Price Discrimination in Input Markets:
Quantity Discounts and Private Information[*]

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We consider a monopolistic supplier's optimal choice of wholesale tariffs when downstream firms are privately informed about their retail costs. The existing literature typically ignores the possibility of downstream firms having private information. Under discriminatory pricing, downstream firms that differ in their ex ante distribution of retail costs are offered different tariffs. Under uniform pricing, the same wholesale tariff is offered to all downstream firms. In contrast to the extant literature on third-degree price discrimination with nonlinear wholesale tariffs, we find that banning discriminatory wholesale contracts—the usual legal practice in the EU and US—often is beneficial for consumers and social welfare. This result is shown to be robust even when the upstream supplier faces competition in the form of fringe supply.

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1. INTRODUCTION

According to recent empirical evidence, vertical contracts between manufacturers and retailers are often nonlinear, see Villas-Boas (2009) or Bonnet and Dubois (2010). Moreover, the legal practice in the EU and US is to regard quantity discounts as a justifiable pricing strategy of manufacturers—as long as they are not discriminatory in the sense of applying different conditions to identical transactions with other trading partners.[1] The literature on third-degree price discrimination in intermediate good markets, however, mainly focuses on linear wholesale contracts. The few papers investigating the welfare effects of third-degree price discrimination in intermediate industries that allow for quantity discounts find that a ban on price discrimination reduces welfare.[2] In contrast to this

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[1]Cf. Waelbroeck (2005) and Motta (2009). Regarding the EU, however, this statement has to be qualified in the light of the *Michelin II* judgment from 2003, "because for the first time it is found that a dominant firm cannot even resort to pure (non-individualised) quantity discounts, a practice until then accepted by the Courts." (Motta, 2009, p.29)

[2]Cf. O'Brien and Shaffer (1994), Rey and Tirole (2005), and Inderst and Shaffer (2009).

literature but in line with the usual legal practice, we derive conditions such that banning discriminatory wholesale tariffs improves social welfare as well as consumer surplus.

In our model downstream firms differ in the ex ante distribution of their retail costs, which can be either high or low. These different distributions are known by the manufacturer. If third-degree price discrimination is permitted, the manufacturer offers to downstream firms with different distributions of retail costs a different menu of transfer-quantity pairs. Under uniform pricing, on the other hand, the same menu is offered to all downstream firms. When deciding whether or not to accept the supplier's offer, each downstream firm is privately informed about the realization of its retail cost. Thus, the monopolistic supplier does not only offer nonlinear tariffs to reduce double marginalization but also to screen downstream firms according to the retail efficiency, i.e., wholesale tariffs are such that each downstream firm reveals its retail cost.[3] Irrespective of the pricing regime, a low-cost downstream firm procures the efficient quantity from the integrated structure's point of view. The quantities procured by high-cost downstream firms, in contrast, are distorted downwards. With discriminatory pricing this downward distortion in a particular downstream firm's quantity increases as this downstream firm becomes more likely to produce at low cost. Under uniform pricing, on the other hand, the margin of the quantity distortion is determined by the average probability of all downstream firms to produce at low cost. In general, for a high difference in retail cost between a high-cost and a low-cost downstream firm, it may well be optimal for the manufacturer to serve only low-cost firms in order to cut back information rents. In consequence, under uniform pricing, if the average probability that downstream firms produce at high cost is low, then only low-cost downstream firms are served. Under discriminatory pricing, in contrast, due to downstream firms' heterogeneity with respect to ex ante efficiency, some downstream may also be served even when producing at high cost. If this is the case, then permitting discriminatory wholesale tariffs leads to more markets being served, in the sense that some downstream firms are served under discriminatory but not under uniform pricing if they are high-cost retailers. Here, permitting third-degree price discrimination increases the total quantities sold in expectations and therefore also expected welfare. While the results summarized so far are supportive of the predominant opinion in the existing literature, welfare effects of banning price discrimination are reversed if the differences in retail efficiency between a high-cost and a low-cost downstream firm is relatively low. Here, under either pricing regime each downstream firm is served even if producing at high cost, with the quantity procured by a high-cost retailer under uniform pricing lying strictly between the highest and the lowest quantity procured by a high-cost retailer under discriminatory pricing. In these cases—at least for linear demand—banning price discrimination improves social welfare and consumer surplus, which contrasts the findings in the established literature.

In order to allow for the manufacturer's pricing behavior also causing primary-line injuries in the upstream market, we augment the basic model by assuming that downstream firms can purchase the essential input not only from the manufacturer but also

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[3]A model of vertical relations where downstream firms' costs are stochastic is analyzed by Rey and Tirole (1986). They do not discuss third-degree price discrimination.

from a competitive fringe. Most of our findings are robust toward this kind of upstream competition. In particular, for a wide range of parameter values, a ban on discriminatory wholesale tariffs benefits consumers and improves social welfare.

There has been considerable back and forth in the literature regarding the welfare effects of banning third-degree price discrimination in intermediate-good markets. This literature was initiated by Katz (1987). He shows that permitting price discrimination reduces welfare unless it prevents inefficient backward integration by the downstream chain. DeGraba (1990) extends Katz's model to a long-run analysis where downstream firms can invest into cost reduction. Here, a ban on price discrimination does not only increase welfare in the short run, but also is beneficial in the long run. The intuition behind these results is that the wrong firm—the less efficient one—receives a discount under price discrimination.[4] While the above articles assume that the upstream supplier is an unconstrained monopolist, Inderst and Valletti (2009) and O'Brien (forthcoming) relax this assumption. In Inderst and Valletti the upstream supplier is constrained by the threat of demand-side substitution. Here, the more efficient firm receives a discount under price discrimination. As a result—in the long run—consumers benefit and social surplus increases if price discrimination is permitted and demand is linear. O'Brien assumes that wholesale prices are determined by bilateral negotiations between the supplier and a downstream firm. This also gives rise to circumstances where banning price discrimination is socially harmful.[5]

All the aforementioned articles restrict attention to linear wholesale prices. Thus, with linear wholesale tariffs the welfare results regarding a ban on price discrimination are mixed. Among the few exceptions which consider pricing schemes more complex than linear wholesale prices, in contrast, the predominant opinion is that banning discriminatory wholesale pricing is detrimental for welfare. O'Brien and Shaffer (1994) assume that firms can bargain over the terms of a two-part supply tariff. Banning price discrimination renders retailer bargaining power useless and mitigates the manufacturer's market power. All downstream firms pay higher marginal input prices under uniform pricing than under price discrimination. Thus, a ban on price discrimination is harmful for consumers and reduces total welfare. A similar model is analyzed by Rey and Tirole (2005). Here, the manufacturer has all the bargaining power. They show that "non-discrimination laws [...] reduce consumer surplus and total welfare by enabling the monopolist to commit" (p.32).[6] Inderst and Shaffer (2009) abstract from any commitment problems and assume that the offered two-part tariffs are publicly observable. Focusing on asymmetric downstream firms they show that discriminatory contracts amplify differences in downstream firms' competitiveness. Again, a ban on price discrimination tends to raise all final-good prices and thus to reduce total output. In consequence, banning price discrimination reduces consumer surplus and total welfare. While the above "insight raises [...] serious

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[4]Similar results are obtained by Yoshida (2000) and Valletti (2003).

[5]Mixed welfare results regarding price discrimination in input markets are also obtained by Herweg and Müller (2010).

[6]Building on the Rey-Tirole model and assuming that the supplier competes against a competitive fringe, Caprice (2006) shows that a ban on price discrimination leads to an increase in welfare if the fringe is sufficiently efficient.

concerns about the efficacy of the Robinson-Patman Act" (O'Brien and Schaffer, 1994, p.314) or its analogue in EU competition law, we find that, when downstream firms have private information, the reservation toward discriminatory pricing practices embodied in these legal enactments may well be warranted even if nonlinear pricing schemes are feasible.

The rest of the paper is organized as follows: In Section 2, we introduce our basic model with a monopolistic input supplier. This model is analyzed in Section 3. Section 4 augments the basic model by assuming that the upstream supplier is constrained by the threat of demand-side substitution. We conclude in Section 5. All proofs are relegated to the Appendix.

2. THE MODEL

Consider a vertically related industry where the upstream market is monopolized by firm M. The upstream monopolist produces an essential input that is supplied to the downstream sector. For simplicity, we assume that the upstream firm produces quantity q at constant marginal cost, $K > 0$. There are two downstream firms, $i \in \{1, 2\}$, that can transform one unit of the input into one unit of the final good.

We assume that each downstream firm is a local monopolist, i.e., downstream firms operate in distinct and independent markets. We comment on this assumption below. Downstream markets are identical in size and characterized by the inverse demand function $P(q)$, which is strictly decreasing and twice differentiable where $P > 0$. Moreover, we impose the standard assumption $P'(q) < \min\{0, -qP''(q)\}$ where $P > 0$.[7]

Downstream firm i produces at constant marginal cost and without fixed costs. The marginal cost of production is either high or low, $c_i \in \{c_L, c_H\}$ with $0 \leq c_L < c_H < P(0)$. In all what follows, we assume that $P(0) > c_H + K$, which guarantees that the joint-surplus maximizing quantity of a vertically integrated firm is strictly positive.

Let α_i be the probability that firm i has low marginal cost. We assume that $1 > \alpha_1 > \alpha_2 > 0$, i.e., ex ante firm 1 is more likely to produce at low marginal cost than firm 2. We refer to firm 1 and firm 2 as the efficient and the inefficient downstream firm, respectively. Its type—i.e., its marginal cost of production—is private information of the respective downstream firm. The upstream firm only knows the probability α_i, $i \in \{1, 2\}$, with which downstream firm i is the low-cost type. Instead of assuming private information regarding the retail costs, we could also assume that downstream firms are privately informed about demand conditions. Our results, in particular the structure of the optimal wholesale mechanisms as well as our welfare findings, would also hold under this alternative assumption. [8]

The upstream monopolist can make a take-it-or-leave-it offer to the downstream firms.[9]

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[7]See, for example, Vives (1999).

[8]Suppose both downstream firms produce at marginal cost c_L. Downstream demand is $P(q)$ in the "good" state and $\tilde{P}(q) = P(q) - (c_H - c_L)$ in the "bad" state. The exact demand conditions are private information of the downstream firm that operates in this market. With these assumptions, the profit functions of the downstream firms and the welfare function are exactly the same as the ones we obtain by assuming private information about costs.

[9]The assumption of the upstream supplier having all the bargaining power, "which arguably can be justified

With downstream firms operating in independent markets and with the type space being identical for both downstream firms, without loss of generality, the input supplier offers downstream firm $i \in \{1, 2\}$ a direct mechanism $\Gamma_i = \langle (q_{Li}, t_{Li}), (q_{Hi}, t_{Hi}) \rangle$, that specifies a quantity, $q \in \mathbb{R}_{\geq 0}$, and a transfer from firm i to the upstream supplier, $t \in \mathbb{R}$, for each feasible type announcement.

With this type of wholesale contracts, the question whether or not a downstream firm is forced to sell the whole quantity procured is immediately at hand. We assume free disposal for downstream firms. Thus, when having purchased quantity q' of the input, downstream firm i can produce any quantity $q \in [0, q']$ of the final output at cost $c_i q$.

The sequence of events is as follows: First, nature draws the cost type for each downstream firm $i \in \{1, 2\}$, which thereafter is privately observed. Next, the input supplier makes a take-it-or-leave-it offer to each downstream firm. Under price discrimination the upstream supplier offers each downstream firm a possibly different tariff, whereas under uniform pricing one and the same tariff applies to both firms.[10] A downstream firm either chooses one of the two offered bundles or it rejects the upstream supplier's offer. In case of rejection, the downstream firm obtains its reservation profit, which is normalized to zero. If the downstream firm accepts a quantity-transfer pair (q, t), it decides how much of this acquired input to transform into the final good, and sells the produced output to consumers.

We focus on separate markets in order to isolate the effect of discriminatory wholesale tariffs in the case of asymmetric information from potential competitive effects. From an applied point of view, this restriction also seems justifiable: Besides geographic price discrimination, a case in which separate markets are a natural assumption, the European Commission mostly concerns itself with whether discriminatory pricing causes a *primary-line* or *secondary-line injury*, which are adressed in Article 82 (b) and (c) EC.[11] Article 82(b) does not impose the requirement that a downstream firm has to be placed at a competitive disadvantage in the first place. Application of Article 82(c), on the other hand, calls for a downstream firm to be placed at a disadvantageous position. Recent practice of the EU Commission, however, generally overlooks this requirement when relying on Article 82(c).[12]

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on the grounds that for antitrust purposes the considerations of price discrimination in intermediate-goods markets is primarily relevant if the supplier enjoys a dominant position" (Inderst and Shaffer (2009), p.4) is common in the extant literature. The only exceptions are O'Brien and Shaffer (1994) and O'Brien (2008).

[10] As noted by Inderst and Shaffer (2009), another way to model uniform pricing would be to assume that the manufacturer can offer a menu of tariffs, as long as the same menu is offered to both downstream firms. In our setup the manufacturer cannot benefit from offering a menu of quantity-transfer tariffs, since downstream firms cannot be screened according to their ex ante efficiency.

[11] While the first type of price discrimination involves discrimination on the part of a dominant firm with the objective of excluding rival competitors, the latter type refers to the charging of different prices to downstream competitors thereby placing one or more of them at a competitive disadvantage relative to others.

[12] As criticized by, for example, Geradin and Petit (2005),when dealing with cases involving primary-line injury price discrimination, the EU Commission often relies on Article 82(c) EC—instead of Article 82(b) EC—and usually tends to ignore the requirement that the pricing practice in question has to put one downstream firm at a competitive disadvantage.

3. THE ANALYSIS

Let $q^*(c) = \arg\max_{q \geq 0}\{(P(q) - c)q\}$. It is readily verified that $q^*(\cdot)$ is strictly decreasing in marginal cost c, such that $q^*(c_H) < q^*(c_L)$. Due to free disposal, downstream firm i's maximum profit when faced with tupel (q, t) is $\pi(q, c_i) - t$, where

$$\pi(q, c_i) = [P(\min\{q, q^*(c_i)\}) - c_i]\min\{q, q^*(c_i)\}. \tag{1}$$

Thus, downstream firm i's gross profits $\pi(q, c_i)$ are strictly increasing and strictly concave in q on $[0, q^*(c_i))$ and constant for $q \geq q^*(c_i)$. Moreover, $\pi(q, c_i)$ satisfies the following single-crossing property:

Lemma 1 *For all* $0 \leq q' < q'' \leq q^*(c_L)$, $\pi(q'', c_L) - \pi(q', c_L) > \pi(q'', c_H) - \pi(q', c_H)$.

