Fiscal Policy, Sovereign Default, and Bailouts¹

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This version: February 14, 2013

Abstract

This paper examines fiscal policy without commitment and the effects of bailout loans. We apply a simple closed economy model with distortionary taxation, where the government decides discretionary between full debt repayment or costly default. The government tends to overborrow due to myopia, which aggravates welfare losses originating from the lack of commitment and provides a rationale to constrain sovereign borrowing. We consider bailout loans that are offered at a favorable price and conditional upon minimum primary surpluses. While the government's willingness to accept these offers decreases with the tightness of the fiscal constraint, we find that household welfare can be enhanced under sufficiently large minimum surplus. Yet, under welfare-enhancing bailout offers overborrowing is not resolved as the average stock of public debt tends to increase.

JEL classification: E32; H21; H63.

Keywords: Discretionary fiscal policy, overborrowing, sovereign default, bailout loans, fiscal consolidation

¹We are grateful to Klaus Adam, Felix Bierbrauer, Kai Christoffel, Christian Bayer, Christian Hellwig, Michael Krause, Dirk Krüger, Bartosz Mackowiak and Harald Uhlig for helpful comments and suggestions. Financial support of the DFG SPP 1578 is gratefully acknowledged.

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1 Introduction

In the aftermath of the subprime crisis, several industrialized countries have experienced a dramatic worsening of the fiscal stance, i.e. high public deficits and debt-to-GDP ratios. As a consequence, sovereign default risk, which has previously been viewed as particularly relevant for less developed countries and emerging market economies, has become a serious issue for industrialized countries as well. These countries are traditionally characterized by larger shares of domestically held public debt (see Reinhart and Rogoff, 2011), while expectations of public sector default are largely based on the reluctance of governments to sufficiently lower deficits or to increase surpluses. Given that increasing costs of borrowing tend to aggravate this problem, bailout loans at favorable terms and conditional upon fiscal consolidation plans have recently been offered to members of the European Monetary Union (EMU). Since Fall 2012, these types of bailout loans are supplied by the European Stability Mechanism, a financial institution which is funded by the majority of members of the EMU. In this paper, we examine the effects of this type of bailout mechanism, i.e. bailout loans at a favorable price and conditional upon fiscal constraints, focussing on the following questions:

- *i.*) When are governments willing to accept conditional bailout loans?
- ii.) Can a self-financed bailout mechanism enhance welfare?
- *iii.*) How does the existence of a bailout mechanism affect sovereign borrowing?

To address these questions, we apply a closed economy model with a government that lacks commitment, when raising distortionary taxes, purchasing goods, and borrowing in terms of non-state contingent debt. To account for debt accumulation, we further consider myopia as an additional friction that causes governments to overborrow,³ since lack of commitment alone is known not to be sufficient to explain accumulation of public debt (see Derbotoli and Nunes, 2012). The government further discretely decides between full debt repayment or costly default on domestically held outstanding debt. It thereby faces a trade-off between defaulting, which allows to avoid welfare-reducing tax increases or spending cuts, and repaying debt to avoid costs of default, which are modelled as deadweight resource losses (like in Cole and Kehoe, 2000, or Arellano, 2008).⁴ In contrast to the majority of studies on sovereign default,

³Specifically, we follow Grossman and van Huyck (1988) and assume that the government discounts future periods at a higher rate than society does, for example, due to the possibility of not staying in office forever.

⁴This trade-off is typically negleted in the voluminous literature on sovereign default. Most contributions to this literature follow Eaton and Gersovitz (1981), where fiscal policy is not explicitly modelled when the government might default on foreign debt (see e.g. Aguiar and Gopinath, 2006, or Arellano, 2008). Exceptions

we do not consider foreign lending, such that gains of default arise here mainly by allowing to reduce distortionary taxes or/and to raise utility providing government consumption.⁵ Under sufficiently large default costs, sovereign default will however be welfare-reducing in this framework. Lack of commitment and myopia are then responsible for a non-zero default probability, which provides a rationale for introducing the bailout mechanism.

Fiscal policy in the basic framework without bailout loans can be summarized as follows. The government neither commits itself to a tax/spending plan nor to repay debt. When default costs are prohibitively high, it never decides to default. If the government were not myopic, the time consistent fiscal policy would then be optimal, i.e. it would be identical to the fiscal policy plan under full commitment.⁶ Under non-prohibitive default costs, lack of commitment affects the government's choices and the default option becomes favorable in adverse states, i.e. when outstanding debt is high and income is low. Myopia then enlarges the set of states where the government favors default over debt repayment. Investors internalize the government's default incentives such that the price of government bonds becomes debt elastic. When the state of the economy worsens and public debt increases, the costs of borrowing increase, such that the government optimally relies on raising taxes and/or reducing expenditures to balance the budget. As it trades off the costs of increasing surpluses and the costs of default, it decides to default when the state becomes adverse.

This framework is employed to assess the effects of bailout loans that are offered at a favorable price and conditional upon a minimum primary surplus (or maximum primary deficit) by an institution, which is independent of the government and financed by households paying a lump-sum fee. This institution, which we call the bailout fund, is assumed not to dispose of a superior enforcement technology such that bailout loans are also subject to default risk. The answers to the above-mentioned questions are as follows: i.) We find that the government's willingness to accept the bailout offer decreases with the tightness of the fiscal constraint. ii.) We show that conditional bailout loans can enhance welfare, such that they are voluntarily supported by households. In particular, bailout offers can be welfareenhancing if the fiscal constraint is not too tight, i.e. for sufficiently large minimum surpluses. Yet, welfare is in general not monotonically increasing with minimum surpluses (for example, we find that welfare can decrease for loose fiscal constraints and less impatient governments).

are Cole and Kehoe (2000) and Cuadra et al. (2010), where fiscal instruments are considered for the analysis of sovereign default in small open economies.

⁵Defaulting on non-state contingent debt can further enhance welfare by making debt repayment contingent on the state (see Klaus and Grill, 2012).

⁶This equivalence relies on the assumption that the utility function is quasi-linear (as in Cole and Kehoe, 2000), which further facilitates the analysis of discretionary policy, since it implies risk-neutral investors.

iii.) Under the bailout mechanism, we find that the average level of public debt increases for all cases under consideration. The reason is that the existence of bailout offers tends to reduce the mean default risk premium on public debt in the first place, which makes borrowing for governments more attractive.

The paper builds on the literature on optimal fiscal policy in a closed economy, for example, focussing on non-state contingent debt (see e.g. Aiyagari et al., 2002) or lack of commitment (e.g. Derbotoli and Nunes 2013). Our paper is further related to several studies on sovereign default in small open economies. Adam and Grill (2011) examine sovereign default when the borrower acts under full commitment and show that default can be optimal under large (disaster) shocks or sufficiently high debt positions. Boz (2011) examines how emerging market sovereign borrowers decide between private loans and loans from international financial institutions (IFIs), which are characterized by a superior repayment enforcement. Our paper further shares an explicit specification of fiscal instruments with Cuadra et al. (2010), who show that a government in a small open economy which acts without commitment adjusts consumption taxes and government spending in a procyclical way. Further, Roch and Uhlig (2012) analyze sunspot shocks in an Arellano-type framework and discuss the effects of bailout loans that are offered at fundamental prices.

The remainder of the paper is structured as follows. Section 2 presents the model. Section 3 describes the choice of an optimizing government and the welfare losses due to the lack of commitment and myopia. In Section 4 we introduce bailout loans and present results under bailout offers. Section 5 concludes.

2 The model

In this section, we describe a closed economy model with endogenous production. Households consume, supply working time, and invest in non-state contingent government bonds. The government raises labor income taxes and purchases goods, while it lacks commitment. Debt repayment is modelled as a discrete choice, i.e. public debt is either fully repaid or not at all, where default is assumed to be associated with resource losses, as in Cole and Kehoe (2000) or Arellano (2008). A bailout fund is further assumed to offer loans at a favorable price, conditional upon repayment of outstanding debt and on a minimum primary surplus (maximum primary deficit).

2.1 Private sector

There exists a continuum of infinitely lived and identical households of mass one. Their utility is increasing in consumption c_t and government expenditures g_t , and decreasing in working time l_t . The objective of a representative household is given by

$$E\sum_{t=0}^{\infty}\beta^{t}u\left(c_{t},g_{t},l_{t}\right), \quad \text{with } \beta \in (0,1),$$
(1)

where E denotes the expectations operator based on information at the beginning of period 0 and β denotes the discount factor. The utility function u is twice continuously differentiable in consumption c_t , government spending g_t , and working time l_t , and satisfies $u_c > 0$, $u_{cc} \le 0$, $u_g > 0$, $u_g < 0$, $u_l < 0$ and $u_{ll} < 0$.

