Bargaining with a principal: contracts vs. agreements

João Montez*

Preliminary and incomplete

Abstract

We study a model where a principal bargains bilateraly with N agents under two contracting modes: binding and non-binding contracts or agreements. We find that the unique pairwise stable payoffs coincide respectively with the nucleolus and the Shapley value of related coalitional games. Next we study the distributive effects of contracts and we find that the principal (agents) likes (dislike) non-binding contracts when agents are substitutes—and vice-versa if agents are complements. Finally we study at the bargaining effects of a both a "vertical" and a "horizontal" merger and we argue that contracts can be an important element to identify the bargaining effects of mergers, such as its profitability and possible waterbed effects.

Abstract

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1 Introduction

When one party (the principal) bargains bilaterally with several other parties (the agents), the bilateral benefit of agreement between the principal and each agent often depends on the agreements the principal has with the remaining agents. For example, for a producer the cost of supplying a particular client depends on the orders of other clients, for a retailer the benefit of carrying an additional product depends on agreements with the remaining suppliers, for a product development company the value of a particular patent depends on the cost of other complementary patents and for a firm the benefit of hiring a worker depends on the agreements with other workers.

Parties are expected to take this interdependence into account when bargaining over a mutually beneficial agreement. Contracts are *binding* agreements, the breach of which is sanctioned by law. Yet many economic relationships are however governed by self-enforcing agreements or *non-binding* contracts.[1] What is the role of legally binding contracts on the sharing of the economic surplus? Which players benefit from the enforcement of contracts by a legal system?

In this paper we provide an answer to these questions, in a setting where a principal bargains with multiple agents, by studying the relationship between contracts and bargaining power. The principal is the single player who can coordinate and organize an economic activity. He bargains with each individual agent over a bilateral contract, an agreement for a payoff the principal will leave the agent in exchange for his cooperation. We assume that individual players have no

*London Business School, Sussex Place Regent's Park, London NW1 4SA UK. E-mail: jmontez@london.edu.

[1] While this may reflect players' choice, common motives also include difficulties in specifying the contractual object or courts that are not expected to sanction unilateral termination.. A contract can also be a hollow vessel, specifying some terms of trade but not binding the parties—e.g. it may contain a termination for convenience clause or an indefinite term.

particular bargaining skills or other advantage in *bilateral* negotiations: contracts divide equally the bilateral surplus between the principal and the agent with respect to their outside options.[2] Asymmetries in equilibrium payoffs arise here endogenously—capturing the players' relative bargaining power—from the legal and economic fundamentals. By comparing the outcomes with binding and non-binding contracts we can learn how contracts affect the sharing of the surplus in different economic situation.

The key to the analysis lies in understanding how contracts can determine each parties's outside options, their respective payoffs in case of disagreement. If negotiations fail between the principal and agent i, then agent i receives his outside option which is his reservation payoff; for the principal his outside option is his payoff from contracting with all remaining agents, which needs to be determined endogenously.

If contracts are binding the principal is expected to compensate the remaining agents according to their original contracts. If however contracts are non-binding, and a bilateral negotiation ends in disagreement, contracts with all remaining agents may be renegotiated by either side. So in the latter case the principal's outside option is the principal's payoff following the issuing renegotiation.

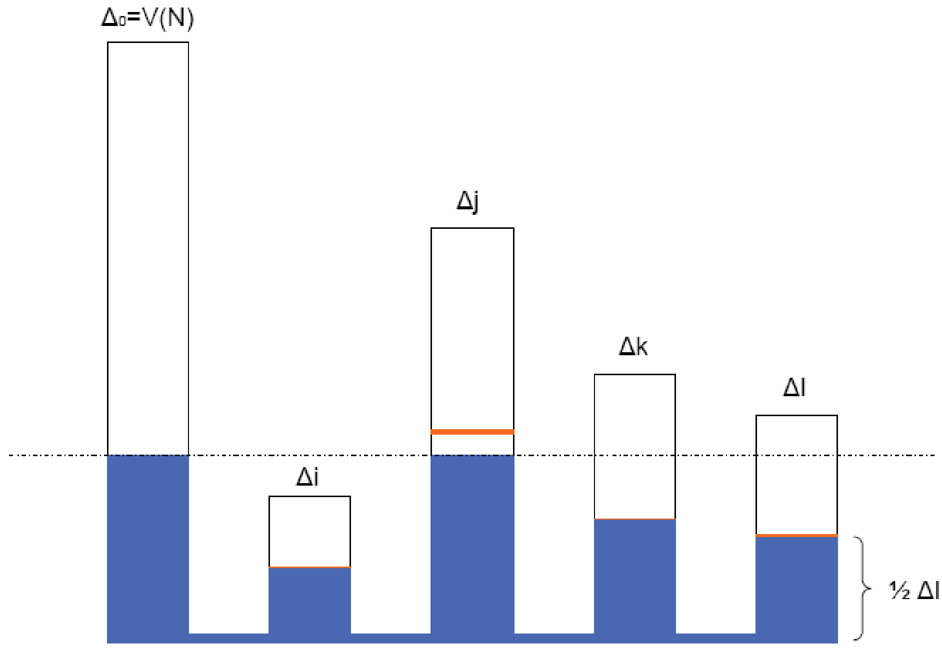
A set of contracted payoffs is *pairwise stable* if no player can gain from renegotiating a bilateral contract given the equal sharing of the bilateral surplus, while taking the contracts with the remaining agents as given. We determine the principal's outside options endogenously in each bilateral bargaining problem and we find that the equilibrium payoffs in each setting can be obtained as solutions to underlying coalitional games. If contracts are non-binding the unique pairwise stable payoff vector coincides with the Shapley value of a game that accounts for the communication structure of our setting: it is a weighted average of each players' marginal contributions to the surplus the principal can achieve with the collaboration of each subgroup of agents.

