Vertical Foreclosure and Risk Aversion

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VERY PRELIMINARY

The Issue

- Our overall project is to understand what are the incentives of a Principal to contract with one (or few) as opposed to many Agents, and what are the efficiency implications of this choice
- Examples: employer/employees, patent holder/licensees, manufacturer/retailers, prize giver/contestants
- This question has obvious relevance from an IO/competition policy perspective: the study of input foreclosure was our motivation

Foreclosure and exclusivity restrictions

- Input foreclosure and clauses which restrict dealership: long-standing debate on whether they could be anticompetitive
- One of the few rationales for (inefficient) vertical foreclosure is Hart-Tirole (1990):
  - When contracts between the upstream firm and each downstream firm are not observable to other downstream firms, the upstream firm cannot commit not to flood the market
  - Foreclosure becomes a commitment device to avoid this problem and restore monopoly profit
- Our project can be seen as starting from HT: is it possible to have foreclosure when the contractual arrangements for all participants are public information? And if so, when could it be inefficient?

Our story, in a nutshell

- If a principal has many agents, each agent's payoff may be affected by the uncertainty created by the action/existence of the other agents
- Each agent may want to be compensated against such uncertainty: the Principal should leave rents to them
- There is a trade-off for the Principal: increasing the number of agents is costly since it raises their uncertainty; but other things being equal, having more agents is better than having fewer (think of product variety, exploring different technological trajectories etc.)
- We propose a particular way of modelling the interaction among agents and the resulting uncertainty. (At the end, we suggest other ways to model the same idea.)

Our formalisation (hidden-knowledge model)

- We introduce a model with downstream cost heterogeneity and private information
- The upstream firm posts a *public* menu of contracts designed for each cost type
- With more downstream firms, the volatility of aggregate revenue goes down but the volatility of individual revenue goes up
- When downstream firms are risk averse, a trade-off emerges for the upstream firm between increasing expected revenue and decreasing rent extraction
- When downstream firms are infinitely risk averse, the upstream firm deals with only one firm

Interpretation of Contracting Assumptions with Two Firms

- We know from Laffont and Tirole (1987) and McAfee and McMillan (1987) that the optimal contract in this environment is a mechanism whose equilibrium features only one firm producing
- Our model captures a situation in which U cannot identify potential participants and organize an auction, but instead publicly posts the terms of trade on the network/platform (similar as Hart-Tirole)
- In terms of observability, our model is equivalent to one in which the upstream firm sets a menu of wholesale rates and fixed fees with subsequent Cournot competition
- Alternatively, one could simply view the firms as dispersed participants independently choosing tariffs and prices only being realized ex post

Structure of Presentation

- Simple two firm model with infinite risk aversion to illustrate ideas
- Preliminary extension to N firms and CARA utility
- Discussion

Model

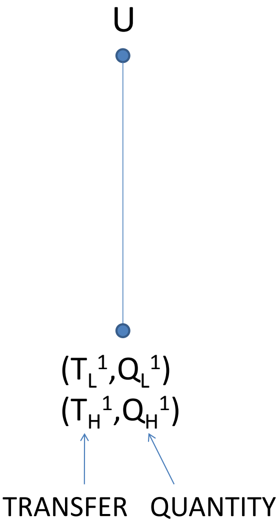
Preferences and Technology

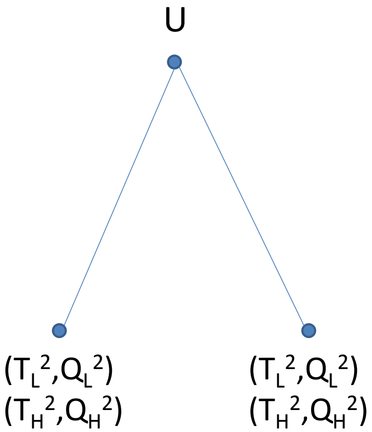
- There is an upstream supplier U that controls access to a market with inverse demand function $P(Q) = 1 - Q$
- Two potential downstream firms 1 and 2
- Firm i produces q_i units of the good at cost $c_i q_i$ where $0 = c_L < c_H < 1$, $c_i \in \{c_L, c_H\}$ with $\Pr[c_i = c_L] = r$, and c_i is iid across firms
- Both downstream firms earn 0 profit if they do not produce
- U is risk neutral