Households can invest in government bonds b_t^h and borrow/lend by issuing/buying oneperiod non-state contingent bonds b_t^{rf} . We assume that households fully commit to repay debt. In period t, household debt is issued at the price $1/R_t^{rf}$ and delivers one unit of the consumption good in period t + 1, such that R_t^{rf} is the risk free rate, which serves as a benchmark to measure the default risk premium on public debt. Government debt is issued at the price $1/R_t$, while the government does not commit to repay one unit of the consumption good in period t + 1. Labor income is taxed at a rate $\tau_t \in (0, 1)$. The ex-post budget constraint reads

$$c_t + \left(b_t^h / R_t\right) + \left(b_t^{rf} / R_t^{rf}\right) \le (1 - \tau_t) w_t l_t + p_t b_{t-1}^h + b_{t-1}^{rf} + \Pi_t - f_t^b,$$
(2)

where Π_t denotes firms' profits, w_t the wage rate, and $f_t^b \geq 0$ a lump-sum fee paid to the bailout fund (see 2.2). Note that p_t indicates whether the government fully repays its outstanding debt, $p_t = 1$, or defaults in period t, $p_t = 0$. Households rationally take into account the possibility of default, where expectations about the repayment probability $E_t (1 - \delta_{t+1}) \geq 0$ depend on the government behavior. Households maximize (1) subject to (2), a no-Ponzi game condition, $\lim_{t\to\infty} (b_t^{rf}/R_t^{rf}) \prod_{i=1}^t 1/R_{t-i}^{rf} \geq 0$, and $b_t^h \geq 0$. The households' first order conditions are given by

$$-u_l(c_t, g_t, l_t) = u_c(c_t, g_t, l_t) (1 - \tau_t) w_t,$$
(3)

$$u_c(c_t, g_t, l_t) = R_t \beta E_t \left[(1 - \delta_{t+1}) \, u_c(c_{t+1}, g_{t+1}, l_{t+1}) \right],\tag{4}$$

$$u_c(c_t, g_t, l_t) = R_t^{rf} \beta E_t u_c(c_{t+1}, g_{t+1}, l_{t+1}),$$
(5)

and the transversality conditions for privately issued bonds, $\lim_{t\to\infty} E(b_t^{rf}/R_t^{rf}) \prod_{i=1}^t 1/R_{i-1}^{rf} = 0$, and government bonds, $\lim_{t\to\infty} E(b_t^h/R_t) \prod_{i=1}^t 1/R_{i-1}^{rf} = 0$.

The final good y_t is produced by identical and infinitely many firms of mass one and is purchased by households and the government for consumption only. Firms are perfectly competitive and their production technology is given by

$$y_t = \Xi \left(a_t, p_t \right) f(l_t), \tag{6}$$

where a_t is a stochastic productivity level satisfying $a_t \in \Upsilon$ and $\pi(a_{t+1}|a_t)$ are the transition probabilities. The productivity factor Ξ_t is weakly increasing in a_t and is adversely affected when the government decides to default in period t, $p_t = 0$. Specifically, we assume that $\Xi(a_{i,t}, 0) \leq \Xi(a_{i,t}, 1)$ for any productivity level $a_{i,t} \in \Upsilon$, such that default triggers temporary resource losses, like in Cole and Kehoe (2000) or Arellano (2008). The specification of default losses in terms of the productivity factor $\Xi(a_t, p_t)$ can be viewed as a short-cut of modelling the adverse effects of sovereign default on financial intermediation, which seems to be the main source of costs that deter sovereign borrowers from defaulting (see, for example, Panizza et al., 2009). For the quantitative analysis (see section 4.2), we follow Arellano (2008) and consider a cost specification which implies that default is associated with relatively higher resource losses in otherwise favorable states. Firms maximize profits taking prices as given and subject to (6), such that labor demand satisfies $w_t = \Xi(a_t, p_t) f'(l_t)$.

2.2 The bailout fund

We consider an independent institution which provides credit to the sovereign borrower in cases where the government is otherwise willing to default. This institution is modelled to account for some main features of the European Stability Mechanism (ESM). Specifically, we assume that it is organized as a fund financed by households, which can further raise revenues by issuing one-period non-state contingent bonds. In contrast to the government, the fund offers bailout loans to the government conditional upon debt repayment and a fiscal constraint. We specify the fund as if it is voluntarily supported by society: Households once and for all decide to support the fund depending on whether the implemented bailout mechanism is welfare enhancing or not. If the plan of the bailout fund indeed turns out to be welfare-enhancing, households authorize the fund to raise lump-sum fees, for example, in the initial period as seed capital, exactly to the amount required to finance the bailout loans. We implicitly assume that the bailout fund can commit to this mechanism.

A bailout loan consists of a one-period loan b_t^b offered at a favorable price conditional upon repayment of previous debt ($p_t = 1$) and conditional upon a constraint on current fiscal choices that is intended to increase the likelihood of repayment and to induce lower future sovereign debt levels. To give a preview, the conditional bailout loan is in principle suited to address two main frictions. Overborrowing induced by the government's myopia can be restrained by imposing the fiscal constraint and the adverse effect of the lack of commitment on bond prices can be reduced by offering a more favorable price.

We do not examine an optimal bailout loan, since this requires to be explicit about the objective of the fund. While it would be obvious to consider that it aims at maximizing household welfare, the goals of institutions that provide bailout loans (like the IMF or the ESM) rather seem to be the avoidance of default and the reduction of public debt. Instead of deriving an optimal policy of the fund we consider a simple form, which allows to disclose parametrically how the terms of the loan affect the government's willingness to accept the bailout offer and its tendency to default and to accumulate debt. In particular, we assume that bailout loans are offered at the risk-free price $q_t^b = 1/R_t^{rf}$ and conditional upon not defaulting on previous debt and on a current surplus $s_t = \tau_t a_t \alpha l_t^{\alpha} - g_t$ being sufficiently large compared to the current debt level,

$$s_t \ge \Psi \cdot b_t,\tag{7}$$

The parameter $\Psi > 0$, which is set by the bailout fund, governs the strength of the fiscal constraint and will be varied in the numerical analysis (see Section 4.3) to disclose when the government is actually willing to accept the bailout loan and how the conditionality affects household welfare. The fiscal constraint (7) implies that the government is neither forced to implement a specific spending plan nor does the bailout impose a particular level (or increase) of the tax rate. The government is still free to optimally choose these instruments while satisfying the constraint on the current primary surplus (7). It should be noted that the bailout mechanism does not require the government to commit to future surpluses and that the bailout fund is not endowed with a specific enforcement technology. Hence, repayment of the bailout loan is not guaranteed and depends on the government's decision to repay debt in the subsequent period. The budget constraint of the bailout fund is thus given by

$$q_t^b b_t^b - p_t b_{t-1}^b = (b_t^{rf} / R_t^{rf}) - b_{t-1}^{rf} + f_t^b.$$

The bailout loan (q_t^b, Ψ) is in general inconsistent with an individually rational behavior of a price-taking investor. According to the optimal investment choice, an investor should only be willing to offer a loan at the price $1/R_t^{rf}$ if full repayment is guaranteed. In particular, a risk-free loan is only consistent with individual rationality if the constraint (7) with Ψ^* leads to a new debt level b_t^* that induces the government to voluntarily decide to repay debt in the subsequent period in all stochastic states of the world. Hence, choosing taxes and government expenditures, τ_t and g_t , in a way that end-of-period debt equals b_t^* can already be realized by borrowing from households, implying that a bailout offer (q_t^b, Ψ^*) would neither affect the government choice nor the equilibrium outcome. To provide a non-trivial analysis, we therefore consider bailout loans with a fiscal constraint (7) that induces a loan size larger than b_t^* .

2.3 The government

The government purchases the amount g_t of the final good, raises labor income taxes, and issues non-state contingent one-period bonds. We assume that the government cannot credibly commitment to its future policy actions. In contrast to Krusell et al. (2005) and Derbotoli and Nunes (2013), who also examine fiscal policy when the government cannot commit to a taxation and spending plan, we do not assume that the government can nevertheless commit to repay debt. Instead, we assume that does not guarantee debt repayment and consider the default decision as a discrete choice, like in the literature on sovereign default (see Eaton and Gersovitz, 1981, or Arellano, 2008).⁷ The government thus acts on a period by period basis by choosing the amount of goods purchases, the tax rate, and whether to fully default on its debt or not. It thereby treats loans from households and from the bailout fund in an identical way. We assume that in each period the government aims at maximizing the sum of discounted household utility over an infinite horizon

$$V_t = \max E_t \sum_{k=0}^{\infty} \widetilde{\beta}^k u\left(c_{t+k}, g_{t+k}, l_{t+k}\right), \qquad (8)$$

while its particular objective might differ from (1) by the government discount factor $\tilde{\beta} \in (0, 1)$. Specifically, we allow for the case where the government discounts future periods at a higher rate than society does (see Grossmann and van Huyck, 1998):

$$\widetilde{\beta} \leq \beta$$
.

When $\tilde{\beta} < \beta$, the government acts in an myopic way, which can be rationalized by assuming that the government faces a constant probability of being in office $\alpha \in (0, 1)$ where $\tilde{\beta} = \alpha \beta$. According to this interpretation, the government's time horizon ends with its term in office, where $1/(1-\alpha)$ measures the expected incumbency.⁸ When the government is myopic, $\tilde{\beta} < \beta$, it will tend to overborrow, i.e. government indebtedness will tend to be larger than for $\tilde{\beta} = \beta$. Hence, lack of commitment and myopia provide two sources of inefficiency that originate in the government's behavior.

 $^{^7\}mathrm{An}$ exception is the analysis of Adam and Grill (2012), who examine continous default decisions under commitment.

⁸Note that this relates to Arellano's (2008) assumption that the inverse of the domestic country's discount factor differs from the world interest rate.

