Localizing Temperature Risk

温度风险

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Weather

PricewaterhouseCoopers Survey 2005 releases the Top 5 sectors in need of financial instruments to hedge weather risk.

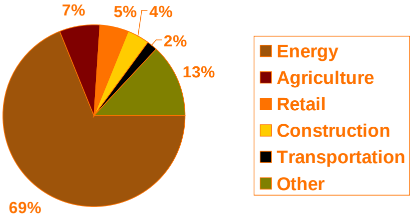


Figure 1: PwC survey 2005 for Weather Risk Management Association

Weather

- Influences our daily lives and choices
- Impact on corporate revenues and earnings
- Meteorological institutions: business activity is weather dependent
  - British Met Office: daily beer consumption gain 10% if temperature increases by 3° C
  - If temperature in Chicago is less than 0° C consumption of orange juice declines 10% on average

Examples

- Natural gas company suffers negative impact in mild winter
- Construction companies buy weather derivatives (rain period)
- Cloth retailers sell fewer clothes in hot summer
- Salmon fishery suffer losses by increase of sea temperatures
- Ice cream producers hedge against cold summers
- Disney World (rain period)

What are Weather Derivatives (WD)?

Hedge weather related risk exposures

- Payments based on weather related measurements
- Underlying: temperature, rainfall, wind, snow, frost

Chicago Mercantile Exchange (CME)

- Monthly/seasonal/weekly temperature Futures/Options
- 24 US, 6 Canadian, 9 European and 3 Asian-Pacific cities
- From 2.2 billion USD in 2004 to 15 billion USD through March 2009

WD market

CME offers weather contracts on 42 cities throughout the world

Weather Derivatives

CME products

- $\text{HDD}(\tau_1, \tau_2) = \int_{\tau_1}^{\tau_2} \max(18^\circ\text{C} - T_t, 0)dt$
- $\text{CDD}(\tau_1, \tau_2) = \int_{\tau_1}^{\tau_2} \max(T_t - 18^\circ\text{C}, 0)dt$
- $\text{CAT}(\tau_1, \tau_2) = \int_{\tau_1}^{\tau_2} T_t dt$, where $T_t = \frac{T_{t,max} + T_{t,min}}{2}$
- $\text{AAT}(\tau_1, \tau_2) = \int_{\tau_1}^{\tau_2} \widetilde{T}_t dt$, where $\widetilde{T}_t = \frac{1}{24} \int_1^{24} T_{t_i} dt_i$ and T_{t_i} denotes the temperature of hour t_i, (also referred to as C24AT index).

Go to pricing approaches

Algorithm

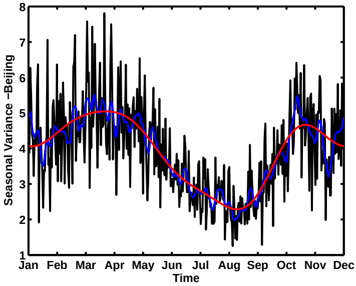
Econometrics

$$
\begin{array}{c}
T_t \\
\downarrow \\
X_t = T_t - \Lambda_t \\
\downarrow \\
X_{t+p} = a^\top X_t + \sigma_t \varepsilon_t \\
\downarrow \\
\hat{\varepsilon}_t = \frac{\hat{X}_t}{\hat{\sigma}_t} \sim \mathbf{N}(0,1)
\end{array}
$$

Fin. Mathematics.

$$
\begin{array}{c}
CAR(3) \\
\downarrow \\
F_{CAT(t,\tau_1,\tau_2)} \\
\downarrow \\
MPR
\end{array}
$$

- How to smooth the seasonal variance curve?
- How close are the residuals to $\mathbf{N}(0,1)$?
- How to price no CME listed cities?



Go to details

Figure 2: Daily empirical variance, seasonal variance $\hat{\sigma}^2_{t,FTSG}$, $\hat{\sigma}^2_{t,LLR}$ using Epanechnikov Kernel and bandwidth $h = 4.49$ for Beijing

Outline

1. Motivation ✓
2. Weather Dynamics
3. Stochastic Pricing of WD
4. Local Temperature Risk

Weather Dynamics: Asian Data

Temperature Market (CME): Tokyo and Osaka

AAT Index

Can we make money?

	Trading Period		Measurement Period		Index	
Code	First-trade	Last-trade	τ_1	τ_2	CME[1]	AAT[2]
F9	20080203	20090202	20090101	20090131	200.2	181.0
G9	20080303	20090302	20090201	20090228	220.8	215.0
H9	20080403	20090402	20090301	20090331	301.9	298.0
J9	20080503	20090502	20090401	20090430	460.0	464.0
K9	20080603	20090602	20090501	20090531	627.0	621.0

Table 1: Osaka AAT contracts listed at CME on 20090130. Source: Bloomberg. CME[1] prices of AAT Futures as listed on CME, [2] AAT index values computed from the realized temperature data.

Temperature: $T_t = X_t + \Lambda_t$

Seasonal function with trend:

$$\Lambda_t = a + bt + \sum_{l=1}^{L} c_l cos \left\{ \frac{2\pi(t - d_l)}{l \cdot 365} \right\} \tag{1}$$

$\hat{a}$: average temperature, $\hat{b}$: global Warming.

City	Period
Berlin	1948-2009
Osaka	1973-2009
Tokyo	1973-2009
Beijing	1973-2009
Taipei	1992-2009
Kaohsiung	1973-2009

Table 2: Daily average temperature data. Source: Bloomberg, Deutsche Wetter Dienst.

