

Suppliers' merger and consumers' welfare

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Abstract

This article explores the consequences of a suppliers' merger on consumers' welfare when the product is sold to consumers by independent retailers competing *à la Cournot*. The literature shows that under the standard assumptions of private contracting and passive beliefs, there is no impact at all. I show that this unintuitive result strongly depends on the implicit assumption that suppliers have infinite capacities of production. Indeed, assuming that suppliers face a capacity constraint and that retailers hold out-of-equilibrium beliefs compatible with this constraint, I show that the merger raises the price on the final market and reduces consumers' welfare.

1 Introduction

Obtaining clearance from the antitrust authorities for a merger to monopoly is certainly a very difficult task for firms operating in countries where strict antitrust rules are rigorously enforced, as they are in the US and the European Union. Such a merger would inevitably lead to an increase in price and, thus, a reduction in consumers' welfare, unless spectacular (and improbable) efficiency gains can be implemented. A ban can thus be expected, unless remedies are negotiated between the firms and the antitrust authority. Divestiture stands as an obvious candidate as remedies in such a case. However, the literature on vertical contracting suggests an alternative, more efficient type of remedies. The solution is to forbid the monopolist to directly sell its product to consumers and to force it to sell through independent retailers. This leads to a complete loss of market power for the monopolist. More precisely, the monopolist captures industry profits, but level of price on the market (and thus of industry profits) is determined solely by the intensity of competition between retailers. Antitrust authority have accumulated a huge experience in dealing with competition issues at the retailing level. That these remedies are not adopted in practice and, as far as I know, are not even discussed, suggests that antitrust

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authorities are skeptical about the theoretical prediction of standard models of vertical contracting like McAfee and Schwartz (1994). Another, closely related, implication of previous results on vertical contracting to which the antitrust authorities would find difficult to adhere is that a merger between duopolistic suppliers has no impact on the final price.

The skepticism of antitrust authorities may find a justification in the caveats that qualify these results. Economists are of course aware of these caveats. In fact, they were first to point at them, like Rey and Tirole (2007). Strangely enough, however, this did not lead to the emergence of models relying on more appealing assumptions and leading to results that could find their way into the practice of antitrust authorities and thus exert an influence on antitrust case law. This article is an attempt to contribute to such an evolution by exploring the consequences of the fact that suppliers don't enjoy an infinite production capacity. This sounds fairly obvious, but the results quoted above rely on the opposite (implicit) assumption of infinite production capacity. I show that this is far from neutral. Indeed, assuming that production costs are null, I show that a merger between capacity constrained duopolistic suppliers strictly increases the price for consumers. This result is obtained by comparing the equilibria of a game with duopolistic capacity constrained suppliers, presented and solved here for the first time, with the equilibria of a game with a monopolistic capacity constrained supplier, presented in a previous paper (Avenel (2010)).

In section 2, I present the theoretical framework of this article. In section 3, I treat the case of infinite production capacity and replicate previous results from the literature by showing that, under the usual assumption of passive beliefs, a merger to monopoly has no impact on the price paid by consumers. In section 4, I consider capacity constrained suppliers. I present in details the resolution of the game with duopolistic suppliers and show that a merger strictly increases the price to consumers. Section 5 concludes.

2 A private bilateral contracting game with constrained suppliers

The industry is composed of two suppliers (S_1 and S_2) and two retailers (R_1 and R_2). Suppliers compete on the intermediate market to supply retailers. Retailers compete à la Cournot on the final market. Consumers' demand is assumed to be linear, with $P(X) = 1 - X$. Suppliers' marginal cost of production is taken equal to zero as long as the output is less than or equal to the production capacity. I denote by \bar{Q}_i the production capacity of S_i . It is exogenous and public knowledge. I will consider both the case where production capacities are infinite and the case where they are finite. Retailing costs are assumed to be null.

The strategic interaction between firms is modelled with the two-stage game $\Gamma(\bar{Q}_1, \bar{Q}_2)$ in which the timing of moves is:

Stage 1 (offers): S_1 and S_2 simultaneously make privately observed offers

to retailers. R_k is offered (q_{k1}, t_{k1}) by S_1 and (q_{k2}, t_{k2}) by S_2 , where q_{ki} is the quantity supplied by S_i to R_k and t_{ki} is the transfer from R_k to S_i .

Stage 2 (acceptances and competition): Each retailer decides whether to accept or reject the offer it received from each supplier. If R_k accepts the contract (q_{ki}, t_{ki}) , it pays t_{ki} and receives q_{ki} . The quantity x_k put on the market by R_k must be less than or equal to the total quantity of product received from suppliers.

It is useful to introduce here some notations for later reference. Let Q^m and π^m denote the output and the profit of a vertically integrated monopolist facing the inverse demand function $P(\cdot)$. Let q^C and π^C denote the output and the profit of a vertically integrated duopolist and $Q^C = 2q^C$ be the total aggregate output of the duopoly. Finally, let $BR(\cdot)$ denotes the best reply of an integrated duopolist to its rival's output, i.e. $BR(x) = \arg \max_y P(x+y) * y$.

Since R_l , $l \neq k$, does not observe (q_{k1}, t_{k1}) and (q_{k2}, t_{k2}) , $\Gamma(\bar{Q}_1, \bar{Q}_2)$ is an extensive form game of imperfect information. Solving Γ thus amounts to determining its (weak) perfect Bayesian equilibria (Mas-Colell, Whinston and Green (1995)). This requires that players' strategies are sequentially rational given beliefs and beliefs are consistent with strategies. A strategy for retailer R_k is a triplet (z_{k1}, z_{k2}, x_k) , where (i) $z_{ki}(q_{k1}, t_{k1}, q_{k2}, t_{k2}) = 1$ if R_k accepts (q_{ki}, t_{ki}) and 0 otherwise and (ii) x_k is a function from $[0, \bar{Q}_1] \times \mathbb{R}^+ \times [0, \bar{Q}_2] \times \mathbb{R}^+$ into $[0, z_{k1} * q_{k1} + z_{k2} * q_{k2}]$. Beliefs for retailer R_k are a quadruplet of functions $(\tilde{q}_{l1}, \tilde{t}_{l1}, \tilde{q}_{l2}, \tilde{t}_{l2})$.¹ Since a deviation by S_j does not convey information on S_i 's strategy (it is not even observed by S_i), \tilde{q}_{li} and \tilde{t}_{li} are functions of (q_{ki}, t_{ki}) . Finally, a strategy for S_i is a quadruplet $(q_{1i}, t_{1i}, q_{2i}, t_{2i}) \in \mathcal{S}_i := \{(q_{1i}, t_{1i}, q_{2i}, t_{2i}) \in (\mathbb{R}^+)^4 : q_{1i} + q_{2i} \leq \bar{Q}_i\}$.

Definition 1 An assessment $\left((q_{ki}, t_{ki}, \tilde{q}_{ki}, \tilde{t}_{ki})_{i=1,2; k=1,2}, (z_{k1}, z_{k2}, x_k)_{k=1,2}\right)$ is a (weak) perfect Bayesian equilibrium of $\Gamma(\bar{Q}_1, \bar{Q}_2)$ if and only if:

- (i) For any $(q_{k1}, t_{k1}, q_{k2}, t_{k2})$,
$$(z_{k1}, z_{k2}) = \arg \max_{(z_1, z_2) \in \{0,1\}^2} \left\{ \max_{x \in [0, z_1 * q_{k1} + z_2 * q_{k2}]} \{P(x + x_l((\tilde{q}_{l1}, \tilde{t}_{l1}, \tilde{q}_{l2}, \tilde{t}_{l2})) * x - z_1 * t_{k1} - z_2 * t_{k2})\} \right\}$$
and $x_k(q_{k1}, t_{k1}, q_{k2}, t_{k2}) = \arg \max_{x \in [0, z_{k1} * q_{k1} + z_{k2} * q_{k2}]} \{P(x + x_l((\tilde{q}_{l1}, \tilde{t}_{l1}, \tilde{q}_{l2}, \tilde{t}_{l2})) * x - z_{k1} * t_{k1} - z_{k2} * t_{k2})\}.$
- (ii) Beliefs are derived from strategies through Bayes' rule whenever possible.
- (iii) $(q_{1i}, t_{1i}, q_{2i}, t_{2i}) = \arg \max_{(q_{1i}, t_{1i}, q_{2i}, t_{2i}) \in \mathcal{S}_i} \{z_{1i}(q_{11}, t_{11}, q_{12}, t_{12}) * t_{1i} + z_{2i}(q_{21}, t_{21}, q_{22}, t_{22}) * t_{2i}\}$

While beliefs along the equilibrium pass are derived from the strategies using Bayes' rule, the equilibrium concept imposes no a priori restriction on out-of-equilibrium beliefs. It is however necessary to specify these beliefs, because of the multiplicity of equilibria. Most contributions in the literature assume that players hold passive beliefs.

