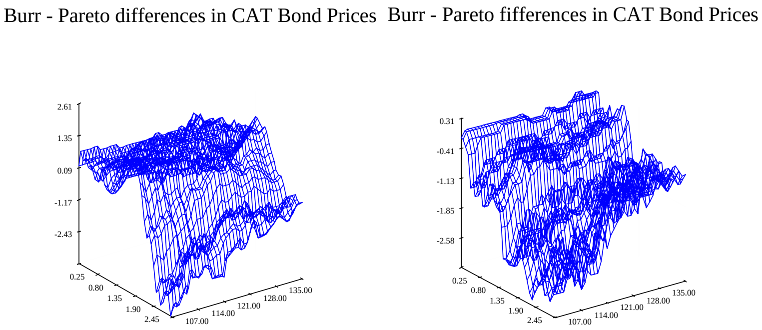


Figure 11: The difference in ZCCB price (left panel) & CCB prices (right panel) in the vertical axis left panel between the Burr & Pareto distributions under a HPP, with respect to the T (horizontal left axis) & D (horizontal right axis) CMX06f.xpl

	Min. (% Principal)	Max. (% Principal)
Diff. ZCB Burr-Pareto	-2.640	0.614
Diff. CB Burr-Pareto	-1.552	0.809
Diff. ZCB-CB Burr	-6.228	-0.178
Diff. ZCB-CB Pareto	-5.738	-0.375

Table 6: Min. & max. of the diff. in the ZCCB-CCB prices in % of P for the Burr-Pareto distributions of the modeled loss

- V_t^1 & V_t^2 decreases as T increases
- V_t^1 & V_t^2 increases as D increases
- F(x) influences the price of the CAT bond
- Modeled loss: no significant impact on ZCCB-CCB prices, but more important than the loss distribution

Conclusion

- Seismic risk can be transfered with CAT bonds
- CAT bonds: No credit risk, high returns and better performance of the portfolio
- Calibration of a Mexican CAT bond:
  1. Parametric trigger (physical parameters): the intensity rates of EQ in $\neq$ parts of the contract vs. real historical
     - N_t is a HPP with intensity λ
  2. Modeled-Index loss trigger considers several variables: the intensity rate of EQ and the level of accumulated losses L_s

References

P. Cizek, W. Härdle, R. Weron (2005)
Statistical Tools for Finance and Insurance
Springer Verlag, Heidelberg. .

Yu. Baryshnikov, A. Mayo, D.R Taylor (2001)
Pricing Cat Bonds
preprint.

K. Burnecki, G. Kukla (2003)
Pricing of Zero-Coupon and Coupon Cat Bonds, Appl. Math
(Warsaw) 30(3): 315-324.

References

W. Dubinsky, D. Laster (2005)
SIGMA: Insurance - link securities
Swiss Re publications.

LIBOR Rate
http://www.fanniemae.com/tools/libor/2006.jhtml

B. Lopez (2003)
Valuación de bonos catastróficos para terremotos en México
http://www.mexder.com.mx/inter/info/mexder/avisos/ValuaciondeBonoscatastroficosparaTerremotosenMexico.pdf

References

- B. Lopez (2006)
  Pricing Catastrophic Bonds for earthquakes in Mexico
  http : //www.edoc.hu − berlin.de/docviews/abstract.php?lang = ger&id = 27524

- Secretaría de Hacienda y Crédito Público Mexico (SHCP) (2004)
  Administración de Riesgos Catastróficos del FONDEN
  Secretaría de Hacienda y Crédito Público.

- Servicio Sismologico Nacional Instituto de Geosifísica UNAM Mexico (SSN) (2006)
  Earthquakes Data Base
  UNAM.