

Unemployment Fluctuations, Match Quality, and the Wage Cyclicalilty of New Hires*

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Abstract

Macroeconomic models often incorporate some form of wage stickiness to help account for employment fluctuations. However, a recent literature calls in to question this approach, citing evidence of new hire wage cyclicalilty from panel data studies as evidence for contractual wage flexibility for new hires, which is the relevant margin for employment volatility. We analyze data from the SIPP and find that the wages for new hires coming from unemployment are no more cyclical than those of existing workers, suggesting wages are sticky at the relevant margin. The new hire wage cyclicalilty found in earlier studies instead appears to reflect cyclical average wage gains of workers making job-to-job transitions, which we interpret as evidence of procyclical match quality for new hires from employment. We then develop a quantitative general equilibrium model with sticky wages via staggered contracting, on-the-job search, and variable match quality, and show that it can account for both the panel data evidence and aggregate labor market regularities. An additional implication of the model is that a sullyng effect of recessions emerges, along the lines originally suggested by Barlevy (2002).

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1 Introduction

Aggregate wage data suggests relatively little variation in real wages as compared to output and unemployment. This consideration has motivated incorporating some form of wage rigidity in quantitative macroeconomic models to help account for business cycle fluctuations, an approach that traces back to the early large scale macroeconometric models and remains prevalent in the recent small scale DSGE models.¹ Such considerations have also motivated the inclusion of wage rigidity in search and matching models of the labor market in the tradition of Diamond, Mortensen and Pissarides. Most notably, Shimer (2005) and Hall (2005) show that the incorporation of wage rigidity greatly improves the ability of search and matching models to account for unemployment fluctuations.²

An influential paper by Pissarides (2009), however, argues that the aggregate data may not provide the relevant measure of wage stickiness: what matters for employment adjustment is the wages of new hires, which need to be disentangled from aggregate measures of wages. In this regard, there is a volume of panel data evidence beginning with Bils (1985) that finds that wages of new hires are substantially more cyclical than those of existing workers. Pissarides interprets the findings of this literature as evidence for a high degree of contractual wage flexibility among new hires, calling into question efforts to incorporate wage rigidity into macroeconomic models.

In this paper, we revisit new hire wage cyclicalities and the associated implications for aggregate unemployment fluctuations. We argue that the interpretation of *wage cyclicalities* as direct evidence of *wage flexibility* ignores confounding cyclical variation in new hire wages that is due to workers moving to better jobs during expansions. We adopt a novel empirical strategy to separate contractual wage flexibility from cyclical match quality. Guided by an existing empirical and theoretical literature, we argue that cyclical changes in match quality should be more prevalent among employed workers. We use a detailed dataset to argue that typical findings of excess wage cyclicalities among new hires can be attributed to cyclical match quality among new hires from employment. We find no evidence of excess wage cyclicalities among new hires from unemployment. We then develop a quantitative macroeconomic model that is able to account for both the aggregate and panel data evidence.

Key to our approach is the observation that the existing empirical literature identifies new hire wage cyclicalities using an econometric specification that does not distinguish be-

¹ See for, example, Christiano, Eichenbaum, and Evans (2005), Smets and Wouters (2007), Gertler, Sala, and Trigari (2008), Galí, Smets, and Wouters (2011), Christiano, Eichenbaum, and Trabandt (2015), and Christiano, Eichenbaum, and Trabandt (2016).

²Gertler and Trigari (2009), Hall and Milgrom (2008), Blanchard and Galí (2010), and Christiano, Eichenbaum, and Trabandt (2016) build on this approach and model the wage setting mechanism in greater detail.

tween new hires from unemployment and those from coming other jobs. First, we note that the key wage for understanding unemployment fluctuations is that of new hires from unemployment, as argued by Haefke, Sonntag, and van Rens (2013). Second, we argue that in pooling these two types of new hires, the prototypical regression conflates possible wage flexibility of new hires with procyclical improvements of match quality for new hires from employment. Drawing upon existing literatures on cyclical upgrading and job-to-job changes, we argue that excess wage cyclicality for workers from employment should be interpreted as evidence of cyclical match quality.

To address these concerns, we construct a unique dataset from the Survey of Income and Program Participation (SIPP) that allows us to separately estimate the wage cyclicality of new hires from unemployment from those making job-to-job transitions. We first show that by pooling the two types of new hires with our data, we can replicate the typical result of the existing literature: new hire wages appear to be more flexible than the wages of continuing workers. When we estimate separate terms for both types of new hires, however, we find no evidence of excess wage cyclicality for new hires coming from unemployment, but substantial evidence of procyclical match improvement for workers making job-to-job transitions.

Then, to drive our point home, we develop a search and matching model with staggered wage contracting, variable match quality, and on-the-job search with endogenous search intensity. We show that the model is consistent with both the aggregate data and the panel data evidence. In particular, while the wages of new hires are sticky within the model, cyclical improvements in match quality generate new hire wage cyclicality, offering the appearance of wage flexibility among new hires. All the three ingredients of the model are critical for achieving this consistency. Further, the cyclical job reallocation within the model generates a quantitatively important sullyng effect of recessions, as originally conceived by Barlevy (2002).

Our results are aligned with a rich literature on earnings growth and job-to-job transitions. Beginning with Topel and Ward (1992), an extensive empirical literature has documented that a large fraction of the wage increases experienced by a given worker occur through job-to-job transitions. Such job movements can be understood as employed workers actively searching for higher paying jobs, along the lines of Burdett and Mortensen (1998). A related theoretical literature has shown that such match improvements are more easily realized during expansions than during recessions (Barlevy, 2002; Menzio and Shi, 2011). In contrast, such job-ladder models offer no systematic prediction for wage changes of workers searching from unemployment, as such workers are predicted to adopt a reservation wage strategy that is not contingent on their most recent wage. We conclude that, while other mechanisms could be in play to generate cyclical match quality for new hires from

unemployment, the existing literature suggests that it should be most apparent for workers making job-to-job changes.

While the interpretation of flexible wages for new hires is still prevalent in the literature, other papers have documented that the addition of finer controls weakens the case for excess wage flexibility for new hires. Gertler and Trigari (2009) provide suggestive evidence that the wage cyclicality of new hires relative to continuing workers is driven by procyclical match quality of job changers. A similar idea is pursued in Hagedorn and Manovskii (2013), who develop indirect measures of match quality to challenge empirical findings of excess new hire wage cyclicality and implicit contracts a la Beaudry and DiNardo (1991). Martins, Solon, and Thomas (2012) use data from Portugal to isolate wage cyclicality of workers newly hired into a fixed set of “entry jobs”; their estimates suggest that new hire wage cyclicality is roughly the same as that for continuing workers.³

In terms of empirical methodology, our paper is closest to Haefke, Sonntag, and van Rens (2013) who examine directly the wage cyclicality of new hires from unemployment. They use cross-sectional data from the CPS and recover point estimates suggestive of excess wage cyclicality of new hires from unemployment, although not statistically significant. We instead use a rich, high-frequency panel data set from the SIPP. The panel aspect of our data permits sharp controls for unobserved heterogeneity and compositional effects. To this end, we find statistically significant evidence that new hires wages from unemployment are no more cyclical than for existing workers. As a corollary, we show that the excess wage cyclicality of new hires recovered by the literature is entirely driven by new hires from employment, raising the possibility that this excess cyclicality is an artifact of cyclical movements in match quality via the job ladder, as opposed to true wage flexibility. Finally, as noted earlier, to support this hypothesis we develop a macroeconomic model with wage rigidity, variable match quality, and on the job-search with endogenous search intensity. We then show that simulated data from the model is consistent with both the aggregate and panel data evidence.

Section 2 provides the new panel data evidence. Section 3 describes the model and Section 4 presents the numerical results. Concluding remarks are in Section 5.

³More precisely, the semi-elasticities of wages to unemployment range from 1.48 to 1.81 for new hires and 1.25 to 1.51 for incumbents, depending on the exact empirical specification. Moreover, while Martins, Solon, and Thomas (2012) find roughly equivalent cyclicality for both continuing workers and new hires within jobs, real wage variation in their data are largely driven by large exchange rate devaluations. This is quite different from the type of real wage variation measured in the U.S., where most real wage variation reflects changes in nominal wages.

2 Data and Empirics

This section presents new evidence on the wage cyclicality of new hires. We do so using a rich new data set. We first show that we are able to replicate the existing evidence showing greater cyclicality of the wages of new hires relative to existing workers. We then proceed to show that there is no evidence of excess wage cyclicality for workers hired from unemployment, but substantial evidence for procyclicality in match quality for job changers. Our evidence is fully consistent with new hires having the same degree of wage flexibility as existing workers. We first describe the data and then move to the estimation.

2.1 Data

We use data from the Survey of Income and Program Participation (SIPP) from 1990 to 2012. The SIPP is administered by the U.S. Census Bureau and is designed to track a nationally representative sample of U.S. households. The SIPP is organized by panel years, where each panel year introduces a new sample of households. Over our sample period the Census Bureau introduced eight panels. The starting years were 1990-1993, 1996, 2001, 2004, and 2008. The average length of time an individual stays in a sample ranges from 32 months in the early samples to 48 in the more recent ones.

Most key features of the SIPP are consistent across panels. Each household within a panel is interviewed every four months, a period referred to as a wave. During the first wave that a household is in the sample, the household provides retrospective information about employment history and other background information for working age individuals in the household. At the end of every wave, the household provides detailed information about activities over the time elapsed since the previous interviews, including job transitions that have occurred within the wave. Although individuals report earnings for each month of the wave, we only use reported earnings from the last month of the wave to accommodate the SIPP “seam effect.”⁴

The SIPP has several features that make it uniquely suited for our analysis. Relative to other commonly used panel data sets, the SIPP follows many more households, follows multiple representative cohorts, and is assembled from information collected at a high frequency (e.g. surveys are every four months as opposed to annually). This high frequency structure of the data is crucial for constructing precise measurements of employment status and wages. In particular, we use job-specific earnings to generate monthly records of job-holding for each individual, allowing us to discern direct job-to-job transitions from job

⁴Specifically, we find that the vast majority of earnings changes for workers employed at the same job continuously across multiple waves occur between waves, as opposed to during a wave. The “seam effect” is discussed in greater detail in the SIPP User’s Guide (U.S. Census Bureau, 2001, 1-6).

transitions with an intervening spell of non-employment.⁵ As the SIPP contains multiple cohorts, at each point in time the sample is always representative of the U.S. population, in contrast to other widely used panel datasets such as the NLSY.

Crucial to our approach is that the SIPP maintains consistent job IDs. Fujita and Moscarini (2013) document that, starting with the 1996 SIPP wave, a single job may be assigned multiple IDs for an identifiable subset of survey respondents. In the appendix, we develop a procedure that exploits a feature of the SIPP employment interview module that allows us to identify jobs that may have been assigned multiple IDs. We find evidence for recall employment, corroborating Fujita and Moscarini’s finding that recalls compose a significant fraction of transitions to employment from non-employment.⁶

The appendix provides further discussion of the data and the construction of the variables we use in the estimation.

2.2 Baseline Empirical Framework

We begin with a simple statistical framework to study the response of individual level wages to changes in aggregate conditions that has been popular in the literature, beginning with Bils (1985).⁷ Let w_{ijt} be the wage of individual i in job j at time t , x_{ijt} individual level characteristics such as education and job tenure as well as a time trend, u_t the unemployment rate, $\mathbb{I}(new_{ijt})$ an indicator variable that equals unity if the worker is a new hire and zero if not, and α_i an individual fixed effect. The measurement equation for wages is then given by

$$\log w_{ijt} = x'_{ijt}\pi_x + \pi_u \cdot u_t + \pi_n \cdot \mathbb{I}(new_{ijt}) + \pi_{nu} \cdot \mathbb{I}(new_{ijt}) \cdot u_t + \alpha_i + e_{ijt} \quad (1)$$

where e_{ijt} is random error term.

The inclusion of the unemployment rate in the regression is meant to capture the influence of cyclical factors on wages, while the interaction of the new hire dummy with the unemployment rate is meant to measure the extra cyclicity of new hires wages. In particular, the coefficient π_u can be interpreted as the semi-elasticity of wages with respect to

⁵Starting with the 1996 panel, respondents report the start and end dates associated with a job. While our measure is highly correlated with the self-reported measure, the self-reported measure is sometimes inconsistent with self-reported activity from other waves—e.g., a worker will report a starting date that corresponds to a prior wave for which the respondent had previously reported being unemployed or employed at a different job. We use our earnings-based measure for all panels to avoid such issues of measurement error and maintain consistency in our analysis of the pre- and post-1996 data.

⁶We do not include these observations as new hires in our analysis; if these workers receive wages that are only as cyclical as “stayers”, they would bias the estimation of wage cyclicity of new hires from unemployment downwards.

⁷Included among the many studies regressing individual level wages on some measure of unemployment as a cyclical indicator are Beaudry and DiNardo (1991); Shin (1994); Solon, Barsky, and Parker (1994); Barlevy (2001); Carneiro, Guimarães, and Portugal (2012); Devereux (2002); Martins, Solon, and Thomas (2012); and Hagedorn and Manovskii (2013).

unemployment, while $\pi_u + \pi_{nu}$ gives the corresponding semi-elasticity for new hires. The key finding in the literature is that π_{nu} is negative (along with π_u), suggesting greater cyclical sensitivity of new hires’ wages.

At this point we make two observations: First, with exception of Haefke, Sonntag, and van Rens (2013), the literature typically does not distinguish between new hires coming from unemployment and those coming from other jobs. Second, since changes in wages of workers making job-to-job transitions include variation in quality across jobs, cyclical movements in job match quality will bias the new hire effect for workers coming from employment. We turn to these issues shortly.

We first show that with our data we can obtain the results in the literature. To obtain consistent coefficient estimates of equation (1), it is necessary to account for the presence of unobserved heterogeneity α_i that may be correlated with observables. The convention in the literature, accordingly, is to use either a first difference or a fixed effects estimator, depending on the properties of the error term. The low serial correlation of the error terms in the exercises we perform suggests that the fixed effects estimator is preferred. However, since Bills (1985) and others used a first difference estimator, we show the results are robust to either approach.⁸

The regressions are based on monthly data.⁹ For comparability to Bills (1985), we only use observations for men between the ages of 20 and 60. Accordingly, unemployment is the prime age unemployment rate. As our measure of hourly wages, we use job-specific earnings. In cases in which an hourly wage is directly available, we use that as our measure. In cases in which an hourly wage is not directly available, we use job-specific earnings divided by the product of job-specific hours per week and job-specific weeks per month. Wages are deflated by a four months average of the monthly PCE. Finally, we define “new hires” as individuals who are in the first four months of their tenure on a job.¹⁰ The appendix provides additional information on variable construction, including the individual level characteristics we use.

Table 1 presents the results. The first column presents the estimates of equation (1) using fixed effects and the second presents estimates using first differences. The results are robust across specifications. Similar to Bills (1985), we find that new hires’ wages are significantly more cyclical than those for existing workers. When estimating the equation in first differences, the semi-elasticity of new hire wages is -1.445 , compared to -0.448 for continuing workers. With fixed effects, the new hire semi-elasticity is estimated to be -1.409 , compared to -0.162 for continuing workers. In both specifications, the semi-

⁸As in Bills (1985), we difference the unemployment rate in the interaction term but not the new hire identifier itself, i.e. $\mathbb{I}(new_{ijt}) \cdot \Delta u_t$ rather than $\Delta [\mathbb{I}(new_{ijt}) \cdot u_t]$.

⁹While we have monthly information, as we noted earlier we only use wage information from the final month of each four month wave to avoid the “seam” effect.

¹⁰Note that given this definition we will only have one wage observation for a new hire since we only use the final month of a four month wave to obtain wage data.

elasticity is significant at the 1% level for continuing workers; and the new hire differential is significant at the 1% level. We find no evidence of serial correlation in the predicted errors, implying that fixed effects are more efficient than first differences. Hence, our preferred estimates come from the fixed effects regression.

While we recover precise coefficient estimates that imply both continuing worker wage cyclicality and a new hire effect, our estimates reveal less cyclicality than most of the existing literature. Using annual NLSY data from 1966-1980, Bills (1985) finds a continuing worker semi-elasticity of 0.6, versus 3.0 for changers. Barlevy (2001) uses both PSID and NLSY through 1993 and recovers a semi-elasticity of 3.0 for job changers. We speculate that our lower estimates are due mainly the high-frequency of our data (every four months as opposed to every year). If workers are on staggered multi-period contracts (as will be the case in the quantitative model we present), then a smaller fraction of wages are likely to be adjusted over a four month interval than would be the case annually. In any case, our quantitative model will generate data consistent with the degree of wage cyclicality suggested by the evidence in Table 1.

2.3 Robustness of the New Hire Effect

We now present evidence that the estimated new hire effect in the literature reflects cyclical wage gains of workers making job-to-job transitions, rather than greater wage flexibility of new hires.

We begin by distinguishing new hires that come from unemployment from those that come from other jobs. As Haefke, Sonntag, and van Rens (2013) emphasize, the hiring margin that is key for generating unemployment volatility in search and matching models with sticky wages is that of workers coming from unemployment, not that of workers making job-to-job transitions. Yet most empirical studies do not distinguish between new hires that are job changers and workers hired from unemployment.¹¹ Accordingly, it is important to isolate the wage behavior for new hires coming from unemployment. To do so, we estimate a variant of (1) that allows for a separate new hire effect for workers coming from non-employment and workers making direct job-to-job transitions:

$$\begin{aligned} \log w_{ijt} = & x'_{ijt}\pi_x + \pi_u \cdot u_t + \pi_{nu}^{ENE} \cdot \mathbb{I}(new_{ijt} \& EN E_{ijt}) \cdot u_t + \pi_{nu}^{EE} \cdot \mathbb{I}(new_{ijt} \& EE_{ijt}) \cdot u_t \\ & + \pi_n^{ENE} \cdot \mathbb{I}(new_{ijt} \& EN E_{ijt}) + \pi_n^{ENE} \cdot \mathbb{I}(new_{ijt} \& EN E_{ijt}) + \alpha_i + e_{ijt}, \end{aligned} \quad (2)$$

¹¹One exception is Hyatt and McEntarfer (2012), who use quarterly data from the LEHD to study the cyclicality of job-to-job changes. While they do not discuss the differential wage cyclicality of new hires from employment and new hires from non-employment, their Figure 2 seems to suggest that earnings changes for workers with 3 to 8 months of nonemployment between jobs are less cyclical than for workers who make job changes within a quarter. Relative to Hyatt and McEntarfer, we use data that contains hourly wages, permits precise measures of job transitions, and also covers a longer time span.

where we use the notation ENE to signify workers with an intervening spell of non-employment and EE to signify workers who made direct job-to-job transitions.¹²

Table 2 presents the results. For robustness, we consider four different measures of what constitutes a new hire from non-employment. In the baseline case presented in the first column we use the broadest measure: all new hires who did not receive a wage in the previous month, independent of how long the unemployment spell. The second column addresses the concern that new hires from nonemployment who have only missed a single month of pay might in fact be job-changers taking a short break between jobs; here, we group such new hires with workers making direct job-to-job transitions. In the third column for ENE transitions we exclude new hires with excessively long unemployment spells, which we consider to be a spell of greater than nine months. We do this to address concerns that such workers may have atypical wage outcomes. Finally, the fourth column addresses at the same time both concerns of long-term unemployment and short break in-between jobs.

As the table shows, for all three specifications, the new hire effect disappears for workers coming from unemployment. The coefficient π_{nu}^{ENE} is not statistically significant in each case. Thus, for new hires coming from unemployment, wages are no more cyclical than those for existing workers. Moreover, while the new hire effect disappears for workers coming from unemployment, we find substantial evidence of procyclical changes in match quality for job changers. Indeed, the coefficient on the job-changer interaction term is higher than the coefficient on the interaction term for the baseline regressions in Table 1, where both types of new hires are pooled together.¹³

Although we expect to obtain more efficient estimates from the fixed effects regressions, it will be convenient to use estimates of the various interaction terms from the first difference regressions when we examine our results with a quantitative model. We proceed identically in Table 3 as in Table 2, except we estimate the regression in first differences. Our results do not change: we find no evidence of wage cyclicity for new hires from unemployment, but recover a negative and statistically significant coefficient on the interaction term for new hires from employment.

We regard the negative coefficient on π_{nu}^{EE} as indicative of procyclical match quality for employed workers. Pissarides (2009) interprets a negative coefficient on the new hire term pooled across all new hires as indicative of greater flexibility in new hire wages. According to this interpretation, the results from Table 2 might suggest that firms have substantial leeway to adjust the wages of new hires from employment but not of new hires from unemployment. First, we note that such an interpretation would still not detract from one of our main

¹²Note that workers making ENE transitions may have gone a full wave without employment. We drop this observation, as the worker is not employed and earns zero wages.