¹ As usual in the literature on vertical contracting, I assume that R_k holds beliefs that put probability one on one strategy of S_i .

Definition 2 Under passive beliefs, when R_k receives the offer (q_{ki}, t_{ki}) from S_i , it believes that S_i offers to R_l the equilibrium offer (q_{li}^*, t_{li}^*) .

Under the common assumptions of a monopolist supplier with infinite production capacity, assuming passive beliefs is reasonable. It is indeed optimal for the supplier to deal with retailers as if they were operating on different markets. Furthermore, in the type of game we consider, passive beliefs are wary beliefs, which reinforces their legitimacy (Rey and Vergé (2004)). Passive beliefs remain reasonable in a model with two unconstrained suppliers (Hart and Tirole (1990)). I assume passive beliefs in game $\Gamma^\infty = \Gamma(\infty, \infty)$.

As soon as a supplier has a finite production capacity, passive beliefs have to be abandoned. I assume that S_i cannot offer more than it is able to produce, namely $q_{1i} + q_{2i} \leq \bar{Q}_i$. When R_1 receives an out-of-equilibrium offer $q_{1i} > \bar{Q}_i - q_{2i}^*$, where q_{2i}^* is the quantity offered by S_i to R_2 in equilibrium, R_1 cannot believe that S_i still offers q_{2i}^* to R_2 . Beliefs must be compatible with suppliers' strategy sets. A modification of the assumption on out-of-equilibrium beliefs is thus necessary when suppliers are capacity constrained. In Avenel (2010) I define full capacity beliefs in the case of a monopolist supplier. Here I complete the definition of full capacity beliefs to take into account the existence of a second supplier. It is useful to first define precisely what an acceptable offer is.

Definition 3 An offer (q_{li}, t_{li}) from S_i to R_l is acceptable given (q_{lj}, t_{lj}) , the offer from S_j to R_l , if and only if:

$$\begin{cases} t_{li} + t_{lj} \leq \max_{x \in [0, q_{li} + q_{lj}]} F(q_{li}, q_{lj}, t_{li}, t_{lj}, x) \\ t_{li} \leq \max_{x \in [0, q_{li} + q_{lj}]} F(q_{li}, q_{lj}, t_{li}, t_{lj}, x) - \max_{x \in [0, q_{lj}]} F(q_{li}, q_{lj}, t_{li}, t_{lj}, x) \\ t_{lj} \leq \max_{x \in [0, q_{li} + q_{lj}]} F(q_{li}, q_{lj}, t_{li}, t_{lj}, x) - \max_{x \in [0, q_{li}]} F(q_{li}, q_{lj}, t_{li}, t_{lj}, x) \end{cases} \text{ or } \begin{cases} t_{li} \leq \max_{x \in [0, q_{li}]} F(q_{li}, q_{lj}, t_{li}, t_{lj}, x) \\ t_{lj} > \max_{x \in [0, q_{li} + q_{lj}]} F(q_{li}, q_{lj}, t_{li}, t_{lj}, x) - \max_{x \in [0, q_{li}]} F(q_{li}, q_{lj}, t_{li}, t_{lj}, x) \\ t_{lj} - t_{li} > \max_{x \in [0, q_{lj}]} F(q_{li}, q_{lj}, t_{li}, t_{lj}, x) - \max_{x \in [0, q_{li}]} F(q_{li}, q_{lj}, t_{li}, t_{lj}, x) \end{cases}$$

where $F(q_{li}, q_{lj}, t_{li}, t_{lj}, x) = P(x + x_k(\tilde{q}_{ki}(q_{li}, t_{li}), \tilde{t}_{ki}(q_{li}, t_{li}), \tilde{q}_{kj}(q_{lj}, t_{lj}), \tilde{t}_{kj}(q_{lj}, t_{lj}))) * x$.

When offered (q_{li}, t_{li}) acceptable given (q_{lj}, t_{lj}) , R_l may accept both contracts, which requires that this leaves it with non-negative profits and that accepting both contracts leads to profits not smaller than accepting only one contract. Alternatively, R_l may accept only (q_{li}, t_{li}) . It is the best thing to do if this leads to a non-negative profit and to a strictly larger profit than either accepting both contracts or accepting (q_{lj}, t_{lj}) only. In this case, (q_{li}, t_{li}) could still be defined as acceptable when R_l is indifferent between (q_{li}, t_{li}) and (q_{lj}, t_{lj}) . However, definition (3) doesn't allow for this, for reasons explained below. In this sense, acceptability as described here is strict acceptability. Note that for any (q_{lj}, t_{lj}) , there is an acceptable (q_{li}, t_{li}) . In fact, any contract with $t_{li} = 0$ is acceptable.

Definition 4 Under full capacity beliefs, when R_k receives the offer (q_{ki}, t_{ki}) from S_i , it believes that S_i offers $(\tilde{q}_{li}(q_{ki}, t_{ki}), \tilde{t}_{li}(q_{ki}, t_{ki}))$ to R_l , where $\tilde{q}_{li}(q_{ki}, t_{ki}) = \bar{Q}_i - q_{ki}$ and $\tilde{t}_{li}(q_{ki}, t_{ki})$ is such that $(\tilde{q}_{li}(q_{ki}, t_{ki}), \tilde{t}_{li}(q_{ki}, t_{ki}))$ is acceptable given that R_l receives from S_j the equilibrium offer (q_{lj}^*, t_{lj}^*) .

The acceptability of (q_{li}, t_{li}) depends on (q_{lj}, t_{lj}) . R_k assumes that S_i makes an acceptable offer to R_l . What this exactly means depends on S_i 's beliefs on (q_{lj}, t_{lj}) . Indeed, (q_{li}, t_{li}) is considered by S_i as acceptable, given S_i 's beliefs on (q_{lj}, t_{lj}) . Consistency with the sequential rationality of S_i requires that S_i believes that S_j plays its equilibrium strategy.

The previous definitions imply that R_k assumes that S_i is not willing to offer R_l a contract such that R_l is indifferent between accepting only (q_{li}, t_{li}) and accepting only (q_{lj}, t_{lj}) . In this situation, R_l will reject (q_{li}, t_{li}) with some probability, a risk that S_i can eliminate by reducing t_{li} by an infinitely small amount. R_k assumes that this is indeed what S_i does.

Another implication of the previous definitions is that full capacity beliefs do not depend on the transfer offered to the retailer. We thus drop transfers as arguments of the beliefs functions from now on.

A merger between S_1 and S_2 will transform the game $\Gamma(\bar{Q}_1, \bar{Q}_2)$ into the game $\mathfrak{G}(\bar{Q}_1 + \bar{Q}_2)$ in which a monopolist supplier with production capacity $\bar{Q}_1 + \bar{Q}_2$ supplies retailers. In fact, $\mathfrak{G}(\bar{Q})$ is simply $\Gamma(\bar{Q}, 0)$. However, for clarity, it is better to introduce specific notations. In $\mathfrak{G}(\bar{Q})$, there is only one supplier, S , supplying R_1 and R_2 . Contract offers are privately observed by retailers. I denote by (q_k, t_k) the contract offered to R_k in stage 1. A strategy for retailer R_k is a pair (z_k, x_k) , where (i) $z_k(q_k, t_k) = 1$ if R_k accepts (q_k, t_k) and 0 otherwise and (ii) x_k is a function from $[0, \bar{Q}] \times \mathbb{R}^+$ into $[0, z_k * q_k]$. Beliefs for retailer R_k are a pair $(\tilde{q}_l, \tilde{t}_l)$ of functions of (q_k, t_k) . Finally, a strategy for S is a quadruplet $(q_1, t_1, q_2, t_2) \in \mathcal{S} := \{(q_1, t_1, q_2, t_2) \in (\mathbb{R}^+)^4 : q_1 + q_2 \leq \bar{Q}\}$.

Definition 5 An assessment $((q_k, t_k, \tilde{q}_k, \tilde{t}_k)_{k=1,2}, (z_k, x_k)_{k=1,2})$ is a (weak) perfect Bayesian equilibrium of $\mathfrak{G}(\bar{Q})$ if and only if:

- (i) For any $(q_{k1}, t_{k1}, q_{k2}, t_{k2})$, $z_k = \arg \max_{z \in \{0,1\}^2} \left\{ \max_{x \in [0, z * q_k]} \{P(x + x_l((\tilde{q}_l, \tilde{t}_l)) * x - z * t_k)\} \right\}$
- and $x_k(q_k, t_k) = \arg \max_{x \in [0, z_k * q_k]} \{P(x + x_l((\tilde{q}_l, \tilde{t}_l)) * x - z_k * t_k)\}$.
- (ii) Beliefs are derived from strategies through Bayes' rule whenever possible.
- (iii) $(q_1, t_1, q_2, t_2) = \arg \max_{(q_1, t_1, q_2, t_2) \in \mathcal{S}} \{z_1(q_1, t_1) * t_1 + z_2(q_2, t_2) * t_2\}$.