Seasonality

Seasonal function with trend:

$$
\begin{aligned}
\hat{\Lambda}_t &= 24.4 + 16 \cdot 10^{-5} t + \sum_{i=1}^{3} \hat{c}_i \cdot \cos\left\{ \frac{2\pi i (t - \hat{d}_i)}{365} \right\} \\
&+ \mathcal{I}(t \in \omega) \cdot \sum_{i=4}^{6} \hat{c}_i \cdot \cos\left\{ \frac{2\pi (i-4)(t - \hat{d}_i)}{365} \right\},
\end{aligned}
$$

with $\mathcal{I}(t \in \omega)$ an indicator for December, January and February.
Seasonal function with a Local linear Regression (LLNR), $t = 1 \cdots 365$ days:

$$
\underset{e,f}{\arg\min} \sum_{t=1}^{365} \left\{ \bar{T}_s - e_s - f_s(t-s) \right\}^2 K\left(\frac{t-s}{h} \right) \tag{2}
$$

Go to details

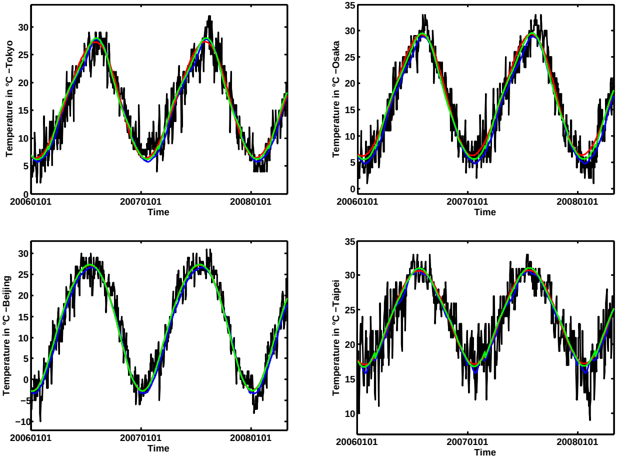


Figure 3: The Fourier truncated, the corrected Fourier and the the local linear seasonal component for daily average temperatures in Tokyo Narita International Airport (upper left), Osaka Kansai International Airport (upper right), Beijing (lower left), Taipei (lower right).

AR(p): $X_{t+p} = \sum_{i=1}^{p} \beta_i X_{t+p-i} + \sigma_t \varepsilon_t$

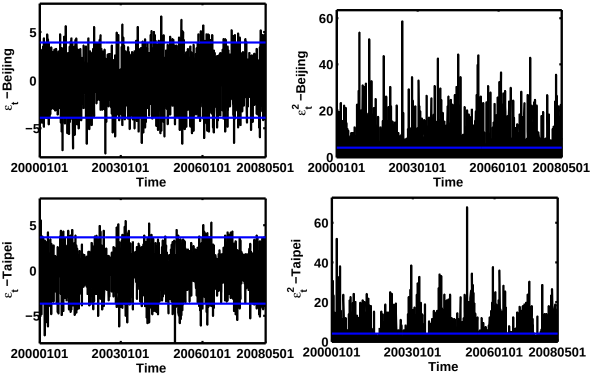


Figure 4: Residuals $\hat{\varepsilon}_t$ (left) and squared residuals $\hat{\varepsilon}_t^2$ (right) of the AR(p) (Beijing (up), Taipei (down)). No rejection of H_0 that the residuals are uncorrelated at 0% significance level, according to the modified Li-McLeod Portmanteau test

Go to details

Seasonal Volatility:

Highly seasonal ACF for squared residuals of AR(p)

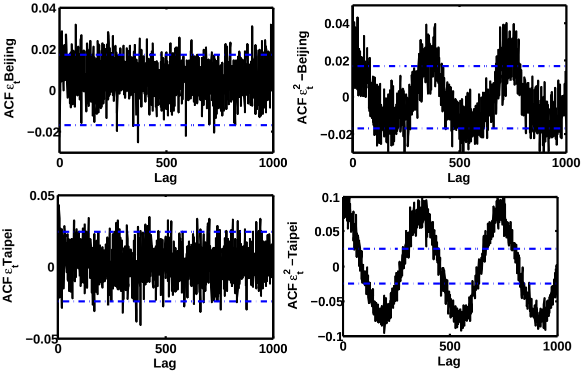


Figure 5: ACF for residuals $\hat{\varepsilon}_t$ (left) and squared residuals $\hat{\varepsilon}_t^2$ (right) of the AR(p) for Beijing (up), Taipei (down).

Calibration of Seasonal Variance: σ_t^2

Calibration of daily variances of residuals AR(3) for 36 years:

- 2 Steps: Fourier truncated series + GARCH(p,q): $\hat{\sigma}^2_{t,FTSG}$

$$\begin{aligned} \hat{\sigma}^2_{t,FTSG} &= c_1 + \sum_{l=1}^{L} \left\{ c_{2l} \cos\left(\frac{2l\pi t}{365}\right) + c_{2l+1} \sin\left(\frac{2l\pi t}{365}\right) \right\} \\ &+ \alpha_1 (\sigma^2_{t-1}\varepsilon_{t-1})^2 + \beta_1 \sigma^2_{t-1} \end{aligned} \tag{3}$$

- 1 Step: Local linear Regression (LLR): $\hat{\sigma}^2_{t,LLR}$

$$\arg\min_{g,h} \sum_{t=1}^{365} \left\{ \hat{\varepsilon}_t^2 - g_s - h_s(t-s) \right\}^2 K\left(\frac{t-s}{h}\right) \tag{4}$$

return

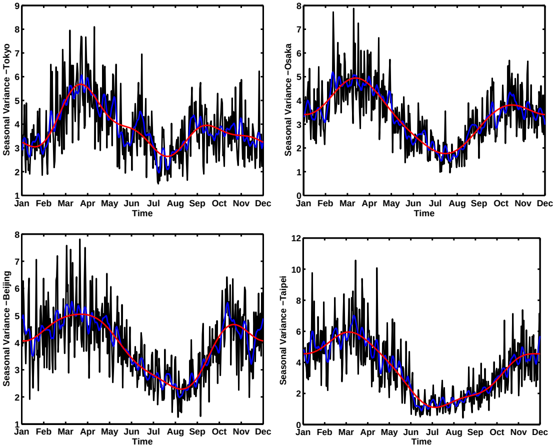


Figure 6: Daily empirical variance, seasonal variance $\hat{\sigma}^2_{t,FTSG}$, $\hat{\sigma}^2_{t,LLR}$ using Epanechnikov Kernel and bandwidth $h = 4.49$ for Tokyo (upper left), Osaka (upper right), Beijing (lower left), Taipei (lower right).