¹³We can reject the null hypothesis that the wage cyclicity for new hires from non-employment equals the wage cyclicity for new hires from employment at the 5% level.

points: the primal wage for studying the volatility of unemployment is that of new hires from unemployment and this wage is no more cyclical than the wage of continuing workers. Second, we find such an interpretation implausible: it is hard to rationalize a bargaining mechanism whereby wages for new hires from employment are flexible, but wages for new hires from unemployment are not. Instead, we interpret our results in line with (i) an empirical literature finding that job changers realize substantial wage gains from switching jobs (Topel and Ward, 1992), and (ii) a theoretical literature arguing that is easier for workers in employment to locate better matches during expansions than recessions (Barlevy, 2002).¹⁴

How does our interpretation map into the regression equation (2)? Note that while the regression specification allows for time-invariant person fixed effects, it does not control for match specific effects that differ across jobs. Suppose that the error term e_{ijt} in the regression equation (2) takes the form

$$e_{ijt} = q_{ij} + \varepsilon_{ijt} \quad (3)$$

where q_{ij} represents unobserved match quality. If workers find better matches when the unemployment rate is low – or similarly, if the share of workers moving from bad to good matches of total job flows procyclical – then changes in e_{ijt} across jobs should be correlated with the change in the unemployment rate due to cyclical changes in q_{ij} :

$$Cov(\Delta q_{ij}, \Delta U_t) < 0. \quad (4)$$

It follows that, among new hires from employment, the error term e_{ijt} will be correlated with the unemployment rate U_t in differences (if the regression is estimated in first-differences) and mean deviations (if the regression is estimated in fixed effects). As a consequence, the estimated coefficient intended to identify the excess cyclical of new hires wages, π_{nu}^{EE} , will be biased downward. If so, estimates of a negative value of π_{nu}^{EE} would reflect composition bias rather than greater cyclical of new hire wages.

Figure 1 illustrates how procyclical match upgrading for job changers may bias estimates of new hire wage cyclical. The figure portrays cyclical wage variation across two jobs: a good match and a bad match. For each match, the solid and dotted lines are the wage with and without cyclical effects, where within each type of match, wages are only modestly cyclical. Consider, however, a worker employed in a bad match through period $\tau + 1$ paying a wage w_B . When the aggregate state turns from recession to expansion at period $\tau + 2$,

¹⁴In the next section, we develop a formal model consistent with this interpretation and show that the model well accounts for aggregate unemployment and wage volatility, as well as the micro findings shown here.

the worker moves to a good match paying wage \bar{w}_G . There are two cyclical sources of the worker’s wage increase: the modest cyclical increase in wages for all workers in good matches (\bar{w}_G vs. \underline{w}_G), but more importantly, the improvement in match quality facilitated by the expansion (w_G vs. w_B). While it is clear that the wage growth of such job changers is more cyclical than that of continuing workers, it is impossible to discern whether any of the excess cyclicity is due to greater wage flexibility for new hires absent appropriate controls for match quality. One possible way to control for match quality would be to introduce fixed effects at the person-job level and identify wage flexibility for new hires from wage variation within a job. However, we do not observe individuals on specific jobs for periods sufficiently long to tightly identify the job-person fixed effect.

We view our regression estimates as conditional moments from the data. In the next section, we develop a model of equilibrium unemployment with on-the-job search, variable match quality, and wage stickiness for new hires. We follow the typical strategy of targeting steady state quantities (such as the average wage growth of new hires from employment and unemployment) and leave the cyclical moments as model outcomes. We find that the model is successfully able to simultaneously match the untargeted micro and macro moments.

Crucially, in our model, new hires wages are no more flexible than those for existing workers; yet data generated from the model will give rise to the appearance of new hire wage flexibility when evaluated by the typical regression from the literature.

3 Model

We model employment fluctuations using a variant of the Diamond, Mortensen, and Pissarides search and matching model. Our starting point is a simple real business cycle model with search and matching in the labor market, similar to Merz (1995) and Andolfatto (1996). As in these papers, we minimize complexity by imposing complete consumption insurance. Our use of the real business cycle model is also meant for simplicity. It will become clear that our central point of how a model with wage rigidity can account for the micro wage evidence will hold in a richer macroeconomic framework.¹⁵

We make two main changes to the Merz/Andolfatto framework. First we allow for staggered wage contracting with wage contracts determined by Nash bargaining, as in Gertler and Trigari (2009). Second, we allow for both variable match quality and on-the-job search with variable search intensity. These features will generate procyclical job ladder effects, in

¹⁵ Similarly, Gertler and Trigari (2009) have investigated the role of staggered Nash bargaining within a real business cycle model with technology shocks as the only driving force. Gertler, Sala, and Trigari (2008) have then verified that the insights on the role of the contracting structure in generating plausible movements in the labor share and the relevant labor market variables carry over to a more general setup that features multiple sources of cyclical fluctuations and additional propagation mechanisms.

the spirit of Barlevy (2002) and Menzio and Shi (2011). As we will show, both these variants will be critical for accounting for both the macro and micro evidence on unemployment and wage dynamics.

3.1 Search, Vacancies, and Matching

There is a continuum of firms and a continuum of workers, each of measure unity. Workers within a firm are either good matches or bad matches. A bad match has a productivity level that is only a fraction ϕ of that of a good match, where $\phi \in (0, 1)$. Let n_t be the number of good matches within a firm that are working during period t and b_t the number of bad matches. Then the firm's effective labor force l_t is the following composite of good and bad matches:

$$l_t = n_t + \phi b_t \tag{5}$$

Firms post vacancies to hire workers. Firms with vacancies and workers looking for jobs meet randomly (i.e., there is no directed search). The quality of a match is only revealed once a worker and a firm meet. Match quality is idiosyncratic. A match is good with probability ξ and bad with complementary probability $1 - \xi$. Hence, the outcome of a match depends neither on ex-ante characteristics of the firm or the worker. Whether or not a meeting becomes a match depends on the realization of match quality and the employment status of the searching worker.

Workers search for jobs both when they are unemployed and when they are employed. Before search occurs, matches are subject to an exogenous separation shock. With probability ν , workers will search on-the-job; absent successful search that generates a new match at a different firm, these workers will remain at the firm for another period. With probability $1 - \nu$, the match is terminated. Workers who are subject to the separation shock and do not successfully find a job by the end of the period will be unemployed at the start of the next period.

There are three general types of searchers: the unemployed, the employed, and the recently separated. We first consider the unemployed. Let $\bar{n}_t = \int_i n_t di$ and $\bar{b}_t = \int_i b_t di$ be the total number of workers who are good matches and who are bad matches, respectively, where firms are indexed by i . The total number of unemployed workers \bar{u}_t is then given by

$$\bar{u}_t = 1 - \bar{n}_t - \bar{b}_t. \tag{6}$$

We assume that each unemployed worker searches with a fixed intensity, normalized at unity. Under our parameterization, it will be optimal for a worker from unemployment to accept both good and matches.

The second type of searchers we consider are those who search on the job. Absent

other considerations, the only reason for an employed worker to search is to find a job with improved match quality.¹⁶ In our setting, the only workers who can improve match quality are those currently in bad matches. We allow such workers to search with variable intensity ς_{bt} . As has been noted in the literature, however, not all job transitions involve positive wage changes (see Tjaden and Wellschmied, 2014). Accordingly, we suppose that workers in good matches may occasionally leave for idiosyncratic reasons, e.g. locational constraints.¹⁷ We assume that these workers search with fixed intensity ς_n and only accept other good matches.

Finally, we assume that the fraction $1 - \nu$ of workers separated during period t search with fixed intensity ς_u . Such workers are either hired by another firm to work in the subsequent period or remain unemployed. As is the case with workers searching from unemployment, workers separated within the period will find it optimal to both accept good or bad matches. We include such flows to be consistent with the observation that workers observed making job-to-job transitions sometimes are observed to take pay cuts; and, as pointed out by Christiano, Eichenbaum, and Trabandt (2016), flows in the data that appear to be job-to-job transitions may in fact be separations immediately followed by successful job search.

We derive the total efficiency units of search effort \bar{s}_t as a weighted sum of search intensity across the three types:

$$\bar{s}_t = \bar{u}_t + \nu(\varsigma_{bt}\bar{b}_t + \varsigma_n\bar{n}_t) + (1 - \nu)\varsigma_u(\bar{n}_t + \bar{b}_t) \quad (7)$$

The first term reflects search intensity of the unemployed; the second term, the search intensity of the employed; the third, the search intensity of workers separated within the period. As we will show, the search intensity of bad matches on the job will be procyclical. Furthermore, the cyclical sensitivity of the efforts of workers in bad matches to find better jobs will ultimately be the source of procyclical movements in match quality and new hire wages.

The aggregate number of matches \bar{m}_t is a function of the efficiency weighted number of

¹⁶Strictly speaking, with staggered wage contracting, workers in good matches may want to search if their wages are (i) sufficiently below the norm and are (ii) not likely to be renegotiated for some time. However, because the fraction of workers likely to be in this situation in our model is of trivial quantitative importance, due to the transitory nature on average of wage differentials due to staggered contracting, we abstract from this consideration.

¹⁷For similar reasons, structural econometric models formulated to assess the contribution of on-the-job search to wage dispersion in a stationary setting often include a channel for exogenous, non-economic job-to-job transitions with wage drops. Examples include Jolivet, Postel-Vinay, and Robin (2006) and Lentz and Mortensen (2012).

searchers \bar{s}_t and the number of vacancies \bar{v}_t , as follows:

$$\bar{m}_t = \sigma_m \bar{s}_t^\sigma \bar{v}_t^{1-\sigma}, \quad (8)$$

where σ is the elasticity of matches to units of search effort and σ_m reflects the efficiency of the matching process.

The probability p_t a unit of search activity leads to a match is:

$$p_t = \frac{\bar{m}_t}{\bar{s}_t} \quad (9)$$

The probability the match is good p_t^n and the probability it is bad p_t^b are given by:

$$p_t^n = \xi p_t \quad (10)$$

$$p_t^b = (1 - \xi) p_t \quad (11)$$

The probability for a firm that posting a vacancy leads to a match q_t^m is given by

$$q_t^m = \frac{\bar{m}_t}{\bar{v}_t} \quad (12)$$

Not all matches lead to hires, however, and hires vary by quality. The probability q_t^n a vacancy leads to a good quality hire and the probability q_t^b it leads to a bad quality one are given by

$$q_t^n = \xi q_t^m \quad (13)$$

$$q_t^b = (1 - \xi) \left(1 - \frac{\nu(\varsigma_{bt} \bar{b}_t + \varsigma_n \bar{n}_t)}{\bar{s}_t} \right) q_t^m \quad (14)$$

Since all workers accept good matches, q_t^n is simply the product of the probability of a match being good conditional on a match, ξ , and the probability of a match, q_t^m . By contrast, since on the job searchers do not accept bad matches, to compute q_t^b we must net out the fraction of searchers who are doing so on the job, $\nu(\varsigma_{bt} \bar{b}_t + \varsigma_n \bar{n}_t)/\bar{s}_t$.

Finally, we can express the expected number of workers in efficiency units of labor that a firm can expected to hire from posting a vacancy, q_t , as

$$q_t = q_t^n + \phi q_t^b \quad (15)$$

It follows that the total number of new hires in efficiency units is simply $q_t \nu_t$.

3.2 Firms

Firms produce output y_t using capital and labor according to a Cobb-Douglas production technology:

$$y_t = z_t k_t^\alpha l_t^{1-\alpha}, \quad (16)$$

where k_t is capital and l_t labor in efficiency units. Capital is perfectly mobile. Firms rent capital on a period by period basis. They add labor through a search and matching process that we describe shortly. The current value of l_t is a predetermined state.

Labor in efficiency units is the quality adjusted sum of good and bad matches in the firm (see equation (5)). It is convenient to define $\gamma_t \equiv b_t/n_t$ as the ratio of bad to good matches in the firm. We can then express l_t as the follow multiple of n_t :

$$l_t = n_t + \phi b_t = (1 + \phi\gamma_t)n_t, \quad (17)$$

where as before, $\phi \in (0, 1)$ is the productivity of a bad match relative to a good one. The labor quality mix γ_t is also a predetermined state for the firm.

The evolution of l_t depends on the dynamics of both n_t and b_t . Let ρ_t^i be the probability of retaining a worker in a match of type $i = n, b$. Letting q_t^i denote the probability of filling a vacancy with a worker leading to a match of type i , we can express the evolution of n_t and b_t as follows:

$$n_{t+1} = \rho_t^n n_t + q_t^n v_t \quad (18)$$

$$b_{t+1} = \rho_t^b b_t + q_t^b v_t \quad (19)$$

where $q_t^i v_t$ is the quantity of type i matches and where equations (13) and (14) define q_t^n and q_t^b . The probability of retaining a worker is the product of the job survival probability ν and the probability the worker does not leave for a job elsewhere ($1 - \varsigma_{it} p_t^n$):

$$\rho_t^i = \nu(1 - \varsigma_{it} p_t^n), \quad i = n, b, \quad (20)$$

It follows from equations (17) and (20) that we can express the survival probability of a unit of labor in efficiency units, ρ_t , as the following convex combination of ρ_t^n and ρ_t^b :

$$\rho_t = \frac{\rho_t^n + \phi\gamma_t \rho_t^b}{1 + \phi\gamma_t} \quad (21)$$

The hiring rate in efficiency units of labor, x_t , is ratio of new hires in efficiency units $q_t v_t$ to the existing stock, l_t

$$x_t = \frac{q_t v_t}{l_t} \quad (22)$$

where the expected number of efficiency weighted new hires per vacancy q_t is given by equation (15). The evolution of l_t is then given by:

$$l_{t+1} = (\rho_t + x_t) l_t \quad (23)$$

It is useful to define $\bar{\gamma}_t^m \equiv (q_t^b v_t) / (q_t^n v_t) = q_t^b / q_t^n$ as the ratio of newly-formed bad to good matches. Then, making use of equations (15), (17), (18), (19) and (22) to characterize how the quality mix of workers $\gamma_t = b_t / n_t$ evolves over time, we obtain:

$$\gamma_{t+1} = \frac{\rho_t^b \gamma_t + q_t^b v_t / n_t}{\rho_t^n + q_t^n v_t / n_t} = \frac{\frac{\gamma_t}{1+\phi\gamma_t} \rho_t^b + \frac{\bar{\gamma}_t^m}{1+\phi\bar{\gamma}_t^m} x_t}{\frac{1}{1+\phi\gamma_t} \rho_t^n + \frac{1}{1+\phi\bar{\gamma}_t^m} x_t} \quad (24)$$

where $1/(1 + \phi\gamma_t)$ is the share of good matches among incumbent workers and $1/(1 + \phi\bar{\gamma}_t^m)$ is the share of good matches among new hires and where $\gamma_t/(1 + \phi\gamma_t)$ and $\bar{\gamma}_t^m/(1 + \phi\bar{\gamma}_t^m)$ are the complementary shares of bad matches.

We now turn to the firm's decision problem. Assume that labor recruiting costs are quadratic in the hiring rate for labor in efficiency units, x_t , and homogeneous in the existing stock l_t .¹⁸ Then let $\Lambda_{t,t+1}$ be the firm's stochastic discount factor (i.e., the household's intertemporal marginal rate of substitution), r_t be the rental rate of capital, and w_t be the wage per efficiency unit of labor. Then the firm's decision problem is to choose capital k_t and the hiring rate x_t to maximize the discounted stream of profits net recruiting costs, subject to the equations that govern the laws of motion for labor in efficiency units l_t and the quality mix of labor γ_t , and given the expected paths of rents and wages. We express the value of each firm $F_t(l_t, \gamma_t, w_t) \equiv F_t$ as

$$F_t = \max_{k_t, x_t} \left\{ z_t k_t^\alpha l_t^{1-\alpha} - \frac{\kappa}{2} x_t^2 l_t - w_t l_t - r_t k_t + \mathbb{E}_t \{ \Lambda_{t,t+1} F_{t+1} \} \right\}$$

subject to equations (23) and (24), and given the values of the firm level states (l_t, γ_t, w_t) and the aggregate state vector. The firm's decision problem is formulated according to the following intra-period timing protocol: (i) realization of aggregate and firm-level shocks, (ii) wage bargaining and production, (iii) realization of match-level separation shocks, and (iv) search and matching. For the time being, we take the firm's expected wage path as given. In Section 3.4 we describe how wages are determined for both good and bad workers.

Given constant returns and perfectly mobile capital, the firm's value F_t is homogeneous in l_t . The net effect is that each firm's choice of the capital/labor ratio and the hiring rate is independent of its size. Let J_t be firm value per efficiency unit of labor and let $\check{k}_t \equiv k_t / l_t$

¹⁸We assume quadratic recruiting costs because we have temporary wage dispersion due to staggered contracts and perfectly mobile capital. With proportional costs, all capital would flow to the low wage firms.

be its capital labor ratio. Then

$$F_t = J_t \cdot l_t \quad (25)$$

with $J_t \equiv J_t(\gamma_t, w_t)$ given by

$$J_t = \max_{\check{k}_t, x_t} \left\{ z_t \check{k}_t^\alpha - \frac{\kappa}{2} x_t^2 - w_t - r_t \check{k}_t + (\rho_t + x_t) \mathbb{E}_t \{ \Lambda_{t,t+1} J_{t+1} \} \right\}. \quad (26)$$

subject to (23) and (24).

The first order condition for capital rental is

$$r_t = \alpha z_t \check{k}_t^{\alpha-1}. \quad (27)$$

Given Cobb-Douglas production technology and perfect mobility of capital, \check{k}_t does not vary across firms.

The first order condition for hiring is

$$\kappa x_t = \mathbb{E}_t \left\{ \Lambda_{t,t+1} \left[J_{t+1} + (\rho_t + x_t) \left[\frac{\partial J_{t+1}}{\partial \gamma_{t+1}} + \frac{\partial J_{t+1}}{\partial w_{t+1}} \frac{\partial w_{t+1}}{\partial \gamma_{t+1}} \right] \frac{\partial \gamma_{t+1}}{\partial x_t} \right] \right\} \quad (28)$$

The expression on the left is the marginal cost of adding worker, and the expression on the right is the discounted marginal benefit. The first term on the right-hand side of (28) is standard: it reflects the marginal benefit of adding a unit of efficiency labor. The second term reflects a “composition effect” of hiring. While the firm pays the same recruitment costs for bad and good workers (in quality adjusted units), bad workers have separate survival rates within the firm due to their particular incentive to search on-the-job. The composition term reflects the effect of hiring on period-ahead composition, and the implied effect on the value of a unit of labor quality to the firm.¹⁹

3.3 Workers

We next construct value functions for unemployed workers, workers in bad matches, and workers in good matches. These value functions will be relevant for wage determination, as we discuss in the next section. Importantly, they will also be relevant for the choice of search intensity by workers in bad matches who are looking to upgrade.

We begin with an unemployed worker: Let U_t be the value of unemployment, V_t^n the value of a good match, V_t^b the value of a bad match, and u_B the flow benefit of unemployment. Then, the value of a worker in unemployment satisfies

$$U_t = u_B + \mathbb{E}_t \left\{ \Lambda_{t,t+1} \left[p_t^n \bar{V}_{t+1}^n + p_t^b \bar{V}_{t+1}^b + (1 - p_t) U_{t+1} \right] \right\}. \quad (29)$$

¹⁹Under our calibration, the effect will be zero, up to a first order. See appendix for details.

where $p_t^n = \xi p_t$, $p_t^b = (1 - \xi)p_t$, p_t is given by (9), and where \bar{V}_{t+1}^n and \bar{V}_{t+1}^b are the average values of good and bad matches at time $t + 1$.²⁰

For workers that begin the period employed, we suppose that the cost of searching as a function of search intensity is given by

$$c(\varsigma_{it}) = \frac{\varsigma_0}{1 + \eta_\varsigma} \varsigma_{it}^{1 + \eta_\varsigma}$$

where $i = b, n, u$. Let w_{it} be the wage of a type i worker, $i = b, n$. The value of a worker in a bad match $V_t^b(\gamma_t, w_t) \equiv V_t^b$ is given by

$$\begin{aligned} V_t^b = & \max_{\varsigma_{bt}} \{ w_{bt} + \tau_t - [\nu c(\varsigma_{bt}) + (1 - \nu)c(\varsigma_u)] \\ & + \mathbb{E}_t \{ \Lambda_{t,t+1} \{ \nu [(1 - \varsigma_{bt} p_t^n) V_{t+1}^b + \varsigma_{bt} p_t^n \bar{V}_{t+1}^n] \\ & + (1 - \nu) [\varsigma_u p_t^n \bar{V}_{t+1}^n + \varsigma_u p_t^b \bar{V}_{t+1}^b + (1 - \varsigma_u p_t) U_{t+1}] \} \} \end{aligned} \quad (30)$$

The flow value is the wage w_{bt} net the expected costs of search plus a term τ_t we describe below. If the worker “survives” within the firm, which occurs with probability ν , he searches with variable intensity ς_{bt} . If he is separated, which occurs with probability $1 - \nu$, he searches with fixed intensity ς_u . The first term in the continuation value is the value of continuing in the match, which occurs with probability $\nu(1 - \varsigma_{bt} p_t^n)$. The second term reflects the value of switching to a good match, which occurs with probability $\nu \varsigma_{bt} p_t^n$. The third term and fourth term reflect the value of being separated but immediately finding a good or bad job. The final term reflects the value of being separated into unemployment.

