Vertical Integration with Complementary Inputs*

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Abstract

We analyze the profitability and welfare consequences of vertical integration when downstream firms deal with suppliers of complementary intermediate goods that exert market power. We show that the results in this setting are markedly different from those of the received literature that deals only with substitute intermediate goods. In particular, vertical integration is not necessarily profitable since the integrated firm faces the problem that the complementary input producer expropriates the higher profits earned by the integrated chain on the downstream market. Interestingly, this effect is particularly strong is the integrated firm is very efficient. We also show that if vertical integration is profitable, foreclosure of downstream rivals is no longer the optimal strategy of the integrated firm. Instead, the integrated firm may set prices even below marginal costs thereby rendering vertical integration pro-competitive, which has profound consequences on antitrust policy.

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1 Introduction

The combination of complementary inputs is a pervasive characteristic of the production process in many industries. Downstream firms usually purchase several intermediate goods from the re-

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spective wholesale markets and employ them to produce their final products. For example, in the information and communications sector many products are based on technological standards and require the use of multiple specialized inputs that are produced by different firms. In addition, high technology products can often only be produced when having access to multiple patents that are owned by different IP holders. All these inputs—and patents—are perfect complements. Another example is the supermarket industry. Here shopping costs on the consumer side induces them to bundle their purchases. This creates a complementarity in the demand of several goods which requires supermarkets to supply a large number of them.

In these industries vertical integration is also a prevalent feature. In the communication industry several handset makers like Nokia or Sony Ericsson develop and produce some parts of their handheld devices on their own while stand alone firms hold essential patents for other technologies, e.g., Qualcomm for transmission of data packages. This can also be observed in the computer manufacturing industry, where manufacturers produce several inputs on their own but buy their microprocessors from Intel and AMD, firms that do not produce computers themselves. Also, supermarket chains often offer private label consumer products but buy other products from specialized firms that are not active in the distribution industry.

Thus, the question arises what the consequences of vertical integration for consumers and welfare are and under which conditions firms find it profitable to integrate given that complementary inputs are required. Surprisingly, although the need of two or more essential inputs is widespread, the received literature so far has almost exclusively focussed on the case where manufacturers need only one input. In particular, the theory of harm behind vertical integration and the resulting conclusions on antitrust policy are based on settings where only input is needed. A prominent idea behind this theory is that with downstream competition it can be difficult for a dominant upstream firm to extract monopoly profit since it cannot commit to restrict its output to the monopoly level. However, via vertical integration, the firm can foreclose its downstream rivals, thereby reducing output and rendering vertical integration profitable but anticompetitive. This idea of raising rivals' costs is brought forward by Salop and Scheffman (1983, 1987), Hart and Tirole (1990), McAfee and Schwartz (1994) and is extensively discussed in the recent survey by Rey and Tirole (2007).[1]

The aim of this paper is to fill the aforementioned gap in the literature. We provide a model with complementary input producers that exert market power vis-a-vis downstream firms but also face competition from producers of substitute goods. In this framework, we assess the profitability

[1]Notice that the use of the term foreclosure to identify such a conduct may be misleading. The legal definition of foreclosure is relatively broad and includes all the strategic practices undertaken by a firm to limit the competitive pressure it faces on the market. Instead, here the term foreclosure is used for the specific practice of excessive pricing at the expenses of competitors.

and the consequences of vertical mergers.

The effects arising from vertical integration in an industry with complementary inputs are largely distinct from those characterizing a model with only substitute intermediate goods, even though they share some similarities. On the one hand, in analogy to the case with substitute inputs, an integrated firm may aim at weakening the position of its downstream rivals via increased input prices (*foreclosure motive*). On the other hand, the integrated chain may be more vulnerable to an *expropriation conduct* by other inputs' producers since these producers are now the only ones exerting market power vis-à-vis the downstream unit of the integrated firm. This leads the integrated firm to lower its wholesale price to the downstream rival to be able to extract more profit from it via the fixed payment. This effect can be so large that the integrated company finds it profitable to sell to the downstream rival at a wholesale price below marginal costs, whereby rendering vertical integration pro-competitive. Vertical integration is nevertheless profitable because of an *information effect*: The downstream unit of the integrated now observes the wholesale price at which its rival competitor bought the input and can optimally react on it via its downstream price. Finally, if the expropriation conduct of the complementary input producer is large, vertical integration is *unprofitable* and firms stay separated. Interestingly, this problem is the more severe, the more efficient the upstream firm is leading to the result that those firms are more likely to stay independent. Therefore, in a model with complementary inputs the incentives to integrate and foreclose the downstream market are threatened by the expropriation behavior undertaken by suppliers of other complementary intermediate goods, which can reverse the results obtained in models with a single input.

More specifically, our framework embeds two upstream firms that provide perfectly complementary products. Each input supplier competes with an alternative and less efficient source (or bypass alternative), and makes secret offers to downstream firms by means of contracts with two-part tariffs. On the downstream market, two firms compete and need both intermediate goods to produce the final good. Finally, suppliers serve downstream firms on order and the latter produce the output good.

In this framework we obtain the following results. First, foreclosure emerges at equilibrium only if the integrated firm is not "too efficient", that is, if the cost advantage over the second source is not too large. If a wholesale firm is not much more efficient than its bypass alternative, then, once integrated, the profit that it can extract from the downstream market via foreclosure is also not overly large. The expropriation problem it faces from the complementary input provider is not a big concern and foreclosure is the optimal strategy. So in this case our model is indeed consistent with the conclusion that vertical integration leads to foreclosure.

However, as the efficiency of an upstream firm over its bypass alternative rises, the expropriation conduct that it would suffer under integration becomes a bigger concern. This is the case because, since market power is on the side of the upstream firms, the complementary input producer extracts as much profit as possible from the integrated firm and is only constrained by the second source for its respective input. Consequently, the merged company prefers to shield part of the rents it can squeeze from the downstream market by lowering the wholesale price it sets to its downstream competitor and levying a higher fixed fee on the competitor. Therefore, foreclosure is no longer necessarily the optimal strategy for the integrated firm. In particular, we show that the fear of this expropriation conduct can lead the integrated firm to set the whole price to its competitor even below marginal costs, thereby rendering vertical integration pro-competitive.

The question arises why vertical integration occurs in the first place if the expropriation conduct is large and aggregate industry profits are even lower than without integration. The reason is that there is a genuine information advantage effect retained by the integrated organization that is not present if a firm stands alone. This is that the downstream unit of the integrated firm knows if its downstream competitor has bought from its upstream division or from the inefficient source; hence, it can taylor its downstream quantity to the competitor's decision and be more aggressive if the competitor purchases from the bypass alternative at higher costs. Via that it can squeeze more profit from the competitor through the fixed fee.[2]

Finally, we also show that firms may abstain from integration when it is less profitable than staying separated. This occurs if the expropriation conduct of the complementary input supplier is high. Interestingly, this result occurs if an upstream firm is "particularly efficient", i.e., its cost advantage over the bypass alternative is large. Indeed, when being highly efficient, an upstream firm can extract a lot of profit from the downstream market if it stays independent. Instead, if the upstream firm integrates, it internally trades the input at marginal costs, whereby losing its power to extract profits from the downstream unit. To the converse, the provider of the second essential input can now fully exploit its power and extract more profits from the integrated chain. Thus, via staying separated, an efficient upstream firm shields some of its profits from the complementary input provider. This prediction is opposite to the one delivered by the received literature, which concludes that vertical mergers are particularly profitable for very efficient firms, see e.g., Rey and Tirole (2007).

The problem of expropriation conduct identified in our analysis can be observed in two recent antitrust cases in the information and communication technology: The EC v. Qualcomm case and the FTC v. Rambus case. Qualcomm and Rambus are stand-alone upstream firms active in the development of Intellectual Property Rights (IPRs). In the first case, Qualcomm has been

[2]This effect is also present in a framework with just one input in which foreclosure is the unique equilibrium. However, it is never effective there because the raising rivals' costs strategy brings the wholesale price to a value at which the fixed payment required by the integrated firm is nil. In our framework, instead, as the concerns for the expropriation conduct rise, the wholesale price set by the integrated chain decreases and the fixed fee it sets increases.

accused by Nokia and other vertically integrated firms, which produce handsets and develop IPRs, to have infringed its obligation to negotiate prices on fair, reasonable and nondiscriminatory terms. Generally, these vertically integrated firms accused Qualcomm to charge an excessive royalty rate for the licensing of IPRs that are essential to the UMTS technology.[3]

In 2006, the FTC found Rambus guilty of having manipulated the works in JEDEC, the Standard Setting Organization that was deciding on the specification of the SDRAM standard.[4] Interestingly, Micron, IBM and other vertically integrate firms claimed that they would have strongly opposed the inclusion of Rambus technology in the standard.

Summarizing, in both cases vertically integrated firms are threatened by stand-alone upstream suppliers that hold essential inputs for downstream production technology, a result that is consistent with the predictions of our model and that affects the conclusions on vertical mergers that are important for antitrust policy.

The rest of the paper is organized as follows. The next Section provides an overview over the related literature. Section 3 sets out the model and Section 4 analyzes the case without integration. Section 5 provides the analysis and the results of the case with a vertical merger. In Section 6 we discuss an extension with public offers and Section 7 concludes.

2 Literature Review

The problem of a dominant upstream firm to be unable to commit to the monopoly quantity when selling via multiple competing downstream firms was first pointed out by Hart and Tirole (1990) and is summarized in the survey by Rey and Tirole (2007). In their framework, upstream firms' offers are made by means of secret contracts and downstream firms adopt passive beliefs to infer the offers received by their competitors when they face out-of-equilibrium offers. In these circumstances, the dominant upstream firm comes across a Coasian commitment problem that limits its ability to extract full monopoly profit and the unique equilibrium is characterized by Cournot quantities, price and profits. We take the same approach as in Rey and Tirole (2007) when modeling the structure of the contracting game between upstream and downstream firms. Consequently, the same commitment problem arises in our framework. Instead, the crucial twist of our framework compared to Rey and Tirole (2007) consists in the presence of producers of complementary inputs, which are rivals in extracting the surplus from downstream manufacturers.

The role of manufacturers' beliefs has been highly debated by the literature on vertical restraints. More specifically, the paradox inherent to the commitment problem was investigated later by

[3]See EC MEMO/07/389, 01/10/2007.

[4]The FTC alleged that the deceptive conduct kept by Rambus allowed the firm to include some of its patented technologies in the final version of the standard. See In the Matter of Rambus Inc., Docket No. 9302.

O'Brien and Shaffer (1992), McAfee and Schwartz (1994) and Marx and Shaffer (2004). The general conclusion from these papers is that via vertical integration the dominant firm is able to restrict its quantity thereby moving closer to the monopoly level which renders vertical integration profitable but anticompetitive.

An important assumption in these settings is that manufacturers have perfect information on the marginal cost of the intermediate goods' suppliers. White (2007) relaxes this hypothesis and introduces incomplete information about upstream firms' costs. She finds that even in a context with incomplete information it is still necessary to specify the downstream firms' beliefs concerning out-of-equilibrium offers made by wholesale firms. She also shows that with upstream marginal costs' uncertainty, vertical integration can result in high-cost types selling to downstream firms at lower prices than they would set if vertically separated and this result is partly due to the kind of equilibrium selection employed.[5]

Baake, Kamecke, and Normann (2004) also show that vertical integration may enhance efficiency and makes it socially preferable to non-integration. In their model, an upstream monopolist can publicly commit to a capacity level before formulating its offers to manufacturers. In this way, the monopolist can partly solve the Coasian conjecture problem, commit to underinvest in capacity and produce at a level that can even be below the monopoly output. Thus, vertical integration can deliver a pro-competitive outcome as output increases to the monopoly level.

The mechanism that leads to non-foreclosure in our model is markedly different from the above two papers. In particular, we show that due to the complementary input provider the integrated firm may have no incentive to engage in foreclosure, but sets the wholesale price to downstream rivals below marginal costs, while in the papers above foreclosure is still optimal but the monopolist produces even less when being unintegrated. In addition, vertical integration is always profitable in these papers while this is no longer true when complementary inputs are important.

There are several other papers that analyze the effects of vertical integration in different set-ups. For example, Ordover, Saloner and Salop (1990) or Chen (2001) consider the case of Bertrand competition between upstream producers with public offers in linear prices. They determine under which conditions vertical integration is profitable and analyze if counter mergers can occur. Choi and Yi (2003) provide a model in which upstream firms can choose to specialize their inputs to the needs of downstream firms and analyze the consequences of vertical integration in this case.

[5]The adoption of incomplete information implies that suppliers may engage in strategic signaling and this leads to a multiplicity of equilibria. In order to eliminate equilibria that are not supported by out-of-equilibrium beliefs, White (2007) focuses on the Cho-Kreps intuitive criterion. This equilibrium features no output distortion for the low-cost types and a downward distortion of the high-cost types' output. Consequently, if the cost difference is low enough, an high-cost non integrated firm produces less than its monopoly output. Clearly, if high types are numerous enough, a policy that eliminates strategic signaling and restores the incentives to set the monopoly output—like vertical integration—improves welfare.