Go to details

ACF of (Squared) Residuals after Correcting Seasonal Volatility

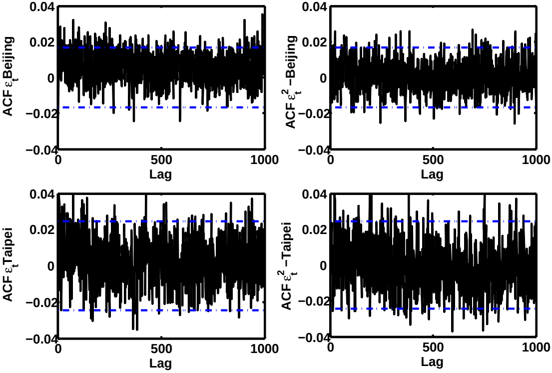


Figure 7: (Left) Right: ACF for temperature (squared) residuals $\frac{\hat{\varepsilon}_t}{\hat{\sigma}_{t,LLR}}$ for Beijing (up), Taipei (down)

City		JB	Kurt	Skew	KG	AD
Tokyo	$\frac{\hat{\varepsilon}_t}{\hat{\sigma}_{t,FTSG}}$	133.26	3.44	-0.10	0.02	8.06
	$\frac{\hat{\varepsilon}_t}{\hat{\sigma}_{t,LLR}}$	148.08	3.44	-0.13	0.02	10.31
Osaka	$\frac{\hat{\varepsilon}_t}{\hat{\sigma}_{t,FTSG}}$	98.49	3.35	-0.11	0.01	4.94
	$\frac{\hat{\varepsilon}_t}{\hat{\sigma}_{t,LLR}}$	94.15	3.34	-0.1	0.01	5.07
Beijing	$\frac{\hat{\varepsilon}_t}{\hat{\sigma}_{t,FTSG}}$	218.81	3.26	-0.28	0.02	15.22
	$\frac{\hat{\varepsilon}_t}{\hat{\sigma}_{t,LLR}}$	209.44	3.23	-0.28	0.02	15.64
Taipei	$\frac{\hat{\varepsilon}_t}{\hat{\sigma}_{t,FTSG}}$	187.89	3.27	-0.39	0.03	13.55
	$\frac{\hat{\varepsilon}_t}{\hat{\sigma}_{t,LLR}}$	177.26	3.28	-0.38	0.03	13.64
Kaohsiung	$\frac{\hat{\varepsilon}_t}{\hat{\sigma}_{t,FTSG}}$	2753.0	4.68	-0.71	0.06	79.93
	$\frac{\hat{\varepsilon}_t}{\hat{\sigma}_{t,LLR}}$	2252.5	4.52	-0.64	0.06	79.18

Table 3: Skewness, kurtosis, Jarque Bera (JB), Kolmogorov Smirnov (KS) and Anderson Darling (AD) test statistics (365 days). Critical value at at 5% significance level is 5.99, at 1% – 9.21.

Residuals ($\frac{\hat{\varepsilon}_t}{\hat{\sigma}_t}$) become normal:

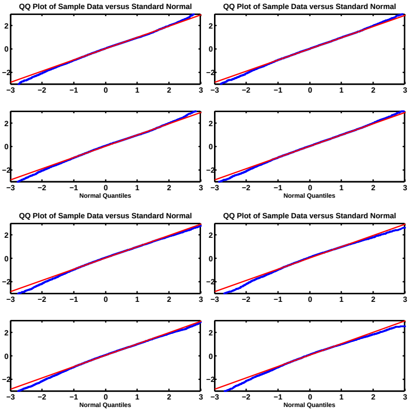


Figure 8: QQ-plot of residuals $\frac{\hat{\varepsilon}_t}{\hat{\sigma}_{t,FTSG}}$ (upper) $\frac{\hat{\varepsilon}_t}{\hat{\sigma}_{t,LLR}}$ (lower) and Normal residuals of Tokyo (upper left), Osaka (upper right), Beijing (lower left), Taipei (lower right)

Temperature Dynamics

Temperature time series:

$$T_t = \Lambda_t + X_t$$

with seasonal function Λ_t. X_t can be seen as a discretization of a continuous-time process AR(p) (CAR(p)).

This stochastic model allows CAR(p) futures/options pricing.

Stochastic Pricing

Ornstein-Uhlenbeck process $\mathbf{X}_t \in \mathbb{R}^p$:

$$d\mathbf{X}_t = \mathbf{A}\mathbf{X}_t dt + \mathbf{e}_p \sigma_t dB_t$$

$\mathbf{e}_k$: kth unit vector in $\mathbb{R}^p$ for $k = 1, ...p$, $\sigma_t > 0$, A: $(p \times p)$-matrix

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & \dots & \dots & 0 & 1 \\ -\alpha_p & -\alpha_{p-1} & \dots & & -\alpha_1 \end{pmatrix}$$

Go to proof details

Stationarity condition

Solution of $\mathbf{X}_t = \mathbf{x} \in \mathbb{R}^p$, $s \geq t \geq 0$:

$$\mathbf{X}_s = \exp\{\mathbf{A}(s-t)\}\mathbf{x} + \int_t^s \exp\{\mathbf{A}(s-u)\}\mathbf{e}_p \sigma_u dB_u$$

is stationarity as long as all the eigenvalues $\lambda_1, \ldots, \lambda_p$ of A have negative real parts, i.e. the variance matrix:

$$\int_0^t \sigma_{t-s}^2 \exp\{A(s)\}\mathbf{e}_p \mathbf{e}_p^\top \exp\left\{A^\top(s)\right\} ds$$

converges as $t \to \infty$.