If contracts are binding it coincides with the nucleolus of an associated bankrupcy problem, in which a player's marginal contribution is his claim to the total surplus, which can be easily obtained as follows. It divides equally each dollar of the surplus by all players until half the lowest marginal contribution is reached. At that moment those agents receiving half their marginal contributions are removed and the next dollar is divided equally among the remaining players. It proceeds in this way, removing those agents who reach half their marginal contributions, until the total surplus is distributed or, if all agents get half their marginal contributions, the remainder goes to the principal. An interesting way to capture these payoffs is to imagine a system of connected containers, one for each player, so that each container has a the height of a players' marginal contribution Δ_i—and a similar base normalized to one. If we close the agents' containers at half their height and introduce an amount of water equal to the surplus $v(M)$ in the system, this water becomes distributed according to the respective payoffs (see picture).

We use these results to study the bargaining effect of the enforcement of the bilateral contracts. We find that these can be captured by the complementarity and substitution of the agents. Agents are substitutes (complements) in the economy if the marginal contribution of agents to the economic surplus decreases (increases) with the presence of additional agents, i.e. if the economic surplus is submodular (supermodular) with respect to agent inclusion.

Relative to non-binding contracts, binding contracts benefit the principal if agents are substitutes, and vice versa if agents are complements. The accompanying intuition reflects two considerations which are present in this bargaining situation. Suppose that agents are substitutes, then presence of each agent i has two effects: it increases the total surplus and reduces

[2] Unlike a large literature which assumes that the principal holds all bargaining power by making take-it-or-leave-it offers to the agents, or vice versa—and which focus on other issues. Our work adds to a small but increasing literature—reviewed below—that studies how bargaining power is determined endogenously.



the marginal contribution of the remaining agents. If contracts are non-binding then each agent i understands that, while his marginal contribution may be low, if he were to leave the game the remaining agents can renegotiate their contracts and ask for a higher payoff since their marginal contributions are now higher. This would not arise if contracts could not be renegotiated, i.e. contracts are binding. The presence of i is therefore more valuable to the principal when contracts are non-binding, and this explains why the principal is in a stronger bargaining position when agents are substitutes and contracts are binding. In the particular case where agents are perfect substitutes the principal can extract the full surplus with binding contracts while he can only do so asymptotically with non-binding contracts.

If agents are complements and i leaves the game then both total surplus and the marginal contributions of the remaining agents are reduced. If contracts are binding then each agent i understands that not only is his marginal contribution high but also that if he were to leave the game the principal's liabilities vis a vis the other agents would remain unchanged—since the agents can insist on being compensated according to the original contracts—and so the surplus reduction would come mostly at the principal's expense. If on the other hand contracts are non-binding, contracts would be rebargained under the understanding that both the marginal contribution and the surplus have been reduced, and so the effect of i's departure is shared by all remaining players. The presence of i is therefore more valuable to the principal when contracts are binding, and this explains why the principal is in a weaker bargaining position when agents are complements and contracts are binding.

The solutions coincide, and contracts are therefore payoff irrelevant, in the particular case where agents are perfect complements—i.e. where each agent is indespensable—since each player then gets an equal share of the total surplus. The solutions also coincide if the agents' contributions are independent, in which case each agent receives half his marginal contribution to the total surplus.

Therefore if the principal is given the strategic choice on which type of contracts to use, he will choose binding contracts when agents are substitutes and non-binding contracts when they are complements. Interestingly these choices leads to pairwise stable outcomes that are stable in a *strong* sense but also that lie at the "center" of the Core of the underlying coalitionall game.

Our final question is: How do contracts change the bargaining effects of a pairwise merger? These could be either the merger of the principal with an agent (vertical merger) or of an agent with an another agent (horizontal merger). If agents are substitutes we find that a vertical merger is profitable when contracts are non-binding but it is payoff irrelevant when contracts are binding. On the other hand if agents are complements a vertical merger is unprofitable with both binding and non-binding contracts.

For horizontal mergers the case of binding contracts is also clear: mergers are profitable when agents are substitutes and unprofitable when agents are complements. The case of non-binding contracts is more complex. Segal (2003) finds that when one uses the Shapley value as a solution to a game then "collusion between two complementary (substitutable) players helps (hurts) players who are indispensable"—and so the effect on the principal can again be related to complementarity—the profitability of a merger and its effect on the remaining agents should depend on how agent complementarity is affected by the presence of the other agents. If agents are substitutes, and agent substitutability is increasing with respect to agent inclusion, the merger hurts the remaining agents. Under these conditions we have a *waterbed effect*. If on the other hand are complements, and complementarity is increasing with respect to agent inclusion, then the merger helps the remaining agents and we have a *reversed waterbed effect*. We here go one step further and show that complementarities also provide sufficient conditions for a horizontal merger to be profitble: a merger is profitable if agents are substitutes and unprofitable if agents are complements.

Therefore we conclude that contracts can therefore be an important element to identify the bargaining effects of mergers, such as its profitbaility and possible waterbed effects.

2 Literature review

to be added

3 The Model

We start by discribing the technology in the first subsection. We then introduce the game and the notion of pairwise stability with equal sharing of the bilateral surplus. Finally we present how in our model contracts determine the bargaining situation by changing the principal's outside options.

3.1 Technology

We consider an economy with $M = 0 \cup N$ players. We call player 0 the principal and players in $N = \{1, ..., n\}$ the agents. These labels are used to reflect our focus on situations in which only player 0 (the principal) can bargain and contract with all remaining players (the agents) but the latter cannot contract among themselves. A generic subset of players is denoted by S with a respective size of $|S|$.

A trading opportunity between the principal and agent i is denoted by $x_i \in X_i$, where X_i is a compact subset of R_+ with $0 \in X_i$—the no-trade element. A general trade vector is denoted by $x = (x_1, .., x_n) \in X_N$ and X_S denotes the set of trading opportunities with the generic subset $S \subseteq N$.