Riordan (1998) considers a model with a dominant firm that has market power in a final and an intermediate good market. He shows that vertical integration of the dominant firm is anti-competitive due to foreclosure although production costs of the dominant fall.[6] In contrast to our set-up, these papers just consider a single input and are not concerned with complementary inputs. In addition, they all show that integrated firms have an incentive to foreclose their downstream rivals.

Finally, papers that consider the case of complementary inputs usually look at markets where upstream firms hold essential patents that are required for the production of a final good, see e.g., Shapiro (2001). However, this literature is not concerned with the consequences of vertical integration. The only exception is Schmidt (2007). He considers a model in which each patent holder is monopolist for its patent while there are several downstream firms competing on the product market. Patent holder compete via public contracts. Schmidt (2007) shows that vertical integration leads to foreclosure of rival downstream firms and to a reduction of output although the integrated firm produces more due to the avoidance of double-marginalization. In contrast to his model, we consider the case of private contracts and allow for a richer market structure in the upstream market where a (less efficient) competitor exists for each input. As mentioned, in this set-up we obtain starkly different results to the previous literature.

3 The Model

There are two downstream firms, denoted by D_1 and D_2, that produce a homogeneous good: To produce one unit of the output good each downstream firm needs one unit of two input goods (or intermediate goods). In other words, the two input goods are perfect complements and used in fixed proportions for the production of the final good. In the following, we denote the output of firm D_i by q_i, $i = 1, 2$.

Each input good j is produced by two firms, U_j and $\hat{U}_j$, $j = A, B$. Firm U_j is assumed to be more efficient than firm $\hat{U}_j$, i.e., it produces input j at a marginal cost of c_j, while firm $\hat{U}_j$ incurs a marginal cost of production given by $\hat{c}_j > c_j$. The inefficient source needs not to be just one firm; one can also interpret it as a fringe of firms that produce the input j using an inferior technology. The framework is given in Figure 1.

[FIGURE 1 ABOUT HERE]

[6] Nocke and White (2007) analyze the effects of vertical integration on the sustainability of collusion between upstream firm in a repeated game framework. They show that vertical integration facilitates collusion because, after integration, non integrated upstream firms have a smaller incentive to deviate since they can no longer sell via the downstream unit of the integrated firm.

Downstream firms face a downward sloping inverse demand function $p = P(q_1 + q_2)$. As will become clear below, the downstream firms play a Bertrand-Edgeworth game of price competition with capacity constraints à la Kreps and Scheinkman (1983). We follow Kreps and Scheinkman (1983) by assuming that the inverse demand function is concave, $P''(q_1+q_2) \leq 0$. This assumption is stricter than necessary, i.e., all our results also hold if $P''(q_1+q_2)$ is slightly convex. However, the assumption simplifies some of the proofs and we therefore maintain it throughout the analysis.

The game proceeds as follows:

1. In the *first stage* each upstream firm U_j and $\hat{U}_j$ makes a take-it-or-leave-it offer to each D_i consisting of two-part tariffs, which are denoted by $T_{U_j}^{D_i} = w_{U_j}^{D_i} x_{U_j}^{D_i} + F_{U_j}^{D_i}$ ($T_{\hat{U}_j}^{D_i} = w_{\hat{U}_j}^{D_i} x_{\hat{U}_j}^{D_i} + F_{\hat{U}_j}^{D_i}$ for firms $\hat{U}_j$), where $x_{U_j}^{D_i}$ ($x_{\hat{U}_j}^{D_i}$) denotes the quantity of input j that D_i buys from U_j ($\hat{U}_j$). The offer game proceeds as follows. The offers for input j and $-j$ are made in sequential order. First, the pair of firms U_j and $\hat{U}_j$ simultaneously make an offer to D_i, $i = 1, 2$. Then, U_{-j} and $\hat{U}_{-i}$ simultaneously make an offer to D_i. To ensure equal bargaining power between the input providers we assume that each pair of upstream firms has equal probability of being first. We assume that U_{-j} and $\hat{U}_{-j}$ know that they are the second pair to offer but they do not observe the offers made in the first stage. After having observed all offers, D_i decides whether (and from which firm) to buy the intermediate goods, orders input quantities of goods j and $-j$, and pays the respective tariffs.
2. In the *second stage*, each downstream firm transforms the intermediate goods into output, observes the output of its rival and sets its price on the product market.

The equilibrium concept we employ is perfect Bayesian equilibrium. Given the quantity purchased in the first stage, in the second stage downstream firms transform their purchased input units to output. We assume that if firms purchase an amount of inputs A and B in the viable range, it is optimal for them to transform all units into output. The price in the second stage of the game is then given by $P(q_1 + q_2)$.

As for the first stage, the game is solved under the assumption that upstream firms supply on order and that wholesale contracts are secret. The latter assumption implies that each D_i observes all contracts it is offered by the upstream firms, but not the contracts that are offered to D_{-i}. In particular, by using the common agency taxonomy, we restrict our attention to a bidding game with *passive (out-of-equilibrium) beliefs*. The assumption of passive beliefs implies that if a downstream firm faces an out-of-equilibrium offer by a supplier, it does not revise its beliefs concerning the offers made to its rival. More precisely, passive beliefs imply that a downstream firm D_i presumes that, regardless of the offer received by a supplier, its downstream rival D_{-i} produces the candidate equilibrium quantity.[7]

[7]See Rey and Tirole (2007) for an extensive discussion on the role of beliefs in settings with secret contracts.

Some remarks on our game structure are in order: First, we suppose that the offer game takes place sequentially, i.e., first input j's providers make an offer, then input $-j$'s suppliers move. The reason for this assumption is that in a setting with complementary inputs a continuum of equilibria exists if upstream firms were to make offers simultaneously. In all these equilibria, upstream firms split the downstream profit in a different way. We circumvent this problem by employing the sequential time structure that allows us to derive a unique equilibrium. Note that our equilibrium is also an element of the continuum of equilibria that arise under simultaneous offers. In particular, it is the equilibrium in which upstream firms split the profits of downstream firms evenly, up to their relative efficiency over the bypass alternatives.

Second, downstream firms decide from which firm to buy the input goods after having observed all upstream suppliers' offers. We choose this structure because it gives D_i the largest flexibility. In particular, D_i knows if it can make weakly positive profits only after observing all offers.[8]

Third, we assume that the second pair to offer, U_{-j} and $\hat{U}_{-j}$, does not observe the contract offers of the first pair. This is a reasonable assumption since in reality negotiations are secret. Hence, input suppliers do not observe the offers of other input suppliers. The assumption is also consistent with our assumption that downstream firms do not observe the offers to their rivals.

Before solving the model, it is useful to introduce some additional notation. In particular, in the following we shall denote by Q^c and Π^c the Cournot profit of one manufacturer if both downstream firms face a marginal cost of production given by $c_A + c_B$. Hence,

$$q^c = \arg\max_q \{[P(q + q^c) - c_A - c_B]q\},$$

$$\Pi^c = [P(2q^c) - c_A - c_B]q^c.$$

Finally, we impose that the bypass alternatives are indeed effective in constraining the market power of U_j, $j = 1, 2$, that is $\hat{c}_j$ is low enough, so that the downstream firms' threat to buy from the alternative sources matters. More specifically, we impose that the following holds:

Assumption 1.

$$q_i^c = \arg\max_q \{[P(q + q_{-i}^c) - \hat{c}_j - c_{-j}]q\} > 0,$$

where

$$q_{-i}^c = \arg\max_q \{[P(q + q_i^c) - c_A - c_B]q\}.$$

Assumption 1 implies that the quantity of D_i when resorting to $\hat{U}_j$ is positive, so that its respective profit is positive as well.

[8]If D_i is required to accept upstream firm's offers sequentially, the problem can arise that in the second round the efficient upstream firm exploits this via offering such a large fixed fee that the sum of the fixed fees exceeds the downstream profit of D_i. Nevertheless, D_i will accept this second-round offer because its losses would be larger when rejecting the offer and receiving a lower downstream profit. This is the case because it has to pay the fixed fee to the first upstream firm. Our game structure rules out that such a situation can occur.

4 Set-up without Integration

In this section, we present the case in which no firm is vertically integrated. Here we obtain the following result:

Proposition 1.
The perfect Bayesian equilibrium of the game exhibits the following properties:

- The equilibrium quantities are given by $q_1 = q_2 = q^c$.
- The per-unit price in any wholesale contract is given by $w_{U_j}^{D_i} = c_j$, with $i = 1, 2$ and $j = A, B$, that is, each upstream firm offers a per-unit price equal to its marginal costs to each downstream firm.
- If U_j is the first to offer, it proposes a fixed fee that is given by $F_{U_j}^{D_i} = \Pi^c - \max_q\{[P(q + q^c) - \hat{c}_j - c_{-j}]q$ to downstream firm D_i.
- If U_j is the second to offer, it proposes a fixed fee that is given by $F_{U_j}^{D_i} = \max_q\{[P(q+q^c) - c_j - \hat{c}_{-j}]q - \max[\max_q\{[P(q + q^c) - \hat{c}_j - \hat{c}_{-j}]q, 0]$ to downstream firm D_i.

Proof See the appendix.

The perfect Bayesian equilibrium of the game without integration features the same commitment problem that arises in Rey and Tirole (2007); both downstream firms buy the inputs from the efficient upstream firms at marginal cost and produce respective Cournot quantities. Since a downstream firm does not observe the contract offers to its rival and holds passive beliefs, upstream firms cannot commit to sell the monopoly quantity.

The presence of second sources also constrains the ability of upstream firms to extract profits from the downstream firms. From Proposition 1 it is evident that the fixed fees are larger the more efficient U_j and U_{-j} are relative to the bypass alternatives. If an upstream firm U_j is the first to propose a contract to a downstream firm, it extracts the Cournot profit from the downstream market minus the profit that the downstream firm would get when buying from the bypass alternative. Thus, the fixed fee is increasing in $\Delta_j = \hat{c}_j - c_j$, i.e., it is the larger the more efficient U_j is. If instead U_j is the second to propose the contract, it must take into account that D_i can also reject the offers of both efficient firms and resort to the offers of the bypass alternatives. Thus, in this case U_j proposes as a fixed fee the profit that U_{-j} had left in its first offer minus the profit that D_i can ensure when buying from the bypass alternatives. Naturally, since U_j and U_{-j} are the efficient firms, in equilibrium they supply to both downstream firms while $\hat{U}_A$ and $\hat{U}_B$ stay inactive.

Before analyzing the profitability of a vertical merger between firms U_j and D_i, we have to determine the profits that U_j and D_i receive when staying independent. From Proposition 1 it is evident that the profit of D_i in case of non-integration is given by $\max\left[\max_q\{[P(q+q^c) - \hat{c}_j - \hat{c}_{-j}]q, 0\right]$. Determining the profit of U_j—and recognizing that U_j is the first to offer to both downstream firms with probability 1/2—we obtain that its expected profit under non-integration is given by

$$\Pi^c + \max_q\{[P(q+q^c) - c_j - \hat{c}_{-j}]q\} - \max_q\{[P(q+q^c) - \hat{c}_j - c_{-j}]q\} - \max\left[\max_q\{[P(q+q^c) - \hat{c}_j - \hat{c}_{-j}]q\}, 0\right].$$

Thus, the sum of profits of U_j and D_i is given by

$$\Pi^c + \max_q\{[P(q+q^c) - c_j - \hat{c}_{-j}]q\} - \max_q\{[P(q+q^c) - \hat{c}_j - c_{-j}]q\}. \tag{1}$$

We can now use this value to determine the profitability of a vertical merger.

5 Vertical Merger between U_A and D_1

Suppose that U_A and D_1 are integrated and the other firms are independent. The new framework is given in Figure 2.

[FIGURE 2 ABOUT HERE]

The integrated firm trades the input good internally at marginal cost. This assumption is standard in the literature (see e.g., Ordover, Saloner and Salop, 1990, Chen, 2001, or Choi and Yi, 2001) and is justified by the fact that pricing at marginal cost is (ex-post) the optimal strategy for the integrated firm. Even if it would like to credibly commit to outsiders that the internal wholesale price is above marginal costs, it cannot do so, since the firm has an incentive to secretly change the price afterwards. It can do so via exchanging payments between the upstream and the downstream unit, which is unobservable to outsiders.

As in the case without integration, the firms delivering the inputs still have all the bargaining power; this now implies that U_B makes a take-it-or-leave-it-offer to the newly integrated firm. In other words, vertical integration does not imply a change of the bargaining power positions, it only changes the strategic position of the newly integrated firm, which now maximizes its joint profit.

We first determine the optimal wholesale price that the integrated firm charges to D_2. As is well-known from Rey and Tirole (2007), in the absence of suppliers of the complementary good, the integrated firm finds it optimal to soften downstream product market competition via foreclosure. This means that the integrated firm sets its downstream rival's marginal costs of production to the highest possible value, i.e., $w_{D_2}^{U_A} = \hat{c}_A$. We will now analyze if this is still true if there are complementary inputs.