A worker in the bad match chooses the optimal search intensity ς_{bt} according to (30), satisfying

$$\varsigma_0 \varsigma_{bt}^{\eta_\varsigma} = \mathbb{E}_t \left\{ \Lambda_{t,t+1} p_t^n \left(\bar{V}_{t+1}^n - V_{t+1}^b \right) \right\} \quad (31)$$

Search intensity varies positively with the product of the likelihood of finding a good match, p_t^n , and the net gain of doing so, i.e. the difference between the value of good and bad matches. One can see from equation (31) how the model can generate procyclical search intensity by workers in bad matches. The probability of finding a good match will be highly procyclical and the net gain roughly acyclical. Thus, the expected marginal gain from search will be highly procyclical, leading to procyclical search intensity.

²⁰Technically, the average value of employment in the continuation value of U_t should be that of a new hire rather than the unconditional one. However, Gertler and Trigari (2009) show that the two are identical up to a first order. Hence, we use the simpler formulation for clarity. In particular, the unconditional average value for a type i match is $\bar{V}_{t+1}^i = \int V_{t+1}^i dG_{t+1}$, where G denotes the joint distribution of wages and composition, while the average value conditional on being a new hire is given by $\bar{V}_{x,t+1}^i = \int V_{t+1}^i(x_t/\bar{x}_t) dG_t$, where $\bar{x}_t = \int x_t dG_t$. Since w , γ and x in the steady state are identical across firms, $\bar{V}_{x,t+1}^i = \bar{V}_{t+1}^i$ up to a first order.

If there is dispersion of wages among bad matches due staggered contracting, then search intensities can differ across these workers. To simplify matters, we assume that the family provides an insurance scheme that smooths out search intensities across its family members, much in the same way it offers consumption insurance. In particular, we assume that there is a transfer scheme that insures that the sum of the wage and the transfer equals the average wage across matches, \bar{w}_{bt} . In particular, $\tau_t = (\bar{w}_{bt} - w_{bt})$, which implies $w_{bt} + \tau_t = \bar{w}_{bt}$. With the transfer, the discounted marginal benefit to search (the right side of equation (31)) does not depend on worker-specific characteristics, so that $V_t^b = \bar{V}_t^b$. Search intensity is thus the same across all workers in bad matches.

The value of a worker in a good match $V_t^n(\gamma_t, w_t) \equiv V_t^n$ is analogous to the value function for a bad match.

$$\begin{aligned}
V_t^n &= w_{nt} - [\nu c(\varsigma_n) + (1 - \nu)c(\varsigma_u)] \\
&\quad + \mathbb{E}_t\{\Lambda_{t,t+1}\{\nu[(1 - \varsigma_n p_t^n)V_{t+1}^n + \varsigma_n p_t^n \bar{V}_{t+1}^n] \\
&\quad + (1 - \nu)[\varsigma_u p_t^n \bar{V}_{t+1}^n + \varsigma_u p_t^b \bar{V}_{t+1}^b + (1 - \varsigma_u p_{t+1})U_{t+1}]\}\} \tag{32}
\end{aligned}$$

One key difference is that on-the-job search intensity is fixed for good matches. Note that up to a first order, however, there are zero expected gains from search given that workers in good matches only move to other good matches. Hence, we rule out variable search by workers in good matches without loss of generality.

We assume that the the search intensities of good and bad matches are identical in steady state (i.e., $\varsigma_{bt} = \varsigma_n$ in steady state). As we show in Appendix B, this greatly simplifies the analysis of the firm's problem, as it implies that the average retention rates of workers in good and bad matches are the same. In the absence of direct evidence of the broader relation of job quality and match retention, we interpret this as a neutral assumption.²¹

3.4 Nash Wage

As in GT, workers and firms divide the joint match surplus via staggered Nash bargaining. For simplicity, we assume that the firm bargains with good workers for a wage. Bad workers then receive the fraction ϕ of the wage for good workers, corresponding to their relative productivity. Thus if w_t is the wage for a good match within the firm, then ϕw_t is the wage for a bad match. It follows that w_t corresponds to the wage per unit of labor quality. We note that this simple rule for determining wages for workers in bad matches approximates

²¹One study of job tenure and match quality over the business cycle is Bowlus (1995), who uses job tenure as a proxy for match quality. When she allows controls for starting wages, she finds no systematic relation of match quality and tenure to aggregate conditions. Her conclusions are consistent with a central implication of our model: "Workers take these mismatched jobs during recessions and then move on when times are better" (p. 346).

the optimum that would come from direct bargaining. It differs slightly due mainly to differences in duration of good and bad matches with firms. The gain from imposing this simple rule is that we need only characterize the evolution of a single type of wage. Importantly, in bargaining with good workers, firms also take account of the implied costs of hiring bad workers.

Our assumptions are equivalent to having the good workers and firms bargain over the wage per unit of labor quality w_t . For the firm, the relevant surplus per worker is J_t , derived in section 3.2 (equation (26)). For good workers, the relevant surplus is the difference between the value of a good match and unemployment:

$$H_t = V_t^n - U_t \tag{33}$$

As in GT, the expected duration of a wage contract is set exogenously. At each period, a firm faces a fixed probability $1 - \lambda$ of renegotiating the wage. With complementary probability, the wage from the previous period is retained. The expected duration of a wage contract is then $1/(1 - \lambda)$.²² Workers hired in between contracting periods receive the prevailing firm wage per unit of labor quality w_t . Thus in the model there is no new hire effect: Adjusting for relative productivity the wages of new hires are the same as for existing workers.

Let w_t^* denote the wage per unit of labor quality of a firm renegotiating its wage contract in the current period.²³ The wage w_t^* is chosen to maximize the Nash product of a unit of labor quality to a firm and a worker in a good match, given by

$$H_t^\eta J_t^{1-\eta} \tag{34}$$

subject to

$$w_{t+1} = \begin{cases} w_t & \text{with probability } \lambda \\ w_{t+1}^* & \text{with probability } 1 - \lambda \end{cases} \tag{35}$$

where w_{t+1}^* is the wage chosen in the next period if the parties are able to re-bargain and where η is the households relative bargaining power.

Let $H_t^* \equiv H_t(\gamma_t, w_t^*)$ and $J_t^* \equiv J_t(\gamma_t, w_t^*)$ (where $H_t \equiv H_t(\gamma_t, w_t)$ and $J_t \equiv J_t(\gamma_t, w_t)$).

²²We use the Calvo formulation of staggered contracting for convenience, since it does not require keeping track of the distribution of remaining time on the contracts. We expect very similar results from using Taylor contracting, where contracts are of a fixed duration. An advantage with Taylor contracting is that wages are less likely to fall out of the bargaining set, since with Calvo a small fraction of firms may not adjust wages for a long time. Nonetheless, given that the broad insights from Calvo and Taylor contracting are very similar, we stick with the simpler Calvo formulation.

²³We suppress the dependence of w^* and similar objects on the firm's composition in the notation.

Then the first order condition for w_t^* is given by²⁴

$$\eta \frac{\partial H_t^*}{\partial w_t^*} J_t^* = (1 - \eta) \left(-\frac{\partial J_t^*}{\partial w_t^*} \right) H_t^* \quad (36)$$

where

$$\frac{\partial H_t^*}{\partial w_t^*} = 1 + \nu (1 - \varsigma_n p_t^n) \lambda \mathbb{E}_t \left\{ \Lambda_{t,t+1} \frac{\partial H_{t+1}}{\partial w_t^*} \right\} \quad (37)$$

and

$$\frac{\partial J_t^*}{\partial w_t^*} = -1 + (\rho_t + x_t) \lambda \mathbb{E}_t \left\{ \Lambda_{t,t+1} \frac{\partial J_{t+1}}{\partial w_t^*} \right\}. \quad (38)$$

Under multi-period bargaining, the outcome depends on how the new wage settlement affects the relative surpluses in subsequent periods where the contract is expected to remain in effect. The net effect, as shown in GT, is that up a first order approximation the contract wage will be an expected distributed lead of the target wages that would arise under period-by-period Nash bargaining, where the weights on the target for period $t + i$ depend on the likelihood the contract remains operative, λ^i .

In general, the new contract wage will be a function of the firm level state γ_t (the ratio of bad to good matches), as well as the aggregate state vector. However, given our assumptions that steady state search intensities are the same for good and bad matches and that wages are proportional to productivity, w_t^* is independent of γ_t in the first order approximation. Accordingly, to a first order, we can express the evolution of average wages \bar{w}_t as

$$\bar{w}_t = (1 - \lambda) w_t^* + \lambda \bar{w}_{t-1}$$

where $1 - \lambda$ is the fraction of firms that are renegotiating and λ is the fraction that are not and where the average wage per unit of labor quality is defined by

$$\bar{w}_t = \int_{w,\gamma} w dG_t(\gamma, w)$$

with $G_t(\gamma, w)$ denoting the time t fraction of units of labor quality employed at firms with wage less than or equal to w and composition less than or equal to γ . (See the appendix for details.)

3.5 Households: Consumption and Saving

We adopt the representative family construct, following Merz and Andolfatto, allowing for perfect consumption insurance. There is a measure of families on the unit interval, each with

²⁴For simplicity, we omit additional terms in the expression for $\partial H^*/\partial w^*$ that will be zero up to a first order. See the appendix for details.

a measure one of workers. Before making allocating resources to per-capita consumption and savings, the family pools all wage and unemployment income. Additionally, the family owns diversified stakes in firms that pay out profits. The household can then assign consumption \bar{c}_t to members and save in the form of capital \bar{k}_t , which is rented to firms at rate r_t and depreciates at the rate δ .

Let Ω_t be the value of the representative household. Then,

$$\Omega_t = \max_{\bar{c}_t, \bar{k}_{t+1}} \{ \log(\bar{c}_t) + \beta \mathbb{E}_t \Omega_{t+1} \} \quad (39)$$

subject to

$$\begin{aligned} \bar{c}_t + \bar{k}_{t+1} + \frac{s_0}{1 + \eta_\varsigma} \left\{ \left[\nu \varsigma_n^{1+\eta_\varsigma} + (1 - \nu) \varsigma_u^{1+\eta_\varsigma} \right] \bar{n}_t + \left[\nu \bar{\varsigma}_{bt}^{1+\eta_\varsigma} + (1 - \nu) \varsigma_u^{1+\eta_\varsigma} \right] \bar{b}_t \right\} \\ = \bar{w}_t \bar{n}_t + \phi \bar{w}_t \bar{b}_t + (1 - \bar{n}_t - \bar{b}_t) u_B + (1 - \delta + r_t) \bar{k}_t + T_t + \Pi_t, \end{aligned} \quad (40)$$

and

$$\bar{n}_{t+1} = \bar{\rho}_t^n \bar{n}_t + \xi p_t \bar{s}_t \quad (41)$$

$$\bar{b}_{t+1} = \bar{\rho}_t^b \bar{b}_t + \xi \tilde{\gamma}_t^m p_t \bar{s}_t \quad (42)$$

where Π_t are the profits from the household's ownership holdings in firms and T_t are lump sum transfers from the government.²⁵

The first-order condition from the household's savings problem gives

$$1 = (1 - \delta + r) \mathbb{E}_t \{ \Lambda_{t,t+1} \} \quad (43)$$

where $\Lambda_{t,t+1} \equiv \beta \bar{c}_t / \bar{c}_{t+1}$.

3.6 Resource Constraint, Government Policy, and Equilibrium

The resource constraint states that the total resource allocation towards consumption, investment, vacancy posting costs, and search costs is equal to aggregate output:

$$\begin{aligned} \bar{y}_t = \bar{c}_t + \bar{k}_{t+1} - (1 - \delta) \bar{k}_t \\ + \frac{\kappa}{2} \int_i x_t^2 l_t di + \frac{s_0}{1 + \eta_\varsigma} \left(\left[\nu \varsigma_n^{1+\eta_\varsigma} + (1 - \nu) \varsigma_u^{1+\eta_\varsigma} \right] \bar{n}_t + \left[\nu \bar{\varsigma}_{bt}^{1+\eta_\varsigma} + (1 - \nu) \varsigma_u^{1+\eta_\varsigma} \right] \bar{b}_t \right). \end{aligned} \quad (44)$$

²⁵Chodorow-Reich and Karabarbounis (2016) show the addition of non-separable utility from leisure can greatly increase the difficulty of generating sufficient unemployment volatility when the model is calibrated to match the estimated cyclical volatility of the opportunity cost of employment. For simplicity we do not include non-separable utility from leisure, but in ongoing work we show that our model with staggered wage contracting is robust to this critique.

The government funds unemployment benefits through lump-sum transfers:

$$T_t + (1 - \bar{n}_t - \bar{b}_t) u_B = 0. \quad (45)$$

A recursive equilibrium is a solution for (i) a set of functions $\{J_t, V_t^n, V_t^b, U_t\}$; (ii) the contract wage w_t^* ; (iii) the hiring rate x_t ; (iv) the subsequent period's wage rate w_{t+1} ; (v) the search intensity of a worker in a bad match ς_{bt} ; (vi) the rental rate on capital r_t ; (vii) the average wage and hiring rates, \bar{w}_t and \bar{x}_t ; (viii) the capital labor ratio \check{k}_t ; (ix) the average consumption and capital, \bar{c}_t and \bar{k}_{t+1} ; (x) the average employment in good and bad matches, \bar{n}_t and \bar{b}_t ; (xi) the density function of composition and wages across workers $dG_t(\gamma, w)$; and (xii) a transition function $Q_{t,t+1}$. The solution is such that (i) w_t^* satisfies the Nash bargaining condition (36); (ii) x_t satisfies the hiring condition (28); (iii) w_{t+1} is given by the Calvo process for wages (35); (iv) ς_{bt} satisfies the first-order condition for search intensity of workers in bad matches (31); (v) r_t satisfies (27); (vi) $\bar{w}_t = \int_{w,\gamma} w dG_t(\gamma, w)$ and $\bar{x}_t = \int_{w,\gamma} x dG_t(\gamma, w)$; (vii) the rental market for capital clears, $\check{k}_t = \bar{k}_t / (\bar{n}_t + \phi \bar{b}_t)$; (viii) \bar{c}_t and \bar{k}_{t+1} solve the household problem (39); (ix) \bar{n}_t and \bar{b}_t evolve according to (41) and (42); (x) the evolution of G_t is consistent with $Q_{t,t+1}$; (xi) $Q_{t,t+1}$ is defined in the appendix.

3.7 New Hire Wages and Job-to-Job Flows

Here we describe how our model is able to capture the panel data evidence on new hire wage cyclicality, despite new hires' wages being every bit as sticky as those for existing workers (conditional on match quality). To do that, we derive an expression for the average wage growth of job changers that permits to interpret the semi-elasticity of job changers' wage to changes in unemployment that is implied by the model.

The model includes two types of job-to-job transitions: those due to on-the-job search and those due to separations followed by search and finding of a new job within the same period, with no spell of unemployment between the jobs. Since workers searching on the job only accept good matches, the first type of transitions leads to bad-to-good and good-to-good job flows. The second type of transitions instead leads to the full range of job-to-job flows. For example, the bad-to-good job flow consists of two components: the first due to on-the-job search, $\nu \bar{\varsigma}_{bt} \xi p_t \bar{b}_t$; the second from match separation and successful job-finding within the period, $(1 - \nu) \varsigma_u \xi p_t \bar{b}_t$.

Let \bar{g}_t^w denote the average wage growth of continuing workers, \bar{g}_t^{EE} the average wage growth of new hires who are job changers, and c_t^w the component of \bar{g}_t^{EE} due to compositional effects (i.e. changes in match quality across jobs). Further, let $\delta_{BG,t}$ be the share of flows moving from bad to good matches out of total job flows at time t and let $\delta_{GB,t}$ be the share moving from good to bad matches. Then to a first order (see the appendix for details) we

can express average wage growth for changers:

$$\bar{g}_t^{EE} = \tilde{g}^{EE} + (1 - \omega)\bar{g}_t^w + \omega\hat{c}_t^w \quad (46)$$

with

$$\bar{g}_t^w = \hat{w}_t - \hat{w}_{t-1} \quad (47)$$

$$\hat{c}_t^w = \pi_{BG}\hat{\delta}_{BG,t-1} - \pi_{GB}\hat{\delta}_{GB,t-1} \quad (48)$$

where \hat{z} denotes log deviations of variable z from steady state and $\omega \in [0, 1]$ is the steady state share of average job changer wage growth that is due to changes in match quality. As shown in the appendix, the parameters ω , π_{BG} , and π_{GB} are all positive and are functions of model primitives.

Equation (46) indicates that average wage growth for job changers is a convex combination of average wage growth for existing workers and a composition component. Absent the composition effect (i.e. if $\omega = 0$), average wage growth for job changers would look no different than for continuing workers. With the composition effect present, however, cyclical variation of the composition of new match quality enhances the relative volatility of job changers wages.

In particular, the cyclical composition effect \hat{c}_t^w varies positively with the share in total job flows of workers moving from bad to good matches, $\delta_{BG,t-1}$, and negatively with the share moving from good to bad, $\delta_{GB,t-1}$. As we have discussed, the search intensity by workers in bad matches, \bar{s}_{bt} , is highly procyclical, leading to $\delta_{BG,t-1}$ being procyclical and $\delta_{GB,t-1}$ countercyclical. The dynamics of the shares also depends on the average firm composition, $\bar{\gamma}_t$, determining the relative stocks of bad and good matches available to make a job-to-job transition. During expansions composition slowly improves ($\bar{\gamma}_t$ decreases) so that over time less workers in bad matches remain available to make a bad-to-good transition and more workers in good matches can make a good-to-bad transition. Specifically, after substituting the expressions for the flow shares (see the appendix for details), the compositional component can be rewritten as

$$\hat{c}_t^w = \pi_\gamma\hat{\gamma}_{t-1} + \pi_\varsigma\hat{\varsigma}_{bt-1} \quad (49)$$

where the parameters π_γ and π_ς are positive and functions of model primitives. In the next section, we show that the net effect of procyclical search intensity and countercyclical composition is that \hat{c}_t^w is procyclical, i.e. the composition effect on job changers' enhance wage growth in good times and weakens it in bad times. In this way the model can produce the kind of cyclical movements in match quality that can lead to estimates of new hire

wage cyclicality that suffer from the kind of composition bias we discussed in Section 2. We demonstrate this concretely in the next section by showing that data generated from the model will generate estimates of a new hire effect on wages for job changers, even though new hires’ wages have the exact same cyclicality as for existing workers.

4 Results

In this section we present some simulations to show how the model can capture both the aggregate evidence on unemployment fluctuations and wage rigidity and the panel data evidence on the relative cyclicality of new hires’ versus continuing workers’ wages. We first describe the calibration before turning to the results.

4.1 Calibration

We adopt a monthly calibration. There are 16 parameters in the model for which we must select values. We calibrate 10 of the parameters using external sources. Five of the externally calibrated parameters are common to the macroeconomics literature: the discount factor, β ; the capital depreciation rate, δ ; the “share” of labor in the production technology, α ; and the autoregressive parameter and standard deviation for the productivity process, ρ_z and σ_z . Our parameter choices are standard: $\beta = 0.99^{1/3}$, $\delta = 0.025/3$, $\alpha = 1/3$, $\rho_z = 0.95^{1/3}$, and $\sigma_z = 0.0075$.^{26,27}

Five more parameters are specific to the search literature. Our choice of the matching function elasticity with respect to searchers, σ , is 0.4, guided by the estimates from Blanchard and Diamond (1989).²⁸ We set the worker’s bargaining power η to 0.5, as in GT. We normalize the matching function constant, σ_m , to 1.0. We set the elasticity of search costs, η_σ , to 0.5. This is in the range of values estimated by Lise (2013), who obtains separate parameter estimates of 0.249 and 0.168, and Christensen et al. (2005), who estimate an elasticity of 1.19. We choose λ to target the average frequency of wage changes. Taylor (1999) argues that medium to large-size firms adjust wages roughly once every year; this is validated by findings from microdata by Gottschalk (2005), who concludes that wages are adjusted roughly every year. We consider two values of this parameter: a conservative

²⁶Note that, in contrast to the frictionless labor market model, the term α does not necessarily correspond to the labor share, since the labor share will in general depend on the outcome of the bargaining process. However, because a wide range of values of the bargaining power imply a labor share just below α , here we simply follow convention by setting $\alpha = 2/3$.