Let $q^{JS}(c) = \arg\max_{q \geq 0}(P(q) - c)q - Kq$ denote the optimal quantity produced by a vertically integrated structure comprising of the upstream supplier and a downstream firm with marginal cost c. Under the imposed assumptions we have $0 < q^{JS}(c) < q^*(c)$. Since $q^{JS}(\cdot)$ is strictly decreasing in marginal cost c, it holds that

$$q^{JS}(c_H) < \min\{q^*(c_H), q^{JS}(c_L)\} < q^*(c_L). \tag{2}$$

3.1. Discriminatory Offers

Suppose the M is not restricted to offering the same wholesale tariffs to both downstream firms. Since downstream firms operate in independent markets, M solves two independent maximization problems. Thus, when contracting with a downstream firm that produces at low costs with probability α, M offers this firm a wholesale mechanism $\Gamma = \langle (q_L, t_L), (q_H, t_H) \rangle$ that maximizes expected upstream profits,

$$\Pi^D(q_L, q_H, t_L, t_H) = \alpha[t_L - Kq_L] + (1 - \alpha)[t_H - Kq_H]. \tag{3}$$

subject to the constraints that Γ is truthful and individually rational.

The wholesale mechanism is truthful if and only if it satisfies both the incentive compatibility constraint for the low-cost type, (IC$_L$), and the incentive compatibility constraint for the high-cost type, (IC$_H$). Formally,

$$\pi(q_L, c_L) - t_L \geq \pi(q_H, c_L) - t_H, \tag{IC$_L$}$$

$$\pi(q_H, c_H) - t_H \geq \pi(q_L, c_H) - t_L. \tag{IC$_H$}$$

Moreover, the mechanism has two satisfy the individual rationality constraints, i.e., for each cost type the designated quantity transfer tuple has to yield nonnegative profits. The individual rationality constraints for the low-cost type and the high-cost type are

$$\pi(q_L, c_L) - t_L \geq 0, \tag{IR$_L$}$$

$$\pi(q_H, c_H) - t_H \geq 0, \tag{IR$_H$}$$

respectively.

Implications of free disposal.—An important implication of free disposal is that in the optimum we must have $q_H \leq q^*(c_H)$. To see this, assume the opposite, i.e., the optimal contract stipulates $q'_H > q^*(c_H)$. Then, leaving q_L and transfers t_L and t_H unchanged, M could offer the high-cost type the lower quantity $q^*(c_H)$. First, this change obviously does not affect (IR_L). Moreover, due to free disposal, we have $\pi(q'_H, c_H) = \pi(q^*(c_H), c_H)$, which implies that ($IR_H$) and ($IC_H$) are also left unchanged. Last, this decrease in the quantity offered to the high-cost type strictly relaxes (IC_L) because $\pi(q^*(c_H), c_L) < \pi(q'_H, c_L)$. Thus, all constraints remain satisfied under this new contract, but upstream cost of production is strictly lower than under the original contract, contradicting its optimality. Analogous reasoning reveals that in the optimum we have $q_L \leq q^*(c_L)$.

Implications of incentive compatibility.—Combining and rearranging both incentive compatibility constraints, (IC_L) and (IC_H), yields

$$\pi(q_L, c_L) - \pi(q_H, c_L) \geq t_L - t_H \geq \pi(q_L, c_H) - \pi(q_H, c_H). \tag{4}$$

As usual, incentive compatibility imposes the following monotonicity requirement: in the optimal contract we must have $q_H \leq q_L$. Then, it follows that $\pi(q_L, c_H) \geq \pi(q_H, c_H)$, which in turn implies that under the optimal contract we must have $t_L - t_H \geq 0$.

The implications of free disposal and incentive compatibility are summarized in the following lemma:

Lemma 2 *The optimal contract satisfies the following monotonicity constraint:*

$$q_H \leq \min\{q_L, q^*(c_H)\} \leq q^*(c_L). \tag{MON}$$

Since the incentive compatibility constraints limit only the differences in transfers and not the absolute values, we can conclude that (IR_H) is binding at the optimum. As it is well established, if (IR_H) and (IC_L) are satisfied then (IR_L) also holds. The remaining incentive compatibility constraint, (IC_H), then holds as long as the monotonicity requirement $q_H \leq q_L$ is met.

Hence, the transfers t_H and t_L are uniquely determined by (IR_H) and (IC_L),

$$t_H = \pi(q_H, c_H), \tag{5}$$

$$t_L = \pi(q_L, c_L) - \pi(q_H, c_L) + \pi(q_H, c_H). \tag{6}$$

The upstream supplier's problem consists of choosing quantities q_L and q_H to maximize upstream profits under a discriminatory pricing regime,

$$\Pi^D(q_L, q_H) = \alpha \left\{ [P(q_L) - c_L]q_L - q_H(c_H - c_L) - Kq_L \right\}$$
$$+ (1 - \alpha) \left\{ [P(q_H) - c_H]q_H - Kq_H \right\} \tag{7}$$

subject to the monotonicity requirement $q_H \leq \min\{q_L, q^*(c_H)\} \leq q^*(c_L)$. Setting the partial derivative of $\Pi^D(\cdot)$ with respect to q_L equal to zero immediately yields

$$q_L^D = q^{JS}(c_L). \tag{8}$$

This is the well-known no-distortion-at-the-top result: the low-cost type produces the quantity that maximizes the joint surplus of the integrated structure.

The quantity sold to a high-cost downstream firm is distorted downwards in order to cut back on the information rent paid to a low-cost firm. The downward distortion depends on the probability with which M deals with a low-cost downstream firm. If it is sufficiently unlikely that the downstream firm produces at high cost, then M will offer the high-cost type a quantity equal to zero. Formally, due to strict concavity of upstream revenues with respect to q_H as long as $P(q) > 0$, M will offer the high-cost type a quantity equal to zero if and only if

$$\left. \frac{\partial \Pi^D}{\partial q_H} \right|_{q_H=0} \leq 0 \iff \alpha \geq \hat{\alpha}^D := \frac{P(0) - c_H - K}{P(0) - c_L - K}. \tag{9}$$

For $\alpha < \hat{\alpha}^D$, on the other hand, the optimal quantity sold to the high-cost type, $\hat{q}^D(\alpha)$, is strictly positive and satisfies the following first-order condition:

$$P(\hat{q}^D(\alpha)) - c_H + P'(\hat{q}^D(\alpha))\hat{q}^D(\alpha) = K + \frac{\alpha}{1-\alpha}(c_H - c_L) . \tag{10}$$

Obviously, $\hat{q}^D(\alpha)$ is strictly decreasing in α and $\lim_{\alpha \searrow 0} \hat{q}^D(\alpha) = q^{JS}(c_H)$. Intuitively, as the probability of dealing with a low-cost downstream firm becomes smaller, M chooses the quantity offered to the high-cost type closer to the joint-surplus maximizing quantity $q^{JS}(c_H)$. If, on the other hand, the probability of contracting with a low-cost downstream firm is sufficiently high, then M prefers to offer a zero quantity to the high-cost type, which eliminates information rents and in turn allows M to extract all the surplus from the interaction with a low-cost type. Obviously, the quantities $\hat{q}^D(\alpha)$ and q_L^D satisfy the monotonicity constraint (MON). It is worthwhile to point out that $\hat{\alpha}^D$ approaches 1 if $c_H - c_L$ goes to zero. Put verbally, both cost types are served by the manufacturer if the difference in possible retail costs is not too high.

Proposition 1 *Under discriminatory wholesale tariffs, (i) $q_L^D = q^{JS}(c_L)$ and (ii) $q_H^D(\alpha) = \hat{q}^D(\alpha)$ if $\alpha < \hat{\alpha}^D$ and zero otherwise.*

3.2. Uniform Pricing

Suppose third-degree price discrimination in the intermediate good market is banned. In this case, M has to offer the same menu to both downstream firms, i.e., $\Gamma_1 = \Gamma_2$. Since this restriction leaves the set of incentive compatibility and individual rationality constraints unchanged, all the above considerations—Lemma 2 in particular—also apply in this situation. Therefore, M chooses quantities q_L and q_H in order to maximize upstream profits,

$$\Pi^U(q_L, q_H) = \alpha_\Sigma \left\{ [P(q_L) - c_L]q_L - q_H(c_H - c_L) - Kq_L \right\}$$
$$+ (2 - \alpha_\Sigma) \left\{ [P(q_H) - c_H]q_H - Kq_H \right\}, \tag{11}$$

where $\alpha_\Sigma := \alpha_1 + \alpha_2$. Differentiation of $\Pi^U(\cdot)$ with respect to q_L reveals that the no-distortion-at-the-top result carries over to a nondiscriminatory pricing regime,

$$q_L^U = q^{JS}(c_L) . \tag{12}$$

Analogous reasoning to the discriminatory pricing regime reveals that the quantity offered to the high-cost type decreases in the overall probability of contracting with a low-cost downstream firm, α_Σ. Once this probability exceeds a certain threshold, M prefers to offer a zero quantity to the high type. Formally, M offers a zero quantity to high-cost downstream firms if and only if

$$\left.\frac{\partial \Pi^U}{\partial q_H}\right|_{q_H=0} \leq 0 \iff \alpha_\Sigma \geq \hat{\alpha}^U := 2\hat{\alpha}^D \tag{13}$$

For $\alpha_\Sigma < \hat{\alpha}^U$, the quantity offered to high-cost types, $\hat{q}^U(\alpha_\Sigma)$, satisfies

$$P(\hat{q}^U(\alpha_\Sigma)) - c_H + P'(\hat{q}^U(\alpha_\Sigma))\hat{q}^U(\alpha_\Sigma) = K + \frac{\alpha_\Sigma}{2 - \alpha_\Sigma}(c_H - c_L) . \tag{14}$$

Note that $\hat{q}^U(\alpha_\Sigma)$ is strictly decreasing in α_Σ and $\lim_{\alpha_\Sigma \searrow 0} \hat{q}^U(\alpha_\Sigma) = q^{JS}(c_H)$. Thus, the monotonicity constraint is satisfied. In order to summarize the above observations we define

$$\hat{\alpha}_1^U(\alpha_2) := \hat{\alpha}^U - \alpha_2.$$

Proposition 2 *Under a nondiscriminatory wholesale tariff, (i) $q_L^D = q^{JS}(c_L)$, and (ii) $q_H^U(\alpha_\Sigma) = \hat{q}^U(\alpha_\Sigma)$ if $\alpha_1 < \hat{\alpha}_1^U(\alpha_2)$ and zero otherwise.*

3.3. Welfare

Having identified the optimal menus for M to offer under price discrimination and uniform pricing, respectively, we now turn to the welfare implications of banning price discrimination. First, we observe the following:

Lemma 3 $q_H^D(\alpha_1) \leq q_H^U(\alpha_\Sigma) \leq q_H^D(\alpha_2) < q^{JS}(c_H)$.

We define welfare as the sum of consumers' and producers' surpluses. Gross consumer surplus in market i is given by $CS_i = \int_0^{q_i} P(z)dz$. With wholesale payments and final-good prices being welfare-neutral transfers, welfare amounts to $W = CS_1 + CS_2 - (c_1 + K)q_1 - (c_2 + K)q_2$. Thus, under a pricing regime in which firm $i \in \{1, 2\}$ is offered a menu that consists of quantities $q_L(\alpha_i, \alpha_{-i})$ and $q_H(\alpha_i, \alpha_{-i})$ together with the corresponding transfers, expected welfare equals

$$E[W] = \sum_{i=1}^2 \left\{ \alpha_i \left[\int_0^{q_L(\alpha_i, \alpha_{-i})} P(z)dz - (c_L + K)q_L(\alpha_i, \alpha_{-i}) \right] \right.$$
$$\left. + (1 - \alpha_i) \left[\int_0^{q_H(\alpha_i, \alpha_{-i})} P(z)dz - (c_H + K)q_H(\alpha_i, \alpha_{-i}) \right] \right\}. \tag{15}$$

Let the difference in expected welfare between the discriminatory pricing regime and the uniform pricing regime be $\Delta W := E[W^D] - E[W^U]$. Since there is no-distortion-at-the-top under either regime, ΔW depends only on the quantities produced by high-cost

retailers. Formally,

$$
\Delta W := \Delta W(\alpha_1, \alpha_2)
$$

$$
= \sum_{i=1}^{2} (1 - \alpha_i) \left[\int_{q_H^U(\alpha_1 + \alpha_2)}^{q_H^D(\alpha_i)} P(z)dz - (c_H + K)(q_H^D(\alpha_i) - q_H^U(\alpha_1 + \alpha_2)) \right]. \quad (16)
$$

With the difference in welfare depending only on the quantities offered to high-cost
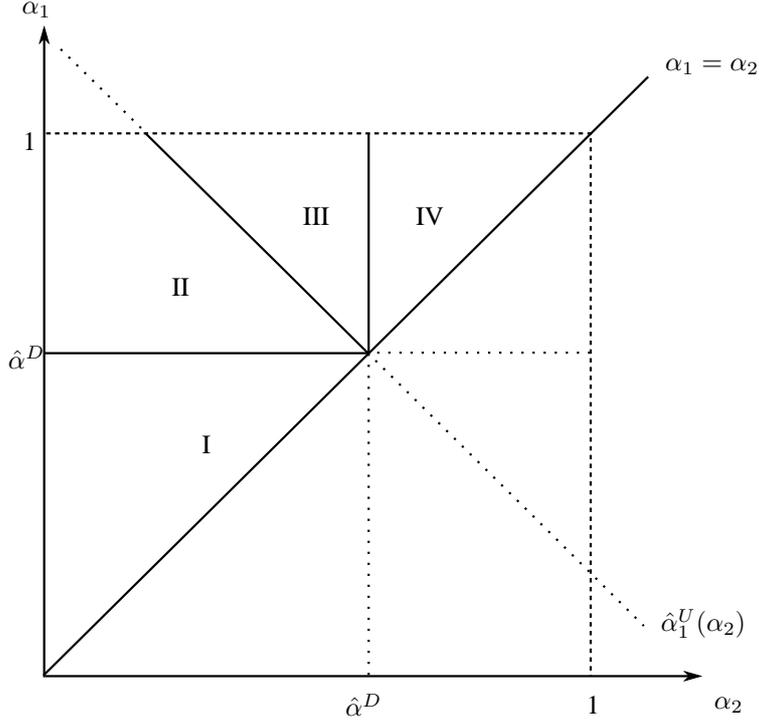
Figure 1: Welfare comparison without demand-side substitution.

downstream firms, from Propositions 1 and 2 it follows that we have to consider the following four cases, as depicted in Figure 1:

(I) $0 < q_H^D(\alpha_1) < q_H^U(\alpha_\Sigma) < q_H^D(\alpha_2)$, which holds if $\alpha_2 < \alpha_1 < \hat{\alpha}^D$;

(II) $0 = q_H^D(\alpha_1) < q_H^U(\alpha_\Sigma) < q_H^D(\alpha_2)$, which holds if $\alpha_2 < \hat{\alpha}^D \leq \alpha_1 < \hat{\alpha}_1^U(\alpha_2)$;

(III) $0 = q_H^D(\alpha_1) = q_H^U(\alpha_\Sigma) < q_H^D(\alpha_2)$, which holds if $\alpha_2 < \hat{\alpha}^D \leq \hat{\alpha}_1^U(\alpha_2) \leq \alpha_1$;

(IV) $0 = q_H^D(\alpha_1) = q_H^U(\alpha_\Sigma) = q_H^D(\alpha_2)$, which holds if $\hat{\alpha}^D \leq \alpha_2 < \alpha_1$.