To do so we first need to characterize the optimization problem of the integrated firm with respect to $w_{U_A}^{D_2}$. We start with the case in which $U_A - D_1$ is first in negotiating with D_2. The inefficient source $\hat{U}_A$ is willing to offer a contract of $w_{\hat{U}_A}^{D_2} = \hat{c}_1$ and $F_{\hat{U}_A}^{D_2} = 0$. Taking this into account, the maximization problem of $U_A - D_1$ is given by

$$\begin{aligned} \max_{w_{U_A}^{D_2}} \quad & \max_q\{(P(q + q_2^c(w_{U_A}^{D_2})) - c_A - \hat{c}_B)q\} + x_{U_A}^{D_2}(w_{U_A}^{D_2})(w_{U_A}^{D_2} - c_A) + F_{U_A}^{D_2}, \\ s.t. \quad & \max_q\{(P(q + q_1^c(w_{U_A}^{D_2})) - w_{U_A}^{D_2} - c_B)q\} - F_{U_A}^{D_2} \geq \max_q\{(P(q + q_1^c(\hat{c}_A)) - \hat{c}_A - c_B)q\}, \end{aligned} \tag{2}$$

where

$$q_2^c(w_{U_A}^{D_2}) = \arg\max_q\{[P(q + q_1^c(w_{U_A}^{D_2})) - w_{U_A}^{D_2} - c_B]q\},$$
$$q_1^c(w_{U_A}^{D_2}) = \arg\max_q\{[P(q + q_2^c(w_{U_A}^{D_2})) - c_A - c_B]q\},$$

while $q_1^c(\hat{c}_A)$ is defined by

$$q_1^c(\hat{c}_A) = \arg\max_q\{[P(q + q_2^c(\hat{c}_A) - c_A - c_B]q\},$$

with

$$q_2^c(\hat{c}_A) = \arg\max_q\{[P(q + q_1^c(\hat{c}_A)) - \hat{c}_A - c_B]q\}.$$

Let us explain this problem in more detail. First, differently to the case without integration, $U_A - D_1$ takes into account that it is operating on the downstream market. Here it raises a rent equal to $\max_q\{(P(q + q_2^c(w_{U_1}^{D_2})) - c_A - \hat{c}_B)q\}$. This is the case because by the same token as in Proposition 1, firm U_B sets input prices to the downstream firms of $w_{U_B}^{D_1} = w_{U_B}^{D_2} = c_B$, implying that the integrated firm's downstream profit is given by $\max_q\{(P(q+q_2^c(w_{U_A}^{D_2})) - c_A - c_B)q\}$. However, U_B extracts part of this profit via its fixed fee, which it sets equal to $F_{U_B}^{D_1} = \max_q\{(P(q+q_2^c(w_{U_A}^{D_2})) - c_A - c_B)q\} - \max_q\{(P(q + q_2^c(\hat{c}_1)) - c_A - \hat{c}_B)q\}$. Thus, the integrated firm's operating profit is $\max_q\{(P(q + q_2^c(\hat{c}_1)) - c_A - \hat{c}_B)q\}$. In addition, the upstream unit of the integrated firm receives as a profit from D_2 that equals the margin of its wholesale price over marginal costs times the quantity that D_2 buys, denoted by $x_{U_A}^{D_2}$, plus the fixed fee.

The constraint on the fixed fee faced by the integrated firm is that D_2 accepts the offer of U_A only in case D_2 can ensure itself weakly larger profits from accepting U_A's offer than from buying the input from $\hat{U}_A$ at a price of $\hat{c}_A$ and a fixed fee of zero. This explains the constraint. Note that, contrary to the case of no integration, in the latter case the integrated firm observes that D_2 does not buy from it. Therefore, the downstream unit D_1 can adjust its quantity accordingly by producing $q_1^c(\hat{c}_A)$ instead of $q_1^c(w_{U_A}^{D_2})$.

Since the constraint is binding at the optimum, we can solve for the fixed fee $F_{D_2}^{U_A}$ and insert it in the optimization problem (2) to get

$$\begin{aligned} \max_{w_{U_A}^{D_2}} \quad & \max_q\{(P(q + q_2^c(w_{U_A}^{D_2})) - c_A - \hat{c}_B)q\} + q_2^c(w_{U_A}^{D_2})(w_{U_A}^{D_2} - c_A) + \\ & + \max_q\{(P(q + q_1^c(w_{U_A}^{D_2})) - w_{U_A}^{D_2} - c_B)q\} - \max_q\{(P(q + q_1^c(\hat{c}_A)) - \hat{c}_A - c_B)q\}, \end{aligned} \tag{3}$$

where we used the fact that $q_2^c(w_{U_A}^{D_2}) = x_{U_A}^{D_2}(w_{U_A}^{D_2})$.

Now we turn to the case in which the integrated firm is the second to negotiate with D_2. The optimization problem of $U_A - D_1$ is then the following:

$$\begin{aligned}
\max_{w_{U_A}^{D_2}} \quad & \max_q\{(P(q+q_2^c(w_{U_A}^{D_2})) - c_A - \hat{c}_B)q\} + x_{U_A}^{D_2}(w_{U_A}^{D_2})(w_{U_A}^{D_2} - c_A) + F_{U_A}^{D_2}, \\
s.t. \quad (i) \quad & \max_q\{(P(q+q_1^c(w_{U_A}^{D_2})) - w_{U_A}^{D_2} - c_B)q\} - F_{U_A}^{D_2} \geq \max_q\{(P(q+q_1^c(w_{U_A}^{D_2})) - \hat{c}_A - c_B)q\}, \\
(ii) \quad & \max_q\{(P(q+q_1^c(w_{U_A}^{D_2})) - w_{U_A}^{D_2} - c_B)q\} - F_{U_A}^{D_2} - F_{U_B}^{D_2} \geq \max\left[\max_q\{(P(q+q_1^c(\hat{c}_A)) - \hat{c}_A - \hat{c}_B])q\}, 0\right].
\end{aligned}$$

Constraint (i) is the same as in (2). It ensures that D_2 prefers to buy from the integrated firm rather than from $\hat{U}_A$ given that it accepts the offer from U_B. Constraint (ii) is new. It implies that D_2's profit when accepting the offers from U_A and U_B is larger than the maximum of the profits when either accepting the offers from $\hat{U}_A$ and $\hat{U}_B$, which is $\max q\{(P(q+q_1^c(\hat{c}_A)) - \hat{c}_A - \hat{c}_B])q\}$, or when rejecting all offers—which gives a profit of zero. U_A optimally sets $F_{U_A}^{D_2}$ such that the tighter constraint of the two holds with equality. Therefore, the value of $F_{U_A}^{D_2}$ is equal to

$$\begin{aligned}
\min\Bigg[&\max_q\{(P(q+q_1^c(w_{U_A}^{D_2})) - w_{U_A}^{D_2} - c_B)q\} - \max_q\{(P(q+q_1^c(w_{U_A}^{D_2})) - \hat{c}_A - c_B)q\}, \qquad (4)\\
&\max_q\{(P(q+q_1^c(w_{U_A}^{D_2})) - w_{U_A}^{D_2} - c_B)q\} - \max\left[\max_q\{(P(q+q_1^c(\hat{c}_A)) - \hat{c}_A - \hat{c}_B])q\}, 0\right] - F_{U_B}^{D_2}\Bigg].
\end{aligned}$$

Plugging this expression into the optimization problem of the integrated firm, we obtain

$$\begin{aligned}
\max_{w_{U_A}^{D_2}} \quad & \max q\{(P(q+q_2^c(w_{U_A}^{D_2})) - c_A - \hat{c}_B)q\} + q_2^c(w_{U_A}^{D_2})(w_{U_A}^{D_2} - c_A) + \\
& + \min\Bigg[\max_q\{(P(q+q_1^c(w_{U_A}^{D_2})) - w_{U_A}^{D_2} - c_B)q\} - \max_q\{(P(q+q_1^c(\hat{c}_A)) - \hat{c}_A - c_B)q\}, \\
& \max_q\{(P(q+q_1^c(w_{U_A}^{D_2})) - w_{U_A}^{D_2} - c_B)q\} - \max\left[\max_q\{(P(q+q_1^c(\hat{c}_A)) - \hat{c}_A - \hat{c}_B])q\}, 0\right] - F_{U_B}^{D_2}\Bigg] \qquad (5)
\end{aligned}$$

Comparing (3) with (5), it is evident that the terms involving $w_{U_A}^{D_2}$ are the same in both problems. This implies that, independent of the integrated firm being the first or the second to negotiate with D_2, $w_{U_A}^{D_2}$ is the same and is given by

$$\begin{aligned}
\arg\max_{w_{U_A}^{D_2}} \quad & \max q\{(P(q+q_2^c(w_{U_A}^{D_2})) - c_A - \hat{c}_B)q\} + q_2^c(w_{U_A}^{D_2})(w_{U_A}^{D_2} - c_A) \qquad (6)\\
& + \max_q\{(P(q+q_1^c(w_{U_A}^{D_2})) - w_{U_A}^{D_2} - c_B)q\}.
\end{aligned}$$

Clearly, $w_{U_A}^{D_2}$ is bounded by $\hat{c}_A$ since otherwise it is always profitable for D_2 to buy from $\hat{U}_A$. The above analysis is summarized by the following proposition:

Proposition 2.
The wholesale price $w_{U_A}^{D_2}$ that the integrated firm charges to D_2 is given (6) if the solution to (6) is smaller than $\hat{c}_A$. Otherwise, $w_{U_A}^{D_2} = \hat{c}_A$.

The proposition shows that foreclosure is no longer necessarily the optimal strategy for the integrated firm. The reason is the following: Due to the complementary input provider U_B, the integrated firm does no longer obtain the full profits it makes in the downstream market. Instead, as the first term in (6) shows, it just receives the profit that it would get when buying from the second source, $\hat{U}_B$, while the rest is extracted by U_B. As a result of this *expropriation* of U_B, as $U_A - D_1$ raises its wholesale price to $w_{U_A}^{D_2} = \hat{c}_A$, its downstream profit increases but this profit is squeezed by U_B via the fixed fee. Therefore, as a reaction to the expropriation threat, the integrated firm finds it optimal to lower $w_{U_A}^{D_2}$, thereby leaving more profit to D_2. Via doing so, the integrated firm can shield some of its profits from the hands of U_B and shift them to D_2 where it can extract them to some extent.

The downstream unit of the integrated firm now knows if its downstream competitor has bought from U_A or the inefficient source for input A, $\hat{U}_A$. Thus, it can tailor its downstream quantity to the competitor's decision and produce a different quantity if the competitor has bought from $\hat{U}_A$ than if it has bought from its upstream unit. As a consequence, the integrated firm can extract more from D_2: If D_2 buys at the higher input price $\hat{c}_A$, $U_A - D_1$ reacts by increasing its quantity. This in turn induces D_2 to lower its quantity, thereby leaving less profit to D_2. Thus, this *information effect* inherently gives the integrated firm an advantage in extracting profits from D_2.[9]

It is instructive to see how $w_{U_A}^{D_2}$ changes if the competitive advantage of the complementary input provider U_B varies. This advantage is expressed by $\hat{c}_B$, i.e., if the second source gets more inefficient, implying that $\hat{c}_B$ rises, U_B can extract more profits from downstream firms. Suppose first that $c_B = \hat{c}_B$, which implies that U_B has no market power. In this case, (6) is the same as in Rey and Tirole (2007). Since there is no expropriation threat by U_B, the integrated firm obtains all of its downstream revenue. As a consequence, it receives the largest profits if the downstream market moves closer to monopoly in which D_2 produces a smaller amount. This implies that it is optimal for $U_A - D_1$ to follow a foreclosure strategy and set $w_{U_A}^{D_2} = \hat{c}_A$.

However, if $\hat{c}_B$ rises from c_B, the expropriation problem becomes relevant. As the next proposition shows, the integrated firm then lowers $w_{U_A}^{D_2}$ continuously to shift more rent to D_2.

Proposition 3.
Suppose that foreclosure is not optimal. Then, the wholesale price that the integrated firm charges from D_2 is strictly decreasing in the market power of the complementary input provider U_B.

[9]This effect is also present in Rey and Tirole (2007), but it emerges in a more effective fashion in our set-up because the integrate firm may depart from foreclosure at equilibrium.

Proof See the appendix.

The preceding analysis implies that the anticompetitive consequences of vertical integration are less drastic when complementary inputs are needed. In particular, the effect may even be completely reversed if the incentive for the integrated firm to lower the wholesale price becomes so large that it find it profitable to price below marginal costs. From the last proposition it is evident that such a case arises if $\hat{c}_B$ is large enough.

So far, we were not concerned with the question under which conditions vertical integration is indeed profitable. This is important to determine e.g., if cases in which vertical integration is pro-competitive can indeed occur because a merger is nevertheless profitable for U_A and D_1. Therefore, we will now deal with this question.