²⁷The parameter σ_z is chosen to target the standard deviation of output,

²⁸This value lays slightly outside the range of values identified by Pissarides and Petrongolo (2001) and well below the value estimated by Shimer (2005). Note that in these papers, only the unemployed search and enter the matching function, while searchers in our model comprise both unemployed and employed workers. When we simulate data from our model and estimate the matching function elasticity under the assumption that only the unemployed search, we recover an elasticity in excess of 0.6.

value of $\lambda = 8/9$, implying that wages are renegotiated on average every 3 quarters, and a second value of $\lambda = 11/12$, implying an average duration between negotiations of one year. The parameter values are given in Table 4.

The remaining six parameters are jointly calibrated to match model-relevant moments measuring aggregate labor flows, individual-level wage dynamics, and the value of leisure.²⁹ We calibrate the inverse productivity premium, ϕ ; the probability that a new match is good, ξ ; the hiring cost parameter, κ ; the scale parameter of the search cost, ς_0 ; the separation probability, $(1 - \nu)$; and the flow value of unemployment, u_B , to match six moments: the average wage change of workers making E-E transitions in our data; average wage loss of workers making an E-N-E transition in our data; the U-E probability; the E-E probability; the E-U probability; and the relative value of non-work. Although there is not a one-to-one mapping of parameters to moments, there is a sense in which the identification of particular parameters are more informed by certain moments than others. We use this informal mapping to provide a heuristic argument of how the various parameters are identified.

We calibrate ϕ to target the average wage change of workers making direct job-to-job transitions in our data, 4.8% (see Table 3, column 1); holding everything constant, a higher ϕ implies a smaller (positive) average percentage wage increase for job changers. We recover $\phi = 0.65$. We calibrate ξ to match the average wage loss of workers making an E-N-E transition, 5.1% (see Table 3, column 1). Holding fixed the inverse productivity premium ϕ and the steady state value of γ , a lower ξ corresponds to a lower probability of finding a good match from unemployment; and hence, a lower ξ generates a larger wage loss for workers making E-N-E transitions. We recover $\xi = 0.02$.

We calibrate the separation probability $(1 - \nu)$ to match the empirical E-U probability of 0.034. Note that separated workers have the opportunity to find a new job and avoid unemployment. Hence, the E-U in the model equals $(1 - \nu)(1 - \varsigma_u \tilde{p})$, implying $(1 - \nu) = 0.06$ (where \tilde{z} denotes steady state of a variable z). The hiring cost parameter κ determines the resources that firms place into recruiting, and hence, influences the probability that a worker finds a job. We set the steady state job finding probability \tilde{p} to match the monthly U-E transition probability, 0.44; and then calibrate κ to be consistent with \tilde{p} . We restrict $\varsigma_u = \varsigma_n = \tilde{\varsigma}_b$ and note that a higher search cost implies a lower E-E probability. We calibrate ς_i to match an E-E probability of 0.029; we obtain $\varsigma_0 = 0.03$.³⁰

We interpret the flow value of unemployment u_B as capturing both unemployment insurance and utility of leisure. We calibrate u_B to target a relative value of nonwork to work activity \bar{u}_T equal to 0.71 as in Hall and Milgrom (2008). In our setting, the relative

²⁹The joint calibration can be interpreted as a method of moments estimation with a diagonal weighting matrix of ones.

³⁰The values for the E-U and U-E probabilities are from Shimer (2012). The value for the EE probability is from Menzio and Shi (2011).

value of nonwork activities satisfies

$$\bar{u}_T = \frac{u_B + \frac{s_0}{1+\eta_\kappa} \left[\nu \varsigma_i^{1+\eta_\kappa} + (1-\nu) \varsigma_u^{1+\eta_\kappa} \right]}{\tilde{a} + (\kappa/2) \tilde{x}^2},$$

where $\tilde{a} = (1-\alpha) \tilde{y}/\tilde{l}$. Note that the value of nonwork includes saved search costs from on-the-job search and the value of work includes saved vacancy posting costs. Finally, when taking the model to the data, we assume that workers employed in bad matches suffer a lower disutility from labor. We capture this feature by adding to their surplus a term proportional to u_B , with scaling factor $(1-\phi)$. This makes the period surplus from unemployment in bad match proportional to the period surplus in a good match: $\phi w + (1-\phi)u_B - u_B = \phi(w - u_B)$.

The full list of parameter values and targeted moments are given in 5. Having fully calibrated the model, we now evaluate whether it provides an accurate description of aggregate and individual-level dynamics. We first test the ability of the model to match the cyclical properties of aggregate unemployment and wages. Second, we assess the ability of the model to generate the correct relative cyclicalities in wage growth for job changers versus continuing workers.

4.2 Model Simulations of Aggregate and Panel Data Evidence

We first explore whether the model provides a reasonable description of labor market volatility. In particular, we compare the model implications to quarterly U.S. data from 1964:1 to 2013:2. We take quarterly averages for monthly series in the data. Given that the model is calibrated to a monthly frequency, we take quarterly averages of the model simulated data series.

We measure output y as real output in the nonfarm business sector. The wage w is average per worker earnings of production and non-supervisory employees in the private sector, deflated with the PCE. Total employment $n + b$ is measured as all employees in the nonfarm business sector. Unemployment u is civilian unemployment 16 years and older. Vacancies v are a composite help-wanted index computed by Barnichon (2010) combining print and online help-wanted advertising. The data and model output are detrended with an HP filter with the conventional smoothing parameter.

To explore the how the model works to capture the aggregate data, we first compute impulse responses to a one percent shock to productivity. The solid line is the response of the baseline model with staggered wage contracting and the dashed line is the model with period-by-period Nash bargaining. The model with wage rigidity produced an enhanced response of output and the various labor market variables, relative to the flexible wage

case. This result is standard in the literature dating back to Shimer (2005) and Hall (2005) and in close keeping with Gertler and Trigari (2009), who use a similar model of staggered wage contracting, but without job-to-job transitions. We see that the addition of job-to-job transitions does not alter the main implications of wage rigidity for aggregate dynamics.

We then compute a variety of business cycle moments obtained from stochastic simulation obtained from feeding in a random sequence of productivity shocks. We do not mean to suggest that productivity shocks are the main business cycle driving forces. Rather, the simple real business cycle model offers a convenient way of studying the model implications for unemployment and wage dynamics.

We first consider the model implications of an impulse response to a one percent increase in productivity. The plots are given in Figure 2. To highlight the role of staggered wage contracting, we plot the model generated output for the staggered case ($\lambda = 8/9$) and the flexible wage case ($\lambda = 0$). Under period-by-period contracting, the model implications are reminiscent of those of the standard Nash bargaining model discussed by Hall (2005) and Shimer (2005). Wages immediately increase following a technology shock, whereas employment, unemployment, and vacancy posting respond only gradually and moderately. In the case with staggered contracting, the pattern is reversed: wages adjust gradually and only modestly, whereas there are larger changes in employment, unemployment, and vacancies. We also find a greater increase in the job-finding probability under staggered bargaining. Additionally, we see that for both period-by-period and staggered bargaining, the stock of workers in good matches increases while the stock of workers in bad matches decreases; however, the quantitative magnitude of the change is greater for the economy with staggered bargaining.

Table 6 compares the various business cycle statistics and measures of labor market volatility generated by the model with the data. The top panel gives the empirical standard deviations, autocorrelations, and correlations with output of output, wages, employment, unemployment, and vacancies. All standard deviations are normalized relative to output. The bottom panels compute the same statistics using the model. We simulate the model for recontracting on average every three quarters ($\lambda = 8/9$), every four quarters ($\lambda = 11/12$), and continuous recontracting ($\lambda = \infty$).

Overall, the model does a reasonable job of accounting for the relative volatility of unemployment (5.12 and 4.72 in the model with $\lambda = 11/12$ and $\lambda = 8/9$ versus 5.74 in the data) and for wages (0.43 and 0.47 versus 0.48). As is common in the literature, the model understates the volatility of employment; here, the absence of a labor force participation margin is relevant. Consistent with Shimer (2005) and Hall (2005), the wage inertia induced by staggered contracting is critical for the ability of the model to account for the volatility of unemployment. This result is robust to allowing for on-the-job search and procyclical

match quality.

We next turn to the model’s ability to account for the panel data evidence, and we simulate the model to generate time series for unemployment rates and wages of new hires and continuing workers. We use the simulated data to perform two validation checks. First, we estimate equation (1), where we estimate a single term for new hire wage cyclicality. Second, we estimate equation (2), where we allow separate terms for new hires from unemployment and non-employment. Both equations are estimated in first differences.

Results for the first exercise are given in Table 7, where we compare the results from the panel data (the first column) with those obtained from data from our model with wage contracts fixed for three quarters on average (the second column), four quarters on average (the third column), and flexible wages (the fourth column). The calibrated models with staggered contracting generate wage semi-elasticities similar to the coefficient estimates from the SIPP (aside from the slightly low wage elasticities for continuing workers in the 4Q calibration). The estimated excess wage cyclicality for new hires, however, is an artifact of cyclical composition bias, as wages for new hires in the model are no more flexible than wages of continuing workers. In the last column we explore the implications of period-by-period Nash bargaining for wage determination. Although the model generates a new hire effect, the estimated wage elasticities are too large. Thus, to account for the panel data estimates it is necessary to have not only procyclical movements in new hires’ match quality but also some degree of wage inertia as, for example, produced by staggered multi-period contracting.

Table 8 gives results for the second exercise, where we estimate separate terms for new hires from unemployment and employment. The results show that the excess wage cyclicality of new hires in the model is driven by those coming from employment. Notice that, although the average match quality of new hires from unemployment is acyclical in the model, we still recover modest excess cyclicality from the coefficient estimates. This excess sensitivity also reflects an indirect compositional effect, though one that is both different in nature and quantitatively smaller than the one underlying job changers wage cyclicality. While we relegate further discussion of this effect to Appendix B.6, we note that the *ENE* coefficient from the model simulated data falls within a one standard error confidence band of the estimates from Table 3.

Figures 3 and 4 illustrate how compositional effects influence wage dynamics. We repeat the experiment of a one percent increase in TFP. Figure 3 then reports impulse responses for labor in efficiency units, good matches, bad matches and job flows between good and bad matches. In the wake of the boom, labor quality increases. Underlying this increase is a rise in good matches and a net fall in bad matches. The rise in good matches is due in part to good matches being hired out of unemployment. But it is mostly due to an increase

in the job flow share of workers moving from bad to good matches and a decline in the reverse flow share, as the two bottom left panels indicates. This pattern in the net flows also leads to a net decline in bad matches.³¹

Figure 4 decomposes the response of new hires' wage growth into the part due to the growth of contracts wages and the part due to compositional effects, using equations (46), (47), and (48). The solid line in the top panel is total new hires' wage growth, the dashed line is the part due to composition, and the dotted line is average contract wage growth. As the figure illustrates, most of the new hires' wage response is due to compositional effects. The bottom panel then relates the compositional effect mainly to the increase in the share of job flows moving from bad to good matches.

Finally, while our motivation for introducing procyclical job reallocation is to account for the panel data evidence, we note that it also generates interesting implications for the cyclical behavior of productivity. In particular, total factor productivity in the model depends on the allocation of workers between good and bad matches. To see this, we take the production function (16) and the definition of labor quality (17) to obtain an expression for how productivity depends on the quality composition, measured by $\gamma_t = b_t/n_t$:

$$\begin{aligned} y_t &= z_t k_t^\alpha (n_t + \phi b_t)^{1-\alpha} \\ &= z_t \left(\frac{1 + \phi \gamma_t}{1 + \gamma_t} \right)^{1-\alpha} k_t^\alpha (n_t + b_t)^{1-\alpha} \end{aligned}$$

where the term $z_t \left(\frac{1 + \phi \gamma_t}{1 + \gamma_t} \right)^{1-\alpha}$ is the effective level of TFP. Loglinearizing this term yields the effect of cyclical reallocation on cyclical productivity:

$$\hat{z}_t - (1 - \alpha) \frac{1}{1 + \gamma} \frac{1 - \phi}{1 + \phi \gamma} \hat{\gamma}_t$$

Since $\hat{\gamma}_t$ is countercyclical, the effect of labor reallocation on productivity is procyclical.

In Figure 5 we report the response of the endogenous component of productivity e_t to a one percent increase in the exogenous component z_t , where \hat{e}_t can be expressed as

$$\hat{e}_t = -(1 - \alpha) \frac{1}{1 + \gamma} \frac{1 - \phi}{1 + \phi \gamma} \hat{\gamma}_t.$$

The endogenous component has a substantial and highly persistent effect on productivity, as the top panel suggests. The bottom panel shows the effect on output: the improvement in aggregate match quality due to the reallocation of labor leads to a similar increase in output. Hence, the experiment implies a sizeable impact for labor reallocation on both the

³¹In gross term there are bad matches due to workers being hired from unemployment; however, the behavior of the job-to-job flows swamps this effect.

initial increase and subsequent decay to output.

5 Concluding Remarks

We present panel data evidence suggesting that the excess cyclicality of new hires' wages relative to existing workers may be an artifact of compositional effects in the labor force that have not been sufficiently accounted for in the existing literature. We reinforce this point by developing a model of aggregate unemployment that generates quantitative implications consistent with both macro and micro data. In the model, new hires' wages are the same as continuing workers of the same match productivity; but, as we find in our estimates from panel data, new hire wages appear to be more cyclical due to the procyclicality of job quality in new matches. Our bottom line: it is reasonable for macroeconomists to continue to make use of wage rigidity to account for economic fluctuations. The focus should be on how best to model wage rigidity rather than whether it is appropriate to model at all.

Finally, our model of unemployment fluctuations with staggered wage contracting differs from much of the DSGE literature in allowing a channel for procyclical job-to-job transitions. For many purposes, it may be fine to abstract from this additional channel. However in major recessions like the recent one, a slowdown in job reallocation is potentially an important factor for explaining the overall slowdown of the recovery. A recent study by Haltiwanger, Hyatt, and McEntarfer (2015) provides evidence that the rate of job-to-job transitions has not recovered relative to the overall job-finding rate in the current recovery. Our model provides a hint about how the slowdown in job reallocation might feedback into other economic activity, by reducing overall total factor productivity. It might be interesting to explore these issues and consider other factors, such as financial market frictions, that have likely hindered the reallocation process in the recent recession.

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Table 1: “Standard regression” (e.g. Bils, 1985) and the new hire effect

	1990-2012 sample	
	(1)	(2)
Unemployment rate	-0.162*** (0.0582)	-0.448*** (0.0920)
Unemp. rate · $\mathbb{I}(\text{new})$	-1.247*** (0.2477)	-0.997** (0.4465)
$\mathbb{I}(\text{new})$	-0.010*** (0.0014)	0.011*** (0.0037)
Estimator	Fixed Effects	First Differences
No. observations	379,104	321,397

Robust standard errors in parenthesis

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 2: Job changers (EE) vs. new hires from unemployment (ENE), fixed effects

	1990-2012 sample			
	(1)	(2)	(3)	(4)
UR	-0.160*** (0.0582)	-0.160*** (0.0582)	-0.159*** (0.0582)	-0.159*** (0.0582)
UR · $\mathbb{I}(\text{new} \ \& \ EE)$	-1.921*** (0.4696)	-1.927*** (0.4403)	-1.920*** (0.4696)	-1.926*** (0.4403)
UR · $\mathbb{I}(\text{new} \ \& \ ENE)$	-0.326 (0.5086)	0.120 (0.5636)	-0.487 (0.5616)	0.005 (0.6353)
UR · $\mathbb{I}(\text{new} \ \& \ LTU)$	- -	- -	0.963 (1.1325)	0.964 (1.1325)
$\mathbb{I}(\text{new} \ \& \ EE)$	0.007*** (0.0021)	0.004* (0.0020)	0.007*** (0.0021)	0.004* (0.0020)
$\mathbb{I}(\text{new} \ \& \ ENE)$	-0.031*** (0.0027)	-0.035*** (0.0031)	-0.026*** (0.0029)	-0.029*** (0.0034)
$\mathbb{I}(\text{new} \ \& \ LTU)$	- -	- -	-0.067*** (0.0087)	-0.067*** (0.0087)
$P(\pi_{nu}^{EE} = \pi_{nu}^{ENE})$	0.019	0.004	0.046	0.011
Unemp. spell for ENE	0+	1+	(0, 9]	(1, 9]
No. observations	375,649	375,649	375,649	375,649
No. of fixed effects	56,878	56,878	56,878	56,878

Robust standard errors in parenthesis

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 3: Job changers (EE) vs. new hires from unemployment (ENE), first differences

	1990-2012 sample			
	(1)	(2)	(3)	(4)
UR	-0.415*** (0.0912)	-0.410*** (0.0912)	-0.397*** (0.0911)	-0.395*** (0.0911)
UR · $\mathbb{I}(\text{new} \ \& \ EE)$	-1.556** (0.6609)	-1.523** (0.6068)	-1.540** (0.6609)	-1.510** (0.6068)
UR · $\mathbb{I}(\text{new} \ \& \ ENE)$	-0.289 (0.6364)	-0.267 (0.6990)	-0.748 (0.7497)	-0.743 (0.8593)
UR · $\mathbb{I}(\text{new} \ \& \ LTU)$	- -	- -	-0.067 (1.1398)	-0.068 (1.1400)
$\mathbb{I}(\text{new} \ \& \ EE)$	0.048*** (0.0045)	0.041*** (0.0043)	0.048*** (0.0045)	0.040*** (0.0043)
$\mathbb{I}(\text{new} \ \& \ ENE)$	-0.051*** (0.0062)	-0.068*** (0.0072)	-0.039*** (0.0065)	-0.053*** (0.0076)
$\mathbb{I}(\text{new} \ \& \ LTU)$	- -	- -	-0.174*** (0.0197)	-0.175*** (0.0197)
$P(\pi_{nu}^{EE} = \pi_{nu}^{ENE})$	0.163	0.171	0.424	0.463
Unemp. spell for ENE	0+	1+	(0, 9]	(1, 9]
No. observations	318,771	318,771	318,771	318,771

Robust standard errors in parenthesis

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 4: Calibration

Parameter values	
Discount factor	β 0.997 = $0.99^{1/3}$
Capital depreciation rate	δ 0.008 = $0.025/3$
Production function parameter	α 0.33
Technology autoregressive parameter	ρ_z 0.983 = $0.95^{1/3}$
Technology standard deviation	σ_z 0.0075
Elasticity of matches to searchers	σ 0.4
Bargaining power parameter	η 0.5
Matching function constant	σ_m 1.0
Search cost elasticity	η_ς 0.5
Renegotiation frequency	λ 0.92 8/9 or 11/12 (3 or 4 quarters)

Table 5: Jointly calibrated parameters

Parameter	Description	Value	Target
ϕ	Inverse productivity premium	0.65	Average E-E wage increase (4.8%)
ξ	Prob. of good match	0.02	Average E-N-E wage decrease (5.1%)
κ	Hiring cost parameter	102.87	U-E probability (0.44)
ς_0	Scale parameter of search cost	0.03	E-E probability (0.029)
$1 - \nu$	Separation probability	0.06	E-U probability (0.034)
u_B	Flow value of unemployment	2.67	Relative value, non-work (0.71)

Table 6: Aggregate statistics

	y	w	$n + b$	u	v
U.S. Economy, 1964:1-2013:02					
Relative St. Dev.	1.00	0.48	0.64	5.74	6.38
Autocorrelation	0.88	0.88	0.94	0.92	0.92
Correlation with y	1.00	0.57	0.79	-0.87	0.91
Model Economy, $\lambda = 8/9$ (3 quarters)					
Relative St. Dev.	1.00	0.47	0.36	4.72	11.41
Autocorrelation	0.86	0.95	0.88	0.88	0.85
Correlation with y	1.00	0.73	0.94	-0.94	0.98
Model Economy, $\lambda = 11/12$ (4 quarters)					
Relative St. Dev.	1.00	0.43	0.40	5.12	11.86
Autocorrelation	0.87	0.96	0.89	0.89	0.86
Correlation with y	1.00	0.67	0.94	-0.94	0.98
Model Economy, $\lambda = \infty$ (Flex wages)					
Relative St. Dev.	1.00	0.64	0.26	3.40	9.37
Autocorrelation	0.85	0.81	0.91	0.91	0.87
Correlation with y	1.00	1.00	0.88	-0.88	1.00

Table 7: Wage semi-elasticities: All new hires

Semi-elasticities of wages w/r.t. unemployment				
	SIPP	Model, 3Q	Model, 4Q	Model, flex
UR	-0.45	-0.37	-0.24	-0.92
UR · I(new)	-1.00	-0.94	-0.88	-1.91

Table 8: Wage semi-elasticities: EE vs. ENE

Semi-elasticities of wages w/r.t. unemployment				
	SIPP	Model, 3Q	Model, 4Q	Model, flex
UR	-0.42	-0.37	-0.24	-0.92
UR · I(new & EE)	-1.56	-1.61	-1.47	-3.57
UR · I(new & ENE)	-0.29	-0.66	-0.65	-1.10