In case (IV), with M never serving a high-cost downstream firm irrespective of the pricing regime, we trivially have $\Delta W = 0$. Therefore, in what follows we focus on cases (I) - (III), i.e., we restrict attention to $\alpha_2 < \hat{\alpha}^D$.

Proposition 3 *(i) If $\alpha_2 < \hat{\alpha}^D \leq \hat{\alpha}_1^U(\alpha_2) \leq \alpha_1$, then $\Delta W > 0$. (ii) If $\alpha_2 < \hat{\alpha}^D < \alpha_1 < \hat{\alpha}_1^U(\alpha_2)$, then ΔW is strictly increasing in α_1.*

In case (III), part (i) of Proposition 3, a ban of price discrimination is detrimental for welfare. This observation, in a sense, confirms the findings by O'Brien and Shaffer (1994) and Inderst and Shaffer (2009): banning third-degree price discrimination reduces total welfare if the upstream supplier can make use of pricing schemes more sophisticated than linear wholesale pricing. According to part (ii) of Proposition 3, however, once we enter case (II), the difference in expected welfare between the two pricing regimes decreases as the probability of firm 1 to be of the low-cost type decreases. This finding suggests that banning price discrimination—even with more sophisticated pricing schemes being at the input supplier's disposal—may be welfare improving.

What is the intuition behind the welfare results presented in Proposition 3? In case (III), both the probability of firm 1 producing at low cost (α_1) and the overall probability of dealing with a low-cost downstream firm (α_Σ) are relatively high. In consequence, in order to cut back on information rents, M assigns a zero quantity to firm 1 in the case it produces at high cost under price discrimination and to high-cost downstream firms in general under uniform pricing. Put differently, high-cost downstream firms are never served except for firm 2 under a discriminatory pricing regime. In this sense, price discrimination leads to more markets being served (in expectation), which turns out to be beneficial for welfare, thereby supporting the classic Chicago school argument against non-discrimination clauses. In case (II), on the other hand, both markets are always served under uniform pricing, whereas under price discrimination market 1 is served only when firm 1 produces at low cost. Here, however, banning price discrimination—and thus ensuring that both markets are served irrespective of downstream technology—is not necessarily beneficial from a welfare point of view. The reason is that both markets always being served under uniform pricing comes at the cost of a more severe downward distortion in the quantity sold to firm 2 in the case of high costs. Since the upstream supplier trades off minimizing information rents paid to low-cost types versus maximizing the surplus generated with high-cost types, the downward distortion in quantity q_H^U increases as it becomes more likely that firm 1 is a low-cost firm. Thus, in case (II), for high values of α_1 we would expect welfare to be higher under price discrimination than under uniform pricing. If α_1 is low, on the other hand, then the negative effect of banning price discrimination on q_H^U is small and the positive effect of more markets being served should outbalance. While not to be obtained in general, as we will show next, this conjecture holds true for a linear demand function.

3.4. An Application with Linear Demand

Suppose the inverse demand function is linear, $P(q) = \max\{0, 1 - q\}$, and assume that $c_H + K < 1$. In this case, it is readily verified that $q_H^{JS}(c_H) = \frac{1 - c_H - K}{2}$, $q_H^D(\alpha) = \max\{0, q_H^{JS}(c_H) - \frac{\alpha}{1-\alpha}\frac{c_H - c_L}{2}\}$, and $q_H^U(\alpha_\Sigma) = \max\{0, q_H^{JS}(c_H) - \frac{\alpha_\Sigma}{2-\alpha_\Sigma}\frac{c_H - c_L}{2}\}$. Gross consumer surplus in market $i \in \{1, 2\}$ is $CS(q_i) = q_i - \frac{1}{2}q_i^2$. A linear inverse demand function allows us to rewrite the difference in expected welfare and consumer surplus as

$$\Delta W = \sum_{i=1}^{2}(1 - \alpha_i)(q_H^D(\alpha_i) - q_H^U(\Sigma))\left[(1 - c_H - K) - \frac{q_H^D(\alpha_i) + q_H^U(\Sigma)}{2}\right] . \quad (17)$$

and

$$\Delta CS = \sum_{i=1}^{2}(1 - \alpha_i)(q_H^D(\alpha_i) - q_H^U(\Sigma))\left[1 - \frac{q_H^D(\alpha_i) + q_H^U(\Sigma)}{2}\right],\qquad(18)$$

respectively. Tedious but straightforward calculations then yield the following result.

Proposition 4 *Suppose* $P(q) = \max\{0, 1-q\}$, $c_H + K < 1$, *and* $\alpha_2 < \hat{\alpha}^D$. *Let* $\alpha_1^W(\alpha_2)$ *and* $\alpha_1^{CS}(\alpha_2)$ *be implicitly defined by* $\Delta W(\alpha_1^W(\alpha_2), \alpha_2) \equiv 0$ *and* $\Delta CS(\alpha_1^{CS}(\alpha_2), \alpha_2) \equiv 0$, *respectively. Then,*

(i) $\Delta W < 0$ *for* $\alpha_1 < \alpha_1^W(\alpha_2)$ *and* $\Delta CS < 0$ *for* $\alpha_1 < \alpha_1^{CS}(\alpha_2)$;

(ii) $\Delta W > 0$ *for* $\alpha_1 > \alpha_1^W(\alpha_2)$ *and* $\Delta CS > 0$ *for* $\alpha_1 > \alpha_1^{CS}(\alpha_2)$.

Moreover, $\alpha_1^{CS}(\alpha_2) < \alpha_1^W(\alpha_2)$ *for all* $\alpha_2 \in [0, \hat{\alpha}^D)$.

In summary, there exist unique cutoffs, $\alpha_1^W(\alpha_2)$ and $\alpha_1^{CS}(\alpha_2)$, below which banning price discrimination strictly improves welfare and consumer surplus, respectively. As is proved rigorously in the Appendix, these thresholds, which pass through $(\alpha_1, \alpha_2) = (\hat{\alpha}^D, \hat{\alpha}^D)$, are strictly decreasing in α_2 with a slope strictly between -1 and 0, as is illustrated in Figure 2 with regard to welfare. Note that—according to the final statement of Proposition 4—the set of (α_1, α_2) pairs for which banning price discrimination increases welfare is strictly larger than the set of (α_1, α_2) pairs for which a ban on price discrimination increases consumer surplus.
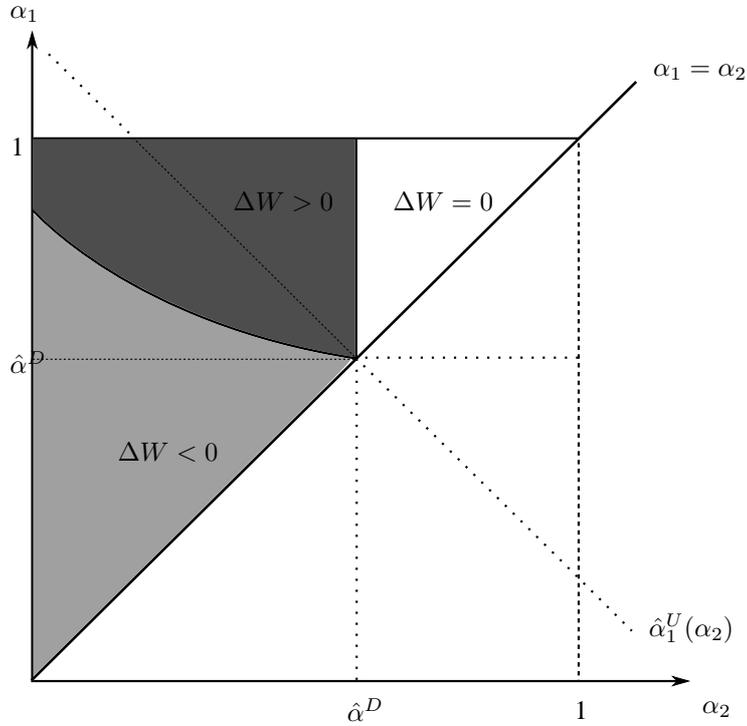

Figure 2: Welfare comparison with linear demand and without demand-side substitution.

Part (i) of Proposition 4 is in contrast to findings in the extant literature on third-degree price discrimination under nonlinear wholesale tariffs: without private information of

downstream firms, a ban on price discrimination is found to unambiguously reduce welfare and consumer surplus if the manufacturer is not restricted to linear prices. Inderst and Shaffer (2009), for instance, consider a manufacturer who is perfectly informed about the retail costs of two asymmetric downstream firms. This manufacturer offers each downstream firm a different two-part tariff under price discrimination, but is restricted to offer only a single two-part tariff under uniform pricing. In this framework, a ban on price discrimination reduces consumer surplus and welfare. For the case of separate markets—Proposition 6 of Inderst and Shaffer—if price discrimination is permitted the optimal two-part tariffs maximize the profits of the integrated structure. Put differently, both marginal wholesale prices equal the manufacturer's marginal production costs and the manufacturer extracts the whole generated profits via the franchise fee since there is no asymmetric information. Under uniform pricing the manufacturer faces a trade-off between efficiency and rent extraction, which leads him to a marginal wholesale price above his marginal cost of production. As a result, both downstream firms acquire a quantity even lower than the optimal quantities from the integrated structurer's point of view, reducing consumer surplus and welfare.[13] Our finding shows that the strong welfare result of Inderst and Shaffer is an artifact of the symmetric information case. Suppose α_2 is close to zero and α_1 is close to one but below $\alpha_1^W(\cdot)$. In this scenario, downstream firm 1 is very likely to be a low-cost firm and downstream firm 2 is very likely to be a high-cost firm.[14] Thus, this scenario is close to the separate markets case analyzed by Inderst and Shaffer. Nevertheless, according to Proposition 4, when downstream firms have private information, a ban on price discrimination increases consumer surplus and welfare, which is the complete opposite to the finding of Inderst and Shaffer (2009). In this sense, introducing only "little" asymmetric information fundamentally alters previous welfare results.

What do we learn from Proposition 4 for a case-based approach regarding discriminatory wholesale tariffs? If the regulation authority needs not be overly concerned about the possibility of one or the other market not being served under either pricing regime, then—at least for linear demand—banning price discrimination is socially desirable irrespective of whether a welfare or a consumer standard is applied. Put differently, if the demand function is sufficiently linear in the relevant range of prices, then the typical practice in the EU and the US—where it is perceived as illegal to apply different wholesale conditions for identical transactions with different trading partners—often protects consumers and improves welfare. Note that the area where a ban on price discrimination improves welfare and consumer surplus is quite large if the differences in retail cost between a high and a low-cost firm is relatively low, i.e., $\hat{\alpha}^D$ is large.

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[13]In the model of Inderst and Shaffer (2009) the restriction on two-part tariffs is without loss of generality for the case of price discrimination but under uniform pricing the manufacturer could do better with a more complex wholesale tariff. A ban on price discrimination would still be welfare harming, however, the low-cost firm would produce the same amount under both pricing regimes.

[14]Since $\alpha_1^W(0) < 1$, the likelihood of firm 1 being a low-cost firm cannot be arbitrarily close to 1. Note, however, that α^D, which increases as the difference in possible retail costs decreases, imposes a lower bound for $\alpha_1^W(0)$.

3.5. Continuous Distribution of Downstream Firms' Production Costs

In this section, we assume that the marginal cost of production of downstream firm $i \in \{1, 2\}$ is continuously distributed, i.e., $c \in [c_L, c_H] \equiv \mathcal{C}$ with $0 \leq c_L < c_H$. Firm 1's cost are ex ante distributed according to c.d.f. $F(c)$ and density $f(c) > 0$ for all $c \in \mathcal{C}$. The marginal cost of firm 2 is distributed according to c.d.f. $G(c)$ and density $g(c) > 0$ for all $c \in \mathcal{C}$. We assume that the cost distributions of the two firms are different in the sense that there exist values of $c \in \mathcal{C}$ such that $F(c)/f(c) \neq G(c)/g(c)$. The two ex ante distributions are known by the upstream supplier. The upstream supplier offers downstream firm i a direct mechanism $\Gamma_i \equiv \langle (q_i(c), t_i(c)) \rangle_{c \in \mathcal{C}}$ that specifies for each feasible type announcement a quantity $q_i(c) \in \mathbb{R}_{\geq 0}$ and a transfer $t_i(c)$ from firm i to the input supplier. If price discrimination is banned, then the two mechanisms have to be identical, i.e., $\Gamma_1 = \Gamma_2$. In order to keep the analysis simple, we abstract from upstream cost and set $K = 0$. Moreover, we focus on linear demand $P(q) = \max\{1 - q, 0\}$, since a main purposes of this continuous cost case is to demonstrate the robustness of our welfare findings. The manufacturer's expected profit is given by:

$$\Pi = \int_{c_L}^{c_H} t_1(c) f(c) \, dc + \int_{c_L}^{c_H} t_2(c) g(c) \, dc \tag{19}$$

As before the manufacturer has to satisfy the individual rationality and incentive compatibility constraints, i.e., for all $i \in \{1, 2\}$ and $c \in \mathcal{C}$:

$$q_i(c)[1 - q_i(c) - c] - t_i(c) \geq 0 \tag{IR}$$

$$c \in \arg\max_{\tilde{c} \in \mathcal{C}}\{q_i(\tilde{c})[1 - q_i(\tilde{c}) - c] - t_i(\tilde{c})\}. \tag{IC}$$

If price discrimination is banned than the manufacturer has to satisfy the additional constraint $\Gamma_1 = \Gamma_2$. It is worthwhile to notice, that for the manufacturer it is more profitable to contract with low downstream types. This implies that the usual monotonicity constraint, which is necessary in order to satisfy incentive compatibility, here requires that $q_i(c)$ and $t_i(c)$ are non-increasing. As it is well-known, with out assumptions on the type distribution the monotonicity constraint may be binding which makes the analysis by far more complicated. In this respect we impose the following restriction.

Assumption 1 *For all $c \in \mathcal{C}$ it holds that $F(c)/f(c)$, $G(c)/g(c)$, and $[F(c)+G(c)]/[f(c) + g(c)]$ are non-decreasing.*

Assumption 1 is not the usual monotone hazard rate property, because here lower types are the better types. Moreover, we need to impose an assumption on the joint distribution in order to guarantee that also under uniform pricing the monotonicity constraint is always slack. The Assumption 1 is obviously satisfied if both density functions are with weakly decreasing densities.[15] Moreover, we focus on cases where—irrespective of the pricing regime—the upstream supplier serves all types of downstream firms. The following assumption guarantees that this is the case under the optimal mechanisms.

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[15]Note that relabeling types such that higher types correspond to lower costs would allow to impose the standard monotone hazard rate property.

Assumption 2 *It is assumed that* $c_H < 1 - [\min\{f(c_H), g(c_H)\}]^{-1}$.

Let superscript D and U denote the pricing regime: price discrimination and uniform pricing, respectively. Now, we are prepared to state the first result of this section.

Lemma 4 *Given Assumptions 1 and 2. Suppose $q_i^D(c) < q_j^D(c)$ for $i, j \in \{1, 2\}$ and $i \neq j$, then $q^U(c) \in (q_i^D(c), q_j^D(c))$.*

Put verbally, one market benefits from price discrimination whereas the other market is harmed compared to uniform pricing for a given cost realization. Nevertheless, for linear demand we obtain a clear welfare result.