Agent i's payoff is $\phi_i(x_i, t_i) = u_i(x_i) - t_i$ where $u_i(x)$ denotes his gross monetary payoff from trade x_i and $t_i \in R$ represents a monetary transfers from agent i to the principal—depending on the applications t_i can be either positive or negative. We normalize the agents' no-trade or default options to zero i.e. $u_i(0) = 0$ for all $i \in N$. Let $(a_i)_{i\in N}$ be the generic notation for vectors. The principal's payoff is similarly defined as $\phi_0(x, (t_i)_{i\in N}) = u_0(x) + \sum_i t_i$ with $\phi_0(0, 0) = 0$, i.e. his no-trade payoff is also zero-normalized.

We assume that there are gains from trade, i.e. there exists at least an $x \in X_N$ such that $u(x) > 0$ where $u(x) = \sum_{i\in M} u_i(x)$ represents the total economy surplus when x is implemented—transfers simply cancel out.[3] For future reference let the function $v : 2^M \to R$ denote the maximum surplus a subset of players can achieve by trading (the set of all functions is denoted by V). Since a subset of players can only trade if the principal is present, the net surplus a subset of players S can achieve by cooperation is

$$v(S) = \begin{cases} \max\limits_{x\in X_S} u(x) \text{ if } 0 \in S \\ \quad 0 \text{ if } 0 \notin S \end{cases}.$$

Let $v(\emptyset) = 0$, then the pair (M, v) defines a transferable utility coaltional game that accounts for the communication structure of the setting. We denote the marginal contribution of a agent i to S by $\triangle_i v(S) \equiv v(S \cup i) - v(S \backslash i)$. The principal is an *indispensable* player since $\triangle_0 v(S) = v(S)$ for all $S \subseteq M$.

3.2 The game

We consider the following timing: In the first-stage the principal and each individual agent bargain bilaterally over a contract (x_i, t_i) that awards agent i a payoff ϕ_i^1 in exchange for i's collaboration in the second-stage. Let S denote the set of players who move to the second-stage with a contract—agents who fail to reach an agreement in the first-stage are removed from the game and receives their no-trade payoffs.

In the second-stage no further bargaining takes place, trade is decided and the gains from trade among the members of S are distributed. If the principal's liabilities to the agents in S are lower than $v(S)$ then each agent is paid ϕ_i^1 and the principal is left with the remainder. If instead the sum of the principal's liabilities exceeds $V(S)$ than the gains are distributed to the members of $S\backslash 0$ taking into account their first-stage contracts—so the principal's liability is limited to his zero-normalized outside option. Formally, in the second-stage the gains from cooperation are shared according to an allocation rule $f : (\phi_i^1)_{i\in N} \times V \to R^M$ such that:

$$\textstyle\sum_{i\in S} f_i = v(S),\ \phi_i^1 \geq f_i \geq 0 \text{ for all } i \in S\backslash 0 \text{ and } f_0 = \left[v(S) - \sum_{i\in S\backslash 0} \phi_i^1\right]_+,$$

where $\theta_+ = \max\{0, \theta\}$.[4]

We assume that individual players have no particular bargaining skills or advantage in bilateral bargaining and so the bilateral gains from the first-period agreement are shared equally between the principal and each agent. With this assumption asymmetries in the equilibrium payoff arise endogenously from the contracts and economic fundamentals and reflect each players' relative power in the game—measured by the share each player appropriates of the total economic surplus.

[3] We often write ϕ_i rather than the more cumbersome $\phi_i(x_i, t_i)$.

[4] The allocation rule can be given the following non-formal interpretation. In the second-stage player 0 can act as a principal, making standard take-it-or-leave-it offers (x_i, t_i) to all those agents $i \in S$, while each agent $i \in S$ can use the original contract as an outside option. So if he is offered a contract with $\phi_i(x_i, t_i) < \phi_i^1$, he may take the principal to court. If the principal cannot leave agent i his reservation payoff then the principal becomes subject to the oversight of a court—as in bankruptcy laws such as the Chapter 11—and the benefits from its reorganization are shared among the members of S.

Each first-stage bilateral bargaining problem can be described by all possible payoffs ϕ_0 and ϕ_i from agreement and the disagreement payoffs d_0^i and 0 for the principal and the agent respectively. Given our symmetry assumption the outcome of bilateral bargaining satisfies

$$\phi_0^2 = d_0^i + \frac{1}{2}\left[\phi_i^2 + \phi_0^2 - d_0^i\right] \geq 0 \Leftrightarrow \phi_0^2 - d_0^i = \phi_i^2 \geq 0. \tag{1}$$

This condition is satisfied by most solution concepts for two-player games and in particular the Nash bargaining solution. The previous elements define what we refer to as the principal-agent game associated to (M, v).

As an equilibrium concept we use the notion of pairwise stability, a situation in which no individual player can improve his payoff in bilateral equal-sharing bargaining and does not wish to break-down negotiations—while taking the principals' disagreement payoff as given. Take a principal-agent game associated to the game (M, v):

Definition 1. The payoffs $\phi_N^* = (\phi_i^*)_{i\in M}$ are *pairwise stable* with respect to $d_0 = (d_0^i)_{i\in N}$ if

$$\phi_0^* - d_0^i = \phi_i^* \geq 0 \text{ for all } i \in N.$$

This notion is similar to the stability notion used by Stole and Zwiebel (1996) but it doesn't impose a particular structure on the principal's outside options. To complete the model specification we need to describe how the principal's outside options d_0^i are determined. This provides the distinction between a binding and non-binding contract setting, the issue we address in the next subsection

3.3 Contracts

If contracts are binding, and a bilateral negotiation fails, the principal has to compensate the remaining agents according to their original contracts, which determines his outside option. Suppose there is a a pairwise stable vector of payoffs ϕ_N^*. The principal's disagreement point when bargaining with each individual agent $i \in N$ is then determined by the allocation rule f while using the remaining agents' payoffs in ϕ_N^* as given, i.e.

$$d_0^i = \left[v(M\backslash i) - \textstyle\sum_{j\in N\backslash i} \phi_j^*\right]_+ .$$

So, with binding contracts, to computation of the bilateral surplus between the principal and each agent $i \in N$ uses v and ϕ_N^*.