Information

- c_i is private information for firm i

Contracts





CRUCIAL: CHOICE FROM MENU
NOT OBSERVED BY OTHER FIRM

Cases

- Case 1: Downstream firms are risk neutral
- In the standard model with no downstream cost heterogeneity, the upstream firm would be indifferent between contracting with one or two firms (either way it would get the monopoly profit)
- In our model, contracting with two firms is beneficial for the upstream firm because it smooths out volatility in total revenue

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- Case 2: Downstream firms infinitely risk averse (and no price discrimination)
- Now the upstream firm has to leave rents to the downstream firms, and prefers to deal with one

Maximization Problem

Let $\overline{Q}_N = rQ_L^N + (1-r)Q_H^N$. The maximization problem for U is

$$\max_{(Q_L^N, T_L^N),(Q_H^N, T_H^N)} N\left[rT_L^N + (1-r)T_H^N\right]$$

$$\text{s.t. } Q_H^N\left[1 - Q_H^N - \mathbb{1}(N=2)\overline{Q}_2 - c_H\right] - T_H^N = 0 \qquad \text{(PCH)}$$

$$Q_L^N\left[1 - Q_L^N - \mathbb{1}(N=2)\overline{Q}_2 - c_L\right] - T_L^N =$$

$$Q_H^N\left[1 - Q_H^N - \mathbb{1}(N=2)\overline{Q}_2 - c_L\right] - T_H^N \qquad \text{(ICL)}$$

which rewrites as the unconstrained problem of choosing (Q_L^N, Q_H^N) to maximize

$$\overbrace{NrQ_L^N\left[1 - Q_L^N - \mathbb{1}(N=2)\overline{Q}_2 - c_L\right]}^{\text{Expected Profits}} - \overbrace{Nr(c_H - c_L)Q_H^N}^{\text{Information Rent}} +$$

$$N(1-r)Q_H^N\left[1 - Q_H^N - \mathbb{1}(N=2)\overline{Q}_2 - c_H\right].$$

Contracts as Production Shares

It is useful to rewrite the choice variables in the following way:

$$NrQ_L^N = x_N \overline{Q}_N$$
$$N(1-r)Q_H^N = (1-x_N)\overline{Q}_N$$

- x_N is the fraction of expected output produced on the low cost side of the market
- $x = r$ corresponds to both cost types producing the same amount
- $x = 1$ corresponds to excluding the high cost type

The Benefit of Increasing Low Cost Production

Expected production costs are

$$c_L x_N \overline{Q}_N + c_H (1 - x_N) \overline{Q}_N$$

and expected information rents are

$$Nr(c_H - c_L) Q_H^N \longrightarrow \frac{r}{1-r}(1 - x_N)(c_H - c_L)\overline{Q}_N.$$

So the marginal benefit of raising x_N on the margin is

$$\frac{(c_H - c_L)\overline{Q}_N}{1 - r}.$$

The Cost of Increasing Low Cost Production: One Firm

Total expected revenue with one firm is

$$x_1\overline{Q}_1\left(1-\frac{x_1\overline{Q}_1}{r}\right)+(1-x_1)\overline{Q}_1\left(1-\frac{(1-x_1)\overline{Q}_1}{1-r}\right)$$

whose derivative with respect to x_1 is

$$\underbrace{\overline{Q}_1\left(1-\frac{x\overline{Q}_1}{r}\right)-\overline{Q}_1\left(1-\frac{(1-x_1)\overline{Q}_1}{1-r}\right)}_{\text{Effect 1}}\underbrace{-\frac{x_1\overline{Q}_1^2}{r}+\frac{(1-x)\overline{Q}_1^2}{1-r}}_{\text{Effect 2}}$$

1. Sales increase for low cost types and fall for high cost types, but price received by former is smaller
2. Price decrease for low cost types and rise for high cost types, but quantity produced by latter is smaller

Total marginal cost is $\dfrac{2(x_2-r)\overline{Q}_1^2}{r(1-r)}$.