Following the literature on sovereign default, the government's default decision is assumed to include no new debt issuance in the same period. Hence, in each period the government trades off to repay outstanding debt ($p_t = 1$) and to issue new debt, which implies raising sufficiently large surpluses, or to fully default on outstanding debt ($p_t = 0$) and not to borrow in the same period, implying zero primary surpluses. We can summarize the government's budget constraint as

$$p_t \cdot [(b_t/R_t) - b_{t-1}] = -s_t$$
, where $p_t \in \{0, 1\}$. (9)

where public debt b_t is either held by households $b_t = b_t^h$ or by the bailout fund $b_t = b_t^b$ if the government accepts the conditional bailout loan. Subsequent to a period where the government has defaulted, it will regain access to credit, which differs from the assumptions of permanent or temporary autarky that are typically made in the literature on sovereign default in small open economies (see Eaton and Gersovitz, 1981, or Arellano, 2008), such that direct costs of default are only due to resource costs modelled according to (6). This assumption, which we view as more suited for a closed economy set-up, facilitates the welfare analysis of sovereign default.

To account for the property that the government's maximization problem consists of a discrete choice, we introduce V_t^c as the maximum value under full repayment of debt (regardless whether it is held by households or the bailout fund), V_t^d as the maximum value under default, and V_t^b the maximum value when the government accepts the bailout loan. Then, the discrete default choice of the optimizing government is given by

$$p_t = \begin{cases} 1 \text{ if } V_t^d \le \min\{V_t^c, V_t^b\} \\ 0 \text{ if } V_t^d > \max\{V_t^c, V_t^b\} \end{cases},$$
(10)

The government's choice will be characterized by the maximum achievable value V_t :

$$V(b_{t-1}, a_t) = \max\{V^c(b_{t-1}, a_t), V^d(a_t), V^b(b_{t-1}, a_t)\}.$$
(11)

When the government decides to accept a bailout loan, $V_t^b > \max\{V_t^c, V_t^d\}$, it has to choose taxes and spending to satisfy (7) which – together with the government budget constraint, $q_t^b b_t^b + s_t = b_{t-1}$, implies the end-of-period stock of debt to equal $b_t^b \leq \kappa_t b_{t-1}$, where $\kappa_t = 1/(\Psi + q_t^b)$ is the borrowing ratio that relates the maximum amount of new debt the government is allowed to issue under the conditional bailout program to the level of initially outstanding debt. To give a preview, under all parameterizations under considerations the fiscal constraint is found to hold with equality, $b_t^b = \kappa_t b_{t-1}$.

2.4 Equilibrium

We are interested in analyzing Markov-Perfect equilibria, where expectations of private agents and the behavior of the optimizing government are consistent and where the government moves first in each period. The state of the economy can be summarized by the beginning-ofperiod stock of government bonds b_{t-1} and the exogenous productivity level a_t . Let $\Lambda^d(b_{t-1})$ be the set of values for the exogenous state a_t , where the government prefers to default compared to bailout or full repayment, $\mathcal{F}(b_{t-1})$ the set of values for the exogenous state a_t for which the government prefers to accept the bailout loan, and $\Theta(b_{t-1})$ the set of values for the exogenous state a_t where the government prefers to fully repay debt:

$$\Lambda(b_{t-1}) = \left\{ a_t \in \Upsilon : V^d(a_t) > \max\{V^b(a_t, b_{t-1}), V^c(a_t, b_{t-1})\} \right\},$$
(12)
$$F(b_{t-1}) = \left\{ a_t \in \Upsilon : V^b(a_t, b_{t-1}) > \max\{V^d(a_t), V^c(a_t, b_{t-1})\} \right\},$$

$$\Theta(b_{t-1}) = \left\{ a_t \in \Upsilon : V^c(a_t, b_{t-1}) > \max\{V^d(a_t), V^b(a_t, b_{t-1})\} \right\}.$$

The expected default rate is thus given by $E_t \delta_{t+1} = \sum_{a_{t+1} \in \Lambda(b_t)} \pi(a_{t+1}|a_t)$. Let further q_t denote the period t price of government bonds, which is either equals $1/R_t$ if the government borrows from households or $q_t^b = 1/R_t^{rf}$ if it accepts a conditional bailout loan, $q_t \in \{1/R_t, 1/R_t^{rf}\}$. Using that goods and asset markets clear, $y_t = c_t + g_t$, the private sector equilibrium behavior can then be summarized as follows:

$$-u_l(c_t, g_t, l_t) = \Xi (a_t, p_t) f'(l_t) u_c(c_t, g_t, l_t) (1 - \tau_t), \qquad (13)$$

$$q_t u_c(c_t, g_t, l_t) = \beta E_t \left[(1 - \delta_{t+1}) \, u_c(c_{t+1}, g_{t+1}, l_{t+1}) \right], \tag{14}$$

$$c_t + g_t = \Xi \left(a_t, p_t \right) f(l_t), \tag{15}$$

$$E_t (1 - \delta_{t+1}) = 1 - \sum_{a_{t+1} \in \Lambda(b_t)} \pi(a_{t+1}|a_t),$$
(16)

and the transversality condition for privately held government bonds. As described in Section 2.2, we specify the behavior of the bailout fund in an exogenous way. It offers loans only if the government favors defaulting to repaying outstanding debt $V^d(a_t) > V^c(a_t, b_{t-1})$, while the price of the bailout loan is set equal to the risk-free price and Ψ is varied over a wide range of values. The government has to satisfy

$$g_t - \tau_t \Xi(a_t, p_t) f'(l_t) l_t = \begin{cases} q_t b_t - b_{t-1} \,\forall a_t \in \Theta(b_{t-1}) \cup F(b_{t-1}) \\ 0 \quad \forall a_t \in \Lambda(b_{t-1}) \end{cases},$$
(17)

where $b_t = b_t^h$ or $b_t = b_t^b$ if it accepts the bailout loan. As described above, the government aims at maximizing its objective (see 8) by choosing τ_t , g_t , and b_t on a period by period basis. The choice for these instruments further depends on the government's discrete choice to repay debt, to default, or to accept the bailout loans, which is associated with the values $V^c(b_{t-1}, a_t)$, $V^d(a_t)$, or $V^b(b_{t-1}, a_t)$. The government choice thus leads to $V(b_{t-1}, a_t)$ as described in (8) and (11). For the full repayment case, the optimal government choice satisfies $p_t = 1$ and

$$V^{c}(b_{t-1}, a_{t}) = \max_{c_{t}, l_{t}, \tau_{t}, g_{t}, b_{t}} \left\{ u\left(c_{t}, g_{t}, l_{t}\right) + \widetilde{\beta} \sum_{a_{t+1}} V(b_{t}, a_{t+1}) \pi(a_{t+1}|a_{t}) \right\}$$
(18)
s.t. (13)-(16), and $g_{t} - \tau_{t} \Xi(a_{t}, 1) f'(l_{t}) l_{t} = q_{t} b_{t} - b_{t-1}.$

In the default case, $p_t = 0$, the government cannot borrow in period $t, b_t = 0$, while it regains access to the credit market in period t + 1. Hence, its problem can be written as

$$V^{d}(a_{t}) = \max_{c_{t}, l_{t}, \tau_{t}, g_{t}} \left\{ u\left(c_{t}, g_{t}, l_{t}\right) + \widetilde{\beta} \sum_{a_{t+1}} V(0, a_{t+1}) \pi(a_{t+1}|a_{t}) \right\}$$
s.t. (13), (15), and $g_{t} = \tau_{t} \Xi\left(a_{t}, 0\right) f'(l_{t}) l_{t}$,
(19)

where the continuation value accounts for access to the credit market subsequent to the default period. The government's problem under bailout loans can be summarized as

$$V^{b}(b_{t-1}, a_{t}) = \max_{c_{t}, l_{t}, \tau_{t}, g_{t}} \left\{ u\left(c_{t}, g_{t}, l_{t}\right) + \widetilde{\beta} \sum_{a_{t+1}} V(b_{t-1}/\left(\Psi + \beta\right), a_{t+1}) \pi(a_{t+1}|a_{t}) \right\}, \quad (20)$$

s.t. (13), (15), and $\tau_{t} a_{t} \alpha l_{t}^{\alpha} - g_{t} = b_{t-1}/(1 + q_{t}^{b}/\Psi),$

where q_t^b and Ψ are taken as given. The maximum value $V(b_{t-1}, a_t)$ is then given by (11). Notably, the government takes the bailout option fully into account, regardless of previous default or bailout decisions. An equilibrium can then be defined as follows:

Definition 1 A Markov perfect equilibrium under conditional bailout loans is a set of policy functions $c_t = c(b_{t-1}, a_t)$, $l_t = l(b_{t-1}, a_t)$, $q_t = q(b_{t-1}, a_t)$, $b_t = b(b_{t-1}, a_t)$, $\tau_t = \tau(b_{t-1}, a_t)$, $g_t = g(b_{t-1}, a_t)$, $E_t \delta_{t+1} = \delta(b_{t-1}, a_t)$, a discrete decision $p_t = d(b_{t-1}, a_t)$, as well as $V(b_{t-1}, a_t)$, $V^d(a_t)$, $V^c(b_{t-1}, a_t)$, and $V^b(b_{t-1}, a_t)$ satisfying (11), (13)-(17), (18), (19), and (20).

3 Fiscal policy choices

In this section, we examine the problem of the government acting without commitment. We demonstrate how default and myopia alter the government's choice and show that both tend to reduce household welfare, which provides a rationale for the bailout mechanism. We describe the behavior of the government, with a particular focus on the case where default costs are prohibitively high.