Temperature Futures Price

$\exists$ Q_θ pricing so that:

$$F_{(t,\tau_1,\tau_2)} = \mathsf{E}^{Q_\theta}\left[Y|\mathcal{F}_t\right] \tag{5}$$

where Y equals the payoff of the temperature index and by Girsanov theorem:

$$B_t^\theta = B_t - \int_0^t \theta_u du$$

is a Brownian motion for $t \leq \tau_{\max}$. θ: a real valued, bounded and piecewise continuous function (market price of risk)

Temperature Dynamics under Q_θ

Under Q_θ:

$$d\mathbf{X}_t = (\mathbf{A}\mathbf{X}_t + \mathbf{e}_p \sigma_t \theta_t)dt + \mathbf{e}_p \sigma_t dB_t^\theta \tag{6}$$

with explicit dynamics, for $s \geq t \geq 0$:

$$\begin{aligned} \mathbf{X}_s &= \exp\{\mathbf{A}(s-t)\}\mathbf{x} + \int_t^s \exp\{\mathbf{A}(s-u)\}\mathbf{e}_p \sigma_u \theta_u du \\ &\quad + \int_t^s \exp\{\mathbf{A}(s-u)\}\mathbf{e}_p \sigma_u dB_u^\theta \end{aligned} \tag{7}$$

CAT Futures

For $0 \leq t \leq \tau_1 < \tau_2$:

$$
\begin{aligned}
F_{CAT(t,\tau_1,\tau_2)} &= \mathrm{E}^{Q_\theta}\left[\int_{\tau_1}^{\tau_2} T_s ds | \mathcal{F}_t\right] \\
&= \int_{\tau_1}^{\tau_2} \Lambda_u du + \mathbf{a}_{t,\tau_1,\tau_2}\mathbf{X}_t + \int_t^{\tau_1} \theta_u \sigma_u \mathbf{a}_{t,\tau_1,\tau_2}\mathbf{e}_p du \\
&+ \int_{\tau_1}^{\tau_2} \theta_u \sigma_u \mathbf{e}_1^\top \mathbf{A}^{-1}\left[\exp\left\{\mathbf{A}(\tau_2 - u)\right\} - I_p\right]\mathbf{e}_p du \qquad (8)
\end{aligned}
$$

with $\mathbf{a}_{t,\tau_1,\tau_2} = \mathbf{e}_1^\top \mathbf{A}^{-1}\left[\exp\left\{\mathbf{A}(\tau_2 - t)\right\} - \exp\left\{\mathbf{A}(\tau_1 - t)\right\}\right]$, $I_p : p \times p$ identity matrix

Benth et al. (2007)

Local Temperature Risk

Normality of ε_t requires estimating the function $\theta(t) = \{\Lambda_t, \sigma_t^2\}$ with $t = 1 \cdots 365$ days, $j = 0 \cdots J$ years. Recall:

$$
\begin{aligned}
X_{t,j} &= T_{t,j} - \Lambda_t \\
X_{t,j} &= \sum_{l=1}^{L} \beta_l X_{t-l,j} + \sigma_t \varepsilon_{t,j} \\
\varepsilon_{t,j} &\sim \mathbf{N}(0,1), i.i.d. \qquad (9)
\end{aligned}
$$

where $\mathbf{N}(0,1)$ is the standard normal cdf.

Adaptation Scale

Fix $s \in 1, 2, \ldots, 365$, sequence of ordered weights:
$W_s^{(k)} = (w(s,1,h_k), w(s,2,h_k), \ldots, w(s,365,h_k))^\top$.
Define $w(s,t,h_k) = K_{h_k}(s-t), (h_1 < h_2 < \ldots < h_K)$.
Local likelihood:

$$
\begin{aligned}
\hat{\varepsilon}_{365j+t} &= X_{365j+t} - \sum_{l=1}^{L} \hat{\beta}_l X_{365j+t-l} \\
\tilde{\theta}^k(s) &= \mathop{\arg\min}_{\theta\in\Theta} \sum_{t=1}^{365}\sum_{j=0}^{J} \{\log(2\pi\theta)/2 + \hat{\varepsilon}^2_{t,j}/2\theta\} w(s,t,h_k), \\
&\stackrel{\text{def}}{=} \mathop{\arg\min}_{\theta\in\Theta} -L(W_s^{(k)}, \theta)
\end{aligned}
$$

Local Temperature Residuals

$$
\begin{aligned}
\tilde{\theta}^k(s) &= \sum_{t,j} \hat{\varepsilon}^2_{t,j} w(s,t,h_k) / \sum_{t,j} w(s,t,h_k) \\
&= \sum_{t} \hat{\varepsilon}^2_{t} w(s,t,h_k) / \sum_{t} w(s,t,h_k) \\
\textit{with} \qquad \hat{\varepsilon}_t &\stackrel{\mathrm{def}}{=} (J+1)^{-1} \sum_{j=0}^{J} \hat{\varepsilon}^2_{t,j}.
\end{aligned}
$$

For Taipei, daily average temperature 19730101 – 20081230, $J = 36$.

Mirror Observations

To avoid the boundary problem, use mirrored observations:

Assume $h_K < 365/2$, then the observations look like $\hat{\varepsilon}^2_{-364}, \hat{\varepsilon}^2_{-363}, \ldots, \hat{\varepsilon}^2_{0}, \hat{\varepsilon}^2_{1}, \ldots, \hat{\varepsilon}^2_{730}$, where

$$
\begin{aligned}
\hat{\varepsilon}^2_t &\stackrel{\text{def}}{=} \hat{\varepsilon}^2_{365+t}, -364 \leq t \leq 0 \\
\hat{\varepsilon}^2_t &\stackrel{\text{def}}{=} \hat{\varepsilon}^2_{t-365}, 366 \leq t \leq 730
\end{aligned}
$$

Parametric Exponential Bounds

$$
\begin{aligned}
L(W^{(\tilde{k})}, \theta, \theta^*) &\stackrel{\text{def}}{=} L(W^{(k)}, \tilde{\theta}) - L(W^{(k)}, \theta^*) = N\mathcal{K}(W^{(k)}, \tilde{\theta}, \theta^*) \\
\mathcal{K}(W^{(k)}, \theta, \theta^*) &= -\{\log(\theta/\theta^*) + 1 - \theta^*/\theta\}/2
\end{aligned}
$$

where $\mathcal{K}\{\theta, \theta^*\}$ is the Kullback Leibler divergence between θ and θ^* and $N = 365 \times J$. For any $\mathfrak{z} > 0$,

$$
\begin{aligned}
\mathrm{P}_{\theta^*}\{L(W^{(k)}, \tilde{\theta}, \theta^*) > \mathfrak{z}\} &\leq 2\exp(-\mathfrak{z}) \\
\mathrm{E}_{\theta^*}\, L(W^{(k)}, \tilde{\theta}, \theta^*)^r &\leq \mathfrak{r}_r,
\end{aligned}
$$

where $\mathfrak{r}_r = 2r \int_{\mathfrak{z} \geq 0} \mathfrak{z}^{r-1} \exp(-\mathfrak{z}) d\mathfrak{z}$.