The definition of passive beliefs is unchanged, up to notations.

Definition 6 Under passive beliefs, when R_k receives the offer (q_k, t_k) from S , it believes that S offers to R_l the equilibrium offer (q_l^*, t_l^*) .

With an upstream monopolist, it is easier to define full capacity beliefs. R_l is willing to accept a contract if it leaves it a non-negative profit given its beliefs.

Definition 7 An offer (q_l, t_l) from S to R_l is acceptable if and only if: $t_l \leq \max_{x \in [0, q_l]} P(x + x_k(\tilde{q}_k(q_l, t_l), t_k(q_l, t_l))) * x$.

Definition 8 Under full capacity beliefs, when R_k receives the offer (q_k, t_k) from S , it believes that S offers $(\tilde{q}_l(q_k, t_k), \tilde{t}_l(q_k, t_k))$ to R_l , where $\tilde{q}_l(q_k, t_k) = \bar{Q} - q_k$ and $\tilde{t}_l(q_k, t_k)$ is such that $(\tilde{q}_l(q_k, t_k), \tilde{t}_l(q_k, t_k))$ is acceptable.²

Given the previous definitions, asserting the impact of a suppliers' merger on consumers' welfare amounts to comparing the perfect Bayesian equilibria of $\Gamma(\bar{Q}_1, \bar{Q}_2)$ and $\mathfrak{G}(\bar{Q}_1 + \bar{Q}_2)$. In section 3, I do this comparison in the special case where suppliers are not capacity constrained, i.e. $\bar{Q}_1 = \bar{Q}_2 = +\infty$. In section 4, I consider the case where both firms are constrained, but still have a rather large production capacity, that is $Q^C < \bar{Q}_i < +\infty$ for $i = 1, 2$. Each supplier is able to produce strictly more than the output of a Cournot duopoly of vertically integrated retailers.

3 Unconstrained suppliers' merger and consumer welfare

Γ^∞ is similar to a game considered in Hart and Tirole (1990). There are however some differences. In particular, Hart and Tirole assume that the upstream marginal cost is high, while I assume costless production. Their assumption ensures that retailers put on the market all the product received from suppliers. With costless production, retailers may retain part of the product. I deal with this issue in the resolution of Γ^∞ .

I start with a characterization of the equilibrium on the final market in a perfect Bayesian equilibrium with passive beliefs of Γ^∞ . I also characterize profit-sharing between suppliers and retailers in equilibrium.

Lemma 1 In a perfect Bayesian equilibrium with passive beliefs of Γ^∞ , $x_k = q^C$ and $t_{ki} = 0$ for $k = 1, 2$ and $i = 1, 2$.

Proof. Assume that $(x_1, x_2) \neq (q^C, q^C)$. Necessarily, for some $k \in \{1, 2\}$, $x_k < q^C$. Furthermore, $x_l \leq BR(x_k)$, with $l \neq k$. R_k would increase its profits by increasing x_k . If (q_{ki}, t_{ki}) is rejected, then it is profitable for S_i to deviate by reducing t_{ki} so that the contract becomes acceptable for R_k . If (q_{k1}, t_{k1}) and (q_{k2}, t_{k2}) are accepted and $q_{k1} + q_{k2} < q^C$, then S_1 can increase profits by offering $(q_{k1} + \varepsilon, t_{k1} + \eta)$, for some small, positive ε and η . Thus, in equilibrium, $(x_1, x_2) = (q^C, q^C)$.

Assume that $t_{ki} > 0$ for some i and some k . If (q_{ki}, t_{ki}) is accepted by R_k , then S_j , $j \neq i$, can increase its profits by deviating from (q_{kj}, t_{kj}) to

²This definition of full capacity beliefs differs from the definition in Avenel (2010) in which $\tilde{t}_l(q_k, t_k)$ is the *highest* acceptable transfer. Indeed, this aspect of the definition doesn't change the results and its extension to the duopoly case raises many technical problems.

$(q_{kj} + q_{ki}, t_{kj} + t_{ki} - \varepsilon)$. If (q_{ki}, t_{ki}) is rejected by R_k , then either S_j can increase profits by increasing slightly t_{kj} (for $t_{kj} = 0$) or S_i can increase profits by reducing t_{ki} below t_{kj} so that (q_{ki}, t_{ki}) becomes acceptable (for $t_{kj} > 0$). ■

The outcome of the strategic interaction between firms in the industry is fairly competitive. In fact, suppliers are engaged in a very tough competition that drives transfers to zero and makes it impossible for them to restrict retailers' output on the final market. The equilibrium on the final market is identical to the equilibrium that would result from competition between vertically integrated retailers. The comparison is all the more relevant as retailers keep all of their profits for themselves. While lemma (1) characterizes equilibria, it doesn't establish the existence of an equilibrium. I now turn to this point and deal with it by presenting an assessment and showing that it is an equilibrium.

Definition 9 *The assessment E^∞ is characterized by (i) for any $(k, i) \in \{1, 2\}^2$, $(q_{ki}, t_{ki}) = (q^C, 0)$, (ii) for any (q_{ki}, t_{ki}) , $(\tilde{q}_{li}(q_{ki}, t_{ki}), \tilde{t}_{li}(q_{ki}, t_{ki})) = (q^C, 0)$ and (iii) for any (q_{ki}, q_{kj}) , $i \neq j$, (z_{k1}, z_{k2}, x_k) is defined as follows:*

(a) $(q_{ki}, q_{kj}) \in \{(q_i, q_j) \in [0, q^C]^2 : q_i + q_j \geq q^C\}$: $(z_{ki}, z_{kj}, x_k) = (1, 1, q^C)$ for $t_{ki} + t_{kj} \leq \pi^C$, $t_{kj} \leq \pi^C - P(q^C + q_{ki})q_{ki}$ and $t_{ki} \leq \pi^C - P(q^C + q_{kj})q_{kj}$. $(z_{ki}, z_{kj}, x_k) = (1, 0, q_{ki})$ for $t_{ki} \leq P(q^C + q_{ki})q_{ki}$, $t_{kj} > \pi^C - P(q^C + q_{ki})q_{ki}$ and $t_{ki} - t_{kj} \leq P(q^C + q_{ki})q_{ki} - P(q^C + q_{kj})q_{kj}$.³ $(z_{ki}, z_{kj}, x_k) = (0, 0, 0)$ for $t_{ki} > P(q^C + q_{ki})q_{ki}$, $t_{kj} > P(q^C + q_{kj})q_{kj}$ and $t_{ki} + t_{kj} > \pi^C$.

(b) $(q_{ki}, q_{kj}) \in \{(q_i, q_j) \in [0, q^C]^2 : q_i + q_j < q^C\}$: $(z_{ki}, z_{kj}, x_k) = (1, 1, q_{ki} + q_{kj})$ for $t_{ki} + t_{kj} \leq P(q_{ki} + q_{kj} + q^C)(q_{ki} + q_{kj})$, $t_{ki} \leq P(q_{ki} + q_{kj} + q^C)(q_{ki} + q_{kj}) - P(q_{kj} + q^C)q_{kj}$ and $t_{kj} \leq P(q_{ki} + q_{kj} + q^C)(q_{ki} + q_{kj}) - P(q_{ki} + q^C)q_{ki}$. $(z_{ki}, z_{kj}, x_k) = (1, 0, q_{ki})$ for $t_{ki} \leq P(q_{ki} + q^C)q_{ki}$, $t_{kj} > P(q_{ki} + q_{kj} + q^C)(q_{ki} + q_{kj}) - P(q_{ki} + q^C)q_{ki}$ and $t_{ki} - t_{kj} \leq P(q_{ki} + q^C)q_{ki} - P(q_{kj} + q^C)q_{kj}$. $(z_{ki}, z_{kj}, x_k) = (0, 0, 0)$ for $t_{ki} + t_{kj} > P(q_{ki} + q_{kj} + q^C)(q_{ki} + q_{kj})$, $t_{ki} > P(q_{ki} + q^C)q_{ki}$ and $t_{kj} > P(q_{kj} + q^C)q_{kj}$.

(c) $(q_{ki}, q_{kj}) \in \{(q_i, q_j) \in [q^C, +\infty) \times [0, q^C]\}$: $(z_{ki}, z_{kj}, x_k) = (1, 1, q^C)$ for $t_{ki} + t_{kj} \leq \pi^C$, $t_{kj} \leq 0$ and $t_{ki} \leq \pi^C - P(q^C + q_{kj})q_{kj}$. $(z_{ki}, z_{kj}, x_k) = (1, 0, q^C)$ for $t_{ki} \leq \pi^C$, $t_{kj} > 0$ and $t_{ki} - t_{kj} \leq \pi^C - P(q^C + q_{kj})q_{kj}$. $(z_{ki}, z_{kj}, x_k) = (0, 1, q_{kj})$ for $t_{kj} \leq P(q^C + q_{kj})q_{kj}$, $t_{ki} - t_{kj} \geq \pi^C - P(q^C + q_{kj})q_{kj}$ and $t_{ki} > \pi^C - P(q^C + q_{kj})q_{kj}$. $(z_{ki}, z_{kj}, x_k) = (0, 0, 0)$ for $t_{ki} > \pi^C$, $t_{kj} > P(q^C + q_{kj})q_{kj}$ and $t_{ki} + t_{kj} > \pi^C$.