The Cost of Increasing Low Cost Production: Two Firms

Total expected revenue with two firms is

$$x_2\overline{Q}_2\left(1-\frac{x_2\overline{Q}_2}{2r}-\frac{\overline{Q}_2}{2}\right)+(1-x_2)\overline{Q}_2\left(1-\frac{(1-x_2)\overline{Q}_2}{2(1-r)}-\frac{\overline{Q}_2}{2}\right)$$

whose derivative with respect to x_2 is

$$\overbrace{\overline{Q}_2\left(1-\frac{x_2\overline{Q}_2}{2r}-\frac{\overline{Q}_2}{2}\right)-\overline{Q}_2\left(1-\frac{(1-x_2)\overline{Q}_2}{2(1-r)}-\frac{\overline{Q}_2}{2}\right)}^{\text{Price Difference is Smaller than with One Firm}}$$

$$\underbrace{-\frac{x_2\overline{Q}_2^2}{2r}+\frac{(1-x_2)\overline{Q}_2^2}{2(1-r)}.}_{\text{Price Change is Less than with One Firm}}$$

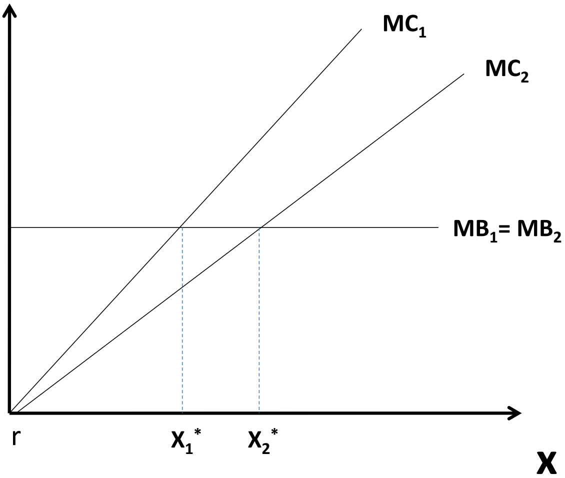
Total marginal cost is $\dfrac{(x_2-r)\overline{Q}_2^2}{r(1-r)}$.

Result

Result

The upstream firm can earn higher expected profit with two downstream firms.

Comparison



Proof

Proof.

Let $\pi_N(x_N, \overline{Q}_N)$ be profits from contracting with N firms. One can express this as

$$\pi_N(r, \overline{Q}_N) + \overline{Q}_N \int_r^{x_N} \left[\frac{c_H - c_L}{1-r} - \frac{2(v-r)\overline{Q}_N}{Nr(1-r)} \right] dv.$$

Let $\overline{Q}_1^*$ and x_1^* represent optimal quantities for the contract with one firm. By contracting with two firms and setting $\overline{Q}_2 = \overline{Q}_1^*$ and $x_2 = x_2^*$ profits are

$$\pi_2(r, \overline{Q}_1^*) + \overline{Q}_1^* \int_r^{x_1^*} \left[\frac{c_H - c_L}{1-r} - \frac{(v-r)\overline{Q}^*}{r(1-r)} \right] dv \geq$$

$$\pi_1(r, \overline{Q}_1^*) + \overline{Q}_1^* \int_r^{x_1^*} \left[\frac{c_H - c_L}{1-r} - \frac{2(v-r)\overline{Q}^*}{r(1-r)} \right] dv = \pi_1(x_1^*, \overline{Q}_1^*).$$

since $\pi_2(r, \overline{Q}_1^*) = \pi_1(r, \overline{Q}_1^*)$. $\square$

Comments

- In the linear case, expected revenue is $\mathbb{E}[\,Q(1-Q)\,] = \mathbb{E}[\,Q\,] - \mathbb{E}\left[\,Q^2\,\right]$
- Contracting with two firms is useful because it insures the upstream firm against output volatility
- Expected benefit in this model is $\mathbb{E}\left[\,\int_0^Q (1-v)dv\,\right] = \mathbb{E}[\,Q\,] - \frac{1}{2}\mathbb{E}\left[\,Q^2\,\right]$
- One can employ the exact same logic as above to show that a social planner would also choose to include two firms
- At the same time, the presence of two firms increases downstream firms' profit uncertainty
- To study this effect we now assume downstream firms are infinitely risk averse

Expected Revenue with Infinite Risk Aversion

With infinite risk aversion each firm is only willing to enter the market if it pays a transfer equal to the worst possible profit realization. The expected revenue that the upstream firm can extract from downstream now equals