Figure 1: New hires from employment and cyclical composition bias

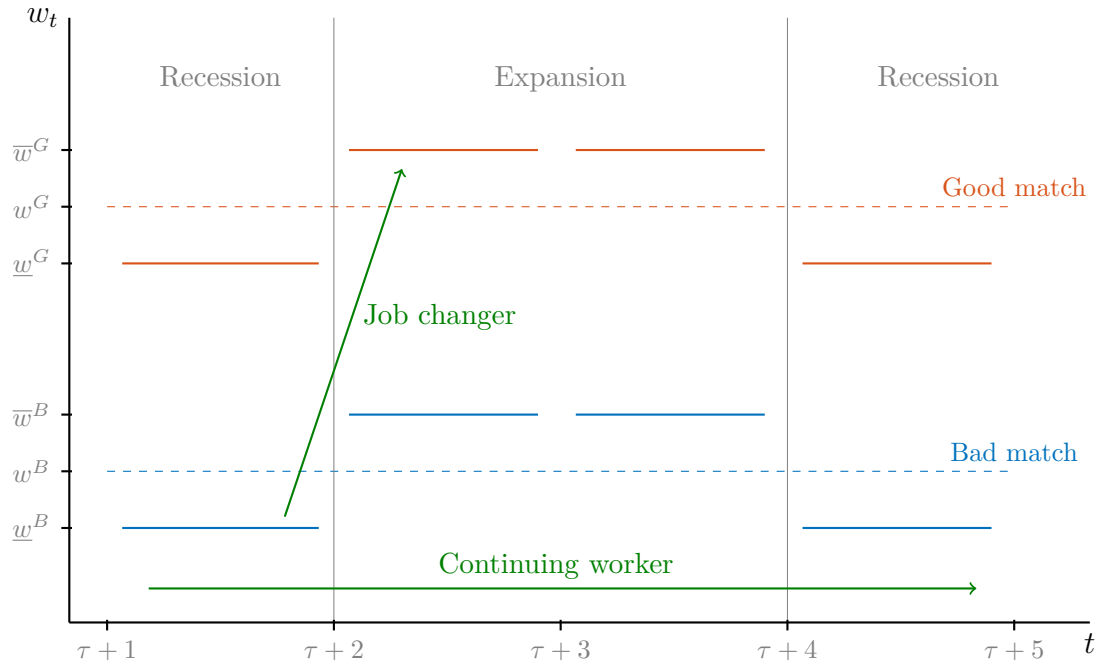


Figure 2: Impulse responses to productivity shock

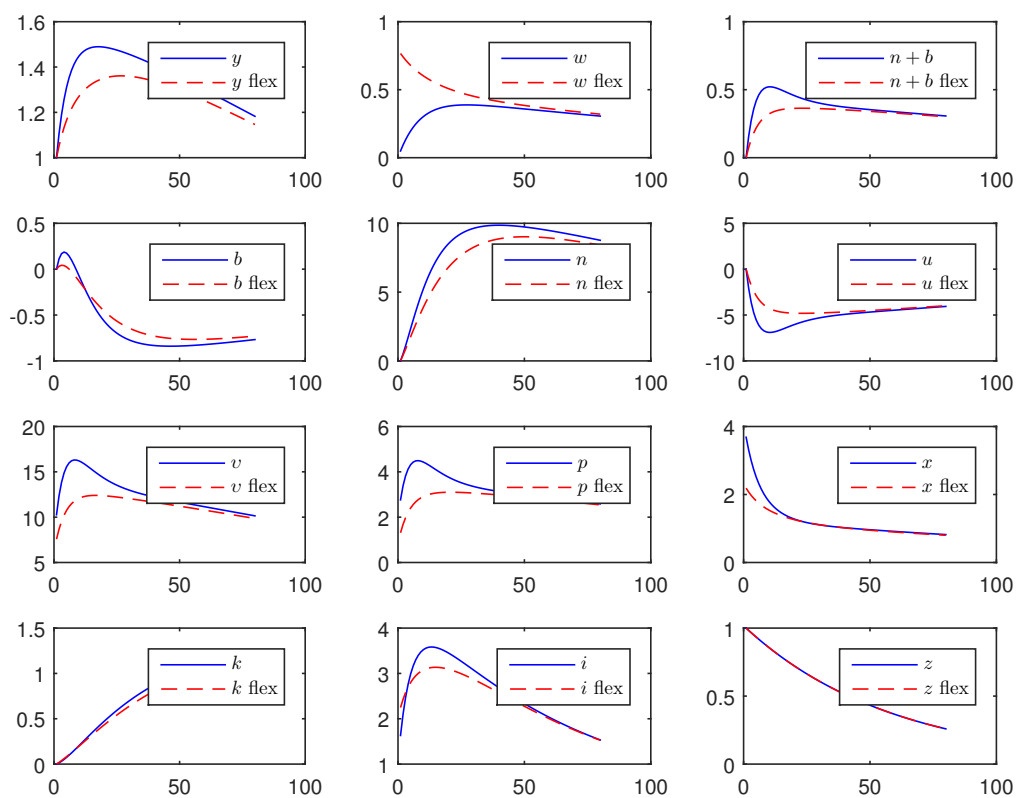


Figure 3: Labor market composition and job flows

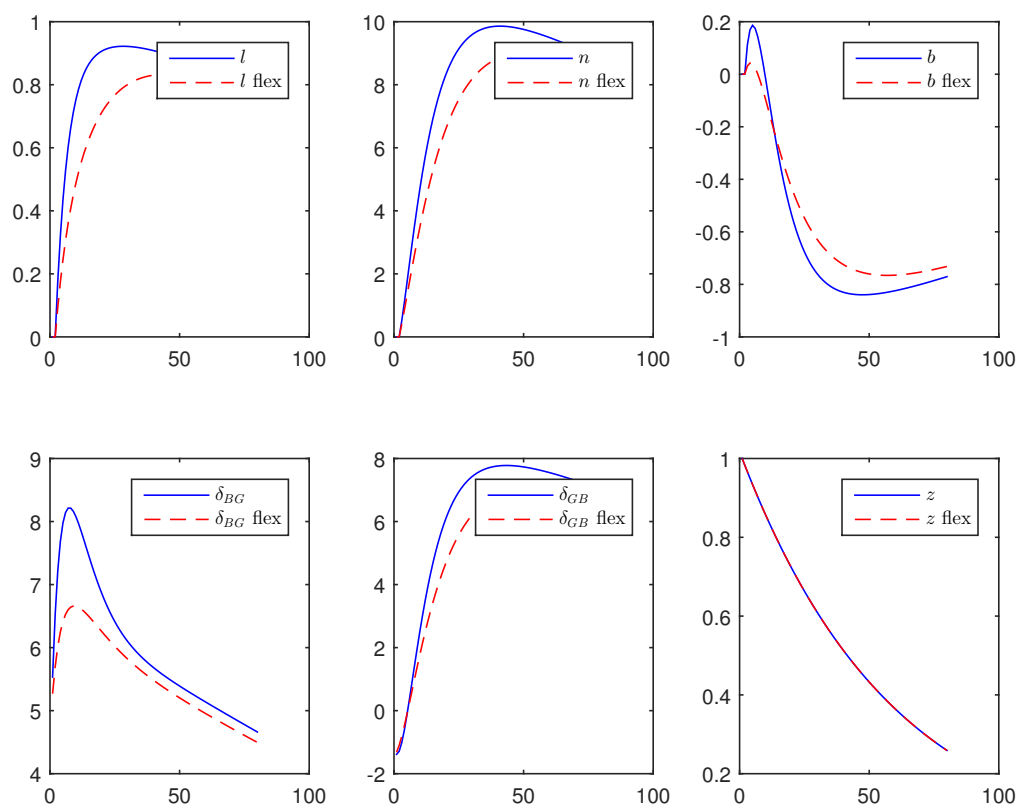


Figure 4: Wage growth and components

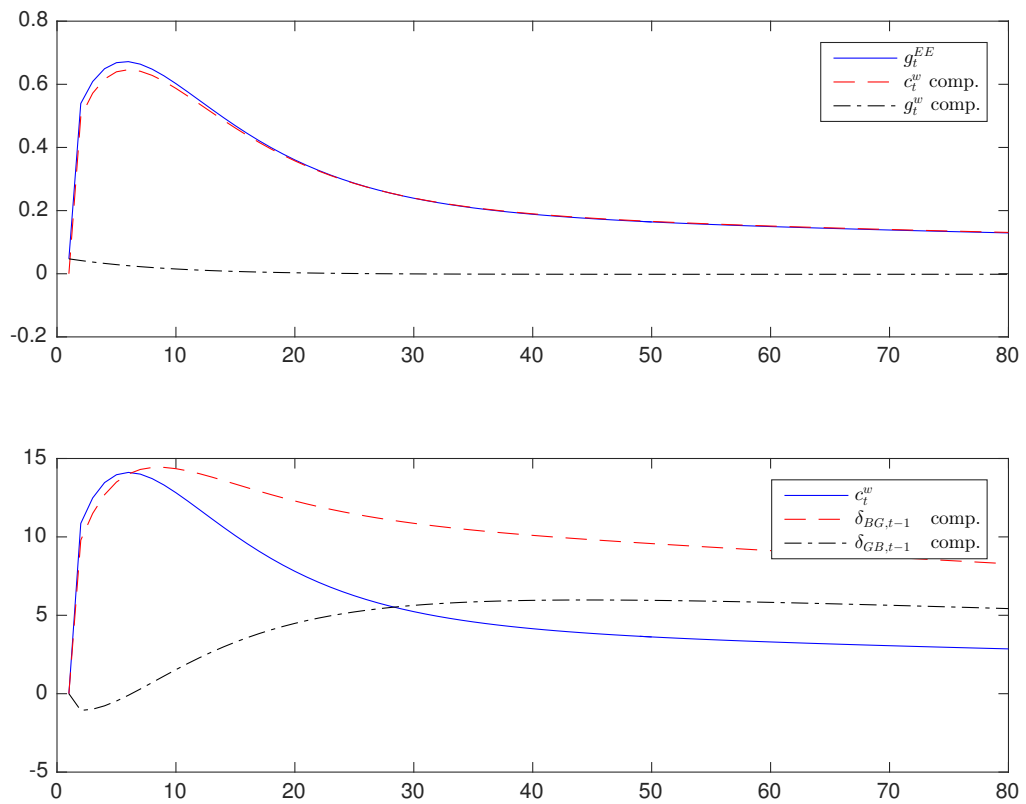
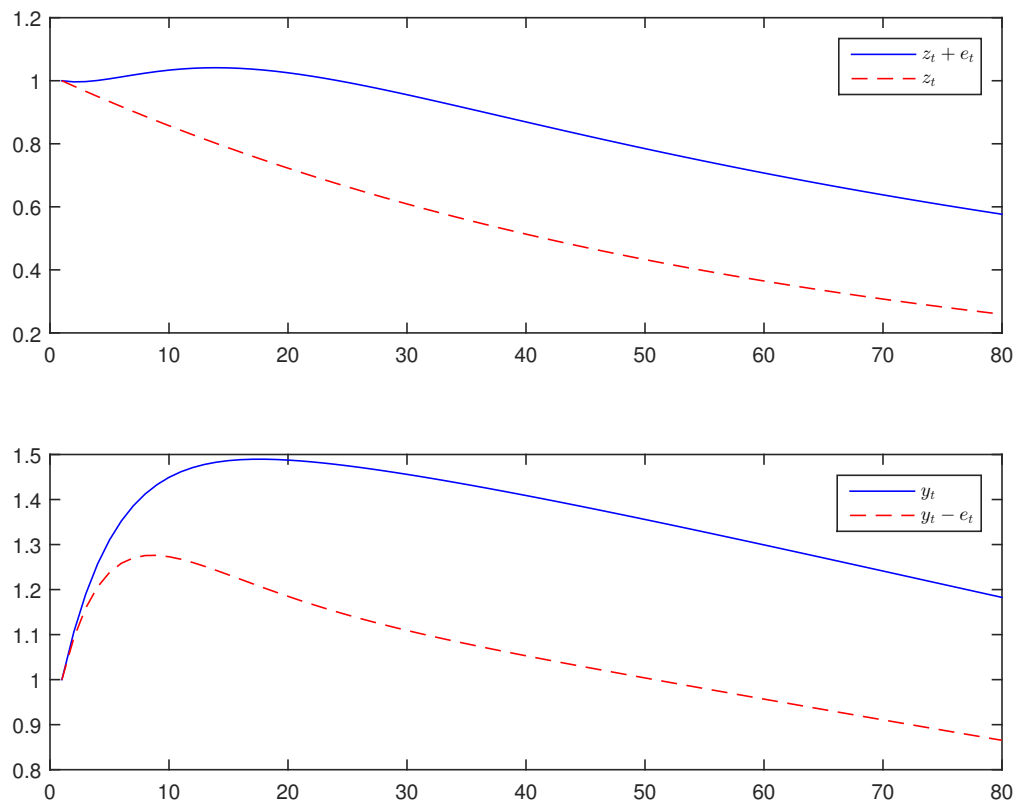


Figure 5: TFP, productivity, and output



A Data appendix

We use data from the Survey of Income and Program Participation (SIPP) from 1990 to 2012. The SIPP is administered by the U.S. Census Bureau and is designed to track a nationally representative sample of U.S. households. The SIPP is organized by panel years, where each panel year introduces a new sample of households. From 1990 to 1993, the Census Bureau would introduce a new panel on an annual basis, where each panel is administered for a period of 32 to 40 months. Hence for certain years in the early 1990s, data is available from multiple panels, each consisting from around 15,000 to 24,000 households. Starting in 1996, the Census changed the structure of the survey to follow contiguous panels. Since the redesign, new panels have been introduced in 1996, 2001, 2004, and 2008. For each of these panels, the Census has followed a larger sample of households (e.g. 40,188 in 1996) over a longer period.

Survey respondents are interviewed every four months on activity since the previous interview, a period referred to as a *wave*. However, some information (including for employment) is available at different frequencies within a wave. For example, the SIPP provides weekly measures of employment status, monthly measures of earnings, and job identifiers are constant for the entire period of the wave. As described in the main text, we combine monthly earnings records specific to each job to discern the pattern of job flows and sources of earnings over the wave.

The SIPP has several advantages relative to other commonly used panel data sources such as the PSID or the NLSY. Relative to the PSID, the SIPP follows a larger number of households, is nationally representative, and has more frequent observations. For the purposes of this paper, the PSID also suffers the disadvantage that it is difficult to identify wage earnings with a particular job in years where multiple jobs are held. Relative to the NLSY, the SIPP follows a larger number of households, but more importantly, multiple cohorts. Relative to both surveys, the SIPP suffers the disadvantage that it follows any particular individual for a shorter overall duration. But as mentioned before, the SIPP collects rich retrospective information that gets around problems of left-censoring: in particular, we observe start dates for jobs held during the first wave but started prior to the first interview (including the 1990 to 1993 panels). We then use our earnings-based measures of job transitions to determine the following sequence of jobs spells for the rest of the sample.³²

³²For each wave, the survey contains fields for up to two jobs. The survey maintains longitudinally consistent job IDs for each individual and tracks certain job-specific characteristics at a monthly frequency, including earnings. We follow the procedure detailed by Stinson (2003) to correct inconsistent job identification variables for the 1990 to 1993 panels. We use monthly earnings data within waves to determine at which job the individual is working and for what months the individual is working at each potential job. From these data, we determine within a wave whether an individual made a job transition; and whether the

A.1 Variables and sample selection

Following [Bils \(1985\)](#), we only consider males between the ages of 20 and 60. We drop observations for individuals who are disabled, self-employed, serving in the armed forces, or enrolled in school full-time. We use the monthly employment status recode variable to identify and drop observations where an individual reports not working for the entire month. We drop observations where an individual works less than 10 or greater than 100 hours a week. We also drop observations where the wage is top-coded or below the minimum wage. All observations are associated with a job-specific wage. As such, we drop observations where a worker is working at multiple jobs. Such observation may either reflect a job-to-job transition or multiple job-holding; but in either case, it is difficult to determine which observation should be included in the estimation.

We use hourly wages as our measure of earnings. In some instances, SIPP includes hourly wages and total monthly earnings. In cases where the hourly wage is directly available, we use that as our measure of wages. In cases where the hourly wage is not available, we construct a measure of implied hourly wages from monthly earnings divided by the product of weeks worked and hours worked per week. For the 1990 to 1993 waves, all of these variables are job-specific. Starting with the 1996 panel, the measure for weeks worked is no longer job-specific. We instead construct a measure of weeks worked from weeks with job minus weeks absent from work. Note that the implied hourly wage measure is subject to greater measurement error at the beginning of a job, when an individual does not necessarily spend a full month working at a job. In such cases, we use the second observation as the “new hire” wage. There is no considerable change for the fixed effects regression if we do not apply this correction, but many of the coefficients are not statistically significant for the first-differences regression, including for new hires from employment. We deflate wages using a four-month average of the PCE. Covariates include four indicators for educational attainment, separate indicators for union coverage and marital status, a quadratic in job tenure, and a time trend. We use combined weights across panels, applying the method recommended by in the SIPP User’s Guide ([U.S. Census Bureau, 2001](#)). We use monthly prime-aged male unemployment.

A.2 Identifying recalls

The SIPP maintains job-specific longitudinally consistent employment information over waves for which an individual reports non-zero employment. For such case, the SIPP maintains the same job identifier for a given job, allowing users to distinguish new jobs from “recalls” (to adopt the terminology of [Fujita and Moscarini 2013](#)). [Table A.1](#) gives

job transition was characterized by an intervening period of non-employment.

an example employment history of an individual who works at a job, spends four months in non-employment, but returns to the same job. The SIPP correctly records that the individual returned to the job that she left.

But starting in 1996, the SIPP resets employment records for individuals who are without employment for an entire wave. If individuals return to a previously held job after spending an entire wave in non-employment, the SIPP will incorrectly record the individual as starting a new job. Hence, a single job can be given multiple job identifiers. Table A.2 gives a sample employment history of an individual who works at a job, spends an entire wave out of work, and then returns to the same job. As in the previous example, the individual spends four months not working; but because those four months happen to fall over the entirety of a wave, the job is given a new identifier when the individual returns to work. For such individuals, we could mistakenly label a recall to be a transition across separate jobs.

We exploit an additional source of information recorded by the SIPP to identify potential recalls. Every time that a distinct job identifier is associated with an individual, the survey also adds a start date. This is indicated by the box around “start date” in the third row of table A.2. When we observe a start date that falls before the date that the SIPP purges job identifiers, we have a good indication that the “new job” is in fact a recall.

To what extent do respondents report the date that they began the job, inclusive of employment gaps, versus the date that they last began a contiguous employment spell? We note that the survey question recording start dates is explicitly designed to identify the start date to be the former of the two, as it is designed to distinguish jobs that began within the wave from jobs that began before the wave.

For example, in the 1996 panel, respondents are asked “Did [FIRST AND LAST NAME] begin [HIS HER] employment with [NAME OF EMPLOYER] at some time between [MONTH1] 1st and today?” (variable STRTJB). If individuals respond in the affirmative, they are asked about the month and day within the wave that the job began (STRTREFP). Otherwise, they are asked to give their “BEST estimate” of the year, month, and date that the job began (variables STRTMONJB, STRTJYR, STRTJMTH).³³

To identify potential recalls, we apply the following criterion: for individuals with an incomplete employment record – e.g. respondents who have spent a complete wave in non-employment – we consider any job with a start date prior to the period of non-employment (the date at which the SIPP purges internal employment records) as a potential recall, and we do not count the individual as a new hire.

We illustrate our criterion in tables A.3 and A.4. In table A.3, we observe an individual

³³See the 1996 Panel Wave 02 Questionnaire at <http://www.census.gov/content/dam/Census/programs-surveys/sipp/questionnaires/1996/SIPP%201996%20Panel%20Wave%2002%20-%20Core%20Questionnaire.pdf>

work a wave at Job A, spend an entire wave in non-employment, and then start work at Job B in wave 3. The start date of Job B is before the “gap date”, and hence, it is more likely that Job B is the same as Job A. Hence, we do not consider the individual as a new hire at Job B. In table A.4, we similarly observe an individual work at Job A, spend a wave in non-employment, and then work at Job B; however, the start date for job B in this instance is after the gap date, and hence, we consider the worker to be a new hire in wave 3.

We apply the gap date criterion with two small additions: first, for a subset of job dissolutions, workers report the cause of the dissolution. If the worker reports that he left the pre-gap job to take another job, we do preclude the possibility that the post-gap job is a recall to the first job. Second, if the start date at a post gap job is missing or statistically imputed, we identify the job as a potential recall and do not count the worker as a new hire.

Table A.1: Two separate employment spells, one job, correct IDs. Job ID preserved across contiguous employment spells because individual reports employment for each wave.

Wave	Time Period	Recorded Job ID	Recorded Start Date	Employment within wave	Actual Job ID
1	01/96-04/96	A	09/95	M1-M4	A
2	05/96-08/96	A	09/95	M1	A
3	09/96-12/96	A	09/95	M2-M4	A

Table A.2: Two separate employment spells, one job, incorrect IDs. Job ID information is lost when individual spends an entire wave without employment. At wave 3, the job is incorrectly coded as being a new job and the start date is asked again.