LMS Procedure

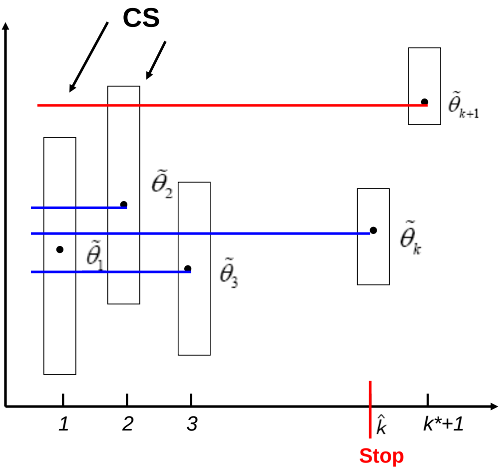
Construct an estimate $\hat{\theta} = \hat{\theta}(s)$, on the base of $\tilde{\theta}_1(s), \tilde{\theta}_2(s), \ldots, \tilde{\theta}_K(s)$.

- Start with $\hat{\theta}_1 = \tilde{\theta}_1$.
- For $k \geq 2$, $\tilde{\theta}_k$ is accepted and $\hat{\theta}_k = \tilde{\theta}_k$ if $\tilde{\theta}_{k-1}$ was accepted and

$$L(W^{(k)}, \tilde{\theta}_\ell, \tilde{\theta}_k) \leq \mathfrak{z}_\ell, \ell = 1, \ldots, k-1$$

  $\hat{\theta}_k$ is the the latest accepted estimate after the first k steps.

Illustration



"Propagation" Condition

A bound for the risk associated with first kind error:

$$\mathsf{E}_{\theta^*} \frac{|L(W^{(k)}, \tilde{\theta}_k, \hat{\theta}_k)|^r}{\mathfrak{r}_r} \leq \alpha \tag{10}$$

where $k = 1, \ldots, K$ and $\mathfrak{r}_r$ is the parametric risk bound.

Sequential Choice of Critical Values

- Consider first only $\mathfrak{z}_1$ letting $\mathfrak{z}_2 = \ldots = \mathfrak{z}_{K-1} = \infty$. Leads to the estimates $\hat{\theta}_k(\mathfrak{z}_1)$ for $k = 2, \ldots, K$.
- The value $\mathfrak{z}_1$ is selected as the minimal one for which

$$\sup_{\theta^*} \mathsf{E}_{\theta^*} \frac{|L\{W^{(k)}, \tilde{\theta}_k, \hat{\theta}_k(\mathfrak{z}_1)\}|^r}{\mathfrak{r}_r} \leq \frac{\alpha}{K-1}, k = 2, \ldots, K.$$

- Set $\mathfrak{z}_{k+1} = \ldots = \mathfrak{z}_{K-1} = \infty$ and fix $\mathfrak{z}_k$ lead the set of parameters $\mathfrak{z}_1, \ldots, \mathfrak{z}_k, \infty, \ldots, \infty$ and the estimates $\hat{\theta}_m(\mathfrak{z}_1, \ldots, \mathfrak{z}_k)$ for $m = k+1, \ldots, K$. Select $\mathfrak{z}_k$ s.t.

$$\sup_{\theta^*} \mathsf{E}_{\theta^*} \frac{|L\{W^{(k)}, \tilde{\theta}_m, \hat{\theta}_m(\mathfrak{z}_1, \mathfrak{z}_2, \ldots, \mathfrak{z}_k)\}|^r}{\mathfrak{r}_r} \leq \frac{k\alpha}{K-1},$$

$m = k+1, \ldots, K$.

Critical Values

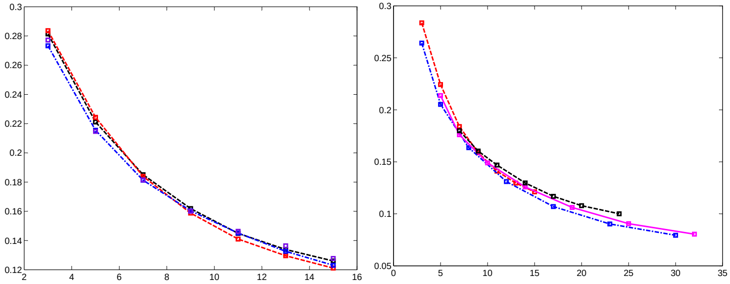


Figure 9: Simulated CV with $\theta^* = 1$, $r = 0.5$, $MC = 5000$ with $\alpha = 0.3$, 0.5, 0.8 (left), with different bandwidth sequences (right).

Bound for Critical Values

Suppose $0 < \mu \leq h_{k-1}/h_k \leq \mu_0 < 1$.

Let $\theta(.) = \theta^*$, for all $t \in (0, 365)$. There is a constant $a_0 > 0$ depending on r and μ_0, μ, s.t.

$$\mathfrak{z}_k = a_0 \log K + 2 \log(nh_k/\alpha) + 2r \log(h_K/h_k)$$

ensures (10).

"Small Modeling Bias" (SMB)

Condition:

$$\begin{aligned}\Delta(W^{(k)},\theta) &= \sum_{t=1}^{365}\mathcal{K}\{\theta(t),\theta\}\mathbf{1}\{w(s,t,h_k)>0\} \\ &\leq \Delta, \forall k < k^* \end{aligned} \tag{11}$$

k^* is the maximum k satisfying the SMB condition.