Proposition 5 *Given Assumptions 1 and 2. A ban on price discrimination improves total welfare, i.e., $\Delta W < 0$.*

4. DEMAND-SIDE SUBSTITUTION

In this section, we posit that the upstream supplier is no longer an unconstrained monopolist. As was recently shown by Inderst and Valletti (2009) and Caprice (2005), the implications of price discrimination in input markets for pricing decisions and welfare may be reversed if the assumption of a monopolistic input supplier is relaxed. We augment our basic model by assuming that downstream firms can purchase the essential input not only from the manufacturer but also from an alternative source. As we will show, the main effect of this outside option is to shift rents from the manufacturer to the downstream firms. As a result, by and large, our findings are robust toward relaxing the assumption of a monopolistic input supplier.

Following Katz (1987) and Inderst and Valletti (2009), we suppose that a downstream firm, when rejecting the supplier's offer, can turn to an alternative source of input supply. How profitable this switch to the alternative supply is for a particular downstream firm depends on its efficiency in production. If a firm with marginal cost $c \in \{c_L, c_H\}$ acquires its input from the alternative supply, then its profits are $\pi^A(c)$ with $0 \leq \pi^A(c_H) < \pi^A(c_L)$.[16,17]

For notational convenience, let $\pi_H^A := \pi^A(c_H)$ and $\pi_L^A := \pi^A(c_L)$. In the following, we assume that the alternative source of input supply is not too attractive in the sense that the joint surplus generated by M and either type of downstream firm exceeds that downstream firm's profit obtained under the alternative supply.

Assumption 3 *For all $c \in \{c_L, c_H\}$ it holds that: $\pi(q^{JS}(c), c) - Kq^{JS}(c) > \pi^A(c)$.*

We define

$$\phi := \frac{\pi_L^A - \pi_H^A}{c_H - c_L}. \tag{20}$$

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[16]One possible interpretation is that there exists a competitive fringe that produces an input good that is substitutable to the upstream firm's product. The downstream firms can acquire this fringe product at a cost per unit of $\bar{w} > 0$. To switch input suppliers comes for the downstream firms at fixed cost $F \geq 0$. With this interpretation we obtain $\pi^A(c_i) := \max\{0, \max_q[P(q) - c - \bar{w}]q - F\}$

[17]Here, the manufacturer faces a screening problem with a type dependent outside option. This class of problems is thoroughly analyzed, for instance, by Jullien (1996, 2000).

The term ϕ declares how much more a low-cost firm benefits from the alternative input supply than a high-cost firm, relative to the low-cost firm's cost advantage. The case analyzed in Section 3 then corresponds to a special case of the situation where the outside option is equally attractive for both types, i.e., where $\phi = 0$. In order to stick close to our basic model without alternative supply, in all that follows we keep c_L and c_H fixed and assume that any variation in ϕ arises due to changes in π_L^A or π_H^A. For reasons of tractability, we assume that the outside option is not superior, in the sense that under the optimal contract it is never the upward incentive constraint that is binding.[18] Formally, we impose the following assumption:

Assumption 4 $\phi \leq q^{JS}(c_L)$.

Otherwise the model is the same as before. In particular, the upstream firm can still make take-it-or-leave-it offers to downstream firms. Hence, M still maximizes the same objective function subject to the usual incentive compatibility constraints. The individual rationality constraints, however, are not the same as before due to the existence of an alternative source of supply. The individual rationality constraint M has to satisfy in order to contract with the low-cost or the high-cost type, respectively, is:

$$\pi(q_L, c_L) - t_L \geq \pi_L^A , \tag{IR_L^A}$$

$$\pi(q_H, c_H) - t_H \geq \pi_H^A . \tag{IR_H^A}$$

Clearly, if M prefers to serve only one cost type of one or both downstream firms, then welfare results require a specification of the alternative input supply. Therefore, we first identify the optimal contracts under the hypothesis that M serves both downstream firms which allows to draw welfare implications irrespective of the particular form the alternative source of supply takes. Thereafter, we determine under what circumstances M indeed prefers to serve both cost types of downstream firms. At this point, we just provide an outline of the analysis of this case; a more detailed account is found in the Appendix. In order to state the discussion as concise a possible, define α^r, with $r \in \{D, U\}$ denoting the pricing regime, as follows: $\alpha^D = \alpha_i$ for $i \in \{1, 2\}$ under a discriminatory pricing regime, and $\alpha^U = \alpha_\Sigma$ under a uniform pricing regime.[19]

4.1. Serving both cost types of downstream firms

Consider pricing regime $r \in \{D, U\}$, and suppose the upstream supplier M serves both types of downstream firms. Since the incentive constraints are the same as before, the optimal wholesale mechanism still needs to satisfy the monotonicity constraint (MON). Moreover, $t_L \geq t_H$. Even though the problem is similar as before, the usual procedure, i.e., satisfying (IR_H^A) and (IC_L) with equality, possibly is not feasible. This is due to the fact that $\pi_L^A \neq \pi_H^A$. Nevertheless, the following observation is immediate: In the optimum at least one constraint regarding each cost type is binding. If both constraints

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[18]Cf. Tirole (1988, p.154.)

[19]Under price discrimination the manufacturer solves two independent maximization problems; one for each downstream firm $i \in \{1, 2\}$. With a slight abuse of notation, we suppress the subscript i for the discriminatory pricing regime.

regarding one type were slack, then M could increase profits by increasing that type's transfer until one of those constraints bind—thereby relaxing the incentive constraint while not affecting the participation constraint of the other type. Moreover, given monotonicity is satisfied, if one incentive constraint holds with strict equality, then the other incentive constraint also holds.

The following observation will turn out to be quite useful: Suppose the bundle (q_H, t_H) makes the high-cost type just indifferent between picking this bundle or switching to the alternative source of supply, i.e., (IR_H^A) just binds. Next to opting for bundle (q_L, t_L), the low-cost firms has two alternative options: first, switching to the outside option, or secondly, picking the bundle (q_H, t_H) designed for the high-cost type. Compared to the profits of a high-cost firm resulting from these actions, a low-cost firm's profit is higher by $\pi_L^A - \pi_H^A > 0$ in the first case, and $q_H(c_H - c_L) > 0$ in the latter. Since the high-cost type (by hypothesis) is indifferent between these two courses of action, the low-cost firm strictly prefers choosing bundle (q_H, t_H) over switching to the alternative supply whenever $\pi_L^A - \pi_H^A < q_H(c_H - c_L)$, or, equivalently, $q_H > \phi$. Put differently, if $q_H > \phi$, then—with regard to the low-cost type—the incentive compatibility constraint is more pressing than the participation constraint. An analogous argument establishes that the high-cost type's individual rationality constraint is relevant rather than the incentive compatibility constraint whenever $q_L < \phi$.

Suppose that in the optimum the only constraints that have bite are (IR_H^A) and (IC_L). Solving these binding constraints for the transfers t_L^r and t_H^r reveals that, except for being shifted downward due to the fact that there now is a relatively attractive outside option for both cost types of downstream firms, the transfers are the same as in the standard case without alternative supply. In consequence, the allocation implemented is exactly the same as in Section 3: $q_L^r(\alpha^r) = q^{JS}(c_L)$, and $q_H^r(\alpha^r) = \hat{q}^r(\alpha^r)$ for $\alpha^r \leq \hat{\alpha}^r$ while zero otherwise. With (MON) and (IC_R) being satisfied, it remains to check whether (IR_L^A) is also met under this contractual arrangement. According to the above preliminary observation, this is the case whenever $q_H^r(\alpha^r) \geq \phi$. Remember that $\hat{q}^r(\alpha^r) \in [0, q^{JS}(c_H)]$ with $d\hat{q}^r(\alpha^r)/d\alpha^r < 0$ for $\alpha^r \in [0, \hat{\alpha}^r]$. Therefore, ($\text{IR}_L^A$) is satisfied as long as $\phi \leq q^{JS}(c_H)$ and $\alpha \leq \alpha^r(\phi) \in [0, \hat{\alpha}^r]$, where $\alpha^r(\phi)$ is implicitly defined by

$$q_H^r(\alpha^r(\phi)) \equiv \phi. \tag{21}$$

Next, suppose that under the optimal contract only the individual rationality constraints, (IR_H^A) and (IR_L^A), are binding. With both individual rationality constraints binding, the upstream supplier extracts all the surplus and therefore maximizes the joint surplus, irrespectively of whether discriminatory offers are feasible or not. Hence, the quantities implemented are $q_L^r(\alpha^r) = q^{JS}(c_L)$ and $q_H^r(\alpha^r) = q^{JS}(c_H)$. This allocation clearly satisfies the monotonicity constraint (MON). Regarding incentive compatibility under this contractual arrangement, based on the above preliminary observation, (IC_H) is satisfied due to Assumption 4, whereas (IC_L) is met only if $\phi \geq q^{JS}(c_H)$.

Last, for the "intermediate case" of $\phi \leq q^{JS}(c_H)$ and $\alpha > \alpha^r(\phi)$, both individual rationality constraints as well as the low-cost types incentive compatibility constraint turn

out to be binding under the optimal contractual arrangement. While the no-distortion-at-the-top result prevails, i.e., $q_L^r(\alpha^r) = q^{JS}(c_L)$, ($\text{IR}_L^A$) and ($\text{IC}_L$) being equally restrictive yields $q_H^r(\alpha^r) = \phi$ in accordance with our preliminary observation.

The above observations are summarized in the following proposition. Figure 3 illustrates these observations for the discriminatory pricing regime.

Proposition 6 *Suppose Assumptions 3 and 4 hold and that the upstream firm serves both types of downstream firms. The optimal wholesale mechanism under pricing regime $r \in \{D, U\}$ allocates quantities*

(i) $q_L^r(\alpha^r) = q^{JS}(c_L)$ and $q_H^r(\alpha^r) = \hat{q}^r(\alpha^r)$ if $\phi \le q^{JS}(c_H)$ and $\alpha^r \le \alpha^r(\phi)$;

(ii) $q_L^r(\alpha^r) = q^{JS}(c_L)$ and $q_H^r(\alpha^r) = \phi$ if $\phi \le q^{JS}(c_H)$ and $\alpha^r \ge \alpha^r(\phi)$;

(iii) $q_L^r(\alpha^r) = q^{JS}(c_L)$ and $q_H^r(\alpha^r) = q^{JS}(c_H)$ if $q^{JS}(c_H) \le \phi \le q^{JS}(c_L)$.
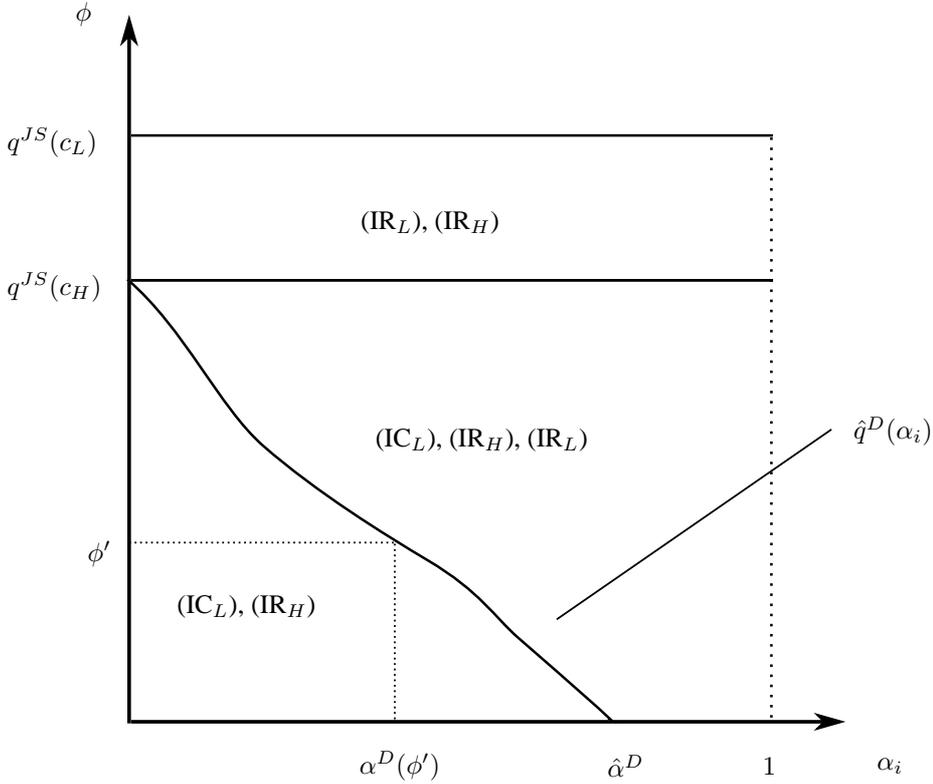
Figure 3: Binding constraints when M serves both types.

4.2. Serving only one type of downstream firm

It remains to investigate what happens if it is profitable for M to serve only one type of downstream firm. Unless stated otherwise, the following observations apply to both pricing regimes.

Clearly, when serving only one type of downstream firm with cost c, the highest possible profit M could hope for would be achieved by offering the joint-surplus-maximizing

quantity $q^{JS}(c)$ and charging a transfer that just ensures participation by that type, $t = \pi(q^{JS}(c), c) - \pi^A(c)$. This observation has two immediate implication. First, for $\phi \in [q^{JS}(c_H), q^{JS}(c_L)]$ it never pays off for M to serve only one type of downstream firm because, according to Proposition 6 (iii), under the optimal contract that serves both cost types each type is offered the respective joint-surplus-maximizing quantity and—with both participation constraints binding—M extracts all the surplus. A second implication is that even for $\phi < q^{JS}(c_H)$ it can never be optimal for M to exclude the low-cost type because this type does not reject the bundle $(q^{JS}(c_H), \pi(q^{JS}(c_H), c_H) - \pi_H^A)$, which makes the high-cost type just break even. Thus, for $\phi < q^{JS}(c_H)$ the upstream supplier will always benefit from serving both types of downstream firms instead of designing a contract that excludes the low-cost type.