We show that for quasi-linear preference, i.e. $u_{cc} = 0$, and with a non-myopic government, the allocation is identical to an optimal fiscal policy chosen by a government acting under commitment (the latter is examined in Appendix A), which implies that lack of commitment per se does not lead to a suboptimal policy in this framework. Under quasi-linear preferences, lack of commitment leads to a policy that differs from the policy under commitment only if there exists a relevant default option, i.e. if default costs are not prohibitively high. For the subsequent numerical analysis we therefore assume that $u_{cc} = 0$, where optimal fiscal policy serves as the natural starting point and welfare losses are solely due to the default option and myopia. It should be noted that the assumption of risk-neutral investors is shared by the majority of studies on sovereign default, while a quasi-linear utility function of domestic households is also applied in Cole and Kehoe's (2000) analysis of sovereign default.

Consider the general case where $u_{cc} \geq 0$. Under full debt repayment, the government problem is summarized by (18). By eliminating consumption and the bond price, it suffices to examine the government's choices of the tax rate, expenditures, working time, and end-of period debt. The first order conditions are then given by

$$\mu_t \Xi_t f'(l_t) = \gamma_t \Xi_t f'(l_t) l_t \tag{21}$$

$$u_{c,t}\gamma_t = u_{g,t} + u_{cc,t}\mu_t \left(u_{l,t}/u_{c,t}^2 \right) + \left[u_{cc,t}\gamma_t \left(b_t/u_{c,t}^2 \right) \beta E_t ((1 - \delta_{t+1}) u_{c,t+1}) \right]$$
(22)

$$u_{c,t}\Xi_t f'_t + u_{l,t} + \mu_t \left(u_{ll,t}/u_{c,t} \right) + \mu_t \left(1 - \tau_t \right) \Xi_t f''(l_t) + \gamma_t \tau_t \Xi_t \left(f'(l_t) + f''(l_t) l_t \right)$$
(23)

$$= u_{cc,t} \mu_t \Xi_t f'_t \left(u_{l,t} / u_{c,t}^2 \right) + \left[u_{cc,t} \gamma_t \Xi_t f'_t \left(b_t / u_{c,t}^2 \right) \beta E_t \left((1 - \delta_{t+1}) \, u_{c,t+1} \right) \right]$$

where $\Xi_t = \Xi(a_t, 1)$ and $E_t(1 - \delta_{t+1}) = 1 - \sum_{a_{t+1} \in \Lambda(b_t)} \pi(a_{t+1}|a_t)$ and μ_t and γ_t denote the multipliers for the constraints $u_{l,t}/[u_{c,t}(\Xi_t f(l_t) - g_t)] + (1 - \tau_t)\Xi_t f'(l_t) = 0$ and $\tau_t \Xi_t f'(l_t) l_t - g_t + \beta E_t \frac{[(1 - \delta_{t+1})u_{c,t+1}(\Xi_{t+1}f(l_{t+1}) - g_{t+1})]}{u_{c,t}(\Xi_t f(l_t) - g_t)} b_t = b_{t-1}$. The government's decision satisfies the conditions (21)-(23) as well as an optimal choice for newly issued debt b_t , where the government accounts for the equilibrium impact of b_t on the expected repayment rate and on the households' consumption decision in the subsequent period.

For the case where the government decides to default, the government's problem is given by (19). The first order conditions for the government's choice of the tax rate, expenditures, and working time are then identical to (21)-(23), expect for $\Xi_t = \Xi(a_t, 0)$ and for the terms in the square brackets in (22) and (23), which account for the policy impact on the bond price and vanish under the default choice, $p_t = 0$. It should further be noted that the government's first order conditions under repayment and default are identical if the marginal utility of consumption is constant, $u_{cc,t} = 0$.

When the government favors a bailout, it has to satisfy two conditions to actually get the bailout loan at the favorable price q_t^b . First, it has repay previous debt $(p_t = 0)$ and, second, it has to raise surpluses in the current period in accordance with the constraint (7). Given that end of period debt then has to satisfy $b_t^b \leq \kappa b_{t-1}$, which will hold with equality, the tax and spending choices are restricted by $\tau_t a_t \alpha l_t^\alpha - g_t = b_{t-1}/(1 + q_t^b/\Psi)$, while the first order conditions with regard to the tax rate and government expenditures are again given by (21) and (22), where $b_t = b_t^b$.

Now suppose that default costs are prohibitively high, $\Xi(a_t, 0) \to 0$. Then, default leads to a situation where no resources are available for private and public consumption, which implies $V^d(b_{t-1}, a_t) \to -\infty$ (see 19)). Hence, the government never opts in favor of default and pays back debt in all states, such that the default set is empty $\Lambda(b_{t-1}) = \emptyset$. Given that the default probability then equals zero, the price of debt q_t equals the inverse of the risk-free rate $1/R_t^{rf}$ (see 4 and 5). This case therefore corresponds to a fiscal policy without commitment and without myopia, where debt is repaid in all states, as for example analyzed in Krusell et al. (2006) or Debortoli and Nunes (2013). The government's choice for newly issued debt can then be described by a first order condition $\Xi(a_t, 0) \to 0$: $\gamma_t[q_t + \beta E_t(\frac{u_{cc,t+1}}{u_{c,t}}\frac{\partial c_{t+1}}{\partial b_t})b_t] = 0$ $\beta E_t \gamma_{t+1}$, where the derivative $\partial c_{t+1} / \partial b_t$ accounts for the impact of debt issued in period t on the consumption decision of households in period t + 1. As shown by Debortoli and Nunes (2013) for $\beta = \beta$ in a closely related environment, public debt then converges to a mean level of zero. To account for public debt accumulation and to allow for a reasonable role of conditional bailouts, we therefore focus – for the quantitative analysis – on the case where the government acts in a myopic way, i.e. $\tilde{\beta} < \beta$, which tends to increase public borrowing (since the market price $1/R_t$ exceeds the government's marginal valuation of debt).

The case of a non-myopic government provides a useful benchmark for the subsequent welfare analysis. In particular, considering the case of non-myopic government behavior allows us to investigate how defaults and myopia lead to welfare losses compared to an optimal policy. It can be shown that the government's choice under lack of commitment is identical to the case of full commitment (see Appendix A), when the utility function is quasi-linear, $u_{cc} = 0$. For this specification, there is no justification for bailout loans or fiscal constraints when the default option is irrelevant and when the government is not myopic, since fiscal policy is then conducted in an optimal way. This property is summarized in the following proposition.

Proposition 1 Suppose that default costs are prohibitively high such that the default set is empty $\Lambda(b_{t-1}) = \emptyset$ and that the government is non-myopic, $\tilde{\beta} = \beta$. The allocation under a fiscal policy without commitment is identical to the optimal allocation under commitment if the utility function satisfies $u_{cc} = 0$.

Proof. See appendix A \blacksquare

If the marginal utility of consumption is constant, $u_{cc} = 0$, the government's first order condition for debt issuance reduces – for $\tilde{\beta} = \beta$ – to $\gamma_t q_t = \beta E_t \gamma_{t+1}$. Together with (4), which reduces to $q_t = \beta$ for $\Lambda(b_{t-1}) = \emptyset$, the latter implies the well-known near random-walk property of fiscal policy under commitment and non-state contingent debt, $\gamma_t = E_t \gamma_{t+1}$. Under $u_{cc} = 0$, the remaining government choices under commitment and without commitment are also identical, as shown in the proof of proposition 1. The assumption of quasi-linearity thus allows isolating the welfare-reducing effects of defaults and myopia from potential welfare losses from lack of commitment per se.

When default costs are not prohibitively high, the default set might be non-empty, $\Lambda(b_{t-1}) \neq \emptyset$. Then, the price of government bonds will in general be a function of public debt, i.e. $\partial q_t / \partial b_t \neq 0$, even if $u_{cc,t} = 0$. It should be noted that government bonds become (at least partially) state contingent under default, which can potentially lead to welfare gains compared to the case where debt is fully repaid and non-state contingent.⁹ For the remainder of the analysis, we consider default costs that are sufficiently large, such that the associated welfare losses exceed the welfare gains due to the default-induced state contingency of government bonds. This scenario then provides a framework that leaves room for welfare improvements via conditional bailout loans (as specified above), which can in principle address both sources of welfare losses, i.e. the default option and myopia, by offering a favorable price (q^b) conditional upon a fiscal constraint (Ψ) (7).

4 Results

This section presents the quantitative results. The first part presents the functional forms for preferences and technology as well as the parametrization of the model. In the second part of this section, we describe how government choices, the equilibrium allocation, and bond prices change with the state of the economy for a benchmark version of the model where no bailout is offered. We further present welfare computations for different parametrization of the benchmark version, which confirm that default and myopia are welfare reducing. In the third part, we introduce conditional bailout offers and compute main statistics of model simulations for fiscal constraints (7) that differ with regard to the parameter Ψ (or κ), which governs the tightness of the constraint.

⁹Adam and Grill (2011) show that these welfare gains can be dominated by default costs even if the latter are relatively small.