(d) $(q_{ki}, q_{kj}) \in \{(q_i, q_j) \in [q^C, +\infty)^2\}$: $(z_{ki}, z_{kj}, x_k) = (1, 1, q^C)$ for $t_{ki} + t_{kj} \leq \pi^C$, $t_{kj} \leq 0$ and $t_{ki} \leq 0$. $(z_{ki}, z_{kj}, x_k) = (1, 0, q^C)$ for $t_{ki} \leq \pi^C$, $t_{kj} > 0$ and $t_{ki} \geq t_{kj}$. $(z_{ki}, z_{kj}, x_k) = (0, 0, 0)$ for $t_{ki} + t_{kj} > \pi^C$, $t_{ki} > \pi^C$ and $t_{kj} > \pi^C$.

Lemma (1) implies that in equilibrium each retailer receives at least q^C . In E^∞ , each retailer receives exactly q^C from each supplier. Both offers are accepted and the retailer puts q^C on the final market. Thus, the retailer covers its entire need for product with what it receives from only one supplier, while

³In case of equality in this last condition, R_k is indifferent between accepting only the contract offered by R_i and accepting only the contract offered by R_j . In this case, we assume the usual tie-breaking rule that with probability 1/2 it chooses one and with probability 1/2 it chooses the other. This plays no role in the proof of the following lemma.

the product received from the other supplier is kept out of the market. It can be either stored or destroyed or returned to the supplier, but in this last case the assumption is that it is returned after the final market is closed. The clothing industry provides examples of such a mechanism. In this article, I don't specify what happens with the product that retailers don't use. They are assumed to get rid of it without a cost.

Lemma 2 E^∞ is a perfect Bayesian equilibrium with passive beliefs of Γ^∞ .

Proof. Whatever offers it receives from the suppliers, R_k believes that R_l is offered $(q^C, 0)$ by both suppliers and, thus, that $(z_{l1}, z_{l2}, x_l) = (1, 1, q^C)$. Sequential rationality for R_k thus requires that (z_{k1}, z_{k2}, x_k) is a best reply to $(z_{l1}, z_{l2}, x_l) = (1, 1, q^C)$. This leads for R_k to the strategy described in E^∞ . As regards suppliers' rationality, given that S_i offers $(q^C, 0)$ to both retailers, there is no profitable deviation for S_j . ■

Lemma (2) is essentially an existence result. It proves that the set of equilibria characterized in lemma (1) is not empty. I briefly discuss why E^∞ is an equilibrium. Each supplier supplies free of charge both retailers with a quantity of product that covers their entire needs. The other supplier is consequently unable to extract any transfer from retailers. It is thus indifferent between supplying both retailers with q^C and supplying them with any other quantity, since production costs are null. This is why suppliers' strategies in E^∞ are rational. As regards retailers, their rationality along the equilibrium path is easy to deal with because they can only accept contracts that require them no paiement at all. Once they accept the contract, they are essentially competing *à la Cournot* with a capacity constraint that is not binding. Of course, retailers' rationality has to be examined not only along the equilibrium path, but also out of equilibrium. Each retailer essentially believes that its competitor will put q^C on the market and makes its decision regarding acceptance of contracts and output on the final market based on this belief.

E^∞ is not the unique equilibrium of Γ^∞ . A supplier may offer more than q^C to a retailer, keeping the transfer at zero. However, in any equilibrium, the outcome on the final market is the same, as well as upstream and downstream profits.

I now turn to the case of a monopolist supplier. \mathfrak{G}^∞ is similar to a game considered in Rey and Tirole (2007) with, again, some differences, in particular on the value of the upstream marginal cost. I proceed as for the duopoly case, starting with a characterization of the outcome on the final market and of profit-sharing between suppliers and retailers.

Lemma 3 In a perfect Bayesian equilibrium with passive beliefs of \mathfrak{G}^∞ , $x_k = q^C$ and $t_k = P(Q^C)q^C$ for $k = 1, 2$.

Proof. Assume that $(x_1, x_2) \neq (q^C, q^C)$. Necessarily, for some $k \in \{1, 2\}$, $x_k < q^C$. Furthermore, $x_l \leq BR(x_k)$, with $l \neq k$. R_k would increase its profits by increasing x_k . If (q_k, t_k) is rejected, then it is profitable for S to deviate by reducing t_k so that the contract becomes acceptable for R_k . If (q_k, t_k) is

accepted and $q_k < q^C$, then S can increase profits by offering $(q_k + \varepsilon, t_k + \eta)$, for some small, positive ε and η . Thus, in equilibrium, $(x_1, x_2) = (q^C, q^C)$.

A consequence of the above analysis is that in equilibrium S offers q^C to each retailers and the contracts are accepted. Given this, each retailer expects his competitor to put q^C on the market. It is thus willing to pay up to $P(Q^C)q^C$ for the quantity q^C . In equilibrium, S raises transfers up to this limit. ■

Lemma (3) formulates the standard result that under passive beliefs a monopolist retailer cannot achieve the monopolization of the final market when it sells its product to consumers through competing retailers. More precisely, that the upstream industry is a monopoly or a duopoly is irrelevant if we consider only the output and the price on the final market. The difference between the two situations appears when one considers profit-sharing between suppliers and retailers. A monopolist supplier is not able to increase industry profits, but it is able to capture them entirely. I now deal with the issue of the existence of an equilibrium.

Definition 10 *The assessment \mathfrak{E}^∞ is characterized by (i) $(q_1, t_1, q_2, t_2) = (q^C, P(Q^C)q^C, q^C, P(Q^C)q^C)$, (ii) for any (q_k, t_k) , $((\tilde{q}_l(q_k), \tilde{t}_l(q_k))) = (q^C, P(Q^C)q^C)$ and (iii) For $0 \leq q_k < q^C$, $(z_k, x_k) = (1, q_k)$ for $t_k \leq P(q_k + q^C)q_k$, $(z_k, x_k) = (0, 0)$ otherwise. For $q^C \leq q_k$, $(z_k, x_k) = (1, q^C)$ for $t_k \leq P(Q^C)q^C$, $(z_k, x_k) = (0, 0)$ otherwise.*

Lemma 4 \mathfrak{E}^∞ is a perfect Bayesian equilibrium with passive beliefs of \mathfrak{G}^∞ .

Proof. Given retailers' beliefs, it is rational for them to accept $(q^C, P(Q^C)q^C)$ and put q^C on the market. The supplier cannot extract a larger transfer than $P(Q^C)q^C$ because this is $\max_q \{P(q + q^C)q\}$ and each retailer believes that its competitor puts q^C on the market. ■

Lemmas (3) and (4) provide a full characterization of what happens on the final market when a monopolist supplier contracts with competing retailers under passive beliefs as it is the case in \mathfrak{G}^∞ . This concludes the analysis of the strategic interaction between suppliers and retailers both with an upstream monopoly and with an upstream duopoly. This makes it possible to conclude on the impact of an horizontal merger between suppliers.

Proposition 1 *When both suppliers have an infinite production capacity and retailers hold passive beliefs, a merger to monopoly between suppliers has no impact on consumers' welfare.*

Proof. The proposition is a straightforward consequence of lemmas (1) to (4). ■

The impact of a merger between the two suppliers is limited to retailers. While industry profits are captured by retailers before the merger, they are captured by the supplier after the merger. However, the monopolist supplier is not able to induce the monopoly outcome on the final market or at least to induce a price above the price observed on the final market before the merger.

4 Constrained suppliers' merger and consumer welfare

In this section, I characterize the equilibrium outcome on the final market for an upstream duopoly and an upstream monopoly. The comparison of both cases leads to a characterization of the impact of a merger on consumers' welfare.