To do so we first need to determine the expected profit of the integrated firm. If $U_A - D_1$ is the first to negotiate with D_2, it receives a profit of (3). Instead, if the integrated firm is second to negotiate with D_2, the expression for its profits is given by

$$\begin{aligned}\Pi^{U_A-D_1}(w_{U_A}^{D_2}) &= \max q\{(P(q+q_2^c(w_{U_A}^{D_2})) - c_A - \hat{c}_B)q\} + q_2^c(w_{U_1}^{D_2})(w_{U_A}^{D_2} - c_A) + \\ &+ \min\Big[\max_q\{(P(q+q_1^c(w_{U_A}^{D_2})) - w_{U_A}^{D_2} - c_B)q\} - \max_q\{(P(q+q_1^c(\hat{c}_A)) - \hat{c}_A - c_B)q\}, \\ &\max_q\{(P(q+q_1^c(w_{U_A}^{D_2})) - w_{U_A}^{D_2} - \hat{c}_B)q\} - \max\Big[\max_q\{(P(q+q_1^c(\hat{c}_1)) - \hat{c}_A - \hat{c}_B])q\}, 0\Big]\Big], \end{aligned} \tag{7}$$

which uses the fact that $F_{U_B}^{D_2} = \max_q\{(P(q+q_1^c(w_{U_A}^{D_2})) - w_{U_A}^{D_2} - c_B)q\} - \max_q\{(P(q+q_1^c(\hat{c}_A)) - w_{U_A}^{D_2} - \hat{c}_B])q\}$ when U_B is first in negotiating with D_2.

Therefore, the expected profit of the integrated firm can be written as

$$\begin{aligned}\Pi^{U_A-D_1}(w_{U_A}^{D_2}) &= \max_q\{(P(q+q_2^c(w_{U_A}^{D_2})) - c_A - \hat{c}_B)q\} + q_2^c(w_{U_A}^{D_2})(w_{U_A}^{D_2} - c_A) \\ &+\frac{1}{2}\left[\max_q\{(P(q+q_1^c(w_{U_A}^{D_2})) - w_{U_A}^{D_2} - c_B)q\} - \max_q\{(P(q+q_1^c(\hat{c}_A)) - \hat{c}_A - c_B)q\}\right] \\ &+\frac{1}{2}\min\Big\{\max_q\{(P(q+q_1^c(w_{U_A}^{D_2})) - w_{U_A}^{D_2} - c_B)q\} - \max_q\{(P(q+q_1^c(\hat{c}_A)) - \hat{c}_A - c_B)q\}, \\ &\max_q\{(P(q+q_1^c(w_{U_A}^{D_2})) - w_{U_A}^{D_2} - \hat{c}_B)q\} - \max\left[\max_q\{(P(q+q_1^c(\hat{c}_A)) - \hat{c}_A - \hat{c}_B])q\}, 0\right]\Big\}. \end{aligned} \tag{8}$$

We can now analyze if U_A and D_1 find it profitable to integrate even in case in which vertical integration would be pro-competitive. The next proposition shows that this can indeed be the case.

Proposition 4.
Suppose that $w_{U_A}^{D_2} = c_A$ after integration. Then, vertical integration is profitable for U_A and D_1 if

$$\max_q\{[P(q+q^c) - \hat{c}_A - c_B]q\} - \max_q\{(P(q+q_1^c(\hat{c}_A)) - \hat{c}_A - c_B)q\} >$$

$$\frac{1}{2}\Big[\Pi^c + \max\Big[\max_q\{(P(q+q_1^c(\hat{c}_A)) - \hat{c}_A - \hat{c}_B])q\}, 0\Big] - \tag{9}$$
$$- \max_q\{[P(q+q^c) - c_A - \hat{c}_B]q\} - \max_q\{(P(q+q_1^c(\hat{c}_A)) - \hat{c}_A - c_B)q\}\Big].$$

Proof See the appendix.

An implication of the result that vertical integration with $w_{U_A}^{D_2} = c_A$ can be profitable is that, by continuity reasons, even a vertical merger after which the wholesale price of the integrated firm is below marginal costs can be profitable. This is the case although industry profits fall compared to the no-integration case since prices of both downstream firms decrease. The reason why vertical integration is nevertheless profitable is the information effect. Although the downstream profit of D_1 is lower, U_A can extract more rent from D_2 since its downstream unit D_1 is informed about the input supplier of D_2, implying that D_1 extends its quantity if D_2's input price is $\hat{c}_A$ instead of $w_{U_A}^{D_2}$. This effect is the stronger, the lower $w_{U_A}^{D_2}$ is.

This intuition can also be seen in condition (9). As shown in the proofs of Propositions 1 and 4, the right-hand side of (9) is positive by Jensen's inequality, due to the fact that the optimized profit function is convex in marginal costs. However, the left-hand side is also positive, due to the information effect. The first term, $\max_q\{[P(q+q^c) - \hat{c}_A - c_B]q\}$, is the rent that U_A has to leave to D_2 in case of no integration. As is evident, the quantity of D_1 is q_1^c in this case since D_1 does not observe D_2's input price. By contrast, the second term, $\max_q\{(P(q+q_1^c(\hat{c}_A)) - \hat{c}_A - c_B)q\}$, is the rent that the integrated firm has to leave to D_2. Here, the downstream unit D_1 reacts to D_2's input price by producing a quantity of $q_1^c(\hat{c}_A) > q_1^c$. Therefore, although the quantities produced on the equilibrium path are unchanged with integration if $w_{U_A}^{D_2} = c_A$, the upstream unit of the integrated can extract more rents from D_2 than without integration, thereby rendering integration potentially profitable. We will later show in a specific example, that there are indeed cases under which vertical integration is pro-competitive, which implies that the competitive effect of vertical integration in case of complementary inputs can be completely reversed.

Another question of interest is under which conditions vertical integration can be unprofitable. In particular, we are interested in relating the profitability of integration to the market power of the input suppliers, expressed by $\hat{c}_j$, $j = A, B$. First, we consider a change in $\hat{c}_B$. From the discussion above, we know that the expropriation threat of U_B gets larger the larger is $\hat{c}_B$. Therefore, one would expect that the larger is the market power of the rival input supplier, the less profitable vertical integration for U_A is. Indeed, this result is confirmed in the next proposition.

Proposition 5.
The profitability of a vertical merger of U_A falls in the market power of U_B.

Proof See the appendix.

Finally, we turn to the market power of the integrating firm U_A. Here the comparative statics are not clear-cut over the whole range of $\hat{c}_A$. However, the result is clear-cut if $\hat{c}_A$ is relatively large.

Proposition 6.
There exists a $\hat{c}_A'$ such that for all $\hat{c}_A \geq \hat{c}_A'$ the profitability of a vertical merger falls in $\hat{c}_A$.

Proof See the appendix.

Proposition 7 implies that if U_A is much more profitable than the bypass alternative for input A, vertical integration may not be profitable. The reason for this result is the following: In case of no integration U_A can extract the profit of D_1 to a large extent because the difference between $\hat{c}_A$ and c_A is large. After integration, U_B is the only one exerting bargaining power vis-à-vis D_1. Therefore, via integrating U_A loses its grip on the profit of the downstream unit since D_1 is now fully vulnerable in the negotiation with U_B. This implies that vertical integration does not pay off if the bypass alternatives are very inefficient.[10]

This last result is markedly different from the conclusion in Rey and Tirole (2007) that vertical integration is particularly profitable for efficient firms. *An interesting—and perhaps counter-intuitive—implication of our analysis is that, in an industry with highly complementary inputs, very efficient firms are less likely to vertically integrate than firms that are only slightly more efficient than their competitors.*

Examples with specific demand functions

In the following, we illustrate our analysis by employing two specific demand functions. First, we use a standard linear demand function and completely characterize the equilibrium conditions for vertical integration to be profitable and determine under which conditions foreclosure is not optimal. Second, we demonstrate that with an exponential demand function vertical integration results in pro-competitive market outcomes.

To begin with, we use the linear demand function $P(q_1 + q_2) = \max\{0, 1 - q_1 - q_2\}$. It is easy to check that Assumption 1 is fulfilled in this case if $\Delta_j < (1 - c_A - c_B)/2$, where $\Delta_j = \hat{c}_j - c_j$. We obtain the following results.

Proposition 7.

[10] Note that if an integrated firm could credibly commit to set its internal wholesale price above marginal costs, this result would not occur. However, since this is impeded by secret internal renegotiation's incentives, vertical integration can be unprofitable.

- If $\Delta_B \in (0, (1-c_A-c_B)/3)$ one has that:
  - If $0 < \Delta_A \leq (1-c_A-c_B-3\Delta_B)/2 \equiv \underline{\Delta}_A$, the integrated firm $U_A - D_1$ optimally follows a foreclosure strategy and sets $w_{D_2}^{U_A} = \hat{c}_A$. Moreover, it is profitable for U_A and D_1 to integrate.
  - Define

    $$\tilde{\Delta}_A \equiv [2(1-c_A-c_B) - 6\Delta_B + \sqrt{[9\Delta_B + 11(1-c_A-c_B)][(1-c_A-c_B) - 3\Delta_B}]/7$$

    and

    $$\tilde{\tilde{\Delta}}_A \equiv 2(1-c_A-c_B) - \sqrt{(1-c_A-c_B)(6\Delta_B + 1 - c_A - c_B) + 9/2\Delta_B^2}.$$

    If $\underline{\Delta}_A < \Delta_A \leq \min\{\tilde{\tilde{\Delta}}_A, \tilde{\Delta}_A\}$, it is profitable for U_A and D_1 to integrate and set $w_{D_2}^{U_A} = (1 + c_A + 2c_B - 3\hat{c}_B)/2 < \hat{c}_A$.
  - If $\min\{\tilde{\Delta}_A, \tilde{\tilde{\Delta}}_A\} < \Delta_A < (1-c_A-c_B)/2$, it is not profitable for U_A and D_1 to integrate.
- If $\Delta_B \in ((1-c_A-c_B)/3, (1-c_A-c_B)/2)$ integration is not profitable.

Proof See the appendix.

First, the proposition shows that if Δ_B were nil, then foreclosure is always more profitable than non-integration.[11] Indeed, as discussed above, if Δ_B is equal to zero then the rent that the provider of the complementary input would be able to extract from the integrated firm collapses, and so does the expropriation problem faced by $U_A - D_1$.

Second, if Δ_A is in a middle range, it is not optimal for U_A to pursue a foreclosure strategy but instead set the per-unit price charged to D_2 only to a value that is smaller than $\hat{c}_A$. Nevertheless, vertical integration is still profitable. This is the case because Δ_B is not too high so that the expropriation effect, although being at work, is not large enough to render vertical integration unprofitable.

Finally, if either Δ_A or Δ_B are large, vertical integration is no longer profitable. If Δ_B is large, the expropriation effect becomes dominating, independent of the value of Δ_A. If instead Δ_B is relatively small but Δ_A is large, firm U_A can shield some profits of D_1 from U_B by staying unintegrated.

An effect that can not occur under linear demand is that vertical integration is both pro-competitive and profitable. In what follows, we consider a numerical example with an exponential demand function that demonstrates that such a case can indeed arise. Suppose that the demand function is given by $P(q_1 + q_2) = k - \exp\{q_1 + q_2\}$. We obtain the following result.

[11] This is the case because from Assumption 1 we know that $\Delta_A \leq (1-c_A-c_B)/2$, and thus at $\Delta_B = 0$ we are always in the first region.

Example 1. Suppose $k = 10$, $c_A = c_B = 0.2$ and $\hat{c}_A = \hat{c}_B = 2.7$. Then one has the following results:

- The sum of the profit of U_A and D_1 under non-integration is equal to 3.607.
- The optimal linear price set by the integrated company $U_A - D_1$ is given by $w_{D_2}^{U_A} = 0.059 < 0.2 = c_A$.
- The profit of the integrated firm $U_A - D_1$ is equal to 3.697.

Therefore, it is profitable for U_A and D_1 to integrate. Importantly, Assumptions 1 is satisfied in this example.

In this example, vertical integration has a pro-competitive effect. As remarked above, the information advantage retained by the integrated company increases as $w_{D_2}^{U_A}$ decreases. In Example 1, this information effect is so large that it is still profitable for U_A and D_1 to integrate although industry profits fall compared to the no-integration case. The intuition behind this result is that the exponential demand function is very concave. The information effect is based on firm j knowing that the input costs of firm $-j$ are higher, which results in firm j producing a larger quantity. But the consequences on the profit of firm $-j$ are more drastic if the demand function is very concave because the increase in q_j leads to a larger price decrease in this case. Thus, in the model the integrated firm benefits to a larger extent from the information effect, the more concave the demand function is. *This result also gives predictions on the conditions under which vertical integration with complementary input providers is likely to be pro-competitive; ceteris paribus, this is the case if the demand function is very concave.*

6 Public offers

In this section, we briefly discuss the case in which offers are public, that is, each downstream firm observes not only the offers to itself but also the ones to its rival. As mentioned by e.g., Rey and Tirole (2007), public offers are less realistic in many circumstances because negotiations often take place privately and hard information about these contracts is relatively difficult to communicate. The analysis can serve as benchmark case to the secret offers case.