The remaining question is whether M might benefit from excluding the high-cost type when $\phi \leq q^{JS}(c_H)$. Given Assumption 4, a high-cost firm always rejects the bundle $(q^{JS}(c_L), \pi(q^{JS}(c_L), c_L) - \pi_L^A)$. Hence, M's profits under pricing regime $r \in \{D, U\}$ from serving only type L are given by

$$\Pi_L^r = \alpha^r \left[\pi(q^{JS}(c_L), c_L) - \pi_L^A - K q^{JS}(c_L) \right] . \tag{22}$$

If, on the other hand, M serves both types of downstream firms, we know that both (IC$_L$) and (IR$_H$) are binding under both pricing regimes for $\phi \leq q^{JS}(c_H)$. With transfers being pinned down by these constraints, the quantities offered correspond to $q_L^r(\alpha^r) = q^{JS}(c_L)$ and $q_H^r(\alpha^r)$ as identified in Proposition 6. Thus, M's profits from serving both types of downstream firms under pricing regime r is

$$
\begin{aligned}
\Pi_{LH}^r = \alpha^r &\left\{ \pi(q^{JS}(c_L), c_L) - (c_H - c_L) q_H^r(\alpha^r) - \pi_H^A - K q^{JS}(c_L) \right\} \\
&+ \delta^r \left\{ \pi(q_H^r(\alpha^r), c_H) - \pi_H^A - K q_H^r(\alpha^r) \right\}. \tag{23}
\end{aligned}
$$

Comparison of (22) and (23) reveals that M prefers to serve only the low-cost type under pricing regime $r \in \{D, U\}$ if

$$\alpha^r (c_H - c_L)(q_H^r(\alpha^r) - \phi) > \delta^r \left[\pi(q_H^r(\alpha^r), c_H) - \pi_H^A - K q_H^r(\alpha^r) \right] . \tag{24}$$

Since $\pi(q_H, c_H) - K q_H$ is strictly increasing in q_H on $[0, q^{JS}(c_H))$, under Assumption 3 there exists a unique quantity between 0 and $q^{JS}(c_H)$ at which the right-hand side (RHS) of (24) equals zero. Let this quantity-threshold be denoted by $\tilde{\phi}$. Formally, $\tilde{\phi}$ is implicitly defined by

$$\pi(\tilde{\phi}, c_H) - \pi_H^A - K \tilde{\phi} \equiv 0. \tag{25}$$

In the Appendix, we show that for $\phi \in [\tilde{\phi}, q^{JS}(c_H)]$ it never pays off for M to exclude the high-cost downstream firm. With ϕ being relatively large, a low-cost downstream firm benefits by far more from procuring the input from the fringe than a high-cost downstream firm. Thus, the rents the manufacturer can extract when contracting with a low-cost type are relatively low. This in turn implies that cutting back on information rents paid to a low-cost type is less important but contracting with a high-cost type is not that unimportant. Hence, it is optimal always to contract with a high-cost downstream firm.

For $\phi \in [0, \tilde{\phi})$, on the other hand, we are closer to the standard case without a fringe supply. While M serves both types of downstream firms when the probability of facing a high-cost type is high, once α^r exceeds a certain threshold, M considers it profitable to serve only the low-cost type. To characterize this threshold formally, fix some $\phi \in [0, \tilde{\phi})$ and consider values of $\alpha^r \in (0, \tilde{\alpha}^r]$, where $\tilde{\alpha}^r$ is implicitly defined by $\hat{q}^r(\tilde{\alpha}^r) = \tilde{\phi}$. Application of the envelope theorem yields

$$\frac{d(\Pi_L^r - \Pi_{LH}^r)}{d\alpha^r}$$
$$= (c_H - c_L)(q_H^r(\alpha^r) - \phi) + \left[\pi(q_H^r(\alpha^r), c_H) - \pi_H^A - Kq_H^r(\alpha^r)\right] > 0, \quad (26)$$

where the inequality follows from the definition of $\tilde{\phi}$ in (25) and $\hat{q}^r(\alpha^r) \geq \tilde{\phi}$ for $\alpha^r \in (0, \tilde{\alpha}^r]$. Since $\Pi_L - \Pi_{LH}^r|_{\alpha^r=0} < 0$ and $\Pi_L - \Pi_{LH}^r|_{\alpha^r=\tilde{\alpha}^r} > 0$, by the intermediate value theorem we know that for any $\phi \in [0, \tilde{\phi})$ there exists a unique value $\tilde{\alpha}^r(\phi) \in (0, \tilde{\alpha}^r)$ such that

$$\Pi_L^r - \Pi_{LH}^r|_{\alpha^r=\alpha^r(\phi)} \equiv 0, \quad (27)$$

which yields the desired characterization of the threshold.

We summarize the observations of this subsection in the following lemma, which is illustrated for a discriminatory pricing regime in Figure 4. In the light-gray shaded area both types of downstream firms are served, whereas in the dark-gray shaded area the high-cost type is excluded.[20]

Lemma 5 *Suppose Assumptions 3 and 4 hold. Under either pricing regime, the low-cost type is never excluded. Under pricing regime $r \in \{D, U\}$, the upstream supplier does not exclude the high-cost type if (i) $\phi \in [\tilde{\phi}, q^{JS}(c_H)]$, or (ii) $\phi \in [0, \tilde{\phi})$ and $\alpha^r \leq \tilde{\alpha}^r(\phi)$.*

Note that the upstream firm's motive for not serving the high-cost type changes as α^r increases: For α^r only slightly above the threshold $\alpha^r(\phi)$ the (IR_L^A) constraint is slack under the optimal contract when serving both firms, so M's incentive for excluding the high-cost type is rooted in the desire to cut back on the information rent paid to the low-cost type. For relatively high values of α^r, on the other hand, (IR_L^A) is binding under the optimal contract when serving both firms; here, exclusion of the high-cost type is rooted in M's desire to avoid making losses from serving this type.

4.3. Welfare under Demand-Side Substitution

What are the welfare effects of banning price discrimination in the presence of an alternative source of input supply? In the light of the observations obtained in the previous sections, we distinguish three cases: (a) $\phi \in [q^{JS}(c_H), q^{JS}(c_L)]$, (b) $\phi \in [\tilde{\phi}, q^{JS}(c_H)]$, and (c) $\phi \in [0, \tilde{\phi})$.

The welfare implications of banning price discrimination in case (a) are trivial: The quantities offered are the same under both pricing regimes, which implies $\Delta W = 0$.

---

[20]As becomes obvious from (24), the threshold $\tilde{\alpha}^r(\phi)$ depends on both π_L^A and π_H^A. In order to depict the locus of this threshold in the (α^r, ϕ)-space, in Figure 4 it is implicitly assumed that variations in ϕ are due to changes of either π_L^A or π_H^A.
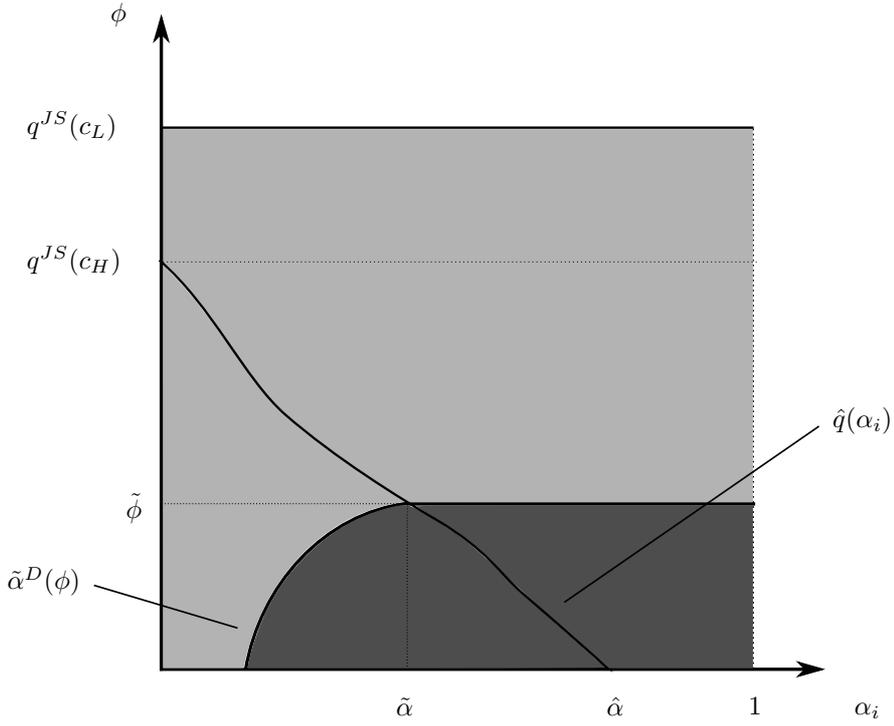
Figure 4: M's decision which types to serve

Next consider case (b). Here, while M still prefers to serve both types of each downstream firm under either regime and offers $q^{JS}(c_L)$ to any low-cost downstream firm, the quantity offered to a high-cost downstream firm depends on both the pricing regime and its ex ante efficiency. More precisely, under price discrimination firm i, when producing at high cost, is offered quantity $q_H^D(\alpha_i) = \hat{q}^D(\alpha_i)$ if $\alpha_i \leq \alpha^D(\phi)$ and quantity $q_H^D(\alpha_i) = \phi$ otherwise. Under uniform pricing M offers $q_H^U(\alpha_\Sigma) = \hat{q}^U(\alpha_\Sigma)$ if $\alpha_\Sigma \leq \alpha^U(\phi)$, or equivalently

$$\alpha_1 \leq \alpha^U(\phi) - \alpha_2 =: \alpha_1^U(\alpha_2; \phi), \tag{28}$$

and $q_H^U(\alpha_\Sigma) = \phi$ otherwise. Note that $\alpha_1^U(\alpha^D(\phi); \phi) = \alpha^D(\phi)$. This gives rise to four cases similar to the four cases depicted in Figure 1. Since in case IV(2) the quantities offered by M are identical under both pricing regimes, here we obviously have $\Delta W = 0$. The welfare implications for the remaining cases parallel those drawn in the standard model without an alternative source of input supply.

Proposition 7 *Suppose $\phi \in [\tilde{\phi}, q^{JS}(c_H)]$. (i) If $\alpha_2 < \alpha^D(\phi) \leq \alpha_1^U(\alpha_2; \phi) \leq \alpha_1$, then $\Delta W > 0$. (ii) If $\alpha_2 < \alpha^D(\phi) < \alpha_1 < \alpha_1^U(\alpha_2; \phi)$, then ΔW is strictly decreasing in α_1. (iii) If $\alpha_2 < \alpha_1 \leq \alpha^D(\phi)$, then $\Delta W < 0$ for $P(q) = 1 - q$.*

The intuition behind the welfare result of Proposition 7 is basically the same as the one behind Proposition 3. Due to the outside option rents are shifted from the manufacturer to the downstream firms, but this shift does not affect total welfare as long as no downstream firm acquires its input from the fringe supply. Proposition 7 shows that

our previous findings are robust toward relaxing the assumption of an unconstrained manufacturer. In particular, if the potential differences in retail costs are low, then a ban on price discrimination improves welfare at least for linear demand: Here we have $\alpha^D(\phi) = [q^{JS}(c_H) - \phi]/[q^{JS}(c_L) - \phi]$, which approaches 1 if c_H tends to c_L.

Regarding a case-based approach of banning discriminatory wholesale contracts, Proposition 7 provides a justification for the competition policy authority to ban discriminatory wholesale contracts if concerned with a primary-line injury case. Put differently, if a competitor of a dominant manufacturer complains before the competition policy agency that the dominant manufacturer uses discriminatory wholesale tariffs, then the agency often is well advised to condemn this pricing practice. Banning discriminatory wholesale contracts is advisable, however, not to protect competitors of the dominant manufacturer but to protect consumers.

Last, consider case (c). Without further specification of the alternative source of input supply there are only few cases that allow to draw out the welfare implications of banning price discrimination. Remember that under price discrimination the high-cost type of firm i, is offered quantity $q_H^D(\alpha_i) = \hat{q}^D(\alpha_i)$ if $\alpha_i \leq \tilde{\alpha}^D(\phi)$ and is not served otherwise. Under uniform pricing M offers $q_H^U(\alpha_\Sigma) = \hat{q}^U(\alpha_\Sigma)$ if $\alpha_\Sigma \leq \tilde{\alpha}^U(\phi)$, or equivalently

$$\alpha_1 \leq \tilde{\alpha}^U(\phi) - \alpha_2 =: \tilde{\alpha}_1^U(\alpha_2; \phi), \tag{29}$$

but does not serve the high-cost downstream type otherwise. Note that $\alpha_1^U(\tilde{\alpha}^D(\phi); \phi) = \tilde{\alpha}^D(\phi)$. Thus, if $\alpha_2 > \tilde{\alpha}^D(\phi)$, then M does not serve high-cost downstream firms irrespective of the pricing regime. In this case, $\Delta W = 0$. On the other hand, if $\alpha_1 \leq \tilde{\alpha}^D(\phi)$, then M serves high-cost downstream firms irrespective of both the pricing regime and ex ante efficiency, offering $q_H^r(\alpha^r) = \hat{q}^r(\alpha^r)$. In this case, from Proposition 4 we know that $\Delta W < 0$ at the least for linear demand.

In summary, even with an alternative source of input supply, for a broad range of parameter values a ban on discrimination improves welfare.

5. Conclusion

In this paper, we analyze a vertically related industry with asymmetric information between the upstream and the downstream sector. The main purpose is to inquire into the welfare effects of banning price discrimination in intermediate-good markets when nonlinear pricing schemes are feasible. This question is of immediate practical interest because from a legal perspective, quantity discounts per se are commonly regarded as a justifiable pricing strategy of manufacturers or wholesale firms as long as they are not discriminatory in the sense of applying different conditions to identical transactions with other trading partners.

While there has been considerable back and forth in the academic literature regarding the question whether banning price discrimination in intermediate-good markets constitutes a desirable course of policy when wholesale prices are linear, among the few exceptions which consider nonlinear wholesale pricing schemes the predominant opinion is that banning discriminatory wholesale pricing is detrimental for welfare. In contrast

to these findings, which raise "[...] serious concerns about the efficacy of the Robinson-Patman Act" (O'Brien and Schaffer, 1994, p.314) or its analogue in EU competition law, we find that, when downstream firms have private information, the reservation toward discriminatory pricing practices embodied in these legal enactments may well be warranted even if nonlinear pricing schemes are feasible. This result holds irrespective of whether the upstream firm is a monopolistic supplier of the input or has to compete with an alternative source of input supply.

We restrict attention to downstream firms operating in separate markets in order to isolate the effect of discriminatory wholesale tariffs in the case of asymmetric information from potential competitive effects. Moreover, modeling downstream competition would raise the following concerns: Regarding the information structure, does each downstream firm know its competitor's cost type or only its own? In the former case, the upstream firm can use a mechanism that severely punishes both downstream firms if their reports regarding their own their competitor's cost types do not match, thereby revealing the downstream firms' private information without cost. With this type of mechanism being feasible under both pricing regimes, there is no scope for analyzing the welfare effects of banning price discrimination. If, on the other hand, downstream firms know only their own cost types, then the quantity offered to a downstream firm may nevertheless depend on both downstream firms' reports. Under price discrimination, for example, the upstream firm now has eight quantities and eight transfers to specify which raises analytical complexity. Besides being by far less tractable, contracts with a firm's transfer depending on quantities procured by both firms seem hard to reconcile with observed practice. But even constricting contracts such that a firm's transfers and quantities depend only on its own type does not circumvent the question whether a firm know's its competitor's type before or after accepting the upstream firm's offer, i.e., whether ex ante or ex post participation constraints matter. Regarding all these issues, an investigation of the welfare effects of banning price discrimination with downstream competition seems beyond the scope of this paper and is left for future research.