4.1 Functional forms and calibration

Given that the government faces a discrete choice (to repay debt or to default), the model is solved numerically applying value function iteration. Technical details can be found in Appendix C. We introduce the following specifications for the household preferences u_t , for the default costs Ξ_t , and the production technology $y_t = \Xi_t f(l_t)$:

$$u(c_t, g_t, l_t) = \psi c_t + [(g_t^{1-\sigma} - 1)/(1-\sigma)] - \vartheta l_t^{1+\eta}/(1+\eta),$$
(24)

$$\Xi_t = \begin{cases} a_t & \forall a_t \in \Theta(b_{t-1}) \cup F(b_{t-1}) \\ h(a_t) < a_t & \forall a_t \in \Lambda(b_{t-1}) \end{cases}.$$
(25)

$$f(l_t) = l_t^{\alpha},\tag{26}$$

The utility function is assumed to be quasi-linear and satisfies, $u_{cc,t} = 0$, such that the results apply that have been derived in the previous section (see proposition 1). The deterministic part of the production function (26) is standard, while the impact of stochastic productivity levels needs some further discussion. According to the specification of the effective productivity Ξ_t (see 25), the productivity level a_t together with $f(l_t)$ determines total output in states where the government decides to fully repay debt, $a_t \in \Theta(b_{t-1})$. If the government defaults in a particular state $a_t^i \in \Lambda(b_{t-1})$), total output equals $h(a_t^i)f(l_t)$, which is less than $a_t^i f(l_t)$. Hence, (25) induces aggregate resource costs of default. We follow Arellano (2008) and assume that these costs of default are relatively more severe in favorable states. Like in Arellano (2008), we apply this type of cost structure to induce incentives for the government to default in less favorable productivity states, which is consistent with empirical evidence (see Tomz and Wright, 2007).¹⁰ Further details on the fiscal policy choice for the functional forms (24)-(26) are given in Appendix B.

Throughout the numerical analysis, we set the technology and preference parameters { α , β , σ , η } to values which are standard in the business cycle literature: $\alpha = 2/3$, $\sigma = 2$, $\eta = 2$, and $\beta = 0.99$ (implying an annual risk-free rate 4.1%). The remaining parameters are chosen to match average government shares and output dynamics of a set of European countries which are highly indebted and for which sovereign default has recently become an issue. Specifically, we consider average statistics for Greece, Italy, Portugal, and Spain. We set the autocorrelation of productivity to 0.9 and choose the innovation variance σ_{ε}^2 such that the realized standard deviation of HP-filtered log output from stochastically simulated model runs conforms with the standard deviation of HP-filtered log real quarterly GDP for

¹⁰In order to allow for sufficiently high costs of default so that default becomes a relatively rare event, we truncate productivity in default states at the smallest productivity level in the productivity set Υ .

the countries that we consider (using an HP-filter smoothing parameter 1600). The data is taken from the OECD National Accounts Statistics for the period 1970.I-2011.I and are seasonally adjusted by the publishing institution. The standard deviations of detrended log output range from 0.013 to 0.027 with a mean value of 0.019 which we use as a calibration target for our model.

Using data from Eurostat's annual national accounts for the sample period 1995 to 2010, the government share is measured as the ratio of government consumption over GDP (referring to total government at all federal levels). The average value for the government share for our sample of countries is about 0.19. This value is applied as a calibration target, which is associated with an average tax rate under optimal fiscal policy that amounts to 0.28. This endogenous value is comparable to an empirical measure of an average tax rate. When social security contributions are excluded, the ratio of total tax revenues over GDP amounts to 0.23 for the group of countries we consider. When taking social security contributions into account, the empirical figures are higher. Overall, our model calibration therefore has realistic implications for both the average government share (which is targeted directly) and the average tax rate.

The final parameter to be specified is the degree of myopia by the government, $\hat{\beta}$. Since this parameter directly impacts on the borrowing behavior of the government it has a direct effect on the realized default probability in the model. To facilitate comparability to the existing literature on sovereign default, we choose with $\tilde{\beta} = 0.9$ a baseline value (which correspond to an expected incumbency of 11 years) that leads to a default probability of roughly 3%, like in Arellano (2008). To illustrate the effects of government myopia in the model, we will further consider the values $\tilde{\beta} = 0.96$ and $\tilde{\beta} = 0.86$ (implying expected incumbencies of 33 and 7.6 years). According to this interpretation, the government's time horizon ends with its term in office, where $1/(1 - \alpha)$

4.2 Sovereign default without bailout loans

To see how the bailout mechanism can be rationalized and to disclose the underlying decisions of the government, we focus in this section on the case where no bailout loans are offered to the government. Figure 1 displays equilibrium objects as policy functions, i.e. as functions of the two state variables, productivity a and initial debt b.¹¹ Panel (a) shows the combinations of states (a, b) for which the government chooses to default, i.e. for which $V^d > V^c$. Default

¹¹To lighten the notation, we drop the time index and define $a = a_t$, $a' = a_{t+1}$, $b = b_{t-1}$ and $b' = b_t$ (as well as $q = q_t$, $g = g_t$, $\tau = \tau_t$, $l = l_t$)

is preferred when debt increases and when productivity declines, as in Arellano (2008) and related studies. In order to ease exposition using two-dimensional figures, we draw the following panels showing policy functions for two specific values of productivity which we refer to as "low" (dashed line) and "high" (solid line) productivity. These two productivity levels are also marked in panel (a). Panel (b) shows the value functions V(a, b) for high (solid) and low (dashed) values of current productivity a. In contrast to the value function for the case of high productivity, the value function for low productivity is kinked due to a switch from full repayment V^c to default V^d , where V^d does not depend on b. Panel (c) shows the equilibrium bond price q(b', a), where q is a function of next period debt b'. For the high-productivity case, the government does not decide to default even when debt is high. Accordingly, the government is able to borrow almost always at the risk-free price β even for high values of b'. When productivity is low, by contrast, the bond price begins to decline already at lower levels of next period's debt. In fact, the bond price approaches zero when the government wants to issue very high levels of debt, reflecting high default risk. Issuing bonds at a low price implies that the total amount of resources borrowed, i.e. the value of debt, $q(a,b') \cdot b'$ shrinks as q declines (see panel (d)). Apparently, the government will never find it optimal to issue more debt than at the peak of the $q(a, b') \cdot b'$ curve.

The actual borrowing decision b'(a, b) of the government is displayed in panel (e). Next period's debt b' is increasing in b and in the value of productivity. The dashed line corresponding to the low-productivity case shows that the government borrows only for relatively low levels of outstanding debt b. For higher levels of debt the government defaults (for states where default occurs the plot of the borrowing function ends since b' = 0). Panel (f) shows that the interest rate (1/q) - 1 is increasing in debt due to increasing risk of default, as in related studies. The dashed line shows that the interest rate rises sharply when the economy approaches the default region. Panel (g) and (h) further show that the government tends to implement a higher primary surplus by raising tax rates and reducing spending, when debt takes higher values and for lower values of productivity. The reason is that default risk is higher in a less favorable state of the economy, such that the costs of borrowing increase for the government (see panel (c)). When the government decides to default, it is able to substantially lower tax rates and to increase spending as the debt burden is eliminated through default. Finally, panel (i) shows that output tends to decrease with debt due to the increase in primary surpluses. The kink in output (see dashed line) results from the productivity loss under default due to $h(a_t)$. Hence, the government tends to increase surpluses and to borrow less when the productivity state worsens, while defaults typically occurs in very adverse



Figure 1: Default states and selected policy functions

productivity states.¹²

We further use stochastic model simulations to assess the welfare effects of default and myopia, which are the only reasons why fiscal policy might differ from a optimizing policy under commitment, given that preferences are quasi-linear (see Section 3). On the one hand, the default option causes the price of debt falls with the debt level, such that the government's lack of commitment can become relevant for prices and the allocation. On the other hand, there are direct costs induced by the default decision, since we assumed that default is associated with real resource losses. Both properties tend to reduce welfare compared to the case of optimal fiscal policy, while welfare losses are further aggravated when the government

¹²The same pattern can be observed when the government is more myopic, $\tilde{\beta} = 0.84$, where borrowing and interest rate tend to be higher (see Appendix D).

is myopic. It should however be noted that default introduces an incomplete state contingency of debt which can potentially be exploited by the government in a welfare enhancing way.

We simulate N = 5,000 economies for T = 1,000 periods each. We initialize each economy starting with zero debt and mean productivity, and consider an additional burn-in period of 500 periods, which will not be used for the calculations. We approximate welfare (1) by computing the unconditional expectation of the sum of discounted household utility over the 1,000 periods, where the expectations are evaluated by averaging over the 5,000 samples, i.e. we compute $W = E \sum_{i=0}^{1000} \beta^i u (c_{t+i}, g_{t+i}, l_{t+i})$ and presents welfare differences ΔW in steady state consumption equivalents, $\Delta c^{st.st.} = \frac{1-\beta}{\psi} \Delta W$. We compare welfare under the benchmark economy with welfare in a reference economy without myopia and with prohibitively large default costs. To facilitate a reasonable comparison, the reference case is constructed by artificially keeping the value of debt constant at the mean level of debt in the benchmark economy.¹³ For the latter, we get a welfare measure of W = 626.98 and for the reference economy we get W = 627.52, leading the welfare loss of 8.6% stemming from default and myopia.¹⁴

4.3 Effects of bailout loans

For the numerical analysis of bailout loans, we simulate the economy with non-prohibitive default costs as defined above (see definition 1). The government and the private sector take the bailout offer fully into account in all states of the economy. To demonstrate the impact of the bailout offers, we vary the tightness of the fiscal constraint (7), which is crucial for the welfare and consolidation effects of the bailout:¹⁵ On the one hand, a tighter fiscal constraint, i.e. a higher Ψ or a lower $\kappa = 1/(\Psi + \beta)$, makes fiscal consolidation more likely, which tends to reduce the welfare costs originating from overborrowing and lack of commitment. On the other hand, they tend to increase welfare costs induced by higher contemporaneous surpluses, i.e. higher distortionary taxes and/or lower government expenditures.