A non-binding contract can on the other be renegotiated unilaterally in the first-stage. So if negotiations with agent i breaks-down the principal's outside option is instead the principal's payoff following the issuing renegotiation, which is in turn a pairwise stable payoffs in the (sub)game $(M\backslash i, v)$—the outside options in this issuing game are in turn pairwise stable payoffs of the (sub)games $(M\backslash i\backslash j, v)$, and so forth. So with non-binding contracts, to computation of the bilateral surplus between the principal and each agent $i \in N$ requires obtaining by induction pairwise stable payoffs ϕ_S^* of every (sub)game (S, v).

To summarize, the distinction between binding and non-binding contracts lies on how the remaining agents are to be paid if negotiations fail with agent i. With binding contracts payoffs are based on the *vector* ϕ_N^* itself, whether negotiations succeed or not. In the case of non-binding contracts the remaining agents are paid according to the to the *function* $\phi^* : (S, v) \to R^S$, i.e. using ϕ_N^* if negotiations succeed and $\phi_{N\backslash i}^*$ if they don't. In both cases we are looking for payoff vectors that are individually rational, efficient and pairwise consistent with respect to a rule that generates the vector of outside options d_0.

In the next section we show that if d_0 is obtained using the binding contract rule the solution is unique and coincides with the Nucleolus of a claim problem where the amount to be divided is $V(N)$ and each players' claim is $\triangle_i v(N)$, i.e. this is the unique *vector* which is itself pairwise stable. If d_0 is obtained using the binding contract rule the solution is also unique and coincides with the Shapley value of the game (M, v), i.e. this is the unique such *function* that is pairwise stable.[5]

4 Pairwise stable outcomes

For the remainder of the paper we will distinguish the pairwise stable payoffs with binding and non-binding contracts by the labels ϕ_N^B and ϕ_N^A respectively.

4.1 Binding contracts

If ϕ_N^B is a pairwise stable payoff vector then, since all players can guarantee a nonnegative payoff, it is individually rational and

$$\sum_M \phi_i^B = v(M).$$

In addition $\phi_0^B - d_0^i = \phi_i^B \geq 0$ for all $i \in N$, which means that $\phi_i > 0$ for some $i \in S$ and since $d_0^i \geq 0$ it must also be the case that $\phi_0^B > 0$.

We now look at a typical bilateral bargaining problem. Since

$$\phi_0^B = v(M) - \sum_N \phi_i^B \text{ and } d_0^i = \left[v(M \backslash i) - \sum_{N \backslash i} \phi_j^B \right]_+ ,$$

the gains from trade with agent i are less or equal to the marginal contribution of i, i.e.

$$\phi_i^B + \phi_0^B - d_0^i \leq \triangle_i v(M).$$

Moreover by (1) it must be that $\phi_0^B \geq \phi_i^B$ for all $i \in N$ and

$$\begin{aligned} \phi_0^B &= \phi_i^B = \frac{v(M) - \sum_{N \backslash i} \phi_j^B}{2} \text{ if } d_0^i = 0 \\ \phi_i^B &= \frac{\triangle_i v(M)}{2} \text{ and } \phi_0^B = v(M) - -\sum_N \phi_i^B \text{ if } d_0^i > 0 \end{aligned}$$

and so

$$\phi_i^B = \min \left\{ \phi_0^B, \frac{\triangle_i v(M)}{2} \right\} \text{ for all } i \in N. \tag{2}$$

It follows that with binding contracts there is at most one contract equilibrium payoff vector ϕ_N^B. Suppose there were two distinct vectors ϕ_N^B and $\phi_N^{B\prime}$. By Pareto Optimality we could then find at least one agent i such that

$$\phi_0^B \geq \phi_0^{B\prime} \text{ and } \phi_i^B < \phi_i^{B\prime},$$

which contradicts (2).

[5] The latter is reminiscent of Hart and Mas-Colells' (1989) characterization of the Shapley value. The distinction is that, while for general games consistency is required for all subgroup of players, pairwise consistency is sufficient for the case of principal agent problems

We are then looking for the single individually rational, pareto optimal vector, that satisfies (2) and, if each agent gets half his marginal contribution, it gives the remainder to the principal. Formally

$$\begin{aligned} \text{if } \sum_M \frac{\triangle_i v(M)}{2} \quad &\geq \quad v(M) \;\Rightarrow\; \phi_i^B = \min\left\{\lambda, \frac{\triangle_i v(N)}{2}\right\} \;\forall i \in N^0 \\ \text{with } \lambda \quad &: \quad \sum_M \min\left\{\lambda, \frac{\triangle_i v(M)}{2}\right\} = v(M) \end{aligned} \tag{3}$$

This expression can be recognized as the constrained equal award rule for half claims of a bankruptcy problem where the amount to be divided is $v(M)$ and each player claim is his marginal contribution to M. In addition

$$\text{if } \sum_M \frac{\triangle_i v(M)}{2} < v(M) \;\Rightarrow\; \phi_i^B = \frac{\triangle_i v(M)}{2} \text{ and } \phi_0^B = \triangle_0 v(M) - \lambda \text{ with } \lambda = \sum_M \frac{\triangle_i v(M)}{2} \tag{4}$$

The second expression on the other hand satisfies the constrained equal losses rule for half claims since

$$\lambda \leq \max_N \left\{\frac{\triangle_i v(M)}{2}\right\}.$$

So ϕ_N^B satisfies the Talmud rule, and so, following Aummann and Maschler (1985) we also have

Proposition 1. There exists an unique ϕ_N^B. It coincides with the Nucleolus of a claim problem where the amount to be divided is $v(M)$ and each players' claim is $\triangle_i v(M)$.