Property:

For any estimate $\tilde{\theta}_k$ and θ satisfying SMB, it holds:

$$\mathrm{E}_{\theta(.)}\log\{1+|L(\tilde{\theta}_k,\theta)|^r/\mathfrak{r}_r\} \leq \Delta + 1$$

"Stability" Property

Stability: the attained quality of estimation during "propagation" can not get lost at further steps.

$$L(W^{(k^*)}, \tilde{\theta}_{k^*}, \hat{\theta}_{\hat{k}})\mathbf{1}\{\hat{k} > k^*\} \leq \mathfrak{z}_{k^*}$$

"Oracle" Property

Theorem

Let $\Delta(W^{(k)}, \theta) \leq \Delta$ *for some* $\theta \in \Theta$ *and* $k \leq k^*$. *Then*

$$
\begin{aligned}
E_{\theta(.)} \log\left\{1 + \frac{|L(W^{(k^*)}, \tilde{\theta}_{k^*}, \theta)|^r}{\mathfrak{r}_r(W^{(k^*)})}\right\} &\leq \Delta + 1 \\
E_{\theta(.)} \log\left\{1 + \frac{|L(W^{(k^*)}, \tilde{\theta}_{k^*}, \hat{\theta})|^r}{\mathfrak{r}_r(W^{(k^*)})}\right\} &\leq \Delta + \alpha \\
&+ \log\{1 + \frac{\mathfrak{z}_k^*}{\mathfrak{r}_r}\}
\end{aligned}
$$

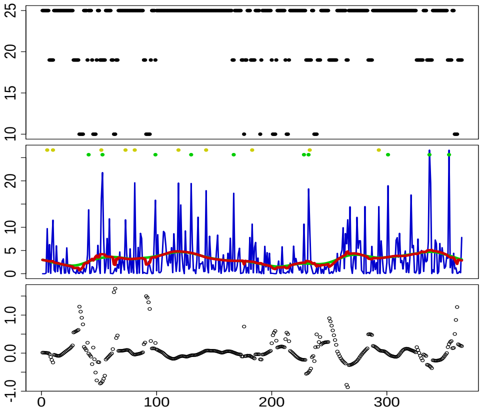


Figure 10: Bandwidth sequences (upper), fixed bandwidth curve; adaptive bandwidth curve for squared residuals (blue middle); difference between adaptive and fixed bandwidth (lower), Beijing, $\alpha = 0.3$, $r = 0.5$

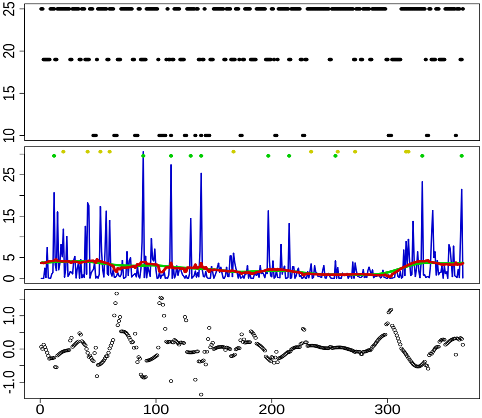


Figure 11: Bandwidth sequences (upper), fixed bandwidth curve; adaptive bandwidth curve for squared residuals (blue middle); difference between adaptive and fixed bandwidth (lower), Kaoshiung, $\alpha = 0.3$, $r = 0.5$

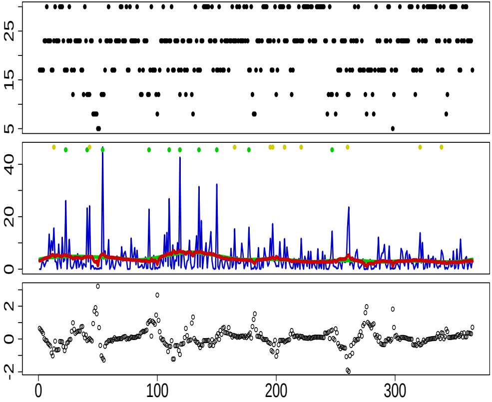


Figure 12: Bandwidth sequences (upper), fixed bandwidth curve; adaptive bandwidth curve for squared residuals (blue middle); difference between adaptive and fixed bandwidth (lower), Berlin, $\alpha = 0.3$, $r = 0.5$

Go to details

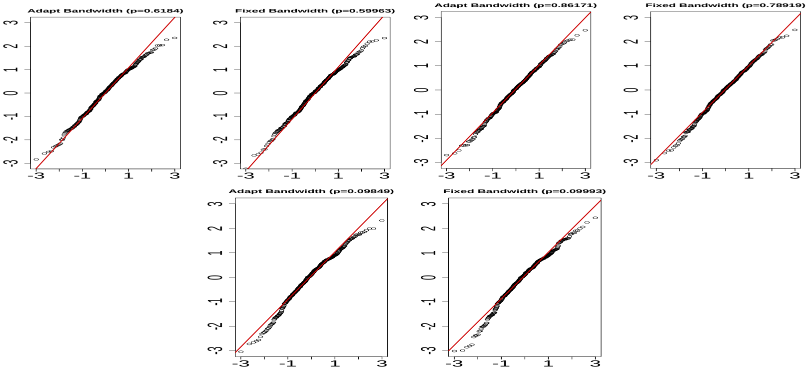


Figure 13: Q-Q Plot for squared residuals from Berlin, Kaoshiung, Beijing

Normalized Residual

city	bandwidth	KS	JB	AD
Beijing	FB	0.3741	0.0233	0.0077
	AB	0.3961	0.0204	0.0073
	ABS	0.4086	0.0204	0.0073
Kaoshiung	FB	0.0213	2.4e-15	1.4e-06
	AB	0.0616	2.4e-03	1.4e-03
	ABS	0.0618	2.4e-03	1.4e-03
Berlin	FB	0.5991	0.1052	0.0952
	AB	0.6181	0.0662	0.0412

Table 4: P-value of normality tests for fixed bandwidth curve (FB), adaptive bandwidth curve (AB), adaptive smoothed bandwidth curve (ABS)

Smoothed Bandwidth

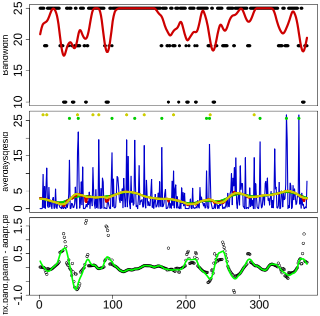


Figure 16: Bandwidth with smoothed curve (upper panel), volatility estimation with fixed bandwidth, smoothed adaptive bandwidth, adaptive bandwidth (middle panel), difference between smoothed adaptive and fixed bandwidth estimator(lower panel).