Wave	Time Period	Recorded Job ID	Recorded Start Date	Employment within wave	Actual Job ID
1	01/96-04/96	A	09/95	M1-M4	A
2	05/96-08/96	–	–	none	–
3	09/96-12/96	B	09/95	M1-M4	A

Table A.3: Two separate employment spells, “gap date” falls after reported job start date for job “B”. Rule out wave 3 job as “new hire”.

Wave	Time Period	Recorded Job ID	Recorded Start Date	Employment within wave	Actual Job ID	Gap date
1	01/96-04/96	A	09/95	M1-M4	A	–
2	05/96-08/96	–	–	none	–	05/96
3	09/96-12/96	B	09/95	M1-M4	A	05/96

Table A.4: Two separate employment spells, “gap date” is prior to reported job start date for job “B”.
 Count wave 3 job as “new hire”.

Wave	Time Period	Recorded Job ID	Recorded Start Date	Employment within wave	Actual Job ID	Gap date
1	01/96-04/96	A	09/95	M1-M4	A	–
2	05/96-08/96	–	–	none	–	05/96
3	09/96-12/96	B	08/96	M1-M4	A	05/96

B Model appendix

We now derive the log-linear equations that describe the first-order dynamics of the model. Most of the derivations are standard or are similar to those in Gertler and Trigari (2009). In the first section we derive loglinear expressions related to hiring and bargaining. Relative to GT, where the only firm-specific state variable was wages, here we must also keep track of composition. As might be expected, there is a non-trivial interplay between composition and wages at the firm level. Composition is inherited from the previous period and influences the wage through the Nash wage bargain; the wage influences next-period composition through hiring. We introduce a simplifying steady state restriction that lends analytic and computational tractability to the analysis. We first state six results that will simplify the derivation of the log-linear equations. We establish how these properties will be used to show that the “composition effect” of hiring - wherein firms vary the hiring rate to vary next-period composition - is zero up to a first order. We then go over the relevant equations for determining the Nash wage: the worker and firm surpluses, certain derivatives of the surpluses, and the Nash first order condition. We then derive recursive log-linear expressions for the firm and worker surpluses. We derive similar expressions for the derivatives of surpluses, which act as discount factors that differ across firms and worker in the determination of the Nash wage under staggered contracting. Then, we prove the steady-state results that we repeatedly invoke in deriving recursive log-linear expressions for the worker and firm surplus and discount factors and to linearize the composition term in hiring. In the second section, we derive log-linear expressions for the wage growth of job changers and the shares of job-to-job flows. In the third section, we derive a loglinear expression for the wage growth of workers making an employment to employment transition with an intervening spell of non-employment. There, we also discuss the indirect composition effect relevant for the wage cyclicality of new hires from unemployment. In the last section, we tie up a final loose end and define the operator mapping the distribution function from period t to period $t + 1$.

B.1 Some useful results

Labor force composition affects firms through the average retention rate of an efficiency unit of labor. Let \tilde{z} denotes the steady state of variable z . Assume that $\varsigma_n = \tilde{\varsigma}_b$, so that $\tilde{\rho}_n = \tilde{\rho}_b = \tilde{\rho}$ (i.e., retention rates of good and bad workers are the same in steady state). Then we obtain the following results:

1. $Var_t(\gamma_t) = 0$ up to a first order
2. $(\partial\gamma_{t+1}/\partial x_t)|_{ss} = 0$

3. $(\partial \rho_t / \partial \gamma_t) |_{ss} = 0$
4. $(\partial w_t^* (\gamma_t) / \partial \gamma_t) |_{ss} = 0$
5. $(\partial J_t (\gamma_t, w_t) / \partial \gamma_t) |_{ss} = 0$
6. $(\partial H_t (\gamma_t, w_t) / \partial \gamma_t) |_{ss} = 0$

These results guarantee that composition evolves as though it were an aggregate state variable. We use these results to derive recursive log-linear equations for the worker and firm surpluses, log-linear equations for the worker and firm discount factors, and to prove that the “composition effect” of hiring is zero up to a first order.

We will invoke these results in the following subsections and then prove them at the end of the section.

B.2 Hiring equation

In the main text, we derive the first order condition for hiring. Given that next period wage equals this period wage w_t with probability λ and next period contract wage $w_{t+1}^* (\gamma_{t+1})$ with probability $1 - \lambda$, we can write the hiring condition at a firm with composition γ_t and wage w_t as

$$\begin{aligned} \kappa x_t (\gamma_t, w_t) &= \mathbb{E}_t \{ \Lambda_{t,t+1} [\lambda J_{t+1} (\gamma_{t+1}, w_t) + (1 - \lambda) J_{t+1} (\gamma_{t+1}, w_{t+1}^* (\gamma_{t+1}))] \} \\ &\quad + \omega_t (\gamma_t, w_t), \end{aligned}$$

where the second term represents a retention motive in hiring:

$$\begin{aligned} \omega_t (\gamma_t, w_t) &= [\rho_t (\gamma_t) + x_t (\gamma_t, w_t)] \times \\ &\quad \mathbb{E}_t \left\{ \Lambda_{t,t+1} \left[\lambda \frac{\partial J_{t+1} (\gamma_{t+1}, w_t)}{\partial \gamma_{t+1}} + (1 - \lambda) \frac{\partial J_{t+1} (\gamma_{t+1}, w_{t+1}^* (\gamma_{t+1}))}{\partial \gamma_{t+1}} \right. \right. \\ &\quad \left. \left. + (1 - \lambda) \frac{\partial J_{t+1} (\gamma_{t+1}, w_{t+1}^* (\gamma_{t+1}))}{\partial w_{t+1}^* (\gamma_{t+1})} \frac{\partial w_{t+1}^* (\gamma_{t+1})}{\partial \gamma_{t+1}} \right] \frac{\partial \gamma_{t+1}}{\partial x_t} \right\}. \end{aligned}$$

The firm cares about composition for the implied retention rate of a unit of labor quality (represented by the first two terms in square brackets) and through possible effects of firm composition on future renegotiated wages (the third term).

Since we will prove that $\partial J / \partial \gamma$, $\partial w^* / \partial \gamma$, $\partial \gamma' / \partial x$ are all equal to 0 in the steady state, it follows that up to a first order $\omega_t (\gamma_t, w_t) = 0$.

B.3 Staggered Nash bargaining

Consider the problem of a renegotiating firm and workers in good matches.³⁴ We can write the surplus of workers in good matches $H_t(\gamma_t, w_t^*(\gamma_t))$ as

$$\begin{aligned} H_t(\gamma_t, w_t^*(\gamma_t)) &= w_t^*(\gamma_t) - u_B - [\nu c(\varsigma_n) + (1 - \nu) c(\varsigma_u)] \\ &\quad + \mathbb{E}_t \left\{ \Lambda_{t,t+1} \left[\nu \varsigma_n p_t^n \bar{H}_{t+1} - (1 - (1 - \nu) \varsigma_u) p_t \bar{H}_{t+1}^a \right] \right\} \\ &\quad + \nu (1 - \varsigma_n p_t^n) \mathbb{E}_t \left\{ \Lambda_{t,t+1} [\lambda H_{t+1}(\gamma_{t+1}, w_t^*(\gamma_t)) \right. \\ &\quad \left. + (1 - \lambda) H_{t+1}(\gamma_{t+1}, w_{t+1}^*(\gamma_{t+1}))] \right\} \end{aligned}$$

with

$$\bar{H}_t^a \equiv \xi \left(\bar{V}_t^n - U_t \right) + (1 - \xi) \left(\bar{V}_t^b - U_t \right).$$

Similarly, we can write firm surplus $J_t(\gamma_t, w_t^*(\gamma_t))$ as

$$\begin{aligned} J_t(\gamma_t, w_t^*(\gamma_t)) &= a_t - w_t^*(\gamma_t) - \frac{\kappa}{2} x_t(\gamma_t, w_t^*(\gamma_t))^2 \\ &\quad + [\rho_t(\gamma_t) + x_t(\gamma_t, w_t^*(\gamma_t))] \times \\ &\quad \mathbb{E}_t \left\{ \Lambda_{t,t+1} [\lambda J_{t+1}(\gamma_{t+1}, w_t^*(\gamma_t)) \right. \\ &\quad \left. + (1 - \lambda) J_{t+1}(\gamma_{t+1}, w_{t+1}^*(\gamma_{t+1}))] \right\} \end{aligned}$$

with

$$\begin{aligned} \kappa x_t(\gamma_t, w_t^*(\gamma_t)) &= \mathbb{E}_t \left\{ \Lambda_{t,t+1} [\lambda J_{t+1}(\gamma_{t+1}, w_t^*(\gamma_t)) \right. \\ &\quad \left. + (1 - \lambda) J_{t+1}(\gamma_{t+1}, w_{t+1}^*(\gamma_{t+1}))] \right\} \\ &\quad + \omega_t(\gamma_t, w_t^*(\gamma_t)) \end{aligned}$$

and

$$a_t \equiv (1 - \alpha) z_t \check{k}_t^\alpha.$$

The first order condition for Nash bargaining is

$$\eta \frac{\partial H_t(\gamma_t, w_t^*(\gamma_t))}{\partial w_t^*(\gamma_t)} J_t(\gamma_t, w_t^*(\gamma_t)) = (1 - \eta) \left(- \frac{\partial J_t(\gamma_t, w_t^*(\gamma_t))}{\partial w_t^*(\gamma_t)} \right) H_t(\gamma_t, w_t^*(\gamma_t))$$

where the $\partial H^*/\partial w^*$ and $-\partial J^*/\partial w^*$ act as cumulative discount factors applied by the worker

³⁴Note that the outcome of the bargaining problem will generally depend on labor composition within the firm. Since $Var_t(\gamma_t) = 0$ up to a first order, it is of no matter for studying the first-order model dynamics.

and the firm to value the contract wage stream. The worker discount factor is given by

$$\frac{\partial H_t(\gamma_t, w_t^*(\gamma_t))}{\partial w_t^*(\gamma_t)} = 1 + \nu(1 - s_n p_t^n) \lambda \mathbb{E}_t \left\{ \Lambda_{t,t+1} \frac{\partial H_{t+1}(\gamma_{t+1}, w_t^*(\gamma_t))}{\partial w_t^*(\gamma_t)} \right\} + \psi_t(\gamma_t, w_t^*(\gamma_t))$$

with

$$\begin{aligned} \psi_t(\gamma_t, w_t^*(\gamma_t)) = & \nu(1 - s_n p_t^n) \mathbb{E}_t \left\{ \Lambda_{t,t+1} \left[\lambda \frac{\partial H_{t+1}(\gamma_{t+1}, w_t^*(\gamma_t))}{\partial \gamma_{t+1}} \right. \right. \\ & + (1 - \lambda) \frac{\partial H_{t+1}(\gamma_{t+1}, w_{t+1}^*(\gamma_{t+1}))}{\partial \gamma_{t+1}} \\ & + (1 - \lambda) \frac{\partial H_{t+1}(\gamma_{t+1}, w_{t+1}^*(\gamma_{t+1}))}{\partial w_{t+1}^*(\gamma_{t+1})} \frac{\partial w_{t+1}^*(\gamma_{t+1})}{\partial \gamma_{t+1}} \left. \right] \times \\ & \left. \frac{\partial \gamma_{t+1}}{\partial x_t} \frac{\partial x_t(\gamma_t, w_t^*(\gamma_t))}{\partial w_t^*(\gamma_t)} \right\} \end{aligned}$$

Since we will prove that $\partial H/\partial \gamma$, $\partial w^*/\partial \gamma$, $\partial \gamma'/\partial x$ are all equal to 0 in the steady state, it follows that up to a first order $\psi_t(\gamma_t, w_t^*(\gamma_t)) = 0$.

The firm discount factor is given by

$$\begin{aligned} \frac{\partial J_t(\gamma_t, w_t^*(\gamma_t))}{\partial w_t^*(\gamma_t)} = & -1 + [\rho_t(\gamma_t) + x_t(\gamma_t, w_t^*(\gamma_t))] \lambda \times \\ & \mathbb{E}_t \left\{ \Lambda_{t,t+1} \frac{\partial J_{t+1}(\gamma_{t+1}, w_t^*(\gamma_t))}{\partial w_t^*(\gamma_t)} \right\} \end{aligned}$$

where we have used the fact that $J_t(\gamma_t, w_t^*(\gamma_t))$ is maximized with respect to $x_t(\gamma_t, w_t^*(\gamma_t))$, so that in taking the derivative with respect to $w_t^*(\gamma_t)$, we can hold $x_t(\gamma_t, w_t^*(\gamma_t))$ fixed at its optimal value.

B.3.1 Surplus of workers in good matches

We now develop a first order approximation of the Nash condition. We start by loglinearizing $H_t(\gamma_t, w_t^*(\gamma_t))$. We will establish that composition is first-order equivalent across firms, that is, $Var_t(\gamma_t) = 0$. This permits us to drop composition as a separate argument of the value function and the contract wage and allow it to be captured by the aggregate state. We can

then write

$$\begin{aligned}
H_t(w_t^*) &= w_t^* - u_B - [\nu c(\varsigma_n) + (1 - \nu) c(\varsigma_u)] \\
&\quad + \mathbb{E}_t \left\{ \Lambda_{t,t+1} \left[\nu \varsigma_n p_t^n \bar{H}_{t+1} - (1 - (1 - \nu) \varsigma_u) p_t \bar{H}_{t+1}^a \right] \right\} \\
&\quad + \nu (1 - \varsigma_n p_t^n) \mathbb{E}_t \{ \Lambda_{t,t+1} H_{t+1}(w_{t+1}^*) \} \\
&\quad + \nu (1 - \varsigma_n p_t^n) \lambda \mathbb{E}_t \{ \Lambda_{t,t+1} [H_{t+1}(w_t^*) - H_{t+1}(w_{t+1}^*)] \}
\end{aligned}$$

Loglinearizing, we obtain:

$$\begin{aligned}
\hat{H}_t(w_t^*) &= \left(\tilde{w} / \tilde{H} \right) \hat{w}_t^* \\
&\quad + (\nu - \tilde{\rho}) \beta \mathbb{E}_t \left\{ \hat{p}_t + \hat{\Lambda}_{t,t+1} + \hat{H}_{t+1} \right\} \\
&\quad - (1 - (1 - \nu) \varsigma_u) \tilde{\rho} \beta \left(\tilde{H}^a / \tilde{H} \right) \mathbb{E}_t \left\{ \hat{p}_t + \hat{\Lambda}_{t,t+1} + \hat{H}_{t+1}^a \right\} \\
&\quad + \tilde{\rho} \beta \mathbb{E}_t \left\{ -\frac{\nu - \tilde{\rho}}{\tilde{\rho}} \hat{p}_t + \hat{\Lambda}_{t,t+1} + \hat{H}_{t+1}(w_{t+1}^*) \right\} \\
&\quad + \tilde{\rho} \lambda \beta \left(\tilde{w} / \tilde{H} \right) \tilde{\varepsilon} \mathbb{E}_t \{ \hat{w}_t^* - \hat{w}_{t+1}^* \}
\end{aligned}$$

where

$$\tilde{\varepsilon} = \frac{\partial H_t(w_t^*)}{\partial w_t^*} \Big|_{ss} = \frac{1}{1 - \tilde{\rho} \lambda \beta}$$

and

$$\hat{H}_{t+1}^a = \left(\xi \frac{\tilde{H}}{\tilde{H}^a} \right) (\hat{V}_t^n - \hat{U}_t) + \left(1 - \xi \frac{\tilde{H}}{\tilde{H}^a} \right) (\hat{V}_t^b - \hat{U}_t).$$

Further simplifying gives

$$\begin{aligned}
\hat{H}_t(w_t^*) &= \left(\tilde{w} / \tilde{H} \right) [w_t^* + \tilde{\rho} \lambda \beta \tilde{\varepsilon} \mathbb{E}_t \{ \hat{w}_t^* - \hat{w}_{t+1}^* \}] \\
&\quad - (1 - (1 - \nu) \varsigma_u) \tilde{\rho} \beta \left(\tilde{H}^a / \tilde{H} \right) \mathbb{E}_t \left\{ \hat{p}_t + \hat{\Lambda}_{t,t+1} + \hat{H}_{t+1}^a \right\} \\
&\quad + (\nu - \tilde{\rho}) \beta \mathbb{E}_t \left\{ \hat{\Lambda}_{t,t+1} + \hat{H}_{t+1} \right\} \\
&\quad + \tilde{\rho} \beta \mathbb{E}_t \left\{ \hat{\Lambda}_{t,t+1} + \hat{H}_{t+1}(w_{t+1}^*) \right\}.
\end{aligned}$$

B.3.2 Surplus of firms

Combining the expression of the firm surplus with the hiring condition and using the same notation as for the worker surplus in good matches, i.e., dropping composition as a separate

argument of the value function and the contract wage, we can write the firm surplus as

$$\begin{aligned} J_t(w_t^*) &= a_t - w_t^* + \frac{\kappa}{2} x_t (w_t^*)^2 \\ &\quad + \rho_t \mathbb{E}_t \{ \Lambda_{t,t+1} J_{t+1}(w_{t+1}^*) \} \\ &\quad + \lambda \rho_t \mathbb{E}_t \{ \Lambda_{t,t+1} [J_{t+1}(w_t^*) - J_{t+1}(w_{t+1}^*)] \} \end{aligned}$$

where we have dropped the term $\omega_t(\gamma_t, w_t^*(\gamma_t))$ that is zero up to a first order.

Loglinearizing yields

$$\begin{aligned} \hat{J}_t(w_t^*) &= \left(\tilde{a}/\tilde{J} \right) \hat{a}_t - \left(\tilde{w}/\tilde{J} \right) \hat{w}_t^* + \left(\kappa \tilde{x}^2/\tilde{J} \right) \hat{x}_t(w_t^*) \\ &\quad + \tilde{\rho}\beta \mathbb{E}_t \left\{ \hat{\rho}_t + \hat{\Lambda}_{t,t+1} + \hat{J}_{t+1}(w_{t+1}^*) \right\} \\ &\quad - \lambda \tilde{\rho}\beta \left(\tilde{w}/\tilde{J} \right) \tilde{\mu} \mathbb{E}_t \left\{ \hat{w}_t^* - \hat{w}_{t+1}^* \right\}, \end{aligned}$$

where

$$\tilde{\mu} = \frac{\partial J_t(w_t^*)}{\partial w_t^*} \Big|_{ss} = \frac{1}{1 - \lambda\beta}.$$

Rearranging further,

$$\begin{aligned} \hat{J}_t(w_t^*) &= \left(\tilde{a}/\tilde{J} \right) \hat{a}_t - \left(\tilde{w}/\tilde{J} \right) \left[\hat{w}_t^* + \lambda \tilde{\rho}\beta \tilde{\mu} \mathbb{E}_t \left\{ \hat{w}_t^* - \hat{w}_{t+1}^* \right\} \right] \\ &\quad + \left(\kappa \tilde{x}^2/\tilde{J} \right) \hat{x}_t(w_t^*) \\ &\quad + \tilde{\rho}\beta \mathbb{E}_t \left\{ \hat{\rho}_t + \hat{\Lambda}_{t,t+1} + \hat{J}_{t+1}(w_{t+1}^*) \right\}. \end{aligned}$$

B.3.3 Worker and firm discount factors

Following the same simplifications as for the worker and firm surpluses, we can write the firm and worker discount factors as:

$$\begin{aligned} \frac{\partial H_t(w_t^*)}{\partial w_t^*} &= 1 + \nu (1 - \varsigma_n p_t^n) \lambda \mathbb{E}_t \left\{ \Lambda_{t,t+1} \frac{\partial H_{t+1}(w_t^*)}{\partial w_t^*} \right\} \\ \frac{\partial J_t(w_t^*)}{\partial w_t^*} &= -1 + [\rho_t + x_t(w_t^*)] \lambda \mathbb{E}_t \left\{ \Lambda_{t,t+1} \frac{\partial J_{t+1}(w_t^*)}{\partial w_t^*} \right\} \end{aligned}$$

where we have dropped from the worker discount factor the term $\psi_t(\gamma_t, w_t^*(\gamma_t))$ that is 0 up to a first order. Now define the variables $\epsilon_t(w)$ and $\mu_t(w)$ as follows:

$$\epsilon_t(w) \equiv \frac{\partial H_t(w)}{\partial w}, \quad \mu_t(w) \equiv -\frac{\partial J_t(w)}{\partial w}$$

We can then write the worker and firm discount factors in bargaining as

$$\begin{aligned}\epsilon_t &= 1 + \nu(1 - \varsigma_n p_t^n) \lambda \mathbb{E}_t \{ \Lambda_{t,t+1} \epsilon_{t+1} \} \\ \mu_t(w_t^*) &= -1 + [\rho_t + x_t(w_t^*)] \lambda \mathbb{E}_t \{ \Lambda_{t,t+1} \mu_{t+1}(w_t^*) \}\end{aligned}$$

and their corresponding loglinear expressions as

$$\begin{aligned}\hat{\epsilon}_t &= \tilde{\rho} \beta \lambda \mathbb{E}_t \{ \hat{\Lambda}_{t,t+1} + \hat{\epsilon}_{t+1} \} - (\nu - \tilde{\rho}) \beta \lambda \hat{p}_t \\ \hat{\mu}_t(w_t^*) &= \beta \lambda [\tilde{\rho} \hat{\rho}_t + (1 - \tilde{\rho}) \hat{x}_t(w_t^*)] + \beta \lambda \mathbb{E}_t \{ \hat{\Lambda}_{t,t+1} + \hat{\mu}_{t+1}(w_t^*) \}\end{aligned}$$

To derive a recursive expression for $\hat{\mu}_t(w_t^*)$, first note that we have

$$\mathbb{E}_t \{ \hat{\mu}_{t+1}(w_t^*) - \hat{\mu}_{t+1}(w_{t+1}^*) \} = -(1 - \tilde{\rho}) (\lambda \tilde{\mu}) (\beta \lambda \tilde{\mu}) (\tilde{w}/\tilde{J}) \mathbb{E}_t \{ \hat{w}_t^* - \hat{w}_{t+1}^* \}$$

Combining, we obtain

$$\begin{aligned}\hat{\mu}_t(w_t^*) &= \beta \lambda [\tilde{\rho} \hat{\rho}_t + (1 - \tilde{\rho}) \hat{x}_t(w_t^*)] + \beta \lambda \mathbb{E}_t \{ \hat{\Lambda}_{t,t+1} + \hat{\mu}_{t+1}(w_{t+1}^*) \} \\ &\quad - \beta \lambda (1 - \tilde{\rho}) (\lambda \tilde{\mu}) (\tilde{w}/\tilde{J}) (\beta \lambda \tilde{\mu}) \mathbb{E}_t \{ \hat{w}_t^* - \hat{w}_{t+1}^* \}.\end{aligned}$$

B.4 Derivation of steady-state results

We now derive the steady-state results invoked at the beginning of the appendix.