In words, to obtain ϕ_N^B we divide each dollar of the surplus equally by all players until half the lowest marginal contribution is reached. At that moment those agents receiving half their marginal contributions are removed and the next dollar is divided equally among the remaining players. It proceeds in this way, removing those agents who reach half their marginal contributions, until the total surplus is distributed or, if all agents get half their marginal contributions, the remainder goes to the principal—see also the discussion in the introduction on how to obtain the solution from a system of connected containers.

Pairwise stability is therefore a natural and simple concept that in the case of binding contracts selects a single solution for every game. This concept considers the optimalitty of each bilateral bargaining in isolation but it does not alow the principal to break-down several negotiations simultaneously. It may however be profitable for the monopolist to abandon a subset of agents, and sign the equilibrium contracts with the remaining agents. Let ϕ_N^B be *strongly pairwise stable* if in adition it is unprofitable for the principal to leaves a subset of agents without contracts while paying the remainder according the original ones, i.e. it satisfies $\phi_0^B \geq v(S) - \sum_{S\backslash 0} \phi_j^B$ for all $S \subseteq M$ s.t. $0 \in S$. We have

Lemma 1. ϕ_N^B is strongly pairwise stable if and only if it lies the core of the game (M, v).

Proof: The core of a game is given by the set of payoff vectors ϕ_N such that $\phi_N \geq 0$, $\sum_M \phi_i = v(N)$ and $\sum_S \phi_i \geq v(S)$ for all $S \subset N$. Since $v(S) > 0$ only if $0 \in S$, the above statement follows from the condition for strong pariwise stability.

Since the solution ϕ_N^B is unique, this additional requirement cannot be used as a refinement but is an interesting property that provides additional validation for its use in games in which it is satisfied.

4.2 Non-binding contracts

Consider now a payoff vector ϕ_N^A that is pairwise stable with non-binding contracts. All players can guarantee a nonnegative payoff so the vector is individually rational and Pareto Optimal. Suppose that in each bilateral bargaining problem between the principal and agent i if the gains from trade are positive, i.e. $\phi_i^A + \phi_0^A - d_0^i \geq 0$, then $\phi_i^A = \phi_0^A - d_0^i$. Adding over all $i \in N$ we have

$$\sum_N \phi_i^A = N\phi_0^A - \sum_N d_0^i. \tag{5}$$

As we discussed above, the principal's payoff if the negotiations with agent i breaks-down is itself the principal's payoff if he is contracting with only $N\backslash i$, i.e his pairwise stable payoff in the (sub)game $(N\backslash i, v)$. Using the following notation $d_0^i = \phi_0^A(N\backslash i)$ and $\phi_0^A(N\backslash i) = 0$ and since ϕ_N^A is PO we have

$$\sum_M \phi_i^A = V(N) \text{ and } (5) \Rightarrow \sum_M \triangle_i \phi_0^A(N) = V(N)$$

The same reasoning holds if the principal is negotiating only with a subset of agents S, so the function $\phi_0^A : (S, v) \to R$ is a potential function. In addition, from (1) we have $\phi_i^A = \triangle_i \phi_0^A(N)$ for all $i \in M$.

Hart and Mas-Colell (1989) show that for any coaltional game the marginal contribution of a player to its unique Potential function coincides with its Shapley value—since here the principal is an indispensable player his payoff is also the potential itself. The game (M, v) is monotonic, i.e. $v(T) \geq v(T')$ if $T' \subseteq T$, so the potential is non-decreasing and therefore the gains from trade in bilateral negotiations are non-negative. It follows:

Proposition 2. There exists an unique ϕ_N^A and it coincides with the the Shapley value of the game (M, v).

In words, with non-binding contracts the pairwise stable payoffs are a weighted average of a player's marginal contributions to all possible subsets of M. For the purpose of the discussion below it helps to remind an interesting way to capture the Shapley value. Imagine that the players are ordered randomly, with each of the possible $|M|!$ orderings being equally likely. If a player is placed after a set of players S then he is paid $\triangle_i v(S)$. The Shapley value is simply the expectation of this taken over all random orderings.

Because the potential of (M, v) is non-decreasing, the principal does not wish to break-down negotiations simultaneously with several agents to get the outcome of the renegotiated contracts in the issuing game. So ϕ_N^A is by construction pairwise stable in a strong sense, even if it does not lie in the core. Also, as each player is paid its marginal contribution to the potential, and since here the principal is an indispensable player his payoff is also the potential itself, it follows that $\phi_i^A \leq \phi_0^A$ for all $i \in N$.

5 Binding vs non-binding contracts

The analysis from the previous section applies to any game but to compare the payoffs we need to impose some structure on the economic fundamentals. In this paper we study in detail the cases where agents are either complements or substitutes. To introduce formally these concepts we use the second-order difference operator

$$\Delta_{ij}^2 v(S) \equiv \triangle_j \left[\triangle_i v(S)\right].$$

We consider throughout the paper that $n \geq 2$. Agents i and j are *substitutes in S* if the marginal contribution of i to S is decreased by the presence of j, i.e. i and j are substitutes if

$$\Delta_{ij}^2 v(S) = v(S \cup i \cup j) - v(S \backslash i \cup j) - v(S \backslash j \cup i) + v(S \backslash i \backslash j) \leq 0.$$

Agents i and j are *complements in S* if the opposite is true. Since the principal is indispensable he is also a complement in all $S \subseteq N$.