The goal of this section is to demonstrate in a simple way that vertical integration is never profitable in case of public offers. The reason is that the upstream firms can extract as much as possible from the downstream firms already under non-integration, taking into account the constraint that downstream firms can buy from the bypass alternatives. Thus, foreclosure is not necessary to increase industry profits, and so vertical integration cannot improve the profits of an upstream firm. However, the integrated firm's problem of expropriation conduct by the complementary input provider remains. As a consequence, vertical integration yields weakly lower profits to the integrated firms and, therefore, does not occur in equilibrium.

To show this intuition in a simple way, we concentrate our analysis to the case in which upstream firms do not charge wholesale prices that are below marginal costs. This simplifies the analysis without affecting the main point.

Consider the same framework as above but now suppose that offers are public, that is, each downstream firm before deciding which offer to accept can not only observe to offers made to itself but also the one made to the rival. Suppose first that there is no integration. The goal of the upstream firms is to maximize industry profits in order to extract these profits from the downstream firms, given the alternative sources. The easiest way to do so is to offer per-unit prices of $w_{U_A}^{D_i} = c_A$ and $w_{U_B}^{D_i} = c_B$ to firm D_i and very high wholesale prices to firm D_{-i}. If there are no alternative sources, D_i would then buy the monopoly quantity and each upstream firm receives expected profits of half of the monopoly profit. However, firm D_{-i} would buy from the alternative sources in this case. Therefore, it is optimal for the upstream firms to serve D_{-i} themselves at wholesale prices of $w_{U_A}^{D_{-i}} = \hat{c}_A$ and $w_{U_B}^{D_{-i}} = \hat{c}_B$. This implies that downstream firms play an asymmetric Cournot game in the downstream market in which they produce quantities of

$$q(c) = \arg\max_q \{(P(q + q(\hat{c})) - c_A - c_B)q\}$$

and

$$q(\hat{c}) = \max\left[\arg\max_q \{(P(q + q(c)) - \hat{c}_A - \hat{c}_B)q\}, 0\right].$$

Via inducing these quantities, the upstream firms are as close as possible to the monopoly profit.

As a consequence, we have that the fixed fees to firm D_{-i} are nil, while the fixed fees to firm D_i are given by

$$F_{U_j}^{D_i} = \max_q \{(P(q + q(\hat{c})) - c_j - c_{-j})q\} - \max_q \{(P(q + q(\hat{c})) - \hat{c}_j - c_{-j})q\}, \tag{10}$$

in case firm U_j is the first to offer to D_i, and by

$$F_{U_j}^{D_i} = \max_q \{(P(q + q(\hat{c})) - c_j - \hat{c}_{-j})q\} - \max\left[\max_q \{(P(q + q(\hat{c})) - \hat{c}_j - \hat{c}_{-j})q\}, 0\right], \tag{11}$$

in case firm U_j is the second to offer to D_i.[12]

[12] If upstream firms had the possibility to set wholesale prices below marginal costs, it can be profitable for them to do so under some circumstances. The reason is that $q(\hat{c})$ is then more likely to be zero and so upstream firms have to leave a smaller rent to downstream firms. However, the quantity in the downstream market then becomes biased as well which has a profit reducing effect. To determine which of these effects dominates is a tedious matter because it depends on the particular shape of the demand function and the cost differences. Thus, we confine our analysis to this simpler case. As will become clear from the next paragraph, the result on the profitability of vertical mergers is not affected by this assumption.

Now, let us look the case of a vertical merger between U_j and D_i. Since D_{-i} is operating with the highest possible marginal costs in case of no integration, such a vertical merger cannot raise industry profits because they are already as close as possible to the monopoly profits. In addition, there is no information effect either, since D_{-i} buys input i at a price of $\hat{c}_j$ in case of no integration already, and D_i knows this. The only effect that is present is that U_{-j} can now extract more profits from D_i than in case of no integration since U_j has no more bargaining power on D_i. Overall, the result is that the merged firm would be put in difficulty as much as in a framework with secret offers due to the expropriation conduct of the complementary input provider but the degree of downstream competition is not affected by the vertical merger. As a consequence, a vertical merger cannot be profitable.

7 Conclusion

This paper analyzed the profitability and consequences of vertical integration in a model where downstream firms need complementary inputs, and these inputs are supplied by producers that exert market power vis-á-vis downstream firms. We showed that the presence of the complementary input supplier gives rise to an expropriation conduct that is not present in the case when only one input is necessary for production. A consequence of this is that an integrated organization may not find it optimal to foreclose its downstream rival because the complementary input supplier can then extract large profits from the integrated firm. Instead, via setting a lower wholesale price to the downstream rival, the integrated firm shields some downstream profits from the expropriation conduct. We show that this effect can be so large that the integrated company sets a wholesale price below marginal costs, which has profound consequences for antitrust policy in these kind of industries. Although aggregate industry profit falls when pricing below marginal cost, vertical integration is profitable because the downstream unit of the integrated can now observe from which upstream firm the rival buys, taylor its quantity accordingly and extract more profits from the same rival. Finally, and in contrast to previous analysis, we find that vertical mergers are more likely to be unprofitable for very efficient upstream firms because for them the expropriation conduct of the complementary input producer is most harmful.

We restricted our attention to the case in which there is just one vertical merger. However, in our set-up it is also natural to consider the case of a counter-merger between U_B and D_2. In particular, it is interesting to analyze if the first merger increases or decreases the incentives for a second merger. This can give new insights on the conditions under which an asymmetric outcome can arise in an industry, in which some firms stay separated while others are integrated. Such an analysis would also show how the new effects identified in this paper—e.g., the expropriation conduct and the information effect—play out in case both chains are integrated and how that affects output prices and welfare.

Another direction for future research is to consider the case of Bertrand competition in the downstream market. In our analysis we focussed on the case of Cournot competition—in line with Rey and Tirole (2007)—which implies that firms' strategy variables are strategic substitutes. It is also natural to consider the opposite case of strategic complements, for example, via analyzing a model with differentiated Bertrand competition as in O'Brien and Shaffer (1992). It is of interest how the problem of being expropriated, that drives many of our results, is attenuated once the mode of competition in the downstream market is changed and if our results are robust to this extension. We plan to do that in the future.

A Appendix

Proof of Proposition 1

We first show that upstream firm U_j sets the per-unit price equal to marginal costs when making an offer to a downstream firm D_i.

We solve the game by backward induction. Thus, we start with the *second stage*, the downstream stage. Since contract offers to a downstream firm D_i, $i = 1, 2$, are not observable to the rival firm D_{-i} and downstream firms hold passive conjectures, D_i expects D_{-i} to produce the candidate equilibrium quantity q_{-i} independent of the contract offers it receives.[13] Therefore, if D_i accepts offers such that its input costs are w_j for input j and w_{-j} for input $-j$, due to the one-to-one technology it will produce a quantity q_i that is given by

$$q_i \;=\; \arg\max_q \;\{(P(q + q_{-i}) - w_j - w_{-j})\, q\}\,. \tag{12}$$

In the following of this proof, for simplicity we denote the downstream profit $(P(q_i + q_{-i}) - w_j - w_{-j})\, q_i$ by $\Pi_i(q_i(w_j,\, w_{-j}), q_{-i})$.

Now we turn to the *first stage*, the offer game. Again we proceed by backward induction.

Assume that U_j is the second to offer to D_i. Since $\hat{U}_j$ is less efficient than U_j, it is willing to offer a contract of $w^{D_i}_{\hat{U}_j} = \hat{c}_j$ and $F^{D_i}_{\hat{U}_j} = 0$. Since U_j is the second to offer, it faces two constraints. First, it has to set its tariff such that D_i prefers to buy the input from U_j and not from $\hat{U}_j$, given that it also accepts the offer from the provider of input $-j$ in the first stage. Second, U_j's tariff has to be such that D_i does not prefer to buy from both bypass alternatives. Therefore, U_j's optimization problem can be written as

$$\begin{aligned}
\max_{w^{D_i}_{U_j}} \quad & x^{D_i}_{U_j}(w^{D_i}_{U_j}, E[w^*_{-j}])(w^{D_i}_{U_j} - c_j) + F^{D_i}_{U_j} \\
s.t. \quad (i) \quad & \Pi_i(q_i(w^{D_i}_{U_j}, E[w^*_{-j}]), q_{-i}) - E[F^*_{-j}] - F^{D_i}_{U_j} \geq \Pi_i(q_i(\hat{c}_j, E[w^*_{-j}]), q_{-i}) - E[F^*_{-j}] \\
(ii) \quad & \Pi_i(q_i(w^{D_i}_{U_j}, E[w^*_{-j}]), q_{-i}) - E[F^*_{-j}] - F^{D_i}_{U_j} \geq \max\left[\Pi_i(q_i(\hat{c}_j, \hat{c}_{-j}), q_{-i}), 0\right].
\end{aligned}$$

Here, $E[w^*_{-j}]$ denotes the wholesale price at which D_i buys input $-j$ and $E[F^*_{-j}]$ denotes the fixed fee that i pays for input $-j$. Firm U_j receives as a profit from D_i the margin of its wholesale price over marginal costs times the quantity that D_i buys, denoted by $x^{D_i}_{U_j}$, plus the fixed fee. Constraint (i) states that the profit of D_i from accepting the offer of U_j must be weakly larger than accepting the offer of $\hat{U}_j$ given that D_i pays a wholesale price of w^*_{-j} to the provider of input $-j$. Constraint (ii) states that D_i's profit when accepting the offers from U_j and the provider of

[13]The concept of passive conjectures is introduced by by Hart and Tirole (1990) in a setting with substitute inputs.

input $-j$ is larger than the maximum of the profits when either accepting the offers from $\hat{U}_j$ and $\hat{U}_{-j}$, which is $\Pi_i(q_i(\hat{c}_j, \hat{c}_{-j}), q_{-i})$, or when rejecting all offers—which gives a profit of zero.

Since D_i does not observe the offer made to D_{-i} and holds passive beliefs, we can treat q_{-i} as a constant in the above maximization problem.

Assume first that constraint (i) *is the binding one.* It is optimal for U_j to set $F_{U_j}^{D_i}$ as large as possible, which implies

$$F_{U_j}^{D_i} = \Pi_i(q_i(w_{U_j}^{D_i}, E[w_{-j}^*]), q_{-i}) - \Pi_i(q_i(\hat{c}_j, E[w_{-j}^*]), q_{-i}).$$

The maximization problem can then be written as

$$\max_{w_{U_j}^{D_i}} \; x_{U_j}^{D_i}(w_{U_j}^{D_i}, E[w_{-j}^*])(w_{U_j}^{D_i} - c_j) + \Pi_i(q_i(w_{U_j}^{D_i}, E[w_{-j}^*]), q_{-i}) - \Pi_i(q_i(\hat{c}_j, E[w_{-j}^*]), q_{-i}). \tag{13}$$

The last term is independent of $w_{U_j}^{D_i}$. Because of the envelope theorem, the effect of a change of q_i in response to a change in $w_{U_j}^{D_i}$ on the profit of D_i is zero. Thus, differentiating (13) with respect to $w_{U_j}^{D_i}$ gives

$$(w_{U_j}^{D_i} - c_j)\frac{\partial x_{U_j}^{D_i}}{\partial w_{U_j}^{D_i}} + x_{U_j}^{D_i} - q_i = 0.$$

Since the downstream transformation technology is one-to-one and downstream firms transform all input to output we have that $q_i = x_{U_j}^{D_i}$. Moreover, given that $(\partial x_{U_j}^{D_i}/\partial w_{U_j}^{D_i}) < 0$, we obtain $w_{U_j}^{D_i} = c_j$, i.e. U_j optimally sets the per-unit price equal to marginal cost.

Suppose now that constraint (ii) *is the binding one.* It is optimal for U_j to set $F_{U_j}^{D_i}$ as to satisfy the following condition:

$$F_{U_j}^{D_i} = \Pi_i(q_i(w_{U_j}^{D_i}, E[w_{-j}^*]), q_{-i}) - E[F_{-j}^*] - \max\left[\Pi_i(q_i(\hat{c}_j, \hat{c}_{-j}), q_{-i}), 0\right].$$

In this case, the maximization problem can be rewritten as

$$\max_{w_{U_j}^{D_i}} \; x_{U_j}^{D_i}(w_{U_j}^{D_i}, E[w_{-j}^*])(w_{U_j}^{D_i} - c_j) + \Pi_i(q_i(w_{U_j}^{D_i}, E[w_{-j}^*]), q_{-i}) - E[F_{-j}^*] - \max\left[\Pi_i(q_i(\hat{c}_j, \hat{c}_{-j}), q_{-i}), 0\right]. \tag{14}$$

As in problem (13), the last term is independent of $w_{U_j}^{D_i}$. Again, invoking the envelope theorem, the effect of a change of q_i in response to a change in $w_{U_j}^{D_i}$ on the profit of D_i is zero. Thus, differentiating (14) with respect to $w_{U_j}^{D_i}$ gives

$$(w_{U_j}^{D_i} - c_j)\frac{\partial x_{U_j}^{D_i}}{\partial w_{U_j}^{D_i}} + x_{U_j}^{D_i} - q_i = 0.$$

Using the same arguments as above, we have that $w_{U_j}^{D_i} = c_j$.