A. PROOFS OF PROPOSITIONS AND LEMMAS

Proof of Lemma 1:
First, suppose $q' < q'' < q^*(c_H)$. Then

$$
\begin{aligned}
&\pi(q'', c_L) - \pi(q', c_L) > \pi(q'', c_H) - \pi(q', c_H) \\
&\iff [P(q'') - c_L]q'' - [P(q') - c_L]q' > [P(q'') - c_H]q'' - [P(q') - c_H]q' \\
&\iff (c_H - c_L)(q'' - q') > 0 \\
&\iff q' < q''.
\end{aligned}
\tag{A.1}
$$

Next, suppose $q' < q^*(c_H) \leq q'' \leq q^*(c_L)$. Then

$$\pi(q'', c_L) - \pi(q', c_L) > \pi(q'', c_H) - \pi(q', c_H) = \pi(q^*(c_H), c_H) - \pi(q', c_H)$$

$$\iff \quad [P(q'') - c_L]q'' - [P(q') - c_L]q' > [P(q^*(c_H)) - c_H]q^*(c_H) - [P(q') - c_H]q'$$

$$\iff [P(q'') - c_L]q'' - [P(q^*(c_H)) - c_L]q^*(c_H)$$
$$+ [P(q^*(c_H)) - c_L]q^*(c_H) - [P(q') - c_L]q'$$
$$> [P(q^*(c_H)) - c_H]q^*(c_H) - [P(q') - c_H]q'$$

$$\iff \quad \pi(q'', c_L) - \pi(q^*(c_H), c_L) + (c_H - c_L)(q^*(c_H) - q') > 0.$$

(A.2)

where the last inequality holds by $q' < q^*(c_H) \leq q'' \leq q^*(c_L)$ and $\pi(q, c_L)$ being strictly increasing in q on $q \in [0, q^*(c_L))$.

Last, suppose $q^*(c_H) \leq q' < q'' \leq q^*(c_L)$. Then

$$\pi(q'', c_L) - \pi(q', c_L) > \pi(q'', c_H) - \pi(q', c_H)$$
$$= \pi(q^*(c_H), c_H) - \pi(q^*(c_H), c_H) \qquad \text{(A.3)}$$
$$= 0$$

holds since $\pi(q, c_L)$ is strictly increasing in q on $q \in [0, q^*(c_L))$. ∎

Proof of Lemma 3:

Inspection of the first-order conditions (10) and (14), together with the definition of $q^{JS}(c)$ and the fact that

$$\alpha_2 < \alpha_1 \implies \frac{\alpha_2}{1 - \alpha_2} < \frac{\alpha_1 + \alpha_2}{2 - \alpha_1 - \alpha_2} < \frac{\alpha_1}{1 - \alpha_1}, \qquad \text{(A.4)}$$

immediately implies $\hat{q}^D(\alpha_1) < \hat{q}^U(\alpha_1 + \alpha_2) < \hat{q}^D(\alpha_2) < q^{JS}(c_H)$. The desired statement then follows from Propositions 1 and 2. ∎

Proof of Proposition 3:

We prove each part of the proposition in turn. To cut back on notation, we will make use of the following notation: $q^D_{Hi} := q^D_H(\alpha_i)$ for $i \in \{1, 2\}$, and $q^U_H := q^U_H(\alpha_1 + \alpha_2)$.

(i) With $\alpha_2 < \hat{\alpha}^D \leq \hat{\alpha}^U_1(\alpha_2) \leq \alpha_1$, i.e., in case (III), we have $q^D_{H1} = q^U_H = 0 < q^D_{H2} = \hat{q}^D(\alpha_2)$. According to (16), the difference in expected welfare under the two pricing regimes is

$$\Delta W = (1 - \alpha_2) \left[\int_0^{q^D_{H2}} P(z)dz - (c_H + K)q^D_{H2} \right]. \qquad \text{(A.5)}$$

The desired result then follows from the first-order condition (10) together with $P'(\cdot) < 0$ whenever $P(\cdot) > 0$:

$$P(q^D_{H2}) - (c_H + K) = -P'(q^D_{H2})q^D_{H2} + \frac{\alpha}{1 - \alpha}(c_H - c_L) > 0$$
$$\implies \left[P(q^D_{H2}) - (c_H + K) \right] q^D_{H2} > 0 \qquad \text{(A.6)}$$
$$\implies \int_0^{q^D_{H2}} P(z)dz - (c_H + K)q^D_{H2} > 0$$

(ii) With $\alpha_2 < \hat{\alpha}^D < \alpha_1 < \hat{\alpha}_1^U(\alpha_2)$, i.e., in case (II), we have $q_{H1}^D = 0 < q_H^U = \hat{q}^U(\alpha_1 + \alpha_2) < q_{H2}^D = \hat{q}^D(\alpha_2)$. According to (16),

$$
\begin{aligned}
\frac{d\Delta W}{d\alpha_1} = &-\left[\int_{q_H^U}^{0} P(z)dz + (c_H + K)q_H^U\right] \\
&+ (1 - \alpha_1)\left[\frac{dq_{H1}^D}{d\alpha_1}P(0) - \frac{dq_H^U}{d\alpha_1}P(q_H^U) - (c_H + K)\left(\frac{dq_{H1}^D}{d\alpha_1} - \frac{dq_H^U}{d\alpha_1}\right)\right] \\
&+ (1 - \alpha_2)\left[-\frac{dq_H^U}{d\alpha_1}P(q_H^U) + (c_H + K)\frac{dq_H^U}{d\alpha_1}\right]
\end{aligned}
$$
(A.7)

First, note that $\frac{dq_{H1}^D}{d\alpha_1} = 0$. Next, remember that $q_H^U = \hat{q}^U(\alpha_1 + \alpha_2)$ is defined by (14). This implies that $\frac{dq_H^U}{d\alpha_1} < 0$. Moreover, with $P'(\cdot) < 0$ whenever $P(\cdot) > 0$, from (14) it follows that

$$
P(q_H^U) - (c_H + K) = -P'(q_H^U)q_H^U + \frac{\alpha_1 + \alpha_2}{2 - (\alpha_1 + \alpha_2)}(c_H - c_L) > 0
$$
$$
\implies \int_0^{q_H^U} P(z)dz - (c_H + K) > 0. \quad (A.8)
$$

Taken together, these observations allow us to conclude that

$$
\begin{aligned}
\frac{d\Delta W}{d\alpha_1} = &\left[\int_0^{q_H^U} P(z)dz - (c_H + K)q_H^U\right] \\
&- (2 - (\alpha_1 + \alpha_2))\frac{dq_H^U}{d\alpha_1}\left[P(q_H^U) - (c_H + K)\right] > 0 \quad (A.9)
\end{aligned}
$$

which establishes the desired result. ∎

Proof of Proposition 4:
With ΔW being given by (17), we consider in turn each of the three relevant cases identified in the main text : (I) $\alpha_2 < \alpha_1 < \hat{\alpha}^D$; (II) $\alpha_2 < \hat{\alpha}^D \leq \alpha_1 < \hat{\alpha}_1^U(\alpha_2)$; and (III) $\alpha_2 < \hat{\alpha}^D < \hat{\alpha}_1^U(\alpha_2) \leq \alpha_1$. To cut back on notation, we will make use of the following notation: $q_{Hi}^D := q_H^D(\alpha_i)$ for $i \in \{1, 2\}$, and $q_H^U := q_H^U(\alpha_\Sigma)$, $q_H^{JS} := q^{JS}(c_H)$, and $\Delta_c : c_H - c_L$.

(I) With $\alpha_2 < \alpha_1 < \hat{\alpha}^D$ we have $q_{Hi}^D = q_H^{JS} - \frac{\alpha}{1-\alpha}\frac{\Delta_c}{2}$ and $q_H^U = q_H^{JS} - \frac{\alpha_1 + \alpha_2}{2 - (\alpha_1 + \alpha_2)}\frac{\Delta_c}{2}$. It follows that

$$
q_{Hi}^D - q_H^U = \left[\frac{\alpha_i + \alpha_j}{2 - (\alpha_i + \alpha_j)} - \frac{\alpha_i}{1 - \alpha_i}\right]\frac{\Delta_c}{2} = \frac{\alpha_j - \alpha_i}{[2 - (\alpha_i + \alpha_j)](1 - \alpha_i)}\frac{\Delta_c}{2} \quad (A.10)
$$

and

$$
\begin{aligned}
q_{Hi}^D + q_H^U &= 2q_H^{JS} - \left[\frac{\alpha_i + \alpha_j}{2 - (\alpha_i + \alpha_j)} + \frac{\alpha_i}{1 - \alpha_i}\right]\frac{\Delta_c}{2} \\
&= 2q_H^{JS} - \left[\frac{\alpha_j - \alpha_i + 2\alpha_i[2 - (\alpha_i + \alpha_j)]}{[2 - (\alpha_i + \alpha_j)](1 - \alpha_i)}\right]\frac{\Delta_c}{2} \quad (A.11)
\end{aligned}
$$

First, note that

$$\sum_{i=1}^{2}(1-\alpha_i)(q_H^D(\alpha_i)-q_H^U(\Sigma))\left[-(c_H+K)\right]$$

$$=-(c_H+K)\left\{(1-\alpha_1)\frac{\alpha_2-\alpha_1}{[2-(\alpha_1+\alpha_2)](1-\alpha_1)}\frac{\Delta_c}{2}+(1-\alpha_2)\frac{\alpha_1-\alpha_2}{[2-(\alpha_1+\alpha_2)](1-\alpha_2)}\frac{\Delta_c}{2}\right\}$$

$$=-(c_H+K)\left\{\frac{\alpha_2-\alpha_1}{[2-(\alpha_1+\alpha_2)]}\frac{\Delta_c}{2}+\frac{\alpha_1-\alpha_2}{[2-(\alpha_1+\alpha_2)]}\frac{\Delta_c}{2}\right\}$$

$$=0,$$

such that

$$\Delta W=\Delta CS=\sum_{i=1}^{2}(1-\alpha_i)(q_H^D(\alpha_i)-q_H^U(\Sigma))\left[1-\frac{q_H^D(\alpha_i)+q_H^U(\Sigma)}{2}\right],\quad\text{(A.12)}$$

After substituting (A.10) and (A.11), some further manipulation of this expresion yields

$$\Delta W=\Delta CS=-\frac{(\alpha_1-\alpha_2)^2(\Delta_c)^2}{8(2-\alpha_\Sigma)(1-\alpha_1)(1-\alpha_2)},\quad\text{(A.13)}$$

which obviously is strictly negative.

(II) With $\alpha_2<\hat{\alpha}^D\le\alpha_1<\hat{\alpha}_1^U(\alpha_2)$, we have $q_{H1}^D=0$, $q_{H2}^D=q_H^{JS}-\frac{\alpha_2}{1-\alpha_2}\frac{\Delta_c}{2}$ and $q_H^U=q_H^{JS}-\frac{\alpha_1+\alpha_2}{2-(\alpha_1+\alpha_2)}\frac{\Delta_c}{2}$. The difference in expected welfare thus equals

$$\Delta W=(1-\alpha_2)q_{H2}^D\left\{1-\frac{1}{2}q_{H2}^D-(c_H+K)\right\}$$

$$-(2-\alpha_\Sigma)q_H^U\left\{1-\frac{1}{2}q_H^U-(c_H+K)\right\}.\quad\text{(A.14)}$$

Let $\alpha_1^W(\alpha_2)$ be implicitly defined by

$$\Delta W(\alpha_1^W(\alpha_2),\alpha_2)\equiv 0.\quad\text{(A.15)}$$

Differentiation of A.15 with respect to α_2 reveals that

$$\frac{d\alpha_1^W(\alpha_2)}{d\alpha_2}\left[-(2-\alpha_\Sigma)\frac{dq_H^U}{d\alpha_\Sigma}\left\{1-q_H^U-(c_H+K)\right\}+q_H^U\left\{1-\frac{1}{2}q_H^U-(c_H+K)\right\}\right]$$

$$=-(1-\alpha_2)\frac{dq_{H2}^D}{d\alpha_2}(\left\{1-q_{H2}^D-(c_H+K)\right\}+q_{H2}^D\left\{1-\frac{1}{2}q_{H2}^D-(c_H+K)\right\}$$

$$+(2-\alpha_\Sigma)\frac{dq_H^U}{d\alpha_\Sigma}\left\{1-q_H^U-(c_H+K)\right\}-q_H^U\left\{1-\frac{1}{2}q_H^U-(c_H+K)\right\}\quad\text{(A.16)}$$

Substituting $q_{H2}^D=q_H^{JS}-\frac{\alpha_2}{1-\alpha_2}\frac{\Delta_c}{2}$ and $q_H^U=q_H^{JS}-\frac{\alpha_\Sigma}{2-\alpha_\Sigma}\frac{\Delta_c}{2}$ and noting that $\frac{dq_{H2}^D}{d\alpha_2}=$

$-\frac{1}{(1-\alpha_2)^2}\frac{\Delta_c}{2}$ and $\frac{dq_H^U}{d\alpha_\Sigma} = -\frac{2}{(2-\alpha_\Sigma)^2}\frac{\Delta_c}{2}$ yields

$$
\frac{d\alpha_1^W(\alpha_2)}{d\alpha_2}\left[\frac{2}{2-\alpha_\Sigma}\frac{\Delta_c}{2}\left\{q_H^{JS} + \frac{\alpha_\Sigma}{2-\alpha_\Sigma}\frac{\Delta_c}{2}\right\}\right.
$$
$$
\left. + \left\{q_H^{JS} - \frac{\alpha_\Sigma}{2-\alpha_\Sigma}\frac{\Delta_c}{2}\right\}\left\{\frac{3}{2}q_H^{JS} + \frac{1}{2}\frac{\alpha_\Sigma}{2-\alpha_\Sigma}\frac{\Delta_c}{2}\right\}\right]
$$
$$
= \left[\frac{1}{1-\alpha_2}\frac{\Delta_c}{2}(\left\{q_H^{JS} + \frac{\alpha_2}{1-\alpha_2}\frac{\Delta_c}{2}\right\}\right.
$$
$$
\left. + \left\{q_H^{JS} - \frac{\alpha_2}{1-\alpha_2}\frac{\Delta_c}{2}\right\}\left\{\frac{3}{4}q_H^{JS} + \frac{1}{2}\frac{\alpha_2}{1-\alpha_2}\frac{\Delta_c}{2}\right\}\right]
$$
$$
- \left[\frac{2}{2-\alpha_\Sigma}\frac{\Delta_c}{2}\left\{q_H^{JS} + \frac{\alpha_\Sigma}{2-\alpha_\Sigma}\frac{\Delta_c}{2}\right\}\right.
$$
$$
\left. + \left\{q_H^{JS} - \frac{\alpha_\Sigma}{2-\alpha_\Sigma}\frac{\Delta_c}{2}\right\}\left\{\frac{3}{2}q_H^{JS} + \frac{1}{2}\frac{\alpha_\Sigma}{2-\alpha_\Sigma}\frac{\Delta_c}{2}\right\}\right]
$$

$$(A.17)$$

A first important observation is that each term in square brackets is strictly positive. This immediately implies that $d\alpha_1^W(\alpha_2)/d\alpha_2 > -1$. Moreover, all the terms with q_H^{JS} on the RHS of A.17 cancel out, which allows us to rewrite A.17 as follows:

$$
\frac{d\alpha_1^W(\alpha_2)}{d\alpha_2}\left[\frac{2}{2-\alpha_\Sigma}\frac{\Delta_c}{2}\left\{q_H^{JS} + \frac{\alpha_\Sigma}{2-\alpha_\Sigma}\frac{\Delta_c}{2}\right\}\right.
$$
$$
\left. + \left\{q_H^{JS} - \frac{\alpha_\Sigma}{2-\alpha_\Sigma}\frac{\Delta_c}{2}\right\}\left\{\frac{3}{2}q_H^{JS} + \frac{1}{2}\frac{\alpha_\Sigma}{2-\alpha_\Sigma}\frac{\Delta_c}{2}\right\}\right]
$$
$$
= \left(\frac{\Delta_c}{2}\right)^2\left\{\frac{\alpha_2(2-\alpha_2)}{(1-\alpha_2)^2} - \frac{\alpha_\Sigma[4-\alpha_\Sigma]}{(2-\alpha_\Sigma)^2}\right\}
$$

$$(A.18)$$

Straightforward manipulation of the RHS yields

$$
\left(\frac{\Delta_c}{2}\right)^2\left\{\frac{\alpha_2(2-\alpha_2)}{(1-\alpha_2)^2} - \frac{\alpha_\Sigma[4-\alpha_\Sigma]}{(2-\alpha_\Sigma)^2}\right\}
$$
$$
= \frac{1}{2}\left(\frac{\Delta_c}{2}\right)^2\frac{\alpha_\Sigma^2 - 4\alpha_\Sigma + 4\alpha_2(2-\alpha_2)}{(1-\alpha_2)^2(2-\alpha_\Sigma)^2}. \quad (A.19)
$$