Lemma 5 *In a perfect Bayesian equilibrium with full capacity beliefs of $\Gamma(\bar{Q}_1, \bar{Q}_2)$, with $\bar{Q}_1 > Q^C$ and $\bar{Q}_2 > Q^C$, $x_1 + x_2 > Q^m$.*

Proof. Full capacity beliefs imply that in equilibrium all contracts are accepted. Indeed, R_k believes that S_1 offers R_l a contract that is acceptable given S_2 's equilibrium offer and that S_2 offers R_l a contract that is acceptable given S_1 's equilibrium offer. R_k thus believes that when R_l receives equilibrium offers from both suppliers, it accepts both. This beliefs has to be correct in equilibrium. As a consequence, the total quantity transferred to retailers is $\bar{Q}_1 + \bar{Q}_2$. It is easy to check that for any $x \in [0, \bar{Q}_1 + \bar{Q}_2]$, $BR(x) \leq \bar{Q}_1 + \bar{Q}_2 - x$ and $x + BR(x) \geq Q^m$. In equilibrium, $x_2 = \min(BR(x_1), q_{21} + q_{22})$. $x_2 = BR(x_1) \Rightarrow x_1 + x_2 = x_1 + BR(x_1) \geq Q^m$. $x_2 = q_{21} + q_{22} \Rightarrow x_1 = \min(BR(q_{21} + q_{22}), \bar{Q}_1 + \bar{Q}_2 - q_{21} - q_{22}) = BR(q_{21} + q_{22})$ and $x_1 + x_2 = q_{21} + q_{22} + BR(q_{21} + q_{22}) \geq Q^m$. In a PBE with full capacity beliefs, $x_1 + x_2 \geq Q^m$. I now show that $x_1 + x_2 \neq Q^m$. The sequential rationality of retailers allows for $x_1 + x_2 = Q^m$ only if one of the retailers (say R_1) receives $q_{11} + q_{12} = \bar{Q}_1 + \bar{Q}_2$ and the other (R_2) receives $q_{21} + q_{22} = 0$. Suppliers' strategies should then be $(q_{11}, t_{11}, q_{21}, t_{21}) = (\bar{Q}_1, 0, 0, 0)$ and $(q_{12}, t_{12}, q_{22}, t_{22}) = (\bar{Q}_2, 0, 0, 0)$. Note that $t_{11} = t_{12} = 0$ because R_1 can reject the offer of one of the suppliers without any impact on its market output. Consider the following deviation for S_2 : $\sigma'_2 = (q'_{12}, t'_{12}, q'_{22}, t'_{22}) = (0, 0, \bar{Q}_2, \varepsilon)$, where $\varepsilon > 0$ is infinitely small. The deviation is profitable for S_2 if and only if it is accepted by R_2 . Full capacity beliefs for R_2 are $(\tilde{q}_{11}(q_{21}), \tilde{t}_{11}(q_{21})) = (\bar{Q}_1, 0)$ and $(\tilde{q}_{12}(q_{22}), \tilde{t}_{12}(q_{22})) = (0, 0)$. (z_{21}, z_{22}, x_2) must be a best reply to $x_1(\bar{Q}_1, 0, 0, 0)$. If R_2 accepts (q'_{22}, t'_{22}) , rational sequentiality implies $x_2 = x'_2$, with: $x'_2 = \arg \max_{x_2 \leq \bar{Q}_2} P(x_2 + x_1(\bar{Q}_1, 0, 0, 0))x_2$. Obviously, $z_{11}(\bar{Q}_1, 0, 0, 0) = z_{12}(\bar{Q}_1, 0, 0, 0) = 1$. R_1 believes that S_2 offers R_2 the quantity \bar{Q}_2 and a transfer η that is acceptable given $(q_{21}, t_{21}) = (0, 0)$. R_1 chooses $x_1(\bar{Q}_1, 0, 0, 0)$ so as to maximize $P(x_2(0, 0, \bar{Q}_2, \eta) + x_1)x_1$ subject to $x_1 \leq \bar{Q}_1$. Note that $x_2(0, 0, \bar{Q}_2, \eta)$ is precisely x'_2 , so that $x_1(\bar{Q}_1, 0, 0, 0) = \arg \max_{x_1 \leq \bar{Q}_1} P(x'_2 + x_1)x_1$ and $x'_2 = x_1(\bar{Q}_1, 0, 0, 0) = q^C$. Accepting (q'_{22}, t'_{22}) , R_2 thus makes a profit equal to $P(Q^C)q^C - \varepsilon > 0$ for ε infinitely small. σ'_2 is thus a profitable deviation for S_2 and there is no equilibrium such that $x_1 + x_2 = Q^m$. ■

In equilibrium, suppliers produce at full capacity and offer their total output to retailers through contracts that are accepted. As a consequence, the retailing sector is a Cournot duopoly in which one firm has a production capacity of q and the other has a production capacity of $\bar{Q}_1 + \bar{Q}_2 - q$. With \bar{Q}_1 and \bar{Q}_2 strictly larger than the output of an unconstrained Cournot duopoly, this cannot lead to

an output strictly lower than the monopoly output. It can lead to the monopoly output if both suppliers offer their total output to the same retailer. However, this is not an equilibrium because suppliers would deviate from this strategy profile. Indeed, the supplied retailer pays no transfer to the suppliers, while the other retailer is willing to pay a positive transfer to get access to the product. If a PBE with full capacity beliefs of $\Gamma(\bar{Q}_1, \bar{Q}_2)$ exists, it is characterized by a price paid by consumers that is strictly below the monopoly price. I now address the question of the existence of an equilibrium. The first step is to define an assessment that will afterwards be proved to be an equilibrium.

Definition 11 *The assessment $E(\bar{Q}_1, \bar{Q}_2)$ is characterized by (i) $(q_{11}, t_{11}, q_{21}, t_{21}) = (q^C, 0, \bar{Q}_1 - q^C, 0)$ and $(q_{12}, t_{12}, q_{22}, t_{22}) = (\bar{Q}_2 - q^C, 0, q^C, 0)$, (ii) for any (q_{ki}, q_{kj}) , $i \neq j$, $(\tilde{q}_{li}, \tilde{t}_{li}, \tilde{q}_{lj}, \tilde{t}_{lj}) = (\bar{Q}_i - q_{ki}, 0, \bar{Q}_j - q_{kj}, 0)$ and (iii) for any (q_{ki}, q_{kj}) , $i \neq j$, (z_{k1}, z_{k2}, x_k) is defined as follows:*

(a) $(q_{ki}, q_{kj}) \in \{(q_i, q_j) \in [0, q^C]^2 : q_i + q_j > q^C\}$: $(z_{ki}, z_{kj}, x_k) = (1, 1, q^C)$ for $t_{ki} + t_{kj} \leq \pi^C$, $t_{kj} \leq \pi^C - P(q^C + q_{ki})q_{ki}$ and $t_{ki} \leq \pi^C - P(q^C + q_{kj})q_{kj}$. $(z_{ki}, z_{kj}, x_k) = (1, 0, q_{ki})$ for $t_{ki} \leq P(q^C + q_{ki})q_{ki}$, $t_{kj} > \pi^C - P(q^C + q_{ki})q_{ki}$ and $t_{ki} - t_{kj} \leq P(q^C + q_{ki})q_{ki} - P(q^C + q_{kj})q_{kj}$. $(z_{ki}, z_{kj}, x_k) = (0, 0, 0)$ for $t_{ki} > P(q^C + q_{ki})q_{ki}$, $t_{kj} > P(q^C + q_{kj})q_{kj}$ and $t_{ki} + t_{kj} > \pi^C$.

(b) $(q_{ki}, q_{kj}) \in \{(q_i, q_j) \in [0, q^C]^2 : q_i + q_j \leq q^C\}$: $(z_{ki}, z_{kj}, x_k) = (1, 1, q_{ki} + q_{kj})$ for $t_{ki} + t_{kj} \leq P(q_{ki} + q_{kj} + BR(q_{ki} + q_{kj}))(q_{ki} + q_{kj})$, $t_{ki} \leq P(q_{ki} + q_{kj} + BR(q_{ki} + q_{kj}))(q_{ki} + q_{kj}) - P(q_{kj} + BR(q_{ki} + q_{kj}))q_{kj}$ and $t_{kj} \leq P(q_{ki} + q_{kj} + BR(q_{ki} + q_{kj}))(q_{ki} + q_{kj}) - P(q_{ki} + BR(q_{ki} + q_{kj}))q_{ki}$. $(z_{ki}, z_{kj}, x_k) = (1, 0, q_{ki})$ for $t_{ki} \leq P(q_{ki} + BR(q_{ki} + q_{kj}))q_{ki}$, $t_{kj} > P(q_{ki} + q_{kj} + BR(q_{ki} + q_{kj}))(q_{ki} + q_{kj}) - P(q_{ki} + BR(q_{ki} + q_{kj}))q_{ki}$ and $t_{ki} - t_{kj} \leq P(q_{ki} + BR(q_{ki} + q_{kj}))q_{ki} - P(q_{kj} + BR(q_{ki} + q_{kj}))q_{kj}$. $(z_{ki}, z_{kj}, x_k) = (0, 0, 0)$ for $t_{ki} + t_{kj} > P(q_{ki} + q_{kj} + BR(q_{ki} + q_{kj}))(q_{ki} + q_{kj})$, $t_{ki} > P(q_{ki} + BR(q_{ki} + q_{kj}))q_{ki}$ and $t_{kj} > P(q_{kj} + BR(q_{ki} + q_{kj}))q_{kj}$.