Now, let us consider the problem of U_j when it is the first to offer to D_i. As above, $\hat{U}_j$ offers $w_{\hat{U}_j}^{D_i} = \hat{c}_j$ and $F_{\hat{U}_j}^{D_i} = 0$. Moreover, U_j knows that U_{-j}, which is here the second to approach D_i, offers a wholesale price of $w_{U_{-j}}^{D_i} = c_{-j}$. The maximization problem with respect to $w_{U_j}^{D_i}$ is given by

$$\begin{aligned} \max_{w_{U_j}^{D_i}} \quad & x_{U_j}^{D_i}(w_{U_j}^{D_i}, c_{-j})(w_{U_j}^{D_i} - c_j) + F_{U_j}^{D_i} \\ s.t. \quad & \Pi_i(q_i(w_{U_j}^{D_i}, c_{-j}), q_{-i}) - F_{U_j}^{D_i} \geq \Pi_i(q_i(\hat{c}_j, c_{-j}), q_{-i}). \end{aligned} \tag{15}$$

Here U_j faces the constraint that D_i accepts the offer of U_j only in case D_i can ensure itself weakly larger profits from accepting U_j's offer than from buying the input from $\hat{U}_j$ at a price of $\hat{c}_j$ and a fixed fee of zero.

The problem in (15) is isomorphic to the one of U_j when it takes turn as second in approaching D_i and constraint (i) is binding. Therefore, the firm's maximization problem reduces to

$$\max_{w_{U_j}^{D_i}} x_{U_j}^{D_i}(w_{U_j}^{D_i}, c_{-j})(w_{U_j}^{D_i} - c_j) + \Pi_i(q_i(w_{U_j}^{D_i}, c_{-j}), q_{-i}) - \Pi_i(q_i(\hat{c}_j, c_{-j}), q_{-i}). \tag{16}$$

Hence, differentiating (16) with respect to $w_{U_j}^{D_i}$ gives

$$(w_{U_j}^{D_i} - c_j)\frac{\partial x_{U_j}^{D_i}}{\partial w_{U_j}^{D_i}} + x_{U_j}^{D_i} - q_i = 0,$$

which, again, leads to $w_{U_j}^{D_i} = c_j$.

Summarizing, both downstream firms face marginal costs of $c_A + c_B$. Therefore, the maximization problem of downstream firm i is given by

$$\max_q \left\{(P(q + q_{-i}) - c_A - c_B)\, q\right\}, \ i = 1, 2.$$

It thus follows that each downstream firm produces the Cournot quantity for marginal costs of $c_A + c_B$, that is,

$$q_1 = q_2 = q^c = \arg\max_q \left\{(P(q + q^c) - c_A - c_B)\, q\right\}.$$

Finally, we turn to the determination of the fixed fees. From above, it is evident that if U_j is the first to offer to D_i, it sets a fixed fee of

$$F_{U_j}^{D_i} = \Pi_i(q^c, q^c) - \Pi_i(q_i(\hat{c}_j, c_{-j}), q^c). \tag{17}$$

To the contrary, if U_j is the second to offer to D_i, the fixed fee depends on constraint (i) or (ii) being the tighter one. Since it is optimal for U_j to set $w_{U_j}^{D_i} = c_j$, a binding constraint (i) can be written as

$$F_{U_j}^{D_i} = \Pi_i(q^c, q^c) - \Pi_i(q_i(\hat{c}_j, c_{-j}), q^c). \tag{18}$$

Turning to the second constraint we know that if U_{-j} is the first to offer its fixed fee is given by

$$F_{U_{-j}}^{D_i} = \Pi_i(q^c, q^c) - \Pi_i(q_i(c_j, \hat{c}_{-j}), q^c).$$

Inserting this into a binding constraint (ii) and rearranging, we obtain

$$F_{U_j}^{D_i} = \Pi_i(q_i(c_j, \hat{c}_{-j}), q^c) - \max\left[\Pi_i(q_i(\hat{c}_j, \hat{c}_{-j}), q^c), 0\right]. \tag{19}$$

To determine which of the two constraints is tighter, we have to compare the right-hand sides of (18) and (19). Subtracting the right-hand side of (19) from the right-hand side of (18) and rearranging, we obtain that the right-hand side of (18) is larger than the one of (19) if

$$\Pi_i(q^c, q^c) + \max\left[\Pi_i(q_i(\hat{c}_j, \hat{c}_{-j}), q^c), 0\right] > \Pi_i(q_i(\hat{c}_j, c_{-j}), q^c) + \Pi_i(q_i(c_j, \hat{c}_{-j}), q^c). \tag{20}$$

Now suppose that $\Pi_i(q_i(\hat{c}_j, \hat{c}_{-j}), q^c) > 0$. Then (20) writes as

$$\Pi_i(q^c, q^c) + \Pi_i(q_i(\hat{c}_j, \hat{c}_{-j}), q^c) > \Pi_i(q_i(\hat{c}_j, c_{-j}), q^c) + \Pi_i(q_i(c_j, \hat{c}_{-j}), q^c). \tag{21}$$

The first term on the left-hand side of (21) is the profit of firm D_i given that its marginal costs are $c_A + c_B$ while the second term on the left-hand side is the profit of firm D_i given that its marginal costs are $\hat{c}_A + \hat{c}_B$. Instead, the two terms on the right-hand side of (21) represent firm D_i's profit given that its marginal costs are $\hat{c}_A + c_B$ and $c_A + \hat{c}_B$, respectively.[14] By Jensen's inequality, (21) is fulfilled if the profit function of D_i is convex in marginal costs. Now, differentiating Π_i with respect to marginal costs $C := c'_A + c'_B$ and using the envelope theorem, we obtain

$$\frac{\partial \Pi_i}{\partial C} = -q_i < 0$$

and

$$\frac{\partial^2 \Pi_i}{\partial C^2} = -\frac{\partial q_i}{\partial C} > 0.$$

Thus, Π_i is convex in marginal costs and (21) holds. The only difference between (20) and (21) is that in (20) the second term is given by the maximum between $\Pi_i(q_i(\hat{c}_j, \hat{c}_{-j}), q^c)$ and 0 while in (21) it is just $\Pi_i(q_i(\hat{c}_j, \hat{c}_{-j}), q^c)$. Therefore, the left-hand side of (20) is weakly larger than the one of (21) implying that (20) is fulfilled as well. This implies that the fixed fee given by (18) is larger than the one given by (19) and constraint (ii) is the tighter one. Thus, if firm U_j is the second to offer it sets a fixed fee that is given by (19).

To conclude, we have that if firm U_j is the first to offer to D_i, it proposes a contract in which the wholesale price is given by $w_{U_j}^{D_i} = c_j$ and the fixed fee is given by $F_{U_j}^{D_i} = \Pi_i(q^c, q^c) - \Pi_i(q_i(\hat{c}_j, c_{-j}), q^c) = \Pi^c - \max_q \left\{ \left(P(q + q^c) - \hat{c}_j - c_{-j}\right) q \right\}$. Instead, if firm U_j is the second to offer to

[14] Note that in each of the four terms in (21) firm D_{-i} produces a quantity of q^c.

D_i, it proposes a contract in which the wholesale price is again given by $w_{U_j}^{D_i} = c_j$ and the fixed fee is given by $F_{U_j}^{D_i} = \Pi_i(q_i(c_j, \hat{c}_{-j}), q^c) - \max\left[\Pi_i(q_i(\hat{c}_j, \hat{c}_{-j}), q^c), 0\right] = \max_q \left\{\left(P(q+q^c) - c_j - \hat{c}_{-j}\right) q\right\} - \max\left[\max_q \left\{\left(P(q+q^c) - \hat{c}_j - \hat{c}_{-j}\right) q\right\}, 0\right]$. ■

Proof of Proposition 3

If foreclosure is not optimal, i.e., $w_{U_A}^{D_2} < \hat{c}_A$, there is an interior solution to (6), which implies that (6) is concave. Deriving the first-order condition for (6) and using the Implicit Function Theorem, we obtain that

$$\text{sign}\left\{\frac{\partial w_{U_A}^{D_2}}{\partial \hat{c}_B}\right\} = \text{sign}\left\{\frac{\partial q_2^c(w_{U_A}^{D_2})}{\partial w_{U_A}^{D_2}} \frac{\partial q}{\partial \hat{c}_B}\left(P'(q + q_2^c(w_{U_A}^{D_2})) + P''(q + q_2^c(w_{U_A}^{D_2}))q\right)\right\},$$

where $q = \arg\max_q\{(P(q + q_2^c(w_{U_A}^{D_2})) - c_A - \hat{c}_B)q\}$.

We know that $\partial q_2^c(w_{U_A}^{D_2})/\partial w_{U_A}^{D_2} < 0$ and $\partial q/\partial \hat{c}_B < 0$, implying that the sign of $\partial w_{U_A}^{D_2}/\partial \hat{c}_B$ equals the sign of $P'(q + q_2^c(w_{U_A}^{D_2})) + P''(q + q_2^c(w_{U_A}^{D_2}))q$. But since $P'(\cdot) < 0$ and $P''(\cdot) \leq 0$, we obtain that the sign is negative. ■

Proof of Proposition 4

From (1) we know that the sum of the profits of U_A and D_1 without integration is given by

$$\Pi^c + \max_q\{[P(q + q^c) - c_A - \hat{c}_B]q\} - \max_q\{[P(q + q^c) - \hat{c}_A - c_B]q\}. \tag{22}$$

To determine the expected profit with integration, we need determine if

$$\max_q\{(P(q + q_1^c(w_{U_A}^{D_2})) - w_{U_A}^{D_2} - c_B)q\} + \max\left[\max_q\{(P(q + q_1^c(\hat{c}_A)) - \hat{c}_A - \hat{c}_B])q\}, 0\right] \tag{23}$$

is smaller or larger than

$$\max_q\{(P(q + q_1^c(w_{U_A}^{D_2})) - w_{U_A}^{D_2} - \hat{c}_B)q\} + \max_q\{(P(q + q_1^c(\hat{c}_A)) - \hat{c}_A - c_B)q\} \tag{24}$$

at $w_{U_A}^{D_2} = c_A$. Due to the convexity of the optimized profit function in costs, we know from the proof of Proposition 1 that if $q_1^c(\hat{c}_A) = q^c$, the expression in (23) is larger than the one in (24). However, we know that $q_1^c(\hat{c}_A) > q^c$. Now let us determine how both expressions change with a change in $q_1^c(\hat{c}_A)$. Here we obtain that the derivative of (23) with respect to $q_1^c(\hat{c}_A)$ is given by $q'P'(q+q_1^c(\hat{c}_A)) < 0$, with $q' = \max\left[\arg\max_q\{(P(q' + q_1^c(\hat{c}_A)) - \hat{c}_A - c_B)q\}, 0\right]$, while the derivative of (24) with respect to $q_1^c(\hat{c}_A)$ is given by $q''P'(q'' + q_1^c(\hat{c}_A)) < 0$, with $q'' = \arg\max_q\{(P(q + q_1^c(\hat{c}_A)) - \hat{c}_A - c_B)q\}$. But since $q'' > q'$ and the since the demand function is concave, we have $q''P'(q''+q_1^c(\hat{c}_A)) < q'P'(q+q_1^c(\hat{c}_A))$. Therefore, the fact that $q_1^c(\hat{c}_A) > q^c$ has a larger negative effect on (24) than on (23). As a consequence, we obtain that (24) is smaller than (23) at $w_{U_A}^{D_2} = c_A$, which implies that the profit of $U_A - D_1$ at $w_{U_A}^{D_2} = c_A$ is given by

$$\begin{aligned} \Pi^{U_A - D_1}(c_A) &= \max_q\{(P(q + q_2^c(c_A)) - c_A - \hat{c}_B)q\} \\ &+\tfrac{1}{2}\left[\max_q\{(P(q + q_1^c(c_A)) - c_A - c_B)q\} - \max_q\{(P(q + q_1^c(\hat{c}_A)) - \hat{c}_A - c_B)q\}\right] \\ &+\tfrac{1}{2}\left[\{\max_q(P(q + q_1^c(c_A)) - c_A - \hat{c}_B)q\} - \max\left[\max_q\{(P(q + q_1^c(\hat{c}_A)) - \hat{c}_A - \hat{c}_B])q\}, 0\right]\right] \quad . \end{aligned} \tag{25}$$

Subtracting (22) from (25) and rearranging we obtain the that vertical integration is profitable if (9) holds. ■

Proof of Proposition 5

The sum of the profits of U_A and D_1 without integration is given by

$$\Pi^c + \max_q\{[P(q + q^c) - c_A - \hat{c}_B]q\} - \max_q\{[P(q + q^c) - \hat{c}_A - c_B]q\}.$$

Taking the derivative of with respect to $\hat{c}_B$, we obtain that it is given by $-q'$, with $q' = \arg\max_q\{[P(q+ q^c) - c_A - \hat{c}_B]q\}$.