Since $\alpha_\Sigma^2 - 4\alpha_\Sigma + 4\alpha_2(2-\alpha_2) < 0$ if and only if $\alpha_1 \in (\alpha_2, 4-3\alpha_2)$, the RHS of (A.18) is strictly negative. Therefore, with the term in square brackets on the LHS of (A.18) being strictly positive, we must have

$$
\frac{d\alpha_1^W(\alpha_2)}{d\alpha_2} < 0. \quad (A.20)
$$

Taken together, the above observations imply

$$
\frac{d\alpha_1^W(\alpha_2)}{d\alpha_2} \in (-1, 0). \quad (A.21)
$$

Last, note that if $\alpha_1^W(\alpha_2) = \alpha_2$, then $\alpha_1^W(\alpha_2) = \alpha_2 = \hat{\alpha}^D$. To see this, note that for $\alpha_1 = \alpha_2$ we have $q_{H2}^D = q_H^U$, and in consequence

$$\Delta W = -(1-\alpha_2)q_{H2}^D \left\{ 1 - \frac{1}{2}q_{H2}^D - (c_H + K) \right\}$$

$$= -(1-\alpha_2)q_{H2}^D \left\{ \frac{3}{2}q_H^{JS} + \frac{1}{2}\frac{\alpha_2}{1-\alpha_2}\frac{\Delta_c}{2} \right\}. \quad \text{(A.22)}$$

With $q_H^{JS} > 0$, for $\Delta W = 0$ we must have $q_{H2}^D = 0$, or equivalently, $\alpha_2 = \hat{\alpha}^D$. Together with $\frac{d\alpha_1^W(\alpha_2)}{d\alpha_2} \in (-1, 0)$ this last observation implies $\alpha_1^W(\alpha_2) \in (\hat{\alpha}^D, \hat{\alpha}_1^U(\alpha_2))$. The result then follows immediately from Proposition 3 (ii).

Next, consider the change in expected consumer surplus,

$$\Delta CS := \Delta CS(\alpha_1, \alpha_2) = (1-\alpha_2)\left[q_{H2}^D - \frac{1}{2}(q_{H2}^D)^2 \right] - (2 - \alpha_\Sigma)\left[q_H^U - \frac{1}{2}(q_H^U)^2 \right] \quad \text{(A.23)}$$

Differentiation of (A.23) with respect to α_1 reveals that consumer surplus decreases as α_1 decreases,

$$\frac{d\Delta CS}{d\alpha_1} = \frac{d\alpha_\Sigma}{d\alpha_1}\left[q_H^U - \frac{1}{2}(q_H^U)^2 \right] - (2 - \alpha_\Sigma)\left[1 - q_H^U \right] \frac{dq_H^U}{d\alpha_\Sigma}\frac{d\alpha_\Sigma}{d\alpha_1}$$

$$= q_H^U\left[1 - \frac{1}{2}q_H^U \right] + \left[1 - q_H^U \right]\frac{\Delta_c}{2 - \alpha_\Sigma}$$

$$> 0$$

Let $\alpha_1^{CS}(\alpha_2)$ be defined implicitly by

$$\Delta CS(\alpha_1^{CS}(\alpha_2), \alpha_2) \equiv 0. \quad \text{(A.24)}$$

Implicit differentiation of (A.24) with respect to α_2 yields

$$\frac{d\alpha_1^{CS}(\alpha_2)}{d\alpha_2}\left[q_H^U - \frac{1}{2}(q_H^U)^2 - (2 - \alpha_\Sigma)\left[1 - q_H^U \right]\frac{dq_H^U}{d\alpha_\Sigma} \right]$$

$$= -q_H^U\left[1 - \frac{1}{2}q_H^U \right] + (2 - \alpha_\Sigma)\left[1 - q_H^U \right]\frac{dq_H^U}{d\alpha_\Sigma}$$

$$+ q_{H2}^D\left[1 - \frac{1}{2}q_{H2}^D \right] - (1 - \alpha_2)\left[1 - q_{H2}^D \right]\frac{dq_{H2}^D}{d\alpha_2}. \quad \text{(A.25)}$$

Note that the term in square brackets on the LHS of (A.25) s strictly positive, which implies that $d\alpha_1^{CS}(\alpha_2)/d\alpha_2 > -1$. Substituting $q_{H2}^D = q_H^{JS} - \frac{\alpha_2}{1-\alpha_2}\frac{\Delta_c}{2}$ and $q_H^U = q_H^{JS} - \frac{\alpha_\Sigma}{2-\alpha_\Sigma}\frac{\Delta_c}{2}$, $\frac{dq_{H2}^D}{d\alpha_2} = -\frac{1}{(1-\alpha_2)^2}\frac{\Delta_c}{2}$, and $\frac{dq_H^U}{d\alpha_\Sigma} = -\frac{2}{(2-\alpha_\Sigma)^2}\frac{\Delta_c}{2}$ allows us to rewrite the RHS of (A.25) as

$$- \left\{ q_H^{JS} - \frac{\alpha_\Sigma}{2 - \alpha_\Sigma}\frac{\Delta_c}{2} \right\}\left\{ 1 - \frac{1}{2}q_H^{JS} + \frac{1}{2}\frac{\alpha_\Sigma}{2 - \alpha_\Sigma}\frac{\Delta_c}{2} \right\}$$

$$- \left\{ 1 - q_H^{JS} + \frac{\alpha_\Sigma}{2 - \alpha_\Sigma}\frac{\Delta_c}{2} \right\}\frac{2}{2 - \alpha_\Sigma}\frac{\Delta_c}{2}$$

$$+ \left\{ q_H^{JS} - \frac{\alpha_2}{1 - \alpha_2}\frac{\Delta_c}{2} \right\}\left\{ 1 - \frac{1}{2}q_H^{JS} + \frac{1}{2}\frac{\alpha_2}{1 - \alpha_2}\frac{\Delta_c}{2} \right\}$$

$$+ \left\{ 1 - q_H^{JS} + \frac{\alpha_2}{1 - \alpha_2}\frac{\Delta_c}{2} \right\}\frac{1}{1 - \alpha_2}\frac{\Delta_c}{2}. \quad \text{(A.26)}$$

Again, all the terms containing q_H^{JS} cancel out and the RHS of (A.25) reduces to

$$\frac{1}{2}\left(\frac{\Delta_c}{2}\right)^2 \frac{\alpha_\Sigma^2 - 4\alpha_\Sigma + 4\alpha_2(2 - \alpha_2)}{(1 - \alpha_2)^2(2 - \alpha_\Sigma)^2}. \tag{A.27}$$

Note that (A.27) is identical to (A.19). Comparison of the LHSs of (A.18) and (A.25) then reveals that $0 > d\alpha_1^{CS}(\alpha_2)/d\alpha_2 > d\alpha_1^W(\alpha_2)/d\alpha_2$, and thus $\alpha_1^W(\alpha_2) > \alpha_1^{CS}(\alpha_2)$ for all $\alpha_2 < \hat{\alpha}^D$.

(III) $\Delta W < 0$ follows from Proposition 3 (i). With $q_{H1}^D = q_H^U = 0 < q_{H2}^D$, $\Delta CS < 0$ follows immediately from (18).

Taken together, the above observations establish the desired result. ∎

Proof of Proposition 6:

In order to analyze both pricing regimes in one go, we define α^r, with $r \in \{D, U\}$ denoting the pricing regime, as denoted in the text: $\alpha^D = \alpha_i$ for $i \in \{1, 2\}$ under a discriminatory pricing regime, and $\alpha^U = \alpha_\Sigma$ under a uniform pricing regime.[21] The upstream supplier's maximizes its objective function

$$\Pi = \alpha^r[t_L - kq_L] + \delta^r[t_H - kq_H] \tag{A.28}$$

subject to (IR$_H^A$), (IC$_H$), (IR$_H^L$), and (IC$_L$). If discriminatory offers are allowed, then $\delta^D = 1 - \alpha_i$ with regard to downstream firm $i \in \{1, 2\}$. Under uniform wholesale tariffs, we have $\delta^U = 2 - \alpha_\Sigma$.

First, consider the relaxed optimization problem where, under pricing regime $r \in \{D, U\}$, M maximizes the above objective (A.28) subject only to (IR$_H^A$) and (IC$_L$). Obviously, for a given allocation (q_L, q_H), in the optimum transfers are chosen to make both constrains bind:

$$
\begin{aligned}
t_H^r &= \pi(q_H, c_H) - \pi_H^A, \\
t_L^r &= \pi(q_L, c_L) - \pi(q_H, c_L) + \pi(q_H, c_H) - \pi_H^A.
\end{aligned}
$$

Except for being shifted downward by the amount π_H^A, the transfers are the same as in the standard case without alternative supply. In consequence, the optimal allocation is the same as in Section 3: $q_L^r(\alpha^r) = q^{JS}(c_L)$, and $q_H^r(\alpha^r) = \hat{q}^r(\alpha^r)$ for $\alpha^r \leq \hat{\alpha}^r$ and zero otherwise. With the allocation satisfying the monotonicity constraint (MON), (IC$_H$) is satisfied trivially because (IC$_L$) holds with equality. Thus, this allocation and the associated transfers solve M's original problem as long as the (IR$_L^A$) constraint is satisfied, or, equivalently, as long as

$$\pi(q^{JS}(c_L), c_L) - t_L^r \geq \pi_L^A \iff \phi \leq q_H^r(\alpha^r). \tag{A.29}$$

Recall that $\hat{q}^r(\alpha^r)$ is a strictly decreasing function with $\hat{q}^r(0) = q^{JS}(c_H)$ and $\hat{q}^r(\hat{\alpha}^r) = 0$. In consequence, (IR$_L^A$) holds under the above wholesale mechanism if $\phi \leq q^{JS}(c_H)$ and $\alpha^r \leq \alpha^r(\phi) \in [0, \hat{\alpha}^r]$, where $\alpha^r(\phi)$ is implicitly defined as

$$q_H^r(\alpha^r(\phi)) \equiv \phi. \tag{A.30}$$

---

[21]Under price discrimination the manufacturer solves two independent maximization problems; one for each downstream firm $i \in \{1, 2\}$. With a slight abuse of notation, we suppress the subscript i for the discriminatory pricing regime.

Note that existence and uniqueness of $\alpha^r(\phi)$ follow from the intermediate value theorem together with $\hat{q}^r(\alpha^r)$ being a continuous, strictly decreasing function on $[0, \hat{\alpha}^r]$.

Next, consider the relaxed problem where, under pricing regime $r \in \{D, U\}$, M maximizes the objective function (A.28) subject only to (IR$_H^A$) and (IR$_L^A$). Again, for a given allocation (q_L, q_H), in the optimum transfers are chosen to make both constrains bind:

$$t_L^r = \pi(q_L, c_L) - \pi_L^A \tag{A.31}$$

$$t_H^r = \pi(q_H, c_H) - \pi_H^A \tag{A.32}$$

Inserting these transfers into (A.28) reveals that M's goal is to maximize the joint surplus. Hence, the quantities implemented are $q_L^r(\alpha^r) = q^{JS}(c_L)$ and $q_H^r(\alpha^r) = q^{JS}(c_H)$. Obviously, the above wholesale mechanism satisfies the monotonicity constraint (MON). For this solution to the relaxed problem also to be a solution to the original problem, it needs to be checked that the mechanism is also incentive compatible. The incentive constraint of the low-cost firm, (IC$_L$), is satisfied if

$$\pi(q^{JS}(c_L), c_L) - t_L \geq \pi(q^{JS}(c_H), c_L) - t_H \iff q^{JS}(c_H) \leq \phi. \tag{A.33}$$

A high-cost firm truthfully reveals its type, i.e. (IC$_H$) is satisfied, if

$$\pi(q^{JS}(c_H), c_H) - t_H \geq \pi(q^{JS}(c_L), c_H) - t_L \iff q^{JS}(c_L) \geq \phi. \tag{A.34}$$

Thus, for $\phi \in [q^{JS}(c_H), q^{JS}(c_L)]$ the above wholesale mechanism is optimal under the original problem.