B.4.1 The dynamics and cross-sectional variation of composition

Recall the expression for the retention rates of good and bad workers:

$$\rho_t^i = \nu(1 - \varsigma_{it} p_t^n), \quad i = n, b.$$

Because the steady state search intensities ς_n and $\tilde{\varsigma}_b$ are equal, so too are the steady state retention rates:

$$\tilde{\rho}^n = \tilde{\rho}^b. \tag{50}$$

Now, recall the expression for the dynamics of composition:

$$\gamma_{t+1} = \frac{\rho_t^b \gamma_t / (1 + \phi \gamma_t) + x_t \tilde{\gamma}_t^m / (1 + \phi \tilde{\gamma}_t^m)}{\rho_t^n / (1 + \phi \gamma_t) + x_t / (1 + \phi \tilde{\gamma}_t^m)}.$$

Evaluating at the steady state, gives

$$\tilde{\gamma} = \tilde{\gamma}^m. \tag{51}$$

We use (50) and (51) to establish the first three useful results stated at the beginning of the model appendix.

Derive the log-linear equation for the evolution of composition:

$$\widehat{\gamma}_{t+1} = \widetilde{\rho}\widehat{\gamma}_t + (1 - \widetilde{\rho})\widehat{\gamma}_t^m - (\nu - \widetilde{\rho})\widehat{\zeta}_{bt}.$$

Hence, up to a first order, the dynamics of composition are not driven by any firm-specific variable, implying that composition evolves equally at all firms and independently of the individual firm's history of wages and composition. In particular, starting from steady state, the time path of composition across firms is first-order equivalent, so that

$$\widehat{\gamma}_t = \widehat{\gamma}_t,$$

and hence, $Var_t(\gamma_t) = 0$ up to a first order.

Take the derivative of γ_{t+1} with respect to x_t and evaluate at steady state:

$$\left. \frac{\partial \gamma_{t+1}}{\partial x_t} \right|_{ss} = \left[\frac{(1 + \phi\gamma_t)(1 + \phi\bar{\gamma}_t^m)}{[\rho_t^n(1 + \phi\bar{\gamma}_t^m) + x_t(1 + \phi\gamma_t)]^2} (\bar{\gamma}_t^m \rho_t^n - \gamma_t \rho_t^b) \right] \Big|_{ss} = 0,$$

Now recall the expression of the survival probability of a unit of labor quality:

$$\rho_t = \frac{\rho_t^n + \phi\gamma_t \rho_t^b}{1 + \phi\gamma_t}.$$

Take the derivative with respect to γ_t and evaluate at steady state:

$$\left. \frac{\partial \rho_t}{\partial \gamma_t} \right|_{ss} = \left. \frac{\phi(\rho_t^b - \rho_t^n)}{(1 + \phi\gamma_t)^2} \right|_{ss} = 0.$$

B.4.2 Effect of composition on contract wage, and firm and worker values

We show that $\partial w_t^*(\gamma_t)/\partial \gamma_t = 0$ in steady state. In doing so, we also show that in the steady state $\partial J_t/\partial \gamma_t = 0$ and $\partial H_t/\partial \gamma_t = 0$.

Define

$$\begin{aligned} F_t(\gamma_t, w_t^*(\gamma_t)) &\equiv \eta \epsilon_t(\gamma_t, w_t^*(\gamma_t)) J_t(\gamma_t, w_t^*(\gamma_t)) \\ &\quad - (1 - \eta) \mu_t(\gamma_t, w_t^*(\gamma_t)) H_t(\gamma_t, w_t^*(\gamma_t)) \end{aligned}$$

Since $F_t(\gamma_t, w_t^*(\gamma_t)) = 0$ by the surplus sharing condition, we have

$$\frac{\partial w_t^*(\gamma_t)}{\partial \gamma_t} = -\frac{\partial F_t(\gamma_t, w_t^*(\gamma_t))/\partial \gamma_t}{\partial F_t(\gamma_t, w_t^*(\gamma_t))/\partial w_t^*(\gamma_t)}$$

from the implicit function theorem.

The term $\partial F_t(\gamma_t, w_t^*(\gamma_t))/\partial \gamma_t$ satisfies

$$\begin{aligned} \frac{\partial F_t(\gamma_t, w_t^*(\gamma_t))}{\partial \gamma_t} &= \eta \frac{\partial \epsilon_t(\gamma_t, w_t^*(\gamma_t))}{\partial \gamma_t} J_t(\gamma_t, w_t^*(\gamma_t)) \\ &\quad + \eta \epsilon_t(\gamma_t, w_t^*(\gamma_t)) \frac{\partial J_t(\gamma_t, w_t^*(\gamma_t))}{\partial \gamma_t} \\ &\quad - (1 - \eta) \frac{\partial \mu_t(\gamma_t, w_t^*(\gamma_t))}{\partial \gamma_t} H_t(\gamma_t, w_t^*(\gamma_t)) \\ &\quad - (1 - \eta) \mu_t(\gamma_t, w_t^*(\gamma_t)) \frac{\partial H_t(\gamma_t, w_t^*(\gamma_t))}{\partial \gamma_t}. \end{aligned}$$

We will show that at steady state $\partial H/\partial \gamma$, $\partial J/\partial \gamma$, $\partial \epsilon/\partial \gamma$ and $\partial \mu/\partial \gamma$ are all proportional to $\partial w^*(\gamma)/\partial \gamma$, so that $\partial w^*(\gamma)/\partial \gamma = 0$ at steady state.

B.4.2.1 Effect of composition on worker surplus For any composition γ_t and wage w_t , we can write $\partial H_t(\gamma_t, w_t)/\partial \gamma_t$ as follows

$$\begin{aligned} \frac{\partial H_t(\gamma_t, w_t)}{\partial \gamma_t} &= \nu(1 - s_n p_t^n) \mathbb{E}_t \left\{ \Lambda_{t,t+1} \left[\lambda \frac{\partial H_{t+1}(\gamma_{t+1}, w_t)}{\partial \gamma_{t+1}} \right. \right. \\ &\quad + (1 - \lambda) \frac{\partial H_{t+1}(\gamma_{t+1}, w_{t+1}^*(\gamma_{t+1}))}{\partial \gamma_{t+1}} \\ &\quad \left. \left. + (1 - \lambda) \frac{\partial H_{t+1}(\gamma_{t+1}, w_{t+1}^*(\gamma_{t+1}))}{\partial w_{t+1}^*(\gamma_{t+1})} \frac{\partial w_{t+1}^*(\gamma_{t+1})}{\partial \gamma_{t+1}} \right] \times \right. \\ &\quad \left. \left[\frac{\partial \gamma_{t+1}}{\partial \gamma_t} + \frac{\partial \gamma_{t+1}}{\partial x_t} \frac{\partial x_t}{\partial \gamma_t} \right] \right\} \end{aligned}$$

Evaluating at steady state, using $\partial \gamma'/\partial x = 0$ and rearranging:

$$\frac{\partial H}{\partial \gamma} \left(1 - \rho \beta \frac{\partial \gamma'}{\partial \gamma} \right) = \rho \beta (1 - \lambda) \frac{\partial H}{\partial w} \frac{\partial w^*(\gamma)}{\partial \gamma} \frac{\partial \gamma'}{\partial \gamma}$$

The steady state value of $\partial H/\partial \gamma$ is proportional to the steady state value of $\partial w^*(\gamma)/\partial \gamma$; hence, if $\partial w^*(\gamma)/\partial \gamma$ is equal to zero at steady state, so is $\partial H/\partial \gamma$.

B.4.2.2 Effect of composition on firm surplus For any composition γ_t and wage w_t , we can write $\partial J_t(\gamma_t, w_t)/\partial\gamma_t$ as

$$\begin{aligned} \frac{\partial J_t(\gamma_t, w_t)}{\partial\gamma_t} &= \frac{\partial\rho_t(\gamma_t)}{\partial\gamma_t} \times \\ &\mathbb{E}_t \left\{ \Lambda_{t,t+1} \left[\lambda J_{t+1}(\gamma_{t+1}, w_t) + (1-\lambda) J_{t+1}(\gamma_{t+1}, w_{t+1}^*(\gamma_{t+1})) \right] \right\} \\ &+ [\rho_t(\gamma_t) + x_t(\gamma_t, w_t)] \times \\ &\mathbb{E}_t \left\{ \Lambda_{t,t+1} \left[\lambda \frac{\partial J_{t+1}(\gamma_{t+1}, w_t)}{\partial\gamma_{t+1}} \right. \right. \\ &+ (1-\lambda) \frac{\partial J_{t+1}(\gamma_{t+1}, w_{t+1}^*(\gamma_{t+1}))}{\partial\gamma_{t+1}} \\ &\left. \left. + (1-\lambda) \frac{\partial J_{t+1}(\gamma_{t+1}, w_{t+1}^*(\gamma_{t+1}))}{\partial w_{t+1}^*(\gamma_{t+1})} \frac{\partial w_{t+1}^*(\gamma_{t+1})}{\partial\gamma_{t+1}} \right] \frac{\partial\gamma_{t+1}}{\partial\gamma_t} \right\} \end{aligned}$$

where we have used the fact that $J_t(\gamma_t, w_t)$ is maximized with respect to $x_t(\gamma_t, w_t)$, so that in taking the derivative with respect to γ_t , we can hold $x_t(\gamma_t, w_t)$ fixed at its optimal value.

Evaluating at steady state (using $\partial\rho/\partial\gamma = 0$) gives

$$\frac{\partial J}{\partial\gamma} \left(1 - \beta \frac{\partial\gamma'}{\partial\gamma} \right) = \beta (1 - \lambda) \frac{\partial J}{\partial w} \frac{\partial w^*(\gamma)}{\partial\gamma} \frac{\partial\gamma'}{\partial\gamma}$$

The steady state value of $\partial J/\partial\gamma$ is proportional to the steady state value of $\partial w^*(\gamma)/\partial\gamma$.

B.4.2.3 Effect of composition on worker discount factor For any composition γ_t and wage w_t , we can write $\partial\epsilon_t(\gamma_t, w_t)/\partial\gamma_t$ as

$$\begin{aligned} \frac{\partial\epsilon_t(\gamma_t, w_t)}{\partial\gamma_t} &= \nu (1 - \varsigma_n p_t^n) \lambda \mathbb{E}_t \left\{ \Lambda_{t,t+1} \frac{\partial\epsilon_{t+1}(\gamma_{t+1}, w_t)}{\partial\gamma_{t+1}} \frac{\partial\gamma_{t+1}}{\partial\gamma_t} \right\} \\ &+ \nu (1 - \varsigma_n p_t^n) \mathbb{E}_t \left\{ \Lambda_{t,t+1} \left[\lambda \frac{\partial H_{t+1}(\gamma_{t+1}, w_t)}{\partial\gamma_{t+1}} \right. \right. \\ &+ (1-\lambda) \frac{\partial H_{t+1}(\gamma_{t+1}, w_{t+1}^*(\gamma_{t+1}))}{\partial\gamma_{t+1}} \\ &\left. \left. + (1-\lambda) \frac{\partial H_{t+1}(\gamma_{t+1}, w_{t+1}^*(\gamma_{t+1}))}{\partial w_{t+1}^*(\gamma_{t+1})} \frac{\partial w_{t+1}^*(\gamma_{t+1})}{\partial\gamma_{t+1}} \right] \times \right. \\ &\left. \frac{\partial x_t(\gamma_t, w_t)}{\partial w_t} \frac{\partial(\partial\gamma_{t+1}/\partial x_t)}{\partial\gamma_t} \right\} \end{aligned}$$

where, for simplicity, we have used already that $\partial\gamma'/\partial x = 0$ at steady state. Evaluate at steady state:

$$\frac{\partial\epsilon}{\partial\gamma} \left(1 - \rho\beta\lambda \frac{\partial\gamma'}{\partial\gamma} \right) = \rho\beta \left[\frac{\partial H}{\partial\gamma} + (1-\lambda) \frac{\partial H}{\partial w} \frac{\partial w^*(\gamma)}{\partial\gamma} \right] \frac{\partial x}{\partial w} \frac{\partial(\partial\gamma'/\partial x)}{\partial\gamma}$$

which is proportional to $\partial w^*(\gamma)/\partial\gamma$ since $\partial H/\partial\gamma$ is proportional to $\partial w^*(\gamma)/\partial\gamma$.

B.4.2.4 Effect of composition on firm discount factor For any composition γ_t and wage w_t , we can write $\partial\mu_t(\gamma_t, w_t)/\partial\gamma_t$ as

$$\begin{aligned} \frac{\partial\mu_t(\gamma_t, w_t)}{\partial\gamma_t} &= \left[\frac{\partial\rho_t(\gamma_t)}{\partial\gamma_t} + \frac{\partial x_t(\gamma_t, w_t)}{\partial\gamma_t} \right] \lambda \mathbb{E}_t \left\{ \Lambda_{t,t+1} \frac{\partial J_{t+1}(\gamma_{t+1}, w_t)}{\partial w_t} \right\} \\ &\quad + [\rho_t(\gamma_t) + x_t(\gamma_t, w_t)] \lambda \mathbb{E}_t \left\{ \Lambda_{t,t+1} \frac{\partial\mu_{t+1}(\gamma_{t+1}, w_t)}{\partial\gamma_{t+1}} \frac{\partial\gamma_{t+1}}{\partial\gamma_t} \right\} \end{aligned}$$

Evaluating at steady state,

$$\frac{\partial\mu}{\partial\gamma} \left(1 - \lambda\beta \frac{\partial\gamma'}{\partial\gamma} \right) = \frac{\partial x}{\partial\gamma} \lambda\beta \frac{\partial J}{\partial w}$$

Now consider $x_t(\gamma_t, w_t)$ from the hiring condition and calculate $\partial x_t(\gamma_t, w_t)/\partial\gamma_t$.

We have

$$\begin{aligned} \kappa \frac{\partial x_t(\gamma_t, w_t)}{\partial\gamma_t} &= \mathbb{E}_t \left\{ \Lambda_{t,t+1} \left[\lambda \frac{\partial J_{t+1}(\gamma_{t+1}, w_t)}{\partial\gamma_{t+1}} \right. \right. \\ &\quad \left. \left. + (1-\lambda) \frac{\partial J_{t+1}(\gamma_{t+1}, w_{t+1}^*(\gamma_{t+1}))}{\partial\gamma_{t+1}} \right. \right. \\ &\quad \left. \left. + (1-\lambda) \frac{\partial J_{t+1}(\gamma_{t+1}, w_{t+1}^*(\gamma_{t+1}))}{\partial w_{t+1}^*(\gamma_{t+1})} \frac{\partial w_{t+1}^*(\gamma_{t+1})}{\partial\gamma_{t+1}} \right] \frac{\partial\gamma_{t+1}}{\partial\gamma_t} \right\} \\ &\quad + \frac{\partial\omega_t(\gamma_t, w_t)}{\partial\gamma_t} \end{aligned}$$

with

$$\begin{aligned} \frac{\partial\omega_t(\gamma_t, w_t)}{\partial\gamma_t} &= [\rho_t(\gamma_t) + x_t(\gamma_t, w_t)] \times \\ &\quad \mathbb{E}_t \left\{ \Lambda_{t,t+1} \left[\lambda \frac{\partial J_{t+1}(\gamma_{t+1}, w_t)}{\partial\gamma_{t+1}} \right. \right. \\ &\quad \left. \left. + (1-\lambda) \frac{\partial J_{t+1}(\gamma_{t+1}, w_{t+1}^*(\gamma_{t+1}))}{\partial\gamma_{t+1}} \right. \right. \\ &\quad \left. \left. + (1-\lambda) \frac{\partial J_{t+1}(\gamma_{t+1}, w_{t+1}^*(\gamma_{t+1}))}{\partial w_{t+1}^*(\gamma_{t+1})} \frac{\partial w_{t+1}^*(\gamma_{t+1})}{\partial\gamma_{t+1}} \right] \frac{\partial(\partial\gamma_{t+1}/\partial x_t)}{\partial\gamma_t} \right\} \end{aligned}$$

where, for simplicity, to write $\partial\omega_t(\gamma_t, w_t)/\partial\gamma_t$ we have used already that $\partial\gamma'/\partial x = 0$ at steady state.

Evaluating at steady state,

$$\kappa \frac{\partial x}{\partial \gamma} = \beta \left[\frac{\partial J}{\partial \gamma} + (1 - \lambda) \frac{\partial J}{\partial w} \frac{\partial w^*(\gamma)}{\partial \gamma} \right] \frac{\partial \gamma'}{\partial \gamma} + \frac{\partial \omega}{\partial \gamma}$$

with

$$\frac{\partial \omega}{\partial \gamma} = \beta \left[\frac{\partial J}{\partial \gamma} + (1 - \lambda) \frac{\partial J}{\partial w} \frac{\partial w^*(\gamma)}{\partial \gamma} \right] \frac{\partial (\partial \gamma' / \partial x)}{\partial \gamma}$$

Thus, the steady state value of $\partial x / \partial \gamma$ is proportional to the steady state value of $\partial w^*(\gamma) / \partial \gamma$. This implies that the steady state value of $\partial \mu / \partial \gamma$ is also proportional to $\partial w^*(\gamma) / \partial \gamma$.

B.4.2.5 Effect of composition on contract wage Since all the terms comprising F are proportional to $\partial w^*(\gamma) / \partial \gamma$, we have that $\partial w^*(\gamma) / \partial \gamma$ is equal to zero.

B.5 Wage growth of job changers

In this section, we derive expressions for the flow shares of the various types of job-to-job flows; and we derive an expression for the average wage growth of job changers.