Figure 2 presents default states and equilibrium objects as policy functions under bailout loans for a ratio of newly issued and initial debt $\kappa = b_t/b_{t-1}$ equal to $\kappa = 0.5$. Panel (a) shows that bailout loans are accepted by the government (marked with the light green area) only if productivity is low and debt is at the boundary between full repayment and default. In case of the high productivity level (see solid lines), the bailout loans are not attractive

¹³The reason for this strategy is that the economy exhibits a near random walk behavior under prohibitively large default cost, which cannot be analyzed with the computational methods applied in this paper.

¹⁴It should be noted that the allocation for the reference case is already less favorable the allocation under the optimal policy under commitment, since debt is assumed not to be adjusted over time.

¹⁵Note that the fiscal constraint is binding for all cases under consideration.



Figure 2: Default states and selected policy functions under bailout loans with $\kappa = 0.5$

for the government and never accepted. Bailout loans with a tight fiscal constraint that restricts borrowing to 50% of current outstanding debt tend to be less attractive when the government is more indebted. Even though bailout loans allow debt to be issued at favorable price, a highly indebted government prefers to default, which allows to fully cut down the debt burden. The panels (g) and (h) further show that accepting the bailout loan (bold part of the lines) hardly changes the tax policy and government spending pattern compared to lower debt levels, while the interest rate sharply drops by construction (see panel (f)).

Figure 3 shows the policy functions for a less restrictive fiscal constraint, $\kappa = 0.7$. The bailout loans are again only accepted at the boundary to the default states, while they are now realized over the entire range of public debt. Therefore, bailout states now become relevant for both productivity levels considered in Figure 3. Overall, these figures show that bailout loans are accepted by the government in situations where it, otherwise, would tend



Figure 3: Default states and selected policy functions under bailout loans with $\kappa = 0.7$

to default. Thus, the government accepts bailout loans and the associated conditionality when interest rates are high (see panels (f)) and they are already forced to reduce borrowing due to a strong decline in debt prices. Thus, realized bailout loans do not lead to a fiscal consolidation and instead enable the government to partly over-roll debt in adverse states.¹⁶

Figure 4 presents simulation results of the economy (for three degrees of myopia, $\tilde{\beta} \in \{0.84, 0.9, 0.96\}$) under bailout offers that differ with regard to the tightness of the fiscal constraint κ . The simulations are conducted as described in Section 4.2. The solid lines refer to the benchmark case, $\tilde{\beta} = 0.9$. For tight fiscal constraints, i.e. for low values for κ , the government is less willing to accept conditional bailout loans due to the costs induced by high taxation and low government spending. (The case $\kappa = 0$ coincides with the case

¹⁶These effects are even more pronounced for $\kappa = 0.8$ (see Appendix D).



Figure 4: Sample statistics from model simulations with bailout loans

examined in the previous section, where bailout loans were not offered.) Higher values of κ , which indicate a less restrictive fiscal constraint, are potentially favored by the government, which is reflected by the bailout probability that monotonically increases with κ . Given that the bailout loan is offered at a favorable price, it allows to roll-over a larger amount of debt for higher values of κ . This however implies that the mean level of public debt also increases (monotonically) with less restrictive fiscal constraints. Since increased levels of debt tend to raise the government's default incentives, the default probability is a non-monotonic function of κ . The default probability first falls and then recovers when mean debt strongly increases, i.e. for $\kappa > 0.8$. Notably, the welfare gain relative to the case without bailout offers ΔW monotonically increases with κ for $\tilde{\beta} = 0.9$.

The dotted lines in Figure 4 further show the case where the government is even more myopic, $\tilde{\beta} = 0.84$. The overall pattern is qualitatively identical to the pattern for the benchmark case, $\tilde{\beta} = 0.90$. Intuitively, the welfare gains of bailout offers are larger when the government is more myopic, since the inefficiency due to overborrowing is more pronounced. The level of mean debt, the default probability, and the mean spread are all larger due to higher government impatience and its increased willingness to borrow. Correspondingly, the mean price of debt is lower than in the benchmark case. Under these circumstances, the

bailout offers are more effective, mainly by raising the mean price to a larger relative amount than for $\tilde{\beta} = 0.90$.

The dashed lines in Figure 4 further show the results for a less myopic government, $\tilde{\beta} = 0.96$. While the bailout probability as well as the mean price and the mean level of debt exhibits the same over all pattern, the default probability and the relative welfare measure behave differently. For values of κ that exceed 0.7, the mean price of debt falls and default probability increases even up to values that are larger than for the case without bailout loans. Correspondingly, welfare can fall when bailout loans are offered at loose fiscal constraints. Here, access to bailout loans allows the government to raise mean debt by even more than 30%, such that the impact on the default probability is reserved. Hence, the enhanced ability to roll-over debt under bailout offers can therefore aggravate the inefficiency induced by overborrowing if fiscal constraints are too loose.

5 Conclusion

In this paper, we aim at assessing the rationale for conditional bailout loans that are offered to indebted government that are not committed to fully repay debt. For this, we analyze fiscal policy under discretion in a closed economy framework, where the government faces a trade-off between defaulting on domestically held debt, which allows to avoid welfare-reducing tax increases or spending cuts, and avoiding resource costs of default. The government tends to overborrow due to myopia, which aggravates welfare losses originating from the lack of commitment and potentially justifies constraining sovereign borrowing. In this environment, we examine the effectiveness of conditional bailout loans, which have recently been offered by the European Stability Mechanism of the European Union.

In particular, we consider bailout loans that are offered at a risk-free interest rate and conditional upon minimum primary surpluses. We further assume that bailout loans are not associated with superior enforcement, which implies that bailout loans are inconsistent with individual rationality, and we assume that they are (lump-sum) financed by taxpayers. Nevertheless, we find that welfare can be enhanced under bailout offers, if the associated fiscal constraint is not too tight. Overall, bailout loans allow to over-roll debt at a favorable price, such that the level public debt tends to increase. Hence, overborrowing is not curbed by the conditionality of the bailout loans, such that bailout offers with very loose fiscal constraints can even revert the welfare result, i.e. they can lead to welfare losses compared to the case without bailout loans.

6 References

Adam, K., and Grill, M., 2012, Optimal Sovereign Debt Default, unpublished manuscript, University of Mannheim.

Arellano, C., 2008, Default Risk and Income Fluctuations in Emerging Economies, American Economic Review, 98, 690-712.

Aguiar, M. and G. Gopinath, 2006, Defaultable Debt, Interest Rates and the Current Account, Journal of International Economics 69, 64-83.

Aiyagari, S.R., R., Marcet, T.J. Sargent, and J. Seppälä, 2002, Optimal Taxation without State-Contingent Debt, Journal of Political Economy 110, 1220-1254.

Boz, E., 2011, Sovereign Default, Private Sector Creditors, and the IFIs, Journal of International Economics 83, 70-82

Chow, C., and J. Tsitsiklis, 1991, An Optimal One-Way Multigrid Algorithm for Discrete-Time Stochastic Control, IEEE Transactions on Automatic Control 36, 898-914.

Cole, H. L. and T.J. Kehoe, 2000, Self-fulling Debt Crises, Review of Economic Studies 67, 91-116.

Cuadra, G., J., Sanchez, and H., Sapriza, 2010, Fiscal Policy and Default Risk in Emerging Markets, Review of Economic Dynamics 13, 452-469.

Derbotoli D. and R. Nunes, 2013, Lack of Commitment and the Level of Debt, Journal of the European Economic Association, forthcoming.

Eaton, J. and M. Gersovitz, 1981, Debt with Potential Repudiation: Theoretical and Empirical Analysis, Review of Economic Studies 48, 289-309.

Grossman, H., and J.B. van Huyck, 1988, Sovereign Debt as a Contingent Claim: Excusable Default, Repudiation, and Reputation, American Economic Review 78, 1088-1097.

Klein, P., Krusell, P. and J.-V. Ríos-Rull, 2008, Time-Consistent Public Policy, Review of Economic Studies 75, 789-808.

Krusell, P., Fernando M., and J.-V. Ríos-Rull, 2006, Time-Consistent Debt, unpublished manuscript, Institute for International Economic Studies.

Lucas, R.E. and N.L. Stokey, 1983, Optimal Fiscal and Monetary Policy in an Economy without Capital, Journal of Monetary Economics 12, 55-93.

Panizza, U., Sturzenegger, F. and J. Zettelmeyer, 2009, 'The Economics and Law of Sovereign Debt and Default', Journal of Economic Literature 47, 651–698.

Pouzo, D., 2010, Optimal Taxation with Endogenous Default under Incomplete Markets, unpublished manuscript, UC Berkeley. Roch, F., and H. Uhlig, 2012, The Dynamics of Sovereign Debt Crises and Bailouts, unpublished manuscript, University of Chicago.

Tauchen, G., 1986, Finite State Markov Chain Approximations to Univariate and Vector Autoregressions, Economics Letter 20, 177-181.

Tomz, M. and M. L. J. Wright, 2007, Do Countries Default in Bad Times?, Journal of the European Economic Association 5, 352-360.