(c) $(q_{ki}, q_{kj}) \in \{(q_i, q_j) \in (q^C, \bar{Q}_i] \times [0, q^C] : q_i + q_j \leq \bar{Q}_1 + \bar{Q}_2 - q^C\}$: $(z_{ki}, z_{kj}, x_k) = (1, 1, q^C)$ for $t_{ki} + t_{kj} \leq \pi^C$, $t_{kj} \leq 0$ and $t_{ki} \leq \pi^C - P(q^C + q_{kj})q_{kj}$. $(z_{ki}, z_{kj}, x_k) = (1, 0, q^C)$ for $t_{ki} \leq \pi^C$, $t_{kj} > 0$ and $t_{ki} - t_{kj} \leq \pi^C - P(q^C + q_{kj})q_{kj}$. $(z_{ki}, z_{kj}, x_k) = (0, 1, q_{kj})$ for $t_{kj} \leq P(q^C + q_{kj})q_{kj}$, $t_{ki} - t_{kj} \geq \pi^C - P(q^C + q_{kj})q_{kj}$ and $t_{ki} > \pi^C - P(q^C + q_{kj})q_{kj}$. $(z_{ki}, z_{kj}, x_k) = (0, 0, 0)$ for $t_{ki} > \pi^C$, $t_{kj} > P(q^C + q_{kj})q_{kj}$ and $t_{ki} + t_{kj} > \pi^C$.

(d) $(q_{ki}, q_{kj}) \in \{(q_i, q_j) \in (q^C, \bar{Q}_i]^2 : q_i + q_j \leq \bar{Q}_1 + \bar{Q}_2 - q^C\}$: $(z_{ki}, z_{kj}, x_k) = (1, 1, q^C)$ for $t_{ki} + t_{kj} \leq \pi^C$, $t_{kj} \leq 0$ and $t_{ki} \leq 0$. $(z_{ki}, z_{kj}, x_k) = (1, 0, q^C)$ for $t_{ki} \leq \pi^C$, $t_{kj} > 0$ and $t_{kj} \geq t_{ki}$. $(z_{ki}, z_{kj}, x_k) = (0, 0, 0)$ for $t_{ki} + t_{kj} > \pi^C$, $t_{ki} > \pi^C$ and $t_{kj} > \pi^C$.

(e) $(q_{ki}, q_{kj}) \in \{(q_i, q_j) \in (q^C, \bar{Q}_i]^2 : q_i + q_j > \bar{Q}_1 + \bar{Q}_2 - q^C\}$: $(z_{ki}, z_{kj}, x_k) = (1, 1, BR(\bar{Q}_1 + \bar{Q}_2 - q_{ki} - q_{kj}))$ for $t_{ki} + t_{kj} \leq P(\bar{Q}_1 + \bar{Q}_2 - q_{ki} - q_{kj} + BR(\bar{Q}_1 + \bar{Q}_2 - q_{ki} - q_{kj}))BR(\bar{Q}_1 + \bar{Q}_2 - q_{ki} - q_{kj})$, $t_{kj} \leq 0$ and $t_{ki} \leq 0$. $(z_{ki}, z_{kj}, x_k) = (1, 0, BR(\bar{Q}_1 + \bar{Q}_2 - q_{ki} - q_{kj}))$ for $t_{ki} + t_{kj} \leq P(\bar{Q}_1 + \bar{Q}_2 - q_{ki} - q_{kj} + BR(\bar{Q}_1 + \bar{Q}_2 - q_{ki} - q_{kj}))BR(\bar{Q}_1 + \bar{Q}_2 - q_{ki} - q_{kj})$, $t_{kj} > 0$ and $t_{kj} \geq t_{ki}$. $(z_{ki}, z_{kj}, x_k) = (0, 0, 0)$ for $t_{ki} + t_{kj} > P(\bar{Q}_1 + \bar{Q}_2 - q_{ki} - q_{kj} + BR(\bar{Q}_1 + \bar{Q}_2 - q_{ki} - q_{kj}))BR(\bar{Q}_1 +$

$$\overline{Q}_2 - q_{ki} - q_{kj}), t_{ki} > P(\overline{Q}_1 + \overline{Q}_2 - q_{ki} - q_{kj} + BR(\overline{Q}_1 + \overline{Q}_2 - q_{ki} - q_{kj}))BR(\overline{Q}_1 + \overline{Q}_2 - q_{ki} - q_{kj}) \text{ and } t_{kj} > P(\overline{Q}_1 + \overline{Q}_2 - q_{ki} - q_{kj} + BR(\overline{Q}_1 + \overline{Q}_2 - q_{ki} - q_{kj}))BR(\overline{Q}_1 + \overline{Q}_2 - q_{ki} - q_{kj}).$$

Although the full description of retailers' strategies necessitates a rather long definition of any assessment, what happens along the equilibrium path in $E(\overline{Q}_1, \overline{Q}_2)$ is simple. Each retailer is offered by each supplier a quantity at least equal to its output in the unconstrained Cournot equilibrium. Retailers accept these offers. The equilibrium on the final market is similar to that resulting from unconstrained Cournot competition. I now show that this assessment is a perfect Bayesian equilibrium with full capacity beliefs of $\Gamma(\overline{Q}_1, \overline{Q}_2)$.

Lemma 6 $E(\overline{Q}_1, \overline{Q}_2)$ is a perfect Bayesian equilibrium with full capacity beliefs of $\Gamma(\overline{Q}_1, \overline{Q}_2)$.

Proof. *Full capacity beliefs* For any (q_{k1}, q_{k2}) , $(\tilde{q}_{l1}, \tilde{t}_{l1}, \tilde{q}_{l2}, \tilde{t}_{l2}) = (\overline{Q}_1 - q_{k1}, 0, \overline{Q}_2 - q_{k2}, 0)$. This corresponds to full capacity beliefs because S_1 's equilibrium offer to R_l is either $(q^C, 0)$ or $(\overline{Q}_1 - q^C, 0)$, with $\overline{Q}_1 - q^C > q^C$. In both cases, an offer from S_2 to R_l is acceptable if and only if $t_{l2} = 0$. Similarly, an offer from S_1 to R_l is acceptable if and only if $t_{l1} = 0$.

Consistency of R_k 's beliefs Full capacity beliefs are consistent with the suppliers' equilibrium strategies.

Sequential rationality for suppliers Consider S_1 .⁴ Given that $(q_{12}, t_{12}, q_{22}, t_{22}) = (\overline{Q}_2 - q^C, 0, q^C, 0)$, $x_1 = x_2 = q^C$ for any $(q_{11}, t_{11}, q_{21}, t_{21})$ and neither R_1 nor R_2 is willing to pay a strictly positive transfer to S_1 . There is thus no profitable deviation for S_1 .

Sequential rationality for R_k (a) $(q_{ki}, q_{kj}) \in \{(q_i, q_j) \in [0, q^C]^2 : q_i + q_j > q^C\} \Rightarrow x_l = q^C \Rightarrow \pi_k(z_{ki}, z_{kj}, x_k) = P(q^C + x_k)x_k - z_{ki} * t_{ki} - z_{kj} * t_{kj}$. $(z_{ki}, z_{kj}) = (1, 1) \Rightarrow x_k = q^C$ and $\pi_k = \pi^C - t_{ki} - t_{kj}$. $(z_{ki}, z_{kj}) = (1, 0) \Rightarrow x_k = q_{ki}$ and $\pi_k = P(q^C + q_{ki})q_{ki} - t_{ki}$. Comparing the values of π_k proves that R_k 's strategy in $E(\overline{Q}_1, \overline{Q}_2)$ is sequentially rational for these values of (q_{ki}, q_{kj}) .

(b) $(q_{ki}, q_{kj}) \in \{(q_i, q_j) \in [0, q^C]^2 : q_i + q_j \leq q^C\} \Rightarrow x_l = BR(q_{ki} + q_{kj}) \Rightarrow \pi_k(z_{ki}, z_{kj}, x_k) = P(BR(q_{ki} + q_{kj}) + x_k)x_k - z_{ki} * t_{ki} - z_{kj} * t_{kj}$.

(c) $(q_{ki}, q_{kj}) \in \{(q_i, q_j) \in (q^C, \overline{Q}_i] \times [0, q^C] : q_i + q_j \leq \overline{Q}_1 + \overline{Q}_2 - q^C\} \Rightarrow x_l = q^C \Rightarrow \pi_k(z_{ki}, z_{kj}, x_k) = P(q^C + x_k)x_k - z_{ki} * t_{ki} - z_{kj} * t_{kj}$.

(d) $(q_{ki}, q_{kj}) \in \{(q_i, q_j) \in (q^C, \overline{Q}_i]^2 : q_i + q_j \leq \overline{Q}_1 + \overline{Q}_2 - q^C\} \Rightarrow x_l = q^C \Rightarrow \pi_k(z_{ki}, z_{kj}, x_k) = P(q^C + x_k)x_k - z_{ki} * t_{ki} - z_{kj} * t_{kj}$.