B.5.1 Job-to-job flows

We have two types of job-to-job transitions: those due to on-the-job search and those due to separations followed by search and finding of a new job within the same period. The latter are initiated by a match separation shock, but there is no spell of unemployment between the jobs. We have the following job-to-job flows:

$$\begin{aligned} \text{Bad to good : } & [(1 - \nu) \varsigma_u + \nu \bar{\varsigma}_{bt}] \xi p_t \bar{b}_t \\ \text{Good to bad : } & (1 - \nu) \varsigma_u (1 - \xi) p_t \bar{n}_t \\ \text{Bad to bad : } & (1 - \nu) \varsigma_u (1 - \xi) p_t \bar{b}_t \\ \text{Good to good : } & [(1 - \nu) \varsigma_u + \nu \varsigma_n] \xi \bar{p}_t \bar{n}_t \end{aligned}$$

Summing over the flows we obtain total job flows as:

$$[(1 - \nu) \varsigma_u (1 + \bar{\gamma}_t) + \nu \xi (\varsigma_n + \bar{\varsigma}_{bt} \bar{\gamma}_t)] \bar{p}_t \bar{n}_t$$

The shares of flows over total flows then are defined as:

$$\begin{aligned}\delta_{BB,t} &= \frac{(1-\nu)\varsigma_u(1-\xi)\bar{\gamma}_t}{(1-\nu)\varsigma_u(1+\bar{\gamma}_t) + \nu\xi(\varsigma_n + \bar{\varsigma}_{bt}\bar{\gamma}_t)} \\ \delta_{BG,t} &= \frac{[(1-\nu)\varsigma_u + \nu\bar{\varsigma}_{bt}]\xi\bar{\gamma}_t}{(1-\nu)\varsigma_u(1+\bar{\gamma}_t) + \nu\xi(\varsigma_n + \bar{\varsigma}_{bt}\bar{\gamma}_t)} \\ \delta_{GB,t} &= \frac{(1-\nu)\varsigma_u(1-\xi)}{(1-\nu)\varsigma_u(1+\bar{\gamma}_t) + \nu\xi(\varsigma_n + \bar{\varsigma}_{bt}\bar{\gamma}_t)} \\ \delta_{GG,t} &= \frac{[(1-\nu)\varsigma_u + \nu\varsigma_n]\xi}{(1-\nu)\varsigma_u(1+\bar{\gamma}_t) + \nu\xi(\varsigma_n + \bar{\varsigma}_{bt}\bar{\gamma}_t)}\end{aligned}$$

We also define the component of the share of bad-to-good flows due to on-the-job search, $\delta_{BGS,t}$, given by:

$$\delta_{BGS,t} = \frac{\nu\bar{\varsigma}_{bt}\xi\bar{\gamma}_t}{(1-\nu)\varsigma_u(1+\bar{\gamma}_t) + \nu\xi(\varsigma_n + \bar{\varsigma}_{bt}\bar{\gamma}_t)}$$

B.5.2 Average wage growth of job changers

Let \bar{g}_t^w denote the average wage growth of continuing workers and \bar{g}_t^{EE} the average wage growth of workers making and employment-to-employment transition.

Up to a first order, \bar{g}_t^{EE} can be written as:

$$\begin{aligned}\bar{g}_t^{EE} &= \delta_{BB,t-1} \log\left(\frac{\phi\bar{w}_t}{\phi\bar{w}_{t-1}}\right) + \delta_{GG,t-1} \log\left(\frac{\bar{w}_t}{\bar{w}_{t-1}}\right) + \\ &\quad \delta_{BG,t-1} \log\left(\frac{\bar{w}_t}{\phi\bar{w}_{t-1}}\right) + \delta_{GB,t-1} \log\left(\frac{\phi\bar{w}_t}{\bar{w}_{t-1}}\right)\end{aligned}$$

Simplifying, we obtain:

$$\bar{g}_t^{EE} = \bar{g}_t^w + c_t^w$$

with

$$\bar{g}_t^w = \log\left(\frac{\bar{w}_t}{\bar{w}_{t-1}}\right)$$

and

$$c_t^w = (-\log\phi)(\delta_{BG,t-1} - \delta_{GB,t-1})$$

Thus, average wage growth of new hires that are job changers equals average wage growth of continuing workers plus a composition component measuring the change in match quality among job changers. The composition component equals 0 if match quality is homogeneous ($\phi = 1$).

Loglinearizing the average gross wage growth of job changers, we obtain:

$$\bar{g}_t^{EE} = \tilde{g}^{EE} + \frac{1}{1 + \tilde{c}^w} \bar{g}_t^w + \frac{\tilde{c}^w}{1 + \tilde{c}^w} \tilde{c}_t^w$$

Loglinearizing the compositional effect, we obtain:

$$\hat{c}_t^w = \frac{1}{\tilde{\delta}_{BG} - \tilde{\delta}_{GB}} \left(\tilde{\delta}_{BG} \hat{\delta}_{BG,t-1} - \tilde{\delta}_{GB} \hat{\delta}_{GB,t-1} \right)$$

with

$$\begin{aligned} \hat{\delta}_{BG,t} &= \frac{1}{1 + \tilde{\gamma}} \hat{\gamma}_t + \frac{1 - \tilde{\delta}_{BG}}{\tilde{\delta}_{BG}} \tilde{\delta}_{BGS} \hat{\varsigma}_{bt} \\ \hat{\delta}_{GB,t} &= -\frac{\tilde{\gamma}}{1 + \tilde{\gamma}} \hat{\gamma}_t - \tilde{\delta}_{BGS} \hat{\varsigma}_{bt} \end{aligned}$$

Rearranging, we find the expression relating the composition effect to variable search intensity of workers in bad matches and firm average composition:

$$\hat{c}_t^w = \frac{\tilde{\delta}_{BG} + \tilde{\gamma} \tilde{\delta}_{GB}}{(1 + \tilde{\gamma})(\tilde{\delta}_{BG} - \tilde{\delta}_{GB})} \hat{\gamma}_{t-1} + \frac{1 - (\tilde{\delta}_{BG} - \tilde{\delta}_{GB})}{(\tilde{\delta}_{BG} - \tilde{\delta}_{GB})} \tilde{\delta}_{BGS} \hat{\varsigma}_{bt-1}$$

B.6 Excess wage cyclicality for new hires from unemployment

In our numerical results, we recover an indirect composition effect that lends additional cyclicality to the wage growth of new hires from unemployment. This effect arises from the countercyclical and slow-moving dynamics of inverse composition, and the countercyclical but faster-moving dynamics of unemployment.

Consider the peak of an expansion, when unemployment is at its lowest level. Although unemployment is now increasing in its return to steady state, average match quality is high and still improving. Thus, new hires from unemployment will have had particularly high wages on their previous job, implying larger-than-average wage reductions upon reemployment; but also, the prevailing unemployment rate when they are hired is higher than that when they lost their previous job.³⁵ Therefore, there is a negative correlation between wage growth across jobs and changes in the unemployment rate across jobs for new hires from unemployment, generating excess wage cyclicality.

Hence, there are two composition effects associated with a boom. First, there is an immediate improvement in match quality for workers searching on the job, correlated with declining unemployment rates. This is the primary composition effect that is the focus of the paper. Second, there is the effect discussed above, where a decline in wage growth across jobs for new hires from unemployment is correlated with increasing unemployment rates as the economy returns to steady state.³⁶ While the second composition effect generates excess

³⁵Recall that, because match quality is *iid*, new hires from unemployment do not directly benefit from an increase in average match quality. The only relevance of the increase in match quality to such workers is of the average match quality of their previous job.

³⁶Similar implications for wages have been found for other search models. See the discussion in Davis and

cyclicality in the wages of new hires from unemployment - and illustrates yet another channel by which cyclical match composition may generate spurious evidence of wage flexibility for new hires - the effect is small compared to that of the composition effect for workers actively searching on-the-job.

We provide a more detailed analysis of wage cyclicality for new hires from unemployment below.

B.6.1 ENE wage growth

We develop a recursive expression for the *average last wage of the unemployed* at time t , denoted \bar{w}_t^l . Key in what follows will be the within-period timing protocol: first, there is production and wage bargaining; then separation and search take place.

The expression for \bar{w}_t^l is:

$$\bar{w}_t^l = (1 - \omega_t^u) \bar{w}_{t-1}^l + \omega_t^u \bar{w}_{t-1}^a, \quad (52)$$

where ω_t^u is fraction of unemployed workers at time t who were newly unemployed at time $t - 1$ (employed in $t - 1$, separated in $t - 1$ and unable to find a new match post-separation), given by

$$\omega_t^u = \frac{(1 - u_{t-1})(1 - \nu)(1 - \varsigma_u p_{t-1})}{u_t}, \quad (53)$$

and where $1 - \omega_t^u$ is the fraction of unemployed workers at time t who were unemployed at the start of period $t - 1$ and failed to find a job in $t - 1$, given by

$$1 - \omega_t^u = \frac{u_{t-1}(1 - p_{t-1})}{u_t}. \quad (54)$$

Since there is no selection of unemployed workers into employment, the unemployed in t who were unemployed for the entire period $t - 1$ will have an average last wage in t equal to \bar{w}_{t-1}^l . At the same time, there is no selection of employed into unemployment, implying that unemployed workers in t who were newly unemployed workers in $t - 1$ will have an average last wage in t equal to the average wage per worker in $t - 1$, denoted \bar{w}_{t-1}^a . In turn, the time t average wage per worker is given by

$$\bar{w}_t^a = \frac{1 + \phi \bar{\gamma}_t}{1 + \bar{\gamma}_t} \bar{w}_t, \quad (55)$$

where \bar{w}_t is the average contract wage per efficiency unit of labor and $\bar{\gamma}_t$ is inverse composition. Note that the average wage exhibits excess cyclicality relative to the average contract wage. The quality of jobs incorporated in the average wage improves in booms, raising the

von Wachter (2011) of Burgess and Turon (2010), p. 46.

average wage as inverse composition $\bar{\gamma}_t$ decreases.

The average wage of new hires from unemployment at time t , denoted \bar{w}_t^{ENE} , is given by

$$\bar{w}_t^{ENE} = (\xi + (1 - \xi)\phi) \bar{w}_t. \quad (56)$$

Note that \bar{w}_t^{ENE} exhibits no excess cyclicity relative to continuing workers: the composition of new hires from unemployment across good or bad matches is acyclical (as the probability ξ that a match is good is constant) and all new hires receive the prevailing per-efficiency unit contract wage.

Then, keeping in mind the within-period time protocol, the relevant wage growth for new hires from unemployment can be written as

$$\bar{g}_t^{ENE} = \log \bar{w}_t^{ENE} - \log \bar{w}_{t-1}^l, \quad (57)$$

where the subscript $t - 1$ indicates that the relevant last period wage is that of workers who were unemployed during the production stage of period $t - 1$ but found a job at the end of the period.³⁷

We obtain the elasticity of newly hired workers from unemployment regressing their wage growth \bar{g}_t^{ENE} on the relevant unemployment difference, denoted $\Delta^{ENE} \bar{u}_t$. The relevant change in unemployment for a new hire out of unemployment at time t is

$$\Delta^{ENE} \bar{u}_t = \bar{u}_t - \bar{u}_{t-1}^l, \quad (58)$$

where \bar{u}_t^l is the *average last unemployment rate while employed for the currently unemployed*, denoted \bar{u}_t^l and given by a similar recursive expressions as for the last wage:

$$\bar{u}_t^l = (1 - \omega_t^u) \bar{u}_{t-1}^l + \omega_{t-1}^u \bar{u}_{t-1}. \quad (59)$$

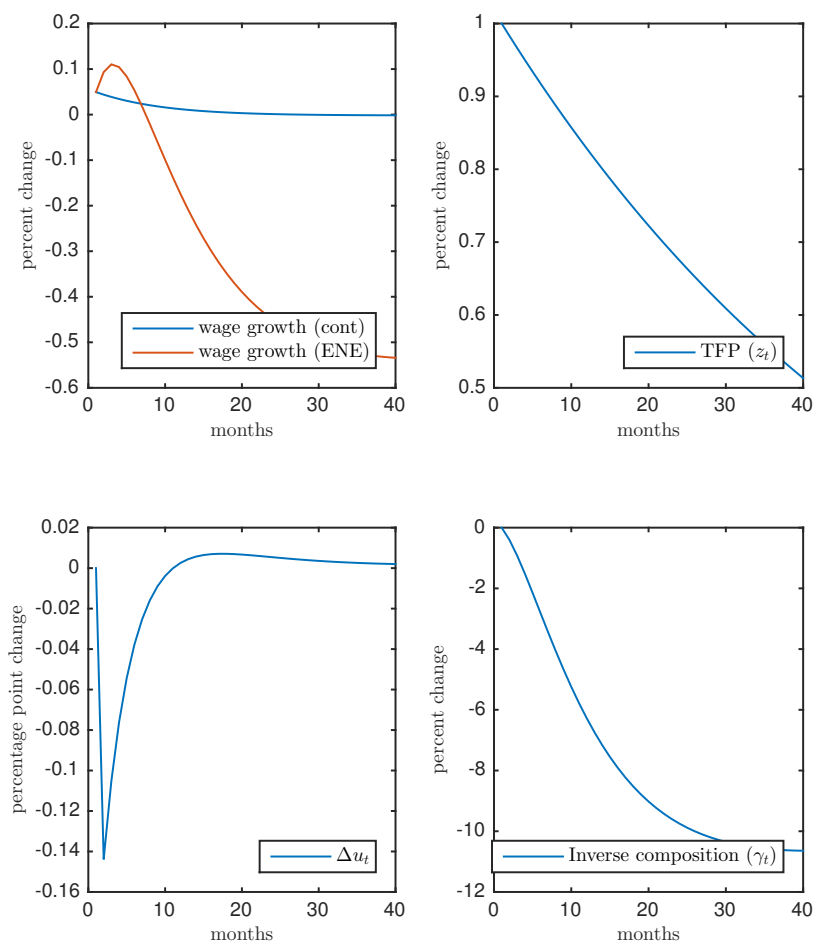
The first term on the right-hand side represents the contribution of workers who entered period $t - 1$ unemployed and remained unemployed for the entire period. The second term represents the contribution of workers who were newly unemployed in time $t - 1$.

B.6.2 ENE wage cyclicity

To explain the indirect compositional effect associated with the ENE wage cyclicity in a transparent manner and to make the parallel with the regression framework easier, we make two simplifying assumptions. First, we focus on workers who are unemployed for a single period, that is, we set $\omega_t^u = 1$ in equations (52) and (59), which gives us the following

³⁷From equation (52), we see that the most recent average wage is \bar{w}_{t-2}^a , associated with workers who lost their job in $t - 2$ and were unemployed for a single period.

Figure B.1: Wage growth for new hires from unemployment



amended system for equations (57) and (58):

$$\bar{g}_t^{ENE} = \log \bar{w}_t^n - \log \bar{w}_{t-2}^a \quad (60)$$

$$\Delta^{ENE} \bar{u}_t = \bar{u}_t - \bar{u}_{t-2} \quad (61)$$

Then, we shift the lagged structure one period, replacing the terms with subscript $t - 2$ with a subscript $t - 1$ to obtain³⁸:

$$\bar{g}_t^{ENE} = \log \bar{w}_t^n - \log \bar{w}_{t-1}^a \quad (62)$$

$$\Delta \bar{u}_t = \bar{u}_t - \bar{u}_{t-1} \quad (63)$$

The amended system behaves similarly to the original one, but allows for a clearer explanation of the excess cyclicality of wages of new hires from unemployment. Substituting (55) and (56) in the expression for \bar{g}_t^{ENE} and rearranging we obtain an expression that emphasizes the relation with the wage growth of continuing workers, \bar{g}_t^w , as follows:

$$\bar{g}_t^{ENE} = \bar{g}_t^w + \log(\xi + (1 - \xi)\phi) - \log\left(\frac{1 + \phi\tilde{\gamma}_{t-1}}{1 + \tilde{\gamma}_{t-1}}\right) \quad (64)$$

where $\bar{g}_t^w = \log(\bar{w}_t/\bar{w}_{t-1})$. This makes clear that the composition incorporated in the subtracted average wage, the last term, changes the cyclicality of wages for new hires from unemployment relative to continuing workers. Note that \bar{g}_t^{ENE} varies positively with $\tilde{\gamma}_{t-1}$. Loglinearizing equation (64) we obtain our final system

$$\bar{g}_t^{ENE} = \tilde{g}^{ENE} + \frac{1}{1 + \tilde{g}^{ENE}} \bar{g}_t^w + \frac{\tilde{\gamma}(1 - \phi)}{(1 + \tilde{g}^{ENE})(1 + \phi\tilde{\gamma})(1 + \tilde{\gamma})} \hat{\tilde{\gamma}}_{t-1} \quad (65)$$

$$\Delta \bar{u}_t = \bar{u}_t - \bar{u}_{t-1} \quad (66)$$

Both $\hat{\tilde{\gamma}}_{t-1}$ and $\bar{u}_t - \bar{u}_{t-1}$ are countercyclical: crucially, however, they are countercyclical over different frequencies. We see in Figure B.1 that the transition dynamics of unemployment are faster than those of TFP, while the transition dynamics of composition are slower than TFP. We see that $\bar{u}_t - \bar{u}_{t-1}$ decreases sharply in response to a positive productivity shock, but then starts to increase as unemployment returns to steady state. At the point at which $\bar{u}_t - \bar{u}_{t-1}$ starts rising, however, inverse composition $\hat{\tilde{\gamma}}_{t-1}$ is still decreasing. Hence, $\bar{u}_t - \bar{u}_{t-1}$ and $\hat{\tilde{\gamma}}_{t-1}$ can be either negatively or positively correlated following a productivity shock depending on the frequencies over which the correlation is computed.

³⁸The logic of this substitution is that the dynamics from $t - 2$ to t are similar to those from $t - 1$ to t , but in this way we can make the parallel with the regression framework easier.

Compute the regression coefficient β_{ENE} from the regression

$$\bar{g}_t^{ENE} = \alpha_{ENE} + \beta_{ENE}\Delta\bar{u}_t + \varepsilon_t, \quad (67)$$

where β_{ENE} is the semi-elasticity of wage growth of new hires from unemployment with respect to unemployment. We can show the following:

$$\begin{aligned} \beta_{ENE} &= \frac{\text{cov}(\bar{g}_t^{ENE}, \Delta\bar{u}_t)}{\text{var}(\Delta\bar{u}_t)} \\ &= \frac{\omega_{\bar{g}^w} \text{cov}(\bar{g}_t^w, \Delta\bar{u}_t) + \omega_\gamma \text{cov}(\hat{\gamma}_{t-1}, \Delta\bar{u}_t)}{\text{var}(\Delta\bar{u}_t)} \\ &= \omega_{\bar{g}^w} \beta_w + \omega_\gamma \beta_\gamma \end{aligned} \quad (68)$$

where $\omega_{\bar{g}^w}$ and ω_γ are the coefficients on \bar{g}_t^w and $\hat{\gamma}_{t-1}$ in equation (65), β_w is the semi-elasticity of the wage growth of continuing workers with respect to composition,

$$\bar{g}_t^w = \alpha_w + \beta_w \Delta\bar{u}_t + \varepsilon_t, \quad (69)$$

and β_γ is the estimated coefficient from the regression of $\hat{\gamma}_t$ on Δu_t ,

$$\hat{\gamma}_t = \alpha_\gamma + \beta_\gamma \Delta\bar{u}_t + \varepsilon_t. \quad (70)$$

Hence, the wages of new hires from unemployment are *more cyclical* than the wages of continuing workers if $\omega_{\bar{g}^w} \beta_w + \omega_\gamma \beta_\gamma < \beta_w$. Given that under our calibration $\omega_{\bar{g}^w} < 0$, $\omega_\gamma > 0$, and $\beta_\gamma < 0$, this condition is satisfied. Thus, despite the fact that wages of new hires from unemployment are no more flexible than those of continuing workers and that the composition of new hires from unemployment across good and bad jobs is acyclical, wage growth of workers making ENE transitions (approximated by β_{ENE}) is more cyclical with respect to $\Delta\bar{u}_t$ than the wages of continuing workers. This is entirely due to selection of workers based on the *previous wage*, which is decreasing in inverse composition, $\bar{\gamma}_t$. This selection is due to the slower transition dynamics of composition compared to unemployment.

We can directly show the role of composition in generating excess cyclicity in ENE wages in the model by shutting down composition in the model simulation. Once we do this, the excess cyclicity of ENE wages goes from 0.65 to nearly zero (0.06).

B.7 Transition function

We now define the law of motion for the distribution function, G_t . Let C and W be the sets of possible compositions and wages. Define the Cartesian product of the worker/firm

state space to be $S \equiv C \times W$ with σ -algebra Σ with typical subset $\mathcal{S} = (C \times W)$. Define the transition function $Q_{\mathbf{s}, \mathbf{s}'}((\gamma, w), C \times W)$ as the probability that an individual retained or hired by a firm characterized by (γ, w) transits to the set $C \times W$ next period when the aggregate state transits from \mathbf{s} to \mathbf{s}' . Then $Q_{\mathbf{s}, \mathbf{s}'}$ satisfies

$$\begin{aligned} Q_{t,t+1}((\gamma_t, w_t), C \times W) &= \mathbb{I}(\gamma_{t+1}(\gamma_t, w_t) \in C) \\ &\times \left[(1 - \lambda) \mathbb{I}(w_t^*(\gamma_{t+1}(\gamma_t, w_t)) \in W) \frac{x_t(\gamma_t, w_t) + \rho_t(\gamma_t, w_t)}{\bar{x}_t + \bar{\rho}_t} \right. \\ &\left. + \lambda \mathbb{I}(w_t \in W) \frac{x_t(\gamma_t, w_t) + \rho_t(\gamma_t, w_t)}{\bar{x}_t + \bar{\rho}_t} \right] \end{aligned}$$

where $\mathbb{I}(\cdot)$ is the indicator function. Then,

$$G_{t+1}(C \times W) = \int_{(\gamma_t, w_t) \in C \times W} Q_{t,t+1}((\gamma_t, w_t), C \times W) dG_t(\gamma_t, w_t).$$