Agents are *substitutes* if and only if the marginal contribution of any subset of agents decreases with the presence of additional agents, i.e. if v is submodular with respect to agent inclusion—the set of subsets can be ordered by inclusion so as $S \leq T$ if and only if $S \subset T$. This is the case if and only if any two agents i and $j \in N$ are substitutes in S for all $S \subseteq M$ (Topkis, 1978). Agents are *complements* if instead v is supermodular with respect to agent inclusion.

If agents are symmetric then the value of the game depends on the number but not the identities of agents, i.e. $v(|S|)$ if $0 \in S$, and agents are complements if the value is convex in the number of agents—and substitutes if the value is concave.

Agents are *perfect complements* in N if and only if all agents are indispensable, i.e. $\Delta_i v(S) = 0$ for all $S \subset M$, and so

$$\Delta_{ij}^2 v(S) = 0 \text{ for all } S \subset N \text{ and } \Delta_{ij}^2 v(N) = v(N).$$

So (M, v) is also pure bargaining game. Agents i is a *perfect substitute* to j if $\triangle_j v(S) = 0$ if $i \in S$ for all $S \subseteq N$ so $\triangle_{ij} v(S) = - \triangle_j v(S)$. An agent is independent if his the marginal contribution of agent i does not depend on the presence of other agents, i.e. if $\Delta_{ij}^2 v(S) = 0$ for all $S \subseteq M \ni 0$. So agents are *independent* if (M, v) is "linear" with respect to agent inclusion.

5.1 Substitute agents

Take the set of agents N and suppose agents are ordered randomly in a sequence. Let k denote the position of an agent in the sequence. For any ordering, the value $v(M)$ is equal to the sum of the marginal contribution of each agent i to the set of the proceeding agents and the principal, i.e.

$$v(M) = \sum_N \triangle_k v(\{0, .., k-1\}).$$

If agents are substitutes we have that

$$v(M) \leq \sum_N \triangle_i v(M).$$

This inequality is reversed if the agents are complements.

Since $\triangle_0 v(M) = v(M)$ it follows from (4) that

$$\phi_i^B = \frac{\triangle_i v(M)}{2} \text{ for all } i \in N \text{ and } \phi_0^B = v(M) - \sum_N \frac{\triangle_i v(M)}{2}. \tag{6}$$

In the case of substitutes ϕ_N^B is also strongly pairwise stable, i.e. it lies in the Core. Moreover it is its center.

Recall that the Shapley value of a player is the expectation over all random orderings of that player's marginal contribution to the value of the players proceeding him in each ordering. In half the possible orderings the principal is placed after an agent i and so his contribution is zero. For the remaining half of the orderings, since the marginal contributions of the agents are decreasing with respect to agent inclusion, we have $\triangle_i v(S) \geq \triangle_i v(M)$. The sum of the probabilities associated to these marginal contributions is one half. This, together with (6), gives that

Proposition 3. If agents are substitutes then binding contracts harm the agents and benefit the principal, i.e.

$$\phi_i^A \geq \phi_i^B \text{ for all } i \in N \Rightarrow \phi_0^A \leq \phi_0^B$$

(the inequality is strict if $\Delta_{ij}^2 v(S) < 0$ for some $S, i, j \in N$).

Consider now the particular case where every agent is a perfect substitute to any other agent. In this case we have that $\triangle_i v(S) = 0$ for all $i, S \subseteq N$ if $S \neq \{0, i\}$, and $\triangle_i v(S) = v(\{0, i\}) = v(M)$ otherwise. So

$$\phi_N^B : \ \phi_i^B = 0 \text{ for all } i \in N \text{ and } \phi_0^B = v(M).$$

So with binding contracts the principal can extract the full surplus, which is the single Core allocation.

On the hand, of the $|M|!$ possible random orderings of the players there are only $|M-2|!$ that start by $0, i, ..$ In these cases the contribution of i is $v(M)$, and in all the others it is zero. We thus have

$$\phi_N^A : \ \phi_i^A = \frac{1}{|M|\,|M-1|} v(M) \text{ for all } i \in N \text{ and } \phi_0^B = \frac{|M-1|}{|M|} v(M).$$

These payoffs lie outside the Core. It is however reasonable to expect that with non-binding contracts each agent will receive a small part of the surplus because by leaving the game each can leave the principal in a weaker position when renegotiating the contracts. So the principal has to pay each of them to keep his options open. The principal does nevertheless approach full extraction when the number of agents is large.

5.2 Complement agents

It on the other hand agents are complements then v is supermodular with respect to player inclusion and the game (M, v) is therefore a convex game. So its Shapley value, ϕ_N^A, always lies in the core—in fact it is its center of gravity. On the other hand ϕ_N^B may not lie on the core.

Recall from the previous section that if agents are complemets we have

$$v(M) \geq \sum_N \triangle_i v(M),$$

and so if contracts are binding (3) applies—the constrained equal award rule for half claims. If every agent's marginal contribution to M is high, i.e. $|M| \min \{\triangle_i v(M)\} \geq V(M)$, then each player gets an equal share of the surplus. If on the other hand there are some agents with small marginal contributions to M, i.e. if $|M| \min \{\triangle_i v(M)\} < V(M)$ then there are some $i \in N$ that are receive $\triangle_i v(M)/2$. The other players receive an equal share of the remainder.

If contracts are non-binding we have that $\phi_i^A = V(M)/|M|$ for all $i \in M$ if and only if agents are perfect complements—i.e. pure bargaining games—since in that case there are $|M-1|!$ of the $|M|!$ possible orderings in which each agent's marginal contribution is $v(M)$ and contributions are increasing with respect to agent inclusion.