Last, consider the relaxed problem where, under pricing regime $r \in \{D, U\}$, M maximizes the objective function (A.28) subject to three constraints, (IR$_H^A$), (IR$_L^A$), and (IC$_L$). Clearly, for $\phi \leq q^{JS}(c_H)$ and $\alpha^r \leq \alpha^r(\phi)$, on the one hand, and for $\phi \in [q^{JS}(c_H), q^{JS}(c_L)]$, on the other hand, the solution to this problem is given by the solution to the respective less heavily constrained optimization problem considered before, where only two of the constraints were binding in the optimum. For $\phi < q^{JS}(c_H)$ and $\alpha^r > \alpha^r(\phi)$, however, in the optimum all three constraints must be binding. Thus, transfers under pricing regime $r \in \{D, U\}$ as functions of the implemented allocation (q_L, q_H) are given by:

$$t_H^r = \pi(q_H, c_H) - \pi_H^A \tag{A.35}$$

$$t_L^r = \pi(q_L, c_L) - \pi_L^A \tag{A.36}$$

$$t_L^r - t_H^r = \pi(q_L, c_L) - \pi(q_H, c_L). \tag{A.37}$$

Solving the above equations (A.35)–(A.37) for q_H yields

$$q_H^r(\alpha^r) = \frac{\pi_L^A - \pi_H^A}{c_H - c_L} = \phi. \tag{A.38}$$

With q_H being fixed by (A.38), the upstream supplier chooses q_L in order to maximize

$$t_L^r - kq_L = \pi(q_L, c_L) - \pi_L^A - kq_L , \tag{A.39}$$

which is achieved by $q_L^r(\alpha^r) = q^{JS}(c_L)$. The above allocation clearly satisfies the monotonicity constraint (MON), and (IC$_H$) trivially holds because (IC$_L$) is satisfied with equality. Thus, the above wholesale mechanism also is a solution to the original problem for $\phi < q^{JS}(c_H)$ and $\alpha^r > \alpha^r(\phi)$. This establishes the desired result. ∎

Proof of Lemma 5:
We first prove Part (i). First, consider the case $\phi \in [\tilde{\phi}, q^{JS}(c_H)]$. Under pricing regime $r \in \{D, U\}$, according to Proposition 6 (ii), for $\alpha^r \geq \alpha^r(\phi)$ the optimal quantity to offer when serving the high-cost type is $q_H^r(\alpha^r) = \phi$. In consequence, the left-hand side (LHS) of (24) equals zero, whereas the RHS is (at least weakly) positive, i.e., M does not exclude the high-cost type. If $\alpha^r < \alpha^r(\phi)$, then—according to Proposition 6 (i)—the optimal quantity to offer when serving a high-cost downstream firm is $q_H^r(\alpha^r) = \hat{q}^r(\alpha^r) \geq \phi$. To see that M prefers to serve both types of downstream firms in this case as well, suppose that—while leaving the quantity to a low-cost firm unchanged—M could offer $q_H = \phi$ to a high-cost downstream firm (instead of $\hat{q}^r(\alpha^r)$) together with tariffs chosen such that (IR$_H^A$) and (IC$_L$) bind. Since $q_H = \phi$, (IR$_L^A$) is satisfied with equality. With this contractual menu, the LHS of (24) obviously equals zero, whereas the RHS is (at least weakly) positive since $\phi \geq \tilde{\phi}$, i.e., M prefers serving both types of downstream firms with this alternative allocation over serving only the low-cost type. Clearly, M's profits under the optimal contractual menu for serving both typs of downstream firms as identified in Proposition 6 (i) cannot be lower than profits under this altered allocation. In summary, under pricing regime $r \in \{D, U\}$, for $\phi \in [\tilde{\phi}, q^{JS}(c_H)]$ we have $\Pi_L^r \geq \Pi_{LH}^r$ irrespective of α^r, i.e., M will always serve both types of downstream firms.

Regarding part (ii) it remains to show that M prefers to serve only the low-cost type for $\phi < \tilde{\phi}$ and $\alpha^r > \tilde{\alpha}^r$. If $\alpha^r \in (\tilde{\alpha}^r, \alpha^r(\phi))$, then $\phi < \hat{q}^r(\alpha^r) < \tilde{\phi}$, which implies that the LHS of (24) is strictly positive whereas the RHS of (24) is strictly negative, i.e., M prefers to serve only the low-cost type of downstream firm. If $\alpha^r \geq \alpha^r(\phi)$, then $q_H^r(\alpha^r) = \phi$. Since $\phi < \tilde{\phi}$, the left-hand side (LHS) of (24) equals zero, whereas the RHS is strictly negative. Thus, M prefers to exclude the high-cost type in this case as well, which establishes the desired result. ∎

Proof of Proposition 7:
(i) For $\alpha_2 < \alpha^D(\phi) \leq \alpha_1^U(\alpha_2; \phi) \leq \alpha_1$, we have $q_{H1}^D = q_H^U = \phi < q_{H2}^D = \hat{q}^D(\alpha_2)$. According to (16), the difference in expected welfare amounts to

$$\Delta W = (1 - \alpha_2) \left\{ \int_\phi^{\hat{q}^D(\alpha_2)} P(z)dz - (c_H + K)\left[\hat{q}^D(\alpha_2) - \phi\right] \right\} \qquad (A.40)$$

Thus, $\Delta W > 0$ if and only if

$$\int_0^{\hat{q}^D(\alpha_2)} P(z)dz - (c_H + K)\hat{q}^D(\alpha_2) > \int_0^\phi P(z)dz - (c_H + K)\phi. \qquad (A.41)$$

To see that this inequality indeed is satisfied, note that the function $\int_0^q P(z)dz - (c_H + K)q$ attains its maximum at q^* which is implicitly characterized by $P(q^*) = c_H + K$. Comparing this last expression with the first-order condition characterizing $\hat{q}^D(\alpha_2)$ in

(10) immediately implies $\hat{q}^D(\alpha_2) < q^*$. Since the function $\int_0^q P(z)dz - (c_H + K)q$ is strictly concave in q whenever $P > 0$, the result follows from $\phi < \hat{q}^D(\alpha_2)$.

(ii) If $\alpha_2 < \alpha^D(\phi) < \alpha_1 < \alpha_1^U(\alpha_2; \phi)$, then $q_{H1}^D = \phi < q_H^U = \hat{q}^U(\alpha_\Sigma) < q_{H2}^D = \hat{q}^D(\alpha_2)$. The difference in expected welfare then is

$$\Delta W = (1 - \alpha_1) \left\{ \int_{\hat{q}^U(\alpha_\Sigma)}^{\phi} P(z)dz - (c_H + K)\left[\phi - \hat{q}^U(\alpha_\Sigma)\right] \right\}$$
$$+ (1 - \alpha_2) \left\{ \int_{\hat{q}^U(\alpha_\Sigma)}^{\hat{q}^D(\alpha_2)} P(z)dz - (c_H + K)\left[\hat{q}^D(\alpha_2) - \hat{q}^U(\alpha_\Sigma)\right] \right\} \quad \text{(A.42)}$$

Differentiation with respect to α_1 yields

$$\frac{d\Delta W}{d\alpha_1} = -\left\{ \int_{\hat{q}^U(\alpha_\Sigma)}^{\phi} P(z)dz - (c_H + K)\left[\phi - \hat{q}^U(\alpha_\Sigma)\right] \right\}$$
$$- (2 - (\alpha_1 + \alpha_2))\frac{\hat{q}^U(\alpha_\Sigma)}{d\alpha_1}\left[P(\hat{q}^U(\alpha_\Sigma)) - (c_H + K)\right] \quad \text{(A.43)}$$

Note that $\phi < \hat{q}^U(\alpha_\Sigma) < q^*$, where q^* was defined in the proof of part (i) and the second inequality follows from (14). The same reasoning as in the proof of part (i) implies $-\left\{ \int_{\hat{q}^U(\alpha_\Sigma)}^{\phi} P(z)dz - (c_H + K)\left[\phi - \hat{q}^U(\alpha_\Sigma)\right] \right\} > 0$. Since (14) also implies that $P(\hat{q}^U(\alpha_\Sigma)) - (c_H + K) = P'(\hat{q}^U(\alpha_\Sigma))\hat{q}^U(\alpha_\Sigma) + \frac{\alpha_\Sigma}{2 - \alpha_\Sigma}(c_H - c_L) > 0$, the desired result follws from $d\hat{q}^U(\alpha_\Sigma)/d\alpha_1 < 0$.

(iii) Follows immediately from the proof of Proposition 4. ∎

B. Proofs for Subsection 3.5

We analyze the upstream supplier's screening problem for the continuous distribution of downstream types. First, the implications of the constraints for the optimal wholesale tariff are analyzed. Note that neither the individual rationality constraints nor the incentive compatibility constraints depend on the pricing regime. To cut back on notation we suppress the subscript i indicating the downstream firm. Define

$$V(c) \equiv q(c)[1 - q(c) - c] - t(c). \quad \text{(B.1)}$$

Using a revealed preference argument for types $c, \hat{c} \in C$ and $\hat{c} > c$ we obtain

$$q(c) \geq \frac{V(c) - V(\hat{c})}{\hat{c} - c} \geq q(\hat{c}). \quad \text{(B.2)}$$

The above chain of inequalities implies that $V'(c) = -q(c)$ except for points of discontinuity. Moreover, from (B.2) we immediately obtain the following result.

Lemma 6 *The incentive compatible quantity and transfer schedules, $q(c)$ and $t(c)$, are non-increasing.*

Using the insights from above, the transfer $t(c)$ can be stated as

$$t(c) = q(c)[1 - q(c) - c] - \int_c^{c_H} q(z) \, dz, \quad \text{(B.3)}$$

since $V(c) = V(c_H) - \int_c^{c_H} q(z)\, dz$ and $V(c_H) = 0$ in the optimum, i.e., the participation constraint is binding for the highest-cost type.

Discriminatory Offers.—Suppose the manufacturer is not restricted to non-discriminatory offers. Thus, the manufacturer solves to isolated maximization problems. We analyze the contracting problem with downstream firm 1. After using integration by parts, the up-stream suppliers problem can be stated as follows:

Program D1:

$$\max_{\langle q(c)\rangle_{c\in\mathcal{C}}} \int_{c_L}^{c_H} \left(q(c)[1 - q(c) - c] - q(c)\frac{F(c)}{f(c)} \right) f(c)\, dc$$

$$\text{subject to: } q(c) \text{ is non-increasing}$$

From the first-order condition, obtained by point-wise maximization and ignoring the monotonicity constraint, we obtain:

$$q_1^D(c) = \frac{1}{2}\left[1 - c - \frac{F(c)}{f(c)} \right]. \tag{B.4}$$

By Assumptions 1 and 2, the quantity schedule $q_1^D(c)$ is strictly decreasing and assigns a positive quantity to all types.

The contracting problem with downstream firm 2—Program D2—can be solved by same reasoning. The optimal quantity schedule in this case is

$$q_2^D(c) = \frac{1}{2}\left[1 - c - \frac{G(c)}{g(c)} \right]. \tag{B.5}$$

Uniform Pricing.—Now, the upstream supplier is restricted to offer the same wholesale tariff to both downstream firms. The supplier maximizes

$$\int_{c_L}^{c_H} t(c)[f(c) + g(c)]\, dc, \tag{B.6}$$

subject to the (IC) and (IR) constraints. Since the constraints are the same as under price discrimination, the incentive compatible transfer schedule is still characterized by (B.3). In order to rewrite the upstream supplier's profit, we use integration by parts. Note that

$$\int_{c_L}^{c_H} \int_c^{c_H} q(z)\, dz[f(c) + g(c)]\, dc = \int_{c_L}^{c_H} q(c)[F(c) + G(c)]\, dc. \tag{B.7}$$

Under uniform pricing, the upstream supplier faces the following problem.

Program U:

$$\max_{\langle q(c)\rangle_{c\in\mathcal{C}}} \int_{c_L}^{c_H} \left(q(c)[1 - q(c) - c] - q(c)\frac{F(c) + G(c)}{f(c) + g(c)} \right) [f(c) + g(c)]\, dc$$

$$\text{subject to: } q(c) \text{ is non-increasing}$$

Suppose the monotonicity constraint is non-binding, then using point-wise maximization we obtain

$$q^U(c) = \frac{1}{2}\left[1 - c - \frac{F(c) + G(c)}{f(c) + g(c)}\right]. \tag{B.8}$$

By Assumptions 1 and 2, the quantity schedule $q^U(c)$ is strictly decreasing and assigns a positive quantity to all types. Now, we can proof Lemma 4.

Proof of Lemma 4:

Suppose $q_1^D(c) < q_2^D(c)$. According to the lemma, $q_1^D(c) < q^U(c)$ which is equivalent to

$$1 - c - \frac{F(c)}{f(c)} \quad < \quad 1 - c - \frac{F(c) + G(c)}{f(c) + g(c)} \tag{B.9}$$

$$\iff \quad \frac{F(c)}{f(c)} \quad > \quad \frac{G(c)}{g(c)}. \tag{B.10}$$

By hypothesis that $q_1^D(c) < q_2^D(c)$, the inequality (B.10) is fulfilled. By a similar reasoning it can be shown that $q^U(c) < q_2^D(c)$, which completes the proof. ∎

Welfare.—Suppose price discrimination is permitted. The expected welfare is then given by

$$\mathbb{E}[W^D] = \int_{c_L}^{c_H} \left[q_1^D(c) - (1/2)(q_1^D(c))^2 - cq_1^D(c)\right] f(c)\, dc$$
$$+ \int_{c_L}^{c_H} \left[q_2^D(c) - (1/2)(q_2^D(c))^2 - cq_2^D(c)\right] g(c)\, dc. \tag{B.11}$$

Inserting (B.4) and (B.5) into the above expression for expected welfare yields

$$\mathbb{E}[W^D] = \frac{3}{8}\left\{ \int_{c_L}^{c_H} \left[f(c) - cf(c) - F(c)\right]\left[1 - c + \frac{F(c)}{3f(c)}\right]\, dc \right.$$
$$\left. + \int_{c_L}^{c_H} \left[g(c) - cg(c) - G(c)\right]\left[1 - c + \frac{G(c)}{3g(c)}\right]\, dc \right\}. \tag{B.12}$$

If price discrimination is banned, the expected welfare amounts to

$$\mathbb{E}[W^U] = \int_{c_L}^{c_H} \left[q^U(c) - (1/2)(q^U(c))^2 - cq^U(c)\right](f(c) + g(c))\, dc \tag{B.13}$$

By using the explicit expression for the implemented quantities (B.8), the expected welfare under uniform pricing can be written as

$$\mathbb{E}[W^U] = \frac{3}{8}\left\{ \int_{c_L}^{c_H} \left[f(c) - cf(c) - F(c)\right]\left[1 - c + \frac{F(c) + G(c)}{3[f(c) + g(c)]}\right]\, dc \right.$$
$$\left. + \int_{c_L}^{c_H} \left[g(c) - cg(c) - G(c)\right]\left[1 - c + \frac{F(c) + G(c)}{3[f(c) + g(c)]}\right]\, dc \right\}. \tag{B.14}$$

Let the expected change in welfare from a regime shift away from uniform pricing to price discrimination be $\Delta W := \mathbb{E}[W^D] - \mathbb{E}[W^U]$.

Proof of Proposition 5:

The expected change in welfare is

$$\Delta W = \frac{1}{8} \left\{ \int_{c_L}^{c_H} f(c) \left[1 - c - \frac{F(c)}{f(c)} \right] \left[\frac{F(c)g(c) - G(c)f(c)}{f(c)[f(c) + g(c)]} \right] \, dc \right. \\ \left. + \int_{c_L}^{c_H} g(c) \left[1 - c - \frac{G(c)}{g(c)} \right] \left[\frac{G(c)f(c) - F(c)g(c)}{g(c)[f(c) + g(c)]} \right] \, dc \right\}. \quad (B.15)$$

Simplifying the above expression yields

$$\Delta W = \frac{1}{8} \left\{ \int_{c_L}^{c_H} \left[\frac{F(c)g(c) - G(c)f(c)}{f(c) + g(c)} \right] \left[\frac{G(c)}{g(c)} - \frac{F(c)}{f(c)} \right] \, dc \right\} \quad (B.16)$$

$$= -\frac{1}{8} \int_{c_L}^{c_H} \frac{[F(c)g(c) - G(c)f(c)]^2}{f(c)g(c)[f(c) + g(c)]} \, dc < 0, \quad (B.17)$$

which establishes the desired result. ∎

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