Tokyo & Osaka AAT Future Prices

		Trading Period		Measurement Period		Index		
City	Code	First-trade	Last-trade	τ_1	τ_2	CME[1]	F_{AAT}	$F_{AAT,loc}$
Osaka	F9	20080203	20090202	20090101	20090131	200.2	181.0	183.0
	G9	20080303	20090302	20090201	20090228	220.8	215.0	210.5
	H9	20080403	20090402	20090301	20090331	301.9	298.0	295.3
	J9	20080503	20090502	20090401	20090430	460.0	464.0	463.6
	K9	20080603	20090602	20090501	20090531	627.0	621.0	615.4
Tokyo	J9	20080503	20090502	20090401	20090430	450.0	454.0	455.2
	K9	20080603	20090602	20090501	20090531	592.0	681.0	595.6

Table 5: Osaka AAT contracts listed at CME on 20090130. Source: Bloomberg. CME[1] prices of AAT Futures as listed on CME, [2] AAT index values computed from the realized temperature data and with adaptive bandwidth $F_{AAT,loc}$

Conclusions and further work

- Temperature risk stochastics closer to Wiener process when applying adaptive methods
- Better estimates of λ_t and σ_t lead to fair price -> pure MPR

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Localizing Temperature Risk

温度风险

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Appendix A

Li-McLeod Portmanteau Test– modified Portmanteau test statistic Q_L to check the uncorrelatedness of the residuals:

$$Q_L = n \sum_{k=1}^{L} r_k^2(\hat{\varepsilon}) + \frac{L(L+1)}{2n},$$

where r_k, $k = 1, \ldots, L$ are values of residuals ACF up to the first L lags and n is the sample size. Then,

$$Q_L \sim \chi^2_{(L-p-q)}$$

Q_L is χ^2distributed on $(L - p - q)$ degrees of freedom where p,q denote AR and MA order respectively and L is a given value of considered lags.

WD pricing models

1. General Equilibrium Theory for incomplete markets: Indifference, Marginal, Market Clearing
2. Pricing via no arbitrage arguments: adequate equivalent martingale measure

return

$\mathbf{X}_t$ can be written as a Continuous-time AR(p) (CAR(p)):

For $p = 1$,

$$dX_{1t} = -\alpha_1 X_{1t} dt + \sigma_t dB_t$$

For $p = 2$,

$$\begin{aligned} X_{1(t+2)} &\approx (2-\alpha_1)X_{1(t+1)} \\ &+ (\alpha_1 - \alpha_2 - 1)X_{1t} + \sigma_t(B_{t-1} - B_t) \end{aligned}$$

For $p = 3$,

$$\begin{aligned} X_{1(t+3)} &\approx (3-\alpha_1)X_{1(t+2)} + (2\alpha_1 - \alpha_2 - 3)X_{1(t+1)} \\ &+ (-\alpha_1 + \alpha_2 - \alpha_3 + 1)X_{1t} + \sigma_t(B_{t-1} - B_t) \end{aligned}$$

Proof $CAR(3) \approx AR(3)$

Let

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\alpha_3 & -\alpha_2 & -\alpha_1 \end{pmatrix}$$

- use $B_{t+1} - B_t = \varepsilon_t$
- assume a time step of length one $dt = 1$
- substitute iteratively into X_1 dynamics

Proof $CAR(3) \approx AR(3)$:

$$\begin{aligned}
X_{1(t+1)} - X_{1(t)} &= X_{2(t)}dt \\
X_{2(t+1)} - X_{2(t)} &= X_{3(t)}dt \\
X_{3(t+1)} - X_{3(t)} &= -\alpha_1 X_{1(t)}dt - \alpha_2 X_{2(t)}dt - \alpha_3 X_{3(t)}dt + \sigma_t \varepsilon_t \\
X_{1(t+2)} - X_{1(t+1)} &= X_{2(t+1)}dt \\
X_{2(t+2)} - X_{2(t+1)} &= X_{3(t+1)}dt \\
X_{3(t+2)} - X_{3(t+1)} &= -\alpha_1 X_{1(t+1)}dt - \alpha_2 X_{2(t+1)}dt \\
&\quad -\alpha_3 X_{3(t+1)}dt + \sigma_{t+1}\varepsilon_{t+1} \\
X_{1(t+3)} - X_{1(t+2)} &= X_{2(t+2)}dt \\
X_{2(t+3)} - X_{2(t+2)} &= X_{3(t+2)}dt \\
X_{3(t+3)} - X_{3(t+2)} &= -\alpha_1 X_{1(t+2)}dt - \alpha_2 X_{2(t+2)}dt \\
&\quad -\alpha_3 X_{3(t+2)}dt + \sigma_{t+2}\varepsilon_{t+2}
\end{aligned}$$