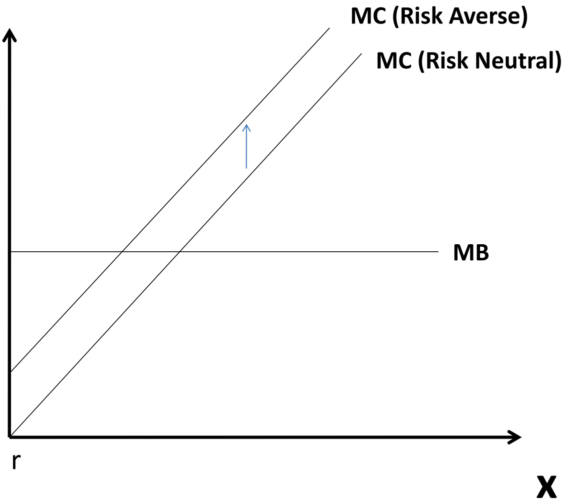
$$x\overline{Q}\left(1-\frac{x\overline{Q}}{r}\right)+(1-x)\overline{Q}\left(1-\frac{x\overline{Q}}{2r}-\frac{(1-x)\overline{Q}}{2(1-r)}\right)$$

whose marginal change with x is

$$\frac{(x-r)\overline{Q}^2}{r(1-r)}+\frac{\overline{Q}^2}{2r}=\text{Increase in Revenue Volatility}+$$

$$\text{Decrease in Rent Extraction}$$

The Effect of Risk Aversion



Foreclosure

Result

When downstream firms are infinitely risk averse, the upstream firm cannot increase profit by contracting with two firms.

Proof.

Suppose the upstream firm contracts some $\overline{Q}_2$ and x_2 with two firms. $\pi_1(x_2, \overline{Q}_2) > \pi_2(x_2, \overline{Q}_2)$ if

$$\int_r^x \left[\frac{c_H - c_L}{1-r} - \frac{2(v-r)\overline{Q}}{r(1-r)}\right] dv > \int_r^x \left[\frac{c_H - c_L}{1-r} - \frac{(v-r)\overline{Q}}{r(1-r)} - \frac{\overline{Q}}{2r}\right] dv$$

which holds since

$$\frac{(x-r)\overline{Q}}{2r} - \int_r^x \frac{(v-r)\overline{Q}}{r(1-r)} dv = \frac{(x-r)\overline{Q}}{2r} - \frac{\overline{Q}}{2r}\frac{(x-r)^2}{1-r} \geq 0.$$

□

Comments

- The foreclosure decision by the upstream firm is efficient in the sense that social welfare is higher under the optimal contract with one firm than the optimal contract with two firms under infinite risk aversion
- This is clearly true within our specific model; moreover, we still haven't worked out the exact intuition
- The role of the no price discrimination assumption is still unclear, both for foreclosure and for efficiency
- There may be better way of organizing the market. The upstream firm could:
  - Contract production but do the selling itself
  - Segment the downstream market into exclusive territories
  - Pay a fixed wage to downstream firms and ask for all profit

Extending the Model

- The model above is the simplest way of illustrating our idea in the context of a vertical market
- We now want to look at the robustness of the insights to two natural extensions:
  - N firms
  - Less extreme forms of risk aversion (CARA utility)
- The basic intuitions of the simple model appear to hold, but we have not fully worked out the results
- We will again consider in turn the case of risk neutral and risk averse downstream firms

The Benefit of Adding Additional Firms

- One can easily show aggregate revenue volatility can be further reduced by adding additional firms
- Just as before we can write

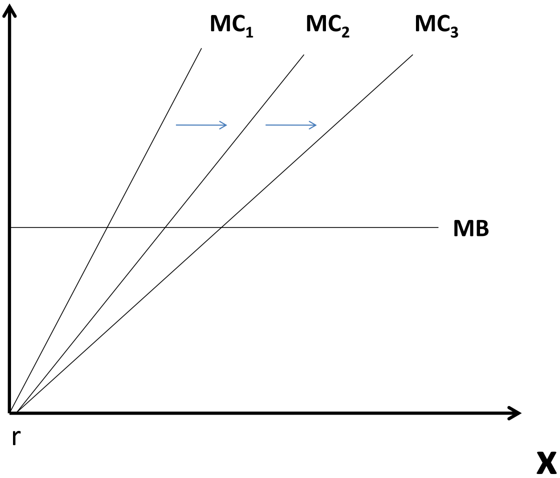
$$NrQ_L^N = x_N\overline{Q}_N$$
$$N(1-r)Q_H^N = (1-x_N)\overline{Q}_N$$