A Appendix to Section 3

Proof of proposition 1. We first examine the problem of a government which aims at maximizing household welfare under commitment, including full debt repayment $p_t =$ $1 \forall t \geq 0$. Given that public debt is non-state contingent, there does not exist a single implementability constraint (see Aiyagari et al., 2002). The problem of a government can then be summarized by considering the equilibrium conditions as constraints and a set of implementability constraints for all periods t, or equivalently as

$$\max_{l_t, g_t, \tau_t, b_t} \quad E \sum_{t=0}^{\infty} \beta^t u_t \quad s.t.$$
(27)

$$0 = u_{l,t} / \left[u_{c,t} \left(\Xi_t f(l_t) - g_t \right) \right] + (1 - \tau_t) \Xi_t f'(l_t)$$
(28)

$$0 = \tau_t \Xi_t f'(l_t) l_t - g_t + \beta E_t \frac{u_{c,t+1} \left(\Xi_{t+1} f(l_{t+1}) - g_{t+1}\right)}{u_{c,t} \left(\Xi_t f(l_t) - g_t\right)} b_t - b_{t-1}$$
(29)

and the transversality condition for government bonds, where $\Xi_t = \Xi(a_t, 1)$. The first order conditions for the tax rate, government spending, working time, and debt are given by

$$0 = \mu_t \Xi_t f'(l_t) - \gamma_t \Xi_t f'(l_t) l_t, \ \forall t \ge 0,$$
(30)

$$0 = -u_{c,t} + u_{g,t} + \mu_t u_{cc,t} \left(u_{l,t} / u_{c,t}^2 \right) - \gamma_t + \gamma_t u_{cc,t} \beta E_t \frac{u_{c,t+1}}{u_{c,t}^2} b_t - \gamma_{t-1} \frac{u_{c,t-1}}{u_{c,t-1}} b_{t-1}, \forall t \ge 1,(31)$$

$$0 = -u_{c,0} + u_{g,0} + \mu_t u_{cc,0} \left(u_{l,0} / u_{c,0}^2 \right) - \gamma_0 + \gamma_0 u_{cc,0} \beta E_0 \frac{u_{c,0+1}}{u_{c,0}^2} b_0,$$
(32)

$$0 = u_{c,t} \Xi_t f'_t + u_{l,t} + \mu_t \left(u_{ll,t}/u_{c,t} \right) - \mu_t u_{cc,t} \Xi_t f'_t \left(u_{l,t}/u_{c,t}^2 \right) + \mu_t \left(1 - \tau_t \right) \Xi_t f''(l_t), \tag{33}$$

$$+\gamma_{t}\tau_{t}\Xi_{t}\left(f'(l_{t})+f''(l_{t})l_{t}\right)-\gamma_{t}u_{cc,t}\beta E_{t}\Xi_{t}f'_{t}\frac{u_{c,t+1}}{u_{c,t}^{2}m}b_{t}+\gamma_{t-1}\Xi_{t}f'_{t}\frac{u_{cc,t}}{u_{c,t-1}}b_{t-1}, \forall t \ge 1,$$

$$0=u_{c}_{0}\Xi_{0}f'_{0}+u_{t}_{0}+u_{0}(u_{t}_{0})-u_{0}u_{cc}_{0}\Xi_{0}f'_{0}(u_{t}_{0})u_{c}^{2})+u_{0}(1-\tau_{0})\Xi_{0}f''(l_{0})$$
(34)

$$= u_{c,0} \pm 0 f_0 + u_{l,0} + \mu_0 (u_{l,0} - u_{c,0}) - \mu_0 u_{cc,0} \pm 0 f_0 (u_{l,0} - u_{c,0}) + \mu_0 (1 - r_0) \pm 0 f_0 (0)$$

$$+ \gamma_0 \tau_0 \Xi_0 \left(f'(l_0) + f''(l_0) l_0 \right) - \gamma_0 u_{cc,0} \beta E_0 \Xi_0 f_0' \frac{u_{c,0+1}}{u_{c,0}^2} b_0,$$

$$0 = \gamma_0 u_{cc,0} \beta E_0 \gamma_0 - \gamma_0 u_{cc,0} \beta E_0 \Xi_0 f_0' \frac{u_{c,0+1}}{u_{c,0}^2} b_0,$$

$$(35)$$

$$0 = \gamma_t q_t - \beta E_t \gamma_{t+1}, \ \forall t \ge 0, \tag{35}$$

where λ_t and γ_t denote the multiplier on (28) and (29). An equilibrium allocation for an optimizing government acting under commitment is a thus set of sequences $\{\lambda_t, \gamma_t, g_t, l_t, \tau_t, c_t, b_t\}_{t=0}^{\infty}$ satisfying (28)-(35), $\Xi_t f(l_t) = c_t + g_t$, $\lim_{t\to\infty} E(b_t/R_t) \prod_{i=1}^t (1/R_{i-1}) = 0$, given $b_{-1} > 0$. The optimal policy under commitment is time inconsistent if households are risk averse, $u_{cc} < 0$, which can immediately be seen from a comparison of (31) and (32) as well as of (33) and (34). If $u_{cc} = 0$, the price for government bond equals β (see 4). Then, (29) simplifies to

$$\tau_t \Xi_t f'(l_t) l_t - g_t + \beta b_t = b_{t-1}, \tag{36}$$

such that there is no relevant forward-looking equilibrium condition that serves as a constraint for the government problem (see 27). Then, (31), (32), (33), and (34) reduce to the following two conditions that apply for all periods $t \ge 0$:

$$\gamma_t = u_{g,t} - u_{c,t},\tag{37}$$

$$-u_{l,t} = u_{c,t} \Xi_t f'_t + \mu_t \left(u_{ll,t} / u_{c,t} \right) + \mu_t \left(1 - \tau_t \right) \Xi_t f''(l_t) + \gamma_t \tau_t \Xi_t \left(f'(l_t) + f''(l_t) l_t \right), \quad (38)$$

where $\Xi_t = \Xi(a_t, 1)$, implying that the optimal policy under commitment is time consistent. Now consider the governments problem without commitment satisfying (11). If default costs are prohibitively high, $\Xi(a_t, 1) \to 0$, private and public consumption satisfy $c_t \to 0$ and $g_t \to 0$ under the default choice $\delta_t = 0$, implying $u(c_t, g_t, l_t) \to -\infty$ by (24) and $V^d(b_{t-1}, a_t) \to -\infty$ by 19), such that the default set is empty, $\Lambda(b_{t-1}) = \emptyset$. The choice of a government that lacks commitment is then characterized by (21)-(23) and (??). When the government is non-myopic $\tilde{\beta} = \beta$ and households are risk-neutral, $u_{cc,t} = 0$, these conditions reduce to (28), (30), (36), (37), (38) and

$$u_{g,t} - u_{c,t} = \sum_{a_{t+1} \in \Upsilon} \left(u_{g,t+1} - u_{c,t+1} \right) \pi(a_{t+1}|a_t)$$
(39)

which are identical to the conditions describing fiscal policy under commitment for $u_{cc,t} = 0$ (see above). This establishes the claims made in the proposition.

B Appendix to Section 4

Under risk-neutrality, $u_{cc} = 0$, the government's optimality conditions for the tax rate, expenditures and working time under full repayment (21)-(23) are identical to the conditions under default, which are given by

$$0 = \mu_t \Xi_t f'(l_t) - \gamma_t \Xi_t f'(l_t) l_t \tag{40}$$

$$0 = u_{g,t} - u_{c,t} - \gamma_t + u_{cc,t} \mu_t \left(u_{l,t} / u_{c,t}^2 \right)$$
(41)

$$0 = u_{c,t} \Xi_t f'_t + u_{l,t} + \mu_t \left(u_{ll,t} / u_{c,t} \right) + \mu_t \left(1 - \tau_t \right) \Xi_t f''(l_t) + \gamma_t \tau_t \Xi_t \left(f'(l_t) + f''(l_t) l_t \right)$$
(42)
$$- u_{cc,t} \mu_t \Xi_t f'_t \left(u_{l,t} / u_{c,t}^2 \right)$$

where $\Xi_t = \Xi(a_t, 0)$ and μ_t and γ_t denote the multipliers for (28) and $\tau_t \Xi_t f'(l_t) l_t = g_t$. Further using $-u_{l,t}/[u_{c,t}(\Xi_t f(l_t) - g_t)] = (1 - \tau_t) \Xi_t f'(l_t)$ and (21) to eliminate working time and the multiplier γ_t and μ_t , shows that its optimal instrument choice is characterized by

$$u_{c,t}\left(\frac{u_{ll,t}l_t}{u_{l,t}}\left(1-\tau_t\right) - \frac{f''(l_t)l_t}{f'(l_t)}\right) = u_{g,t}\left(\frac{u_{ll,t}l_t}{u_{l,t}}\left(1-\tau_t\right) - \frac{f''(l_t)l_t}{f'(l_t)} - \tau_t\right)$$
(43)

Under full repayment, the government's choice is further constrained by $\tau_t \Xi_t f'(l_t) l_t - g_t + \beta E_t \frac{[(1-\delta_{t+1})u_{c,t+1}(\Xi_{t+1}f(l_{t+1})-g_{t+1})]}{u_{c,t}(\Xi_t f(l_t)-g_t)} b_t = b_{t-1}$, which reduces to $\tau_t \Xi_t f'(l_t) l_t - g_t + q_t b_t = b_{t-1}$. Hence, we can summarize the equilibrium under lack of commitment in terms of working time, government spending, the tax rate, the bond price q_t , consumption, and end-of-period debt as time invariant functions of the state variables a_t and b_{t-1} , $l(b_{t-1}, a_t)$, $g(b_{t-1}, a_t)$, $r(b_{t-1}, a_t)$, $c(b_{t-1}, a_t)$, and $b(b_{t-1}, a_t)$, satisfying (13),

$$q(b_{t-1}, a_t) = \beta \left[1 - \sum_{a_{t+1} \in \Lambda(b(b_{t-1}, a_t))} \pi(a_{t+1} | a_t) \right],$$