(e) $(q_{ki}, q_{kj}) \in \{(q_i, q_j) \in (q^C, \overline{Q}_i]^2 : q_i + q_j > \overline{Q}_1 + \overline{Q}_2 - q^C\} \Rightarrow x_l = \overline{Q}_1 + \overline{Q}_2 - q_{ki} - q_{kj} \Rightarrow \pi_k(z_{ki}, z_{kj}, x_k) = P(\overline{Q}_1 + \overline{Q}_2 - q_{ki} - q_{kj} + x_k)x_k - z_{ki} * t_{ki} - z_{kj} * t_{kj}$.

■

Given that suppliers charge transfers equal to zero, it is rational for retailers to accept the contracts. Then, retailers are competing on the final market with each of them able to produce the unconstrained Cournot equilibrium output.

⁴The proof is similar for S_2 .

They actually choose this level of output. No supplier can profitably deviate because its rival is offering (for free) enough product to each retailer for retailers not to need its product. This is for the equilibrium path. However, proving that $E(\bar{Q}_1, \bar{Q}_2)$ is a perfect Bayesian equilibrium requires considering sequential rationality of retailers out of the equilibrium path. If for example R_1 receives quantities of product corresponding to most of the production capacity of both suppliers, it believes that R_2 was offered (and accepted) a small quantity of product corresponding to the remaining production capacities. Upon reception of this small quantity, R_2 believes that R_1 receives the large quantity that it indeed received. Crossing best replies for retailers leads to R_2 putting on the market all the product received from suppliers and R_1 playing its best reply to this quantity, keeping out of the market part of the product. Proving that $E(\bar{Q}_1, \bar{Q}_2)$ is an equilibrium also requires checking that retailers' beliefs are consistent and correspond to our definition of full capacity beliefs. Here, full capacity beliefs are very simple because the only transfer that is acceptable for a retailer given a suppliers' equilibrium offer, regardless of the quantity of product offered, is zero.

$E(\bar{Q}_1, \bar{Q}_2)$ is not the unique PBE with full capacity beliefs of $\Gamma(\bar{Q}_1, \bar{Q}_2)$. An assessment identical to $E(\bar{Q}_1, \bar{Q}_2)$ except for the fact that $(q_{11}, t_{11}, q_{21}, t_{21}) = (q^C + \Delta_1, 0, \bar{Q}_1 - q^C - \Delta_1, 0)$ and $(q_{12}, t_{12}, q_{22}, t_{22}) = (\bar{Q}_2 - q^C - \Delta_2, 0, q^C + \Delta_2, 0)$, with $\Delta_1 \in (0, \bar{Q}_1 - Q^C]$ and $\Delta_2 \in (0, \bar{Q}_2 - Q^C]$ is also an equilibrium of $\Gamma(\bar{Q}_1, \bar{Q}_2)$, leading to the same outcome on the final market. A more difficult question is whether there are other equilibria leading to different market outcomes. It is not considered here. The two previous lemma allow me to claim that equilibria exist and that in any equilibrium, the price on the final market is strictly lower than the monopoly price.

Let us now consider the game resulting from a merger between suppliers. $\mathfrak{G}(\bar{Q}_1 + \bar{Q}_2)$ is similar to a game considered in Avenel (2010).

Lemma 7 *In a perfect Bayesian equilibrium with full capacity beliefs of $\mathfrak{G}(\bar{Q}_1 + \bar{Q}_2)$, $x_1 + x_2 = Q^m$.*

Proof. Full capacity beliefs imply that in equilibrium all contracts are accepted. As a consequence, the total quantity transferred to retailers is $\bar{Q}_1 + \bar{Q}_2$. Since for any $q \in [0, \bar{Q}_1 + \bar{Q}_2]$, $BR(q) \leq \bar{Q}_1 + \bar{Q}_2 - q$ and $q + BR(q) \geq Q^m$, in a PBE with full capacity beliefs, $x_1 + x_2 \geq Q^m$. Now consider an assessment in which $x_1 + x_2 > Q^m$. R_k correctly anticipates a gross profit equal to $P(x_1 + x_2)x_k$ and pays t_k to S . The supplier's profit is thus $P(x_1 + x_2)(x_1 + x_2)$. S can profitably deviate from its strategy in this assessment by offering $(q_1, t_1, q_2, t_2) = (\bar{Q}_1 + \bar{Q}_2, \pi^m, 0, 0)$. Indeed, R_1 believes that it is in a monopoly position and is thus willing to pay a transfer up to the monopoly profit. This deviation is profitable for S because $\pi^m > P(x_1 + x_2)(x_1 + x_2)$. ■

As in the upstream duopoly case, and for the same reason, the output on the final market cannot be strictly less than the monopoly output. However, with an upstream monopolist, it also cannot be strictly more than the monopoly output. The upstream monopolist can avoid this by offering all the product to

one retailer. While this is a profitable deviation from equilibria including too large an output on the final market, it is still unproved that this is an equilibria strategy. I show it below.

Definition 12 *The assessment $\mathfrak{E}(\bar{Q}_1 + \bar{Q}_2)$ is characterized by (i) $(q_1, t_1, q_2, t_2) = (\bar{Q}_1 + \bar{Q}_2, \Pi^m, 0, 0)$, (ii) For $0 \leq q_k \leq q^C$, $(z_k, x_k) = (1, q_k)$ for $t_k \leq P(q_k + BR(q_k))q_k$, $(z_k, x_k) = (0, 0)$ otherwise and $((\tilde{q}_l(q_k), \tilde{t}_l(q_k))) = (\bar{Q}_1 + \bar{Q}_2 - q_k, P(q_k + BR(q_k))BR(q_k))$. For $q^C \leq q_k \leq \bar{Q}_1 + \bar{Q}_2 - q^C$, $(z_k, x_k) = (1, q^C)$ for $t_k \leq P(Q^C)q^C$, $(z_k, x_k) = (0, 0)$ otherwise and $((\tilde{q}_l(q_k), \tilde{t}_l(q_k))) = (\bar{Q}_1 + \bar{Q}_2 - q_k, P(Q^C)q^C)$. For $\bar{Q}_1 + \bar{Q}_2 - q^C \leq q_k \leq \bar{Q}_1 + \bar{Q}_2$, $(z_k, x_k) = (1, BR(\bar{Q}_1 + \bar{Q}_2 - q_k))$ for $t_k \leq P(BR(\bar{Q}_1 + \bar{Q}_2 - q_k) + \bar{Q}_1 + \bar{Q}_2 - q_k)BR(\bar{Q}_1 + \bar{Q}_2 - q_k)$, $(z_k, x_k) = (0, 0)$ otherwise and $((\tilde{q}_l(q_k), \tilde{t}_l(q_k))) = (\bar{Q}_1 + \bar{Q}_2 - q_k, P(\bar{Q}_1 + \bar{Q}_2 - q_k + BR(\bar{Q}_1 + \bar{Q}_2 - q_k))(\bar{Q}_1 + \bar{Q}_2 - q_k))$.*

In $\mathfrak{E}(\bar{Q}_1 + \bar{Q}_2)$, S offers all the product to R_1 . R_1 pays the monopoly profit to S , puts the monopoly output on the final market and breaks even. R_1 keeps part of the product received from S out of the market.

Lemma 8 *$\mathfrak{E}(\bar{Q}_1 + \bar{Q}_2)$ is a perfect Bayesian equilibrium with full capacity beliefs of $\mathfrak{G}(\bar{Q}_1 + \bar{Q}_2)$.*

Proof. *Full capacity beliefs* The issue here is the acceptability of $(\tilde{q}_l(q_k), \tilde{t}_l(q_k))$. For $q_k \in [0, q^C]$, $\tilde{q}_l(q_k) = \bar{Q}_1 + \bar{Q}_2 - q_k \geq \bar{Q}_1 + \bar{Q}_2 - q^C \Rightarrow \tilde{q}_k(\tilde{q}_l(q_k)) = q_k$ and $\tilde{t}_k(\tilde{t}_l(q_k)) = P(q_k + BR(q_k))q_k$. $x_k(\tilde{q}_k(\tilde{q}_l(q_k)), \tilde{t}_k(\tilde{t}_l(q_k))) = q_k$. $\tilde{t}_l(q_k) = P(q_k + BR(q_k))q_k \leq \max_{x \in [0, \bar{Q}_1 + \bar{Q}_2 - q_k]} P(x + q_k)x = P(q_k + BR(q_k))q_k$. $(\tilde{q}_l(q_k), \tilde{t}_l(q_k))$ is

thus acceptable. The proof is similar for $q_k \in (q^C, \bar{Q}_1 + \bar{Q}_2 - q^C] \cup (\bar{Q}_1 + \bar{Q}_2 - q^C, \bar{Q}_1 + \bar{Q}_2]$.