- With risk neutrality, expected revenue (net of information rents) of contracting with N types is

$$x_N\overline{Q}_N\left[1-\frac{x_N\overline{Q}_N}{rN}-\frac{N-1}{N}\overline{Q}_N\right]+(1-x_N)\overline{Q}_N\left[1-\frac{x_N\overline{Q}_N}{(1-r)N}-\frac{N-1}{N}\overline{Q}_N\right]$$

- The marginal change with x_N is -$\frac{2\overline{Q}_N^2}{N}\frac{x_N-r}{r(1-r)}$

N Firms



Exclusion of High Cost Type

- Eventually we reach an N^* at which the marginal cost of increasing x is less than the marginal benefit for all $x \in (r, 1)$
- When the upstream firm can contract with $N \geq N^*$ downstream firms, it excludes the high cost types by setting $(T_H, Q_H) = (0, 0)$ and extracts all the surplus from the low cost types by setting

$$Q_L(1 - Q_L - r(N-1)Q_L - c_L) = T_H.$$

  and solving

$$\max_{Q_L} rQ_L(1 - Q_L - r(N-1)Q_L - c_L).$$

- The optimal output for low cost types is $Q_L^* = \frac{1-c_L}{2(1+r(N-1))}$
- Total profit is therefore

$$\left(\frac{1-c_L}{2}\right)^2 \frac{rN}{1+r(N-1)}.$$

Result

Result

Profits are strictly increasing with N and

$$\lim_{N\to\infty} \pi(N) = \left(\frac{1-c_L}{2}\right)^2.$$

At least for $N \geq N^*$ there are diminishing returns to adding additional firms to the market, so a constant fixed cost of contracting with each firm will limit market size in practice.

Generalization of Risk Aversion

- Suppose downstream firms now have CARA preferences $u(w) = 1 - \exp(-aw)$
- Suppose the upstream firm continues to contract only with the low cost type
- The profit of a low cost firm is the random variable

$$\pi_L^N = Q_L^N \left(1 - Q_L^N - Q_L^N \tilde{Q}_{N-1} - c_L\right) - T_L^N$$

  where $\tilde{Q}_{N-1} \sim \mathrm{B}(N-1, r)$ has a binomial distribution
- $\mathrm{B}(N-1, r)$ is approximately $\mathcal{N}\left[(N-1)r, (N-1)r(1-r)\right]$ for large N
- So the certainty equivalent income needed to satisfy the participation constraint is approximately

$$T_L^N = Q_L^N \left[1 - Q_L^N - r(N-1)Q_L^N - c_L\right] - \frac{a}{2}(N-1)r(1-r)\left(Q_L^N\right)^4$$

Maximization Problem

The firm's maximization problem is (dropping the multiplicative constant Nr)

$$\max_{Q_L^N} Q_L^N \left[1 - Q_L^N - r(N-1)Q_L^N - c_L\right] - \frac{a}{2}(N-1)r(1-r)\left(Q_L^N\right)^4.$$

The optimal quantity is defined by the implicit function

$$(1-c_L) - 2Q_L^{N*}(1 + r(N-1)) - 2a(N-1)r(1-r)\left(Q_L^{N*}\right)^3 = 0.$$

Clearly $\frac{\partial Q_L^{N*}(a)}{\partial a} < 0 \longrightarrow$ for a fixed number of firms, an increase in risk aversion decreases optimal per-firm output.

Going Forward

- Clearly the presence of risk aversion reduces profit by reducing output
- However, we are interested in distortions in market size
- The key question is therefore how risk aversion changes the marginal benefit of adding an additional firm
- We do not know if this always decreases as in the simple model (in which case market size would decrease) or could sometimes increase (in which case market size could increase)

Discussion

- Our main insight is that players' payoff uncertainty increases when there are action externalities in a game
- This is not necessarily new (Green and Stokey 1983 make a similar point in the context of tournaments)
- We believe the new question is how this affects the number of agents a principal wants to contract
- Of course, it is also important to find a reason why a large number of agents are desirable under risk aversion
- This question can go beyond vertical foreclosure:
  - In our IO application, there is a tension between volatility reduction in aggregate revenue and rent extraction
  - In a tournament (or contest) application there might be tension between identifying the most talented person and encouraging effort provision