Now let us look at the expected profit with integration given by (8). Suppose first that

$$\max_q\{(P(q + q_1^c(w_{U_A}^{D_2})) - w_{U_A}^{D_2} - c_B)q\} - \max_q\{(P(q + q_1^c(\hat{c}_A)) - \hat{c}_A - c_B)q\} < \tag{26}$$

$$\max_q\{(P(q + q_1^c(w_{U_A}^{D_2})) - w_{U_A}^{D_2} - \hat{c}_B)q\} - \max\left[\max_q\{(P(q + q_1^c(\hat{c}_A)) - \hat{c}_A - \hat{c}_B])q\}, 0\right].$$

Taking the derivative with respect to $\hat{c}_B$ we obtain that it is given by $-q''$, with $q'' = \max_q\{(P(q+ q_2^c(w_{U_A}^{D_2})) - c_A - \hat{c}_B)q\}$. But from the proof of the last proposition, we know that the inequality (26) can only be fulfilled if $w_{U_A}^{D_2} > c_A$. But this implies that $q_2^c(w_{U_A}^{D_2}) < q^c$ and therefore that $q'' > q'$. As a consequence, if (26) holds, the negative consequence of a increase in $\hat{c}_B$ for U_A is larger in case of integration than in case of no integration.

Now suppose that (26) is not fulfilled. Then the derivative of the expected profit under integration with respect to $\hat{c}_B$ is given by $-q'' - 1/2\,(q''' - \max[q'''', 0])$, where $q''' = \max_q\{(P(q + q_1^c(w_{U_A}^{D_2})) - w_{U_A}^{D_2} - \hat{c}_B)q\}$ and $q'''' = \max_q\{(P(q + q_1^c(\hat{c}_A)) - \hat{c}_A - \hat{c}_B])q\}$. Since $q''' > \max[q'''', 0]$, we again get that for $w_{U_A}^{D_2} > c_A$ the effect of an increase in $\hat{c}_B$ hurts U_A more in case of integration than in case of no integration. Finally, one can show that $1/2\,(q''' - \max[q'''', 0])$ increases by more than q'' as $w_{U_A}^{D_2}$ falls. But this implies that for $w_{U_A}^{D_2} \leq c_A$ we again get $-q'' - 1/2\,(q''' - \max[q'''', 0]) < -q'$, implying again that the effect of an increase in $\hat{c}_B$ hurts U_A more in case of integration than in case of no integration. ■

Proof of Proposition 6

The objective is to take the derivative of the difference between the $\Pi^{U_A-D_1}$ and the sum of Π^{U_A} and Π^{D_1} under non-integration with respect to $\hat{c}_A$. Suppose first that

$$\max_q\{(P(q+q_1^c(w_{U_A}^{D_2}))-w_{U_A}^{D_2}-c_B)q\}-\max_q\{(P(q+q_1^c(\hat{c}_A))-\hat{c}_A-c_B)q\}< \tag{27}$$

$$\max_q\{(P(q+q_1^c(w_{U_A}^{D_2}))-w_{U_A}^{D_2}-\hat{c}_B)q\}-\max\left[\max_q\{(P(q+q_1^c(\hat{c}_A))-\hat{c}_A-\hat{c}_B])q\},0\right].$$

$\Pi^{U_A-D_1}$ is then given by

$$\begin{gathered}\max_q\{(P(q+q_2^c(w_{U_A}^{D_2}))-c_A-\hat{c}_B)q\}+q_2^c(w_{U_A}^{D_2})(w_{U_A}^{D_2}-c_A)\\+\max_q\{(P(q+q_1^c(w_{U_A}^{D_2}))-w_{U_A}^{D_2}-c_B)q\}-\max_q\{(P(q+q_1^c(\hat{c}_A))-\hat{c}_A-c_B)q\},\end{gathered}$$

while the sum of profits under non-integration are equal to

$$\Pi^c-\max_q\{[P(q+q^c)-\hat{c}_A-c_B]q\}+\max_q\{[P(q+q^c)-c_A-\hat{c}_B]q\}.$$

Then, by using the envelope theorem, one obtains that the derivative of this difference is equal to

$$q_2^c(\hat{c}_A)\left(1-P'(q_2^c(\hat{c}_A)+q_1^c(\hat{c}_A))\frac{\partial q_1^c(\hat{c}_A)}{\partial \hat{c}_A}\right)-\tilde{q}(\hat{c}_A)$$

where $q_2^c(\hat{c}_A)=\arg\max_q\{(P(q+q_1^c(\hat{c}_A))-\hat{c}_A-c_B)q\}$ and $\tilde{q}(\hat{c}_A)=\arg\max_q\{[P(q+q^c)-\hat{c}_A-c_B]q\}$.

The first term is positive since $P'(\cdot)<0$ and $\partial q_1^c(\hat{c}_A)/\partial\hat{c}_A<0$, while the second term is negative. However, since $q_1^c(\hat{c}_A)>q^c$ and quantities are strategic substitutes, we know that $q_2^c(\hat{c}_A)<\tilde{q}(\hat{c}_A)$. Moreover, $q_1^c(\hat{c}_A)$ increases with $\hat{c}_A$ while q^c stays unchanged. As a consequence, there exists a $\bar{\hat{c}}_A$ such that $\tilde{q}(\bar{\hat{c}}_A)>0=q_2^c(\bar{\hat{c}}_A)$. Therefore, if $\hat{c}_A=\bar{\hat{c}}_A$, the derivative of the profitability from integration is negative.

In exactly the same way we can show that if the inequality in (27) is reversed, there also exists a value of $\hat{c}_A$ such that for all $\hat{c}_A$ above this value, vertical integration becomes less profitable as $\hat{c}_A$ increases.

Concluding, in both cases it exists a value of $\hat{c}_A$ that we denote by $\hat{c}_A'$ such that, for $\hat{c}_A$ above $\hat{c}_A'$, the profitability of vertical integration decreases in $\hat{c}_A$. ■

Proof of Proposition 7

We first determine the following expression

$$\Pi^c-\max_q\{[P(q+q^c)-\hat{c}_A-c_B]q\}+\max_q\{[P(q+q^c)-c_A-\hat{c}_B]q\}, \tag{28}$$

which gives us the sum of profits of U_A and D_1 under non-integration and linear demand. Starting with Π^c, standard computations yield $q^c = (1 - c_A - c_B)3$ and $\Pi^c = (q^c)^2$.

Next we determine $\max_q\{[P(q+q^c)-c_A-\hat{c}_B]q\}$. We first calculate the value of q' such that

$$q' = \arg\max_q\{[P(q+q^c) - c_A - \hat{c}_B]q\}$$

. In case of linear demand this can be written as

$$\arg\max_q\{[1 - c_A - \hat{c}_B - (1 - c_A - c_B)/3 - q]q\}.$$

It turns out that

$$q' = \frac{2(1 - c_A - c_B) - 3\Delta_B}{6},$$

with $q' > 0$ under our assumption $\Delta_j < (1 - c_A - c_B)/2$, and $\max_q\{[P(q + q^c) - c_A - \hat{c}_B]q\} = (q')^2$.

Finally, we compute $\max_q\{[P(q + q^c) - \hat{c}_A - c_B]q\}$. Defining q'' such that

$$q'' = \arg\max_q\{[1 - \hat{c}_A - c_B - (1 - c_A - c_B)/3 - q]q\},$$

we obtain

$$q'' = \frac{2(1 - c_A - c_B) - 3\Delta_A}{6},$$

and $\max_q\{[P(q + q^c) - \hat{c}_A - c_B]q\} = (q'')^2$.

After rearranging we obtain that the sum of profits of U_A and D_1 under non-integration is equal to

$$\frac{3\Delta_A[4(1 - c_A - c_B) - 3\Delta_A] + [2(1 - c_A - c_B) - 3\Delta_B]^2}{36}. \tag{29}$$

Now, we turn to the computation of the profit under vertical integration. First, we have

$$q_2^c(w_{U_A}^{D_2}) = (1 - 2w_{U_A}^{D_2} - c_B + c_A)/3 = [1 - c_A - c_B - 2(w_{U_A}^{D_2} - c_A)]/3$$

and

$$q_1^c(w_{U_A}^{D_2}) = (1 - 2c_A - c_B + w_{U_A}^{D_2})/3 = (1 - c_A - c_B + w_{U_A}^{D_2} - c_A)/3.$$

Thus,

$$q_2^c(w_{U_A}^{D_2})(w_{U_A}^{D_2} - c_A) = (w_{U_A}^{D_2} - c_A)[1 - c_A - c_B - 2(w_{U_A}^{D_2} - c_A)]/3$$

and

$$\max_q\{[P(q + q_1^c(w_{U_A}^{D_2})) - w_{U_A}^{D_2} - c_B]q\} = (q_2^c(w_{U_A}^{D_2}))^2.$$

Then, we compute the value of $q_1'''(w_{U_A}^{D_2})$, where $q_1'''(w_{U_A}^{D_2})$ is given by

$$q_1'''(w_{U_A}^{D_2}) = \arg\max_q\{[P(q+q_2^c(w_{U_A}^{D_2}))-c_A-\hat{c}_B]q\} = \arg\max_q\{[1-c_A-\hat{c}_B-(1-2w_{U_A}^{D_2}+c_A-c_B)/3-q]q\}.$$

We obtain

$$q_1'''(w_{U_A}^{D_2}) = \frac{2(1-c_A-c_B)+2(w_{U_A}^{D_2}-c_A)-3\Delta_B}{6}$$

and

$$\max_q\{[P(q+q_2^c(w_{U_A}^{D_2}))-c_A-\hat{c}_B]q\} = (q_1'''(w_{U_A}^{D_2}))^2.$$

Finally, to determine the value of $\max_q\{[P(q+q_1^c(\hat{c}_A))-\hat{c}_A-c_B]q\}$ we use the fact that the vertically integrated firm $U_A - D_1$ now knows when D_2 is buying from the bypass alternative and it can react promptly on the product market. Therefore, one has that $q_2^c(\hat{c}_A) = (1-c_A-c_B-2\Delta_A)/3$ and $\max_q\{[P(q+q_1^c(\hat{c}_A))-\hat{c}_A-c_B]q\} = (q_2^c(\hat{c}_A))^2$.

We showed in the main text that the problem of maximization of the integrated firm is given by (6). Inserting the respective expressions into (6) we get

$$\max_{w_{U_A}^{D_2}} \Pi^{U_A-D_1}(w_{U_A}^{D_2}) = \Big[\frac{2(1-c_A-c_B)+2(w_{U_A}^{D_2}-c_A)-3\Delta_B}{6}\Big]^2$$

$$+(w_{U_A}^{D_2}-c_A)\frac{1-c_A-c_B-2(w_{U_A}^{D_2}-c_A)}{3} + \Big[\frac{1-c_A-c_B-2(w_{U_A}^{D_2}-c_A)}{3}\Big]^2.$$

The resulting first-order condition for $w_{U_A}^{D_2}$ is

$$\frac{c_A-3\Delta_B-2w_{U_A}^{D_2}+1-c_B}{9} = 0.$$

The second-order condition is fulfilled and the expression for the optimal value of $w_{U_A}^{D_2}$ is

$$w_{U_A}^{D_2} = c_A + \frac{1-c_A-c_B-3\Delta_B}{2}.$$

Therefore,

$$w_{U_A}^{D_2} \begin{cases} = \hat{c}_A & \text{if} \quad \Delta_A < (1-c_A-c_B)/2 - 3\Delta_B/2 = \underline{\Delta}_A. \\ \\ < \hat{c}_A & \text{otherwise.} \end{cases}$$

We now need to determine the fixed fee $F_{U_A}^{D_1}$. We know that it depends on which expression is the minimum in (4). First, suppose that the minimum is

$$\max_q\{(P(q+q_1^c(w_{U_A}^{D_2}))-w_{U_A}^{D_2}-c_2)q\} - \max_q\{(P(q+q_1^c(\hat{c}_A))-\hat{c}_A-c_2)q\}.$$

Inserting the respective expressions, we obtain that the profit of $U_A - D_1$ is

$$\Pi^{U_A-D_1} = \Big[\frac{2(1-c_A-c_B)+2(w_{U_A}^{D_2}-c_A)-3\Delta_B}{6}\Big]^2 + (w_{U_A}^{D_2}-c_A)\frac{1-c_A-c_B-2(w_{U_A}^{D_2}-c_A)}{3}$$

$$+\Big[\frac{1-c_A-c_B-2(w_{U_A}^{D_2}-c_A)}{3}\Big]^2 - \Big[\frac{1-c_A-c_B-2\Delta_A}{3}\Big]^2. \tag{30}$$

Now suppose Instead, that the minimum in (4) is

$$\max_q\{(P(q+q_1^c(w_{U_A}^{D_2}))-w_{U_A}^{D_2}-\hat{c}_B)q\}-\max\{\max_q\{(P(q+q_1^c(\hat{c}_A))-\hat{c}_A-\hat{c}_B])q\},0\}.$$

Before determining the expected profit of U_A-D_1, we need to determine the maximum of the second term. In case of linear demand, the profit that D_2 gets when buying both inputs from firms $\hat{U}_j$ is equal to

$$\max_q\{[P(q+q_1^c(\hat{c}_A))-\hat{c}_A-\hat{c}_B]q\}=\Big[\frac{2(1-c_A-c_B)-3\Delta_B-4\Delta_A}{6}\Big]^2,$$

with $\arg\max_q\{[P(q+q_A^c(\hat{c}_A))-\hat{c}_A-\hat{c}_B]q\}=(2(1-c_A-c_B)-3\Delta_B-4\Delta_A)/6$. Thus, this quantity is only positive if$\Delta_A\leq(1-c_A-c_B)/2-3\Delta_B/4\equiv\bar{\Delta}_A$.