Consistency of beliefs Full capacity beliefs are consistent with the supplier's equilibrium strategy. In particular, $t_1 = \pi^m$ is acceptable for R_1 .

Sequential rationality for R_k For $0 \leq q_k \leq q^C$, $\tilde{q}_l \geq \bar{Q}_1 + \bar{Q}_2 - q^C$. $(z_l, x_l) = (1, BR(q_k))$ and $(z_k, x_k) = (1, \min(BR(BR(q_k)), q_k)) = (1, q_k) \iff t_k \leq P(q_k + BR(q_k))q_k$. For $q^C \leq q_k \leq \bar{Q}_1 + \bar{Q}_2 - q^C$, $q^C \leq \tilde{q}_l \leq \bar{Q}_1 + \bar{Q}_2 - q^C$. $(z_l, x_l) = (1, q^C)$ and $(z_k, x_k) = (1, q^C) \iff t_k \leq P(Q^C)q^C$. For $\bar{Q}_1 + \bar{Q}_2 - q^C \leq q_k \leq \bar{Q}_1 + \bar{Q}_2$, $0 \leq \tilde{q}_l \leq q^C$. $(z_l, x_l) = (\bar{Q}_1 + \bar{Q}_2 - q_k)$ and $(z_k, x_k) = (1, \min(BR(\bar{Q}_1 + \bar{Q}_2 - q_k), q_k)) = (1, BR(\bar{Q}_1 + \bar{Q}_2 - q_k)) \iff t_k \leq P(\bar{Q}_1 + \bar{Q}_2 - q_k + BR(\bar{Q}_1 + \bar{Q}_2 - q_k))BR(\bar{Q}_1 + \bar{Q}_2 - q_k)$.

Sequential rationality for S I consider all the possible deviations for S and check that none of those is profitable. Without loss of generality, I consider only acceptable offers. If S offers $q_1 \in [0, q^C]$, it can offer any q_2 up to $\bar{Q}_1 + \bar{Q}_2 - q_1 > \bar{Q} - q^C$. For $q_2 \in [0, q^C]$, S gets transfers equal to $t_1 + t_2 = P(q_1 + BR(q_1))q_1 + P(q_2 + BR(q_2))q_2$. This is obviously maximal for $q_1 = q_2 = q^C$, but $P(Q^C)q^C < \Pi^m$. For $q_2 \in [q^C, \bar{Q}_1 + \bar{Q}_2 - q^C]$, $t_1 + t_2 = P(q_1 + BR(q_1))q_1 + P(Q^C)q^C = \frac{1}{2}(1 - q_1)q_1 + (1 - Q^C)q^C$. $\arg \max(t_1 + t_2) = q^C$, but $t_1(q^C) + t_2(q^C) < \Pi^m$. For $q_2 \in [\bar{Q}_1 + \bar{Q}_2 - q^C, \bar{Q}_1 + \bar{Q}_2 - q_1]$, $t_1 + t_2 = P(q_1 + BR(q_1))q_1 + P(BR(\bar{Q}_1 + \bar{Q}_2 - q_2) + \bar{Q}_1 + \bar{Q}_2 - q_2)BR(\bar{Q}_1 + \bar{Q}_2 - q_2) =$

$\frac{1}{2}(1 - q_1)q_1 + \frac{1}{4}(1 - \bar{Q}_1 - \bar{Q}_2 + q_2)^2$. S would set $q_2 = \bar{Q}_1 + \bar{Q}_2 - q_1$ and $t_1 + t_2 = P(q_1 + BR(q_1))q_1 + P(BR(q_1) + q_1)BR(q_1) = \frac{1}{4}(1 - q_1^2) \Rightarrow (q_1, q_2) = (0, \bar{Q}_1 + \bar{Q}_2)$ and $\Pi^S = \Pi^m$. If S offers $q_1 \in [q^C, \bar{Q}_1 + \bar{Q}_2 - q^C]$, it can offer any q_2 up to $\bar{Q}_1 + \bar{Q}_2 - q_1 \in [q^C, \bar{Q} - q^C]$. The case $q_2 \in [0, q^C]$ was examined above (by symmetry). For $q_2 \in [q^C, \bar{Q}_1 + \bar{Q}_2 - q_1]$, $t_1 + t_2 = P(Q^C)Q^C < \Pi^m$. If S offers $q_1 \in [\bar{Q}_1 + \bar{Q}_2 - q^C, \bar{Q}_1 + \bar{Q}_2]$, it can offer any q_2 up to $\bar{Q}_1 + \bar{Q}_2 - q_1 \leq q^C$. $t_1 + t_2 = \frac{1}{4}(1 - \bar{Q}_1 + \bar{Q}_2 + q_1)^2 + \frac{1}{2}(1 - q_2)q_2$. S would choose $q_2 = \bar{Q}_1 + \bar{Q}_2 - q_1$ and $t_1 + t_2 = \frac{1}{4}\left(1 - (\bar{Q}_1 + \bar{Q}_2 - q_1)^2\right)$. This is maximal for $q_1 = \bar{Q}_1 + \bar{Q}_2$, which leads to $t_1 + t_2 = \Pi^m$. ■

Transferring all the product to one retailer is optimal for the supplier because this retailer is willing to pay the monopoly profit to the supplier. There is no way the supplier can extract from retailers more than the monopoly profit. In particular, offering a positive quantity to both retailers would lead to an increase of the output on the final market and a decrease in profits. Because the supplier has a finite production capacity it is able to commit to the supply of only one retailer. In Avenel (2010), I discuss this situation under more general assumptions on marginal cost and production capacity. In the case we consider here, the two previous lemma show that equilibria exist and that in any equilibrium, consumers pay the monopoly price on the final market. The comparison with the upstream duopoly case leads to the following proposition:

Proposition 2 *When both suppliers have a large finite production capacity and retailers hold full capacity beliefs, a merger to monopoly between suppliers reduces consumers' welfare.*

Proof. The proposition is a straightforward consequence of lemmas (5) to (8). ■

Before the merger, the two suppliers face a coordination failure. They could induce the monopoly outcome on the final market, but they are not able to do so in an equilibrium of the non-cooperative game Γ . While I prove that there exists a highly competitive equilibrium in which retailers put on the final market the unconstrained Cournot output, I do not present a full characterization of the set of perfect Bayesian equilibria with full capacity beliefs, even if it is reasonable to suspect that they are quite competitive. It is thus difficult to measure the impact of the merger on the price and thus on consumers' welfare. What I show is that there is an impact and that it is strictly negative. This is a key aspect in the evaluation of a merger by antitrust authorities and on this aspect the difference with the absence of impact of a merger on consumers' welfare when production capacities are infinite and passive beliefs are assumed is striking. As soon as production capacities are finite, there is something to see for antitrust authorities in a merger between suppliers.

5 Conclusion

Central to the analysis of the impact of a merger between suppliers is the ability of a monopolist supplier to eliminate competition between retailers. This is achieved by committing to supply only one retailer. The ability to commit to such a behavior finds its roots in the fact that the supplier's production capacity is finite. It is thus able to offer all of its production to one retailer and, receiving this offer, the retailer knows that the other retailer will not be supplied at all. While this can be done regardless of the marginal cost of production, the assumption that production is costless ensures that this is indeed an equilibrium strategy for the supplier. Duopolists, even if facing a capacity constraint, cannot induce the same equilibrium on the final market because they cannot coordinate on supplying only one retailer. Quite reasonably, I find that a merger solves coordination problems between suppliers and reduces competition.

Relaxing the assumption of costless production is certainly a promising avenue for future research. The ability for a monopolist supplier to implement the monopoly outcome on the final market remains with a positive, small marginal cost of production. The difficulties are more in the duopolistic suppliers' game. Our resolution of this game relies on the fact that transfers are null in equilibrium, but positive marginal costs imply positive transfers in equilibrium. Solving the game with positive equilibrium transfers is more involved and left for future work.

6 References

Avenel, E., 2010, Upstream capacity constraint and the preservation of monopoly power in bilateral vertical contracting, manuscript submitted for publication in the Journal of Industrial economics.

Hart, O. and J. Tirole, 1990, Vertical integration and market foreclosure, Brookings papers on Economic Activity (Microeconomics), 1990, 205-285.

Mas-Colell, A., M. Whinston and J. Green, 1995, Microeconomic theory, Oxford University Press.

McAfee, R.P. and M. Schwartz, 1994, Opportunism in multilateral vertical contracting: Nondiscrimination, exclusivity, and uniformity, American Economic Review, 84(1), 210-230.

Rey, P. and J. Tirole, 2007, Chapter 33: A primer on foreclosure, in Handbook of Industrial Organization, Vol. 3, 2145-2220.

Rey, P. and T. Vergé, 2004, Bilateral control with vertical contracts, RAND Journal of Economics, 35(4), 728-746.