return

City(Period)	$\hat{a}$	$\hat{b}$	$\hat{c}_1$	$\hat{d}_1$	$\hat{c}_2$	$\hat{d}_2$	$\hat{c}_3$	$\hat{d}_3$
Tokyo								
(730101-081231)	15.7415	0.0001	8.9171	-162.3055	-2.5521	-7.8982	-0.7155	-15.0956
(730101-821231)	15.8109	0.0001	9.2855	-162.6268	-1.9157	-16.4305	-0.5907	-13.4789
(830101-921231)	15.4391	0.0004	9.4022	-162.5191	-2.0254	-4.8526	-0.8139	-19.4540
(930101-021231)	16.4284	0.0001	8.8176	-162.2136	-2.1893	-17.7745	-0.7846	-22.2583
(030101-081231)	16.4567	0.0001	8.5504	-162.0298	-2.3157	-18.3324	-0.6843	-16.5381
Taipei								
(920101-081231)	23.2176	0.0002	1.9631	-164.3980	-4.8706	-58.6301	-0.2720	39.1141
(920101-011231)	23.1664	0.0002	3.8249	-150.6678	-2.8830	-68.2588	0.2956	-41.7035
(010101-081231)	24.1295	-0.0001	1.8507	-149.1935	-5.1123	-67.5773	-0.3150	22.2777
Osaka								
(730101-081231)	15.2335	0.0002	10.0908	-162.3713	-2.5653	-7.5691	-0.6510	-19.4638
(730101-821231)	15.9515	-0.0001	9.7442	-162.5119	-2.1081	-17.9337	-0.5307	-18.9390
(830101-921231)	15.7093	0.0003	10.1021	-162.4248	-2.1532	-10.7612	-0.7994	-24.9429
(930101-021231)	16.1309	0.0003	10.3051	-162.4181	-2.0813	-21.9060	-0.7437	-27.1593
(030101-081231)	16.9726	0.0002	10.5863	-162.4215	-2.1401	-14.3879	-0.8138	-17.0385
Kaohsiung								
(730101-081231)	24.2289	0.0001	0.9157	-145.6337	-4.0603	-78.1426	-1.0505	10.6041
(730101-821231)	24.4413	0.0001	2.1112	-129.1218	-3.3887	-91.1782	-0.8733	20.0342
(830101-921231)	25.0616	0.0003	2.0181	-135.0527	-2.8400	-89.3952	-1.0128	20.4010
(930101-021231)	25.3227	0.0003	3.9154	-165.7407	-0.7405	-51.4230	-1.1056	19.7340
Beijing								
(730101-081231)	11.8904	0.0001	14.9504	-165.2552	0.0787	-12.8697	-1.2707	4.2333
(730101-821231)	11.5074	0.0003	14.8772	-165.7679	0.6253	15.8090	-1.2349	1.8530
(830101-921231)	12.4606	0.0002	14.9616	-165.7041	0.5327	14.3488	-1.2630	4.8809
(930101-021231)	13.6641	-0.0003	14.8970	-166.1435	0.9412	16.9291	-1.1874	-4.5596
(030101-081231)	12.8731	0.0003	14.9057	-165.9098	0.7266	16.5906	-1.5323	1.8984

Table 6: Seasonality estimates $\hat{\lambda}_t$ of daily average temperatures in Asia. All coefficients are nonzero at 1% significance level. Data source: Bloomberg.

return

AR(p): $X_{t+p} = \sum_{i=1}^{p} \beta_i X_{t+p-i} + \sigma_t \varepsilon_t$

City	β_1	β_2	β_3
Tokyo(p=3)	0.668	-0.069	0.079
Osaka(p=3)	0.748	-0.143	-0.079
Beijing(p=3)	0.741	-0.071	0.071
Taipei(p=3)	0.808	-0.228	0.063
Kaoshiung(p=3)	0.770	-0.129	0.040

Table 7: Coefficients of AR(p) , Model selection: AIC

return

The long memory diagnosis can be replicated by a short memory process with structural breaks!

Calibration of Seasonal Variance: σ_t^2

	$\hat{c}_1$	$\hat{c}_2$	$\hat{c}_3$	$\hat{c}_4$	$\hat{c}_5$	$\hat{c}_6$	$\hat{c}_7$	α	β
Tokyo	3.91	-0.08	0.74	-0.70	-0.37	-0.13	-0.14	0.09	0.50
Osaka	3.40	0.76	0.81	-0.58	-0.29	-0.17	-0.07	0.04	0.52
Beijing	3.95	0.70	0.82	-0.26	-0.50	-0.20	-0.17	0.03	0.33
Taipei	3.54	1.49	1.62	-0.41	-0.19	0.03	-0.18	0.06	0.33

Table 8: First 7 Coefficients of σ_t^2 and $GARCH(p = 1, q = 1)$. The coefficients in black are significant at 1% level.

return

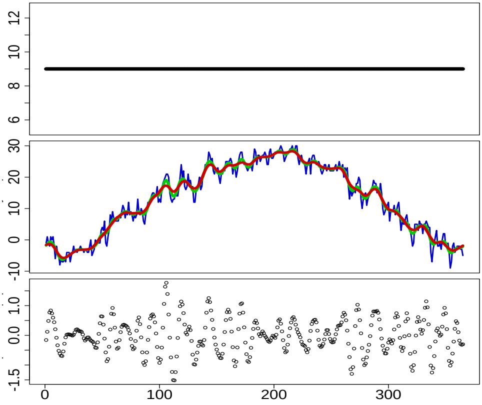

Figure 14: Bandwidth sequences (upper), fixed bandwidth curve; adaptive bandwidth curve for daily temperature difference between adaptive and fixed bandwidth (lower), Beijing, $\alpha = 0.3$, $r = 0.5$

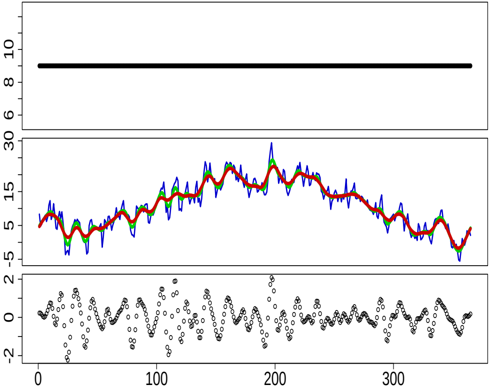

Figure 15: Bandwidth sequences (upper), fixed bandwidth curve; adaptive bandwidth curve for daily temperature ; difference between adaptive and fixed bandwidth (lower), Kaoshiung, $\alpha = 0.3$, $r = 0.5$

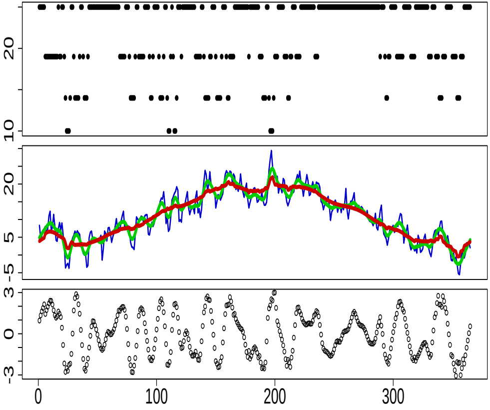

Figure 16: Bandwidth sequences (upper), fixed bandwidth curve; adaptive bandwidth curve for daily temperature ; difference between adaptive and fixed bandwidth (lower), Berlin, $\alpha = 0.3$, $r = 0.5$