We therefore get that, if $\Delta_A\leq\bar{\Delta}_A$, the expected profit of U_A-D_1 equals

$$\Big[\frac{2(1-c_A-c_B)+2(w_{U_A}^{D_2}-c_A)-3\Delta_B}{6}\Big]^2+(w_{U_A}^{D_2}-c_A)\frac{1-c_A-c_B-2(w_{U_A}^{D_2}-c_A)}{3}+$$

$$+\frac{1}{2}\Big\{\Big[\frac{1-c_A-c_B-2(w_{U_A}^{D_2}-c_A)}{3}\Big]^2-\Big[\frac{1-c_A-c_B-2\Delta_A}{3}\Big]^2\Big\}+$$

$$+\frac{1}{2}\Big\{\Big[\frac{2(1-c_A-c_B)-3\Delta_B-4(w_{U_A}^{D_2}-c_A)}{6}\Big]^2-\Big[\frac{2(1-c_A-c_B)-3\Delta_B-4\Delta_A}{6}\Big]^2\Big\}. \qquad (31)$$

Instead, if $\Delta_A>\bar{\Delta}_A$, the value of the expected profits is

$$\Big[\frac{2(1-c_A-c_B)+2(w_{U_A}^{D_2}-c_A)-3\Delta_B}{6}\Big]^2+(w_{U_A}^{D_2}-c_A)\frac{1-c_A-c_B-2(w_{U_A}^{D_2}-c_A)}{3}+ \qquad (32)$$

$$\frac{1}{2}\Big\{\Big[\frac{1-c_A-c_B-2(w_{U_A}^{D_2}-c_A)}{3}\Big]^2-\Big[\frac{1-c_A-c_B-2\Delta_A}{3}\Big]^2+\Big[\frac{2(1-c_A-c_B)-3\Delta_B-4(w_{U_A}^{D_2}-c_A)}{6}\Big]^2\Big\}.$$

We can now derive under which conditions each of the three expected profits, (30), (31) and (32) is relevant. To so we need to determine for conditions for the inequality

$$\max_q\{[P(q+q_1^c(w_{U_A}^{D_2}))-w_{U_A}^{D_2}-c_B]q\}-\max_q\{[P(q+q_1^c(\hat{c}_A))-\hat{c}_A-c_B]q\}\leq$$

$$\max_q\{[P(q+q_1^c(w_{U_A}^{D_2}))-w_{U_A}^{D_2}-\hat{c}_B]q\}-\max\{\max_q\{(P(q+q_1^c(\hat{c}_A))-\hat{c}_A-\hat{c}_B])q\},0\} \qquad (33)$$

to hold. This inequality tells us which of the two terms in (4) is the lower one.

First, if $\Delta_A<(1-c_A-c_B)/2-3\Delta_B/2=\underline{\Delta}_A$, one has that $w_{U_A}^{D_2}=\hat{c}_A$ and (33) is always satisfied. Indeed, at $w_{U_A}^{D_2}=\hat{c}_A$ that both sides in (33) are nil. Therefore, the expected profit of U_A-D_1 is given by (30) evaluated at $w_{U_A}^{D_2}=\hat{c}_A$.

Second, if $\Delta_A \in [(1-c_A-c_B)/2-3\Delta_B/2, (1-c_A-c_B)/2-3\Delta_B/4)$—that is, $\Delta_A \in [\underline{\Delta}_A, \bar{\Delta}_A)$—we get $w_{U_A}^{D_2} < \hat{c}_A$. Then, 33) becomes

$$\left[\frac{1-c_A-c_B-2(w_{U_A}^{D_2}-c_A)}{3}\right]^2 - \left[\frac{1-c_A-c_B-2\Delta_A}{3}\right]^2 \leq$$

$$\left[\frac{2(1-c_A-c_B)-3\Delta_B-4(w_{U_A}^{D_2}-c_A)}{6}\right]^2 - \left[\frac{2(1-c_A-c_B)-3\Delta_B-4\Delta_A}{6}\right]^2,$$

which is satisfied for all $\Delta_A \leq (1-c_A-c_B)/2-3\Delta_B/2 = \underline{\Delta}_A$. This implies that for $\Delta_A \in [\underline{\Delta}_A, \bar{\Delta}_A)$ (31) is the relevant expression for the expected profits of $U_A - D_1$.

Finally, if $\Delta_A \in [\bar{\Delta}_1, (1-c_A-c_B)/2)$, we know from above that $\max[\arg\max_q\{(P(q+q_1^c(\hat{c}_A))-\hat{c}_A -\hat{c}_B])q\}, 0] = 0$. Therefore, (33) becomes

$$\left[\frac{1-c_A-c_B-2(w_{U_A}^{D_2}-c_A)}{3}\right]^2 - \left[\frac{1-c_A-c_B-2\Delta_A}{3}\right]^2 \leq \left[\frac{2(1-c_A-c_B)-3\Delta_B-4(w_{U_A}^{D_2}-c_A)}{6}\right]^2.$$

This inequality is satisfied for all $\Delta_A \leq (1-c_A-c_B)/2-3\sqrt{3}\Delta_B/4 < (1-c_A-c_B)/2-3\Delta_B/4 = \bar{\Delta}_A$. Hence, for $\Delta_A \in [\bar{\Delta}_A, (1-c_A-c_B)/2)$ the expected profit of $U_A - D_1$ is (32).

We can conclude that the threshold below which (30) is relevant is equal to the threshold below which foreclosure is optimal, which is given by $\underline{\Delta}_A = (1-c_A-c_B)/2 - 3\Delta_B/2$.

We now turn to the analysis of the profitability of integration.

For $\Delta_A \leq \underline{\Delta}_A$ foreclosure is optimal. Hence, the expected profit of the integrated firm equals

$$\frac{[2(1-c_A-c_B)+2\Delta_A-3\Delta_B]^2 + 12\Delta_A[(1-c_A-c_B)-2\Delta_A]}{36}.$$

This expression is larger than the profits under non integration (29) in the range of interest, that is, for $\Delta_A \leq \underline{\Delta}_A$.

In the interval $[\underline{\Delta}_A, \bar{\Delta}_A)$ the expected profit under integration is given by (31) evaluated at $w_{D_2}^{U_A} < \hat{c}_A$. Doing so we obtain

$$\frac{4[(1-c_A-c_B)(4\Delta_A-3\Delta_B)-\Delta_A(4\Delta_A+3\Delta_B)]+5(1-c_A-c_B)^2}{36}.$$

By comparing this expression with the profits under non integration, (29), an rearranging, we obtain that integration is profitable if

$$\frac{4\Delta_A[(1-c_A-c_B)-3\Delta_B]+(1-c_A-c_B)^2-7\Delta_A^2-9\Delta_B^2}{36} \geq 0$$

or

$$\Delta_A \leq \frac{2(1-c_A-c_B)-6\Delta_B+\sqrt{[9\Delta_B+11(1-c_A-c_B)][(1-c_A-c_B)-3\Delta_B]}}{7} \equiv \tilde{\Delta}_A.$$

In the interval $[\bar{\Delta}_1, (1-c_A-c_B)/2)$ the expected profit under integration are given by expression (32) evaluated at $w_{D_2}^{U_A} < \hat{c}_A$. In particular, they are equal to

$$\frac{16\Delta_A(1-c_A-c_B-\Delta_A) - 9\Delta_B[4(1-c_A-c_B)-\Delta_B] + 14(1-c_A-c_B)^2}{72}.$$

By comparing this expression with the profit under non integration we obtain the first one is bigger if

$$\frac{-9\Delta_B^2 + 2[\Delta_A^2 - 4\Delta_A(1-c_A-c_B) + 3(1-c_A-c_B)(1-c_A-c_B-2\Delta_B)]}{72} \geq 0$$

or

$$\Delta_A \leq 2(1-c_A-c_B) - \sqrt{\frac{9\Delta_B^2}{2} + (1-c_A-c_B)(1-c_A-c_B+6\Delta_B)} \equiv \tilde{\tilde{\Delta}}_A.$$

Notice that if $\Delta_B < 2(1-c_A-c_B)(2\sqrt{15}-5)/21$ one has that

$$\tilde{\Delta}_1 - \bar{\Delta}_1 \geq 0 \quad \text{and} \quad \tilde{\tilde{\Delta}}_1 - \bar{\Delta}_1 \geq 0.$$

So, if $\Delta_B < 2(1-c_A-c_B)(2\sqrt{15}-5)/21$ then $\bar{\Delta}_A < \min\{\tilde{\Delta}_1, \tilde{\tilde{\Delta}}_A\}$. Instead, if $\Delta_B > 2(1-c_A-c_B)(2\sqrt{15}-5)/21$ then $\bar{\Delta}_A > \max\{\tilde{\Delta}_A, \tilde{\tilde{\Delta}}_A\}$.

Concluding, for $\Delta_B \in (0, 2(1-c_A-c_B)(2\sqrt{15}-5)/21)$ we obtain the following result:

- If $0 < \Delta_A \leq \underline{\underline{\Delta}}_A$, the integrated firm sets $w_{D_2}^{U_A} = \hat{c}_A$ and integration is profitable.
- If $\underline{\Delta}_A < \Delta_A \leq \tilde{\tilde{\Delta}}_1$, the integrated firm sets $c_A < w_{D_2}^{U_A} < \hat{c}_A$ and integration is profitable.
- If $\tilde{\tilde{\Delta}}_A < \Delta_A < (1-c_A-c_B)/2$, the integrated firm would set $w_{D_2}^{U_A} < \hat{c}_A$, but integration is not profitable.

If $\Delta_B \in (2(1-c_A-c_B)(2\sqrt{15}-5)/21), (1-c_A-c_B)/3)$ we obtain the following result

- If $0 < \Delta_A \leq \underline{\underline{\Delta}}_A$, the integrated firm sets $w_{D_2}^{U_A} = \hat{c}_A$ and integration is profitable.
- If $\underline{\Delta}_A < \Delta_A \leq \tilde{\Delta}_A$, the integrated firm sets $w_{D_2}^{U_A} < \hat{c}_A$ and integration is profitable.
- If $\tilde{\Delta}_A < \Delta_A < (1-c_A-c_B)/2$, the integrated firm would set $c_A < w_{D_2}^{U_A} < \hat{c}_A$, but integration is not profitable.

Finally, for $\Delta_B \in ((1-c_A-c_B)/3, (1-c_A-c_B)/2)$ the integrated firm would set $w_{D_2}^{U_A} \leq c_A$, but integration is not profitable.■

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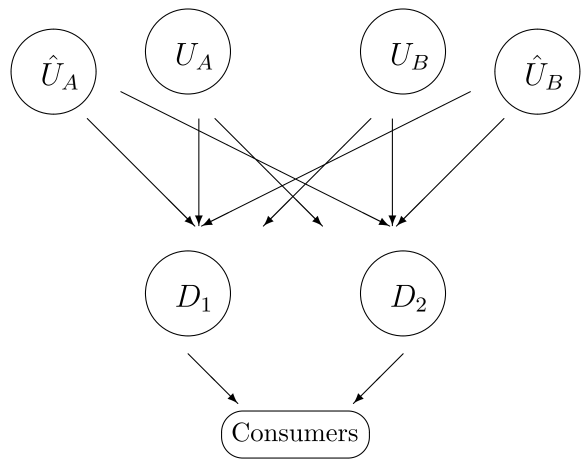

Figure 1: Framework.

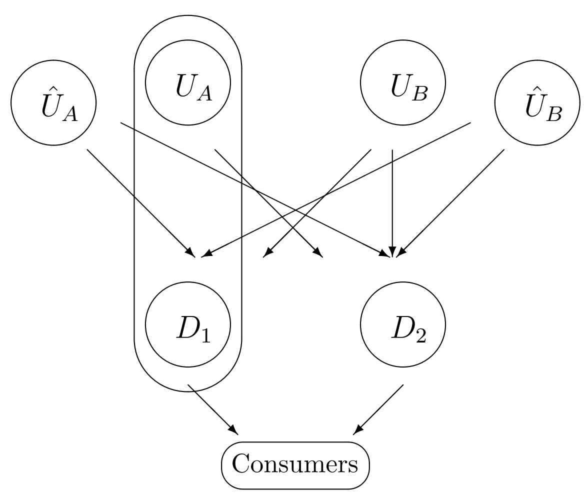

Figure 2: Framework with Integration.