Proposition 4. If agents are perfect complements then payoffs are contract neutral, i.e.

$$\phi_i^A = \phi_i^B = V(M)/|M|.$$

Consider now the case where agents are complements but not perfect complements. Since the principal is an indispensable player, we have that

$$\phi_0^A \geq \phi_i^A \text{ and } \phi_0^A > \frac{v(M) - \sum_S \phi_i^A}{|M| - |S|} \text{ for all } i, S \subseteq N \neq \emptyset. \tag{7}$$

So if $\phi_i^B = V(M)/|M|$ and agents are not perfect complements we have that $\phi_0^A \geq \phi_i^B$ for all $i \in N$ and the inequality is strict for some i. In addition if the marginal contribution of an agent is strictly increasing in the presence of additional agents, i.e. agents are strict complements, and since the sum of the probabilities associated to those orderings the principal is placed before an agent i is one half we have that for any $i \in N$ $\phi_i^A < \triangle_i v(M)/2$—unless if some i is an independent agent, i.e. $\triangle_i v(S) = \triangle_i v(M)$ for all $S \ni 0$. It follow that that if $\phi_i^B \neq V(M)/|M|$ then $\phi_i^B > \phi_i^A$ for all i such that $\phi_i^B = \triangle_i v(M)/2$. Let $S' = \{S : \phi_i^B = \triangle_i v(M)/2\}$, it follows from (7) that

$$\phi_0^B = \frac{v(M) - \sum_{S'} \phi_i^B}{|M| - |S'|} < \phi_0^A \Rightarrow \sum_N \phi_i^B > \sum_N \phi_i^A.$$

Let strong complements mean that agents are neither perfect complements nor are there independent agents. We then have:

Proposition 5. If agents are strong complements then binding contracts harm the principal and benefit the agents as a whole, i.e.

$$\phi_0^A > \phi_0^B \text{ and } \sum_N \phi_i^B < \sum_N \phi_i^A.$$

Finally let us look at the case where all agents are independent. It follows from the independence of marginal contributions that

$$v(M) = \sum_N \triangle_i v(M),\ \phi_0^A = \phi_0^B = \frac{V(M)}{2} \text{ and } \phi_i^A = \phi_i^B = \frac{\triangle_i V(M)}{2} \text{ for all } i \in N.$$

So we have:

Proposition 5. If agents are independent then payoffs are contract neutral, i.e.

$$\phi_i^A = \phi_i^B = \frac{\triangle_i V(M)}{2} \text{ for all } i \in M.$$

6 Pairwise mergers and bargaining power

Our next and final question is: How does a mergers of two players affect bargaining power? A pairwise merger is modelled as an ex-ante contract that gives full control of the resources of both players to a single player without changing the underlying technology, i.e. the game's value function remains unchanged with respect to the resources owned by a group of agents and so it only changes to reflect the changes in resource ownership.

We rely on ours previous results and the work of Segal (2003) to study both vertical (principal-agent merger) and horizontal mergers (agent-agent). We find that accounting for the contractual setting can be an important variable to identify conditions under which we may expect a "bargaining paradox"—i.e. the merger of two players decrease their bargaining power—or a "waterbed effect"—a termed coined in antitrust practice to label those situations in which the merger of two horizontal agents increases their joint bargaining power by simultaneously reducing the bargaining power of all remaining players when bargaining with an indispensable player.[6] Throughout we continue to focus on the case where agents are either complements or substitutes.

[6]The term bargaining paradox was oringiated by Zelton to discribed the simmingly odd effect arising in pure bargaining games. To my knowledge the term waterbed effect was introduced in the economics literature by the work of Majumdar...

6.1 Vertical mergers

We consider first the case of vertical mergers, when 0 acquires agent i so it changes the value function with respect to players' inclusion in the following way:

$$\widehat{v}(S) \left\{ \begin{array}{c} v(S \cup i) \text{ if } 0 \in S \\ 0 \text{ otherwise} \end{array} \right. .$$

Notice that the marginal contributions of agents to all S that includes i, like N, remain unchanged but in the remaining cases it may change.

If contracts are binding and agents are substitutes then each agent still receives half his marginal contribution to $V(N)$ and so this merger leaves the payoffs unchanged. If however agent are complements i's claim is removed but the remaining players' claims remain unchanged. With the same to divide among less claimants, it follows that each agent's payoffs will strictly increase. So the merger is unprofitable.

Consider now the case of non-binding contracts. Segal (2003) finds that when one uses the Shapley value as a solution to a game then "a player helps (hurts) his complements (substitutes) by merging with his indispensable player"—the principal. We again have that a vertical merger is unprofitable when agents are complements but it is profitable when agents are substitutes. The results are summarized in the table below.

		binding	non-binding
complements	i+0	<	<
	j	>	>
substitutes	i+0	=	>
	j	=	<

Payoff changes from a "vertical" merger

Overall we have a *bargaining paradox* with binding contracts: a vertical merger (weakly) decreases the bargaining power of the merging players.

6.2 Horizontal mergers

We now turn to the case of horizontal mergers, when agents i controls those assets that would be owned by j when bargaining with 0. The value function changes in the following way:

$$\underline{v}(S) \left\{ \begin{array}{c} v(S \cup j) \text{ if } i \in S \\ v(S \backslash j) \text{ otherwise} \end{array} \right. .$$

Consider again first the case of binding contracts. If agents are substitutes each agent receives half his marginal contribution. While the merger leaves this unchanged for every agent $z \notin \{i.j\}$, we have that $\triangle_i \underline{v}(M) = v(M) - v(M \backslash i \backslash j) \geq v(M \backslash j) + v(M \backslash i)$—from the definition of substitutes—and so agent i receives a larger share of the surplus due to the merger. So a horizontal merger is profitable to the detriment of the principal since the bargaining power of the remaining agents remains unchanged—so there is no waterbed effect. If agents are complements, agent i has a single claim which is lower than the sum of the claims for i and j if they don't merge. It follows from the constrained equal award rule for half claims that the merger is unprofitable to the merging agents and increases the bargaining power of all other players. So not only is the merger unprofitable but, and more strongly, we have a *reversed waterbed effect*.