(15), (17), (43), and a choice for b_t that satisfies (18) if $a_t \in \Theta(b_{t-1})$, $b_t = \kappa_t b_{t-1}$ if $a_t \in F(b_{t-1})$, or $b_t = 0$ if $a_t \in \Lambda(b_{t-1})$, given $\{a_t\}_{t=0}^{\infty}$. For the functional forms for the utility function $u(c_t, g_t, l_t)$ and the deterministic part of the production function $f(l_t)$ introduced in (24) and (26), condition (43) can the be simplified to

$$\frac{g(a_t, b_{t-1})^{-\sigma}}{\psi} = \frac{1 - \alpha + \eta \left(1 - \tau \left(a_t, b_{t-1}\right)\right)}{1 - \alpha + \eta \left(1 - \tau \left(a_t, b_{t-1}\right)\right) - \tau \left(a_t, b_{t-1}\right)}$$
(44)

while (13), (15), and (17) are given by

$$l(a_t, b_{t-1}) = [(\alpha \psi/\vartheta) (1 - \tau (a_t, b_{t-1})) \Xi_t]^{1/(\eta + 1 - \alpha)},$$
(45)

$$g(a_t, b_{t-1}) = \tau_t a_t \alpha l(a_t, b_{t-1})^{\alpha} + p_t [q(b(a_t, b_{t-1}), a_t) b(a_t, b_{t-1}) - b_{t-1}],$$
(46)

$$c(a_t, b_{t-1}) = \Xi_t l_t^{\alpha} (a_t, b_{t-1}) - g_t(a_t, b_{t-1}), \qquad (47)$$

where $\Xi(a_t, p_t)$ satisfies (25).

C Computation

The model is solved using discrete state space value function iteration. The productivity process is discretized using Tauchen's (1986) algorithm, using an equi-spaced grid with 151 points and a width of ± 3 standard deviations. For the asset space we use 251 equi-spaced grid points. To speed up the numerical solution we apply a multigrid algorithm, see Chow and Tsitsiklis (1991). The algorithm first solves the dynamic programming problem for a coarse grid and then increases the number of grid points until the grid is fine enough. Between the steps of iteration, the solution for the coarser grid is used to generate the initial guess for the value function and the bond price schedule on the new grid by multi-dimensional interpolation.

Algorithm To simplify the notation, we denote $x = x_t$ and $x' = x_{t+1}$ where $x \in \{\tau_t, g_t, a_t, q_t, l_t\}$ and $b = b_{t-1}$ and $b' = b_{t+1}$. We further rewrite the utility function by substituting out consumption and labor by (45) and (47)

$$u(\tau, g, a) = \begin{cases} \psi \left(f(\epsilon a^{\phi} (1 - \tau)^{\phi}) - g \right) + \frac{g^{1 - \sigma} - 1}{1 - \sigma} - [\vartheta/(1 + \eta)] \left(\epsilon a^{\phi} (1 - \tau)^{\phi} \right)^{1 + \eta} & \text{if } p_t = 1\\ \psi \left(f(\epsilon h(a)^{\phi} (1 - \tau)^{\phi}) - g \right) + \frac{g^{1 - \sigma} - 1}{1 - \sigma} - [\vartheta/(1 + \eta)] \left(\epsilon h(a)^{\phi} (1 - \tau)^{\phi} \right)^{1 + \eta} & \text{if } p_t = 0 \end{cases}$$

$$\tag{48}$$

where $\epsilon = (\alpha \psi / \vartheta)^{\phi} > 0$ and $\phi = \frac{1}{\eta + 1 - \alpha} \in (0, 1)$.

Initialize a set of states for a and b, where a is associated with a transition matrix with the probabilities $\pi(a'|a)$ approximating an AR1 process. Consider initial guesses for V_0^d , V_0^c , and V_0^b , and that the value $V_i(a, b)$ is now defined for $i \ge 0$ as

$$V_i(b,a) = \max\{V_i^c(b,a), \max\{V^c(b,a), \max\{V^d(b,a), V^b(b,a) | V^c(b,a) < V^d(b,a)\}\},\$$

and an initial guess for the bond price schedule $q_0 = 1/R_0 = \beta$.

- 1. Consider the <u>no default</u> case: Use q_0 , (44) and (46) to compute the optimal values τ^c and g^c for all combinations of a, b and $b' : \tau^c(a, b, b')$ and $g^c(a, b, b')$.
 - (a) Rewrite current period utility (24) as a function of $\tau^{c}(a, b, b')$ and $g^{c}(a, b, b')$:

$$u^{c}\left(\tau^{c}\left(a,b,b'\right),g^{c}\left(a,b,b'\right),a\right) = \psi\left(f(\epsilon a^{\phi}\left(1-\tau^{c}\left(a,b,b'\right)\right)^{\phi}\right) - g^{c}\left(a,b,b'\right)\right)$$
$$+\frac{g^{c}\left(a,b,b'\right)^{1-\sigma}-1}{1-\sigma} - \left[\vartheta/(1+\eta)\right]\left(\epsilon a^{\phi}\left(1-\tau^{c}\left(a,b,b'\right)\right)^{\phi}\right)^{1+\eta}$$

(b) Insert V_0^c and V_0^d in the RHS of (18), and find an b' for all combinations of a and b that satisfies

$$\max_{b'} \left\{ u^c \left(\tau^c \left(a, b, b' \right), g^c \left(a, b, b' \right), a \right) + \widetilde{\beta} \sum_{a_{t+1}} V_0(b', a') \pi(a' | a_t) \right\},\$$

which delivers the policy function b'(a, b). The associated maximum value is $V_1^c(b, a)$.

2. Consider the <u>default</u> case: Use (44) and (46) to compute the optimal values τ^d and g^d for all a's and $b's : \tau^d(a, b)$ and $g^d(a, b)$.

(a) Rewrite current period utility (48) as a function of $\tau^{d}(a, b)$ and $g^{d}(a, b)$:

$$\begin{aligned} u^{d}\left(\tau^{d}\left(a,b\right),g^{d}\left(a,b\right),a\right) &= \psi\left(f(\epsilon h(a)^{\phi}\left(1-\tau^{d}\left(a,b\right)\right)^{\phi})-g^{d}\left(a,b\right)\right) \\ &+ \frac{g^{d}\left(a,b\right)^{1-\sigma}-1}{1-\sigma}-\left[\vartheta/(1+\eta)\right]\left(\epsilon h(a)^{\phi}\left(1-\tau^{d}\left(a,b\right)\right)^{\phi}\right)^{1+\eta} \end{aligned}$$

(b) Insert V_0^c and V_0^d in the RHS of (19), which gives

$$V_1^d(a,b) = u^d \left(\tau^d(a,b), g^d(a,b), a\right) \\ + \widetilde{\beta} \sum_{a_{t+1}} \left[\mu V_0(b'=0,a') + (1-\mu)V_0^d(a',b'=0)\right] \pi(a'|a).$$

- 3. Consider the <u>bailout-case</u>: Use $q_0 = \beta$, (44) and (46) to compute the optimal values τ^b and g^b for all a's and $b's : \tau^b(a, b)$ and $g^b(a, b)$.
 - (a) Rewrite current period utility (48) as a function of $\tau^{b}(a)$ and $g^{b}(a)$:

$$\begin{split} u^{b}\left(\tau^{b}\left(a,b\right),g^{b}\left(a,b\right),a\right) &= \psi\left(f(\epsilon a^{\phi}\left(1-\tau^{b}\left(a,b\right)\right)^{\phi})-g^{b}\left(a,b\right)\right) \\ &+ \frac{g^{b}\left(a,b\right)^{1-\sigma}-1}{1-\sigma} - \left[\vartheta/(1+\eta)\right]\left(\epsilon a^{\phi}\left(1-\tau^{b}\left(a,b\right)\right)^{\phi}\right)^{1+\eta}. \end{split}$$

(b) Insert V_0^c and V_0^d in the RHS of (20), which gives

$$V_{1}^{b}(a,b) = u^{b}\left(\tau^{b}(a,b), g^{b}(a,b), a\right) + \widetilde{\beta} \sum_{a_{t+1}} V_{0}(b',a')\pi(a'|a),$$

where we used that $b' = b/(\Psi + \beta)$ when the government opts for bailout.

4. For each pair a and b, set d(b, a) = 1 if $V_1^c(b, a) > V_1^d(a, b)$ or if $V_1^b(a, b) > V_1^d(b, a) |V_1^d(a, b) > V_1^c(b, a)$ and d(b, a) = 0 if $V_1^d(a, b) > \max\{V_1^c(b, a), V_1^b(a, b)\}$. Then, collect

$$\Theta(b) = \{a \in \Upsilon : d(b, a) = 1\} \quad \text{and} \quad \Lambda^d(b) = \{a \in \Upsilon : d(b, a) = 0\},\$$

and compute

$$E_t \left[1 - \delta' \right] = \sum_{a' \in \Theta(b')} \pi(a'|a),$$

as well as $q_1(b', a) = \beta E (1 - \delta')$. Further collect

$$F(b) = \left\{ a \in \Upsilon : V_1^b(a, b) > V_1^d(b, a) | V_1^d(a, b) > V_1^c(b, a) \right\}.$$

5. Save $q_1(b', a)$ and $V_1^c(b, a)$, $V_1^d(b, a)$ and $V_1^b(a, b)$ and repeat steps 1-4 until these objects converge.

D Additional figures



Default states and selected policy functions without bailout loans for $\widetilde{\beta}=0.84$



Default states and selected policy functions under bailout loans with $\kappa=0.8$