Consider now the case of non-binding contracts. Segal (2003) finds that when one uses the Shapley value as a solution to a game with an indespensable player—here the principal—then "collusion between two complementary (substitutable) players helps (hurts) players who are indispensable". He also finds that if agents are substitutes, and agent substitutability is increasing with respect to agent inclusion, then a merger is profitable and hurts the remaining agents. Under these conditions we therefore have an *waterbed effect*. If on the other hand are complements, and complementarity is increasing with respect to agent inclusion, then a merger is unprofitable and helps the remaining agents. Therefore under these conditions we have a *anti-waterbed effect*. (Changes is complementarity and substitution are captured by the third-order difference operator, defined as $\Delta^3_{ijk}v(S) = \Delta_k\left[\Delta^2_{ij}v(S)\right]$. If $\Delta^3_{ijk}v(S) \leq$ for all $i,j,k \in N$ and $S \in M$ then agent substitutabiltiy is decreasing with respect to agent inclusion.)

The work of Segal (2003) would sugeest that onlty in some cases, where the payoffs of the other players moved in the same direction, can the profitability of the merger be unambigously asseseed. We find however that submodularity and supermodularity with respect to agent inclusion provide sufficient conditions for the profitability of mergers: horizontal mergers are profitable (unprofitable) if agents are substitutes (complements). Imagine again that the players are ordered randomly, with each of the possible $|M|!$ orderings being equally likely. If a player is placed after a set of players S then he is paid $\triangle_i v(S)$. The Shapley value is simply the expectation of this taken over all random orderings. It follows from Segal (2003) that if agent i merges with agent j then the change in his payoff can be captured by of the the third-difference operator of each player to the subset of players that precced him in the ordering, aggregated over all possible orderings, i.e.

$$-\frac{1}{|M|!}\sum\nolimits_{k\in M\backslash i\backslash j}\sum\nolimits_{\pi\in\Pi:\pi(j)<\pi(k)<\pi(i)}\Delta^3_{ijk}v(\pi^k) \tag{8}$$

where Π denotes the set of orderings of M, $\pi(i)$ the rank of player $i \in M$ in the ordering $\pi \in \Pi$, and $\pi^i = \{j \in M : \pi(j) \leq \pi(i)\}$ denote the set of players that come before i in ordering π including i. Notice that $\Delta^3_{ijk}v(S)$ does not depend on the order of taking differences. For all orderings $\pi \in \Pi$ such that $\pi(i) < \pi(0)$ all terms in the expression above are zero. On the other hand, for all orderings $\pi \in \Pi$ such that $\pi(0) < \pi(j) < \pi(0)$ we have that

$$\Delta^2_{ij}v(\pi^i) - \Delta^2_{ij}v(\pi^i\backslash k) + \Delta^2_{ij}v(\pi^k\backslash k) - \Delta^2_{ij}v(\pi^k\backslash k\backslash k-1) + ... + \Delta^2_{ij}v(\pi^j \cup t) - \Delta^2_{ij}v(\pi^j)$$

where $\pi(i) = \pi(k) + 1$ and $\pi(j) = \pi(t) - 1$. So its sum is simply

$$\Delta^2_{ij}v(\pi^i) - \Delta^2_{ij}v(\pi^j).$$

However, if $\pi(j) < \pi(0) < \pi(0)$ we have that

$$\Delta^2_{ij}v(\pi^i).$$

While the latter sign depends only on the complementarity of the agents, the former cannot be unambigously signed.

Notice however for all ordering with $\pi(0) < \pi(j) < \pi(0)$ such that $\pi(0) \neq 1$ there exist one and only ordering π' that is similar to π except that $\pi(k) = 1$ and $\pi(t) = \pi(j+1)$ are permuted with j and i respectively. The sum of the terms in (8) associated to those two ordering is simply $\Delta^2_{ij}v(\pi^i)$, since $\Delta^2_{ij}v(\pi'^i) = \Delta^2_{ij}v(\pi^j)$ as the set of players precedding t in π and i in π' are the same. In a similar way, for those ordering for which $\pi(0) = 1$ there exists one and only one ordering π' that is similar to π except that 0 and $\pi(t) = \pi(j+1)$ are permuted with j and i respectively. In that case the sum is also $\Delta^2_{ij}v(\pi^i)$. So we can group all orderings in pairs so that each element in (8) is postive if agents are complements and negative if agents are substitutes. This explains why supermodularity and submodularity whith respect to agent inclusion are sufficient conditions for the profitability of mergers. This subsection results are summarized in the table below.

		binding	non-binding
complements	0	>	>
	i+j	<	<
	k	>	? (> if IC)
substitutes	0	<	<
	i+j	>	>
	k	=	? (< if IS)

Payoff changes from an "horizontal" merger

The issue of the profitability of mergers in varied settings has long been the subject of inquery. Harsanyi (1977) called the *joint bargaining paradox* the seemingly odd situation in which a group of players looses bargaining power by bargaining as a group. He observed that in pure-bargaining situations, where all players need to agree to create value and all players are therefore perfect complements, a symmetric solution gives each player a share of 1/n of the surplus. If two players merge we have a symmetric bargaining situation with n-1 players instead and, while two players would receive 2/n when they are independent, they receive only 1/(n-1) if they merge. Basically, a group looses bargaining power because their multiple veto oportunities are reduced to a single one.

Here we find that with both binding and non-binding contracts the joint bargaining paradox applies in this principal agent setting to both vertical and horizontal mergers when agents are complements—but not necessarily perfect complements. On the other hand it also shows that both kinds of mergers are more likely to be profitable if agents are substitutes.

These results also show that the contractual setting may be an important element in the identification of waterbed effects in both theory and practice. The conditions we identify are purely related to the pure bargaining effects. This contrasts with previous research that associates technological changes to changes in the asset ownership, i.e. settings in which the size of the pie depends not only on the assets controlled by a subset of players S but also on the ownership structure itself.

7 Applications

To be added.

8 Conclusion

To be added

9 References

To be added.